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## Techniques for NNLO Higgs production in the Standard Model and the MSSM

R.V. HARLANDER<sup>1</sup> and W.B. KILGORE<sup>2</sup>

<sup>1</sup>*TH Division, CERN, CH-1211 Geneva 23, Switzerland*

<sup>2</sup>*Department of Physics, Brookhaven National Lab, Upton, New York 11973, USA*

New techniques developed in connection with the NNLO corrections to the Higgs production rate at hadron colliders and some recent applications are reviewed.

### 1 Introduction

The NLO corrections for the dominant Higgs production mechanism at hadron colliders,  $gg \rightarrow H$ , amount to about 70% and suffer from rather large scale uncertainties.<sup>1,2,3</sup> The need for the evaluation of the NNLO cross section has resulted in promising new calculational techniques. The first part of this talk will briefly review these techniques. In the second part, we will discuss a recent application, namely the NNLO cross section for MSSM Higgs production in bottom quark fusion.

### 2 Techniques for Higgs production at NNLO

The first calculation of the NNLO prediction for the cross section  $\sigma(pp \rightarrow H + X)$  used the classic approach of computing the amplitudes for virtual and real corrections, squaring them, and integrating over the final state phase space. The two-loop virtual amplitude for  $gg \rightarrow H$  was evaluated<sup>4</sup> using the method of Baikov and Smirnov<sup>5</sup> that maps the occurring integrals to the well-known class of three-loop propagator-type integrals.<sup>6</sup> The phase space integration for the two-loop virtual terms is trivial, resulting in  $\delta(1-x)$ , where  $x = M_H^2/\hat{s}$  and  $\hat{s}$  is the partonic center of mass (c.m.) energy.

The one-loop amplitude for the radiation of a single massless parton has to be interfered with the corresponding tree-level expression, and integrated over the two-particle phase space. Both loop and phase space integration can be performed analytically.

This leaves us with the tree-level contributions for the radiation of two massless partons. The squared amplitude can be obtained straightforwardly with the help of FORM.<sup>7</sup> In the first approach that we are going to describe, the phase space integrals were evaluated in terms of an expansion in  $(1-x)$ . The leading term is called the *soft approximation*<sup>8,9</sup> and is formally of order  $(1-x)^{-1}$ , where the associated divergence as  $x \rightarrow 1$  is parameterized in terms of the distributions  $\delta(1-x)$  and  $[\ln^n(1-x)/(1-x)]_+$ . The higher orders in  $(1-x)$  can be obtained by a systematic expansion of the squared amplitude and the phase space measure.<sup>10</sup> The crucial point is that, independent of the degree of this expansion, one always ends up with the same type of integrals. This classifies the procedure as an algorithm, which can be fully automated.



coupling of bottom quarks. Thus, the  $gg\phi$  coupling may have a significant contribution from virtual bottom quarks ( $\phi$  denotes any of the neutral Higgs bosons in the MSSM). In this case, the NNLO corrections are much harder to evaluate, because the effective-Lagrangian approach of the top quark case is not expected to work.

The main focus here shall be another effect of an enhanced bottom Yukawa coupling, namely the increased rate of associated production of a Higgs boson with a bottom–anti-bottom quark pair. There has been an on-going discussion concerning the proper description of this process.<sup>19,20,21,22</sup> *A priori*, the leading order contribution is, of course, the tree-level process  $gg \rightarrow \phi b\bar{b}$ , where  $\phi$  denotes any of the MSSM Higgs bosons  $h, H, A$ . However, when evaluating the total rate for this process, the integration over small bottom- $p_T$  leads to collinear logarithms of the form  $l_b \equiv \ln(m_b^2/M^2)$ , where  $M$  is a scale of the order of the Higgs boson mass. Since  $m_b \ll M_\phi \sim M$ , these logarithms should be resummed. This is achieved by introducing bottom quark densities and making the process  $b\bar{b} \rightarrow \phi$  the leading order contribution. Schematically, one can write the total cross section in the bottom density approach as follows:

$$\sigma(pp \rightarrow H + X) = \sum_{n=0}^{\infty} (\alpha_s l_b)^{n+2} \left[ c_{n0} + c_{n1} \frac{1}{l_b} + \frac{1}{l_b^2} (c_{n2} + \alpha_s d_{n3} + \alpha_s^2 d_{n4} + \dots) \right]. \quad (2)$$

