

**THERMAL REGAIN FROM DISPLACEMENT OF DUCT LEAKAGE
WITHIN INSULATION**

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May 2002

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ABSTRACT

This report investigates the possibility that energy losses due to air leakage from ducts in small buildings might be reduced if the leaked air is constrained to flow within the insulation to a point upstream or downstream of the initial leakage point. The idea is that the leakage air might warm (or, in the air-conditioning mode, cool) the insulation and thereby retard heat conduction from (or to) the duct. Any such reduction in conductive losses could be credited against the lost energy from the leak itself. Theoretical calculations carried out in this work indicate that such a "thermal regain" effect could recover, in the heating mode, up to half the heat contained in the leaking air, and in the cooling mode, up to 75% of the sensible cooling in the leak. In most actual cases, a smaller amount of regain would be expected.

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EXECUTIVE SUMMARY

In the course of a duct efficiency retrofit, additional insulation may be added to a duct system that is already insulated. For example, a layer of R-4 insulation is added to a duct system that already has R-4 installed. It might happen, either by chance or by design, that the add-on layer, while not stopping duct leaks, causes the leakage air to flow longitudinally for a distance, parallel to the duct, before it finds a way out of the newly added outer layer.

It is plausible that this leakage air might serve a useful function in keeping the insulation layer warmer (or, in the air-conditioning mode, cooler) than it would be in the absence of the leakage. By being held close to the ducts for a while, it might establish an artificially warmer (or cooler, in air conditioning) zone around the ducts. To the extent that this effect would reduce conductive heat transfer through the duct wall, the leakage should be credited with a “thermal regain” in the same way that leakage into the zones containing the ducts is credited with thermal regain when the leakage air warms (or cools) such “buffer zones,” thereby reducing the heating (or cooling) load.

It is suggested that a double layer of insulation that causes leakage air to follow an indirect path may in effect form its own buffer zone. This report investigates whether and to what extent such thermal regain exists. A general equation is derived for the thermal regain fraction (the fraction of heat or cooling effect lost by the leak that is effectively regained by virtue of displacement of the leak within the duct insulation).

The results for typical cases can be simply stated. In the heating mode, the benchmark case sets the temperature of the zone surrounding the ducts 30 °F colder than the house while the air in the ducts is 50 °F warmer than the house if the equipment is a furnace or 30 °F warmer than the house if it uses a heat pump. An upper bound for the thermal regain fraction is 0.40 for the furnace and 0.50 for the heat pump.

In the cooling mode, the benchmark case sets the temperature of the zone surrounding the ducts air in the ducts 10 °F warmer than the house if it is a crawlspace and 40 °F warmer than the house if it is an attic. The air in the ducts is assumed to be 20 °F colder than the house. The upper bound for the thermal regain fraction is 0.37 for crawlspace ducts and 0.75 for attic ducts. These regain fractions apply to the sensible portion of the cooling load only.

Regain fractions that would be obtained in actual practice would usually be lower than these upper bounds, the shortfall depending on the distance between the point where the leak leaves the duct and where it escapes the outer layer of insulation.

It would appear, then, that the strategy outlined above might be especially useful for attic ducts in climates with high cooling loads. This conclusion is strengthened by the consideration that the regain fraction will be higher at design conditions where the attic is hotter. Also, the fraction of the load that is sensible (as opposed to latent) is generally highest at design conditions.

INTRODUCTION

In one type of duct efficiency retrofit, additional insulation is added to a duct system that is already insulated. For example, a layer of R-4 insulation might be added to a duct system that already has R-4 installed. It is possible that -- either by chance or by design -- the add-on layer, while not stopping duct leaks, might cause the leakage air to flow longitudinally for a distance, parallel to the duct, before it finds a way out of the newly added outer layer. This could happen by chance if the outer and inner layers of insulation have seams at different locations. Perhaps more usefully, if such longitudinal displacement of the leakage air turned out to be useful, it might be designed into the makeup of the outer insulation layer intended to be used in the retrofit.

It is plausible that this leakage air might serve a useful function in keeping the insulation layer warmer (or, in the air-conditioning mode, cooler) than it would be in the absence of the leakage. By being held close to the ducts for a while, it might establish an artificially warmer (or cooler, in air conditioning) zone around the ducts. To the extent that this effect would reduce the heat losses from the ducts, the leakage should be credited with a "thermal regain" in the same way that leakage into buffer zones is credited with thermal regain when the leakage air warms (or cools) the buffer zone relative to the temperature it would have in the absence of such duct leakage. The purpose of this report is to investigate whether and to what extent such thermal regain exists.

The model developed below applies to a situation where there are two distinct layers of insulation around the duct, with leakage air moving between them in a longitudinal direction for a distance before it finds its way out from the outer insulation layer. It may also apply approximately where there is a single insulation layer with an air barrier on the outside. Leakage air may pass into the insulation itself and thence longitudinally through the insulation material until it finds an opening in the air barrier through which it can escape.

