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Laurence Littenberg

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Physics Department
Electronic Detector Group
Brookhaven National Laboratory
P.O. Box 5000
Upton, NY 11973-5000
www.bnl.gov

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L. LITTENBERG

Brookhaven National Laboratory, Upton, N.Y. 11973

1. – Introduction

In recent years the study of rare kaon decays has had three primary motivations:

1. The search for physics beyond the Standard Model (BSM). Virtually all attempts to improve on the Standard Model (SM) predict some degree of lepton flavor violation (LFV). Decays such as $K_L \to \mu^\pm e^\mp$ and $K^+ \to \pi^+\mu^+e^-$ have excellent experimental signatures and can consequently be pursued to sensitivities that correspond to extremely high energy scales in models where there can be a tree-level contribution and the only suppression is that of the mass of the exchanged field. There are also theories that predict new particles created in kaon decay or the violation of symmetries other than lepton flavor.

2. The potential of decays that are allowed but that are very suppressed in the SM. In a number of kaon decays, the leading component is a G.I.M.-suppressed[1] one-loop process that is quite sensitive to fundamental SM parameters such as V_{td}. Because of the severe suppression, these decays are also potentially very sensitive to BSM physics.

3. Long-distance-dominated decays can test theoretical techniques such as chiral perturbation theory (χPT) that attempt to account for the low-energy behavior of QCD. Also, information from some of these decays is required to extract fundamental information from certain of the one-loop processes.

This field remains active, as indicated by Table I, that lists the rare decays for which results have emerged in the last few years, as well as those that are under analysis. It is clear that one must be quite selective in a review of moderate length.

TABLE I. – *Rare K decay modes under recent or on-going study.*

$K^+ \to \pi^+\nu\bar{\nu}$	$K_L \to \pi^0\nu\bar{\nu}$	$K_L \to \pi^0\mu^+\mu^-$	$K_L \to \pi^0e^+e^-$
$K^+ \to \pi^+\mu^+\mu^-$	$K^+ \to \pi^+e^+e^-$	$K_L \to \mu^+\mu^-$	$K_L \to e^+e^-$
$K^+ \to \pi^+e^+e^-\gamma$	$K^+ \to \pi^+\pi^0\nu\bar{\nu}$	$K_L \to e^\pm e^\mp\mu^\pm\mu^\mp$	$K^+ \to \pi^+\pi^0\gamma$
$K_L \to \pi^+\pi^-\gamma$	$K_L \to \pi^+\pi^-e^+e^-$	$K^+ \to \pi^+\pi^0e^+e^-$	$K^+ \to \pi^0\mu^+\nu\gamma$
$K_L \to \pi^0\gamma\gamma$	$K^+ \to \pi^+\gamma\gamma$	$K^+ \to \mu^+\nu\gamma$	$K^+ \to e^+\nu e^+e^-$
$K^+ \to \mu^+\nu e^+e^-$	$K^+ \to e^+\nu\mu^+\mu^-$	$K_L \to e^+e^-\gamma$	$K_L \to \mu^+\mu^-\gamma$
$K_L \to e^+e^-\gamma\gamma$	$K_L \to \mu^+\mu^-\gamma\gamma$	$K_L \to e^+e^-e^+e^-$	$K_L \to \pi^0e^+e^-\gamma$
$K_S \to \pi^0e^+e^-$	$K_S \to \pi^0\mu^+\mu^-$	$K_S \to \gamma\gamma$	$K_S \to \pi^0\gamma\gamma$
$K^+ \to \pi^+\mu^+e^-$	$K_L \to \pi^0\mu^\pm e^\mp$	$K_L \to \mu^\pm e^\mp$	$K^+ \to \pi^-\mu^+e^+$
$K^+ \to \pi^-e^+e^+$	$K^+ \to \pi^-\mu^+\mu^+$	$K^+ \to \pi^+X^0$	$K_L \to e^\pm e^\pm\mu^\mp\mu^\mp$
$K^+ \to \pi^+\gamma$	$K_L \to \pi^0\pi^0e^+e^-$	$K_L \to \pi^0\pi^0\nu\bar{\nu}$	$K_L \to \pi^+\pi^-\pi^0e^+e^-$

2. – Beyond the Standard Model

The poster children for BSM probes in kaon decay are LFV processes like $K_L \to \mu e$ and $K^+ \to \pi^+\mu^+e^-$. In principle, these can proceed through neutrino mixing, but the known neutrino mixing parameters limit the rate through this mechanism to a completely negligible level [2]. Thus the observation of LFV in kaon decay would require a new mechanism. Fig.1 shows $K_L \to \mu e$ mediated by a hypothetical horizontal gauge boson X, compared with the kinematically very similar process $K^+ \to \mu^+\nu$ mediated by a W boson.

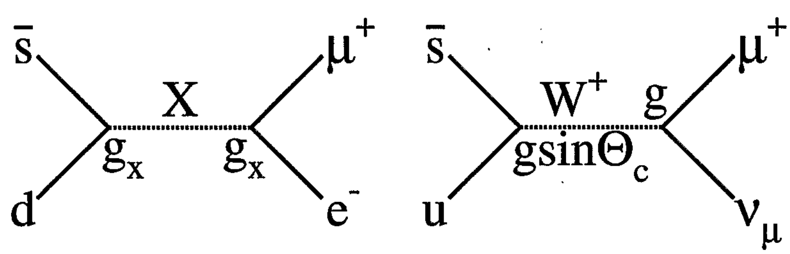

Fig. 1. – Horizontal gauge boson mediating $K_L \to \mu e$, compared with W mediating $K^+ \to \mu^+\nu$.

Using measured values for M_W, the K_L and K^+ decay rates and $B(K^+ \to \mu^+\nu)$, and assuming a $V - A$ form for the new interaction, one can show [3]:

$$M_X \approx 220\,TeV/c^2 \left[\frac{g_X}{g}\right]^{1/4} \left[\frac{10^{-12}}{B(K_L \to \mu e)}\right]^{1/4} \qquad (1)$$

so that truly formidable scales can be probed if $g_X \sim g$ (see also [4]). In addition to this generic picture, there are specific models, such as extended technicolor in which LFV at observable levels in kaon decays is quite natural [5].

There were a number of K decay experiments primarily dedicated to lepton flavor violation at the Brookhaven AGS during the 1990's. These advanced the sensitivity to such processes by many orders of magnitude. In addition, several "by-product" results on LFV and other BSM topics have emerged from the other kaon decay experiments of this period. Rare kaon decay experiments often also yield results on π^0 decays, since these can readily be tagged, *e.g.* via $K^+ \to \pi^+\pi^0$ or $K_L \to \pi^+\pi^-\pi^0$. Table II summarizes the status of work on BSM probes in kaon decay. The relative reach of these processes is best assessed by comparing the partial rates rather than the branching ratios.

