BNL-73586-2005-CP

To appear in the Proceedings of the ICFA-HB2004'04 Workshop
Bensheim, Germany, Oct. 18-22, 2004

Adjusted Field Profile for the Chromaticity Cancellation in FFAG Accelerators*

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Abstract. In an earlier report [1] we have reviewed four major rules to design the lattice of Fixed-Field Alternating-Gradient (FFAG) accelerators. One of these rules deals with the search of the *Adjusted Field Profile*, that is the field non-linear distribution along the length and the width of the accelerator magnets, to compensate for the chromatic behavior, and thus to reduce considerably the variation of betatron tunes during acceleration over a large momentum range. The present report defines the method for the search of the *Adjusted Field Profile*.

Keywords: FFAG Accelerators, Magnet Field, Beam Dynamics, Proton Beams, Chromaticity.
PACS: 29.20.-c

INTRODUCTION

The FFAG ring is made of a number of identical periods shown in Fig. 1. According to one of the rules [1], each period is made of a FDF triplet. There is a reference trajectory that according to another rule [1] corresponds to the injection momentum value p_0. The reference trajectory is made of arcs of circle in the sector magnets F and D with constant curvature, and of straight lines in the drift regions S and g. The opposing exit and entrance planes of the neighboring sector magnets are parallel to each other; thus, the entrance and exit angles of the reference trajectory with each magnet are identically zero. Nevertheless trajectories of other momentum values in a *Non-Scaling Lattice* like the one discussed here, have a more complicate behavior and are not arcs of circle since they cross regions with different bending field. Moreover these trajectories always leave or enter magnets at non-vanishing angles.

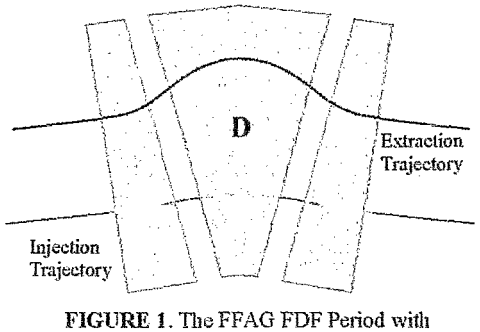

FIGURE 1. The FFAG FDF Period with Inject. (Reference) and Extract. Orbits

EQUATIONS OF MOTION

We adopt a curvilinear coordinate system (x, s, y) where s is the path length along the reference (injection) trajectory, and x, y are respectively the horizontal and vertical displacements from the reference trajectory.

The equations of motion for any particle with momentum p and electric charge q are derived from the (exact) Hamiltonian

$$H = -q A_s / c - (1 + h x) [p^2 - (p_x - q A_x / c)^2 - (p_y - q A_y / c)^2]^{1/2} \quad (1)$$

where c is the light velocity, h = h(s) the curvature of the reference trajectory, and p_x, p_y are the transverse components of the particle momentum canonically conjugated to x, y.

The equations of motion are

* Work performed under the auspices of the U.S. Department of Energy

$$p_x' = -\partial H / \partial x \qquad x' = \partial H / \partial p_x$$
$$p_y' = -\partial H / \partial y \qquad y' = \partial H / \partial p_y \qquad (2)$$

with a prime denoting differentiation with respect to s ($' \equiv d / ds$). Expanding the square root at the r. h. side of Eq. (1) and retaining only up to quadratic order terms give:

$$H = -q A_s / c - (1 + h x) p + (p_x - q A_x / c)^2 / 2 p + (p_y - q A_y / c)^2 / 2 p \qquad (3)$$

after having also dropped higher order terms.

We assume that the magnetic field distribution in the proximity of the reference trajectory is given solely by the longitudinal component A_s of the vector potential, whereas identically $A_x = A_y = 0$. In this case the equations of motion become

$$x'' = (q / pc) \partial A_s / \partial x + h \qquad (4a)$$
$$y'' = (q / pc) \partial A_s / \partial y \qquad (4b)$$

This is correct only as long A_s does not vary with the longitudinal position within the length of one magnet; otherwise a solenoid field component is introduced that should be taken into account.

MAGNETIC FIELD COMPONENTS

With the chosen curvilinear coordinate system, the components B_x and B_y of the magnetic field are

$$(1 + hx) B_y = \partial A_s / \partial x \qquad (5a)$$
$$(1 + hx) B_x = -\partial A_s / \partial y \qquad (5b)$$

and the equations of motion are now

$$x'' = (q / pc) B_y (1 + hx) + h \qquad (6a)$$
$$y'' = -(q / pc) B_x (1 + hx) \qquad (6b)$$

Formally, using complex notation, we can write

$$B = B_y + i B_x \qquad (7)$$
$$z = x + i y \qquad (8)$$

Quite generally

$$B(z) = B_0 + G(z) z \qquad (9)$$

where B_0 is the bending field on the reference trajectory, and $G(z)$. We have

$$B_x = \text{Imaginary}\{B(z)\} \qquad (10a)$$
$$B_y = \text{Real}\{B(z)\} \qquad (10b)$$

