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Non-Scaling Fixed Field Gradient Optimization

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Abstract. Optimization of the non-scaling FFAG lattice for the specific application of the muon acceleration with respect to the minimum orbit offsets, minimum path length and smallest circumference is described. The short muon lifetime requires fast acceleration. The acceleration is in this work assumed to be with super-conducting cavities. This sets up a condition of acceleration at the top of the sinusoidal RF wave.

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INTRODUCTION

A field of the fixed field alternating gradient (FFAG) accelerator has been recently revived. After the Proof of Principle (POP) proton FFAG accelerator was built at the KEK, JAPAN [1], additional designs and machines followed. The original scaling FFAG designs, in early fifties [2], and electron demonstration ring built by MURA group in Wisconsin, USA had been abandoned until resent revival, especially in Japan. The revival comes due to many reasons although advancements in the magnet design and accelerator physics computing tools might be the most important ones. The first part of this report is mostly oriented on muon acceleration, one of the possible applications of an FFAG accelerator, while in the second part a medical application of the non-scaling FFAG for the proton therapy is presented.

Muons have very short lifetime. The fast muon decay requires fast acceleration. The tunes and momentum compaction during acceleration do not change in the scaling while in the non-scaling FFAG design they vary between half-integer and zero in each cell. This is a major difference between the two designs. Due to tunes variations resonances will be crossed. This should not represent a serious problem for muons acceleration as the number of turns is ~8-15 the accelerators is considered. The beam orbits of the scaling FFAG during acceleration are parallel to each other and the size of curvature radius follows the energy. A significant opposite bend (~1/3 of the major bend) is a necessary part of the non-scaling FFAG lattice. The present scaling FFAG designs reduced significantly the maximum orbit offsets with respect to the original designs, but they still require very large magnets. The cyclotrons are more compact, but with enormous size of the magnet with respect to the scaling FFAG. In the non-scaling FFAG, due to much smaller orbit offsets during acceleration, size of the magnets is significantly reduced. In addition, for the same value of the bending field, the non-scaling FFAG has a smaller circumference, due to a smaller size of the opposite bend. In this report the non-scaling FFAG examples are made for muon acceleration in a range between 10 GeV and 20 GeV. The acceleration is assumed by the super-conducting cavities with a frequency of ~200 MHz. Due to difficulty of frequency change in the super-conducting RF the muon acceleration has to be performed at the top of the sinusoidal function. This limitation requires a small difference in the path length for muon beam within the required energy range. A non-linear behavior of the RF wave at the top of the sine function will introduce distortions of the buckets. The smaller the path length differences for different energies, the smaller are distortions of the buckets.

The momentum compaction shows a linear dependence on momentum in the non-scaling, while it has a constant value at the scaling FFAG lattice. As explained later, this implies that the path length

difference has a parabolic dependence on momentum. The momentum compaction is set to be zero around the central momentum of the acceleration range.

The second part of the report shows a possibility of accelerating protons without necessity of accelerating at the top of the RF wave but using the linear region and regular cavities with possibility of frequency variation. The resonance crossing becomes now very important if the time spent at the resonance is long enough.

BASIC PRINCIPLE OF THE NON-SCALING FFAG

The orbit offset, Δx, in the radial direction of an accelerator, has a simple relation to the dispersion function:

$$\Delta x = D_x \frac{\Delta p}{p}, \tag{1}$$

The momentum variation is defined as $\delta=\Delta p/p=(p - p_o)/p_o$. The smaller the dispersion function is the smaller the orbit offset. A motion of particles with a momentum offset with respect to the reference momentum p_o is described, to the lowest order in δ, by the second order inhomogeneous differential equation:

$$D'' + K_x(x)D \cong \frac{1}{\rho}. \tag{2}$$

When there is no bending the dispersion behaves as a harmonic oscillator. The dispersion function in a lattice is best described in the "normalized dispersion space" [3] where the coordinates are defined as:

$$\xi = \frac{D}{\sqrt{\beta}} \quad and \quad \chi = D'\sqrt{\beta} + \frac{\alpha D}{\sqrt{\beta}}. \tag{3}$$