TABLE II. – *Current 90% CL limits on K decay modes violating the SM. The violation codes are "LF" for lepton flavor, "LN" for lepton number, "G" for generation number, [3], "H" for helicity, "N" requires new particle*

Process	Violates	90% CL BR Limit	Γ Limit (sec⁻1)	Experiment	Reference
$K_L \to \mu e$	LF	4.7×10^{-12}	9.1×10^{-5}	AGS-871	[6]
$K^+ \to \pi^+\mu^+e^-$	LF	1.2×10^{-11}	9.7×10^{-4}	AGS-865	[7]
$K^+ \to \pi^+\mu^-e^+$	LF, G	5.2×10^{-10}	4.2×10^{-2}	AGS-865	[8]
$K_L \to \pi^0\mu e$	LF	3.37×10^{-10}	6.4×10^{-3}	KTeV	[9]
$K^+ \to \pi^-e^+e^+$	LN, G	6.4×10^{-10}	5.2×10^{-2}	AGS-865	[8]
$K^+ \to \pi^-\mu^+\mu^+$	LN, G	3.0×10^{-9}	2.4×10^{-1}	AGS-865	[8]
$K^+ \to \pi^-\mu^+e^+$	LF, LN, G	5.0×10^{-10}	4.0×10^{-2}	AGS-865	[8]
$K_L \to \mu^\pm\mu^\pm e^\mp e^\mp$	LF, LN, G	4.12×10^{-11}	8.0×10^{-4}	KTeV	[10]
$K^+ \to \pi^+f^0$	N	5.9×10^{-11}	4.8×10^{-3}	AGS-787	[11]
$K^+ \to \pi^+\gamma$	H	2.3×10^{-9}	1.9×10^{-1}	AGS-949	[12]

This table makes it clear that any deviation from the SM must be highly suppressed. In a real sense the kaon LFV probes have become the victims of their own success. By and large the particular theories they were designed to probe have been forced to retreat to the point where meaningful tests in the kaon system would be very difficult (although there are exceptions[13]). The currently more popular theoretical approaches tend to predict a rather small degree of LFV in kaon decays. For example, although these decays do provide the most stringent limits on strangeness-changing R-violating couplings, the minimal supersymmetric extension of the Standard Model (MSSM) predicts LFV in kaon decay at levels far beyond the current experimental state of the art [14]. Decays such as $K^+ \to \pi^-\mu^+\mu^+$, that violate lepton number as well as flavor, are possible in the MSSM, but even more drastically suppressed([1]).

There have been some recent exceptions to the waning of theoretical interest in kaon

([1]) There are models involving sterile neutrinos in which such processes are conceivably observable[15].

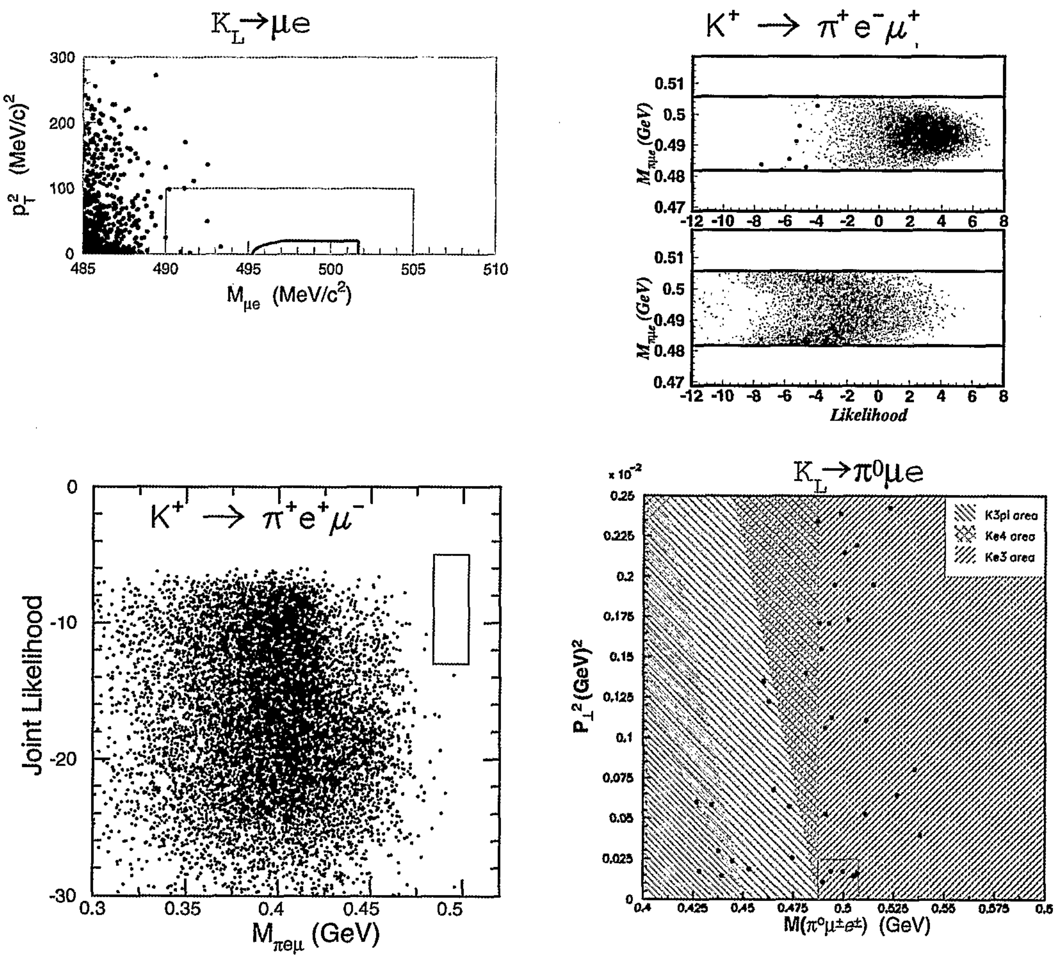

Fig. 2. – Signal planes showing candidates for LFV kaon decays from recent experiments. **Top left:** p_T^2 vs $M_{\mu e}$ from Ref. [6], **Top right:** $M_{\pi\mu e}$ vs Log likelihood from Ref. [7] (upper plot shows signal Monte Carlo, lower plot shows background Monte Carlo, observed events are shown as larger dots on both), **Bottom left:** Joint likelihood vs $M_{\pi\mu e}$ from Ref. [8], and **Bottom right:** $P_\perp^2$ vs $M_{\pi\mu e}$ from Ref. [9].

LFV, including models with extra dimensions [16], but even in an improved motivational climate, there would be barriers to rapid future progress. Although K fluxes significantly greater than those used in the last round of LFV experiments are currently available, the rejection of background is a significant challenge. Fig.2 shows the signal planes of four of the most sensitive LFV searches. It is clear that background either is already a problem or soon would be if the statistical sensitivity of these searches were increased. Thus new techniques will need to be developed to push such searches significantly further.

At the moment no new kaon experiments focussed on LFV are being planned. Interest in probing LFV has largely migrated to the muon sector.

One exception to the poor prospects for dedicated BSM searches in kaon decay is the search for T-violating (out-of-plane) μ^+ polarization in $K^+ \to \pi^0\mu^+\nu$ ([2]). There's a Letter of Intent[18] to continue the work of the current experiment, KEK E246, at the J-PARC facility currently under construction. The proponents seek to make an order of magnitude advance in precision on the measurement of the polarization. Since this is an interference effect the advance is roughly equivalent to that of two orders of magnitude in BR sensitivity. This measurement is quite sensitive to BSM physics, particularly multi-Higgs models including certain varieties of supersymmetry[19, 20, 21, 22, 23].

