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FORMAL REPORT

Proceedings of RIKEN BNL Research Center Workshop Volume 53

Theory Studies for Polarized *pp* Scattering

August – September 2003

Organizers

S. Kretzer and W. Vogelsang

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Preface to the Series

The RIKEN BNL Research Center (RBRC) was established in April 1997 at Brookhaven National Laboratory. It is funded by the "Rikagaku Kenkyusho" (RIKEN, The Institute of Physical and Chemical Research) of Japan. The Center is dedicated to the study of strong interactions, including spin physics, lattice QCD, and RHIC physics through the nurturing of a new generation of young physicists.

During the first year, the Center had only a Theory Group. In the second year, an Experimental Group was also established at the Center. At present, there are seven Fellows and seven Research Associates in these two groups. During the third year, we started a new Tenure Track Strong Interaction Theory RHIC Physics Fellow Program, with six positions in the first academic year, 1999-2000. This program had increased to include ten theorists and one experimentalist in academic year, 2001-2002. With recent graduations, the program presently has eight theorists and two experimentalists. Beginning last year a new RIKEN Spin Program (RSP) category was implemented at RBRC, presently comprising four RSP Researchers and five RSP Research Associates. In addition, RBRC has four RBRC Young Researchers.

The Center also has an active workshop program on strong interaction physics with each workshop focused on a specific physics problem. Each workshop speaker is encouraged to select a few of the most important transparencies from his or her presentation, accompanied by a page of explanation. This material is collected at the end of the workshop by the organizer to form proceedings, which can therefore be available within a short time. To date there are fifty-one proceeding volumes available.

The construction of a 0.6 teraflops parallel processor, dedicated to lattice QCD, begun at the Center on February 19, 1998, was completed on August 28, 1998. A 10 teraflops QCDOC computer in under development and expected to be completed in JFY 2003.

T. D. Lee
November 22, 2002

*Work performed under the auspices of U.S.D.O.E. Contract No. DE-AC02-98CH10886.

CONTENTS

Theory Studies for polarized pp scattering

– Introduction and Overview –

In the past two runs of RHIC, the first measurements with polarized proton beams have been performed. For many years to come, the RHIC spin program will offer exciting physics, exploring QCD and the nucleon in new ways.

The aim of this small workshop was to attract several spin theorists to the center for about two weeks, in order to collaborate with both experimentalists and theorists at RBRC, and to initiate and/or complete studies of relevance to RHIC spin.

A major focus of polarized-pp measurements at RHIC is on measuring the spin-dependent gluon density, Δg. A channel for accessing Δg is high-p_T pion production. The unpolarized cross section for this reaction has been measured by PHENIX and was found in good agreement with a perturbative-QCD based (NLO) calculation. It was a remarkable and exciting coincidence that PHENIX presented also the first results for the spin asymmetry for $\vec{p}\vec{p} \rightarrow \pi^0 X$ during this workshop. This sparked a lot of additional activity and discussion. First steps toward the interpretation of the data were taken. Marco Stratmann and Barbara Jäger (Regensburg University) presented recent work on the NLO calculation of the polarized cross section and the spin asymmetry, setting the stage for future full analysis of the data in terms of Δg. Applications to $\vec{e}\vec{p}$ scattering, very relevant to eRHIC, were also worked out and published during this workshop. Stratmann also discussed the procedure of NLO calculations for the case of transverse polarization in pp scattering.

In the future, RHIC will provide information on the precise flavor structure of the quark polarizations in the nucleon, through parity violation in W-boson production. Our current knowledge on the flavor separation comes so far from semi-inclusive deeply-inelastic scattering performed by SMC and HERMES. The information gathered there is already quite interesting, even though the systematic uncertainties are particularly large, as Elliot Leader (Imperial College) pointed out. He critically reviewed the interpretation of the data in terms of the polarized quark and antiquark distributions.

Jacques Soffer (CPT Marseille) gave an overview on positivity constraints for spin asymmetries and other spin observables. This is of great usefulness for estimating spin effects at RHIC. He also presented recent work on a statistical model for parton distribution functions, leading to predictions that will be testable at RHIC.

Asmita Mukherjee (Dortmund University) presented her work on generalized parton distribution functions and deeply-virtual Compton scattering. Use of light-front QCD allows for a systematic approach also to power-suppressed contributions. During the workshop she also continued ongoing work with Stratmann on NLO calculations to $\vec{p}\vec{p} \rightarrow \pi^0 X$ with transversely polarized protons.

The workshop has been a great success. Significant advances have been made. We are grateful to all participants for coming to the Center, and for their dedicated efforts relating to RHIC-Spin. As always, the level of support provided by Prof. T.D. Lee and his RIKEN-BNL Research Center for this workshop has been magnificent, and we are very grateful for it. We also thank Brookhaven National Laboratory and the U.S. Department of Energy for providing the facilities to hold this workshop. Finally, sincere thanks go to Pamela Esposito for her invaluable help in organizing and running the workshop.

Stefan Kretzer and Werner Vogelsang
RBRC, September 2003

A_{LL} in Pion Production at RHIC [1] and eRHIC[2]

Barbara Jäger

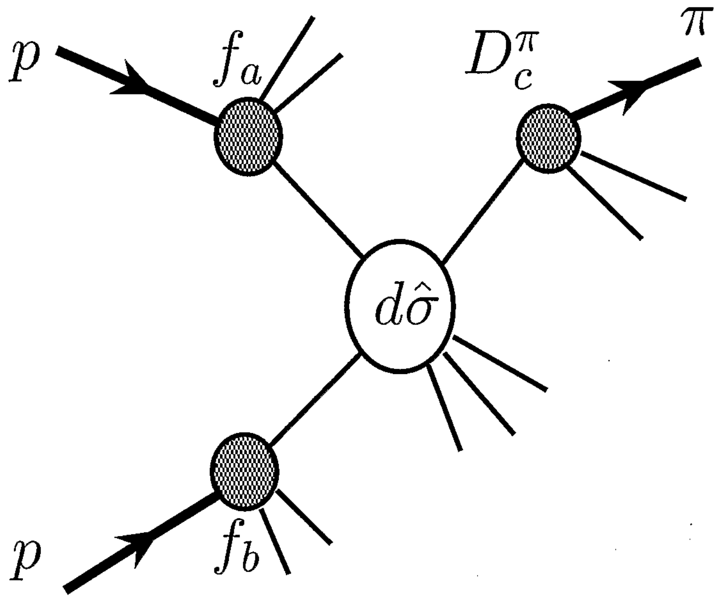
Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany

We present a calculation for single-inclusive large-p_T pion production in collisions of longitudinally polarized protons in next-to-leading order QCD. We choose an approach where fully analytical expressions for the underlying partonic hard-scattering cross sections are obtained. We simultaneously rederive the corresponding corrections to unpolarized scattering and confirm the results existing in the literature. Our results allow the calculation of the double-spin asymmetry A_{LL} for this process at next-to-leading order, which is used at BNL-RHIC to measure the polarization of gluons in the nucleon.

In the second part of the talk a complete next-to-leading order QCD calculation is shown for single-inclusive large-p_T hadron production in longitudinally polarized lepton-nucleon collisions, consistently including "direct" and "resolved" photon contributions. This process could be studied experimentally at a future polarized lepton-proton collider like eRHIC at BNL. We examine the sensitivity of such measurements to the so far completely unknown parton content of circularly polarized photons.

[1]in collaboration with A. Schäfer, M. Stratmann, and W. Vogelsang [Phys. Rev. **D67** (2003), 054005]
[2]in collaboration with M. Stratmann and W. Vogelsang [hep-ph/0309051]

Hadronic Cross Section $d\sigma$



parton distribution functions

parton-to-pion fragmentation function

$$d\sigma^{pp\to\pi X} = \sum_{a,b,c}\int dx_a dx_b dz_c f_a(x_a,\mu_f) f_b(x_b,\mu_f) D_c^\pi(z_c,\mu'_f)$$
$$\times\, d\hat{\sigma}^{ab->cX'}(x_a P_A, x_b P_B, P_\pi/z_c, \mu_f, \mu'_f, \mu_r)$$

partonic cross section

Need for Higher Order Corrections

More Reliable Information

- higher order corrections often large, e.g.:
  - prompt photon production
  - heavy flavors
- closer to experiment (more realistic final state)
- test of perturbative QCD

Beyond QCD

thorough understanding of QCD background

$\updownarrow$

open up ways to search for signatures of new physics

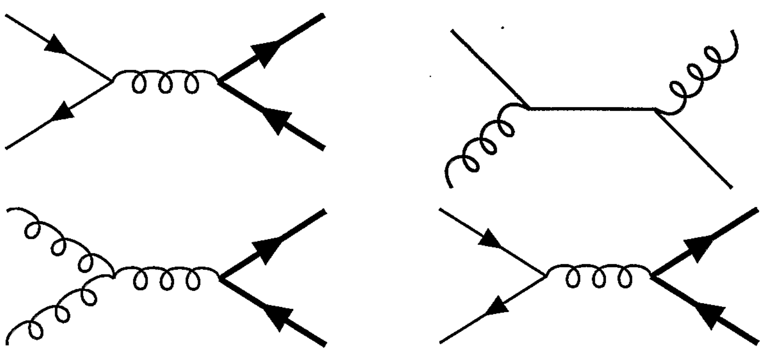
Partonic Cross Section

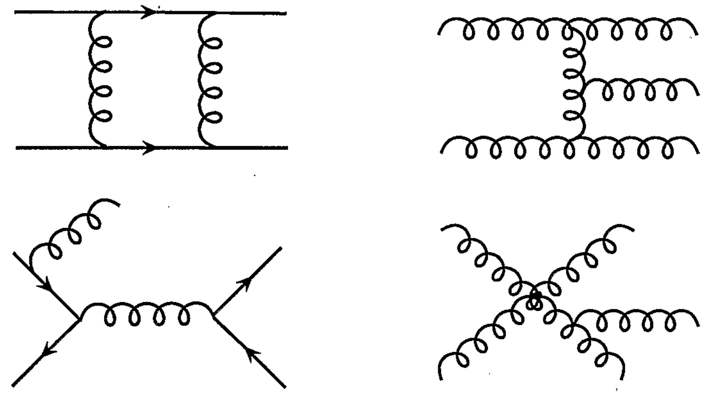
LO and NLO contributions

LO – $\mathcal{O}(\alpha_s^2)$

- all possible tree diagrams for 10 elementary $2 \to 2$ processes:

$$
\begin{array}{lclllcl}
qq' & \to & qX, & \quad & q\bar{q}' & \to & qX, \\
q\bar{q} & \to & q'X, & & qq & \to & qX, \\
q\bar{q} & \to & qX, & & q\bar{q} & \to & gX, \\
qg & \to & qX, & & qg & \to & gX, \\
gg & \to & gX, & & gg & \to & qX.
\end{array}
$$





NLO – $\mathcal{O}(\alpha_s^3)$

- virtual corrections to all $2 \to 2$ diagrams
- $2 \to 3$ diagrams for these and 6 additional processes:

$$
\begin{array}{lclllcl}
qq' & \to & gX, & \quad & q\bar{q}' & \to & gX, \\
qq & \to & gX, & & qg & \to & q'X, \\
qg & \to & \bar{q}'X, & & qg & \to & \bar{q}X.
\end{array}

$\vec{p}\vec{p} \to \pi X$

Numerics

WANTED!! ☞ $$E_\pi \frac{d\Delta\sigma^{\vec{p}\vec{p}\to\pi X}}{d^3p_\pi} = \sum_{a,b,c} \Delta f_a(\mu_f) \otimes \Delta f_b(\mu_f) \otimes D_c^\pi(\mu_f') \otimes E_c \frac{d\Delta\hat{\sigma}^{\vec{a}\vec{b}->cX'}}{d^3p_c}(\mu_f, \mu_f', \mu_r)$$

FFs: *Kniehl, Kramer, Pötter*

convolutions: Monte Carlo integration

unpolarized PDFs: CTEQ5

polarized PDFs: GRSV

$\vec{p}\vec{p} \to \pi X$

Single Pion Inclusive Cross Section

input at $\sqrt{S}$ =200 GeV
(RHIC c.m. energy):
scales: $\mu_r = \mu_f = \mu'_f = p_T$

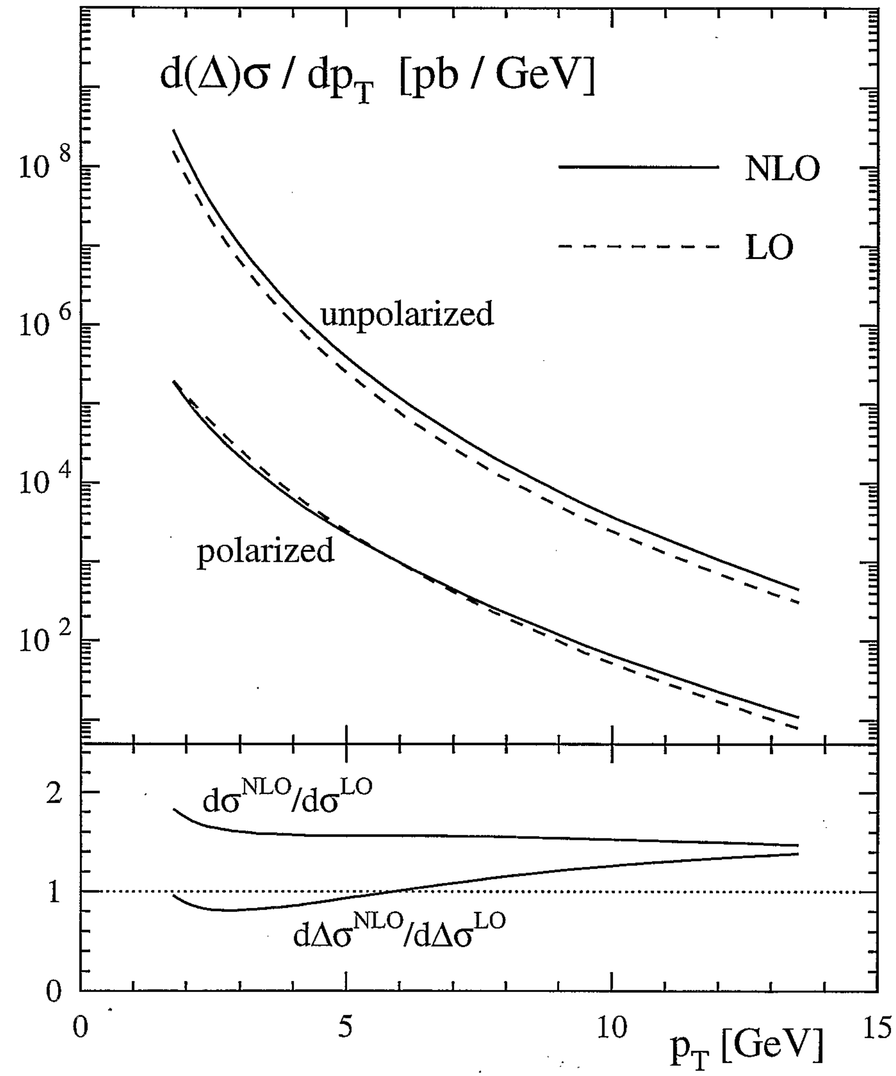
unp. pol.

$LO:$ CTEQ5L GRSVstd.(LO)
KKP(LO), α_S at one loop

$NLO:$ CTEQ5M GRSVstd.(NLO)
KKP(NLO), α_S at two loops

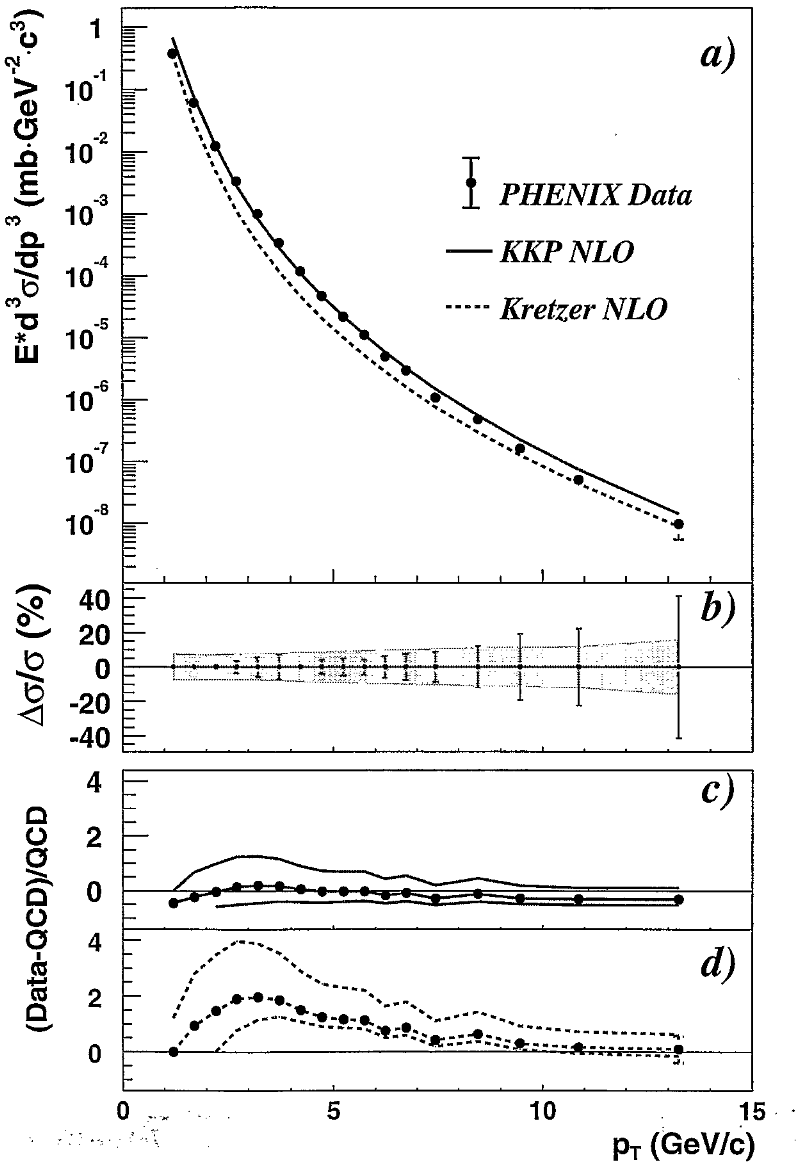
"K-factor": $K = \frac{d(\Delta)\sigma^{NLO}}{d(\Delta)\sigma^{LO}}$

"measure" for importance of NLO corrections



$pp \to \pi X$

Recent Results from PHENIX



data taken from $p + p$ collisions during run-02 *(hep-ex/0304038)*:

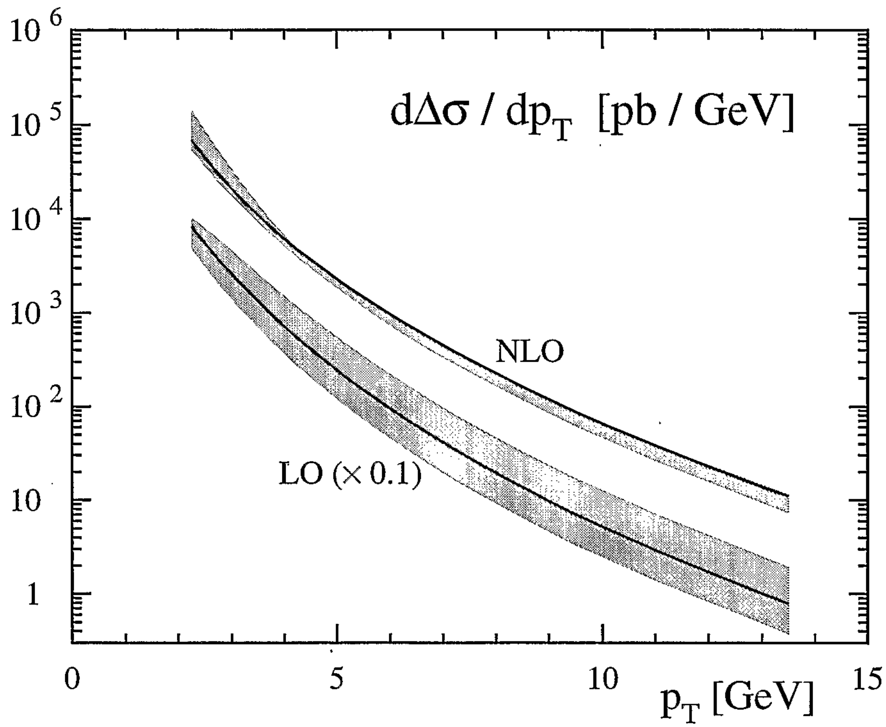
a) data points and NLO predictions for unpolarized differential cross section $E^{\pi}(d^3\sigma/dp_{\pi}^3)$

b) relative statistical errors (points) and systematic errors (bands) of data

c, d) relative difference between data and theoretical predictions

$\vec{p}\vec{p} \to \pi X$

Scale Dependence



recall: motivation

NLO corrections expected to reduce dependence on unphysical scales

↓

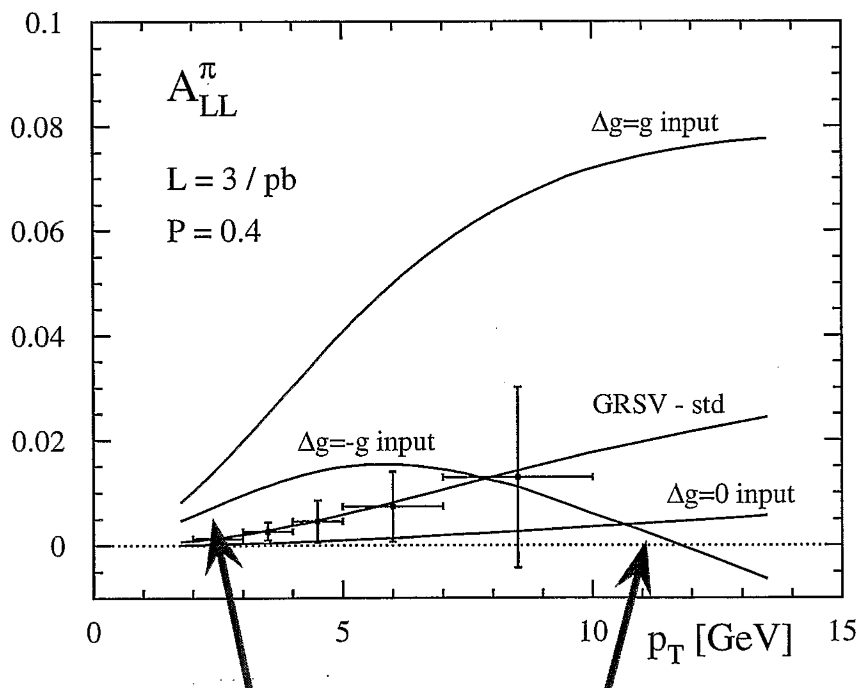
study variation of scales in typical range

$$p_T/2 \leq \mu_r = \mu_f = \mu'_f \leq 2p_T.$$

$\vec{p}\vec{p} \to \pi^0 X$

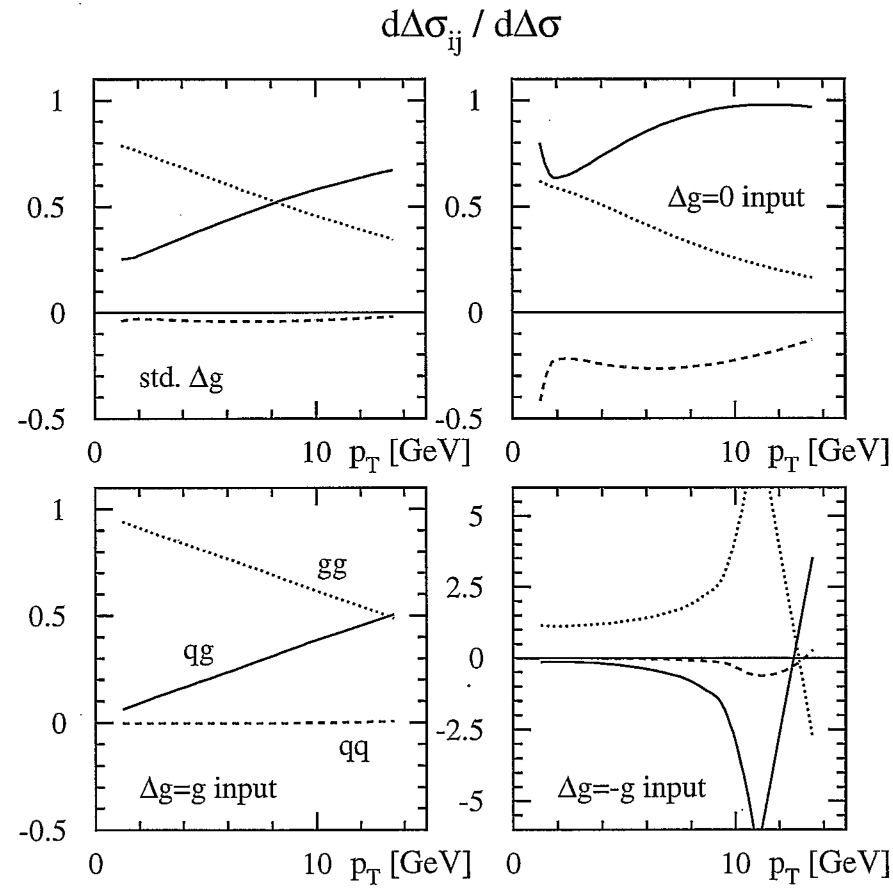
$A_{LL}^{\pi^0}$ ... *surprise?!*

Naohito: "What happens for negative Δg?"



gg dominates

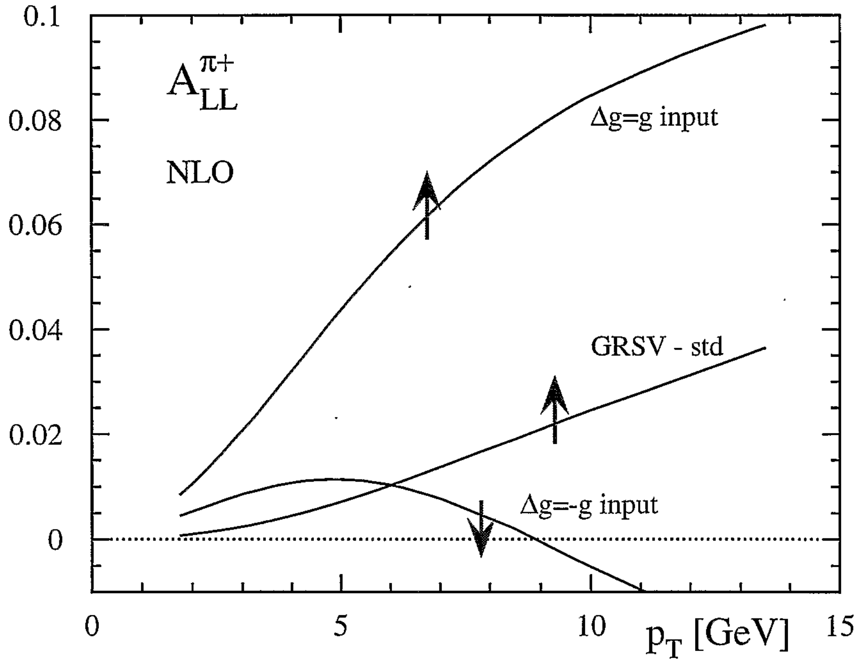
qg takes over

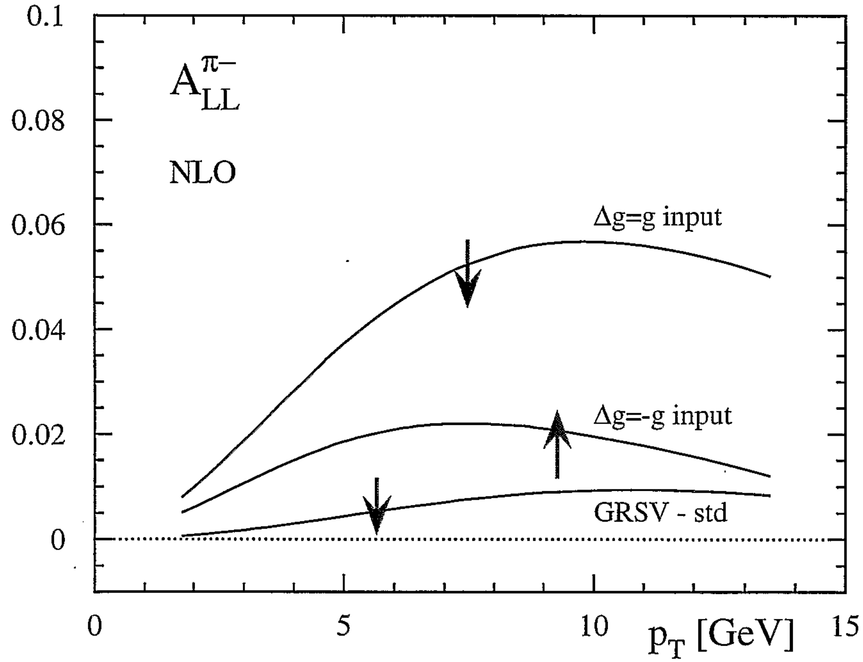


way out: study π^+ and π^-

positive Δg: $A_{LL}^{\pi^+} > A_{LL}^{\pi^0}$
negative Δg: $A_{LL}^{\pi^+} < A_{LL}^{\pi^0}$

positive Δg: $A_{LL}^{\pi^-} < A_{LL}^{\pi^0}$
negative Δg: $A_{LL}^{\pi^-} > A_{LL}^{\pi^0}$

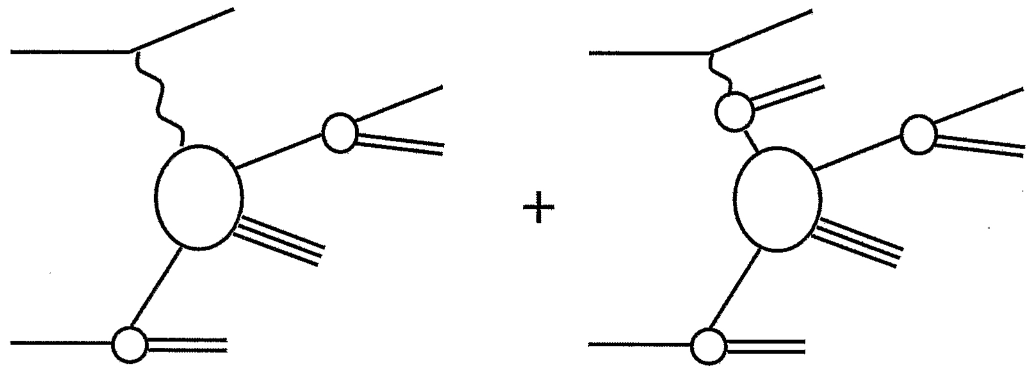




…only at $p_T > 5$ GeV, good
statistics required

Photoproduction of Inclusive Pions

$$\vec{\gamma}\vec{p} \to \pi X$$



- predictions for eRHIC

  …gaining info on parton
  content of the polarized photon
  (completely unmeasured so far)

- LO results promising

  (*Stratmann, Vogelsang*)

$\vec{\gamma}\vec{p} \to \pi X$

Double Spin Asymmetry $A_{LL}^{\pi^0}$

...defined by

$$A_{LL}^{\pi} = \frac{d\Delta\sigma}{d\sigma} = \frac{d\sigma^{++} - d\sigma^{--}}{d\sigma^{++} + d\sigma^{--}}$$

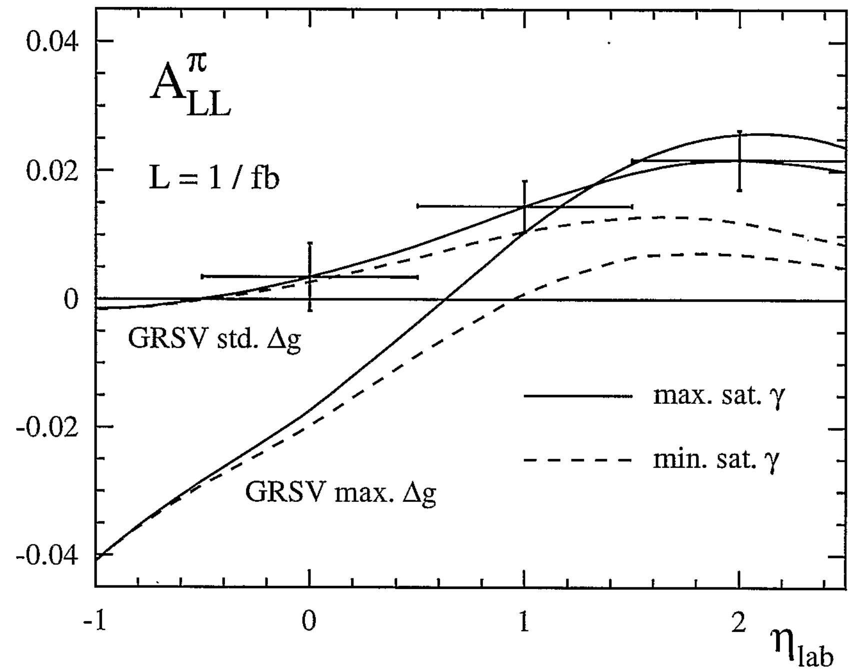
- assume Δg sufficiently known

$\downarrow$

sensitive to Δf^{γ} at large positive rapidities

- expected experimental errors at eRHIC ($\mathcal{P}_{p,e} = 0.7$):

$$\delta A_{LL}^{\pi} \simeq \frac{1}{\mathcal{P}_e \mathcal{P}_p \sqrt{\mathcal{L}\,\sigma_{\text{bin}}}}$$



data should yield information on Δf^{γ} even at rather low luminosities (estimate for $\mathcal{L} = 1$ fb^{-1})

CAN THE POLARIZATION OF THE STRANGE QUARK BE POSITIVE? AND WHY DOES IT MATTER?

Elliot Leader
Imperial College London

The HERMES analysis of their SIDIS data suggests a strange quark polarization marginally positive for medium values of Bjorken-x, whereas all previous analyses of INCLUSIVE DIS have found a strange quark polarization significantly negative.

We argue that it is "almost impossible" for the first moment of the strange quark polarized density to be positive. And we explain why the resolution of this question is so crucial.

CAN THE POLARIZATION OF THE STRANGE QUARK BE POSITIVE?

AND

WHY DOES IT MATTER??

Elliot Leader
Imperial College London

understanding the spin structure of nucleons

1) The problem of flavour separation.

2) The HERMES SIDIS results

3) a) The "almost impossibility" of

$$\int_0^1 \Delta s(x) \geq 0$$

b) Misuse of Bjorken Sum Rule,

4) Lessons for COMPASS, RHIC, ...

[WORK WITH SIDOROV and STAMENOV
Phys. Rev. D67 (2003) 037503]

1) The problem of flavour separation.

In LO

$$g_1^p = \frac{1}{2}\left\{ \frac{4}{9}\left(\Delta u(x) + \Delta \bar{u}(x)\right) + \frac{1}{9}\left(\Delta d(x) + \Delta \bar{d}(x)\right) + \frac{1}{9}\left(\Delta s(x) + \Delta \bar{s}(x)\right)\right\}$$

MANIFESTLY CLEAR: CAN ONLY MEAURE

$$\Delta q(x) + \Delta \bar{q}(x)$$

$\therefore$ IN PRINCIPLE NO INFORMATION ABOUT

$$\Delta \bar{u}(x), \quad \Delta \bar{d}(x), \quad \Delta \bar{s}(x) \text{ vs } \Delta s(x)$$

Convenient to define

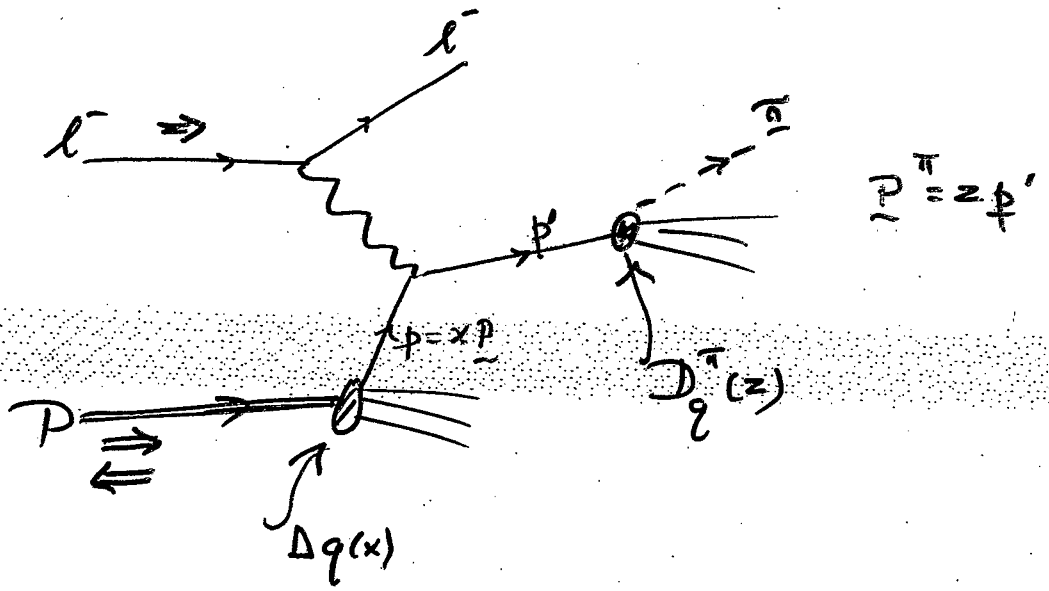
$$\Delta q_3(x) = \left[\Delta u(x) + \Delta \bar{u}(x)\right] - \left[\Delta d(x) + \Delta \bar{d}(x)\right]$$

$$\Delta q_8(x) = \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} - 2\left(\Delta s + \Delta \bar{s}\right)$$

$$\Delta \Sigma(x) = \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s}$$

How can we ever study $\Delta\bar{u}, \Delta\bar{d}$?