The vector χ represents in the thin lens approximation a dipole "kick" where the bending angle of the dipole is equal to a change of the slope of dispersion function $\theta=D_x'$. The normalized dispersion vector rotates around the origin at the places where there is no bending and the angle of that rotation represents the betatron phase. Minimizing the normalized dispersion vector reduces the amplitude of the dispersion function and the effect of dispersion on the particle motion. The "normalized dispersion" within a single non-scaling FFAG cell is presented in Fig. 1. The square of the two normalized dispersion vectors represents the dispersion 'action' H is described as:

$$H(D,D') \equiv \left(\frac{D}{\sqrt{\beta}}\right)^2 + \left(D'\sqrt{\beta} + \frac{\alpha D}{\sqrt{\beta}}\right)^2 = \zeta^2 + \chi^2. \tag{4}$$

In the normalized dispersion space the square rot of the 'action' vector The minimum emittance lattice for the electron storage rings has been previously very well analyzed [4]. A definition of the average value of the dispersion action $<H>$ and conditions for its minimum are presented by equation (5). The result is the lattice with smallest emittance for the electron storage ring:

$$<H> = \frac{1}{L}\int_0^L H(s)\,ds, \qquad \frac{d}{dD_o}<H> = 0, \quad \frac{d}{d\beta_o}<H> = 0, \tag{5}$$

It follows that the smallest average value of the dispersion action H is obtained if the small values of the horizontal betatron function and dispersion are placed in the middle of the bending elements.

The variation of the path length is defined as:

$$\Delta C \equiv \oint_C \sqrt{\left(1+\frac{x}{\rho}\right)^2 + x'^2 + y'^2}\; ds \cong \left[\oint_C \frac{D_x(s)}{\rho} ds\right]\delta \tag{6}$$

Lowering the dispersion function will reduce the path length.

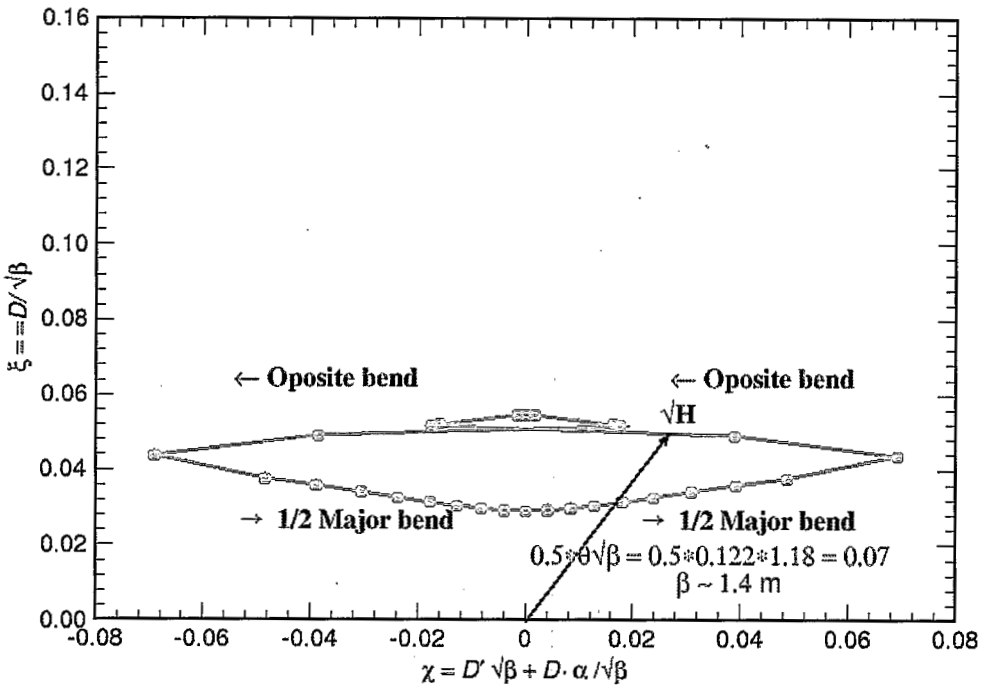

FIGURE 1. The normalized dispersion function within a non-scaling FFAG cell for muon acceleration.