3. – One-loop decays

In the kaon sector experimental effort is now focussed on GIM-suppressed decays in which loops containing weak bosons and heavy quarks dominate or at least contribute measurably to the SM rate. These processes include $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \mu^+\mu^-$, $K_L \to \pi^0\nu\bar{\nu}$, $K_L \to \pi^0 e^+e^-$ and $K_L \to \pi^0\mu^+\mu^-$. In the latter three cases the one-loop contributions violate CP. In $K_L \to \pi^0\nu\bar{\nu}$ this contribution completely dominates the decay[24]. Diagrams for such loops are shown in Fig. 3. Since the GIM-mechanism

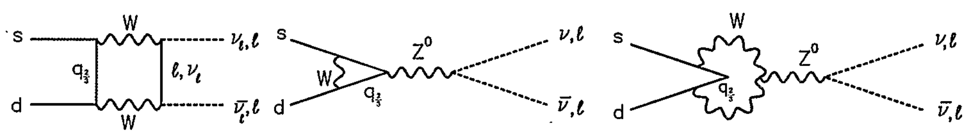

Fig. 3. – Loop contributions to K decays

enhances the contribution of heavy quarks, in the SM these decays are sensitive to the product of couplings $V_{ts}^* V_{td} \equiv \lambda_t$. Although it is perhaps most natural to write the branching ratio for these decays in terms of the real and imaginary parts of λ_t[25, 26], for comparison with results from the B system it is convenient to express them in terms of the Wolfenstein parameters, A, ρ, and η. Fig. 4 shows the relation of these rare kaon decays to the unitarity triangle. The dashed triangle is the usual one derived from $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$ $(\equiv \lambda_u + \lambda_c + \lambda_t)$, the solid one indicates the information available from rare kaon decays. The apex, (ρ, η), can be determined from either triangle, and disagreement between the K and B determinations implies physics beyond the SM. In Fig. 4 the branching ratio closest to each side of the solid triangle can be used to determine the length of that side. The arrows leading outward from those branching ratios point to processes that need to be studied either because they could potentially constitute backgrounds, or because knowledge of them is required to relate the one-loop

([2]) $K\mu3$ is not rare (BR $\sim 3\%$), but the amplitude being probed in this type of experiment is quite small. If the decay went entirely through this amplitude, at the current limit, $|\mathrm{Im}(\xi)| < 0.016$ at 90% C.L.[17] ($\xi \equiv f_-/f_+$ where $f_\pm$ are the form factors of $K\mu3$), this would correspond to a branching ratio of $\sim 10^{-7}$.

process branching ratios to the lengths of the triangle sides. $K_L \to \mu^+\mu^-$, which can in principle determine the bottom of the triangle (ρ), is the decay for which the experimental data is the best but for which the theory is most problematical. $K_L \to \pi^0\nu\bar{\nu}$, which determines the height of the triangle (η) is the cleanest theoretically, but for this mode experiment falls far short of the SM-predicted level. $K^+ \to \pi^+\nu\bar{\nu}$, which determines the hypotenuse, is nearly as clean as $K_L \to \pi^0\nu\bar{\nu}$ and has been observed (albeit with only three events). Prospects for the latter are probably the best of all since it is both clean and already within reach experimentally.

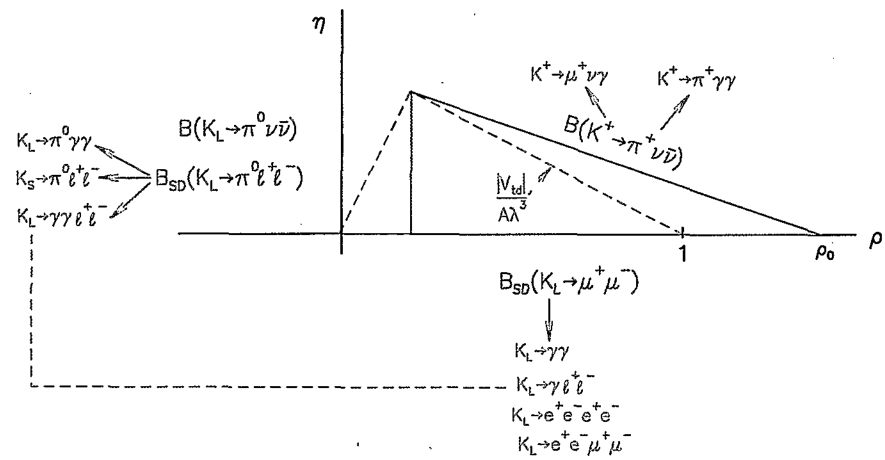

Fig. 4. – K decays and the unitarity plane. The usual unitarity triangle is dashed. The triangle that can be constructed from rare K decays is solid. See text for further details.

3·1. $K^+ \to \pi^+\nu\bar{\nu}$. – From the point of view of theory $K^+ \to \pi^+\nu\bar{\nu}$ is remarkably clean. The often problematical hadronic matrix element can be calculated to $\sim 2\%$ via an isospin transformation from that of $K^+ \to \pi^0 e^+\nu_e$[27]. The hard GIM suppression minimizes QCD corrections and the long-distance contributions to this decay are very small. A recent discussion of the latter with references to previous work can be found in [28].

$K^+ \to \pi^+\nu\bar{\nu}$ is very sensitive to V_{td} - it is actually directly sensitive to the quantity λ_t as can be seen in Eq. 2 [29]:

$$B(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+\left[\left(\mathrm{Im}\lambda_t X(x_t)\right)^2 + \left(\lambda^4 \mathrm{Re}\lambda_c P_c(X) + \mathrm{Re}\lambda_t X(x_t)\right)^2\right] \tag{2}$$

where $\lambda \equiv sin\theta_{Cabibbo}$, $x_t \equiv (m_t/m_W)^2$, $X(x_t)$ and P_c contain the top and charm contributions respectively and will be discussed below, and κ_+ is given by

$$\kappa_+ \equiv r_{K^+}\frac{3\alpha^2 B(K^+\to\pi^0e^+\nu_e)}{2\pi^2 sin^4\theta_W \lambda^2} = (1.48 \pm 0.02) \times 10^{-4}\left[\frac{0.227}{\lambda}\right]^2 \tag{3}$$

The factor 3 in Eq. 3 comes from the summation over neutrino flavors. The appropriate values for α and $sin\theta_W$ to use in this instance are discussed in [30]. The isospin-breaking parameter $r_{K^+} = 0.901$[27].

The Inami-Lim function[31], $X(x_t)$ characterizing the GIM suppression of the top contribution is also given in [30]. For the current measured value of m_t ([3]), $X(x_t) = 1.529 \pm 0.042$ and it is proportional $m_t^{1.15}$ in this region. The QCD correction to this function is $\leq 1\%$.

(4) $$\lambda^4 P_c(X) = \tfrac{2}{3} X_{NL}^e + \tfrac{1}{3} X_{NL}^\tau$$

gives the charm contribution, where the functions X_{NL}^l are those arising from the NLO calculation [32, 29]. The QCD correction leads to a $\sim$ 30% reduction of the charm Inami-Lim function. A recent assessment of the uncertainties in this calculation can be found in Buras *et al.*[30]. They give $\lambda^4 P_c(X) = 9.8 \times 10^{-4}$ with an 18% error where the leading contributors are 12% for the uncertainty in μ_c, the scale parameter, and 8% for the uncertainty in the charm quark mass, $m_c(m_c)$. Very recently, preliminary results of a NNLO calculation were reported[33], in which the former error was reduced to 5% and the total to ~10%. The resultant branching ratio prediction can then be given as:

(5) $$B(K^+ \to \pi^+ \nu\bar{\nu}) = (8.0 \pm 0.4_{m_c} \pm 0.15_{\mu_c} \pm 0.9_{CKM} \pm 0.6_{m_t}) \times 10^{-11}$$

where the last two components, the parametric ones due to uncertainties in the CKM matrix elements and in m_t, will naturally be reduced as data in these areas is improved. This would make a high precision measurement of $B(K^+ \to \pi^+ \nu\bar{\nu})$ very interesting from a BSM point of view.

