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BNL-73216-2004-CP

Large-x Resummations in QCD

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Presented at 2nd HiX Workshop "Structure of the Nucleon at Large Bjorken-x"
Marseille, France
July 26-28, 2004

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Abstract. We give a brief introduction to the resummation of a class of large logarithmic perturbative corrections to partonic hard-scattering cross sections. These corrections occur in deeply-inelastic structure functions at high Bjorken-x, and near partonic threshold in cross sections for large produced invariant mass or transverse momentum, such as the Drell-Yan process or hadronic prompt-photon production. They are associated with soft and/or collinear gluon emission.

INTRODUCTION

Hadronic cross sections at large momentum transfer factorize to leading power in the hard scale into process-independent long-distance pieces (usually parton distributions) and short-distance parts that describe the hard interactions of the partons and are amenable to QCD perturbation theory. For example, for the Drell-Yan reaction $pp \to \mu^+\mu^- X$ at high invariant muon pair mass Q one has:

$$\frac{Q^4 d\sigma_{\rm DY}(\tau)}{dQ^2} = \sum_{a,b=q,\bar{q},g} f_a(x_a,\mu) \otimes f_b(x_b,\mu) \otimes \frac{Q^4 d\hat{\sigma}^{ab}_{\rm DY}\left(\frac{\tau}{x_a x_b}, \frac{Q}{\mu}, \alpha_s(\mu)\right)}{dQ^2}, \quad (1)$$

where $\tau = Q^2/S$, f_a, f_b are the parton densities, $\otimes$ denotes an appropriate convolution, and where the sum is over all contributing partonic channels $a+b \to \mu^+\mu^- + x$, with $d\hat{\sigma}^{ab}_{\rm DY}$ the associated partonic cross section. The latter has the perturbative expansion $d\hat{\sigma}^{ab}_{\rm DY} = d\hat{\sigma}^{ab,(0)}_{\rm DY} + \frac{\alpha_s}{\pi} d\hat{\sigma}^{ab,(1)}_{\rm DY} + \ldots$, corresponding to lowest order (LO) and next-to-leading order (NLO) etc. of perturbation theory. μ is the renormalization/factorization scale. Corrections to the right-hand-side of Eq. (1) are down by inverse powers of Q.

When probed near an exclusive boundary of phase space, the perturbative partonic hard-scattering cross sections acquire large logarithmic corrections arising from incomplete cancellations of soft-gluon effects between virtual and real diagrams. A prominent example are threshold corrections. For the Drell-Yan case above, these are of the form $\alpha_s^n \ln^m(1-z)/(1-z)$, where $m \leq 2n-1$, and become large when $z = Q^2/\hat{s} \to 1$, with $\hat{s}$ the partonic c.m. energy. Sufficiently close to the phase-space boundary, i.e. in the limit of soft and/or collinear radiation, fixed-order perturbation theory is bound to fail. A proper treatment of the cross section requires resummation of the logarithmic corrections to all orders. The techniques for this are well established for many reactions of interest, starting with the Drell-Yan process [1, 2]. Fig. 1 compares the fixed-order (LO, NLO, ...) and the resummed (leading logarithms (LL), next-to-leading logarithms (NLL), and so forth) approaches qualitatively.

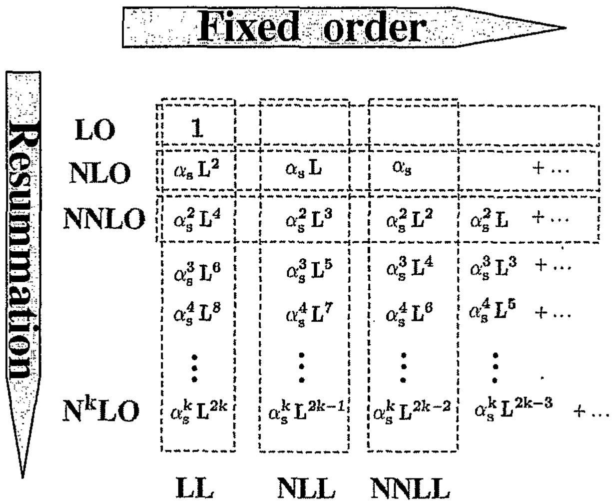

FIGURE 1. Fixed orders in perturbation theory, and resummation. L is a large logarithm near threshold in the appropriate Mellin moment variable; see below.

