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Static Quark Anti-Quark Free and Internal Energy in 2-Flavor QCD and Bound States in the QGP

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Static quark anti-quark free and internal energy in 2-flavor QCD and bound states in the QGP

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We present results on heavy quark free energies in 2-flavour QCD. The temperature dependence of the interaction between static quark anti-quark pairs will be analyzed in terms of temperature dependent screening radii, which give a first estimate on the medium modification of (heavy quark) bound states in the quark gluon plasma. Comparing those radii to the (zero temperature) mean squared charge radii of charmonium states indicates that the J/ψ may survive the phase transition as a bound state, while χ_c and ψ' are expected to show significant thermal modifications at temperatures close to the transition. Furthermore we will analyze the relation between heavy quark free energies, entropy contributions and internal energy and discuss their relation to potential models used to analyze the melting of heavy quark bound states above the deconfinement temperature. Results of different groups and various potential models for bound states in the deconfined phase of QCD are compared.

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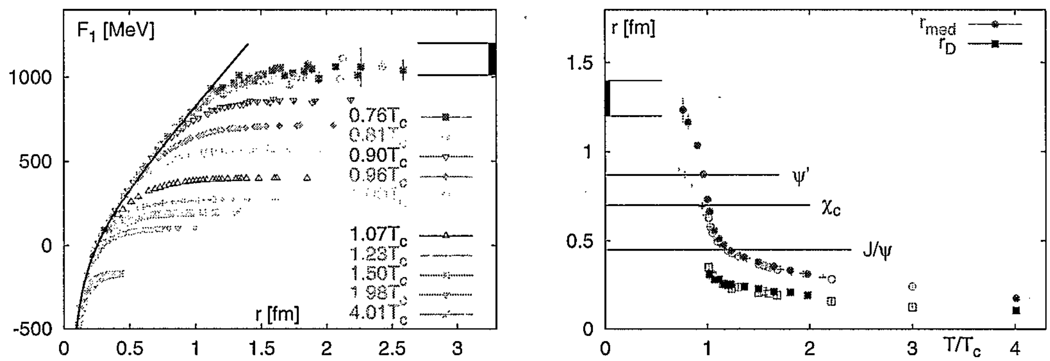

Figure 1: (left) The colour singlet quark anti-quark free energies, $F_1(r,T)$, at several temperatures as function of distance in physical units. Shown are results from lattice studies of 2-flavour QCD (from [1]). The solid line represents the $T=0$ heavy quark potential, $V(r)$. The dashed error band corresponds to the string breaking energy at zero temperature, $V(r_{breaking}) \simeq 1000-1200$ MeV, based on the estimate of the string breaking distance, $r_{breaking} \simeq 1.2-1.4$ fm [2]. (right) The screening radius estimated from the inverse Debye mass, $r_D \equiv 1/m_D$ (N_f=0: open squares, N_f=2 filled squares), and the scale r_{med} (N_f=0: open circles, N_f=2: filled circles, N_f=3: crosses) defined in (2.1) as function of T/T_c. The horizontal lines give the mean squared charge radii of some charmonium states, J/ψ, χ_c and $\psi\prime$ (see also [3, 4]) and the band at the left frame shows the distance at which string breaking is expected in 2-flavor QCD at $T=0$ and quark mass $m_\pi/m_\rho \simeq 0.7$ [2].

1. Introduction

A simple Ansatz to study the possible existence of bound states above the critical temperature is to use effective temperature dependent potentials that model the medium modifications of strong interactions in a quark gluon plasma. To what extend a suitable effective potential at finite temperature can be defined by quark antiquark free or internal energies and furthermore how realistic such (simple) descriptions of bound states in a deconfined medium are is still an open question.
By comparing the screening radii obtained from lattice results on singlet free energies in 2-flavour QCD to the mean squared charge radii we obtain first estimates on the temperatures where charmonium bound states may be influenced by medium effects. In more realistic potential model calculations effective temperature dependent potentials that model medium effects are used in the Schrödinger equation. We present the heavy quark free energies and their contributions, i.e. entropy and internal energy, and discuss the different results obtained using those contributions in potential models.

