RELATIVISTIC HEAVY ION PHYSICS:  
A THEORETICAL OVERVIEW*

D. KHRZEEV
Department of Physics, Brookhaven National Laboratory,  
Upton, New York 11973-5000, USA

This is a mini-review of recent theoretical work in the field of relativistic heavy ion physics.  
The following topics are discussed: initial conditions and the Color Glass Condensate;  
approach to thermalization and the hydrodynamical evolution; hard probes and the properties  
of the Quark-Gluon Plasmas. Some of the unsolved problems and potentially promising directions  
for future research are listed as well.

1 Introduction

In general, theorists get attracted to relativistic heavy ion physics because it is placed at the  
intersection of three different, and equally interesting, directions in contemporary theoretical research:  
i) small $x$, high parton density QCD; ii) non-equilibrium field theory; and iii) phase transitions in strongly interacting matter. Indeed, understanding the evolution of a heavy ion collision requires a working theory of initial conditions, of the subsequent evolution of the produced partonic system, and of the phase transition(s) to the deconfined phase. This mini-review is an attempt to capture some of the recent changes and developments in the theoretical picture of these phenomena which have been triggered by an intense stream of the new data from RHIC.

2 Initial conditions and global observables

2.1 The rôle of coherence

Not so long ago, before the advent of RHIC, it was widely believed that at collider energies the  
total multiplicities will become dominated by hard incoherent processes. The very first data

from RHIC (see\textsuperscript{1} and references therein in this volume) provided a lot of food for new thought: the measured charged hadron multiplicities in \( \text{Au} + \text{Au} \) collisions appeared much smaller than expected on the basis of incoherent superposition of hard processes. Given that any inelastic rescatterings in the final state can only increase the multiplicity\textsuperscript{1}, we have an experimental proof of a high degree of coherence in multi-particle production in nuclear collisions at RHIC energies.

2.2 Semi-classical QCD and hadron multiplicities

Combining the idea of coherence with the parton model, we have to consider the initial parton wave functions of the colliding nuclei as coherent superpositions of the wave functions of the constituent nucleons. Since at small Bjorken \( x \) all of the partons in the nucleus at a fixed transverse coordinate participate in a hard scattering process, this treatment naturally leads to the notion of parton density in the transverse plane \( Q_s^2 \) – a new dimensionful scale of the problem. Once this scale becomes comparable to the resolution scale determined by the kinematics of the hard scattering, the amplitude of the process is severely affected by the coherence. The limit on the parton density is reached when the occupation numbers of the gluon field modes with transverse momenta \( p_T < Q_s \) reach the value \( n_k \sim 1/\alpha_s(Q_s) \), characteristic for classical gauge fields – this is the phenomenon known as "parton saturation"\textsuperscript{2}, leading to a coherent state of gluons – Color Glass Condensate (for reviews, see\textsuperscript{3,4,5,6,7}).

Since the integrated multiplicities are dominated by momenta \( p_T < Q_s \) and parton density in the transverse plane scales as \( Q_s^2 \sim N_{\text{part}}^{1/3} \) (where \( N_{\text{part}} \) is the number of nucleons which participate in the process), Color Glass Condensate leads to a simple prediction\textsuperscript{8} for the centrality dependence of hadron multiplicity in heavy ion collisions:

\[
\frac{dn_{AA}}{d\eta} \sim N_{\text{part}} \ln (N_{\text{part}}).
\]  

Combined with the dependence of the gluon structure function on Bjorken \( x \) known from HERA, which implies \( Q_s^2(x) \sim 1/x^\lambda \), one can generalize this formula to predict the energy, centrality, rapidity, and atomic number dependencies of hadron multiplicities\textsuperscript{8}. Additional information on the dynamics of the collision can be inferred from the numerical lattice simulations\textsuperscript{9}. So far this approach has been quite successful in predicting the multiplicities measured at RHIC; a

\textsuperscript{b}For statistical systems, this is due to the second law of thermodynamics
recent important example is given at Fig. 1 which shows the evolution of centrality dependence with energy in the entire RHIC range between \( \sqrt{s} = 20 \) and 200 GeV. One can see that the shape of the centrality dependence changes very little over a large energy range, in which the perturbative minijet cross section grows by over an order of magnitude. The prediction of the saturation model is seen to agree with the data reasonably well; this indicates the possibility that parton saturation sets in in heavy ion collisions already at moderate energies. We do not expect the method to apply below \( \sqrt{s} = 20 \) GeV however, since at lower energies the coherence length becomes shorter than the nuclear radius.