The conditions for the minimum of <*H*> by the equation (5) had been applied [5] for the non-scaling FFAG lattice constructed by the principle of the minimum emittance electron storage ring. The non-scaling FFAG lattice was used for the muon acceleration in 1999. This first "triplet" design contained: regular bends in the middle of the cell, focusing and defocusing quadrupoles and sextupoles. The lattice solution analyzed in the present report is a result of the significant improvements of the design above. Number of different elements and non-linear sextupole magnets were removed which resulted at the end with only two kinds of magnets. Continuous improvements during the last few years of this design had been reported during many FFAG workshops.

OPTIMIZATION OF THE PARAMETERS

The optimization of the non-scaling FFAG muon acceleration is based on few specific requirements:

1. Super-conducting RF cavities for acceleration. Acceleration by the re circulating linac was found to be very expensive due to high RF cavity cost. Larger number of passages through the

same cavities would reduce the cost. It is assumed that the RF is made of super-conducting cavities and that the fast frequency variation is not available. The acceleration is assumed to be at the top of the RF sine wave. This sets a requirement for the smallest possible difference of the path length for different energies, to allow larger number of turns and smallest distortions of buckets due to non-linear wave function.

2. A requirement of two meters long drift space for a single super-conducting cavity.
3. The path length at the lowest should be equal to the path length at the highest energy.
4. Small orbit offsets will reduce magnet cost due to smaller aperture size.
5. The betatron tunes during acceleration should not cross half or full integer within a single cell.

THE BASIC CELL

The non-scaling FFAG lattice consists of only two types of combined function magnets: in the middle of the cell is a defocusing combined function magnet producing the major bending of the particles. It is surrounded by two focusing combined function opposite bending magnets with a small distance between. The two meters large drift is placed between the two focusing combined function magnets. The betatron functions at the reference momentum and elements are presented in figure 1.

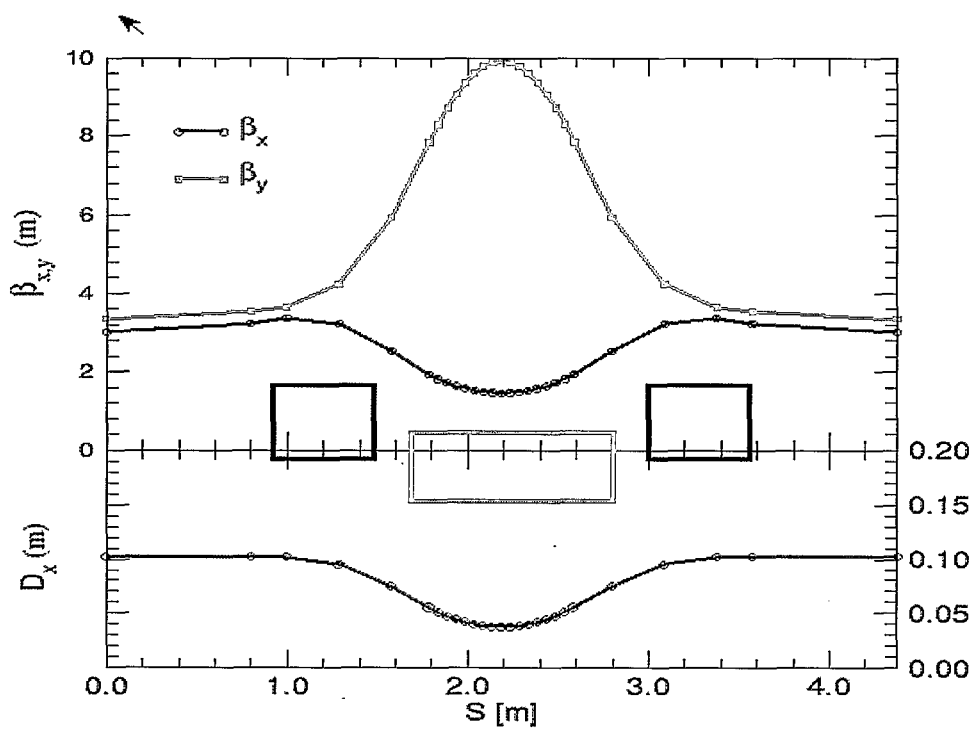

FIGURE 2. The horizontal and vertical betatron functions in the basic cell at the reference momentum. The dispersion function is shown in the lower part of the picture.