CHROMATIC EFFECTS ON THE HORIZONTAL PLANE

Let us consider now the motion on the mid-plane where $y = 0$,

$$x'' = (q / pc)(B_0 + G x)(1 + hx) + h \qquad (11)$$

But the bending condition on the reference orbit is

$$(q B_0 / p_0 c) = -h \qquad (12)$$

Introduce also the relative momentum deviation $p = p_0 (1 + \delta)$ with $\delta = 0$ for the reference (injection) orbit. Neglecting the higher order term $(q / pc) G h x^2$, the horizontal equation of motion (11) reduces to

$$x'' = (q / pc)(h B_0 + G) x + h + q B_0 / pc \cdot$$
$$= -h^2 (1 + G / h B_0) x / (1 + \delta) + h \delta / (1 + \delta) \qquad (13)$$

or, introducing the field index $n = G / h B_0$

$$x'' + h^2 (1 + n) x / (1 + \delta) = h \delta / (1 + \delta) \qquad (14)$$

Like G, also the field index n is in general a function of x (and y). Similarly, in proximity of the mid-plane, the vertical equation of motion is

$$y' - h^2 n y / (1 + \delta) = 0 \qquad (15)$$

Equations (14, 15) can be in general solved numerically. In the special case the field index n is constant, the solution can be found analytically, and the well-known matrix method can be used to estimate the usual lattice functions. Because of the explicit momentum dependence at the denominator of both terms, the solution exhibits strong dispersive and chromatic effects.

FORMULATION OF THE PROBLEM

Let us consider the most general case where the field index is a nonlinear function of both x and s, namely $n = n(x, s)$. We know that, at any location s, for each momentum value δ there is one and only one unique solution $x = x(\delta, s)$, and by *inversion*, topologically, δ can be taken as a function of x and s, namely $\delta = \delta(x, s)$. We pose the following problem: Determine the field distribution, namely $n = n(x, s)$, that compensates (mostly) exactly the momentum dependence of $(1 + \delta)$ at the denominator. We shall write such solution as

$$n(x, s) = n_0 [1 + \delta(x, s)] \quad (16)$$

where n_0 is related to the gradient $G_0 = n_0 h B_0$ on the reference trajectory. If such field distribution is found then the Eq.s (14, 15) of motion reduce simply to

$$x'' + h^2 x/(1+\delta) + h^2 n_0 x = h\delta/(1+\delta) \quad (17a)$$
$$y'' - h^2 n_0 y = 0 \quad (17b)$$

Notice that this approach does not cancel the momentum dependence of the curvature term, nor the constant dispersive term at the r. h. side.

INTEGRATION OF THE EQUATION OF MOTION

The solution of either Eq. (14) or Eq. (17a) can be written as the sum of two contributions

$$x(\delta, s) = x_c(\delta, s) + u(\delta, s) \quad (18)$$

where x_c is a particular solution of the inhomogeneous Eq. (14 or 17a) that has a periodic behavior, and u is the free betatron solution that satisfies the associated homogenous equation. The solution of equation (17a) is straightforward and can be found with the well-known matrix notation. After that, once the closed orbit is found, the solution can be inverted either numerically or symbolically as done by MATHEMATICA [2]. The *inversion* will provide the dependence of the momentum deviation with the radial displacement at any given location s,

$$\delta = \delta(x, s) \quad (19)$$

that we can insert back in the field expression to derive the equation of motion

$$x'' + h^2 \{1 + n_0 [1 + \delta(x, s)]\} x/(1+\delta) = h\delta/(1+\delta) \quad (20)$$

Proceeding backward, this corresponds to the field distribution on the mid-plane given by

$$B(x, s) = B_0 + G_0 [1 + \delta(x, s)] x \quad (21)$$

PROVE OF THE VALIDITY OF THE PROPOSED APPROACH

Equation (20) combined to (21), though nonlinear, can be integrated. The solution is indeed expected to have removed the chromatic behavior caused by the momentum dependence of the magnetic field. The integration of (20) can be attempted in two ways:

(i) Numerical integration by starting with some initial conditions $x_0(\delta)$ and $x_0'(\delta)$ and looking for a solution that closes at the end of the period. To facilitate the search for the closed orbit solution, the initial conditions can be set from the determined solution $x_c(\delta, s)$ of the linear equation (17a).

(ii) By direct substitution of the originally determined solution $x_c(\delta, s)$ of the linear equation (17a) into equation (20), and by recording the relative deviation.

Either of these two methods, as we have successfully demonstrated with the use of MATHEMATICA itself, will have proven the validity of the approach.

CONCLUSIONS

We have applied the method outlined in this report to calculate the *Adjusted Field Profile* for the cancellation of the chromatic behavior in FFAG accelerators adopting *Non-Scaling Lattices*. We have applied the method to three different projects: (1) a 1.5-GeV proton FFAG for Beam Power Upgrade of the Alternating Gradient Synchrotron at Brookhaven National laboratory [4]; (2) a 1-GeV 10-MWatt Proton Driver in an unspecified site [5]; and (3) a 250-MeV proton FFAG as a Medical Facility. We urge the reader to inspect also the other reports in the Reference list to acquire more details about the application and the results of the method described here.

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