2. Screening radii and medium modifications

In Fig. 1 (left) we show results for the heavy quark anti-quark free energies in 2-flavour QCD [1]. While in the limit of short distances $F_1(r,T)$ shows no or only little medium effects, i.e. $F_1(r\to 0) \simeq V(r)$, at large distances the free energies approach temperature dependent constant values, $F_\infty(T) \equiv F_1(r\to\infty,T)$. To characterise distances at which medium effects become important we introduce a screening radius, r_{med}, defined by the distance at which the value of the zero temperature

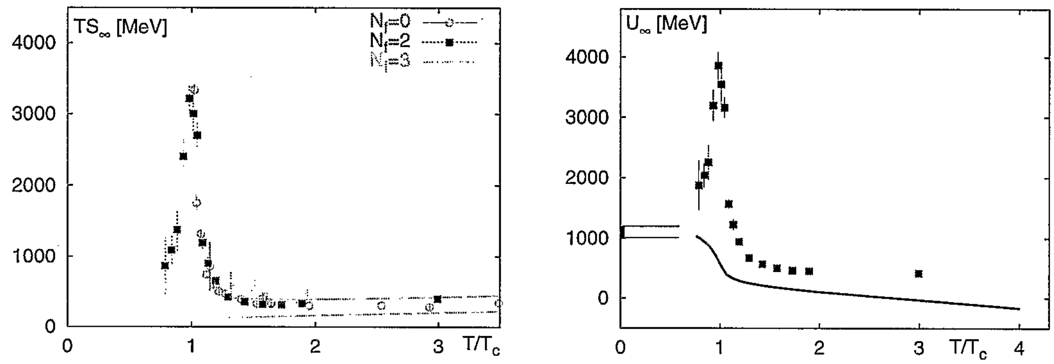

Figure 2: (left) The contribution $TS_\infty(T)$ appearing in the free energy, $F_\infty(T) = U_\infty(T) - TS_\infty(T)$, calculated in 2-flavour QCD as function of T/T_c. We compare our results from 2-flavour QCD to the leading order perturbative result (see [3]). We also show results from 3-flavour QCD for $T \gtrsim 1.1T_c$ [5] and quenched QCD [6]. (right) The internal energy $U_\infty(T)$ versus T/T_c calculated in 2-flavour QCD. The corresponding free energy, $F_\infty(T)$, calculated in 2-flavour QCD is also shown as solid line. We again indicate in this figure the energy at which string breaking is expected to take place at $T = 0$, $V(r_{breaking}) \simeq 1000 - 1200$ MeV (dashed lines), using $r_{breaking} = 1.2 - 1.4$ fm [2].

potential reaches the asymptotic value of the free energies,

$$V(r_{med}) = F_\infty(T). \tag{2.1}$$

The results for r_{med} are compared to the mean squared charge radii (at zero temperature) of typical charmonium states in Fig. 1 (right). The temperature at which these radii are equivalent can give a rough estimate for the onset of thermal effects on the charmonium states. Based on this (simple) picture, J/ψ may survive as a bound state up to temperatures slightly above the deconfinement temperatures, while χ_c and ψ' are expected to show significant thermal modifications already close to the transition.

3. Asymptotic entropy and internal energy

Before analysing the r-dependence of the different contributions to the free energy, we discuss their behaviour at infinite separation. The entropy contribution, $TS_\infty(T)$, and internal energy, $U_\infty(T)$ can be calculated from the asymptotic value of the free energies, $F_\infty(T)$ (see Fig. 2 in [3]) using the thermodynamic relations

$$U_\infty(T) = -T^2 \frac{\partial F_\infty(T)/T}{\partial T} \quad , \quad S_\infty(T) = -\frac{\partial F_\infty}{\partial T}. \tag{3.1}$$

