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IR optics Measurement with linear coupling's action-angle parameterization *

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Abstract

A parameterization of linear coupling in action-angle coordinates is convenient for analytical calculations and interpretation of turn-by-turn (TBT) beam position monitor (BPM) data. We demonstrate how to use this parameterization to extract the twiss and coupling parameters in interaction regions (IRs), using BPMs on each side of the long IR drift region. The example of TBT BPM analysis was acquired at the Relativistic Heavy Ion Collider (RHIC), using an AC dipole to excite a single eigenmode. Besides the full treatment, a fast estimate of beta*, the beta function at the interaction point (IP), is provided, along with the phase advance between these BPMs. We also calculate and measure the waist of the beta function and the local optics.

BACKGROUND

Action-angle parameterization

For general two-dimensional linearly coupled motion, a single particle's motion is represented as [1]

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \mathbf{P} \begin{pmatrix} \sqrt{2J_I}\cos\Phi_I \\ -\sqrt{2J_I}\sin\Phi_I \\ \sqrt{2J_{II}}\cos\Phi_{II} \\ -\sqrt{2J_{II}}\sin\Phi_{II} \end{pmatrix}, \quad (1)$$

where $J_{I,II}$ are the globally constant actions of the two eigenmodes. $\Phi_{I,II}$ are the eigenmode phases.

$\mathbf{P}$ is the transfer matrix from the laboratory coordinate system to the action-angle coordinate system. It can be analytically calculated from the eigenvectors of the one-turn 4×4 transfer matrix. $\mathbf{P}$ can also be defined in terms of Twiss and coupling parameters given by the Edwards-Teng's parameterization. This definition can be inverted to provide Twiss and coupling parameters in terms of $\mathbf{P}$. For example, for eigen mode I,

$$\begin{cases} \beta_I = \frac{p_{11}}{p_{22}} \\ \alpha_I = -\frac{p_{21}}{p_{22}} \\ \gamma_I = \frac{1+\alpha_I^2}{\beta_I} = \frac{p_{21}^2+p_{22}^2}{p_{11}p_{22}} \end{cases}, \quad (2)$$

$$r = \sqrt{p_{11}\,p_{22}} = \sqrt{p_{33}\,p_{44}}\,, \quad (3)$$

$$\mathbf{C} = r\mathbf{P}_{12}\mathbf{P}_{22}^{-1} = -\frac{1-r^2}{r}\mathbf{P}_{11}\mathbf{P}_{21}^{-1}. \quad (4)$$

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Excitation of one eigenmode

To measure Twiss parameters, a single eigenmode motion is excited [2, 3]. In the RHIC, AC dipoles are used for this purpose [4, 5]. For simplicity, Eq. (1) can be rewritten as

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \mathbf{F} \begin{pmatrix} \cos\Phi_I \\ -\sin\Phi_I \\ \cos\Phi_{II} \\ -\sin\Phi_{II} \end{pmatrix}, \quad (5)$$

$$\mathbf{F} = \begin{pmatrix} p_{11}\sqrt{J_I} & 0 & p_{13}\sqrt{J_{II}} & p_{14}\sqrt{J_{II}} \\ p_{21}\sqrt{J_I} & p_{22}\sqrt{J_I} & p_{23}\sqrt{J_{II}} & p_{24}\sqrt{J_{II}} \\ p_{31}\sqrt{J_I} & p_{32}\sqrt{J_I} & p_{33}\sqrt{J_{II}} & 0 \\ p_{41}\sqrt{J_I} & p_{42}\sqrt{J_I} & p_{43}\sqrt{J_{II}} & p_{44}\sqrt{J_{II}} \end{pmatrix}, \quad (6)$$

where $\mathbf{F}$ includes the action information.