DISPLACED LEAKAGE MODEL

A simple model¹ of the displaced leakage situation is illustrated in Figure 1. Heated air passing through a duct reaches a point where a small fraction of this air leaks from the duct. Under the usual assumption, such leakage would pass immediately through any insulation and exit to the space surrounding the duct, at a point immediately adjacent to the leak itself. In the model considered here, the leak passes through a portion of the insulation but is then constrained to move downstream² through an annular region separating an inner and an outer layer of insulation. The leaked air is assumed to pervade the entire annulus, i.e., the flow is radially symmetric about the center line of the duct.

¹ The description that follows is in terms of a heating application, but the derivation goes through for sensible cooling as well as long as the latent portion of the cooling function can be considered as decoupled from the sensible portion.

² The leaked air can also move upstream. This possibility is considered later on.

After traveling thus for a certain distance, the leaked air finds an exit pathway in the outer insulation layer and dissipates into the surrounding space.

Using this model, the question of whether such longitudinal flow of leakage air within the insulation could have a beneficial effect in reducing the net heat losses from the duct is now investigated.

NOMENCLATURE

Referring to Figure 1, the following parameters are defined:

Symbol	Units	Definition
Q_D	ft ³ /h	Airflow rate in the duct, in the domain $0 < x < L$
Q_L	ft ³ /h	Leakage airflow rate
L	ft	Distance of longitudinal displacement of the leak, between exiting the duct and exiting the outer insulation layer
r_1	ft	Effective radius of inner insulation layer
r_2	ft	Effective radius of outer insulation layer
R_1	ft ² -F-h/Btu	R-value of inner insulation layer
R_2	ft ² -F-h/Btu	R-value of outer insulation layer
T_0	°F	Temperature of air in the duct that is entering the region through which the leak is displaced, i.e., the region between where the leak leaves the duct and where it leaves the outer layer of insulation
T_s	°F	Air temperature in space surrounding the duct
T_{in}	°F	Air temperature indoors, in the house
ρC_p	Btu/ft ³ -F	Volume specific heat of air

These parameters are taken to be constants in the analysis. (In particular, changes in air density are ignored.) The values of r_1 and r_2 should strictly speaking be the log-mean radii using the inner and outer surfaces of the inner and outer insulation layers, respectively. As a practical matter, the arithmetic average radius of the inner and outer insulation layers should be sufficiently accurate.

In addition, two variables define the temperature of the air in the duct and of the leakage air at any distance x from the leakage site, which is taken as $x = 0$. That is, the domain of x in the analysis from 0 to L .

$T_D(x)$	°F	Temperature of the air in the duct at point x
$T_L(x)$	°F	Temperature of the leakage air flowing through the insulation at point x

Finally, the functional form of the air temperature in the duct for the case where the leak exits the duct directly, with no longitudinal flow, needs to be evaluated.

$T_{D,dir}(x)$ °F Temperature of the air in the duct at point x for the case where the leak exits the duct and the insulation at the same point ($x=0$).

GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

The approach will be to develop the two simultaneous differential equations that govern the heat flow and then to apply the boundary conditions. This permits $T_D(x)$ and $T_L(x)$ to be evaluated over the entire range of x . Using these results, the heat flow due to the leak is compared with the reduction in conductive losses caused by the longitudinal flow of the leak.

First, two control volumes are defined, a duct control volume comprising the portion of the duct between positions x and $x+dx$, and a leakage control volume comprising the portion of the annulus in which the leakage air flows between the two layers of insulation, also between the positions x and $x+dx$. Next, the heat flow rates are written in terms of the above functions and parameters, as follows:

Heat loss rate via conduction from the air in the duct:

$$2 \pi r_1 / R_1 [T_D(x) - T_L(x)] dx$$

Net heat loss rate from leakage air in the annular region within the insulation:

$$2 \pi r_2 / R_2 [T_L(x) - T_S] dx - 2 \pi r_1 / R_1 [T_D(x) - T_L(x)] dx$$

Heat rate of duct air entering the duct control volume:

$$Q_D \rho C_P T_D(x)$$

Heat rate of duct air leaving the duct control volume:

$$Q_D \rho C_P T_D(x + dx)$$

Heat rate of leakage air entering the leakage control volume:

$$Q_L \rho C_P T_L(x)$$

Heat rate of leakage air leaving the leakage control volume:

$$Q_L \rho C_P T_L(x + dx)$$

Because this is a steady-state model, no buildup or depletion of heat occurs within the control volumes. Therefore, the net heat rate of air flowing into and out of each control volume can be set equal to the net conductive heat loss rate from the control volume:

Duct control volume:

$$Q_D \rho C_P [T_D(x) - T_D(x+dx)] = 2 \pi r_1 / R_1 [T_D(x) - T_L(x)] dx \quad (1)$$

Leakage control volume:

$$Q_L \rho C_P [T_L(x) - T_L(x+dx)] = 2 \pi r_2 / R_2 [T_L(x) - T_S] dx - 2 \pi r_1 / R_1 [T_D(x) - T_L(x)] dx \quad (2)$$