As mentioned above the branching ratio can also be written in terms of the Wolfenstein variables [34] and one finds it is proportional to $A^2 X^2(x_t) \frac{1}{\sigma} \left[(\sigma\bar{\eta})^2 + (\rho_0 - \bar{\rho})^2\right]$ where $\sigma \equiv (1 - \lambda/2)^{-2}$ and $\rho_0 \equiv 1 + \frac{P_c(X)}{A^2 X(x_t)}$. To a good approximation the amplitude is proportional to the hypotenuse of the solid triangle in Fig. 4. This is equal to the vector sum of the line proportional to $V_{td}/A\lambda^3$ and that from (1,0) to the point marked ρ_0. The length $\rho_0 - 1$ along the real axis is proportional to the amplitude for the charm contribution to $K^+ \to \pi^+ \nu\bar{\nu}$. More precisely, $B(K^+ \to \pi^+ \nu\bar{\nu})$ determines an ellipse of small eccentricity in the $(\bar{\rho}, \bar{\eta})$ plane centered at $(\rho_0, 0)$ with axes r_0 and r_0/σ where

(6) $$r_0 \equiv \frac{1}{A^2 X(x_t)} \sqrt{\frac{\sigma B(K^+ \to \pi^+ \nu\bar{\nu})}{5.3 \times 10^{-11}}}.$$

$K^+ \to \pi^+ \nu\bar{\nu}$ has been observed in the E787/949 series of experiments at the BNL AGS. Like all previous experiments E787/949 used stopped K^+. This gives direct access to the K^+ center of mass, and is conducive to hermetic vetoing. The cylindrically

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([3]) For this purpose the correct parameter is $m_t(m_t)$, which is about 10 GeV lower than the pole mass.

symmetric detector, mounted inside a 1 Tesla solenoid [35], is shown in Fig. 5. It sat at the end of a $\sim$ 700 MeV/c positive beamline with two stages of electrostatic separators. This beamline provided an 80% pure beam of $> 10^7$ K^+ per AGS cycle[36]. Beam particles traversed a Cerenkov counter that identified K^+ and π^+ and were tracked by two stations of MWPC's. They were then slowed via dE/dx in a BeO degrader followed by a Cu/scintillator shower counter. Approximately one quarter of the K^+ survived to exit the shower counter and traverse a hodoscope before entering a scintillating fiber stopping target. A hodoscope surrounding the stopping target was used demand a single charged particle leaving the target after a delay of $\sim 0.12\,\tau_K$. The emergent particle was tracked in a low-mass cylindrical drift chamber with momentum resolution $\sim$1%. Additional trigger counters required the particle to exit the chamber radially outward and enter a cylindrical array of scintillators and straw chambers, the "Range Stack" (RS), in which it was required to stop in order for the event to be considered a $K^+ \to \pi^+\nu\bar{\nu}$ candidate. Each RS scintillation counter was read out by phototubes on both ends, allowing a determination of the position of hits along the beam direction via differential timing and pulse height. This facility, along with the pattern of counter pulse heights and the coordinates measured in two layers of straw chambers, determined the range of the stopping particles to $\leq$3%. The detector design minimized "dead" material so that the kinetic energy could also be measured to $\sim$ 3%. Comparison of range, energy and momentum is a powerful discriminator of low energy particle identity. In addition, transient recorder readout of the RS photomultipliers allowed the $\pi^+ \to \mu^+ \to e^+$ decay chain to be used to identify π^+'s. The combination of kinematic and life-cycle techniques can distinguish pions from muons with a misidentification rate of $\mathcal{O}(10^{-8})$. Surrounding the RS was a cylindrical lead-scintillator veto counter array and adjacent to the ends of the drift chamber were endcap photon veto arrays of undoped CsI modules[37]. There were also a number of auxillary veto counters near the beamline as well as a veto in the beamline downstream of the detector.

Monte Carlo estimation of backgrounds was in general not reliable since it was necessary to estimate rejection factors as high as 10^{11} for decays occurring in the stopping target. Instead, methods to measure the background from the data itself were developed, using the primary data stream as well as data from special triggers taken simultaneously. The principles adhered to included:

- To eliminate bias, the signal acceptance region is kept hidden while cuts are developed.
- Cuts are developed on 1/3 of the data (evenly distributed throughout the run) but residual background levels are measured only on the remaining 2/3 after the cuts are frozen.
- "Bifurcated" background calculation. Background sources are identified *a priori.* Two independent high-rejection cuts are developed for each background. Each cut is reversed in turn as the other is studied. After optimization, the combined effect of the cuts can then be calculated as a product.

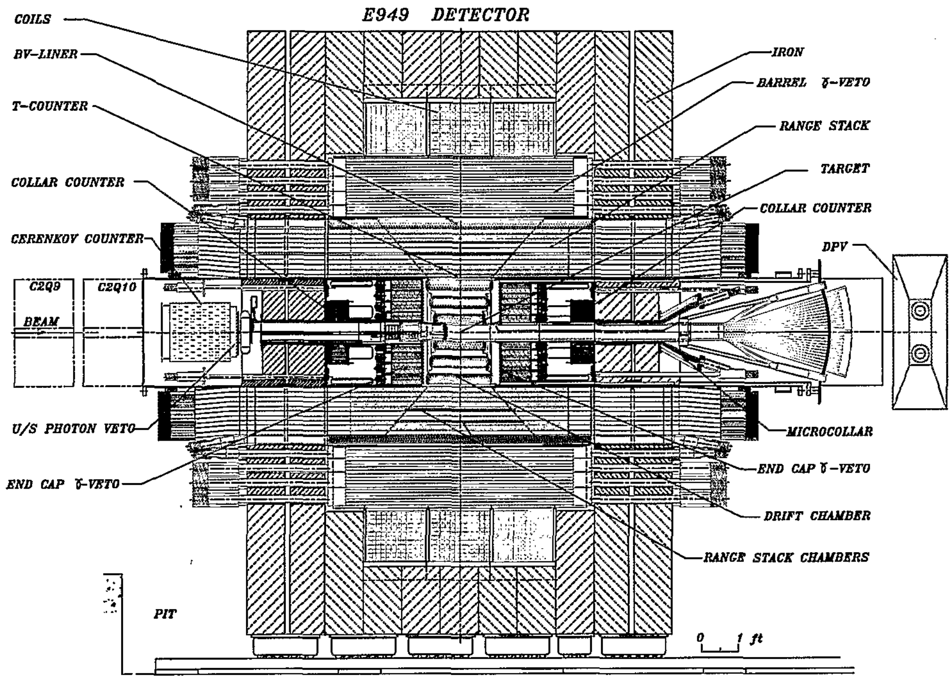

Fig. 5. – E949 detector.

- Cuts are loosened in a controlled fashion to uncover correlations. If any are found, the correlated cuts are applied before the bifurcation instead of after it, and the background determination process repeated.
- Signal and background functions are constructed and used in likelihood analysis
- Background calculations are verified though comparison with data near the signal region.

In this way backgrounds can be reliably calculated at the 10^{-3} to 10^{-2} event level.

All factors in the acceptance besides those of solid angle, trigger and momentum interval were determined from data.

Using these techniques three $K^+ \to \pi^+\nu\bar{\nu}$ events were observed, two by E787 [38, 11] and one by E949 [39]. The combined result was a branching ratio $B(K^+ \to \pi^+\nu\bar{\nu})$= $(1.47^{+1.30}_{-0.89})\times10^{-10}$. This is about twice as high as the prediction of Eq. 5, but statistically compatible with it.

The total background to the two E787 events was measured to be 0.15 of an event and that of the E949 event 0.3 of an event. Thus E787/949 has developed methods to reduce the backgrounds to a level sufficient to make a precise measurement of $K^+ \to \pi^+\nu\bar{\nu}$.

