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An Overview of Neutrino Masses and Mixing in SO(10) Models

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Abstract. We review in this talk various SUSY SO(10) models. Specifically, we discuss how small neutrino masses are generated in and generic predictions of different $SO(10)$ models. A comparison of the predictions of these models for $\sin^2\theta_{13}$ is given.

The flavor problem with hierarchical fermion masses and mixing has attracted a great deal of attention especially since the advent of the atmospheric neutrino oscillation data from Super-Kamiokande indicating non-zero neutrino masses. The non-zero neutrino masses give support to the idea of grand unification based on $SO(10)$ in which all the 16 fermions (including the right-handed neutrinos) can be accommodated in one single spinor representation. Furthermore, it provides a framework in which seesaw mechanism arises naturally. Models based on $SO(10)$ combined with a continuous or discrete flavor symmetry group have been constructed to understand the flavor problem, especially the small neutrino masses and the large leptonic mixing angles. These models can be classified according to the family symmetry implemented in the model. We review in this talk how small masses and large mixing angles in the neutrino sector are generated in SO(10) models, and the unique predictions of each class of models. We also discuss other mechanisms that have been proposed to solve the problem of neutrino masses and mixing. For a more exhaustive list of references and detailed discussion, we refer the readers to our recent review[1] on which this talk is based.

Symmetric textures: This type of models have been considered, for example, in Ref.[2-4]. SO(10) breaks down through the left-right symmetry breaking chain, which ensures the mass matrices are symmetric. The Higgs content of this type of models contains fields in 10, 45, 54, 126 representations, with 10, 126 breaking EW symmetry and generating fermions masses, and 45, 54, 126 breaking SO(10). The mass hierarchy can arise if there is an $SU(2)_H$ symmetry acting non-trivially on the first two generations such that the first two generations transform as a doublet and the third generation transforms as a singlet under $SU(2)_H$, which breaks down at two steps, $SU(2) \overset{\varepsilon M}{\to} U(1) \overset{\varepsilon' M}{\to} nothing$ where $\varepsilon' << \varepsilon << 1$. The mass hierarchy is generated by the Froggatt-Nielsen mechanism which requires the flavon fields acquiring VEV's along the directions specified in Ref.[2-4]. The resulting mass matrices at the GUT scale are given by

$$M_{u,\nu_{LR}} = \begin{pmatrix} 0 & 0 & \langle 10_2^+\rangle\varepsilon' \\ 0 & \langle 10_4^+\rangle\varepsilon & \langle 10_3^+\rangle\varepsilon \\ \langle 10_2^+\rangle\varepsilon' & \langle 10_3^+\rangle\varepsilon & \langle 10_1^+\rangle \end{pmatrix} = \begin{pmatrix} 0 & 0 & r_2\varepsilon' \\ 0 & r_4\varepsilon & \varepsilon \\ r_2\varepsilon' & \varepsilon & 1 \end{pmatrix} M_U \qquad (1)$$

$$M_{d,e}=\begin{pmatrix} 0 & \langle 10_5^- \rangle \varepsilon' & 0 \\ \langle 10_5^- \rangle \varepsilon' & (1,-3)\langle \overline{126}^- \rangle \varepsilon & 0 \\ 0 & 0 & \langle 10_1^- \rangle \end{pmatrix}=\begin{pmatrix} 0 & \varepsilon' & 0 \\ \varepsilon' & (1,-3)p\varepsilon & 0 \\ 0 & 0 & 1 \end{pmatrix} M_D \tag{2}$$

The right-handed neutrino mass matrix is of the same form as $M_{\nu_{LR}}$

$$M_{\nu_{RR}}=\begin{pmatrix} 0 & 0 & \langle \overline{126}_2'^0 \rangle \delta_1 \\ 0 & \langle \overline{126}_2'^0 \rangle \delta_2 & \langle \overline{126}_2'^0 \rangle \delta_3 \\ \langle \overline{126}_2'^0 \rangle \delta_1 & \langle \overline{126}_2'^0 \rangle \delta_3 & \langle \overline{126}_1'^0 \rangle \end{pmatrix}=\begin{pmatrix} 0 & 0 & \delta_1 \\ 0 & \delta_2 & \delta_3 \\ \delta_1 & \delta_3 & 1 \end{pmatrix} M_R \tag{3}$$

Note that, since we use $\overline{126}$-dimensional representations of Higgses to generate the heavy Majorana neutrino mass terms, R-parity is preserved at all energies. The effective neutrino mass matrix is

$$M_{\nu_{LL}}=M_{\nu_{LR}}^T M_{\nu_{RR}}^{-1} M_{\nu_{LR}}=\begin{pmatrix} 0 & 0 & t \\ 0 & 1 & 1+t^n \\ t & 1+t^n & 1 \end{pmatrix} \frac{d^2 v_u^2}{M_R} \tag{4}$$

giving rise to maximal mixing angle for the atmospheric neutrinos and LMA solution for the solar neutrinos. The form of the neutrino mass matrix in this model is invariant under the seesaws mechanism. The value of U_{e3} is predicted to be large, close to the sensitivity of current experiments. This is a consequence of the solar angle being large. The prediction for the rate of $\mu \to e\gamma$ is about two orders of magnitude below the current experimental bound.