Converting differences to derivatives in the usual manner yields the following linked differential equations:

Duct control volume:

$$dT_D(x)/dx = - 2 \pi r_1 / (R_1 Q_D \rho C_P) [T_D(x) - T_L(x)] \quad (3)$$

Leakage control volume:

$$dT_L(x)/dx = - 2 \pi r_2 / (R_2 Q_L \rho C_P) [T_L(x) - T_S] + 2 \pi r_1 / (R_1 Q_L \rho C_P) [T_D(x) - T_L(x)] \quad (4)$$

It is now convenient to rescale the temperatures using T_S as a base, that is, we let $U_D(x) = T_D(x) - T_S$ and $U_L(x) = T_L(x) - T_S$, noting that $dU_D/dx = dT_D/dx$ and $dU_L(x)/dx = dT_L(x)/dx$. Equations 3 and 4 then become:

$$dU_D/dx = - k_0 U_D + k_0 U_L \quad (5)$$

$$dU_L/dx = k_1 U_D - k_3 U_L \quad (6)$$

where $k_0 = 2 \pi r_1 / (R_1 Q_D \rho C_P)$, $k_1 = 2 \pi r_1 / (R_1 Q_L \rho C_P)$, $k_2 = 2 \pi r_2 / (R_2 Q_L \rho C_P)$, and $k_3 = k_1 + k_2$. In later equations, the definition $k_4 = k_0 k_2 / k_3$ will also be used.

Equations 5 and 6 must be solved simultaneously, using appropriate boundary conditions. There are two such boundary conditions in this problem. The first states the temperature of the duct air entering the duct control volume, i.e., that $T_D(0) = T_0$. The second boundary condition states that the leakage air temperature at $x = 0$ is the same as that of the duct air, i.e., $T_L(0) = T_0$ also.

SOLUTION OF THE EQUATIONS

Equations 5 and 6 are simultaneous linear differential equations in two unknowns. The solutions to equations like this involve two exponential functions of x . Each of these exponentials is defined by two constants. A leading coefficient gives a weight to the function as a whole, while a coefficient of x within the argument of the exponential specifies a characteristic distance over which the exponential decays. The derivation of the equations is given in the Appendix. The solutions themselves are displayed below.

$$U_D(x) = U_0 [- k_4 / k_3 \exp (-k_3 x) + (1 + k_4 / k_3) \exp (-k_4 x)] \quad (7)$$

$$U_L(x) = U_0 [k_2/k_3 \exp (-k_3 x) + k_1/k_3 \exp (-k_4 x)] \quad (8)$$

Now we'll go back from the U's to the T's, by adding back T_S:

$$T_D(x) = (T_0 - T_S) [-k_4/k_3 \exp (-k_3 x) + (1 + k_4/k_3) \exp (-k_4 x)] + T_S \quad (9)$$

$$T_L(x) = (T_0 - T_S) [k_2/k_3 \exp (-k_3 x) + k_1/k_3 \exp (-k_4 x)] + T_S \quad (10)$$

The one additional piece of information needed is what happens to the temperature profile of the air in the duct if the leak goes directly out of the duct and the insulation at the same point. That is, what is the functional form of T_{D,dir}(x)? This can be obtained from the above equations by letting Q_L approach zero. This will make k₁ and k₂ (and hence k₃ as well) go to infinity, so care is required.

The first thing to note is that as k₃ approaches infinity, the first terms in Equations 7 and 8 approach zero for all x > 0. However, k₁ / k₂ remains constant at r₁R₂/(R₁r₂) even though its numerator and denominator "blow up." This makes k₄ independent of Q_L also, which further implies that k₄ / k₃ → 0. Putting all these things into the equation for T_D(x) gives the functional form of T_{D,dir}(x):

$$T_{D,dir}(x) = T_S + (T_0 - T_S) \exp (-k_4 x) \quad (11)$$

We now have the tools we need to calculate the heat flow rates and determine the magnitude of the thermal regain.

HEAT FLOWS AND THERMAL REGAIN FRACTION

The strategy for evaluating thermal regain due to longitudinal flow of leakage air within the duct insulation will be to define a control volume surrounding the duct and its insulation, and running from the point where the leak leaves the duct (x=0) to where it leaves the outer layer of insulation (x=L). Note that this is a different control volume from the ones used above to derive Equations 9 and 10.

The next step will be to calculate the relevant heat flow rates, as follows:

Into the control volume from upstream of x=0 (H_{in}):

$$H_{in} = (Q_D + Q_L) \rho C_P (T_0 - T_{in}) \quad (12)$$

From the control volume to downstream of x=L (H_{out}):

$$H_{out} = Q_D \rho C_P [T_D(L) - T_{in}] \quad (13)$$

Initial leakage loss from the duct (H_{leak}):

$$H_{leak} = Q_L \rho C_P (T_0 - T_{in}) \quad (14)$$

Conductive loss from the duct (H_{cond}):

$$H_{\text{cond}} = Q_D \rho C_P [T_0 - T_D(L)] \quad (15)$$

Note that $H_{\text{in}} = H_{\text{out}} + H_{\text{leak}} + H_{\text{cond}}$.