It is possible to use the E787/949 results to extract information on λ_t:

$$0.24 < \quad |\lambda_t|/10^{-3} \quad < 1.1 \qquad (68\%\ CL) \tag{7}$$

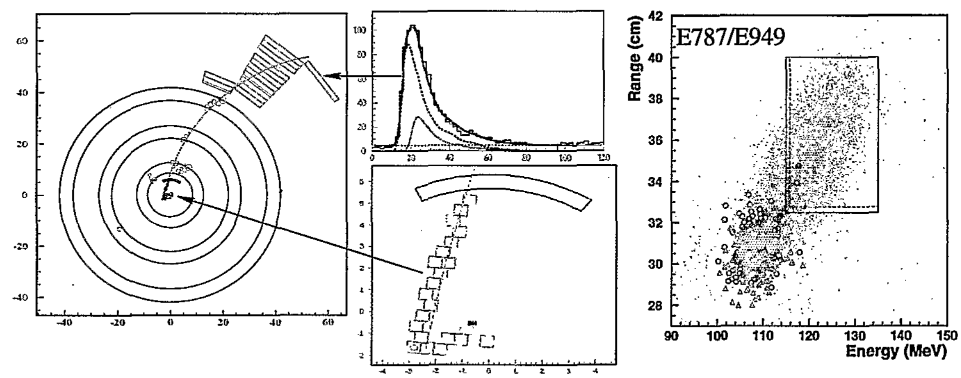

Fig. 6. – **Left:** E949 $K^+ \to \pi^+\nu\bar{\nu}$ event. **Right:** Range vs energy of π^+ for E787/949 samples. The circles are E787 data and the triangles E949 data. The events around $E = 108$ MeV are $K^+ \to \pi^+\pi^0$ background. The simulated distribution of expected signal events is indicated by dots.

$$-0.82 < Re(\lambda_t)/10^{-3} < 1.1 \quad (68\%\ CL) \tag{8}$$

$$Im(\lambda_t)/10^{-3} < 1.1 \quad (90\%\ CL) \tag{9}$$

These limits are not competitive with what can be obtained using the full array of available phenomenological information, but they depend on fewer assumptions. It will be very interesting to compare the large value for $|V_{td}|$ suggested by the E787/949 result with the value that is extracted from $\bar{B}_s - B_s$ mixing when it is finally observed.

From the first observation published in 1997, E787's results for $B(K^+ \to \pi^+\nu\bar{\nu})$ have been rather high with respect to the SM prediction. Although there has never been a statistically significant disagreement with the latter, it has stimulated a number of predictions in BSM theories. Table III lists a selection of such predictions.

The $K^+ \to \pi^+\nu\bar{\nu}$ data also yield an upper limit on the process $K^+ \to \pi^+X^0$ where X^0 is a massless weakly interacting particle such as a familon [40]. For E787 this was $B(K^+ \to \pi^+X^0) < 5.9 \times 10^{-11}$ at 90% CL. The case of $M_{X^0} > 0$ is discussed below.

Fig. 7 left shows the π^+ momentum spectrum from $K^+ \to \pi^+\nu\bar{\nu}$ in the SM, along with the charged track spectra from other kaon decays.

E787/949 is sensitive to the filled-in regions of the $K^+ \to \pi^+\nu\bar{\nu}$ spectrum in Fig. 7. However all the results mentioned so far come from the region on the right ("pnn1"), in which the momentum of the π^+ is greater than than of the π^+ from $K^+ \to \pi^+\pi^0$ (205 MeV/c) . The region on the left ("pnn2") contains a larger portion of the signal phase space, but is more vulnerable to background from $K^+ \to \pi^+\pi^0$. It is relatively easy for the π^+ to lose energy through nuclear interactions. Moreover, there is an unfortunate correlation between nuclear scattering in the stopping target and the relatively weaker photon veto in the beam region. Fig. 7 right illustrates this problem. A K^+ decays with the π^+ pointing downstream (the π^0 must then be pointing upstream). Normally such a decay would not trigger the detector, but here the π^+ undergoes a 90° scatter, loses

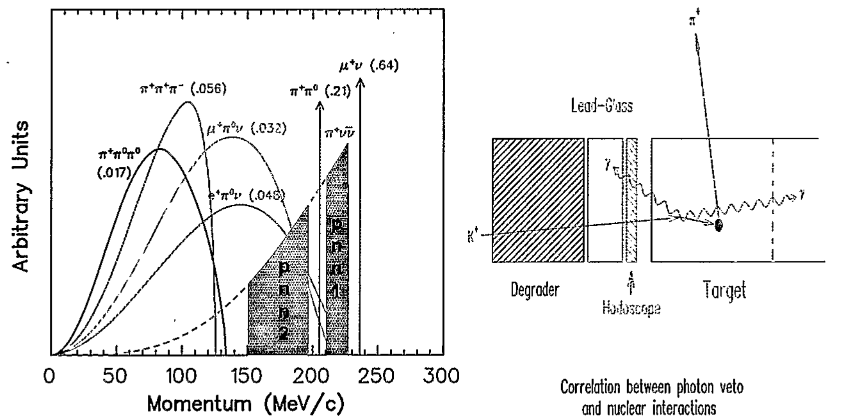

Fig. 7. – **Left**: Center-of-mass momentum spectrum of π^+ from $K^+ \to \pi^+\nu\bar{\nu}$ compared with charged product spectra of the seven most common K^+ decays. Filled areas indicate the portions of the spectrum used in E787/949 analyses. **Right**: Cartoon of limiting background in the pnn2 region of the $K^+ \to \pi^+\nu\bar{\nu}$ spectrum. See text for details.

enough momentum to get into the accepted range and heads for the drift chamber. At the same time the π^0 decays asymmetrically, with the high energy photon heading upstream, to where the veto is least capable and the low energy photon heading downstream, toward

TABLE III. – *BSM predictions for* $B(K^+ \to \pi^+\nu\bar{\nu})$

#	Theory	Ref.
1	MSSM with no new sources of flavor- or CP-violation	[41]
2	General MSSM	[42]
3	Generic SUSY with minimal particle content	[43]
4	SUSY with non-universal A terms	[44]
5	SUSY with broken R-parity	[45, 46]
6	Upper limit from Z' limit given by K mass difference	[47]
7	Topcolor	[48]
8	Topcolor-assisted Technicolor Model	[49]
9	Multiscale Walking Technicolor Model	[50]
10	$SU(2)_L \times SU(2)_R$ Higgs	[51]
11	Four generation model	[52]
12	Leptoquarks	[53]
13	L-R Model with right-handed Z'	[54]
14	Extension of SM to gauge theory of $J = 0$ mesons	[55]
15	Multi Higgs Multiplet Model	[56]
16	Light sgoldstinos	[57]
17	Universal extra dimensions	[58]
18	5-dimensional split fermions	[59]
19	Randell-Sundrum scenario	[60]

another relatively weak veto region. This sequence of events is unlikely, but 20% of K^+ decay to $\pi^+\pi^0$, and one is trying to study a process that happens one in ten billion times. The fact that the same scatter both down-shifts the π^+ momentum and aims the π^0 at the weak veto region confounds the usual product of rejection factors so effective in the pnn1 region. A test analysis using 1996 & 1997 data was undertaken to determine whether methods could be developed to overcome this background. These leaned heavily on exploiting the transient digitized signals from the stopping target target scintillating fibers. At the cost of giving up some acceptance (using only decays later than 0.5 τ_K) one could detect evidence of π^+ scattering occluded by kaon signals in the critical target elements. In this way a single event sensitivity of 7×10^{-10} was achieved with a calculated background of 1.22 events. Fig. 8 left shows the resulting distribution of π^+ kinetic energy and range for surviving candidates. The top left shows the distribution before the final cut on π^+ momentum. The peak at $T_\pi \sim 108$ MeV, $R_\pi \sim 30.5$ cm is due to $K^+ \to \pi^+\pi^0$ events. After the final cut, one event remains, consistent with the background estimation. This yields $B(K^+ \to \pi^+\nu\bar{\nu}) < 2.2 \times 10^{-9}$ at 90% CL [39], consistent with other E787/949 results. This kinematic region is particularly sensitive to possible BSM effects which produce scalar or tensor pion spectra (rather than the vector spectrum given by the SM). One can combine this region with the high momentum region to get 90% CL upper limits of 2.7×10^{-9} and 1.8×10^{-9} for scalar and tensor interactions, respectively. These measurements are also sensitive to $K^+ \to \pi^+X^0$ where X^0 is a hypothetical stable weakly interacting particle or system of particles. Fig. 8 right shows 90% CL upper limits on $B(K^+ \to \pi^+X^)$ together with the previous limit from [61]. The dotted line in the figure is the single event sensitivity. Note that progress in the sensitivity of this kind of search is starting to be impeded by "background" from $K^+ \to \pi^+\nu\bar{\nu}$.