SIDIS:

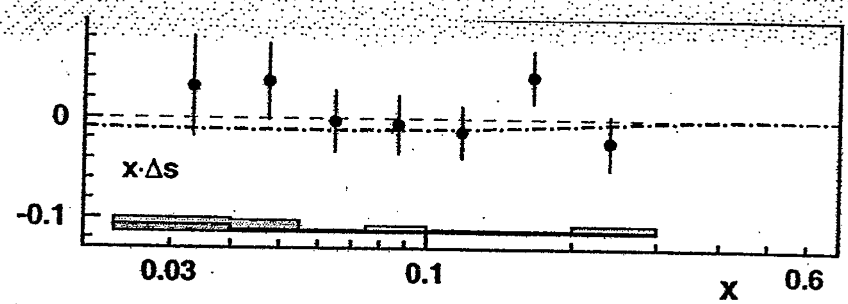


$$\frac{d^2\Delta\sigma}{dx\,dz} \sim \frac{4}{9}\left[\Delta u\, D_u^\pi + \Delta\bar{u}\, D_{\bar{u}}^\pi\right] + \frac{1}{9}\left[\Delta d\, D_d^\pi + \Delta\bar{d}\, D_{\bar{d}}^\pi\right] + \frac{1}{9}\left[\Delta s\, D_s^\pi + \Delta\bar{s}\, D_{\bar{s}}^\pi\right]$$

$\Longrightarrow$ in principle can learn separately about $\Delta\bar{u}$, $\Delta\bar{d}$

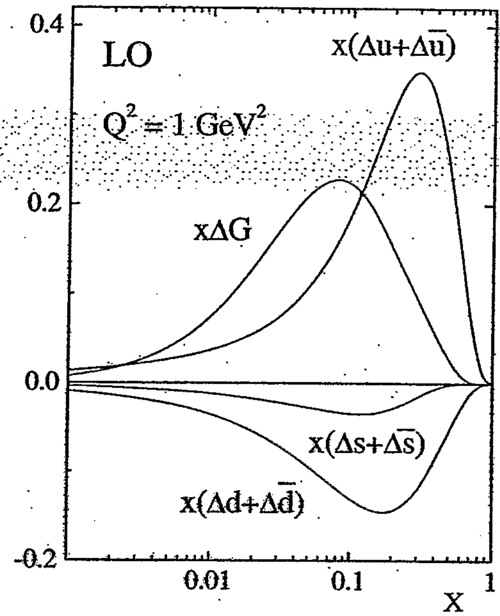
IF we know Fragmentation Functions D_q^π

Some question about how well we know FRAG. FUNCTIONS, BUT GREAT HOPE for near future is polarized SIDIS — HERMES + COMPASS



HERMES : SEMI INCLUSIVE
$\pi^{\pm} + K^{\pm}$

LO ANALYSIS



Inclusive DIS

LO

Is it possible that, because of lack of flavour sensitivity the INCLUSIVE DIS analyses are incorrect ?

$\boxed{\text{NO}}$: IT IS "ALMOST IMPOSSIBLE" FOR THE FIRST MOMENT

$$\delta s(Q^2) \equiv \int_0^1 dx \left[\Delta s(x,Q^2) + \Delta \bar{s}(x,Q^2)\right]$$

TO BE POSITIVE .

In all analyses of DIS, to help with flavour separation, impose Bjorken Sum Rule (come back to this later)

$$a_3 \equiv \int_0^1 \Delta q_3(x,Q^2)\, dx = g_A/g_V = 1{\cdot}2670$$

$\pm\, 0{\cdot}0035$

$$\int_0^1 dx \left\{ (\Delta u + \Delta \bar{u}) - (\Delta d + \Delta \bar{d}) \right\}$$

Usually, also impose:

$$a_8 \equiv \int_0^1 dx\, \Delta q_8(x, Q^2) = 3F - D$$

$$= 0.585 \pm 0.025$$

Based on analysis of HYPERON β-DECAYS, ASSUMING $SU(3)_F$ IS A GOOD SYMMETRY.

No evidence AGAINST $SU(3)_F$ IN THESE DECAYS, BUT CANNOT BE AN EXACT SYMMETRY:

VARIOUS STUDIES SUGGEST BREAKING $\lesssim 10\%$

NEW KTEV EXPT: $\Xi^0 \to \Sigma^+ e^- \bar{\nu}$ SUPPORTS THIS.

WE WILL DOUBLE THE UNCERTAINTY AND INSIST THAT

$$\boxed{0.47 \le a_8 \le 0.70}$$

NOW WRITE

$$\Gamma_1^p(Q^2) = \int_0^1 g_1^p(x, Q^2)\, dx$$

$$= \frac{1}{6}\left\{ \frac{1}{2} a_3 + \frac{5}{6} a_8 + 2\,\delta_s(Q^2) \right\}$$

$$\Longrightarrow$$

$$a_8 = \frac{6}{5}\left\{ 6\,\Gamma_1^p(Q^2) - \frac{1}{2} a_3 - 2\,\delta_s(Q^2) \right\}$$

STRATEGY FOR RHS:

a_3 VERY WELL KNOWN

$\Gamma_1^p(Q^2)$ FROM EXPT (WITH CARE!)

THEN SHOW THAT $\delta_s(Q^2) \not\geq 0$

$\Longrightarrow$ CRAZY VALUE FOR a_8

PROBLEM IS TO GET RELIABLE VALUE FOR Γ_1^p

---- DEPENDS ON EXTRAPOLATION

OF DATA TO $x = 0$ AND $x = 1$

TWO EXTREMES:

Scenario S_1 : ASSUME PQCD AT SMALL x :

(E155 etc) $(Q^2 \approx 5)$

$$\Gamma_1^p = 0.118 \pm 0.004 \pm 0.007 \qquad (S_1)$$

Scenario S_2: ASSUME REGGE AT SMALL x :

(E143 etc) $(Q^2 \approx 3)$

$$\Gamma_1^p = 0.133 \pm 0.003 \pm 0.009 \qquad (S_2)$$

THEN IN

$$a_8 = \frac{6}{5}\left\{6\Gamma_1^p - \frac{1}{2}a_3 - 2\delta_s\right\}$$

$$\delta_s \geqslant 0 \qquad \Longrightarrow$$

$$a_8 \leq 0.089 \pm 0.058 \qquad (S_1)$$

$$a_8 \leq 0.197 \pm 0.068 \qquad (S_2)$$

RECALL FIRM CONVICTION THAT

$$0.47 \leq a_8 \leq 0.70$$

WITH $\pm 20\%$ BREAKING OF $SU(3)_F$.

CONCLUSION :

$$\delta_s \equiv \int_0^1 dx \left[\Delta s + \Delta \bar{s} \right] \geqslant 0$$

$\Longrightarrow$ DRAMATIC BREAKING OF $SU(3)|_F$

MANY TIMES OUTSIDE THE EXPECTED RANGE OF BREAKING.

$\therefore$ IT IS "ALMOST IMPOSSIBLE" FOR δ_s TO BE $\geqslant 0$.

Implications.

1) Either: something is wrong with HERMES

2) Or: our understanding of these reactions is incomplete.

1) HERMES has NOT published their data (14 months!)

???

Is their data incorrect?

Is their analysis of their data incorrect?

Since only hope to learn about $\Delta\bar{u}, \Delta\bar{d}$ in near future is based on SIDIS this is extremely worrying.

2) Possibly the theoretical treatment is incomplete — linked to aspects we are studying here with Anselmino, Murgia and d'Alesio — i.e. the rôle of INTRINSIC TRANSVERSE MOMENTUM

10)

CONCLUSIONS

1) THE GREAT HOPE FOR UNDERSTANDING THE SPIN STRUCTURE OF THE NUCLEON IN THE NEAR FUTURE IS SEMI-INCLUSIVE DIS.

2) UNFORTUNATELY THE HERMES polarized parton densities are unreliable.

3) ASSUMING THE DATA is OK, problem may be use of LO analysis of g_1 without HIGHER TWIST CORRECTIONS. MAY ALSO BE linked to relatively small p_T of the experiment.

4) WHAT ABOUT AN NLO ANALYSIS??

Several groups are ready to do it, but they need the DATA!

So, please, publish or release the data.

Generalized Parton Distributions in Light-front QCD

Asmita Mukherjee

(Dortmund University)

- Introduction
- Light-front Hamiltonian QCD : Motivation and general aspects
- Twist two GPDs
- Twist three GPDs
- Wandzura-Wilczek type relation
- Quark mass effect and genuine twist three
- Summary and discussions

In collaboration with M. Vanderhaeghen

Generalized Parton Distributions in Light-Front QCD

A. Mukherjee[a]

[a] Theoretische Physik IV, Universität Dortmund, D 44221 Germany

Abstract

We investigate the twist-two and twist-three generalized parton distributions in light-front Hamiltonian QCD for a massive dressed quark target. Working in the kinematical region $\xi < x < 1$, we obtain the splitting functions for the evolutions of twist-two quark and gluon distributions in a straightforward way. We discuss the helicity sum rule for a dressed quark. For the twist-three distribution, we find that all contributions are proportional to the quark mass and thus the twist-three distribution is directly related to the chiral symmetry breaking dynamics in light-front QCD. We also show that the off-forward Wandzura-Wilczek relation is violated in perturbative QCD for a massive dressed quark.We calculate the quark mass correction to the WW relation in the off-forward case and show that it is related to $h_1(x)$ in the forward limit. We extract the 'genuine twist three' part of the matrix element in the forward case and verify the Burkhardt-Cottingham and Efremov-Leader-Teryaev sum rules.

Motivation
for using Light-front Hamiltonian QCD

- Wave function representation of relativistic bound states : problems due to quantum fluctuations and Lorentz boost to another frame.

- Light-front : boost generators are kinematical, quantum fluctuations : absent.

- Hadrons are expressed in terms of boost invariant multiparton wave functions.

- A 'field theoretic parton model ' : the partons are massive, on-mass-shell, interacting, have non-zero transverse momenta.

- In light-front gauge, constrained fields are removed. Dynamical fields $\rightarrow$ fermion field and the transverse gluon field.

- Unitarity is manifest.

Light-front Bound State Equation

Hadronic bound states can be expressed in terms of Fock space components

$$
\mid P^+, P^\perp, \lambda\rangle \;=\; \sum_{n,\lambda_i} \int' dx^i d^2\kappa_i^\perp \mid n, x_i P^+, x_i P^\perp + \kappa_i^\perp, \lambda_i\rangle \\
\Phi_n^\lambda(x_i, \kappa_i^\perp, \lambda_i).
$$

$$
x_i = \frac{p_i^+}{P^+}, \kappa_i^\perp = p_i^\perp - x_i P^\perp; \qquad \textstyle\sum_i x_i = 1, \sum_i \kappa_i^\perp = 0.
$$

The states obey the bound state equation

$$
P^- \mid P, \lambda\rangle = \frac{(P^\perp)^2 + M^2}{P^+} \mid P, \lambda\rangle.
$$

Hadron wave functions do not depend on proton momentum $P^+, P^\perp$.

Restrict oneself to fixed-particle sector (boost invariance).

Generalized Parton Distributions

- Universal non-perturbative objects that enter in the description of hard exclusive electroproduction processes.

- Contain a wealth of new informations on the nucleon structure.

- GPDs : inbetween form factors and parton distributions.

Form factors $\rightarrow$ off-forward matrix elements of light-cone local operators.

Parton distributions $\rightarrow$ forward matrix elements of light-cone bilocal operators.

GPD s $\rightarrow$ off-forward matrix elements of light-front bilocal operators.

- Moments of GPDs over parton momentum fraction x : Form factors.

Forward limit : parton distributions.

- Extensive theoretical studies; being probed in on-going experiments $\rightarrow$ HERMES, H1/ZEUS, JLab and Compass.

GPDs in Light-front Hamiltonian QCD

$$
\begin{aligned}
F^{+}_{\lambda\lambda'} &= \int \frac{dz^-}{8\pi} e^{\frac{i}{2}\bar{x}\bar{P}^+ z^-} \langle P'\lambda' \mid \bar{\psi}(-\frac{z^-}{2})\gamma^+ \psi(\frac{z^-}{2}) \mid P\lambda\rangle \\
&= \frac{1}{\bar{P}^+}\bar{U}_{\lambda'}(P')\Big[H_q(\bar{x},\xi,t)\gamma^+ + E_q(\bar{x},\xi,t)\frac{\Delta^+}{2M}\Big]U_\lambda(P),
\end{aligned}
$$

$$
\begin{aligned}
\tilde{F}^{+}_{\lambda\lambda'} &= \int \frac{dz^-}{8\pi} e^{\frac{i}{2}\bar{x}\bar{P}^+ z^-} \langle P'\lambda' \mid \bar{\psi}(-\frac{z^-}{2})\gamma^+\gamma^5 \psi(\frac{z^-}{2}) \mid P\lambda\rangle \\
&= \frac{1}{\bar{P}^+}\bar{U}_{\lambda'}(P')\Big[\tilde{H}_q(\bar{x},\xi,t)\gamma^+\gamma^5 \\
&\qquad + \tilde{E}_q(\bar{x},\xi,t)\frac{\gamma^5\Delta^+}{2M}\Big]U_\lambda(P),
\end{aligned}
$$

$\bar{P}^\mu = \frac{P^\mu + P'^\mu}{2}$, momentum transfer $\Delta^\mu = P'^\mu - P^\mu$.

$P'_\perp = -P_\perp = \frac{\Delta_\perp}{2}$.

Skewedness $\xi = -\frac{\Delta^+}{2\bar{P}^+}$.

Without any loss of generality, we take $\xi > 0$.

$U_\lambda(P)$ is the fermion spinor in light-front representation.

Twist-two GPDs

$$\begin{aligned}\tilde{F}^{+} &= \sqrt{1-\xi^2}\Big[\psi_1^*\psi_1\delta(1-\bar{x}) \\ &+ \sum_{s_1,s_2,\lambda}\int d^2q_\perp \psi^{*\uparrow}_{2s_1,\lambda}(\frac{\bar{x}-\xi}{1-\xi}, q^\perp + \frac{1-\bar{x}}{1-\xi^2}\Delta^\perp) \\ &\chi^\dagger_{s_1}\sigma^3\chi_{s_2}\psi^\uparrow_{2s_2\lambda}(\frac{\bar{x}+\xi}{1+\xi}, q^\perp)\Big].\end{aligned}$$

Jacobi momenta $x_i,\ q_i{}^\perp$;

Light-front wave functions, $\psi_1 = \phi_1,\ \psi_2 = \phi_2\sqrt{P^+}$.

Normalization of state :

$$|\psi_1|^2 = 1 - \frac{\alpha_s}{2\pi}C_f\int_\epsilon^{1-\epsilon} dx\frac{1+x^2}{1-x}log\frac{Q^2}{\mu^2}$$

ψ_2 related to ψ_1 through light-front QCD Hamiltonian;

$$\begin{aligned}\psi^\sigma_{2\sigma_1,\lambda}(x,q^\perp) &= -\frac{x(1-x)}{(q^\perp)^2}T^a\frac{1}{\sqrt{(1-x)}}\frac{g}{\sqrt{2(2\pi)^3}} \\ &\chi^\dagger_{\sigma_1}\Big[2\frac{q^\perp}{1-x} + \frac{\tilde{\sigma}^\perp\cdot q^\perp}{x}\tilde{\sigma}^\perp \\ &- im\tilde{\sigma}^\perp\frac{(1-x)}{x}\Big]\chi_\sigma\epsilon^{\perp *}_\lambda\psi_1.\end{aligned}$$

$\tilde{\sigma}^1 = \sigma^2$ and $\tilde{\sigma}^2 = -\sigma^1$.

Using normalization of the state and the relation between the light-cone wave functions

$$
\begin{aligned}
\tilde{F}^{+} &= \sqrt{1-\xi^2}\Big[\delta(1-\bar{x}) + \frac{\alpha_s}{2\pi} C_f log\frac{Q^2}{\mu^2}\Big(\frac{3}{2}\delta(1-\bar{x}) \\
&\quad + \frac{(1+\bar{x}^2-2\xi^2)}{(1-\bar{x})_+(1-\xi^2)}\Big)\Big].
\end{aligned}
$$

Splitting function:

$$
\tilde{P}_{qq} = C_f \frac{1+\bar{x}^2-2\xi^2}{(1-\bar{x})_+(1-\xi^2)}.
$$

Forward limit :

$$
\begin{aligned}
\tilde{H}(\bar{x},0,0) &= \frac{1}{2}\Big[\delta(1-\bar{x}) + \frac{\alpha_s}{2\pi} C_f log\frac{Q^2}{\mu^2}\Big(\frac{3}{2}\delta(1-\bar{x}) \\
&\quad + \frac{(1+\bar{x}^2)}{(1-\bar{x})_+}\Big)\Big].
\end{aligned}
$$

Used light-front quark spinors.

$g_1(x, Q^2)$ for a dressed quark in perturbation theory.

Helicity flip terms $\rightarrow$ suppressed contributions.

Helicity Sum Rule

Light-front helicity operator expressed in terms of dynamical fields in light front gauge has the same form as in the free theory.

Dynamical fields taken to vanish at the boundary : topologically trivial sector.

$$J^3 = J^3_{fi} + J^3_{fo} + J^3_{gi} + J^3_{go},$$

$$J^3_{fo} = \int dx^- d^2x^\perp \psi^{+\dagger} i(x^1\partial^2 - x^2\partial^1)\psi^+,$$

$$J^3_{fi} = \frac{1}{2}\int dx^- d^2x^\perp \psi^{+\dagger}\Sigma^3\psi^+,$$

$$\begin{aligned} J^3_{go} &= \frac{1}{2}\int dx^- d^2x^\perp [x^1(\partial^+A^1\partial^2A^2 \\ &\quad +\partial^+A^2\partial^2A^2) - x^2(\partial^+A^1\partial^1A^1 + \partial^+A^2\partial^1A^2)], \end{aligned}$$

$$J^3_{gi} = \frac{1}{2}\int dx^- d^2x^\perp (A^1\partial^+A^2 - A^2\partial^+A^1).$$

$$\begin{aligned} \int_0^1 d\bar{x}\bar{x}H_q(\bar{x},0,0) &= 1 - \frac{\alpha_s}{2\pi}C_f log\frac{Q^2}{\mu^2}\frac{4}{3} \\ &= \frac{2}{N}\langle P,\uparrow| J^3_{fi} \mid P,\uparrow\rangle + \frac{2}{N}\langle P,\uparrow| J^3_{fo} \mid P,\uparrow\rangle, \end{aligned}$$

$$\begin{aligned} \int_0^1 d\bar{x}\bar{x}H_g(\bar{x},0,0) &= \frac{\alpha_s}{2\pi}C_f log\frac{Q^2}{\mu^2}\frac{4}{3} \\ &= \frac{2}{N}\langle P,\uparrow| J^3_{gi} \mid P,\uparrow\rangle + \frac{2}{N}\langle P,\uparrow| J^3_{go} \mid P,\uparrow\rangle, \end{aligned}$$

$$\int_0^1 d\bar{x}\bar{x}(H_q(\bar{x},0,0) + H_g(\bar{x},0,0)) = \frac{1}{N}\langle P,\uparrow| 2J^3 \mid P,\uparrow\rangle = 1.$$

Investigating Twist-three GPDs

$$\tilde{F}^{\perp}_{\lambda'\lambda} = \int \frac{dz^-}{8\pi} e^{\frac{i}{2}\bar{P}^+ z^- \bar{x}} \langle P'\lambda' \mid \bar{\psi}(-\frac{z^-}{2})\gamma^{\perp}\gamma^5\psi(\frac{z^-}{2}) \mid P\lambda\rangle.$$

Light-front gauge $A^+ = 0$. Operator involves constrained field :

$$\begin{aligned} O^{\perp 5} &= \bar{\psi}(-\frac{z^-}{2})\gamma^{\perp}\gamma^5\psi(\frac{z^-}{2}) \\ &= \psi^{+\dagger}(-\frac{z^-}{2})\alpha^{\perp}\gamma^5\psi^-(\frac{z^-}{2}) + \psi^{-\dagger}(-\frac{z^-}{2})\alpha^{\perp}\gamma^5\psi^+(\frac{z^-}{2}). \end{aligned}$$

In terms of the dynamical fields:

$$O^{\perp 5} = O^{\perp}_m + O^{\perp}_{k^{\perp}} + O^{\perp}_g,$$

$$O^{\perp}_m = m\Phi^{\dagger}\frac{\sigma^1}{i\partial^+}\Phi + m\Big(\frac{-\sigma^1}{i\partial^+}\Phi^{\dagger}\Big)\Phi.$$

$$\begin{aligned} O^{\perp}_{k^{\perp}} &= \Phi^{\dagger}(-\frac{z^-}{2})(-\partial^2 + i\sigma_3\partial^1)\frac{1}{i\partial^+}\Phi(\frac{z^-}{2}) \\ &+ \Big[(\partial^2 + i\sigma_3\partial^1)\frac{1}{i\partial^+}\Phi^{\dagger}(-\frac{z^-}{2})\Big]\Phi(\frac{z^-}{2}). \end{aligned}$$

$$\begin{aligned} O^{\perp}_g &= g\Phi^{\dagger}(-\frac{z^-}{2})\frac{1}{i\partial^+}(iA^2 + \sigma_3 A^1)\Phi(\frac{z^-}{2}) \\ &+ g\Big[\frac{1}{-i\partial^+}\Phi^{\dagger}(-\frac{z^-}{2})(-iA^2 + \sigma_3 A^1)\Big]\Phi(\frac{z^-}{2}). \end{aligned}$$

Φ is the two-component fermion field.