The minimum of both: the horizontal betatron and dispersion functions are designed to be in the middle of the major bending combined function element. The vertical aperture is defined by the maximum value of the vertical betatron function.

CONDITIONS FOR THE MINIMUM OF <*H*>

The central bending element in the design is a defocusing combined function dipole. The conditions for the minimum of the average value of the <*H*> are shown by the equation (5). The dispersion action ***H*** for the defocusing combined function central magnet is:

$$H(\phi)=\gamma_o D_o^2+\frac{2\alpha_o D_o}{\rho}\sinh\phi-\frac{2\gamma_o D_o}{\rho K}(\cosh\phi-1)+\frac{\beta_o}{\rho^2 K^{3/2}}\sinh\phi(\cosh\phi-1)^2 \quad (7)$$

The D_o' and β_o are dispersion and the horizontal betatron function at the center of the magnet. The average value of the <*H*> function through the half of the combined function dipole is:

$$<H>=\frac{1}{\rho^2 K}\left(\beta_o+\frac{L^2}{\beta_o q^2}\right)\left(\frac{\sinh q}{q}-1\right)-\frac{4\theta L}{\beta_o q^3}\left(D_o+\frac{\theta L}{q^2}\right)\sinh\frac{q}{2}-\frac{\beta_o\theta^2}{q^2}+\frac{1}{\beta_o}\left(D_o+\frac{\theta L}{q^2}\right) \quad (8)$$

The parameter q is defined [4] as $q=L\,K^{1/2}$. Conditions for the smallest <*H*> are:

$$D_o=\frac{2L\theta}{q^3}\sinh\frac{q}{2}.-\frac{L}{q^2}\ and\ \beta_o=\frac{L}{q}\sqrt{\frac{\frac{4}{q^2}\sinh^2\frac{q}{2}-\frac{1}{2}\left(1+\frac{\sinh q}{q}\right)}{1-\frac{1}{2}\left(1+\frac{\sinh q}{q}\right)}} \quad (9)$$

An example of the non-scaling FFAG was produced by applying the conditions provided by the equation (9). This result is presented in TABLE 1.

Table 1. The path lengths, orbit offsets, tune range, D_{xmax} and θ_2/θ_1 dependence on D_{xmin} and β_{xmin} at the center of the bend. First row corresponds to minimum <H>

D_{xmin} (m)	β_{xmin} (m)	H_{max} (cm)	δC_Eo (cm)	δC_Em (cm)	x_{off_min} (mm)	x_{off_max} (mm)	$E_{\cong v_min}$ (GeV)	D_{x_max} (m)	θ_2/θ_1
0.011	**0.180**	**0.148**	**7.11**	**7.15**	**-17.9**	**26.5**	**14.9**	**0.069**	**0.086**
0.012	**0.351**	**0.180**	**7.58**	**7.54**	**-18.7**	**27.8**	**14.5**	**0.071**	**0.095**
0.014	**0.504**	**0.234**	**8.07**	**8.04**	**-19.6**	**29.3**	**13.9**	**0.075**	**0.105**
0.019	**0.791**	**0.386**	**9.36**	**9.61**	**-22.0**	**33.68**	**12.6**	**0.085**	**0.130**
0.024	**1.100**	**0.611**	**11.66**	**11.77**	**-25.7**	**40.7**	**11.1**	**0.103**	**0.158**
0.038	**1.368**	**0.869**	**13.81**	**13.95**	**-29.5**	**47.8**	**10.0**	**0.121**	**0.181**

The path length dependence on the value of the betatron function and dispersion at the middle of the bending element shows a linear dependence. The smallest path length and the smallest orbit offsets correspond to the minimum of the average value of the dispersion action <*H*>. The further the initial conditions for the dispersion and the horizontal betatron function are from the required conditions for the minimum of <*H*> the larger the path length and orbit offsets variation on energy are. Results from the Table 1 show very clearly that the smallest path length differences correspond to the lattice with the smallest value of the average function <*H*> (columns δC_Eo and δC_Emax). The column eighth shows the smallest value of energy with the stable betatron tunes. Conditions for the smallest value of the average

H function have to be compromised to obtain stable tune conditions for the required range of acceleration. The smallest orbit offsets in optimization produce at the same time the smallest difference in the path lengths for the required momentum range, as shown in columns $\delta C_{_Eo}$, $\delta C_{_Emax}$, x_{off_min} and x_{off_max}. The 9th column shows that values of the maximum of dispersion function D_{x_max} at the central energy follows dependence of the momentum offsets and difference in the path length.

