# X-ray Microbeam Diffraction Studies of Si Strips Elastically Loaded in Bending

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Using an x-ray microdiffractometer with capillary focusing optics at the NSLS X20A beamline [1], we have investigated anticlastic bending effects in rectangular, elastically bent (100)-type Si specimens [2-4]. When a specimen is bent along a principal longitudinal direction, bending also occurs in the transverse direction, with an opposite curvature, due to Poisson coupling. Thus, the specimen is bent into an anticlastic, or saddle, shape. We have experimentally verified a quantitative procedure for differentiating between beam and plate structures loaded in bending based on the extent of anticlastic deformation [3]. In this procedure, we use a parameter, which was first introduced in an analysis originated by Searle [5] and expanded on by Ashwell [6], for differentiating between rectangular, constant cross-section beams and plates. This parameter, which we term the Searle parameter ($\beta$), is given by $b^2/(Rt)$, where b, R, and t are the specimen width, specimen thickness, and applied bending radius, respectively. The effect of loading fixture boundary conditions, i.e. rollers used for load application, on the anticlastic curvature in these structures has also been quantified [4].

An *in-situ* four-point bending fixture was mounted on an x-ray microdiffractometer (Figure 1), and (100)-type Si specimens were bent to a known radius of curvature along their lengths. The specimens were step-scanned while monitoring the Si (004) diffraction signal (8.5 keV) with a scintillation detector. Rocking curve scans were performed at each step in order to obtain the local specimen tilt required to maintain the Bragg diffraction condition at different positions on the bent crystals. Consistent with Searle's predictions and Ashwell's model, we found that for $\beta < 5$, anticlastic bending occurred in the transverse, or width, direction (beam behavior), whereas, for $\beta > 60$, anticlastic curvature was neutralized over the majority of the sample width (plate behavior). Figure 2 is a map of the anticlastic specimen tilt across half of the specimen width at various positions along the specimen length for a strip with length (a), width (b), and thickness (t) dimensions of 80 mm, 50 mm, and 0.15 mm, respectively. For this case (R=285mm, $\beta$=58), anticlastic curvature occurred at the edges but was neutralized over the remainder of the specimen width. Interestingly, since $\beta$ depends on the applied bending radius as well as on specimen width and thickness, a single sample can be made to behave as either a beam or a plate by adjusting the amount of bending [3]. For example, if the

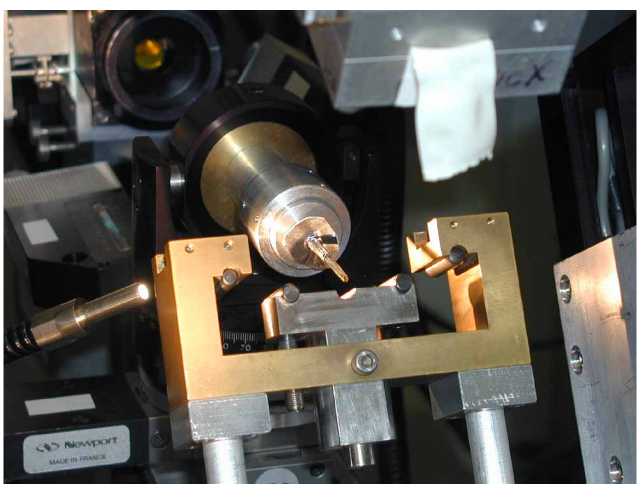

**Figure 1.** *Photograph illustrating the four-point bending fixture (without sample) mounted on the x-ray microdiffractometer with a capillary-focusing optic pointing towards it. Detector slits can be seen at top right.*