In this article, we assume that eigenmode I is more closely related to the horizontal plane than is eigenmode II, whilst eigenmode II is more associated with the vertical plane than eigenmode I. Thus, if only eigenmode I is activated, the elements in the last two columns of $\mathbf{F}$ are zero, If only eigenmode II is activated, the elements in the first two columns of $\mathbf{F}$ are zero. The matrix $\mathbf{F}$ can be obtained from TBT BPM data at a single ring BPM

TBT data at DX BPMs

In each IR of the RHIC, there are two dual-plane BPMs that are close to the IR's DX separation magnets and face to the interaction point. These BPMs are called DX BPMs. There is no other magnet between the two DX BPMs in the IR if we ignore the effect of the detector magnet. Therefore, the TBT angles (x', y') at the two DX BPMs can be determined,

$$\begin{cases} x' = \frac{x_2 - x_1}{2L} \\ y' = \frac{y_2 - y_1}{2L} \end{cases}. \quad (7)$$

$2L$ is the distance between the two DX BPMs; for RHIC, $2L = 16.652$ m. (x_1, y_1) are the BPM position readings at the up-stream DX BPM, (x_2, y_2) are the BPM position readings at the down-stream DX BPM. These angles are constant across the IR drift.

Propagations of optics parameters in the IR drift

Propagation of the Twiss and coupling parameters through the IR drift is used for determination of the beta waist and to provide a fast estimate of β^*. At the β waist,

where $\alpha_{I,II} = 0$,

$$\mathbf{P}_w = \begin{pmatrix} p_{11} & 0 & p_{13} & p_{14} \\ 0 & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & 0 \\ p_{41} & p_{42} & 0 & p_{44} \end{pmatrix}. \quad (8)$$

For simplicity, we assumed that eigenmode I and eigenmode II's β waists are located at the same point. Even if they are not, the following conclusions still hold. $\mathbf{T}_{1\to 2}$ is the 4 x 4 drift transfer matrix from the waist to a point in the IR drift, say point 2. l is the distance from the waist to point 2. Matrix $\mathbf{G}$ defined by [1]'s Eq. (69) is

$$\mathbf{G} = \mathbf{T}_{1\to 2}\mathbf{P}_w = \begin{pmatrix} p_{11} & l\,p_{22} & p_{13} + l\,p_{23} & p_{14} + l\,p_{24} \\ 0 & p_{22} & p_{23} & p_{24} \\ p_{31} + l\,p_{41} & p_{32} + l\,p_{42} & p_{33} & l\,p_{44} \\ p_{41} & p_{42} & & p_{44} \end{pmatrix}. \quad (9)$$

According to Eqs. (78) and (79) in [1], and considering Eq.(4) at the waist, the Twiss parameters at point 2 can be determined with the above $\mathbf{G}$,

$$\begin{cases} \tilde{\beta}_I = \dfrac{p_{11}^2 + l^2 p_{22}^2}{p_{11}p_{22}} = \beta_{w,I} + \dfrac{l^2}{\beta_{w,I}} \\ \tilde{\alpha}_I = -\dfrac{l p_{22}^2}{p_{11}p_{22}} = -\dfrac{l}{\beta_{w,I}} \end{cases}. \quad (10)$$

The subscript w means the parameters are those at the β waist.

Therefore, knowing the Twiss parameters at point 2, the location of the β waist and its β value can be calculated,

$$\beta_{w,I} = \frac{\tilde{\beta}_I}{1 + \tilde{\alpha}_I^2}, \quad (11)$$

$$\delta l_{w,I} = -\tilde{\alpha}_I \beta_{w,I}. \quad (12)$$

$\delta l_{w,I}$ is the longitudinal locations of eigenmode I's waists with respect to point 2.

According to Eqs. (70) and (72) in [1], the phase advance of the eigenmode I from the β waist to point 2 are given by $\mathbf{G}$,

$$\Delta\Phi_I = \tan^{-1}\left(\frac{l\,p_{22}}{p_{11}}\right) = \tan^{-1}\left(\frac{l}{\beta_{w,I}}\right) \quad (13)$$

Assuming that the beta waist is located in the middle of the two DX BPMs, the total phase advance between them is

$$\Delta\Phi_I = 2\tan^{-1}\left(\frac{L}{\beta_{w,I}}\right), \quad (14)$$

where L is the distance from the IR center to the DX BPM. Eq. (14) can be used for a fast estimate of the β_c at the IR's center,

$$\beta_{c,I} = \frac{L}{\tan\frac{\Delta\Phi_I}{2}} \quad (15)$$

One particular advantage of this estimate is that it relies only on BPM TBT phase advance measurements, which are insensitive to BPM gain and offset errors.