BASICS OF THRESHOLD RESUMMATION

For illustration, let us sketch the derivation of the LL threshold resummation for the Drell-Yan process. For rigorous and alternative derivations, see [1, 2, 3, 4]. We start with the first-order corrections $q(p)\bar{q}(\bar{p}) \to \gamma^* g(k)$ when the gluon is soft:

$$|M(p,\bar{p};k)|^2 \approx_{k\to 0} \left|M_{\text{Born}}\right|^2 \cdot (4\pi\alpha_s)\, C_F \frac{2(p\cdot\bar{p})}{(p\cdot k)(\bar{p}\cdot k)} \,. \tag{2}$$

We write the gluon momentum $k = xp + y\bar{p} + k_\perp$ with $x = k\cdot\bar{p}/p\cdot\bar{p} \approx 2k\cdot\bar{p}/Q^2$, $y = k\cdot p/p\cdot\bar{p} \approx 2k\cdot p/Q^2$. One has $k_\perp^2 \approx xyQ^2$. Inserting the gluon phase space yields

$$\int d\Phi_g\, |M(p,\bar{p};k)|^2 \approx 8(4\pi\alpha_s)\, C_F \int \frac{d^{n-1}k}{(2\pi)^3\, 2k_0}\, \frac{1}{k_\perp^2}\, \delta\left(1-z-z_k\right) , \tag{3}$$

where $z = Q^2/\hat{s}$ and $z_k = 2k_0/\sqrt{\hat{s}} = x+y$ is the energy fraction of the emitted gluon. We use dimensional regularization for the infrared and collinear divergencies present in Eq. (3). We now write the δ-function in (3) as an inverse Mellin (or Laplace) transform,

$$\delta\left(1-z-z_k\right) \approx \frac{1}{2\pi i} \int_C dN\, \mathrm{e}^{N(1-z-z_k)} \,, \tag{4}$$

which implies that soft emission (large z / small z_k) corresponds to large N. Then,

$$\int d\Phi_g\, |M(p,\bar{p};k)|^2 \approx \frac{1}{2\pi i} \int_C dN\, \mathrm{e}^{N(1-z)}\, \frac{\alpha_s C_F}{\pi} \int \frac{dx}{x} \int \frac{dk_\perp^2}{\left(k_\perp^2\right)^{1+\varepsilon}}\, \mathrm{e}^{-N\left(x+\frac{k_\perp^2}{Q^2 x}\right)} \,. \tag{5}$$

There are two regions that contribute, $x \leq y \leq 1$ and $y \leq x \leq 1$, where $y = k_\perp^2/Q^2 x$. Thus we will have integrations over $\int_0^1 \frac{dx}{x} \int_0^{x^2 Q^2} dk_\perp^2$ and $\int_0^1 \frac{dx}{x} \int_{x^2 Q^2}^{x Q^2} dk_\perp^2$. Substituting x by y in the second integral shows that the two regions give identical contributions, since the integrand is symmetric in x and y. Hence, adding the contributions from virtual diagrams, the right-hand-side of Eq. (5) becomes

$$\frac{2\alpha_s C_F}{\pi} \int_0^1 \frac{dx}{1-x} \int_0^{Q^2(1-x)^2} \frac{dk_\perp^2}{\left(k_\perp^2\right)^{1+\varepsilon}} \left[\mathrm{e}^{-N\left(1-x+\frac{k_\perp^2}{Q^2(1-x)}\right)} - 1 \right], \tag{6}$$