Lopsided/Asymmetric textures: This type of models have been considered, for example, in Ref.[5-12]. In this case, $SO(10)$ breaks down to SM through the $SU(5)$ breaking chain. The Higgs sector of the model contains $10, 16, 45, 54$, with $<16_{H_1}>$ breaking $SO(10)$ down to $SU(5)$ and $<16_{H_2}>$ breaking the EW symmetry. The lopsided textures arise due to the operator $\lambda(16_i 16_{H_1})(16_j 16_{H_2})$ which gives rise to mass terms for the charged leptons and down quarks, satisfying the SU(5) relation $M_d = M_e^T$. When other operators are included, the lopsided structure of M_e results, provided the coupling λ is of order 1,

$$M_{u,\nu_{LR}}=\begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & (1/3,1)\varepsilon \\ 0 & -(1/3,1)\varepsilon & 1 \end{pmatrix} \cdot m_u \tag{5}$$

$$M_d=\begin{pmatrix} \eta & \delta & \delta' e^{i\phi} \\ \delta & 0 & \lambda+\varepsilon/3 \\ \delta' e^{i\phi} & -\varepsilon/3 & 1 \end{pmatrix} \cdot m_d, \quad M_e=\begin{pmatrix} \eta & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\varepsilon \\ \delta' e^{i\phi} & \lambda+\varepsilon & 1 \end{pmatrix} \cdot m_d. \tag{6}$$

The large mixing in $U_{e,L}$ contributes to large leptonic mixing matrix, leading to the large atmospheric mixing angle. Meanwhile, because large mixing in $U_{e,L}$ corresponds to large mixing in $U_{d,R}$, the CKM mixing angles can be retained small. An unique prediction of the lop-sided models is the large branching ratio for LFV processes, e.g. $\mu \to e\gamma$. By considering neutrino RH Majorana mass term of the following form, large solar mixing angle can arise for some choice of the parameters in $M_{\nu_{RR}}$, leading to LMA solution for solar neutrinos,

$$M_{\nu_{RR}}=\begin{pmatrix} c^2\eta^2 & -b\varepsilon\eta & a\eta \\ -b\varepsilon\eta & \varepsilon^2 & -\varepsilon \\ a\eta & -\varepsilon & 1 \end{pmatrix}\cdot\Lambda_R, \quad M_\nu^{eff}=\begin{pmatrix} 0 & -\varepsilon & 0 \\ -\varepsilon & 0 & 2\varepsilon \\ 0 & 2\varepsilon & 1 \end{pmatrix} m_u^2/\lambda_R. \tag{7}$$

The value for $|U_{e3}|$ is predicted to be small.

Large mixing from RGE's: This class of models have been considered, for example, in [13]. The RG evolution of the effective neutrino mass matrix is given by, $dm_\nu/dt=-\{\kappa_u m_\nu+m_\nu P+P^T m_\nu\}$ where $P\simeq-\frac{1}{32\pi^2}\frac{h_\tau^2}{\cos^2\beta}diag(0,0,1)$, $\kappa_u\simeq\frac{1}{16\pi^2}[\frac{6}{5}g_1^2+6g_2^2-6\frac{h_t^2}{\sin^2\beta}]$. If one assumes nearly degenerate mass pattern and same Majorana CP phases at the GUT scale, the parameters s_{12} and s_{23} are driven to be large, while corrections to m_i are small. Assuming leptonic mixing matrix is identical to the CKM matrix at the GUT scale. Starting with $s_{12}^0\simeq\lambda$, $s_{23}^0\simeq\mathcal{O}(\lambda^2)$, and $s_{13}^0\simeq\mathcal{O}(\lambda^3)$, one obtains $\sin^2 2\theta_{atm}=0.99$, $\sin^2 2\theta_\odot=0.87$, $\sin\theta_{13}=0.08$ at the weak scale. These GUT scale conditions can be understood in Type II seesaw mechanism, with $M_{\nu_{LL}}$ term (due to the coupling to an SU(2) triplet Higgs) dominates: because $M_{\nu_{LL}}\sim I\cdot m_{\nu_{LL}}$ dominates, one obtains the nearly degenerate mass spectrum; as the flavor mixing is due to the usual seesaw term, the mixing angle of the resulting mass matrix is CKM-like. Thus the two conditions needed for enhancing the mixing angles are satisfied. The U_{e3} element is also amplified by the RG flow, with the low energy prediction close to the sensitivity of current experiments.