Further, knowing the coupling matrix $\mathbf{C}$ at one point in the IR drift, the coupling matrix $\tilde{\mathbf{C}}_w$ at the β waist in the IR drift can be obtained. According Eq.(86) in [1],

$$\mathbf{C}_w = \mathbf{M}_d \mathbf{C} \mathbf{M}_d^{-1}, \quad (16)$$

$$\mathbf{M}_d = \begin{pmatrix} 1 & \delta l_w \\ 0 & 0 \end{pmatrix}, \quad (17)$$

where δl_w is the longitudinal drift length from point 2 to the waist.

A similar calculation gives the propagation of eigenmode II.

MEASUREMENTS

In the following, an example is given demonstrating how to use the linear coupling's action-angle parameterization to extract the optics parameters in the IR. BPM data used here was acquired with an AC dipole excitation at eigenmode I's fractional tune frequency

To clarify the data processing procedure, we first calculate the Twiss and coupling parameters at the Blue ring IR6 center. The two dual-plane DX BPMs are rbpm.b-g5 and rbpm.b-g6. The Blue beam circulates from rbpm.b-g5 to rbpm.b-56. The TBT position data at the IR center are given by

$$\begin{cases} x_c = \dfrac{x_1 + x_2}{2} \\ y_c = \dfrac{y_1 + y_2}{2} \end{cases}. \quad (18)$$

Matrix F

With the TBT (x_c, x_c', y_c, y_c') data at Blue ring IRG center, $\mathbf{F}$ is calculated with harmonic analysis,

$$\mathbf{F} = \begin{pmatrix} 1.244 \times 10^{-5} & 0. & 0 & 0 \\ -1.009 \times 10^{-6} & 1.273 \times 10^{-5} & 0 & 0 \\ -6.491 \times 10^{-7} & -9.7441 \times 10^{-7} & 0 & 0 \\ -4.509 \times 10^{-7} & -9.293 \times 10^{-7} & 0 & 0 \end{pmatrix}. \quad (19)$$

In this article, the SI unit system is used.

r and $\sqrt{J_I}$

At JRG center,

$$r\sqrt{J_I} = \sqrt{F_{11}F_{22}} = 1.2583 \times 10^{-5}\,[(\mathrm{m.rad})^{-\frac{1}{2}}]. \quad (20)$$

and

$$\frac{||\mathbf{F}_{21}||}{||\mathbf{F}_{11}||} = \frac{||\mathbf{P}_{21}||}{||\mathbf{P}_{11}||} = \frac{1 - r^2}{r^2} = 0.001035. \quad (21)$$

Therefore,

$$r = 0.9995, \quad (22)$$

$$\sqrt{J_I} = 1.259 \times 10^{-5}\,(\mathrm{m.rad})^{-\frac{1}{2}}. \quad (23)$$

When r is close to 1, the optics is well decoupled locally at that point. J_I is a global constant.

Twiss and coupling parameters

According Eq.(2), from Eq. (19), the eigenmode I's Twiss parameters at the IR6 center are

$$\beta_{c,I} = \frac{p_{11}}{p_{22}} = \frac{F_{11}}{F_{22}} = 0.9771\ \mathrm{m} \tag{24}$$

$$\alpha_{c,I} = -\frac{p_{21}}{p_{22}} = -\frac{F_{21}}{F_{22}} - 0.0793 \tag{25}$$

According to Eqs.(4) and (6), the coupling matrix at the IR6 center can be calculated through **F**,

$$\mathbf{C}_c = \begin{pmatrix} 0.0730 & -0.0765 \\ -0.0422 & 0.0584 \end{pmatrix}. \tag{26}$$

β_I waist determination

Knowing the eigenmode I's Twiss parameters at the IR center, according to Eq. (11), the IR6 β waist can be determined,

$$\beta_{w,I} = 0.9710\ \mathbf{m} \tag{27}$$

$$\delta l_{w,I} = -0.0770\ \mathrm{m}. \tag{28}$$

A negative $\delta l_{w,I}$ signifies the β_I waist is sited upstream with respect to the IR center.