where we have substituted $x \to 1-x$ and suppressed the term $\frac{1}{2\pi i}\int_C dN\, \mathrm{e}^{N(1-z)}$ which just gives the Mellin-inverse. The expression in (6) still has a collinear singularity for $k_\perp \to 0$, which is removed by collinear factorization, i.e., by applying the $\overline{\mathrm{MS}}$ subtraction

$$\frac{2\alpha_s C_F}{\pi} \int_0^1 \frac{dx}{1-x} \left[x^N - 1\right] \int_0^{\mu^2} \frac{dk_\perp^2}{\left(k_\perp^2\right)^{1+\varepsilon}}, \tag{7}$$

where μ is the factorization scale, chosen here for simplicity as Q. Combining Eqs. (6) and (7) one finds that the dominant behavior at large N is contained in the expression

$$\frac{2\alpha_s C_F}{\pi} \int_0^1 \frac{dx}{1-x} \int_{Q^2}^{Q^2(1-x)^2} \frac{dk_\perp^2}{k_\perp^2} \left[\mathrm{e}^{-N(1-x)} - 1\right] = \frac{2\alpha_s C_F}{\pi} \ln^2(\bar{N}) + \mathscr{O}\left(\frac{1}{N}\right), \tag{8}$$

where $\bar{N} = N\mathrm{e}^{\gamma_E}$ with γ_E the Euler constant. When the Mellin inverse is taken, this result yields the term $\alpha_s C_F/\pi \ln(1-z)/(1-z)$ that dominates the first-order Drell-Yan coefficient function at large z. Obviously we could have derived this result directly in "z-space", without using Mellin moments. However, the use of moments is crucial for deriving the resummed cross section. This is because the phase space for multi-gluon emission contains energy-momentum conserving δ-functions connecting components of the gluon momenta. In our case at hand, for emission of n gluons, Eq. (4) turns into

$$\delta\left(1-z-\sum_{i=1}^n z_i\right) \approx \frac{1}{2\pi i}\int_C dN\, \mathrm{e}^{N\left(1-z-\sum_{i=1}^n z_i\right)} = \frac{1}{2\pi i}\int_C dN\, \mathrm{e}^{N(1-z)} \prod_{i=1}^n \mathrm{e}^{-N z_i}, \tag{9}$$

leading to a factorization of the n-gluon phase space under Mellin moments. At the same time, the squared n-soft-gluon matrix element has QED-like factorization properties and is essentially a product of n single-gluon emission factors, times the Born matrix element. As a result, *exponentiation* of the soft-gluon terms occurs in Mellin-moment space, with an exponent that to LL is just the one-loop expression derived in Eq. (8):

$$\hat{\sigma}_{\mathrm{DY}}^{q\bar{q}}(N) \propto \exp\left[2\int_0^1 dx \frac{x^N - 1}{1-x} \int_{Q^2}^{Q^2(1-x)^2} \frac{dk_\perp^2}{k_\perp^2} A(\alpha_s(k_\perp^2))\right], \tag{10}$$

where

$$A(\alpha_s) = \frac{\alpha_s}{\pi} C_F + \left(\frac{\alpha_s}{\pi}\right)^2 \frac{C_F}{2}\left[C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{10}{9} T_R N_F\right]. \tag{11}$$

Here, the first term in A leads to resummation of the LL terms $\alpha_s^k \ln^{2k} N$ in moment space. The second term gives next-to-leading logarithmic contributions of the form $\alpha_s^k \ln^{2k-1} N$ to the cross section. It is evident that the exponent is *positive*, so that resummation will *enhance* the cross section. This is related to the fact that soft-gluon effects are partly already contained in the parton distributions, as shown by the subtraction in (7).