Transversely polarized state :

$$| k^+, k^\perp, s^1\rangle = \frac{1}{\sqrt{2}}(| k^+, k^\perp, \uparrow\rangle \pm | k^+, k^\perp, \downarrow\rangle),$$

$s^1 = \pm m_R.$

Contributions to the matrix element ($\xi < \bar{x} < 1$): Mass dependent part

$$\begin{aligned}
\tilde{F}^1_m &= \frac{m}{\bar{P}+}\Big[\delta(1-\bar{x})\psi_1^*\psi_1 \\
&\quad + \sum_{\sigma,\sigma'}\int d^2q^\perp \frac{\bar{x}}{\bar{x}^2-\xi^2}\psi_2^*(\frac{\bar{x}-\xi}{1-\xi}, q^\perp + \frac{1-\bar{x}}{1-\xi^2}\Delta^\perp) \\
&\quad \chi^\dagger_\sigma \sigma^1 \chi_{\sigma'}\psi_2(\frac{\bar{x}+\xi}{1+\xi}, q^\perp)\Big].
\end{aligned}$$

$$\begin{aligned}
&= \frac{m}{\bar{P}+}\frac{1}{\sqrt{1-\xi^2}}\psi_1^*\psi_1\Big[\delta(1-\bar{x}) \\
&\quad + \frac{\alpha_s}{2\pi}C_f log\frac{Q^2}{\mu^2}\Big(\frac{2\bar{x}(\bar{x}-2\xi^2)}{(1-\bar{x})(\bar{x}^2-\xi^2)}\Big)\Big],
\end{aligned}$$

Transverse momentum dependent part

$$\begin{aligned}
\tilde{F}^1_{k^\perp} &= -i\sum_{\sigma,\sigma'}\int d^2q^\perp \psi_2^*(\frac{\bar{x}-\xi}{1-\xi}, q^\perp + \frac{1-\bar{x}}{1-\xi^2}\Delta^\perp) \\
&\quad \psi_2(\frac{\bar{x}+\xi}{1+\xi}, q^\perp)\frac{q^2}{\bar{P}+}\frac{\xi}{\bar{x}^2-\xi^2} \\
&\quad + \sum_{\sigma,\sigma'}\int d^2q^\perp \psi_2^*(\frac{\bar{x}-\xi}{1-\xi}, q^\perp + \frac{1-\bar{x}}{1-\xi^2}\Delta^\perp) \\
&\quad \chi^\dagger_\sigma \frac{(\sigma^3 q^1)}{\bar{P}+}\chi_{\sigma'}\psi_2(\frac{\bar{x}+\xi}{1+\xi}, q^\perp)\frac{\bar{x}}{\bar{x}^2-\xi^2},
\end{aligned}$$

Using the expression of ψ_2

$$\tilde{F}^1_{k^\perp} = -\frac{m}{\bar{P}^+}\frac{1}{\sqrt{1-\xi^2}}C_f log\frac{Q^2}{\mu^2}\frac{\alpha_s}{2\pi} \frac{(1-\bar{x})(\bar{x}^2+\xi^2+2\bar{x}\xi^2)}{(\bar{x}^2-\xi^2)(1-\xi^2)}.$$

Contribution from the interaction dependent part of the operator

$$\tilde{F}^1_g = C_f log\frac{Q^2}{\mu^2}\frac{\alpha_s}{2\pi}\frac{m}{2\bar{P}^+}\frac{1}{\sqrt{1-\xi^2}}\delta(1-\bar{x})$$

Total contribution :

$$\tilde{F}^1 = \frac{m}{\bar{P}^+}\frac{1}{\sqrt{1-\xi^2}}\Big[\delta(1-\bar{x}) + C_f log\frac{Q^2}{\mu^2}\frac{\alpha_s}{2\pi}\Big(2\delta(1-\bar{x}) + \frac{1+2\bar{x}(1-\xi^2)-\bar{x}^2}{(1-\bar{x})_+(1-\xi^2)}\Big)\Big],$$

- All terms proportional to m.
- Used normalization of state.
- No singularity at $\bar{x} = \xi$.

A. Mukherjee, M. Vanderhaeghen, PRD 67, 085020 (2003).

Summary and Discussions

- Discussed GPDs in terms of overlaps of light-cone wave functions in light-front gauge.

- Tool to investigate higher twist : operator interaction dependent.

- Interrelations between different Fock components of light-cone wave function.

- Simpler state : dressed quark/gluon in perturbation theory.

- Twist-three off-forward matrix element : contributions due to quark mass, quark transverse momentum and quark-gluon interaction are explicitly calculated.

- Quark mass : important.

- Studied WW relation in the off-forward case and the 'genuine twist three' contribution.

STATISTICAL APPROACH TO PARTON DISTRIBUTIONS AND BARYON FRAGMENTATION FUNCTIONS

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Abstract

A global next-to-leading order QCD analysis of unpolarized and polarized structure functions, extrated from deep-inelastic scattering data, is performed with parton distributions constructed in a statistical physical picture of the nucleon. The chiral properties of perturbative QCD imply strong relations between quarks and antiquarks distributions of a given flavor, which are characteristic of this statistical approach. It leads naturally to the Pauli exclusion principle, whose importance is also emphasized, namely $\bar{d}(x,Q^2) > \bar{u}(x,Q^2)$, which results from the fact that the proton contains two u quarks and only one d quark. We obtain a good description, in a broad range of x and Q^2, of all measured structure functions, $F_2^{p,n}(x,Q^2)$, $xF_3^{\nu N}(x,Q^2)$ and $g_1^{p,d,n}(x,Q^2)$ in terms of a rather small number of free parameters. Forthcoming experiments at RHIC-BNL, in particular $pp \to W^{\pm}$, are sensitive tests of the statistical model for the behavior of the $\bar{d}(x)/\bar{u}(x)$ ratio for $x \geq 0.2$ and for the magnitude and sign of $\Delta\bar{u}(x)$ and $\Delta\bar{d}(x)$. We expect $\Delta\bar{u}(x) > 0$, $\Delta\bar{d}(x) < 0$ and $\Delta\bar{u}(x) - \Delta\bar{d}(x) \simeq \bar{d}(x) - \bar{u}(x)$. This new set of parton distributions is also used to make predictions for the charged current structure functions measured at HERA by the H1 and ZEUS Collaborations and also for comparing with very recent RHI-BNL cross section data on π^0 inclusive production from the STAR and PHENIX Collaborations.
Finally, we discuss briefly the extension of this framework to the description of the unpolarized fragmentation functions of the octet baryons, using semi-inclusive deep inelastic scattering and e^+e^- collisions data, which allows some flavor separation.

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RECENT RESULTS IN THE STATISTICAL APPROACH OF POLARIZED PDF AND EXTENSION TO OCTET BARYON FRAGMENTATION FUNCTIONS

(C. BOURRELY, F. BUCCELLA AND J.S. EPJ C23, 487, 2002
MPL A18, 771, 2003

C. BOURRELY AND J.S. CPT-2003/P.4526 PRD68, 014003, 2003)

1_ GENERAL FEATURES OF MODELS OF POL. PDF

2_ THE STATISTICAL APPROACH
THE PATTERN OBTAINED FOR POL. PDF

3_ RESULTS FOR S.F. F_2^p, F_2^d, F_2^n AND g_1^p, g_1^d, g_1^n

4_ EXPECTATIONS FROM RHIC IN $W^\pm$ PRODUCTION

5_ EXTENSION TO BARYON FRAG. FUNCT.

6_ CONCLUSIONS

MODELS FOR POLARIZED PDF

- MOST OF THEM CONSTRUCT THE POL. PDF FROM AN UNPOLARIZED SET (GRV, MRST, ..) i.e.

$$\Delta q_i(x) = h_i(x)\, q_i(x)$$

WHERE $h_i(x)$ IS A POLYNOMIAL FOR EACH FLAVOR AND ALSO FOR $\Delta\bar{q}$ AND ΔG. THIS PROCEDURE INVOLVES 10-12 PARAMETERS TO FIT. IF WE COUNT THE PARAMETERS NEEDED FOR THE UNPOL. PDF, WE END UP, TO DESCRIBE BOTH UNPOL. AND POL. PDF TO ABOUT 20-22 PARAMETERS !!

- IN ADDITION MOST OF THESE MODELS DO NOT PROVIDE A FLAVOR SEPARATION FOR $\bar{q}$ AND $\Delta\bar{q}$ i.e. Y. GOTO ET AL. AAC, PHYS. REV. D62, 034017 (2000)
  E. LEADER ET AL., PHYS. REV. D58, 114028 (1998)
  M. GLÜCK ET AL, PHYS. REV. D53, 4775 (1996)

HOWEVER THEY ARE RECENT ATTEMPTS TO MAKE THIS FLAVOR SEPARATION, EITHER BY USING SEMI-INCLUSIVE DATA, D. DE FLORIAN and R. SASSOT, PHYS. REV. D62 094025 (2000) OR BY MEANS OF A FLAVOR-SYMMETRY BREAKING i.e. M. GLÜCK ET AL. PHYS. REV. D63, 094005 (2001)

OUR MOTIVATION FOR THIS WORK (CPT-2001/P.4269 C. BOURRELY F. BUCCELLA J.S. EJPC 23, 487 2002) IS TO USE THE STATISTICAL APPROACH TO CONSTRUCT q_i, Δq_i, $\bar{q}_i$, $\Delta\bar{q}_i$ WITH VERY SMALL NUMBER OF PARAMETERS AND FLAVOR SEPARATION.

BASIC PROCEDURE TO CONSTRUCT THE PDF IN THE STATISTICAL APPROACH

1. THE NUCLEON IS VIEWED AS A GAS OF MASSLESS PARTONS $(q, \bar{q}, G)$ IN EQUILIBRIUM AT TEMPERATURE T IN A FINITE SIZE VOLUME. SO THE PDF CAN BE ESSENTIALLY PARAMETRIZED BY USUAL FERMI-DIRAC $(q, \bar{q}$ OR BOSE-EINSTEIN (G) DISTRIBUTIONS, AT AN INPUT SCALE Q_0^2

$$p(x) = \left[e^{\left(\frac{x - X_{0p}}{\bar{x}}\right)} \pm 1 \right]^{-1}$$

X_{0p} THERMODYNAMICAL POTENTIAL

$\bar{x}$ $(\sim 2T/M_p)$ UNIVERSAL TEMPERATURE

2. THE BUILDING BLOCKS ARE $q_\pm(x)$ (FOR QUARKS) AND $\bar{q}_\pm(x)$ (FOR ANTIQUARKS), NOT $q\,(\bar{q})$ AND $\Delta q\,(\Delta \bar{q})$.

3. FROM THE QCD CHIRAL PROPERTIES WE HAVE TWO IMPORTANT PROPERTIES WHICH ALLOW TO RELATE q AND $\bar{q}$ DISTRIBUTIONS
   - THE POTENTIAL OF q_λ IS OPPOSITE TO THE POTENTIA OF $\bar{q}_{-\lambda}$ i.e. $X_{0q}^{\lambda} = - X_{0\bar{q}}^{-\lambda}$
   - THE POTENTIAL OF THE GLUON IS ZERO $(G \to q_\lambda \bar{q}_{-\lambda}$

$$X_{0G} = 0$$

4. FROM EXPERIMENTAL DATA WE ANTICIPATE SOME SIMPLE FEATURES
   - $F_2^n(x) / F_2^p(x) \Rightarrow u(x)$ DOMINATES OVER $d(x)$
   - $A_1^p(x) \Rightarrow u_+(x)$ " " $u_-(x)$
   - $A_1^n(x) \Rightarrow d_-(x)$ " " $d_+(x)$

$\Delta u > 0$

$\Delta d < 0$

$$\Rightarrow \quad X_{0u}^{+} > X_{0u}^{-} \quad \text{AND} \quad X_{0d}^{-} > X_{0d}^{+}$$

IN FACT WE HAVE $X^{+}_{0u} > X^{-}_{0d} \gtrsim X^{-}_{0u} > X^{+}_{0d}$
(SEE BELOW)

WHICH LEADS TO IMPORTANT CONSEQUENCES FOR ANTIQUARKS

i) $\bar{d}(x) > \bar{u}(x)$ (FLAVOR SYM. BREAKING IMPLIED BY PAULI EXCLUSION PRINCIPLE)
ALREADY CONFIRMED BY THE VIOLATION OF THE GOTTFRIED S.R. (NMC)

ii) $\Delta\bar{u}(x) > 0$

iii) $\Delta\bar{d}(x) < 0$

ii) AND iii) REMAINED TO BE CHECKED IN HADRONIC COLLISIONS
(IN FACT $\Delta\bar{u} - \Delta\bar{d} \sim \bar{d} - \bar{u}$ i.e. $\bar{u}_+ \sim \bar{d}_+$)

SO CAN PROPOSE A SIMPLE PARAMETRIZATION FOR THESE PDF BASED ON THIS (+)

- ADDITIONAL FACTOR $A X_{0p} x^{b}$ FOR QUARKS AND $\bar{A}(X_{0p})^{-1} x^{2b}$ FOR ANTIQUARKS

- FOR A UNIVERSAL BEHAVIOR AT $x \to 0$ WE ADD A UNIVERSAL DIFFRACTIVE TERM (RELATED TO THE POMERON WHICH MUST BE FLAVOR AND HELICITY INDEPENDANT) TO ALL PDF

FOR STRANGE QUARKS WE TAKE A PARTICULAR CHOICE FOR $s(\bar{s})$ AND $\Delta s(\Delta\bar{s})$

GLUON DISTRIBUTION IS OBTAINED WITH NO FREE PARAMETERS AND WE TAKE $\Delta G(x) = 0$ AT $Q_0^2 = 4\,GeV^2$
TO SUMMARIZE WE HAVE EIGHT FREE PARAMETERS

$\underline{x}$, X^{+}_{0u}, X^{-}_{0u}, X^{+}_{0d}, X^{-}_{0d}, b, $\tilde{b}$ AND $\tilde{A}$

A AND $\bar{A}$ ARE FIXED BY NORMALIZATION CONDITIONS

Polarized Distributions

Quarks:
The density functions are given by [1] :

$$xu^+(x) = \frac{AX^+_{0u}x^b}{\exp[(x - X^+_{0u})/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp[\frac{x}{\bar{x}}] + 1} \quad (1)$$

$$xu^-(x) = \frac{AX^-_{0u}x^b}{\exp[(x - X^-_{0u})/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp[\frac{x}{\bar{x}}] + 1} \quad (2)$$

$$xd^+(x) = \frac{AX^+_{0d}x^b}{\exp[(x - X^+_{0d})/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp[\frac{x}{\bar{x}}] + 1} \quad (3)$$

$$xd^-(x) = \frac{AX^-_{0d}x^b}{\exp[(x - X^-_{0d})/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp[\frac{x}{\bar{x}}] + 1} \quad (4)$$

9 FREE PARAMETERS

$$\begin{aligned}
A &= 1.74938 && (5)\\
b &= 0.40962 \pm 0.00438^{(*)} && (6)\\
\bar{x} &= 0.09907 \pm 0.00110^{(*)} && (7)\\
X^+_{0u} &= 0.46128 \pm 0.00338^{(*)} && (8)\\
X^-_{0u} &= 0.29766 \pm 0.00303^{(*)} && (9)\\
X^+_{0d} &= 0.22775 \pm 0.00294^{(*)} && (10)\\
X^-_{0d} &= 0.30174 \pm 0.00239^{(*)} && (11)\\
\tilde{A} &= 0.08318 \pm 0.00157^{(*)} && (12)\\
\tilde{b} &= -0.25347 \pm 0.00318^{(*)} && (13)
\end{aligned}$$

note:
The temperature $\bar{x}$ is identical for quarks, antiquarks and gluons.

[1] Values marked with an asterisk are free parameters of the model. The input scale is $Q_0^2 = 4\mathrm{GeV}^2$, and $\Lambda(\overline{MS}) = 300\mathrm{MeV}$. The evolution is performed at NLO.