We also need the heat flow rate from the control volume to downstream of $x=L$ under the condition that the leak passes directly out of the insulation as well as the duct at $x=0$. This we will call $H_{\text{out,dir}}$:

$$H_{\text{out,dir}} = Q_D \rho C_P [T_{D,\text{dir}}(L) - T_{\text{in}}] \quad (16)$$

The thermal regain is defined as any benefit, in terms of either recovered heat or heat that is caused not to be lost by virtue of interactions between the leakage air and either the ducts or the building. In this case, the thermal regain, symbol H_{regain} , is just the difference between the heat outflow rate if the leak leaves the system directly and the heat outflow rate when the leak travels longitudinally between the insulation layers:

$$H_{\text{regain}} = H_{\text{out}} - H_{\text{out,dir}} \quad (17)$$

The thermal regain fraction ϕ_{regain} is defined here as the ratio of regained heat due to the specific path of the leak to the heat lost initially by the leak itself. The Greek phi is used here rather than the Latin F because in ASHRAE Standard 152P (ASHRAE 2001), the thermal regain factor F_{regain} is defined differently, i.e., as the fraction of overall conductive plus leakage heat loss from the supply duct or conductive loss from the return duct that is effectively regained. That is, F_{regain} is based on everything that goes on in the entire supply or return duct whereas ϕ_{regain} considers only what happens to a single leak at a particular point in the duct and only as a result of its displacement within the duct insulation. What the leakage does to the buffer zone surrounding the duct is not considered. The regain fraction used here could, however, be incorporated into a broader calculation of F_{regain} for use in Standard 152. With this as preamble, then, ϕ_{regain} is given by:

$$\phi_{\text{regain}} = H_{\text{regain}} / H_{\text{leak}} \quad (18)$$

The next step is to substitute the functions for H_{regain} and H_{leak} into Equation 17 and derive as simple an expression as possible for their ratio. This is done as follows:

$$\begin{aligned} \phi_{\text{regain}} &= H_{\text{regain}} / H_{\text{leak}} \\ &= (H_{\text{out}} - H_{\text{out,dir}}) / H_{\text{leak}} \\ &= \frac{Q_D \rho C_P [T_D(L) - T_{\text{in}}] - Q_D \rho C_P [T_{D,\text{dir}}(L) - T_{\text{in}}]}{Q_L \rho C_P (T_0 - T_{\text{in}})} \\ &= \frac{Q_D [T_D(L) - T_{D,\text{dir}}(L)]}{Q_L (T_0 - T_{\text{in}})} \end{aligned} \quad (19)$$

We now substitute L for x in the functions $T_D(x)$ and $T_{D,dir}(x)$ in Equations 7 and 9, respectively, and then insert these into Equation 19. After a bit of algebra, this simplifies somewhat to:

$$\phi_{\text{regain}} = \frac{Q_D (T_0 - T_s) k_4 [\exp(-k_4 L) - \exp(-k_3 L)]}{Q_L (T_0 - T_{in}) k_3} \quad (20)$$

Next, using the definitions of k_4 and k_3 as given earlier, one can derive:

$$\begin{aligned} k_4 / k_3 &= \frac{Q_L r_1 r_2 R_1 R_2}{Q_D (r_1 R_2 + r_2 R_1)^2} \\ &= \frac{Q_L (r_1/r_2)(R_1/R_2)}{Q_D (r_1/r_2 + R_1/R_2)^2} \end{aligned} \quad (21)$$

which when substituted into Equation 20 yields:

$$\phi_{\text{regain}} = \frac{(T_0 - T_s) (r_1/r_2)(R_1/R_2)}{(T_0 - T_{in}) (r_1/r_2 + R_1/R_2)^2} [\exp(-k_4 L) - \exp(-k_3 L)] \quad (22)$$

Equation 22 may look somewhat complicated, but by breaking the right-hand side into three parts and looking at each one separately, one can get an intuitive feel for what it predicts. Let us first consider the last section of this expression, namely the two exponentials in square brackets:

$$[\exp(-k_4 L) - \exp(-k_3 L)]$$

Of the two “decay constants” k_4 and k_3 , k_3 is a “fast decay” while k_4 is a “slow decay.” The constant k_4 reflects the relaxation of temperature in the duct when the leak goes directly out from the duct and the insulation. It gives the “baseline” behavior of the system, a relatively gradual drop in temperature within the duct. The constant k_3 , in contrast, is governed by the characteristics of the leak. As long as the leak rate Q_L is a small fraction of the total airflow Q_D , then $k_3 \gg k_4$ (cf. Equation 21) and hence the function $\exp(-k_3 x)$ will decay at much smaller values of x than will $\exp(-k_4 x)$.