We note that in QCD the scale $k_\perp$ appears in the strong coupling constant in the exponent in Eq. (10). Because of the singularity of the perturbative coupling at $\Lambda_{\rm QCD}$, the perturbative expansion of the expression shows factorial divergence, which in QCD corresponds to a power-like ambiguity of the series. A closer look, however, shows that the resummed logarithmic terms in N have the form (we define $\alpha_s(Q^2)\ln(N) \equiv \alpha_s L$)

$$\hat{\sigma}_{\rm DY}^{q\bar{q}}(N) \propto \exp\left[\sum_{k=1}^{\infty} \alpha_s^k L^k \left(a_k L + b_k\right) + \mathcal{O}(\alpha_s^{k+1} L^k)\right] , \tag{12}$$

in which the coefficients a_k, b_k have no factorial behavior [5]. The LL terms in the exponent are *single* logarithms of the form $\alpha_s^k L^{k+1}$, which after expansion of the exponential generate the leading *double* logarithms $\alpha_s^k L^{2k}$ in the cross section. The factorial divergence thus appears only at nonleading powers of momentum transfer. This implies that perturbative resummation can suggest [6] the form of nonperturbative, power-suppressed, dynamics. Expanding the full resummed exponential at small $k_\perp$, one finds

$$\exp\left[\frac{2N^2}{Q^2}\int_0^{\lambda^2} dk_T^2 A_a(\alpha_s(k_T)) \ln\left(\frac{Q}{\bar{N}k_T}\right)\right] \to \exp\left[\frac{2C_F}{\pi}\frac{N^2}{Q^2}\left\{g_1 + g_2 \ln\left(\frac{Q}{NQ_0}\right)\right\}\right] \tag{13}$$

with g_1 and g_2 nonperturbative parameters to be determined by comparison to data.

Threshold resummation has been worked out to NLL accuracy for most reactions of current interest; see for example [1, 2, 7, 8, 9, 10], and in some cases to NNLL [11, 12]. If more than two partons are involved in the hard-scattering, exponentiation still occurs but, because of color interferences and correlations, usually comes in the form of sums of exponentials, rather than a single exponential [8, 9]. However, at the LL level, the resummation is always very simple. Each initial parton receives a factor (see Eq. (10))

$$\exp\left[\int_0^1 dx \frac{x^N - 1}{1-x}\int_{Q^2}^{Q^2(1-x)^2} \frac{dk_\perp^2}{k_\perp^2}\frac{C_i \alpha_s(k_\perp^2)}{\pi} + \ldots\right] \to \exp\left[\frac{\alpha_s C_i}{\pi}\ln^2(\bar{N}) + \ldots\right] , \tag{14}$$

where $C_i = C_F$ for (anti)quarks and $C_i = C_A$ for gluons, and where on the right we have for illustration given the simple large-N result obtained for fixed coupling. An "observed" final-state parton (that is, a parton fragmenting into an observed hadron) has the same factor [10]. For an "unobserved" final-state parton, one finds

$$\exp\left[\int_0^1 dx \frac{x^N - 1}{1-x}\int_{Q^2(1-x)^2}^{Q^2(1-x)} \frac{dk_\perp^2}{k_\perp^2}\frac{C_i \alpha_s(k_\perp^2)}{\pi} + \ldots\right] \to \exp\left[-\frac{\alpha_s C_i}{2\pi}\ln^2(\bar{N}) + \ldots\right] . \tag{15}$$

As one can see, this exponent is negative, corresponding to Sudakov-*suppression*, as expected, since for an "unobserved" final-state parton collinear singularities cancel. Also note that for fixed coupling the exponent in Eq. (15) has half the size of that in (14).

RESUMMATIONS FOR VARIOUS PROCESSES

It is instructive to compare the resummed LL exponents for various processes of interest. These will not be sufficient for quantitative calculations (and in fact in the numerical applications discussed below the NLL terms will be included); however, they will indicate where threshold resummation effects will potentially be most important.

- **Drell-Yan $\mathbf{q\bar{q}} \to \gamma^*$:** As shown above,

$$\exp\left[(C_F + C_F)\,\frac{\alpha_s}{\pi}\ln^2(\bar{N}) + \ldots\right] . \tag{16}$$

  Higgs production via gluon-gluon fusion has the same exponent with $C_F \to C_A$.