2.3 High-p_T hadron suppression at forward rapidities, and quantum evolution in the Color Glass Condensate

Parton saturation at transverse momenta \( k_T \leq Q_s \) at sufficiently small \( x \) appears to have non-trivial consequences also for the nuclear dependence of the semi-hard processes. At very small \( x \), when \( \alpha_s \ln(1/x) \sim 1 \), a semi-classical description has to be modified due to the quantum evolution. Small \( x \) evolution introduces anomalous dimension \( \gamma \simeq 1/2 \) in the gluon densities, so that the dependence on the momentum scale \( Q \) is modified, to \( Q^3 \rightarrow Q^\gamma \). Since in the vicinity of the saturation boundary the only dimensionful scale characterizing the system is the saturation momentum \( Q_s^2 \), the cross section of semi-hard scattering should scale as a function of \( Q_s^2/Q^2 \) — it was found that this "geometrical scaling" is consistent with HERA data on deep-inelastic scattering. Combining these two observations with the \( A \) dependence of the saturation momentum \( Q_s^2 \sim A^{1/3} \) we come to the conclusion that at sufficiently small \( x \) and moderate \( k_T \) the nuclear dependence of hard processes in \( AA \) collisions should change from \( S_A Q_s^4 \sim N_{\text{part}}^{4/3} \) (where \( S_A \sim N_{\text{part}}^{3/2} \) is the overlap area) to \( S_A Q_s^{2\gamma} \sim N_{\text{part}} \). In \( pA \) (or \( dA \)) collisions the nuclear dependence is then \( S_A Q_s^{2\gamma} \sim A^{5/6} \), so there has to be a suppression as well. This suppression has also been found in the numerical solution of the Balitsky-Kovchegov equation, as well as inferred for recent work, see also [19,20,21].

The experimental test of these ideas has been performed shortly afterwards — it has been established (for a review, see [43 in this volume]) that at mid-rapidity \( y = 0 \) there is no high \( k_T \) suppression in \( dAu \) data; this means that the suppression observed in \( AuAu \) collisions has to come from the final-state effects, which will be discussed below. The data thus rule out the possibility that \( x \) is small enough for quantum evolution to develop already at mid-rapidity at RHIC. Nevertheless, the presented arguments should apply at sufficiently small \( x \). This is why the data on high \( k_T \) hadron production at forward rapidities giving access to much smaller
values of $x$ were eagerly awaited. The results from the BRAHMS experiment demonstrated a strong suppression of high $k_T$ hadrons; moreover, the centrality dependence appeared consistent with the predicted $R_{AA} \sim N_{\text{part}}^{-1/2}$ scaling, in a dramatic contrast to the increasing Cronin enhancement observed at mid-rapidity. Complementary results have been reported in STAR Collaboration has also reported an observation of a predicted nuclear-dependent weakening of the back-to-back correlations for hadrons separated by several units of rapidity. It will be interesting to check if the suppression extends to charm hadrons at forward rapidities; at mid-rapidity, the first results have been reported. Alternative explanations based on "conventional" shadowing and multiple scattering (for a review, see) are also being explored.