According to Eq. (16), the coupling matrix $\mathbf{C}_{w,I}$ at the eigenmode I's β waist can be calculated from that at the center,

$$\mathbf{C}_{w,I} = \begin{pmatrix} 0.0763 & -0.0752 \\ -0.0422 & 0.0551 \end{pmatrix}. \tag{29}$$

Summary of IR optics parameters

Table 1 lists all IR centers' $\beta_{c,I}$s from the phase advances between the two adjacent **DX** BPMs, according to Eq. (15) . The phase advances between the relevant two DX BPMs also are given. The unit of the phase advances are given in unit of degree.

Table 2 lists the Twiss and coupling parameters at the IR centers from the action-angle parameterization. Since the rbpm.b-g2 BPM vertical data is aberrant, the coupling parameters are not available at IR2 center.

Comparing the $\beta_{c,I}$s at the IR centers from Tables 1 and 2, a big difference in the IR12 is apparent. Looking into the turn-by-turn BPM data **of** the IR12 DX BPMs, I found that the downstream BPM rbpm.b-g12 gave poor horizontal data.

Table 3 lists the locations of $\beta_{w,I}$ waist and its $\beta_{w,I}$, coupling parameters there. $\delta l_{w,I}$ is the longitudinal offset with respect to the IR center.

Table 4 lists the coupling parameter r and eigenmode I's action $\sqrt{J_I}$ in IRs. $\sqrt{J_I}$ is a global constant. The average of $\sqrt{J_I}$ in the IRs in Table 4 is

$$\overline{\sqrt{J_I}} = 1.18 \times 10^{-5}\ (\mathrm{m.rad})^{-\frac{1}{2}}, \tag{30}$$

IRs	IR6	IR8	IR10	IR12	IR2	IR4
$\Delta\Phi_I$	166.695	166.474	147.064	141.788	150.637	119.395
$\beta_{c,I}$	0.9711	0.9874	2.4613	2.8843	2.1815	4.8660

Table 2: Twiss and coupling parameters at the IR centers from the linear coupling's action-angle parametrization

IRs	$\beta_{c,I}$	$\alpha_{c,I}$	c_{11}	c_{12}	c_{21}	c_{22}
IR6	0.9771	0.0793	0.0730	-0.0765	-0.0422	0.0584
IR8	0.9885	0.0335	0.2549	-1.7128	-0.0455	1.0318
IR10	2.4678	0.0536	-0.0192	-0.4945	0.0091	-0.0213
IR12	3.2762	-0.3929	0.0484	-1.2168	0.0337	-0.2773
IR2	2.5006	0.3972				
IR4	4.8918	-0.0844	-0.0854	0.03829	-0.0344	-0.2557

Table 3: Optics parameters at the β_I waist in IRs.

IRs	$\delta l_{w,I}$	$\beta_{w,I}$	c_{11}	c_{12}	c_{21}	c_{22}
IR6	-0.0770	0.9710	0.0763	-0.0752	-0.0422	0.0551
IR8	-0.0331	0.9874	0.2812	-1.9039	-0.0455	1.0318
IR10	-0.1320	2.4607	-0.0204	-0.4955	0.0091	-0.0213
IR4	0.4100	4.8571	-0.1007	-0.0261	-0.0348	-0.2445

Table 4: r and $\sqrt{J_I}$ in IRs.

IRs	r	$\sqrt{J_I}$ [m.mrad]$^{-\frac{1}{2}}$
IR6	0.999	1.26×10^{-5}
IR8	0.911	1.17×10^{-5}
IR10	0.997	1.11×10^{-5}
IR4	0.988	1.09×10^{-5}

which can be used to roughly extract the $r\sqrt{\beta_I}$ at the horizontal BPMs in the arcs,

$$r\sqrt{\beta_I} = \frac{F_{11}}{\overline{\sqrt{J_I}}}. \tag{31}$$

and $\frac{c_{12}}{r}$ at the dual-plane BPMs,

$$\frac{c_{12}}{r} = \frac{F_{32}}{F_{11}}. \tag{32}$$

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