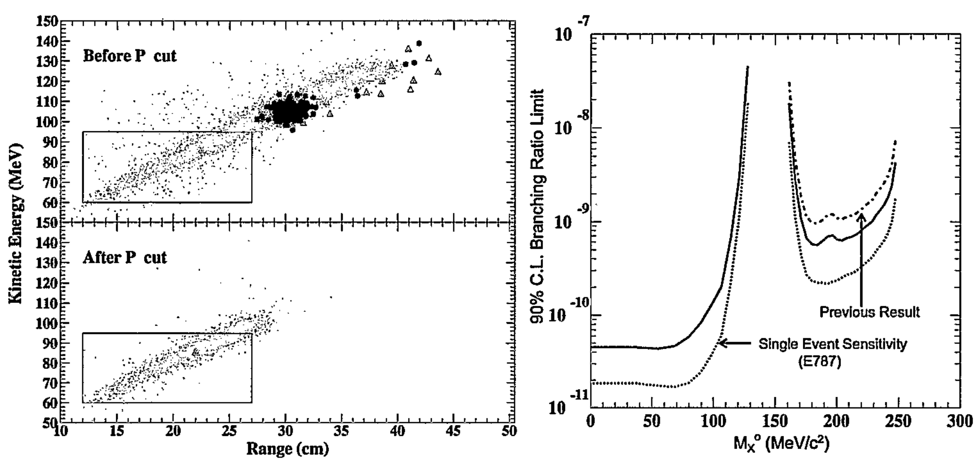

Fig. 8. – **Left**: Signal plane for $K^+ \to \pi^+\nu\bar{\nu}$ analysis of soft π^+ region. **Right**: Limit on $K^+ \to \pi^+X^0$ vs m_X. Vertical lines indicate location of events. For comparison previous limits are indicated as is the single event sensitivity limit of E787.

E949 [62], which ran in 2002 was based on an upgrade of the E787 detector. It was improved in a number of ways with respect to E787: thicker and more complete veto coverage, augmented beam instrumentation, higher capacity DAQ, more efficient trigger counters, upgraded chamber electronics, auxiliary gain monitoring systems, etc. In addition SM sensitivity was anticipated for the pnn2 region ($140 < p_{\pi^+} < 190$ MeV/c). Several of the upgrades were aimed at exploiting this region, and based on the test analysis discussed above, a signal/background of 1:1 is expected for this part of the spectrum. Using the entire flux of the AGS for 6000 hours, E949 was designed to reach a sensitivity of $\sim 10^{-11}$/event. In 2002 the detector operated well at fluxes nearly twice as high as those typical of E787, but unfortunately DOE support of the experiment was terminated after that first run. Aside from the results of the analysis of the pnn2 region that is still continuing, further progress in $K^+ \to \pi^+\nu\bar{\nu}$ will have to come from experiments yet to be mounted.

There are currently two initiatives for future $K^+ \to \pi^+\nu\bar{\nu}$ experiments. One is a J-PARC LOI [63] for a higher-sensitivity stopping experiment. This is very like E787/949 in conception, but with many improvements in detail. These include a lower incident beam momentum (leading to a higher stopping efficiency and a better signal/random rate ratio), higher B-field (leading to better momentum resolution and a more compact geometry), higher granularity (leading to greater rate capability and muon rejection power), brighter scintillators, crystal photon vetoes, etc. The goal of this experiment is to observe 50 events at the SM-predicted level. The possible schedule for such an experiment may become clear later this year when full proposals for J-PARC experiments are expected to be requested. A lower limit may be deduced from the fact that the first hadron beams are scheduled to be available sometime in 2008.

Although the stopped-K^+ technique is now well-understood, and one could be reasonably sure of the outcome of any new experiment of this type, to get really large samples of $K^+ \to \pi^+\nu\bar{\nu}$ ($\geq$100 events), it will almost certainly be necessary to go to an in-flight configuration. There have been a series of attempts to initiate such an experiment, most recently the P326 proposal to CERN [64]. This experiment exploits newly developed tracking technology to allow the use of an extremely intense ($\sim$ 1 GHz) unseparated 75 GeV/c beam. Charged beams of this intensity have been used to search for good-signature kaon decays such as $K^+ \to \pi^+\mu^+e^-$ [65], but P326 is a departure for a poor-signature decay for which high-efficiency vetoing is required.

Fig. 9 shows the layout of the proposed experiment. Protons from the 400 GeV/c SPS will impinge on a 40cm Be target. Positive secondaries with momenta within $\pm 2\%$ of 75 GeV/c will be taken off in the forward direction. The $\sim$5% of K^+ in the beam will be tagged by a differential Cerenkov counter (CEDAR). The 3-momenta of all tracks will be measured in a beam spectrometer with three sets of "GIGATRACKER" detectors (fast Si micro-pixels & micro-mega TPCs). The expected performance is $\sigma_p = 0.3\%$, $\sigma_\theta = 10\mu$r and $\sigma_t = 150$ps. The beamline has been carefully designed to hold the muon halo that contributes to detector random rates to the order of 10MHz. The beam will continue through the apparatus in vacuum. About 10% of the K^+ will decay in a 60m-long fiducial region. The remainder of the beam will be conducted in vacuum out of

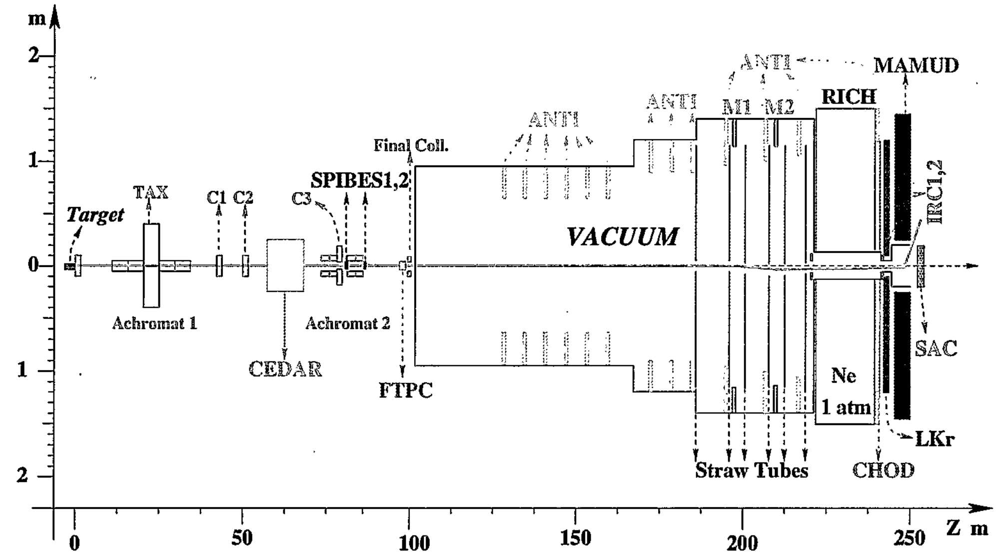

Fig. 9. – P326 detector for $K^+ \to \pi^+ \nu\bar{\nu}$.