Antiquarks
The density functions are given by:

$$x\bar{u}^{+}(x) = \frac{\bar{A}}{X_{0u}^{-}} \cdot \frac{x^{\bar{b}}}{\exp[(x+X_{0u}^{-})/\bar{x}]+1} + \frac{\tilde{A}x^{\bar{b}}}{\exp[\frac{x}{\bar{x}}]+1} \tag{14}$$

$$x\bar{u}^{-}(x) = \frac{\bar{A}}{X_{0u}^{+}} \cdot \frac{x^{\bar{b}}}{\exp[(x+X_{0u}^{+})/\bar{x}]+1} + \frac{\tilde{A}x^{\bar{b}}}{\exp[\frac{x}{\bar{x}}]+1} \tag{15}$$

$$x\bar{d}^{+}(x) = \frac{\bar{A}}{X_{0d}^{-}} \cdot \frac{x^{\bar{b}}}{\exp[(x+X_{0d}^{-})/\bar{x}]+1} + \frac{\tilde{A}x^{\bar{b}}}{\exp[\frac{x}{\bar{x}}]+1} \tag{16}$$

$$x\bar{d}^{-}(x) = \frac{\bar{A}}{X_{0d}^{+}} \cdot \frac{x^{\bar{b}}}{\exp[(x+X_{0d}^{+})/\bar{x}]+1} + \frac{\tilde{A}x^{\bar{b}}}{\exp[\frac{x}{\bar{x}}]+1} \tag{17}$$

$$\bar{A} = 1.90801 \tag{18}$$

$$\bar{b} = 2b = 0.81924 \tag{19}$$

$$xs(x) = x\bar{s}(x) = \tfrac{1}{4}(x\bar{u}(x) + x\bar{d}(x))$$

$$x\Delta s(x) = x\Delta\bar{s}(x) = \tfrac{1}{3}(x\Delta\bar{d}(x) - x\Delta\bar{u}(x))$$

(POOR KNOWLEDGE!! MAKE THIS CHOICE)

Gluon

NO FREE PARAMETERS

$$xG(x) = \frac{A_G x^{b_G}}{\exp[x/\bar{x}]-1} \tag{20}$$

$$A_G = 14.27535 \tag{21}$$

$$b_G = 1 + \tilde{b} = 0.74653 \tag{22}$$

$$x\Delta G(x) = 0 \qquad \text{at} \quad Q_0^2 = 4\text{GeV}^2 \tag{23}$$

Charm
The charm is set to 0 at $Q_0^2 = 4\text{GeV}^2$

$u > d$

$\Delta u = u^+ - u^- > 0$
$\Delta d = d^+ - d^- < 0$

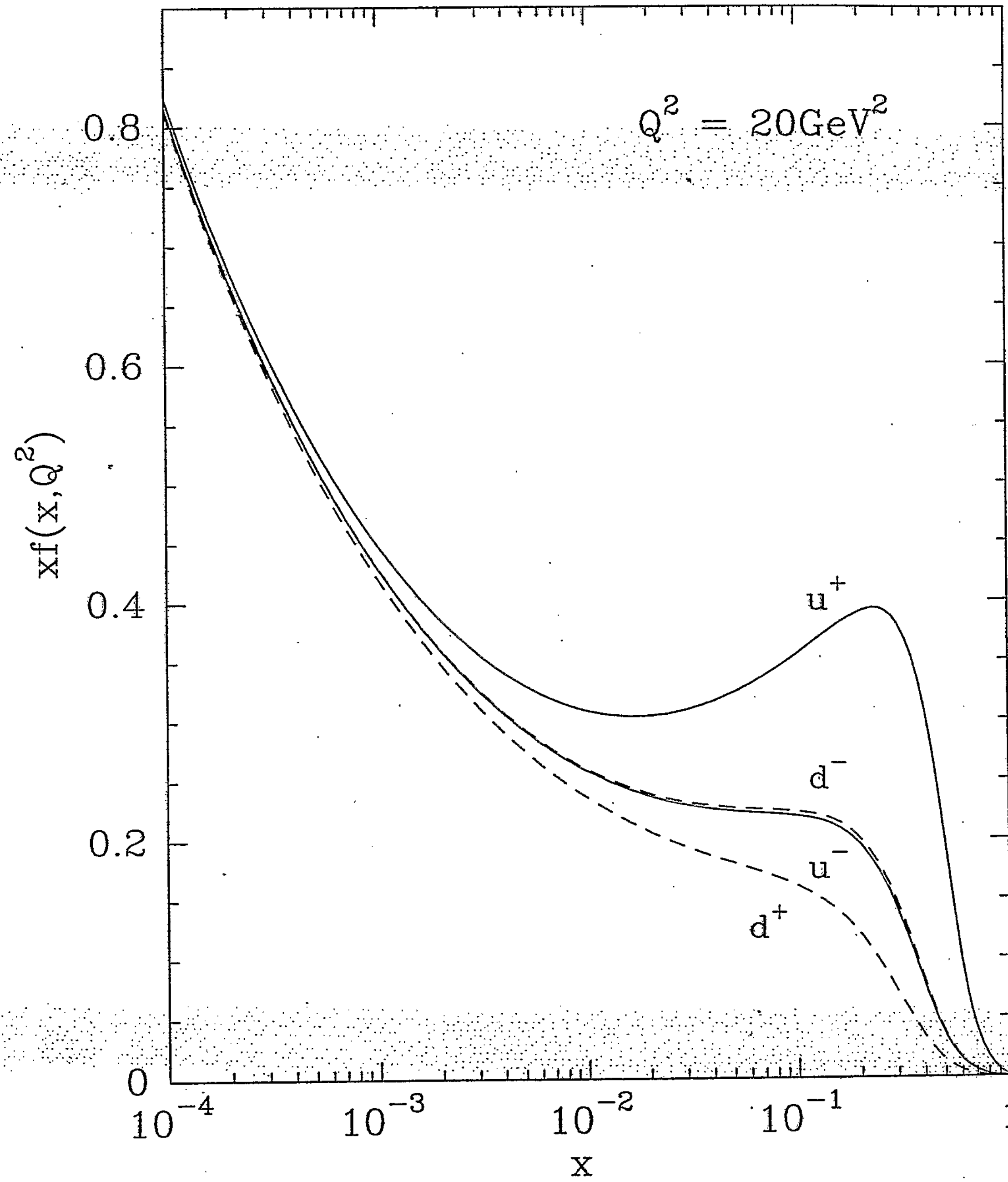


Figure 7: Spin components of quark parton densities .

u_- and d_-

REMAINS AT LARGER Q^2

$\bar{d} > \bar{u}$

$\Delta\bar{u} = \bar{u}^+ - \bar{u}^- > 0$

$\Delta\bar{d} = \bar{d}^+ - \bar{d}^- < 0$

So Bj S.R. RECEIVED A NON-ZERO CONTRIBUTION FROM ANTIQUARKS

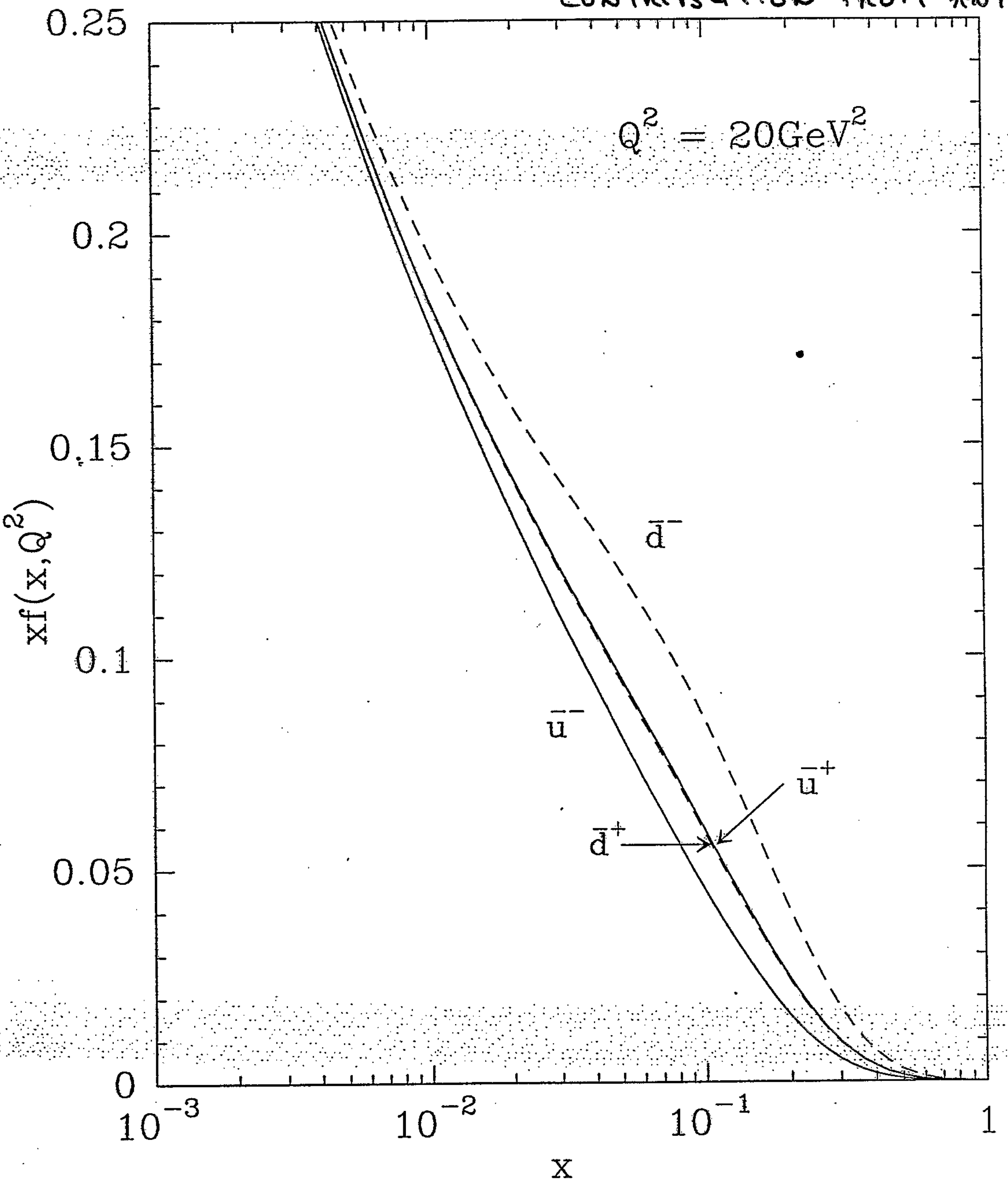


Figure 8: Spin components of antiquark parton densities

$\bar{d}^+ \sim \bar{u}^+$

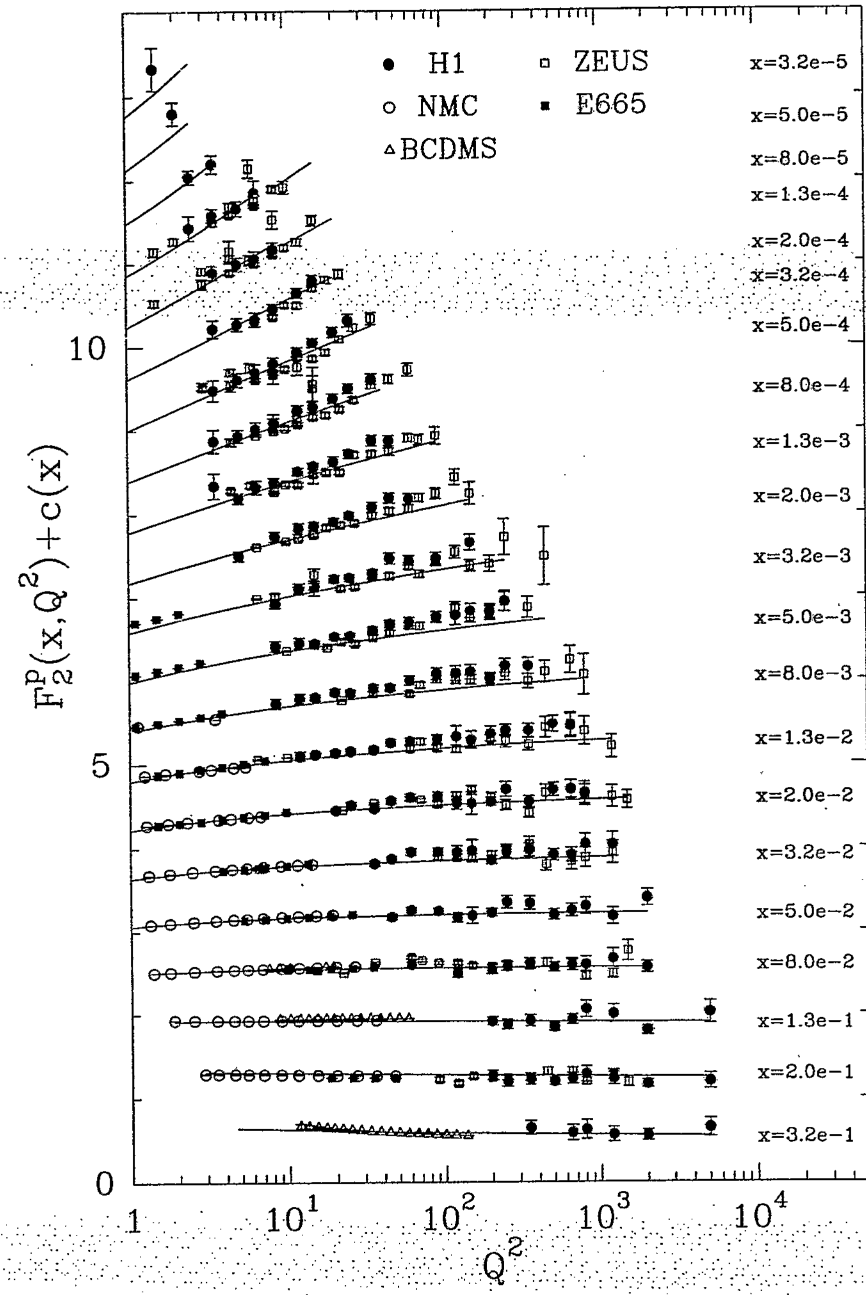
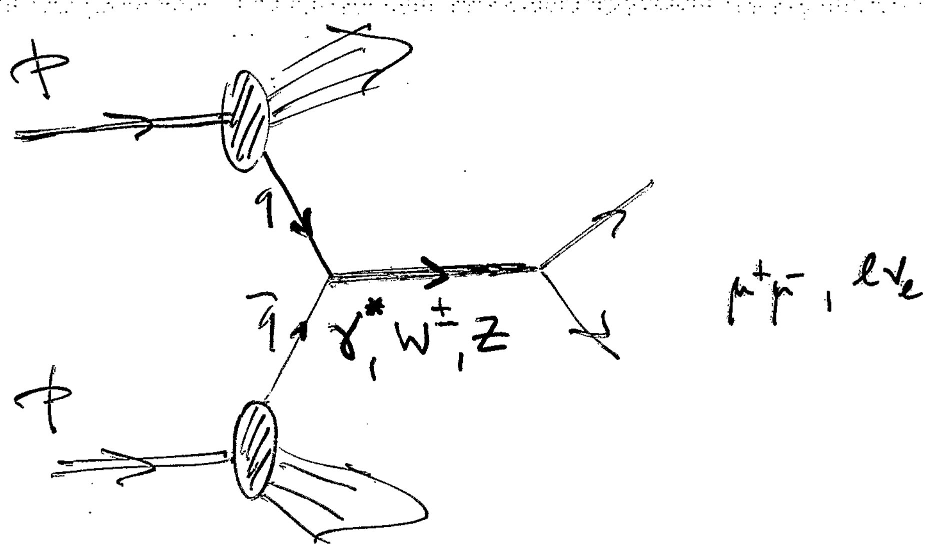
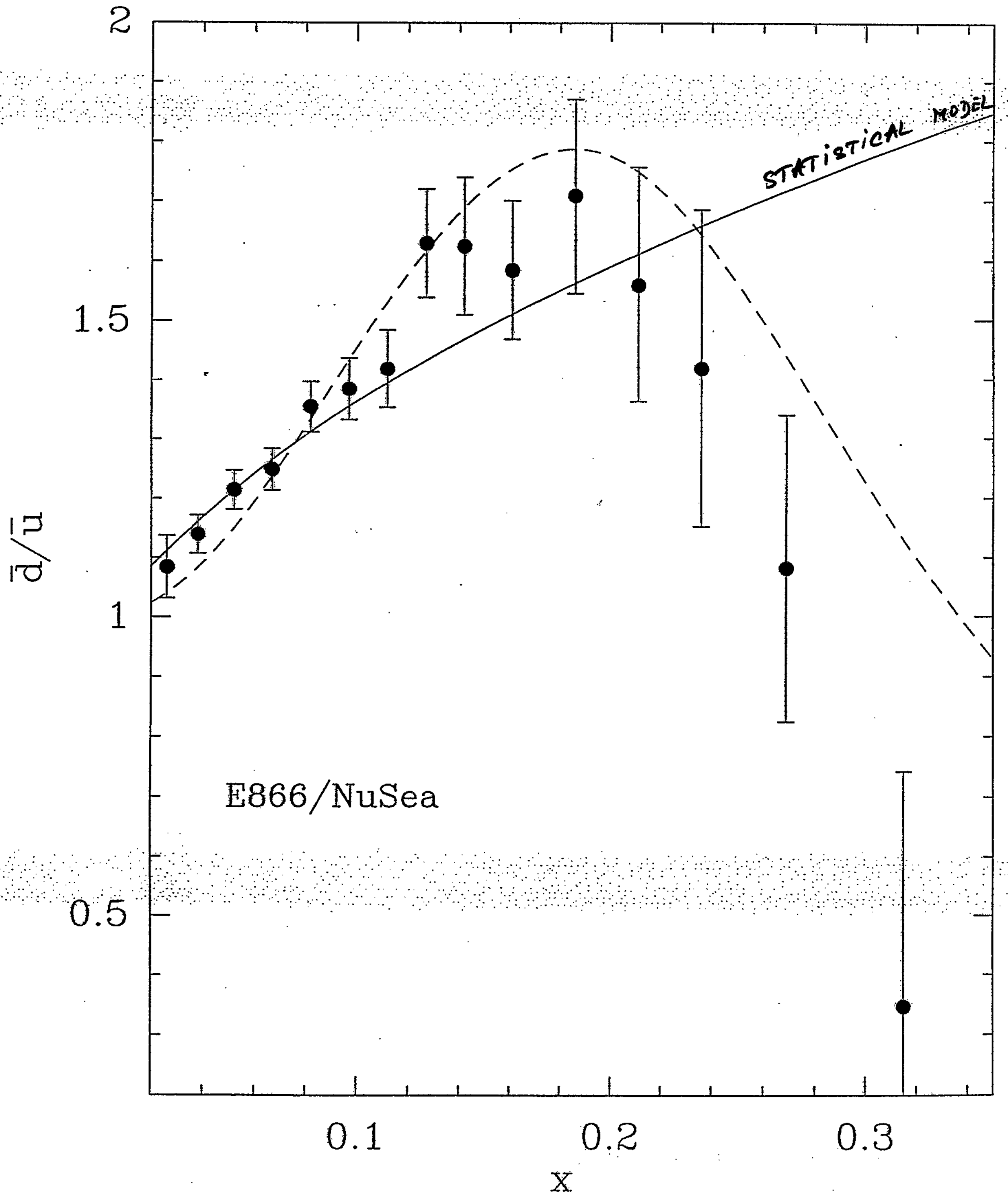


Figure 20: $F_2^p(x, Q^2)$ as function of Q^2 for fixed x, $c(x) = 0.6(i_x - 0.4)$, $i_x = 1 \to x = 0.32$, rebinned data H1, ZEUS, E665, NMC, BCDMS. (Courtesy of R. Voss). (in PDG)

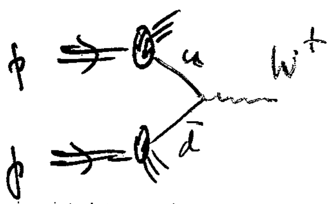
DRELL-YAN PROCESSES



IF $\bar{d} < \bar{u}$
IS PAULI
PRINCIPLE
BREAKING
DOWN ??