From the standpoint of evaluating $[\exp(-k_4 L) - \exp(-k_3 L)]$, values of L can be divided into three ranges. For values of L that are small enough that $k_3 L \ll 1$, both of these exponentials will be very close to unity and hence will nearly cancel, causing the regain fraction nearly to vanish. As L increases, $\exp(-k_3 L)$ will start to decrease while $\exp(-k_4 L)$ remains close to 1.0. In the middle range of values for L such that $k_3 L \gg 1$ while $k_4 L \ll 1$, $[\exp(-k_4 L) - \exp(-k_3 L)]$ approaches its maximum possible value of 1.0.

The third part of the range that is theoretically possible would have $k_3 L$ and $k_4 L$ both much greater than one, but we wouldn't expect this in a practical duct system because this would mean that the duct was losing most of its heat through conduction regardless of whether there was any leakage. Poorly performing as some duct systems are, they usually aren't that bad. In the limiting case where the leak is displaced the entire length of the duct, one would expect $\exp(-k_4 L)$ to be some number reasonably close to 1.0, say in the 0.8 to 1.0 range reflecting commonly found conduction losses, while $\exp(-k_3 L)$ would have decayed nearly all the way to zero.

The middle portion of Equation 20,

$$\frac{(r_1/r_2)(R_1/R_2)}{(r_1/r_2 + R_1/R_2)^2}$$

contains ratios of the effective radii and R-values of the two layers of insulation. Here one can notice that if either R-value is zero, this expression also goes to zero. This makes intuitive sense, in that if $R_1 = 0$ the leakage air is in intimate thermal contact with the air in the duct, and it is as if the leak is just moved down the duct a distance L . On the other hand, if $R_2 = 0$, then the leakage air is in intimate thermal contact with the surrounding space, and it is as if the leak exited the duct directly at $L = 0$. Only when there are finite amounts of insulation on both sides of the path along which the leak is displaced is there a regain effect. The expression is maximized when $R_1/R_2 = r_1/r_2$. Since the values of r_1 and r_2 will usually be comparable (with r_2 slightly greater), this means that we would like to have the R values of the insulation layers nearly equal also.

The front-end portion of Equation 20 is:

$$\frac{T_0 - T_s}{T_0 - T_{in}}$$

This ratio of temperature differences points toward two conclusions:

1. The regain fraction will be largest when the space surrounding the ducts is coldest, because as T_s decreases, $T_0 - T_s$ becomes a larger multiple of $T_0 - T_{in}$. It is theoretically possible to have a regain fraction greater than 1.0 if T_s is low enough, although if realistic values for the duct parameters are inserted, one quickly finds that this doesn't occur unless T_s is in the cryogenic range. So this regain effect may be most important for ducts in unprotected spaces such as vented attics and crawl spaces in cold climates.

2. As long as $T_s < T_{in}$, the regain fraction will be largest when the delivered air temperature is the lowest, because as T_0 decreases, $T_0 - T_{in}$ becomes a smaller fraction of $T_0 - T_s$. This suggests that the regain effect will be greater for heat pumps than for furnaces, since heat pumps usually deliver lower-temperature air than do furnaces.

Putting all of this together, one can derive a simple upper limit for the thermal regain fraction, based on the following:

- The upper limit of possible values of $[\exp(-k_4 L) - \exp(-k_3 L)]$ is 1.0.
- The value of $(r_1/r_2)(R_1/R_2)/(r_1/r_2 + R_1/R_2)^2$ can be expressed as $\rho/(1+\rho)^2$ with $\rho=(r_1/r_2)/(R_1/R_2)$. The maximum value of this expression is 1/4, which occurs when $\rho=1$, i.e., when $r_1/r_2 = R_1/R_2$.

This yields an upper limit value, which we designate $\phi_{\text{regain,max}}$, equal to:

$$0.25 (T_0 - T_s) / (T_0 - T_{\text{in}}) \quad (23)$$

In a heating application, an appropriate benchmark value for this might be obtained using $T_{\text{in}} = 70$ °F as a typical indoor temperature and $T_s = 40$ °F, the latter being within the range of seasonal average crawlspace temperatures in winter across much of the northern U.S. Let us further assume that a furnace delivers air at ~120 °F while the delivered air temperature in a heat-pump system is likely to be ~100 °F.

That is, the buffer zone in which the ducts are located is 30 °F colder than the house and the air in the ducts is 50 °F warmer than the house for a furnace and 30 °F warmer than the house for a heat pump. Benchmark values for $\phi_{\text{regain,max}}$ would be 0.40 for a furnace system and 0.50 for a heat-pump system.

For cooling, any regain from the mechanisms considered here will encompass only the sensible part of the leak. Any latent cooling content will be lost, since it will not affect the temperature distribution surrounding the ducts. Therefore, any regain fraction calculated using these formulas must be multiplied by the sensible heat ratio, or ratio of sensible cooling to the sum of sensible and latent. The value of $\phi_{\text{regain,max}}$ will depend strongly on whether the ducts are in an attic or in a crawlspace.