the detector region. Photons from K^+ decays will be detected in a series of ring vetoes (at wide angles), by an upgraded version of the NA48 liquid Krypton calorimeter (at intermediate angles) and by two dedicated inner veto systems. Charged decay tracks will be momentum-analyzed in two-dipole a straw-tube spectrometer (<1% resolution on pion momentum and 50-60 μr resolution on $\theta_{K\pi}$ are necessary). Downstream of the spectrometer a RICH filled with Ne at 1 atm will help distinguish signal pions from background muons. This is to be followed by a charged particle hodoscope of multigap glass RPC design (100 ps resolution is required). Behind the hodoscope is the "MAMUD" muon veto, a magnetized iron-scintillator sandwich device to complete the pion/muon distinction. Its 5Tm bending power serves to kick the beam out of the way of the small angle photon veto at the back of the detector. Fig. 10 shows the distribution in the square of missing mass recoiling from the assumed π^+ expected in this experiment ([4]) (this plot corresponds to Fig. 7 Left for E787/949). A resolution of $1.1 \times 10^{-3}\,\mathrm{GeV}^2/\mathrm{c}^4$ in this quantity is required to get sufficient signal/background.

The collaboration proposes to build this experiment in time to begin taking data in 2009. A two-year would accumulate ~80 events with a 8:1 signal to background.

3·2. $K_L \to \pi^0 \nu\bar{\nu}$. – $K_L \to \pi^0 \nu\bar{\nu}$ is generally considered to be the most attractive target in the kaon system, since

1. it is direct CP-violating to a very good approximation [24, 66] (in the SM $B(K_L \to$

([4]) The $K^+ \to \mu^+ \nu$ peak is shifted and smeared out by this assumption.

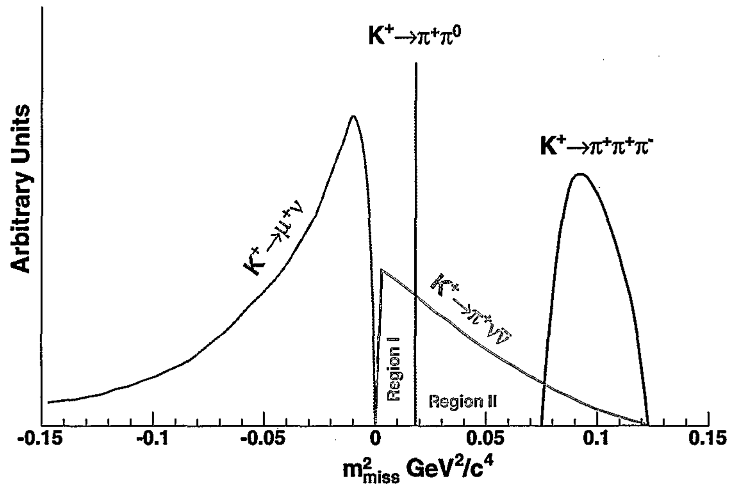

Fig. 10. – Missing mass squared under the pion assumption for P326 (see text).

$\pi^0\nu\bar{\nu}) \propto \eta^2)$ and

2. the rate can be rather precisely calculated in the SM or almost any extension thereof [67].

Like $K^+ \to \pi^+\nu\bar{\nu}$ the hadronic matrix element can be obtained from K_{e3}, but unlike $K^+ \to \pi^+\nu\bar{\nu}$, it has no significant contribution from charm. Consequently, the intrinsic theoretical uncertainty connecting $B(K_L \to \pi^0\nu\bar{\nu})$ to the fundamental short-distance parameters is only about 2%. Note also that in the SM $B(K_L \to \pi^0\nu\bar{\nu})$ is directly proportional to the square of $Im\lambda_t$ and that $Im\lambda_t = -\mathcal{J}/[\lambda(1-\frac{\lambda^2}{2})]$ where $\mathcal{J}$ is the Jarlskog invariant [68]. Thus a measurement of $B(K_L \to \pi^0\nu\bar{\nu})$ determines the area of the unitarity triangles with a precision twice as good as that on $B(K_L \to \pi^0\nu\bar{\nu})$ itself.

$B(K_L \to \pi^0\nu\bar{\nu})$ can be bounded indirectly by measurements of $B(K^+ \to \pi^+\nu\bar{\nu})$ through a nearly model-independent relationship pointed out by Grossman and Nir [69]. The application of this to the E787/949 results yields $B(K_L \to \pi^0\nu\bar{\nu}) < 1.4\times10^{-9}$ at 90% CL. This is far tighter than any extant direct experimental limit. To actually observe $K_L \to \pi^0\nu\bar{\nu}$ at the SM level ($\sim 3\times10^{-11}$), one will need to improve on the current state of the art by some four orders of magnitude.

The first dedicated $K_L \to \pi^0\nu\bar{\nu}$ experiment, KEK E391a [70], mounted at the KEK 12 GeV proton synchrotron, aims to achieve sensitivity comparable to the indirect limit. Although it will not approach the SM level, it will serve as a test for a future more sensitive experiment to be performed at J-PARC [71]. E391a features a carefully designed "pencil" beam [72] with average K_L momentum $\sim$ 2 GeV/c. Fig. 11 shows a layout of the detector.

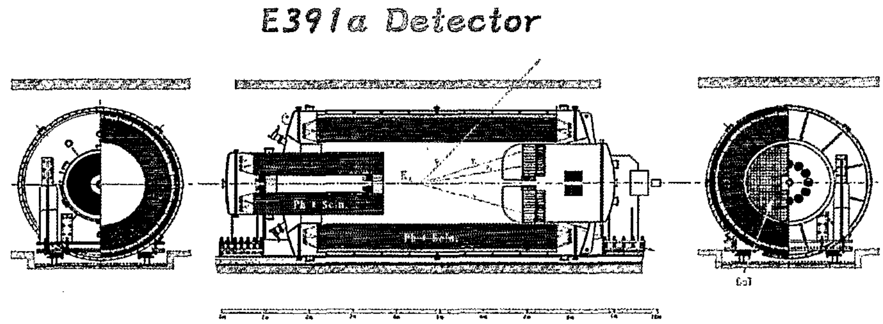

Fig. 11. – KEK E391a detector for $K_L \to \pi^0\nu\bar{\nu}$

A particular challenge of the E391a approach is to achieve extremely low photon veto inefficiency. The photon veto system consists of two cylinders. The inner, more upstream barrel is intended to suppress beam halo and reduce confusion from upstream K_L decays. Roughly 4% of the K_L's decay in the 2.4m fiducial region between the end of the inner cylinder and the charged particle veto in front of the photon detector. Signal photons are detected in a multi-element CsI-pure crystal calorimeter [73]. The entire apparatus operates in vacuum. Physics running began in February 2004. A second run took place early this year, and a third is scheduled for the fall.

In this experiment events with two showers in the calorimeter and no additional activity are examined to determine whether any point along the fiducial section of the beamline results in a reconstructed mass consistent with a π^0. If so, the p_T can then be determined. Cuts are imposed on the shower patterns and energies, the Z_V and the p_T. In addition, events consistent with $\eta \to \gamma\gamma$ were discarded.