The results for two flavour QCD compared to quenched and 3-flavour results are shown in Fig. 2. The results for small temperatures indicate that $TS_\infty(T)$ vanishes in the zero temperature limit while at high temperatures we find a tendency for an increase of $TS_\infty(T)$ with increasing temperature. This behaviour is also as expected from leading order perturbative contribution,

$$S_\infty(T) \simeq \frac{4}{3} \frac{m_D(T)}{T} \alpha(T) + 4 \frac{m_D(T)}{T} \alpha(T) \frac{\beta(g)}{g(T)}, \tag{3.2}$$

$$\text{i.e.} \quad TS_\infty(T) \simeq \mathcal{O}(g^3 T). \tag{3.3}$$

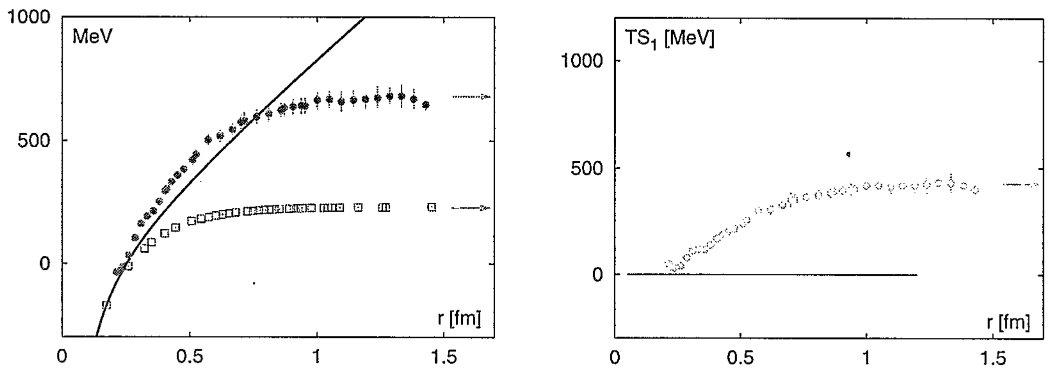

Figure 3: (left) The singlet internal energy, $U_1(r,T)$ (filled circles), calculated from renormalised singlet free energy, $F_1(r,T)$ (open squares), at fixed $T \simeq 1.3T_c$ in 2-flavour lattice QCD compared to $V(r)$ (line) [1, 7]. (right) The corresponding colour singlet quark anti-quark entropy, $TS_1(r,T \simeq 1.3T_c)$, as function of distance calculated from renormalised free energies. The arrows in both figures point at the temperature dependent values of the free and internal energy and entropy at asymptotic large distances, *i.e.* $F_\infty(T) \equiv \lim_{r\to\infty} F_1(r,T)$, $U_\infty(T) \equiv \lim_{r\to\infty} U_1(r,T)$ and $TS_\infty(T) \equiv T\lim_{r\to\infty} S_1(r,T)$.

In contrast to the expected behaviour of the entropy contributions in the limit of small and large temperatures, in the vicinity of the phase transition significant differences are evident. Even at moderate temperatures above T_c the entropy is to a large extend dominated by non-perturbative effects reaching large values around the critical temperature. A similar behaviour is also visible for the internal energies (Fig. 2 (right)) around T_c. A comparison to the free energies (solid line) shows that $U_\infty(T) > F_\infty(T)$ at all temperatures analysed here. We stress here that even at high temperatures the difference between both show that entropy contributions play an important role. At small temperatures, the comparison to the expected value at zero temperature indicates that this value seems to be approached already at rather large temperatures. This is in agreement with the observation that only small temperature effects are visible in the free energies in Fig. 1 (left) below 0.8 T_c.

4. r-dependent entropies and internal energies

We now turn to the discussion of the r-dependent entropy and internal energy contributions calculated by the appropriate relations as (3.1) and (3.1). In Fig. 3 (left) the internal energy at a temperature $T \simeq 1.3$ T_c is compared to the free energy. Both energies approach the zero temperature potential at small distances. Therefore the free energy at small separations is dominated by energy contributions. The results of $TS_1(r,T)$ in Fig. 3 (right) indicate that at intermediate and large distances entropy contributions play an important role leading to an enhancement of the internal energy compared to the free energy. In Fig. 4 we summarise results for $U_1(r,T)$ at various temperatures below (left) and above (right) T_c. The results for $U_1(r,T)$ are larger than $F_1(r,T)$ for all temperatures and show a much steeper slope compared to $F_1(r,T)$. Therefore potential models using $U_1(r,T)$ as an effective T-dependent potential will lead to stronger bound states compared to models using free energies as effective potential.