This equation is to be understood as follows: First of all, one should note that the  $c_{ni}$  and  $d_{ni}$  are not obtained individually for each  $n$ , because the sum over  $n$  is implicit in the parton densities. Including only the leading order process  $b\bar{b} \rightarrow \phi$ , one obtains the contribution from the  $c_{n0}$ . The NLO diagrams contribute terms of order  $1/l_b$  (e.g.,  $gb \rightarrow b\phi$ ) and of order  $\alpha_s$  ( $b\bar{b} \rightarrow \phi$  at 1-loop) with respect to the leading term. Both are contained in the  $c_{n1}$ . At N<sup>n</sup>LO, for  $n \geq 2$ , we have terms of order  $\alpha_s^{n-k}/l_b^k$  w.r.t. LO, where  $k = 0, 1, 2$ . The reason why there can be only two inverse powers of  $l_b$  comes from the fact that there are only two initial state partons. Looking at Eq. (2), it becomes clear why the NNLO plays an exceptional role in this process: It comprises all terms at leading order in  $\alpha_s$ .

The calculation of the process<sup>b</sup>  $pp \rightarrow (b\bar{b})\phi$  in the bottom density approach proceeds in complete analogy to, say, Drell-Yan production of virtual photons. Technical details of the calculation can be found elsewhere.<sup>23</sup>

Fig. 1 (a) shows the factorization scale dependence of the cross section at LO, NLO, and NNLO at the LHC, for a Higgs mass of  $M_H = 120$  GeV at the LHC. We notice several intriguing features: First, in contrast to the LO and NLO result, the NNLO curve has a clearly distinguished point where the derivative is zero, *i.e.* where the sensitivity to  $\mu_F$  is minimal. Second, the NNLO corrections are zero at a point where the NLO corrections are small. Third, this point is close to the point of least sensitivity. And fourth, all these points are compatible with a previous estimate<sup>20,22,21</sup> of the “natural” factorization scale for this process of around  $\mu_F = M_H/4$ . For the Tevatron, the overall picture is essentially the same<sup>23</sup>. These features nicely demonstrate the self-consistency of the bottom density approach and support the general considerations concerning the proper choice of the factorization scale for heavy quark partons.<sup>20,22,21</sup>

Fig. 1 (b) shows the total cross section for  $pp \rightarrow (b\bar{b})H + X$  as a function of the Higgs boson mass up to NNLO, for two different values of the factorization scale, indicating the theoretical uncertainty (the renormalization scale dependence can be neglected).

**Conclusions.** Recent progress in the evaluation of radiative corrections has led to NNLO predictions for Higgs production at hadron colliders, both in the SM and the MSSM. The results are very stable with respect to scale variations and indicate a well-behaved perturbative series.

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<sup>b</sup>We adopt the notation  $(b\bar{b})\phi$  in order to indicate that the bottom quarks may be produced at small transverse momenta and thus escape detection.

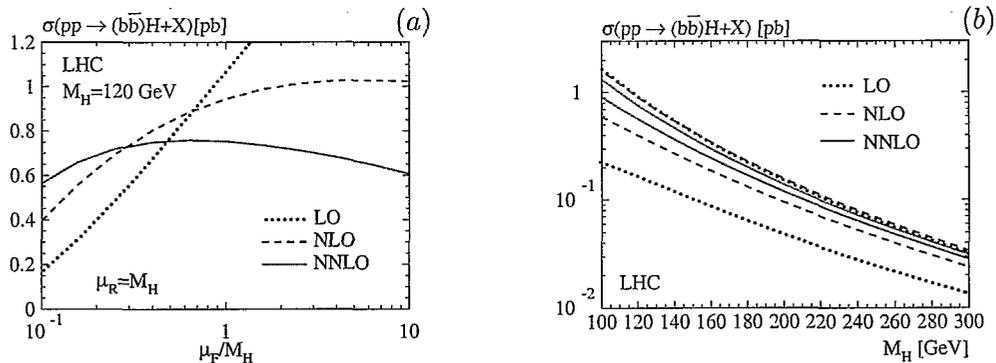


Figure 1:

(a) Factorization scale dependence of the cross section for  $pp \rightarrow (b\bar{b})H + X$  ( $\tan \beta = 1$ ); (b) Total cross section for  $pp \rightarrow (b\bar{b})H + X$  — upper/lower line:  $\mu_F = 0.7M_H/\mu_F = 0.1M_H$  [ $\mu_R = M_H$ ].

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