3 Approach to thermalization, and the role of classical fields

There is by now an ample evidence of the importance of final state interactions in heavy ion collisions. Among the bulk observables, the azimuthal anisotropy of hadron production is a most spectacular evidence of this – indeed, if all of the elementary nucleon-nucleon collisions were independent, the produced hadrons would not be correlated with the nucleus-nucleus reaction plane. The observed azimuthal anisotropy (or the "elliptic flow", in the parlance of the field; see for a review) indicates the existence of a correlation between the geometry of the nucleus-nucleus collision and the momenta of the emitted hadrons. An economical way of describing the evolution of a large number of particles in space and momentum is provided by relativistic hydrodynamics, which transforms the gradients of the initial parton density into the momentum flow of the produced hadrons. Hydrodynamical description is valid when the mean free path of partons is much smaller than the size of the system, i.e. when the system is sufficiently thermalized. The free expansion (or "inflation") of the produced system with time reduces the density gradients, so the magnitude of the elliptic flow crucially depends on the thermalization time when a hydrodynamical calculation is initiated. It appears that to describe the elliptic flow of the observed magnitude, one has to assume that the thermalization time is very short, about $\tau_{\text{therm}} \approx 0.5$ fm (for a review, see). Such a short thermalization time presents a problem both for the traditional perturbative and non-perturbative treatments. Indeed, in perturbative QCD the rescattering amplitudes are suppressed by powers of the coupling $\alpha_s$, so the thermalization time appears long, on the order of 10 fm, which makes the description of the elliptic flow problematic. In non-perturbative approaches, the interactions can be assumed strong, but the typical time scale of an interaction is $\sim 1/\Lambda_{\text{QCD}} \sim 1$ fm, so it is difficult to expect that several interactions needed for thermalization will occur during $\tau_{\text{therm}} \approx$.
0.5 fm. The coherent classical fields present in the Color Glass Condensate (CGC) scenario may eventually provide a solution to this puzzle, since in this case the multi-gluon scattering amplitudes \(A(n \rightarrow m)\) from \(n \sim 1/\alpha_s\) to \(m \sim 1/\alpha_s\) gluons are not suppressed. An approach to thermalization in this scenario was explored in Ref. 36. Recently, an attention was brought also to the role of instabilities in the equilibration process 37. The use of CGC initial conditions of Ref. 8 in a hydrodynamical approach 38 has led to a successful description of the RHIC data. Nevertheless, much work will have to be done to understand the thermalization process.

4 Hydrodynamical evolution: more fluid than water

Since hydrodynamical description relies on the direct use of the equation of state, the data can be used to extract an information on the properties of the medium. It appears that the quark–gluon plasma equation of state as measured on the lattice (for a review, see 39) is successful in describing the data. However the data can tell even more about the properties of the medium, if one considers the influence of viscous corrections on various observables 40. Viscosity of the medium appears to affect the observables in a very significant way; in fact, one can deduce an upper limit 40,34 on the ratio of shear viscosity \(\eta\) to the entropy density \(s\), \(\eta/s \leq 0.1\) – much smaller than the same ratio for the water! Such a small value of viscosity, which reflects the dissipation of energy in a hydrodynamical evolution, contradicts the picture of weakly coupled quark-gluon plasma, and is more indicative of a strongly coupled quark-gluon liquid. A calculation of shear viscosity in the strong coupling regime of QCD is still beyond the reach; however it has been made in \(N = 4\) supersymmetric Yang-Mills theory 41 – the result is a small ratio of \(\eta/s = 1/4\pi\), comparable to the one inferred from RHIC data.

A small value of viscosity in the strongly coupled quark-gluon plasma \(a posteriori\) justifies the use of the approach 8 to hadron multiplicities assuming the proportionality of the number of measured hadrons to the number of the initially produced partons. This assumption would be unnatural if the evolution of the plasma were accompanied by parton multiplication, but is justified if the viscosity is small and evolution of the system is close to isentropic.

5 High \(p_T\) hadron suppression, jet quenching, and heavy quarks

The suppression of high \(p_T\) hadrons in \(Au-Au\) collisions is certainly one of the most spectacular new results at RHIC (for a comprehensive review of the data, see 43 in this volume). Such an effect has not been seen at lower energies; moreover, the results from the \(dAu\) run at RHIC indicate that at pseudo-rapidity \(\eta = 0\) the observed suppression is entirely due to the final state effects, very likely a jet quenching in the quark-gluon plasma (for an overview, see 44,45). Alternative scenarios, e.g. the absorption in a dense hadron gas, seem unlikely in view of the high energy density \(\epsilon \sim 20\ \text{GeV/fm}^3\) (see e.g. 8) achieved in the collisions. Nevertheless, additional experimental checks have to be performed; an important additional test of the jet quenching scenario involves the measurement of the suppression for heavy hadrons containing \(c\) or \(b\) quarks. If the suppression of high \(p_T\) particles is indeed due to the induced radiation of gluons by fast partons, heavy quarks should lose significantly less energy than the light ones due to the "dead cone" effect 46. This prediction seems to be in accord with the first RHIC data, which within the error bars indicate no quenching effect on the spectra of open charm, as inferred from the decay electrons 47; however more precise data are desirable. Several other calculations of the energy loss of heavy partons have been performed (see Refs. 48,49,50 and papers 51,52 in this volume); while they differ in the formalisms used, they all find a reduced energy loss for the heavy quarks. On the other hand, since heavy mesons (\(D, B, \ldots\)) have a typical large size determined by the