- **DIS $\gamma^*\mathbf{q} \to \mathbf{q}$:** Mellin moments are taken in light-cone momentum fractions x.

$$\exp\left[\left(C_F - \frac{C_F}{2}\right)\frac{\alpha_s}{\pi}\ln^2(\bar{N}) + \ldots\right] . \tag{17}$$

- **Prompt-photon production :** for single-inclusive reactions such as prompt-photon production $pp \to \gamma X$, integrated for simplicity over all photon rapidities, a partonic threshold is reached when the center-of-mass energy of the incoming partons is just large enough to produce the photon and the recoiling parton, that is, when $\hat{x}_T \equiv 2p_T^\gamma/\sqrt{\hat{s}} \to 1$. Threshold logarithms occur as $\alpha_s^k \ln^{2k}(1-\hat{x}_T^2)$. Taking Mellin moments in $\hat{x}_T^2$, one obtains the resummed exponents for the two partonic channels:

$$\begin{aligned} q\bar{q} \to \gamma g : \quad & \exp\left[\left(C_F + C_F - \frac{C_A}{2}\right)\frac{\alpha_s}{\pi}\ln^2(\bar{N}) + \ldots\right] , \\ qg \to \gamma q : \quad & \exp\left[\left(C_F + C_A - \frac{C_F}{2}\right)\frac{\alpha_s}{\pi}\ln^2(\bar{N}) + \ldots\right] . \end{aligned} \tag{18}$$

- **Single-inclusive hadron production :** kinematics for the process $pp \to hX$ are similar to the prompt photon case. A major difference is that one final-state parton is "observed" since it fragments into the observed hadron h. One therefore finds extra enhancement for the cross section, due to the factor in Eq. (14). In addition, all $2 \to 2$ QCD subprocesses contribute at Born level, among them $gg \to gg$. Because of the large gluon color charge $C_A = 3$, resummation effects are expected to be large. We only give examples for two subprocesses here; the others follow directly.

$$\begin{aligned} gg \to gg : \quad & \exp\left[\left(C_A + C_A + C_A - \frac{C_A}{2}\right)\frac{\alpha_s}{\pi}\ln^2(\bar{N}) + \ldots\right] , \\ qg \to qg : \quad & \exp\left[\left(C_F + C_A + C_F - \frac{C_A}{2}\right)\frac{\alpha_s}{\pi}\ln^2(\bar{N}) + \ldots\right] . \end{aligned} \tag{19}$$

- **Semi-inclusive DIS $\gamma^*\mathbf{q} \to \mathbf{q}$:** here one has two separate Mellin moments, for the light-cone momentum fractions associated with the initial state and with the fragmentation process. One finds to LL:

$$\exp\left[\frac{C_F\alpha_s}{2\pi}\left(\ln(\bar{N}) + \ln(\bar{M})\right)^2 + \ldots\right] . \tag{20}$$

PHENOMENOLOGICAL EXAMPLES

Among the many processes to which threshold resummation has been applied, we choose three examples.

Higgs production at the LHC. Here the full NNLO fixed-order corrections have been calculated [13], which allow to improve the resummation to NNLL accuracy. Fig. 2 shows results from the study in [12] for the Higgs K-factor. On the left, resummation is done to NLL and compared to the NLO prediction. On the right, one can see the NNLL resummation and the NNLO result. It becomes evident that the NLL resummation yields a fairly good prediction of the NNLO result. NNLL resummation predicts a further, moderate, increase of the cross section over NNLO.

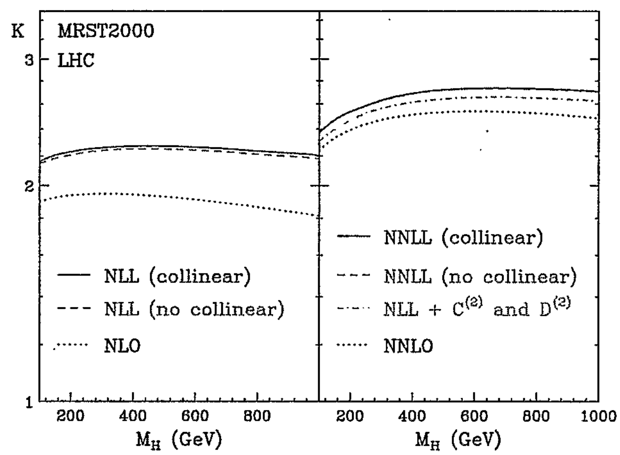

FIGURE 2. Fixed-order (NLO, NNLO) and resummed (NLL, NNLL) predictions for Higgs production at the LHC. Taken from [12].