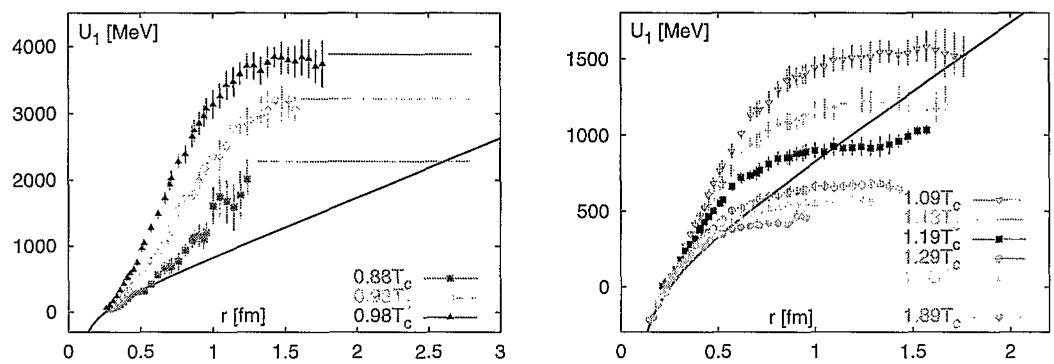

Figure 4: The colour singlet quark anti-quark internal energies, $U_1(r,T)$, at several temperatures below (left) and above (right) the phase transition obtained in 2-flavour lattice QCD. In (left) we also show as horizontal lines the asymptotic values which are approached at large distances and indicate the flattening of $U_1(r,T)$. The solid lines represent the $T=0$ heavy quark potential, $V(r)$ [1, 7].

5. Bound states in potential models

Various potential model calculations in terms of solving the Schrödinger equation using either free energies [8], internal energies [9] or a linear combination of both [10] were recently performed leading to different results for the temperature dependence of binding energies of heavy quark bound states in the quark gluon plasma. Some quarkonium dissociation temperatures obtained by assuming vanishing binding energy are summarised in Tab. 1. Although the results differ, with the smallest dissociation temperatures obtained using $F_1(r,T)$ and the highest using $U_1(r,T)$, they indicate that at least J/ψ may survive the deconfinement transition as a bound state, while the situation for the higher states is still not obvious.

state	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ_b'	Υ''
$E_s^i[GeV]$	0.64	0.20	0.005	1.10	0.67	0.54	0.31	0.20
T_d/T_c	1.1	0.74	0.1-0.2	2.31	1.13	1.1	0.83	0.75
T_d/T_c	~ 1.42	~ 1.05	unbound	~ 3.3	~ 1.22	~ 1.18	-	-
T_d/T_c	1.78-1.92	1.14-1.15	1.11-1.12	$\gtrsim$4.4	1.60-1.65	1.4-1.5	~ 1.2	~ 1.2

Table 1: Estimated dissociation temperatures T_d in units of T_c obtained from potential models using free energies [8] (green), a linear combination of F_1 and U_1 [10] (blue) and internal energies [9] (red) as effective T-dependent potentials.

6. Conclusions

We have compared the screening radii of heavy quark anti-quark pairs in the quark gluon plasma phase to the (zero temperature) mean squared charge radii of charmonium states and found indications that the J/ψ may survive the phase transition as a bound state, while χ_c and ψ' are expected to show significant thermal modifications at temperatures close to the transition.
Beyond this simple approximation of the medium modifications of charmonium bound states above

T_c, we have also analysed the different contributions to the heavy quark free energy and calculated the entropy contributions and internal energy of a static quark anti-quark pair. A comparison of different potential models, using either free energies, internal energies or a combination of both, shows that charmonium states may survive the phase transition and exist as bound states in the quark gluon plasma. This is in (qualitative) agreement with spectral function analyses in quenched QCD [11, 12, 13] and first results obtained in 2-flavour QCD [14]. As the systematic uncertainties in all those analyses are still quite large, up to now it is not clear to which temperatures bound states may exist and which potential models give a realistic description of charmonium or bottomonium systems at high temperatures. Clearly more detailed calculations beyond these simple potential models are needed to clarify the possibility of bound states in the quark gluon plasma as well as their medium modifications and dissociation properties in a deconfined medium. A comparison of the various potential models to (direct) lattice calculations of charmonium correlation and spectral functions may clarify the question if and which potential models lead to the correct description of bound state phenomena within their applicability.

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