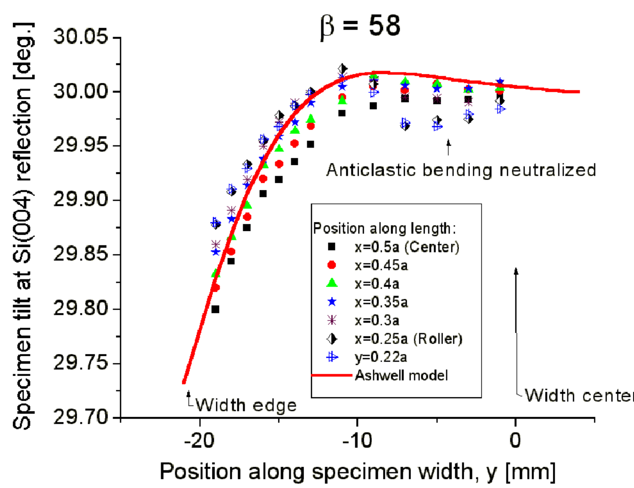


**Figure 2.** *The transverse, or anticlastic, specimen tilt of a bent (100)-type Si plate across half of the specimen width. The separate curves correspond to measurements at different locations along the specimen length. As expected for a plate (β=58), anticlastic curvature is neutralized across the central region of the specimen width.*

sample depicted in Figure 2 was bent less to a radius of curvature of 3000 mm instead of 285 mm, β would be equal to 5.6 (closer to beam regime), and a finite anticlastic curvature would be present across the entire specimen width.

In addition to studying the transition between beam and plate behavior, we have quantified the effect of the rollers that are used for load application on the anticlastic curvature in beams [4]. A beam loaded in pure bending, which corresponds approximately to the four-point bending case, should experience the same anticlastic curvature at all positions along its length where the bending moment is constant. However, the rollers that are used in a four-point fixture, in addition to applying the desired principal curvature along the specimen length, also inhibit the transverse curvature and modify the sample's stress state. An 80 mm x 24 mm x 0.6 mm (100)-type Si specimen was loaded in a four-point configuration (R=1600 mm, β=0.6), and the principal and transverse specimen tilts were measured. It can be shown that the ratio of the anticlastic curvature to the principal curvature in a beam should be equal to Poisson's ratio [7]. Figure 3 is a plot of the experimentally measured Poisson's ratio at various positions along half of the specimen length. The variability in this plot is due to changes in the transverse curvature; although the principal curvature was found to be constant, the anticlastic specimen curvature varied with distance away from the rollers. Only at the mid-length centerline did anticlastic curvature occur unhindered and give the correct value of Poisson's ratio. At other positions closer to the inner roller, the transverse curvature was partially inhibited, and the measured Poisson's ratio was lower than theoretical predictions. For flexural four-point tests used to measure Poisson's ratio, if a region on the sample that is free from these boundary effects is not chosen, errors as large as 46% can occur [4]. We have also observed this boundary effect in three-point bending tests. In this configuration, a single inner roller is located at the center of the sample length, which coincides with the point of maximum stress, or the location on the sample that is typically chosen for measurements. This is also the point where the roller constraint causes the maximum error.

The results highlighted here, beam/plate differentiation and loading fixture boundary effects, have important implications in experiments that employ elastic bending techniques. These include elastic constant and fracture strength determinations of brittle materials as well as synchrotron monochromator applications.

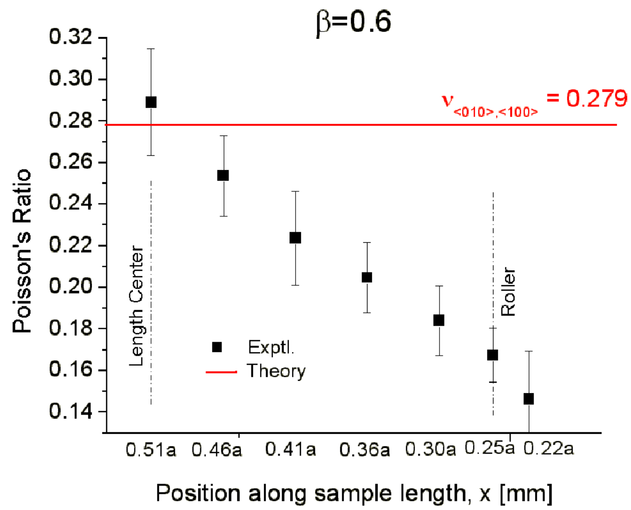


***Figure 3.*** *The ratio of the anticlastic curvature to the principal curvature, or the measured Poisson's ratio, versus position along half of the specimen length. While a transverse curvature is present as expected for a beam (β=0.6), the four-point loading fixture rollers inhibit the anticlastic curvature and thus modify the nominally pure-bending stress state. If measurements are not performed outside the roller constraint region, errors in the measured value of Poisson's ratio will occur.*

## References

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