Semi-inclusive DIS. Here we focus on the "multiplicities" $\frac{d\sigma^{\pi^+ + \pi^-}/dx_B dz}{d\sigma/dx_B}$, where $z = E^{\pi}/\nu$. Fig. 3 (left) shows NLO and resummed results for the kinematic regime relevant to HERMES measurements [14]. Even though we are considering a ratio of cross sections here, resummation effects become important towards larger z and Bjorken-x_B.

Single-inclusive hadron production $pp \to \pi X$. NLO significantly underpredicts [15] data for this process in the fixed-target regime, as may be seen from the lower lines in Fig. (3) (right). On account of Eq. (19), we expect large effects from threshold resummation. Indeed, as the upper lines in the figure show, the agreement with data is substantially improved by NLL resummation, and the scale dependence is strongly reduced. Details will be given elsewhere [17].

ACKNOWLEDGMENTS

I am grateful to C. Bourrely and J. Soffer for their invitation, and to D. de Florian, S. Kretzer, G. Sterman, and M. Stratmann for collaboration on some of the topics presented here. I thank RIKEN, Brookhaven National Laboratory and the U.S. Department of Energy (contract number DE-AC02-98CH10886).

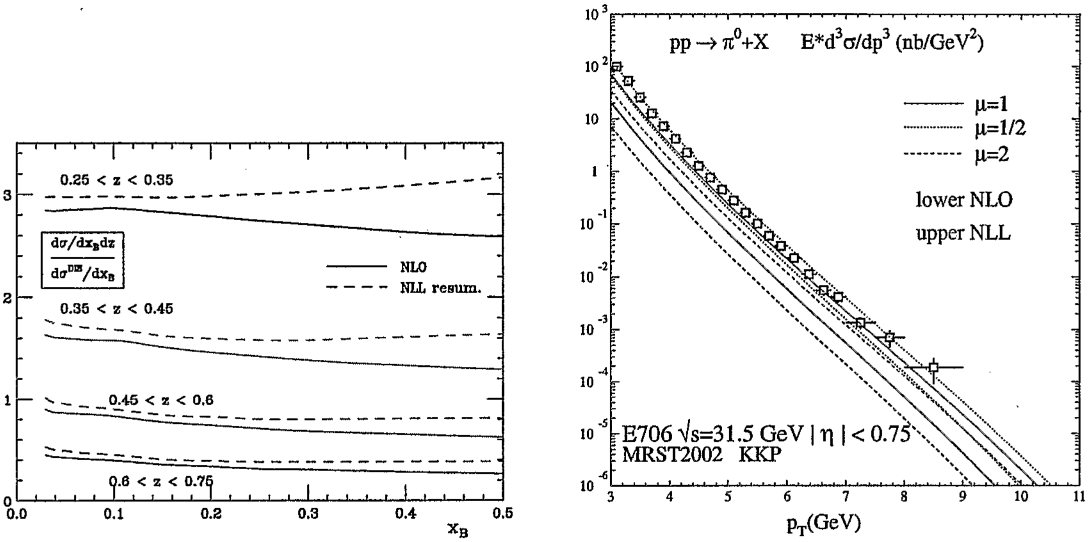

FIGURE 3. Left: Resummation for semi-inclusive DIS in the HERMES kinematic regime. Solid is NLO, dashed NLL resummed. Right: NLO and threshold-resummed results for $pp \to \pi X$ in the fixed-target regime. Data are from E706 [16].

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