Let us assume that the air in the ducts is 20 °F colder than the air in the house, while the buffer zone is 10 °F warmer than the house (crawlspace) or 40 °F warmer than the house (attic). These conditions are close to the ASHRAE Standard 152P seasonal values for Orlando, Florida. Under these conditions, the benchmark values for $\phi_{\text{regain,max}}$ will be 0.38 for crawlspace ducts and 0.75 for attic ducts.

It may be worth noting that for an extreme attic-duct case with 70 °F indoors, 55 °F air in the ducts, and 130 °F air temperature in the attic, $\phi_{\text{regain,max}}$ can actually exceed unity, coming in at 1.25. That is, it is possible for leaky ducts to perform better than non-leaky ones. We hasten to point out, however, that this would be a most unusual situation. The actual regain will usually be much less than one. However, this does illustrate that the proposed strategy may be at its best with attic ducts in the cooling mode. Also, it will be at its best under design conditions when the attic is hottest. It is precisely at such times that duct systems – especially attic ducts in the cooling mode – are generally least efficient.

The exact functional form for ϕ_{regain} can be written as:

$$\phi_{\text{regain}} = [\rho / (1 + \rho)^2] [(T_0 - T_s) / (T_0 - T_{\text{in}})] [\exp(-k_4 L) - \exp(-k_3 L)] \quad (24)$$

The expression $\rho / (1 + \rho)^2$ has a broad maximum around $\rho = 1$, where it equals 0.25. If ρ deviates from unity by a factor of two, i.e., if one insulation layer has twice the R-value of the other, the value of $\rho / (1 + \rho)^2$ drops by only 10%, to 0.22. So as long as the two insulation layers are even roughly comparable, their relative R-values do not influence the thermal regain fraction very much. Moreover, it doesn't matter whether the thicker layer is the inner or outer of the two insulation sheets.

The "difference of exponentials" expression $[\exp(-k_4 L) - \exp(-k_3 L)]$ can fall significantly below its maximum value of 1.0. Obviously, if L is very small one would expect this. A benchmark case had two layers of R-4 insulation wrapped around a 7-inch diameter duct in which 100 cfm of air was flowing. The leak was 5% (or 5 cfm). In this case the value of the difference of exponentials had the following values depending on the linear distance L over which the leak was displaced:

Linear Displacement of Leak, ft	0	2	4	8	16
Value of "Difference of Exponentials"	0	0.34	0.57	0.81	0.93

A table like this indicates that if leaks can be displaced relatively long distances, say 8 ft or more, than most of the theoretical maximum regain might be obtained. If the displacement is very short, say 2 ft, only a small fraction would be garnered. The actual amount of regain will, of course depend in detail on the factors that go into the difference of exponentials (especially the leakage fraction Q_L/Q_D), as well as the extent to which the leakage air actually surrounds the duct, as is assumed in the theoretical calculation.

UPSTREAM DISPLACEMENT OF LEAKAGE FLOW

So far, it has been assumed that the leak, once it leaves the duct and enters the space between the inner and outer layers of insulation, is displaced in the same direction as the flow in the duct itself. It is also possible, of course, that the displacement would be in the opposite direction, against the flow in the duct. The question arises, therefore, whether the direction in which the leakage is displaced would have a significant impact on the thermal regain fraction.

To investigate this case, we set up the problem as shown in Figure 2. The position $x=0$ is still defined as the place where the leak leaves the duct, but now the region where the leak is influencing the outcome is to the left of $x=0$, that is, negative values of x between zero and $-L$.

The heat rates for this case are the same as those given prior to Equation 1 in the section "Governing Equations and Boundary Conditions," with the exception that the heat rates of the leakage air entering and leaving the control volume are reversed. That

is, the heat rate of leakage air entering the leakage control volume is $Q_L \rho C_P T_L(x + dx)$ and the heat rate of leakage air leaving the leakage control volume is $Q_L \rho C_P T_L(x)$. This leaves Equation 1 unchanged, but reverses the sign of the left-hand side of Equation 2, relative to the right-hand side. The governing equations for the upstream leak displacement case are then:

$$Q_D \rho C_P [T_D(x) - T_D(x+dx)] = 2 \pi r_1 / R_1 [T_D(x) - T_L(x)] dx \quad (25)$$

$$Q_L \rho C_P [T_L(x) - T_L(x+dx)] = - 2 \pi r_2 / R_2 [T_L(x) - T_S] dx + 2 \pi r_1 / R_1 [T_D(x) - T_L(x)] dx \quad (26)$$

This leads to equations that are the same as Equations 5 and 6, except for relative minus signs introduced into the latter. That is, we now have:

$$dU_D/dx = - k_0 U_D + k_0 U_L \quad (27)$$

$$dU_L/dx = - k_1 U_D + k_3 U_L \quad (28)$$

with the U 's and k 's defined in the same way as before.