Large mixing from $b-\tau$ unification: This class of models have been considered, for example, in Ref.[14, 15]. It has a minimal Higgs sector which contains $\{10,126,45,54\}$. The following mass relations arise due to SO(10) symmetry, $M_u=f\langle 10\rangle+h\langle\overline{126}\rangle, M_d=f\langle 10\rangle+h\langle\overline{126}\rangle, M_e=f\langle 10\rangle-3h\langle\overline{126}\rangle, M_{\nu_{LR}}=f\langle 10\rangle-3h\langle\overline{126}\rangle$. As there are only one 10 and one 126 Higgs representations, all mass terms are governed by two Yukawa matrices, f and h. The small neutrino masses are explained by the Type II see-saw mechanism with the assumption that the LH Majorana mass term dominates over the usual Type I see-saw term, $M_\nu^{eff}=M_{\nu_{LL}}-M_{\nu_{LR}}M_{\nu_{RR}}^{-1}M_{\nu_{LR}}^T$. The mass terms $M_{\nu_{LL}}$ and $M_{\nu_{RR}}$ are both due to the coupling to $\overline{126}$, leading to $M_{\nu_{LL}}\sim hv_{ew}^2/v_R$ and $M_{\nu_{RR}}\sim hv_R$. In this minimal scheme, we have the following sum-rule $M_\nu^{eff}=c(M_d-M_e)$. The down-type quark and charged lepton mass matrices can be parameterized in terms of Wolfenstein parameter as

$$M_{b,\tau}\sim\begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} m_{b,\tau} \tag{8}$$

For some value of $\tan\beta$ (small values are preferred), the deviation from $b-\tau$ unification at the GUT scale is $m_b(M_{GUT})-m_\tau(M_{GUT})\simeq\mathcal{O}(\lambda^2)m_\tau$ which leads to a bi-large mixing pattern in M_ν. Generic predictions of this model are $\sin^2 2\theta_{23}<0.9$ and $\sin^2 2\theta_{12}>0.9$, making the model testable. The prediction of this model for the value of $|U_{e3}|$ is large. This is a consequence of the atmospheric mixing angle being maximal.

Comparison of models: Predictions of selected models for $\sin\theta_{13}$ are summarized in Table 1. A general observation is the following: (i) large $U_{e3}\sim\mathcal{O}(0.1)$ (can be probed by conventional/superbeam) arises in models with symmetric texture, in models based on anarchy, in models with RG enhanced leptonic mixing, and in minimal models with

TABLE 1. Predictions for $\sin\theta_{13}$ of various models. The upper bound from CHOOZ experiment is $\sin\theta_{13} < 0.24$. First nine models use $SO(10)$. Last two models are not based on $SO(10)$.

Model	family symmetry	solar solution	$\sin\theta_{13}$
Albright-Barr[5]	$U(1)$	LMA	0.014
Babu-Pati-Wilczek[6]	$U(1)$	SMA	5.5×10^{-4}
Blazek-Raby-Tobe[7]	$U(2)\times U(1)^n$	LMA	0.049
Berezhiani-Rossi[8]	$SU(3)$	SMA	$\mathcal{O}(10^{-2})$
Chen-Mahanthappa[4]	$SU(2)$	LMA	0.149
Kitano-Mimura[9]	$SU(3)\times U(1)$	LMA	$\sim\lambda\sim 0.22$
Maekawa[10]	$U(1)$	LMA	$\sim\lambda\sim 0.22$
Raby[11]	3×2 seesaw with $SU(2)_F$	LMA	$\sim m_{\nu_2}/2m_{\nu_3}\sim\mathcal{O}(0.1)$
Ross-Velasco-Sevilla[12]	$SU(3)$	LMA	0.07
Frampton-Glashow[16] -Yanagida	3×2 seesaw	LMA	$\sim m_{\nu_2}/2m_{\nu_3}\sim\mathcal{O}(0.1)$
Mohapatra-Parida[13] -Rajasekeran	RG enhancement	LMA	$0.08-0.10$

approximate $b-\tau$ unification; (ii) intermediate U_{e3} value $\sim(0.05-0.07)$ (can be probed by superbeam) arises in models with asymmetric texture; (iii) small U_{e3} (neutrino factory may be needed) arises in models with lop-sided texture. The mass hierarchy can be probed by long baseline experiments with distance greater than $900Km$, which can then be used to distinguish different models. A general observation is (i) normal hierarchy arise in SO(10) models with $SU(3)_H$, $SU(2)_H$, $U(1)_H$, in minimal SO(10) models with $b-\tau$ unification, and in SO(10) models with 3×2 see-saw; (ii) inverted hierarchy arise in models with $L_e-L_\mu-L_\tau$ horizontal symmetry; (iii) nearly degenerate arises in SO(10) models with RG enhanced lepton mixing and in models with anarchy. This work was supported by US DOE under Grant No. DE-AC02-98CH10886 and DE-FG03-95ER40894.

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