$$R_W(y, M_W^2) = \frac{d\sigma^{W^+}/dy}{d\sigma^{W^-}/dy} = \frac{u(x_a, M_W^2)\,\bar{d}(x_b, M_W^2) + \bar{d}(x_a, M_W^2)\,u(x_b, M_W^2)}{d(x_a, M_W^2)\,\bar{u}(x_b, M_W^2) + \bar{u}(x_a, M_W^2)\,d(x_b, M_W^2)}\Bigg|$$



$$x_a = \frac{M_W}{\sqrt{s}}\,e^{y} \qquad x_b = \frac{M_W}{\sqrt{s}}\,e^{-y}$$

$\sqrt{s} = 500$

$y = 0 \quad x_a = x_b = 0.16$

$\sqrt{s} = 200$

$y = 0 \quad x_a = x_b = 0.40$

$W^-(d\bar{u})$

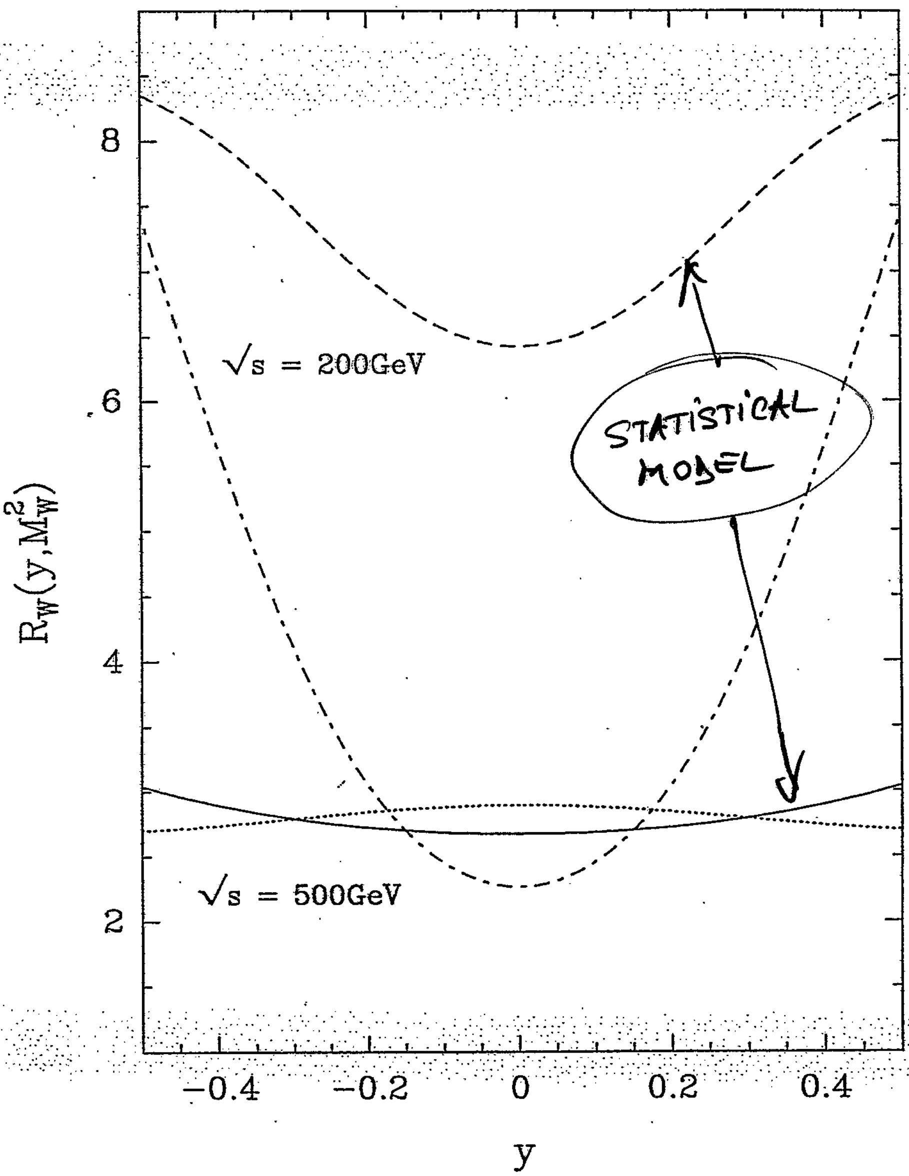


Figure 17: Theoretical calculations for the ratio $R_W(y, M_W^2)$ versus the W rapidity, at two RHIC-BNL energies. Solid curve ($\sqrt{s} = 500$GeV) and dashed curve ($\sqrt{s} = 200$GeV) are the statistical model predictions. Dotted curve ($\sqrt{s} = 500$GeV) and dashed-dotted curve ($\sqrt{s} = 200$GeV) are the predictions obtained using the $\bar{d}(x)/\bar{u}(x)$ ratio from Ref. [36].

PREDICTIONS FOR ASYMMETRIES FOR $W^\pm$, Z PRODUCTION AT RHIC

IN L.O.

$$A_L^{PV}(W^+) = \frac{\Delta u(x_a)\,\bar{d}(x_b) - \Delta\bar{d}(x_a)\,u(x_b)}{u(x_a)\,\bar{d}(x_b) + \bar{d}(x_a)\,u(x_b)}$$

$A_L^{PV}(W^-)$ $(u \leftrightarrow d)$

$A_L^{PV}(Z^0)$ INVOLVES a, b COUPLING BUT SIMILAR EXPRESSION

$$A_{LL}^{PC}(W^+) = -\frac{\Delta u(x_a)\,\Delta\bar{d}(x_b) + \Delta\bar{d}(x_a)\,\Delta u(x_b)}{u(x_a)\bar{d}(x_b) + \bar{d}(x_a)\,u(x_b)}$$

$A_{LL}^{PC}(W^-)$ $(u \leftrightarrow d)$

$A_{LL}^{PC}(Z^0)$

(C. BOURRELY, J.S., PHYS. LETT. B314 132, (1993))

CONCLUDING REMARKS

* WE HAVE PRESENTED A NEW SET OF POL PDF CONSTRUCTED IN THE FRAMEWORK OF A STATISTICAL APPROACH OF THE NUCLEON

* THEY DEPEND ON <u>EIGHT</u> FREE PARAMETERS

  - UNIVERSAL TEMPERATURE $\quad \bar{T} \sim 50$ MeV
  - THERMODYNAMICAL POTENTIALS $\quad 100 < X_{0i} < 200$ MeV

* NEED A PHYSICAL INTERPRETATION (WHY $u^- = d^-$? and $\bar{u}^+ = \bar{d}^+$)

* WE HAVE PERFORMED A NLO GLOBAL ANALYSIS OF UNPOL. AND POL. S.F.

  $F_2^p,\ F_2^d,\ F_3^\nu,\ g_1^p,\ g_1^d,\ g_1^n$

  IN A BROAD KINEMATIC RANGE

* THE STRONG RELATIONSHIP BETWEEN QUARKS AND ANTIQUARKS LEADS NATURALLY TO

  $\bar{d}(x) - \bar{u}(x) > 0$ NEED TO BE CONFIRMED AT RHIC FOR $x > 0.2$, $\sigma(W^+)/\sigma(W^-)$

  $\Delta\bar{u}(x) > 0 \quad \Delta\bar{d}(x) < 0$

  and $\quad \Delta\bar{u}(x) - \Delta\bar{d}(x) \simeq \bar{d}(x) - \bar{u}(x)$ (SINCE $\bar{u}_+(x)$ [illegible] $\bar{d}_+(x)$)

* $W^\pm, Z$ PRODUCTION WITH POL. $\vec{p}$ AT RHIC WILL TEST THAT

  OUR STARTING POINT $\Delta G(x) = 0$ AT $Q_0^2 = 4\ \mathrm{GeV}^2$ MIGHT BE REVISED IN THE FUTURE FROM RHIC DATA

* EXTENSION TO BARYON F.F. WORKS ALSO WELL

THE RELEVANCE OF POSITIVITY IN SPIN PHYSICS

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Abstract

We will emphasize the importance of *positivity* in spin physics, which allows to derive non-trivial model independent constraints on spin observables. These positivity conditions are based on the positivity properties of density matrix or Schwarz inequalities for transition matrix elements in processes involving at least two particles carrying a non-zero spin. We will illustrate this important point by means of several examples chosen in different areas of particle physics, in particular:

- Total cross sections in pure spin states
- Two-body exclusive reactions
- Polarized deep inelastic scattering
- Quark transversity distributions
- Off-shell gluon distributions
- Transverse momentum dependent distributions
- Single-particle inclusive reactions
- Polarized fragmentation functions
- Off-forward parton distributions

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THE RELEVANCE OF POSITIVITY IN SPIN PHYSICS (BNL-RIKKEN, AUG. 28, 2003)

WE WILL SHOW WITH SEVERAL SPECIFIC EXAMPLES, THE RELEVANCE OF POSITIVITY TO PUT MODEL-INDEPENDANT CONSTRAINTS ON SPIN OBSERVABLES

THOSE ARE BASED ON POSITIVITY CONDITIONS OF DENSITY (SCATTERING) MATRIX OR CAUCHY-SCHARZ INEQUALITIES

WE WILL FOLLOW THE WAY THESE IDEAS WERE DEVELOPED OVER THE LAST 30 YEARS OR SO FROM 1970 TO NOWDAYS

POSITIVITY CONSTRAINTS FOR $p^{\uparrow}p \to \Lambda^{\uparrow} X$

(M.G. DONCEL and A. MÉNDEZ, PHYS. LETT. **B 41** (1972) 83)

(THIS REACTION IS DESCRIBED IN TERMS OF SEVEN SPIN OBS.)

LET'S CONSIDER THREE TRANSVERSE SPIN OBSERVABLES

P_Λ Λ POLARIZATION

A_N ANALYZING POWER (OR LEFT-RIGHT ASYM.)

D_{NN} DEPOLARIZATION PARAMETER (SPIN TRANSFER)

POSITIVITY YIELDS THE FOLLOWING CONSTRAINTS WHICH MUST BE SATISFIED FOR ANY KINEMATIC POINT $(x_F, p_T, \sqrt{s})$

① $1 + D_{NN} \geq |P_\Lambda + A_N|$

② $1 - D_{NN} \geq |P_\Lambda - A_N|$

(GIVEN D_{NN} ONE CAN FIND THE ALLOWED REGION IN THE PLANE P_Λ, A_N (AND VICE VERSA)

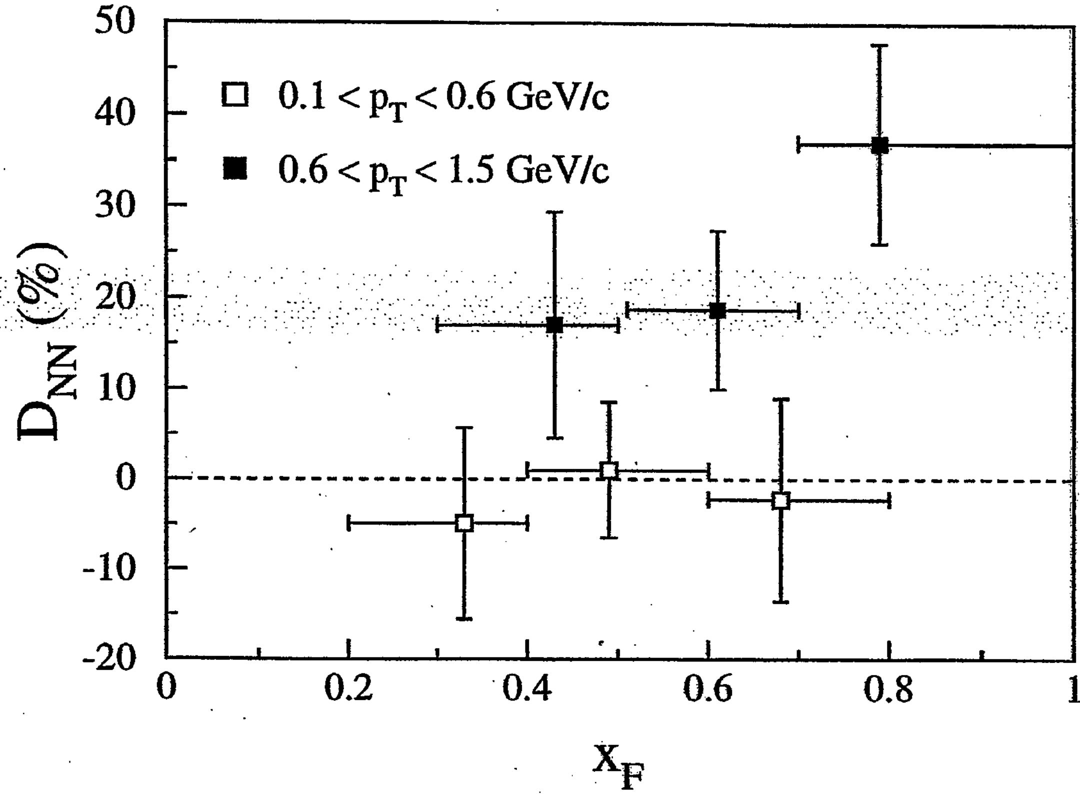
SEE GRAPHS FOR THREE VALUES $D_{NN} = 0, 1/3, 1/2$

FOR $D_{NN} < 0$ SAME REGION BUT MUST EXCHANGE P_Λ AND A_N. IF $D_{NN} \to 1 \Rightarrow P_\Lambda$ AND $A_N \to 0$

E704 SATISFIES POSITIVITY

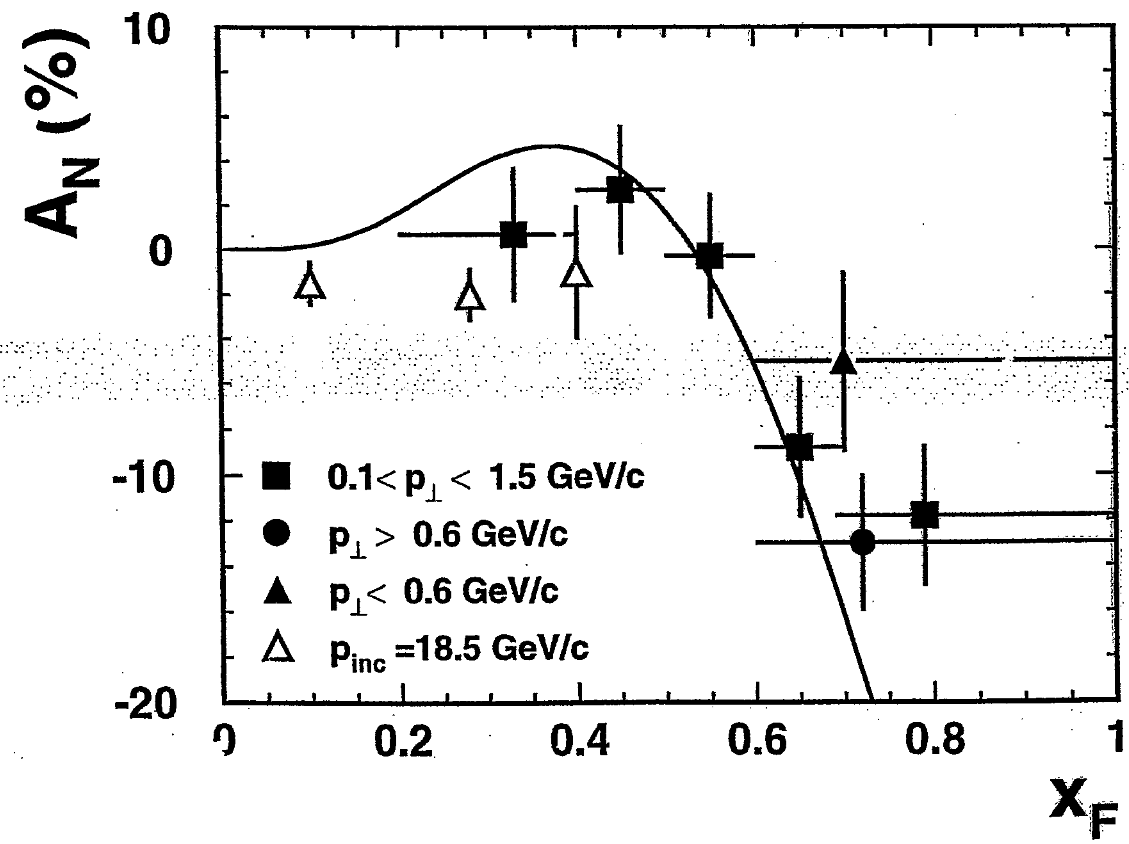
FOR EXAMPLE AT $p_T \sim 1$ GeV/c $x_F \sim 0.8$

$D_{NN} \sim 1/3$ AND THEY HAVE $A_N \sim -10\%$, $P_\Lambda \sim -30\%$



E 704

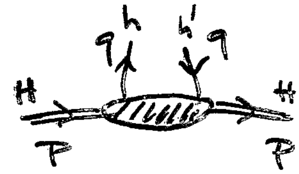
PRL 78 4003 (1997)



E 704

PRL 75 3073 (1995)

POSITIVITY CONSTRAINTS FOR FORWARD PARTON DISTRIBUTIONS



LET'S CONSIDER THE THREE FORWARD QUARK DISTRIBUTIONS

$$q(x)\ ,\quad \Delta q(x)\ ,\quad h_1^q(x)$$

WHICH CAN BE RELATED TO THE FORWARD QUARK-NUCLEON ELASTIC AMPLITUDE

$$q_{h'}(q) + N_H(P) \to q_h(q) + N_{H'}(P)$$

h_1^q CORRESPONDS TO $H=h'=+1/2\ ,\ H'=h=-1/2$

LET'S USE THE MATRIX ELEMENTS

$$a_X^{H,h}(P,q) = \langle N_H(P)\,|\,O\,|\,q_h(q), X\rangle$$

ONE CAN SHOW THAT

$$q_+(x) \equiv \frac{1}{2}\left(q(x)+\Delta q(x)\right)$$

$$= \frac{1}{2}\sum_X\left(|a_X^{+,+}|^2+|a_X^{-,-}|^2\right)$$

$$h_1^q(x) = \frac{1}{2}\sum_X\left[(a_X^{+,+})^*(a_X^{-,-})+(a_X^{+,+})(a_X^{-,-})^*\right]$$

SCHWARZ INEQUALITY LEADS IMMEDIATELY TO

$$2\,|h_1^q(x)| \leq q(x)+\Delta q(x)$$

(J.S. PHYS. REV. LETT. 74 (1995) 1292)

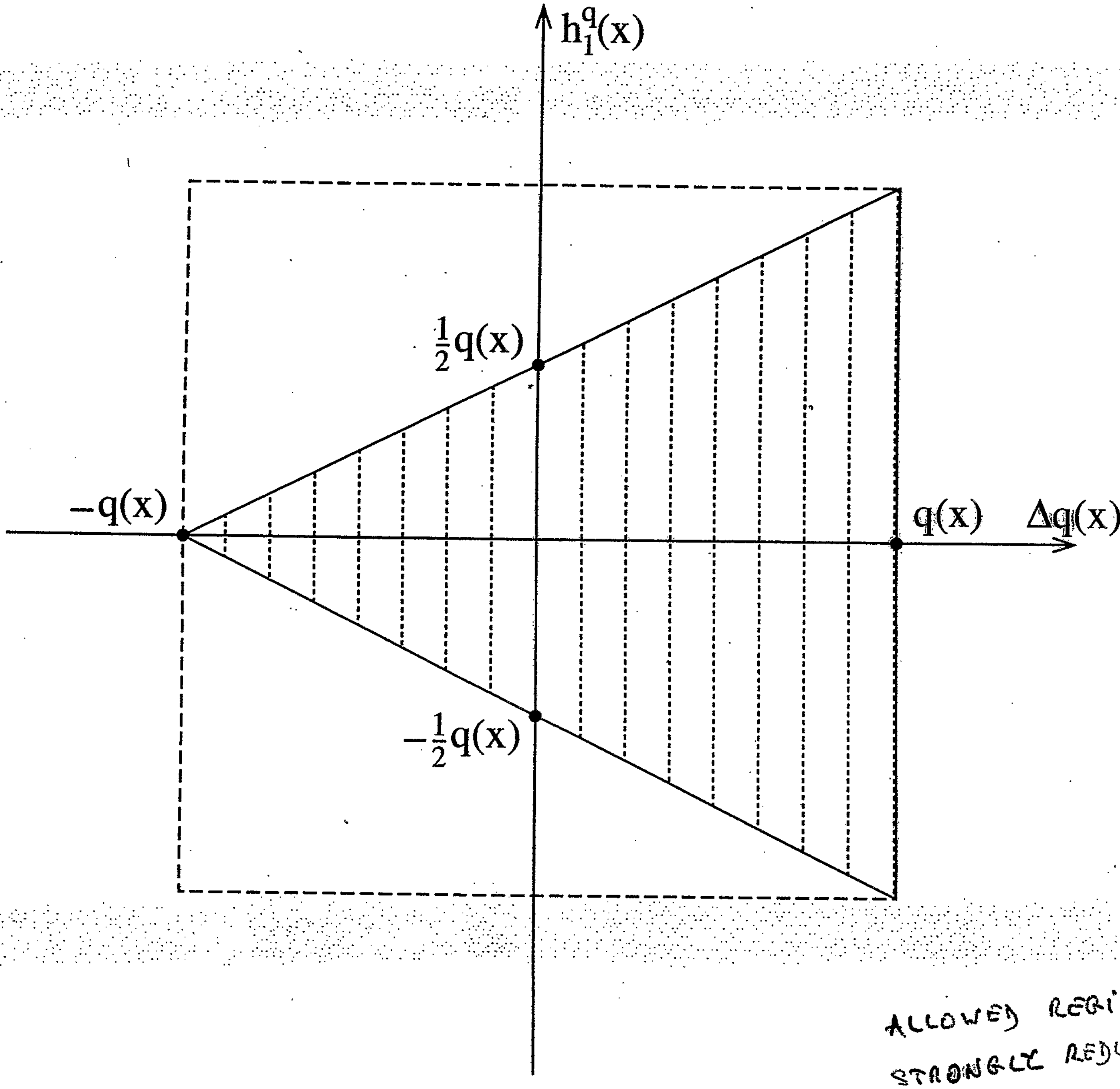
STRONGER THAN THE TRIVIAL ONE $|h_1^q(x)| \leq q(x)$