A comparison of the *'FODO'* and *'DOUBLET'* lattices, constructed by the same magnets, with respect to the lattice described above shows clear advantage of the design presented above. This is according to already presented calculations and comparisons [4].

EQUALIZING THE PARABOLIC PATH LENGTH

The momentum compaction shows close to linear dependence on momentum during acceleration. The momentum compaction is defined as:

$$\alpha_c \equiv \frac{1}{C}\oint \frac{D}{\rho} ds \tag{10}$$

The momentum compaction, as shown in Fig. 3, shows linear dependence on momentum.

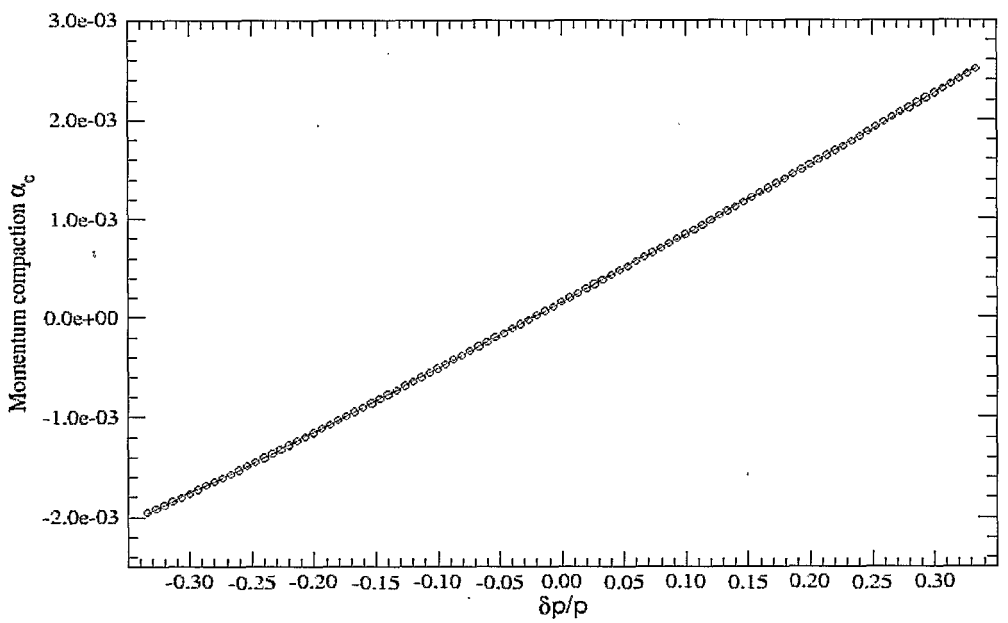

FIGURE 3. The Momentum compaction dependence on momentum.

The analytical solution for the parabolic function dependence of path length on momentum was shown in the previous FFAG workshop [5]. The path length dependence is presented in Fig. 4.