The solution goes along similar lines to the previous case. The location where the leak leaves the duct is still called $x=0$, but now the region of interest ranges from $x=-L$ to $x=0$. The solutions to Equations 27 and 28, obtained in similar manner as in the previous case, are:

$$U_D(x) = U_0 [k_4/k_3 \exp (k_3 x) + (1 - k_4/k_3) \exp (-k_4 x)] \quad (29)$$

$$U_L(x) = U_0 [k_2/k_3 \exp (k_3 x) + k_1/k_3 \exp (-k_4 x)] \quad (30)$$

Notice that the sign of the argument of the first exponential function is positive. This is caused by the fact that the leakage flow is moving opposite to the direction of the flow in the duct.

The one thing here that is quite different is the value of U_0 . The boundary condition in this case must be applied at $x = -L$, namely that $U_D(-L) = T_0 - T_S$, since T_0 is defined as the temperature of air in the duct that is entering the region over which the leak is displaced. Equation 29 then implies:

$$T_0 - T_S = U_0 [k_4/k_3 \exp (-k_3 L) + (1 - k_4/k_3) \exp (k_4 L)] \quad (31)$$

from which U_0 is easily obtained:

$$U_0 = (T_0 - T_S) / [k_4/k_3 \exp (-k_3 L) + (1 - k_4/k_3) \exp (k_4 L)] \quad (32)$$

The expression in square brackets will be close to unity as long as $k_4 \ll k_3$ and $k_4 L \ll 1$ as will normally be the case (the first inequality insured by small leakage, the second by reasonable amounts of insulation). The derivation of the thermal regain fraction will proceed along the same lines as in the forward displacement case considered

earlier. Although there will be slight differences in the calculated results, these will generally be small. The analysis of this case will not be pursued further here.

REFERENCE

ASHRAE 2001. Standard 152P: Method of Test for Determining the Design and Seasonal Efficiencies of Residential Thermal Distribution Systems. American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Atlanta, GA.

APPENDIX. SOLUTION OF EQUATIONS 5 AND 6

Equations 5 and 6 in the main text (repeated here as Equations A-1 and A-2) are simultaneous linear differential equations:

$$dU_D/dx = -k_0 U_D + k_0 U_L \quad (\text{A-1})$$

$$dU_L/dx = k_1 U_D - k_3 U_L \quad (\text{A-2})$$

The solutions will in general contain two exponentials with different constants, so that one will be more rapidly varying (as a function of x) than the other. We write these as:

$$U_D = U_1 \exp(-c_1 x) + (U_0 - U_1) \exp(-c_2 x) \quad (\text{A-3})$$

$$U_L = (U_0 - U_2) \exp(-c_1 x) + U_2 \exp(-c_2 x) \quad (\text{A-4})$$

where $U_0 = T_0 - T_s$, and U_1 , U_2 , c_1 , and c_2 are constants to be determined. (The minus signs in front of c_1 and c_2 are included because decaying exponentials are expected. If that assumption were wrong, the c 's would come out negative in the solution.) The value of U_0 in these equations is fixed by the boundary conditions, which require that at $x=0$, U_D and U_L both equal $T_0 - T_s$.

The values of the other four constants are found by differentiating Equations A-3 and A-4, substituting the resulting functions into Equations A-1 and A-2, and noting that the coefficients of the individual exponentials must separately equate in order for these equations to hold for all values of x in the domain of interest. The substitution process yields the equations:

$$-c_1 U_1 \exp(-c_1 x) - c_2 (U_0 - U_1) \exp(-c_2 x) = -k_0 U_1 \exp(-c_1 x) - k_0 (U_0 - U_1) \exp(-c_2 x) + k_0 (U_0 - U_2) \exp(-c_1 x) + k_0 U_2 \exp(-c_2 x) \quad (\text{A-5})$$

$$-c_1 (U_0 - U_2) \exp(-c_1 x) - c_2 U_2 \exp(-c_2 x) = k_1 U_1 \exp(-c_1 x) + k_1 (U_0 - U_1) \exp(-c_2 x) - k_3 (U_0 - U_2) \exp(-c_1 x) - k_3 U_2 \exp(-c_2 x) \quad (\text{A-6})$$

Separating out the leading coefficients of terms with the same constant in the argument of the exponential (c_1 or c_2) leads to:

$$-c_1 U_1 = -k_0 U_1 + k_0 (U_0 - U_2) \quad (\text{A-7})$$

$$-c_2 (U_0 - U_1) = -k_0 (U_0 - U_1) + k_0 U_2 \quad (\text{A-8})$$

$$-c_1 (U_0 - U_2) = k_1 U_1 - k_3 (U_0 - U_2) \quad (\text{A-9})$$

$$-c_2 U_2 = k_1 (U_0 - U_1) - k_3 U_2 \quad (\text{A-10})$$

We have four equations in four unknowns, so we may hope to find a solution. The equations are not linear in these variables, so the process is not entirely straightforward, but it is possible. The first step is to eliminate c_1 from Equations A-7 and A-9, and c_2 from A-8 and A-10. This yields:

$$0 = k_1 U_1^2 + (k_0 - k_3) (U_0 - U_2) U_1 - k_0 (U_0 - U_2)^2 \quad (\text{A-11})$$

$$0 = k_0 U_2^2 - (k_0 - k_3) (U_0 - U_1) U_2 - k_1 (U_0 - U_1)^2 \quad (\text{A-12})$$