\[A \text{A moderate amount of suppression in the SPS results however cannot be excluded due to uncertainties in the reference pp data.} 42\]
presence of the light quark in their wave functions, in the hadronic absorption mechanism one would expect that heavy mesons interact with about the same probability as the light ones.

Figure 4: Nuclear modification factor for charged hadrons and neutral pions (left and right panels, respectively) in Au + Au collisions at $\sqrt{s} = 200$ GeV; from Ref.43.

6 Heavy quarkonium in hot QCD matter

Ever since it was proposed as a signature of the quark-gluon plasma, the dissociation of heavy quarkonia in hot QCD matter has remained a focal point of vigorous theoretical and experimental studies. The NA38/50 Collaborations at CERN have observed the suppression of $J/\psi$ and $\psi'$, and the current NA60 experiment will significantly extend the existing measurements (for an update on the recent results, see54 in this volume). The first RHIC results have already been reported, in Au+Au55, dAu56, and pp57 collisions. Theoretically, a new insight on the problem has been gained from the recent lattice calculations (see58,59 for an overview) which indicate that the $J/\psi$ and $\eta_c$ survive as bound states in the quark-gluon plasma at least up to the temperatures twice the critical, $2T_c$. This observation is very important in understanding the properties of the strongly coupled quark-gluon plasma discussed above. However, in my opinion, it should not be interpreted as an indication that heavy quarkonia are not suppressed in the quark-gluon plasma unless the temperature is very high; the point is that even if a $zc$ state is bound in a plasma, it can be readily dissociated by the impact of gluons, which have much harder momentum distributions in a deconfined phase. Estimates of the activation rate of quarkonia due to the interaction with the heat bath show that even if $(zc)$ states exist as bound states, their yield can be strongly suppressed. More work has to be done to understand these effects better; on the lattice, a reliable extraction of the thermal widths of heavy quarkonia would be most desirable.

7 Baryon dynamics

The structure of baryons in non-perturbative QCD remains quite puzzling: while in non-relativistic quark model $QQQ$ baryons are not so different from $QQ$ mesons, in the approaches motivated by $1/N_c$ expansion they are drastically different -- in Skyrme picture, for example, they are the topological solitons of the meson fields. A closer look at the quark wave functions of baryons reveals that local gauge invariance requires the presence of novel configurations of gauge field -- so called "baryon junctions". Naively, one expects that at high energies the collision of two relativistic nuclei would not lead to any substantial baryon stopping -- since the valence quarks associated with the baryon number carry a large fraction of the nucleons' momentum, they are hard to stop in a soft process. However the account of non-perturbative baryon junctions leads to a substantial change in this picture, since the baryon number appears to be traced by soft gluons. In perturbation theory, baryon junctions were shown to correspond to
multi-gluon exchanges in higher color representations. Substantial amount of baryon stopping, with the magnitude and rapidity dependence consistent with the baryon junction picture, has been observed at RHIC. The influence of quantum evolution and parton saturation on the $x$ distributions of valence quarks in nuclei has been addressed recently in Ref.

Another exciting observation at RHIC related to baryon dynamics is a strong enhancement of baryon-to-pion ratios at moderate values of transverse momentum (so called $B/\pi$ puzzle) and the larger magnitude of the elliptic flow for baryons. The proposed explanations include the phenomenon of parton coalescence and the interplay of baryon junctions with jet quenching.

Summary

The first years of the experiments at RHIC have changed in a dramatic way the theoretical picture of dense and hot parton systems. The evidence for the existence of new states of QCD matter is mounting, and a consistent description of the observed phenomena has started to emerge. Nevertheless, hot and dense QCD is still in its infancy – and we have every reason to expect new surprises!

I am grateful to the Organizers for the excellent meeting. This work was supported by the U.S. Department of Energy under Contract No. DE-AC02-98CH10886.

References

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.