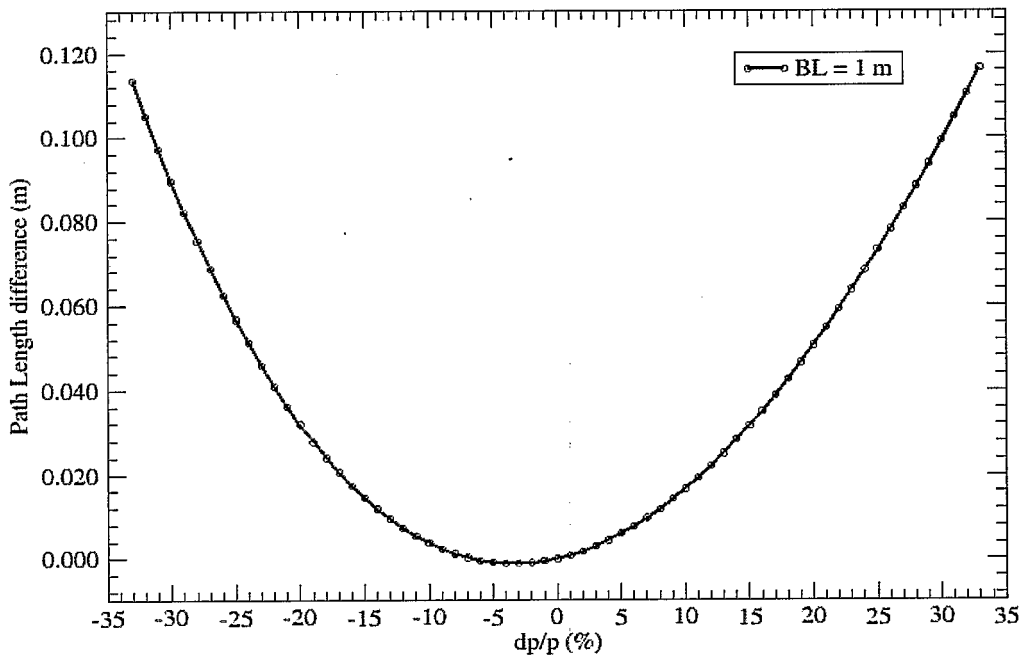

FIGURE 4. Path length difference during acceleration with almost fully optimized values at the ends of parabola.

From the equation (6) the path length could be rewritten as $\Delta C = C\ \alpha_c\ \delta$. As the momentum compaction shows linear dependence on δ as: $\alpha_c \sim k_1\ \delta$, it follows that the path length has a parabolic dependence as: $\Delta C \sim k_2\ \delta^2$.

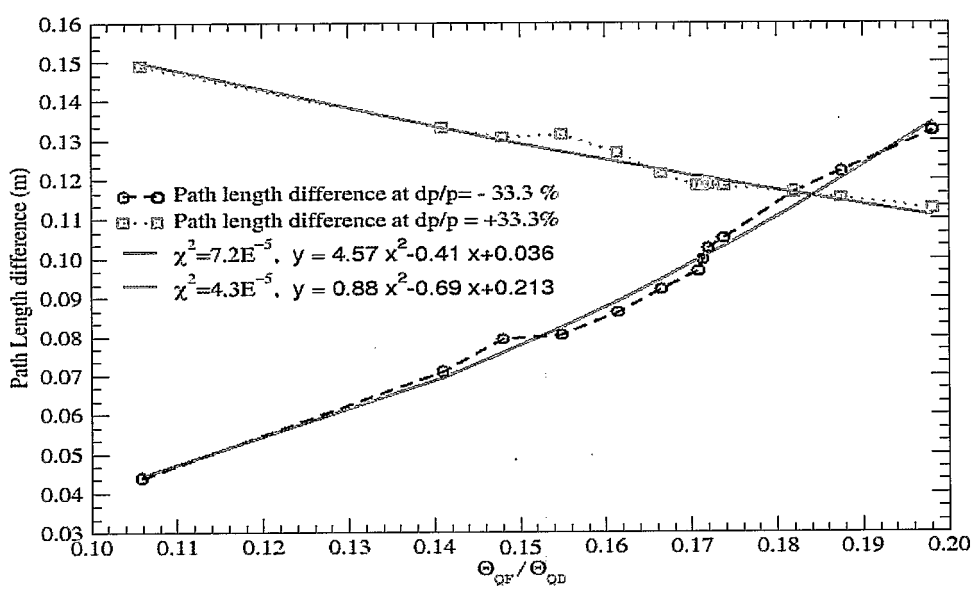

FIGURE 5. Equalizing the path lengths on both sides of the parabola varying the size of the opposite bend.

A bending radius has a negative sign for the focusing combined function magnet. Even when a stable solution for tunes is found for the whole energy range, there might exist a difference in the path length at the lowest with respect to the highest energy. An adjustment for this difference is possible by varying the strength of the opposite bend with respect to the main bend. The ninth column in the TABLE 1 shows a relationship between the opposite of the major bend. This is one of the variables used to make the path length (defined by the equation (6)) at the lowest equal to the one at the highest energy, as shown in Fig. 5.