THIS SIMPLE RESULT IS <u>NOT</u> AFFECTED BY QCD CORRECTIONS UP TO NLO

(W. VOGELSANG, PHYS. REV. D57 (1998) 1886 ;

C. BOURRELY, J.S., O. TERYAEV, [illegible])

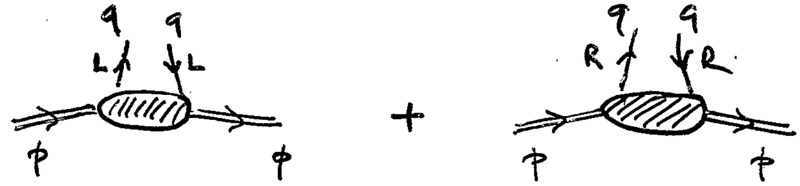
$$\underset{Pq \to Pq}{M} = q(x)\, \mathbb{1} \otimes \mathbb{1} + \Delta q(x)\, \vec{\sigma}.\vec{e}_3 \otimes \vec{\sigma}.\vec{e}_3 + h_1^q(x) \sum_{i=1,2} \vec{\sigma}.\vec{e}_i \otimes \vec{\sigma}.\vec{e}_i$$



ALLOWED REGION
STRONGLY REDUCED

Fig.1

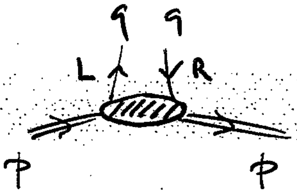
IN TERMS OF DIAGRAMS WE HAVE

$q =$ 

CHIRAL EVEN

$\Delta q =$ " $-$ "

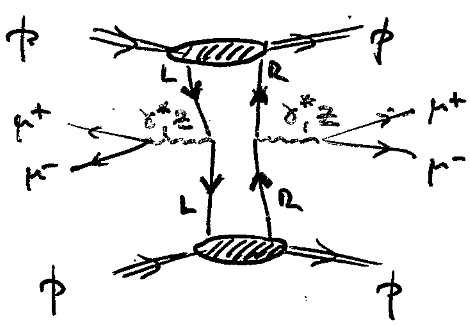
BUT ALSO

$h_1^q =$ 

CHIRAL ODD

(LEFT-RIGHT INT.)

CLEARLY FROM CHIRALITY CONSERVATION h_1^q DECOUPLES FROM DIS <u>BUT</u> IT CAN BE MEASURED IN DRELL-YAN TYPE PROCESSES WITH BOTH INITIAL PROTONS TRANSVERSELY POLARIZED



$$A_{TT}^{\gamma^*} = \hat{a}_{TT} \frac{\sum_q e_q^2 h_1^q(x_a)\, h_1^{\bar{q}}(x_b)}{\sum_q e_q^2 q(x_a)\,\bar{q}(x_b)}$$

ANALOGOUS FOR

$$A_{TT}^{Z} \simeq \frac{\sum_q (a_q^2 - b_q^2)(a_{\bar{q}}^2 - b_{\bar{q}}^2)\, h_1^q h_1^{\bar{q}}}{\sum_q (a_q^2 + b_q^2)(a_{\bar{q}}^2 - b_{\bar{q}}^2)\, q\bar{q}}$$

$A_{TT}^{W^\pm} = 0$ SHOULD VANISH FOR $W^\pm$ PRODUCTION SINCE W IS LEFT-HANDED (NO L-R INT.)

IF $\neq 0$ NEW PHYSICS

SO FAR <u>NO</u> DATA ON $h_1^q(x, Q^2)$ AND HOPE TO GET FIRST DETERMINATION WITH THE POLARIZED pp COLLIDER AT RHIC - BNL (1999 - 2000)

AS ONE DEFINES THE AXIAL CHARGES, FIRST MOMENT OF $\Delta q(x)$

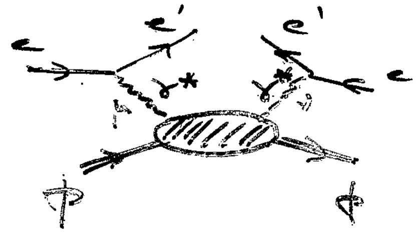
$$\Delta q(Q^2) = \int_0^1 dx \left[\Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2)\right]$$

ONE DEFINES THE TENSOR CHARGES

$$\delta q(Q^2) = \int_0^1 dx \left[h_1^q(x, Q^2) - h_1^{\bar{q}}(x, Q^2)\right]$$

DEEP INELASTIC SCATTERING

$$e p \to e' X$$



$$\frac{d^2\sigma}{dE'\,d\Omega_e} = \frac{4\alpha^2}{Q^4}\,\frac{E'}{E}\,L^{(e)}_{\mu\nu}W^{\mu\nu}$$

$$W_{\mu\nu} = \underset{\underset{W_1,\,W_2}{\uparrow}}{W^{(S)}_{\mu\nu}} + i\,\underset{\underset{G_1,\,G_2}{\uparrow}}{W^{(A)}_{\mu\nu}}$$

THESE FOUR STRUCTURE FUNCTIONS CORRESPOND TO THE FOUR SCATT. AMPLITUDES FOR $\gamma^* p \to \gamma^* p$ WHICH SURVIVE IN THE FORWARD DIRECTION

THEY ARE ALSO RELATED TO THE FOUR TOTAL C.S.

$$\sigma^T_{3/2}\,,\ \sigma^T_{1/2}\,,\ \sigma^L_{1/2}\,,\ \sigma^{TL}_{1/2}$$ ← CAN BE NEGATIVE

UNPOL. DIS LEAD TO $\sigma^T_{3/2} + \sigma^T_{1/2}$ AND $\sigma^L_{1/2}$

POL. DIS TO $\sigma^T_{3/2} - \sigma^T_{1/2}$ AND $\sigma^{TL}_{1/2}$

IN FACT ONE "MEASURES"

$$A_1 = \frac{\sigma^T_{1/2} - \sigma^T_{3/2}}{\sigma^T_{1/2} + \sigma^T_{3/2}} \quad \text{AND} \quad A_2 = \frac{2\,\sigma^{TL}_{1/2}}{\sigma^T_{1/2} + \sigma^T_{3/2}}$$

POSITIVITY CONSTRAINTS (M.G. DONCEL and E. de RAFAEL NUO. CIM. 4A (1971) 363)

$$|A_1| \le 1 \qquad |A_2| \le \sqrt{R} \qquad \left(R = \sigma_L/\sigma_T\right)$$

TRIVIAL NOT TRIVIAL

A_2 HAS BEEN RECENTLY MEASURED BY SLAC AND SMC

$R = \sigma_L / \sigma_T$

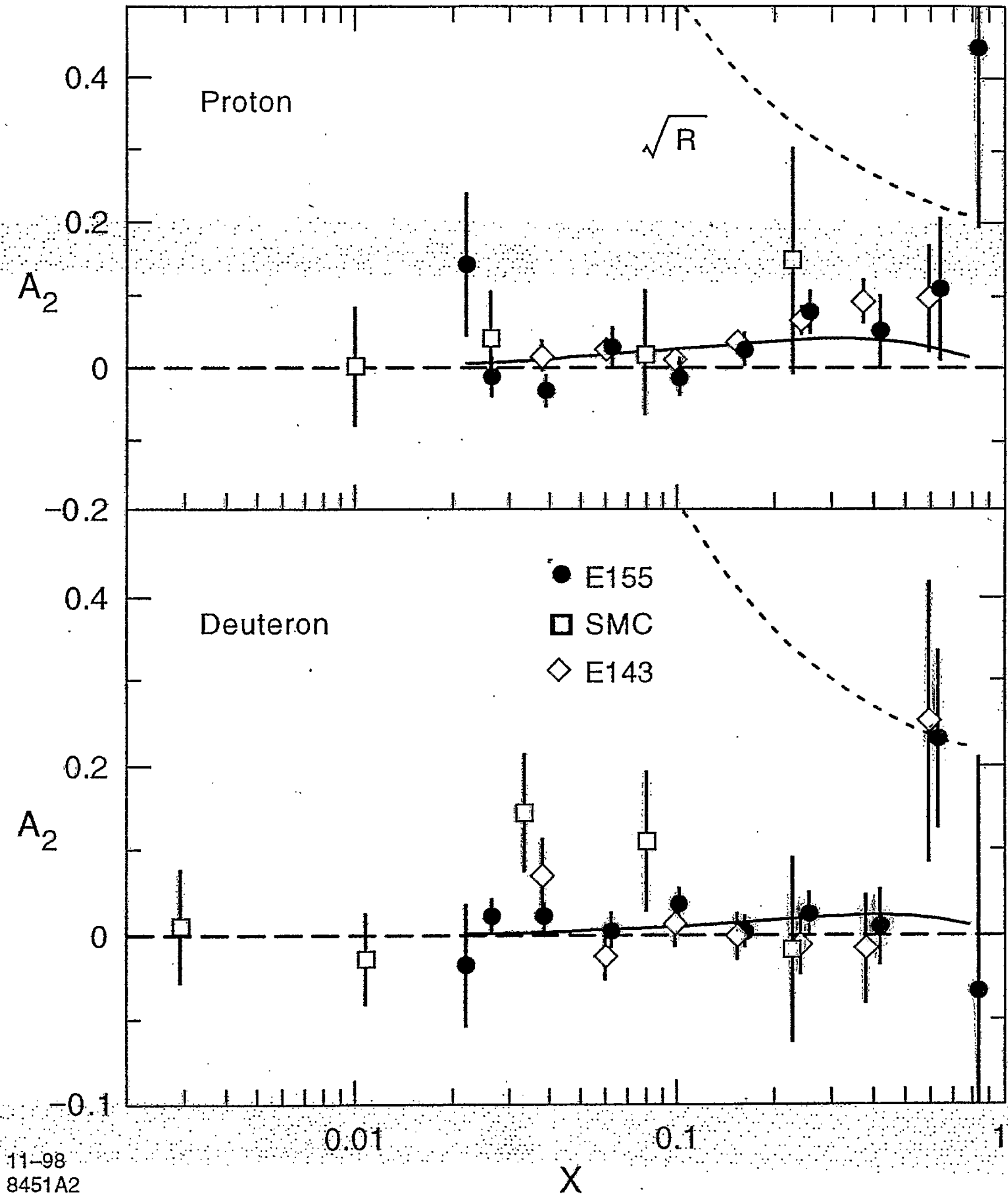


Figure 2: The asymmetries A_2 for proton and deuteron for this experiment (E155) with data from all spectrometers averaged (Table 1). The errors are statistical; the systematic errors are negligible. Also shown are the data from SLAC E143 [7] and SMC[6]. Our A_2^{WW} calculation is shown as the solid line and the $\sqrt{R}$ positivity limit is shown as the dotted curve, evaluated at the average Q^2 for this experiment at each x.

POSITIVITY CONSTRAINTS FOR POL. QUARK DISTRIBUTIONS

(J.S., O. TERYAEV PHYS. LETT. B490 (2000) 106)

REMEMBER THE OLD POSITIVITY BOUND ON A_2, THE TRANSVERSE ASYMMETRY g_T/F_1

$$|A_2| \leq \sqrt{R} \qquad R = \sigma_L/\sigma_T$$

WHICH SEEMS USELESS.

IN FACT IT CAN BE IMPROVED BY THE STRONGER BOUND

$$|A_2| \leq \sqrt{R\left(\frac{1+A_1}{2}\right)}$$

WHERE A_1 IS THE LONGITUDINAL ASYMMETRY

USE A STANDARD CAUCHY-SCHWARZ INEQUALITY FOR THE VARIOUS PHOTON-NUCLEON C.S.

i.e. $$|\sigma_{LT}| \leq \sqrt{\sigma_L \sigma_T^+}$$

SINCE A_1 IS SMALL AND EVEN NEGATIVE FOR NEUTRON WE GET A BETTER CONSTRAINT

THIS CAN BECOME MUCH MORE RELEVANT IF WE USE POSITIVITY FOR EACH FLAVOR BY CONSIDERING PHOTON-QUARK SCATTERING + THE LEADING TWIST APPROXIMATION

POSITIVITY CONSTRAINTS ON INITIAL SPIN OBSERVABLES IN INCLUSIVE REACTIONS

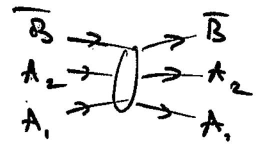
(J.S. CPT-2003/P.4535, CERN-TH/2003-110
<u>hep-ph 0305222</u>, to appear Phys. Rev. Lett)

$$A_1(1/2) + A_2(1/2) \to B + X$$

DON'T OBSERVE SPINS IN FINAL STATE

OBSERVABLES ARE SPIN-DEPENDENT DIFF. C.S. WITH RESPECT TO p_B, i.e. DISCONTINUITIES (W.R.T. M_X) OF THE FORWARD 3-3 AMPLITUDES

$$A_1 + A_2 + \bar{B} \to A_1 + A_2 + \bar{B}$$



(GENERALIZED OPTICAL TH.)

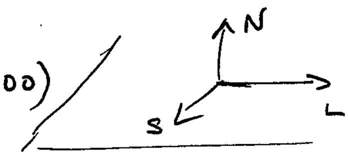
EIGHT OBSERVABLES $\qquad (A_1 A_2 | BX) = f(\sqrt{s}, p_T, y)$

$\sigma_0 = (00|00)$ $\qquad A_{1N} = (N0|00)$

$\qquad A_{2N} = (0N|00)$

$A_{LL} = (LL|00)$, $A_{SS} = (SS|00)$, $A_{NN} = (NN|00)$

$A_{LS} = (LS|00)$, $A_{SL} = (SL|00)$



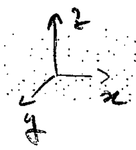
2×2 DENSITY MATRIX OF SPIN-1/2 PARTICLE

$$\rho_i = \frac{1}{2}\left(\mathbb{1}_2 + \vec{e}_i \cdot \vec{\sigma}\right) \qquad i = 1, 2$$

STATE OF POL. OF INCOMING SYSTEM DESCRIBED BY

$$\rho = \rho_1 \otimes \rho_2$$

$$\Rightarrow \qquad \sigma(\vec{e}_1, \vec{e}_2) = \mathrm{Tr}(M\rho)$$



WITH

$$M = \sigma_0 \Big[\mathbb{1}_4 + A_{1N}\, \sigma_{1z} \otimes \mathbb{1}_2 + A_{2N}\, \mathbb{1}_2 \otimes \sigma_{2z} + A_{NN}\, \sigma_{1z} \otimes \sigma_{2z}$$
$$+ A_{LL}\, \sigma_{1x} \otimes \sigma_{2x} + A_{SS}\, \sigma_{1y} \otimes \sigma_{2y} + A_{LS}\, \sigma_{1x} \otimes \sigma_{2y} + A_{SL}\, \sigma_{1y} \otimes \sigma_{2x} \Big]$$

THIS PARAMETRIZATION IS JUSTIFIED SINCE WE HAVE

$$\sigma(\vec{e}_1, \vec{e}_2) = \sigma_0 \Big[1 + A_{1N}\, e_{1z} + A_{2N}\, e_{2z} + A_{NN}\, e_{1z} e_{2z} + A_{LL}\, e_{1x} e_{2x} + A_{SS}\, e_{1y} e_{2y} + A_{LS}\, e_{1x} e_{2y} + A_{SL}\, e_{1y} e_{2x} \Big]$$

M IS A 4×4 (CROSS SECTION) MATRIX WHICH IS HERMITIAN AND POSITIVE

TO DERIVE THE POSITIVITY CONDITIONS SHOULD WRITE THE EXPLICIT EXPRESSION OF M.

AFTER PERMUTING TWO ROWS AND TWO COLUMNS IT REDUCES TO

$$\left(\begin{array}{c|c} M_+ & 0 \\ \hline 0 & M_- \end{array} \right)$$

$$M_\pm = \begin{pmatrix} (1 \pm A_{NN}) \pm (A_{1N} \pm A_{2N}) & (A_{LL} \pm A_{SS}) \mp i (A_{LS} \pm A_{SL}) \\ (A_{LL} \pm A_{SS}) \pm i (A_{LS} \pm A_{SL}) & (1 \pm A_{NN}) \mp (A_{1N} \pm A_{2N}) \end{pmatrix}$$

$\Rightarrow$ TWO STRONGEST CONDITIONS

$$(1 \pm A_{NN})^2 \geq (A_{1N} \pm A_{2N})^2 + (A_{LL} \pm A_{SS})^2 + (A_{LS} \pm A_{SL})^2$$

[IF IN A PROCESS $A_{NN} = \mp 1 \Rightarrow A_{1N} = \pm A_{2N}$, $A_{LL} = \pm A_{SS}$, $A_{LS} = \pm A_{SL}$ AND $1 \geq A_N^2 + A_{LL}^2 + A_{LS}^2$]

SIX WEAKER CONDITIONS

$$1 \pm A_{NN} \geq | A_{1N} \pm A_{2N} |$$

$$1 \pm A_{NN} \geq | A_{LL} \pm A_{SS} |$$

$$1 \pm A_{NN} \geq | A_{LS} \pm A_{SL} |$$

(SHOW GRAPH)

VERY GENERAL, MUST HOLD IN ANY KINEMATICAL REGION $(\sqrt{s}, p_T, y)$ ep, e^+e^-, qq', $q\bar{q}$, ETC....