Equation A-11 is quadratic in U_1 . Its solution in terms of the other variables is:

$$U_1 = (U_0 - U_2) \{ k_3 - k_0 \pm [(k_3 - k_0)^2 + 4 k_1 k_0]^{1/2} \} / (2 k_1) \quad (\text{A-13})$$

One now has to make the decision whether to take the plus or the minus sign in this equation. What should happen is that one sign gives the same set of solutions as the other, with the roles of U_1 and c_1 as opposed to U_2 and c_2 reversed. However, the minus sign provides an easier path, because $k_0 \ll k_1 < k_3$. The first inequality holds as long as the leakage is much smaller than the total duct flow, because $k_0/k_1 = Q_L/Q_D$. The second inequality holds because $k_1 = k_3 - k_2$ and all the k 's are positive numbers.

This permits us to use the approximations $(k_3 - k_0)^2 \gg 4 k_1 k_0$ and $(k_3 - k_0)^2 \approx k_3^2$, which when applied to Equation A-13 lead to:

$$U_1 \approx - (U_0 - U_2) k_0/k_3 \quad (\text{A-14})$$

In view of $k_0 \ll k_3$, this implies that $U_1 \ll U_0$, as long as U_2 isn't too close to U_0 . Let us make this assumption, calculate U_2 from Equation A-12, and then check whether what we have assumed is valid. Equation A-12, then, yields the approximation:

$$0 \approx k_3 U_0 U_2 - k_1 U_0^2 \quad (\text{A-15})$$

which produces $U_2 \approx U_0 k_1/k_3$. Checking our assumption that $U_0 - U_2$ is not close to zero, we calculate $U_0 - U_2 \approx U_0 (1 - k_1/k_3) = U_0 k_2/(k_1 + k_2)$, which will generally be on the order of one-half U_0 as long as the radii and R values for the two layers of insulation are not an order of magnitude different.

The other two coefficients are $(U_0 - U_1)$ and $(U_0 - U_2)$. These can be calculated as $U_0 - U_2 \approx U_0 k_2/k_3$ and $U_0 - U_1 \approx U_0 (1 + k_0 k_2/k_3^2)$, to the same order of accuracy.

It now remains to solve for the c 's. Equation A-7 can be written as:

$$\begin{aligned} c_1 &= k_0 [1 - (U_0 - U_2)/U_1] \\ &\approx k_0 (1 + k_3/k_0) \\ &\approx k_3 \end{aligned} \quad (\text{A-16})$$

Similarly, Equation A-8 can be written:

$$\begin{aligned} c_2 &= k_0 [1 - U_2/(U_0 - U_1)] \\ &\approx k_0 (1 - k_1/k_3) \\ &\approx k_0 k_2/k_3^2 \end{aligned} \quad (\text{A-17})$$

Inserting these constants into Equations A-3 and A-4 leads to Equations 7 and 8.

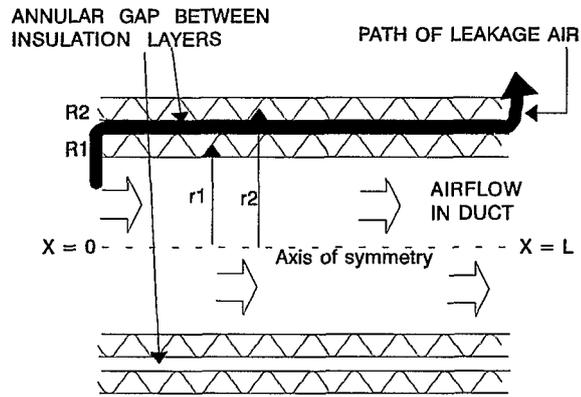


Figure 1. Duct Leakage Displacement Model

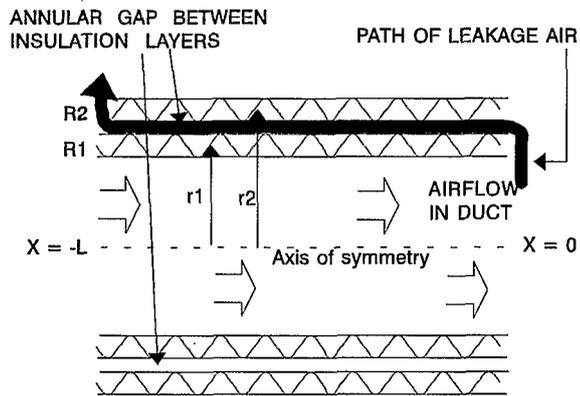


Figure 2. Leakage Displacement for Upstream Flow