ORBITS AT DIFFERENT MOMENTA

Particle motion in the non-scaling FFAG during acceleration start at the beginning with smaller radius of curvature and continue with large one as the energy increases. Orbits are not parallel to each other especially through the central bending part, as shown in Fig. 6. At the center of a drift, reserved for the "cavity", the orbits are parallel and all have zero slope. This is a condition set up by the design: adjust the gradients and drift lengths to get zero slopes of the dispersion and betatron functions. The orbits presented in Fig. 6 are obtained from the results by the Polymorphic Tracking Code -PTC. The orbits on both sides show motion parallel to the orbit corresponding to the reference momentum δ=0. Orbits are not parallel anymore especially at the defocusing combined function dipole, presented at the center of the Fig. 6. This is due to the very strong gradients present at both magnets and due to the edge effects.

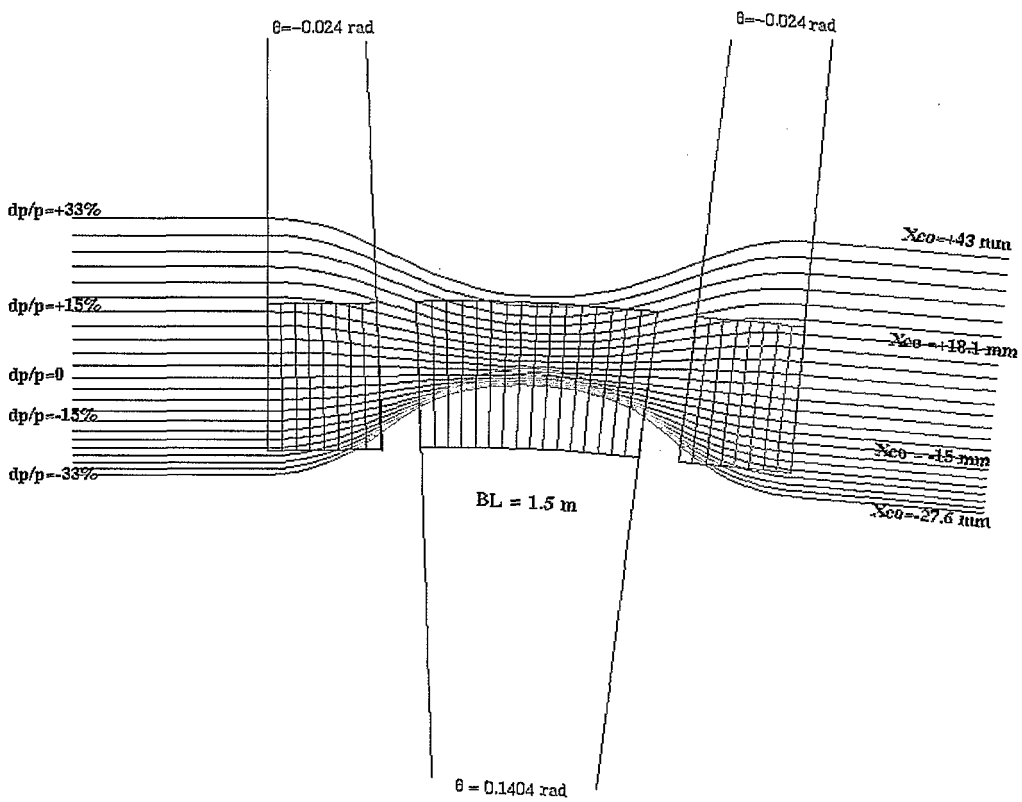

FIGURE 6. Orbits in one of the non-scaling FFAG examples.

CONCLUSIONS

This is a report about optimization with respect to the orbit offsets and path length differences during acceleration of the non-scaling FFAG lattice made for muon acceleration from 10 to 20 GeV. The path lengths need to be of the order of ten centimeters to enable acceleration with 10-16 turns [6]. The normalized dispersion or the dispersion 'action' function H are used to show that the minimum of the average value of $<H>$ produced the smallest orbit offsets and path lengths. It should be noted that for this specific application the super conducting RF cavities are used for acceleration at the top of the sine wave. This is why the orbit offsets minimization is very critical to preserve small distortions of the buckets. The parabolic path length dependence on momentum requires, for optimum conditions, to have equal the largest offsets values. These path length offsets occur at the minimum and maximum values of momentum. This optimization was obtained by variation of the size of the opposite bend. More details about design procedure and non-scaling FFAG lattice are presented elsewhere [6].

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