SPECIFIC CASE OF pp SCATTERING

FOR IDENTICAL INITIAL PARTICLES

$$A_{1N}(y) = A_{2N}(-y)$$

$$A_{LS}(y) = A_{SL}(-y)$$

TWO CONSTRAINTS AMONG FIVE SPIN OBSERVABLES

$$\left[1 \pm A_{NN}(y)\right]^2 \geq \left[A_{1N}(y) \pm A_{1N}(-y)\right]^2 + \left[A_{LL}(y) \pm A_{SS}(y)\right]^2 + \left[A_{LS}(y) \pm A_{SL}(-y)\right]^2$$

So FOR $y = 0$

$$1 + A_{NN}(0) \geq 2\,|A_N(0)|$$

$$1 + A_{NN}(0) \geq 2\,|A_{LS}(0)|$$

IF A_{NN} LARGE AND NEGATIVE $\Rightarrow$ A_N, A_{LS} ~ 0

IF A_{NN} LARGE AND POSITIVE USELESS CONDITIONS

HOWEVER ACCORDING TO QCD CALCULATIONS (BNL-RHIC)

FOR $pp \to \gamma X$ AND $pp \to \begin{matrix} jet + X \\ \pi^0 + X \end{matrix}$ $A_{NN} \sim 0$ (ABSENCE OF h_1 FOR GLUONS IN DOMINANT PROCESS $Gq \to \gamma q$, $Gq \to Gq$)

So ONE GETS

$$|A_N| \leq 1/2 \;, \quad |A_{LS}| \leq 1/2$$

AND ALSO $|A_{LL} + A_{SS}| \leq 1$

So THESE RESULTS ARE STRONG CONSTRAINTS FOR FUTURE DATA AND MODEL BUILDERS

BNL spin discussion

09/09/03

A_{TT} in NLO - a status report

Marco Stratmann

Univ. of Regensburg

work done in collaboration with A. Mukherjee and W. Vogelsang

A_{TT} in NLO - a status report[1]

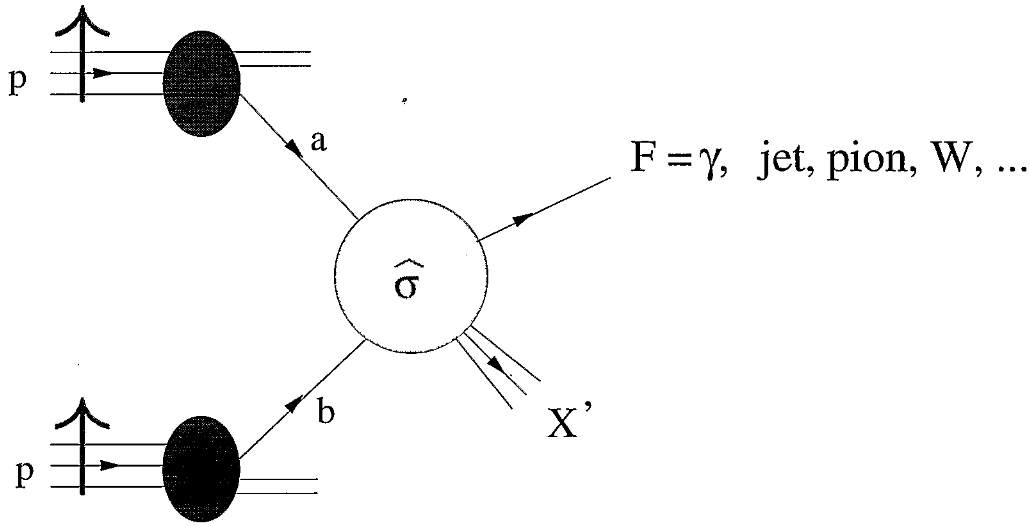
Marco Stratmann

Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany

We present a next-to-leading order QCD calculation of the cross section for isolated large-p_T prompt photon production in collisions of transversely polarized protons. We devise a simple method of dealing with the phase space integrals in dimensional regularization in the presence of the $\cos(2\Phi)$ azimuthal-angular dependence occurring for transverse polarization. Our results allow to calculate the double-spin asymmetry A_{TT}^{γ} for this process at next-to-leading order accuracy, which may be used at BNL-RHIC to measure the transversity parton distributions of the proton.

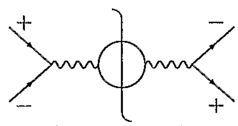
[1]work done in collaboration with A. Mukherjee and W. Vogelsang [Phys. Rev. **D67** (2003), 114006]

A_{TT} at RHIC



$$A_{TT} = \frac{d\sigma^{p^\uparrow p^\uparrow} - d\sigma^{p^\uparrow p^\downarrow}}{d\sigma^{p^\uparrow p^\uparrow} + d\sigma^{p^\uparrow p^\downarrow}} \propto \sum_{a,b} \delta f_a \otimes \delta f_b \otimes \underbrace{\delta\hat{\sigma}^{ab\to FX'}}_{\text{pQCD.}}$$

on the menu:

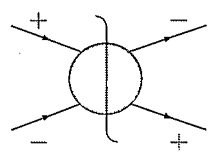
- **Drell-Yan process**  Ralston, Soper; Ji

Cortes, Pire, Ralston; Artru, Mekhfi; Jaffe, Ji

· most suited process: no gluons in LO

· NLO study: meas. suffers from limited $\mu^\pm$ acceptance
nevertheless, appears feasible

Martin, Schäfer, MS, Vogelsang

- **high-p_T prompt photons, hadrons, jets** 

· A_{TT} small due to absence of $g^\uparrow g^\uparrow$ and $q^\uparrow g^\uparrow$ processes

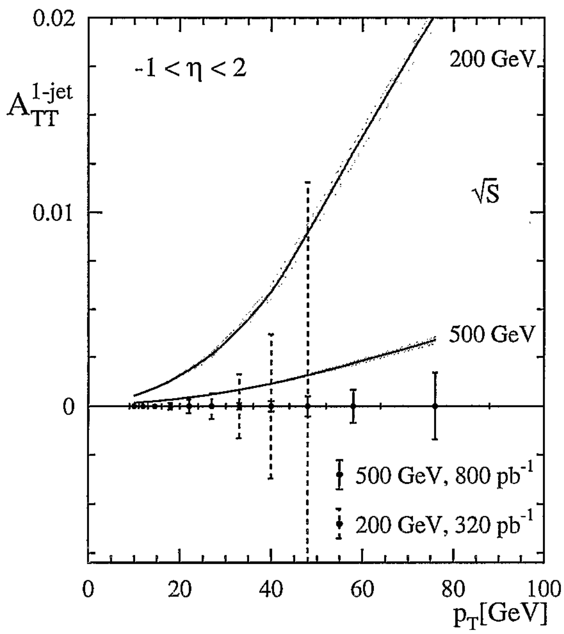
· sizable rates → statistics sufficient even if A_{TT} small

LO results assuming $\delta q(x,\mu_0) \ = \ \frac{1}{2}\,[q(x,\mu_0) + \Delta q(x,\mu_0)\,]$

Soffer, MS, Vogelsang

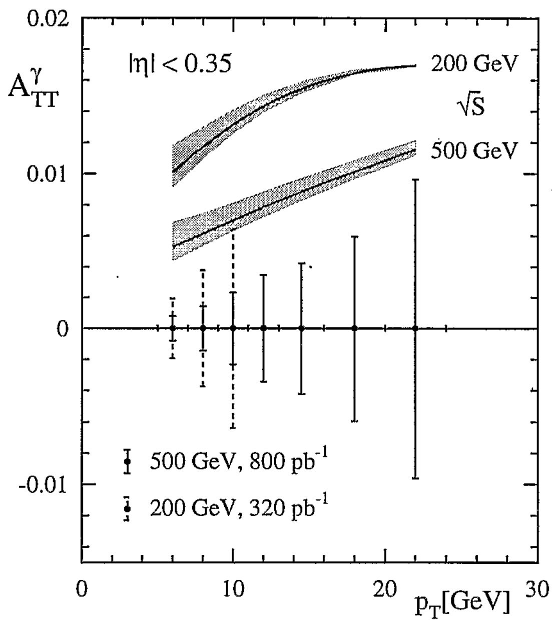
$p^\uparrow p^\uparrow \to \text{jet}\, X$

STAR



$p^\uparrow p^\uparrow \to \gamma X$

PHENIX



however, NLO QCD corrections are in general a must:

(scale dependence, ...) $\rightarrow$ Barbara's talk

further motivation for NLO: "technical challenge"

· general paucity of NLO calculations with transverse spin (until recently: NLO only for incl. DY and evol. kernels)

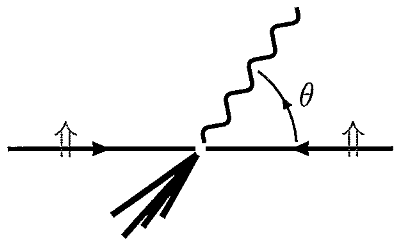
· provide and apply a feasible technique

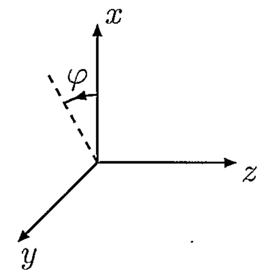
Why transverse spin is more complicated to handle:

long. polarization: spin *aligned* with momentum ✓

trans. polarization: spin = *extra spatial direction*

$\leftrightarrow$ non-trivial azimuthal dep.



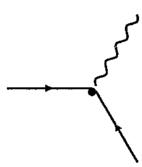


always of the form $$\frac{d^3\delta\sigma}{dp_T d\eta d\Phi} \equiv \cos(2\Phi) \left\langle \frac{d^2\delta\sigma}{dp_T d\eta} \right\rangle$$

$\rightarrow \phi$ integration not appropriate

but standard NLO techniques rely on integrations over

full azimuthal phase space *plus* use of particular

reference frame  Gottfried, Jackson

$\Rightarrow$ difficult to deal with azimuthal angle

(in particular, in $d = 4 - 2\varepsilon$ dimensions) recent progress:

$$\boxed{\text{NLO corrections to } p^\uparrow p^\uparrow \to \gamma X}$$

A. Mukherjee, MS, W. Vogelsang

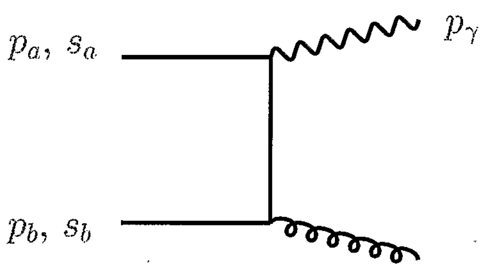
key point: ϕ-dep. always results from covariant expression

$$\mathcal{F}(p_\gamma, s_a, s_b) = \frac{s}{tu}\left[2\,(p_\gamma \cdot s_a)\,(p_\gamma \cdot s_b) \;+\; \frac{tu}{s}\,(s_a \cdot s_b)\right]$$

$$= \cos(2\phi) \text{ in hadronic c.m.s.}$$

$\Rightarrow$ use $\mathcal{F}$ to project out Φ covariantly

LO example: $q\bar{q} \to \gamma g$



$$p_a \cdot s_a \;=\; p_b \cdot s_b \;=\; 0$$
$$s_a^2 = s_b^2 = -1$$

matrix element [use $u(p_a, s_a)\,\bar{u}(p_a, s_a) = \frac{1}{2}\not{p}_a\,[1 + \gamma_5 \not{s}_a], \ldots$]

$$\delta|M|^2 = (ee_q g)^2\,\frac{4C_F}{N_C}\,\frac{s}{tu}\left[2\,(p_\gamma \cdot s_a)\,(p_\gamma \cdot s_b) \;+\; \frac{tu}{s}\,(s_a \cdot s_b)\right]$$

project with $\mathcal{F}$:

$$\frac{1}{\pi}\int d\Omega_\gamma\,\mathcal{F}(p_\gamma, s_a, s_b)\delta|M|^2 = (ee_q g)^2\,\frac{4C_F}{N_C} = \langle\delta|M|^2\rangle \;\checkmark$$

$$\updownarrow$$

terms involving $p_\gamma \cdot s_a\,,\, p_\gamma \cdot s_b$ can be integrated "covariantly" easily generalized to NLO calculation in d dimensions:

(1) multiply any $\delta|M|^2$ with $\mathcal{F}(p_\gamma, s_a, s_b)$

(2) integrate all resulting scalar products with $s_{a,b}$

for example: [up to $\mathcal{O}(\varepsilon)$]

$$\int d\Omega_\gamma \, (p_\gamma \cdot s_a)^2 (p_\gamma \cdot s_b)^2 = \int d\Omega_\gamma \frac{t^2 u^2}{8 s^2} \left[2(s_a \cdot s_b)^2 + s_a^2 s_b^2 \right]$$

(3) arrive at a structure similar to an unpol. $|M|^2$

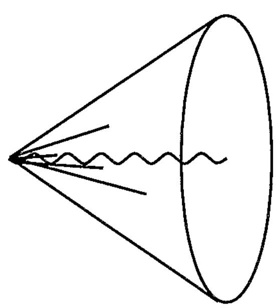
(4) employ *standard techniques* for phase space integr.

(5) restore ϕ dependence afterwards

remarks:

- cancellation of divergencies proceeds as usual
  $\rightarrow$ Barbara's talk

- at colliders, impose "isolation cut" on photon:



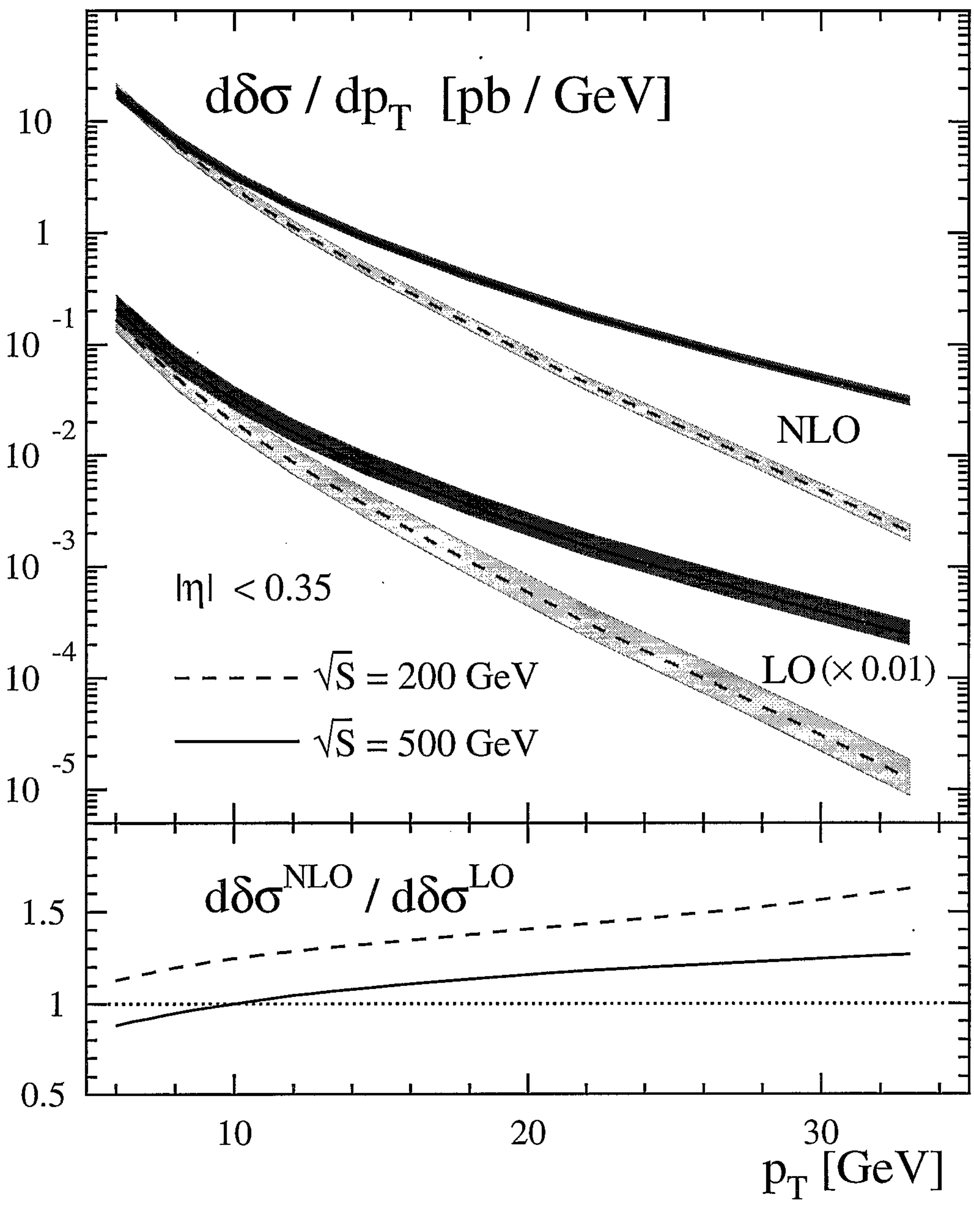
typically, demand $E_{\text{had}} \leq \epsilon E_\gamma$

in $\sqrt{(\Delta\phi)^2 + (\Delta\eta)^2} \leq R$

- for isolation with $\epsilon \propto (r/R)^2$ one can eliminate the fragmentation component altogether Frixione

results for $p^\uparrow p^\uparrow \to \gamma X$:

- improved scale dependence
- reasonable "K-factors"



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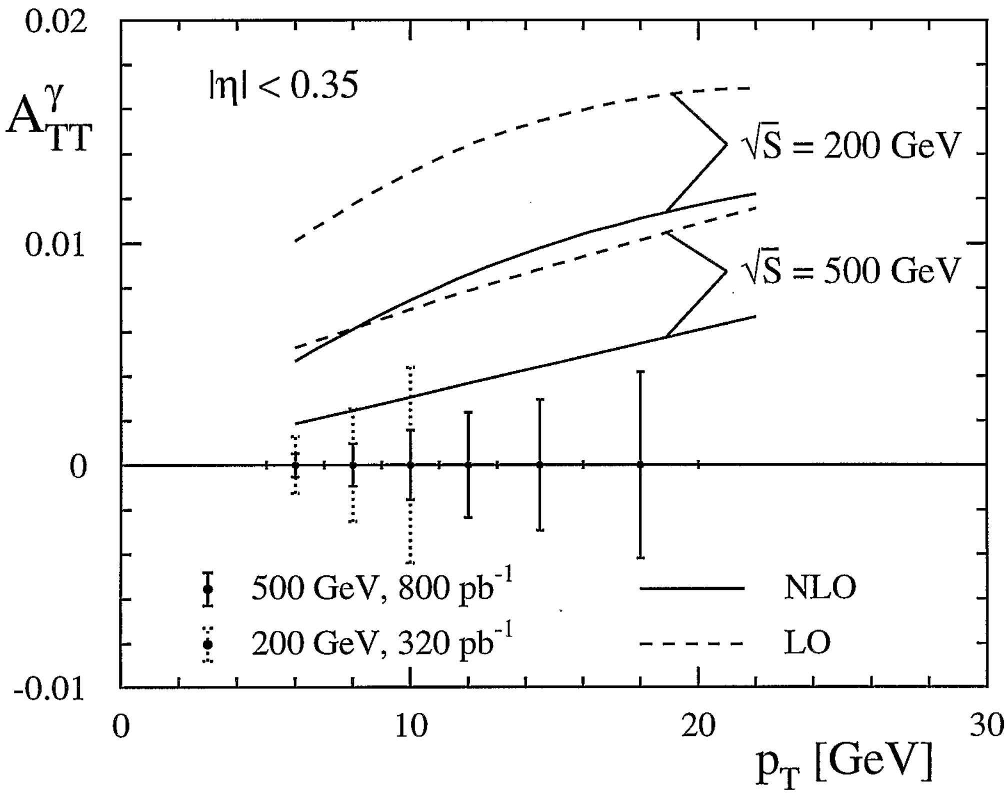
results for A_{TT}:



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work in progress:

- half-way through with $p^{\uparrow}p^{\uparrow} \to \pi X$
- future: $p^{\uparrow}p^{\uparrow} \to$ jet$X$ ...

Theory Studies for Polarized *pp* Scattering

A RIKEN BNL Research Center Workshop
Spring 2002

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RIKEN BNL RESEARCH CENTER

Theory Studies for Polarized *pp* Scattering

August – September 2003

Li Keran

Nuclei as heavy as bulls
Through collision
Generate new states of matter.
T.D. Lee

Speakers:

B. Jäger
E. Leader
A. Mukherjee
J. Soffer
M. Stratmann

Organizers: S. Kretzer and W. Vogelsang