Fig. 12 shows the distribution of residual candidates in p_T vs Z_V, when all other cuts are applied for the first 10% of Run 1. No events were observed in the signal region, with an expected background of 0.03±0.01 event (^{5}). With $1.14 \times 10^9\, K_L$ decays and an acceptance of 0.73%, they obtained a preliminary result of $B(K_L \to \pi^0\nu\bar{\nu}) < 2.86 \times 10^{-7}$ at 90% CL [74].

A different approach is taken by the KOPIO experiment[75] (E926) at BNL, exploiting the intensity and flexibility of the AGS to make a high-flux, low-energy, microbunched K_L beam. The principles of the experiment are illustrated in Fig. 13.

The AGS proton beam will be microbunched at 25 MHz by imposing upon it a train of empty RF buckets as it is extracted from the machine[76]. The AGS's 10^{14} protons will be spilled out over ~4.7s. A neutral beam will be extracted at 42.5° to obtain a

(^{5}) Not all backgrounds have yet been calculated.

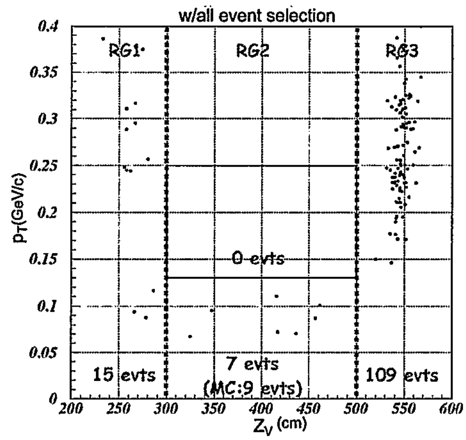

Fig. 12. – p_T vs Z_V E391a events passing all other cuts (from first 10% of Run 1). Signal region is the central rectangle.

spectrum soft enough to permit time-of-flight determination of the K_L velocity. The beam contains about $3 \times 10^8\, K_L$ and 5×10^{10} neutrons per cycle. The large production angle also softens the neutron spectrum so that they (and the K_L) are by and large below threshold for the hadro-production of π^0's. The beam region will be evacuated to 10^{-7} Torr to further minimize such production. With a 10m beam channel and this low energy beam, the contribution of hyperons and K_S to the background will be negligible. The profile of the beam is ribbon-like (4mr $\times$ 90mr) to facilitate collimation of the large aperture and to provide an extra constraint for reconstruction of the decay vertex. All possible quantities are measured: the K_L momentum, the final state photon angles as well as energies and times. Effective kinematic rejection of background is then possible.

The layout of the experiment is shown in Fig. 14. K_L decays from a $\sim$ 3m fiducial region will be accepted ($\sim$ 8% of the K_L decay in this region). Signal photons impinge on a 2 X_0 thick preradiator capable of measuring their direction to $\sim$ 25mrad. An alternating drift chamber/scintillator plane structure will also allow good measurement of the energy deposited in the preradiator. A high-precision shashlyk calorimeter downstream of the preradiator will complete the energy measurement. The photon directions allow the decay vertex position to be determined. This can be required to lie within the beam envelope, eliminating many potential sources of background. Combined with the target position and time of flight information, the vertex information provides a measurement of the K_L 3-momentum so that kinematic constraints as well as photon vetoing are available to suppress backgrounds. The leading expected background is $K_L \to \pi^0\pi^0$, initially eight orders of magnitude larger than the SM-predicted signal. However since π^0's from this background have characteristic signatures in the K_L center of mass, effective kinematic cuts can be applied. Similar remarks pertain to almost all expected backgrounds. This reduces the burden on the photon veto system surrounding the decay region to

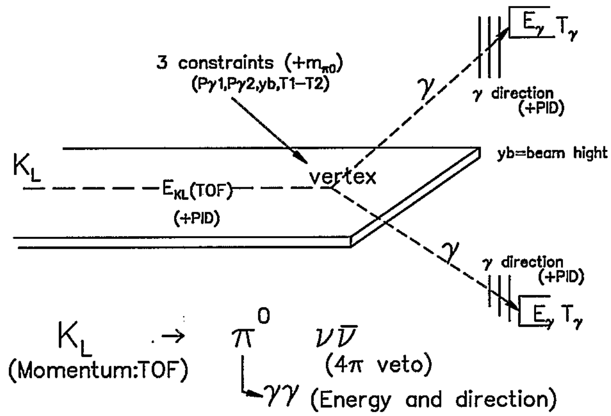

Fig. 13. – Principles of KOPIO $K_L \to \pi^0 \nu\bar{\nu}$ experiment

the point where the hermetic veto techniques proven in E787/949 are sufficient. In fact most of the techniques necessary for KOPIO have been proven in previous experiments or in prototype tests. Fig. 15 shows results on two of the more critical aspects of the experiment.

On the left of the figure is the result of a test of beam microbunching, showing an rms of 265 psec. This is sufficient for KOPIO's purposes, although it still needs to be demonstrated at a 25 MHz repetition rate. Other beam tests have demonstrate the required ≤ 0.001 extraction of protons between the microbunches. On the right is a plot of photon angular resolution obtained with a 6-plane prototype of the preradiator. A tagged beam at the National Synchrotron Light Source (NSLS) provided the photons. A resolution of 25 mrad is observed for 250 MeV photons, in line with GEANT simulation. This resolution is sufficient for KOPIO.

The electromagnetic calorimeter following the preradiator will be a 5m $\times$ 5m array of high resolution shashlyk modules. The required resolution of $3\%/\sqrt(E)$ has been demonstrated in prototypes tested in the NSLS tagged photon beam [77, 78]. The combination of preradiator and calorimeter will measure photon energies to $\sim 2.7\%/\sqrt{E}$.

The upstream veto wall will be 18 r.l. thick lead-scintillator shower counters read out via wavelength-shifting fibers [79]. The cylindrical barrel will be shashlyk of similar structure to those of the calorimeter, but larger transversely and truncated to fit into a cylinder. The demands on the performance of these counters are comparable to that demonstrated in the E787/949 barrel veto that has similar structure. It is also planned to use the barrel shashlyk for positive photon detection. There are a series of downstream vetoes abutting the beam, using plate counter technology similar to that of the upstream wall. Finally it will also be necessary to veto within the beam, which is very challenging but which is facilitated by the low average energy of the beam neutrons. This will be accomplished by a series of lead-aerogel shower counters (the "catcher" veto). For the most part charged particles created by the neutrons are below the Cerenkov threshold of

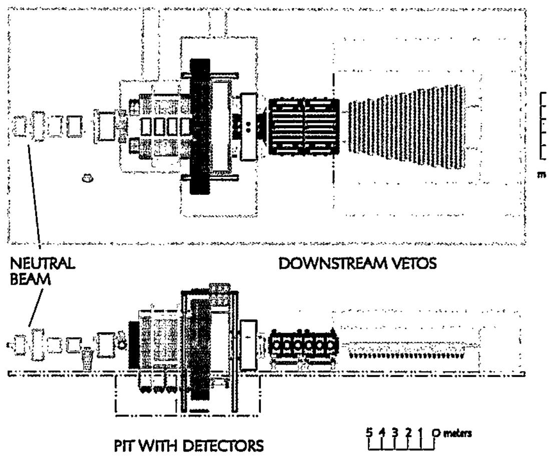

Fig. 14. – Layout of the KOPIO detector.

the aerogel and are so invisible to these counters.

Another very important element is charged particle vetoing needed to eliminate backgrounds such as $K_L \to \pi^0\pi^+\pi^-$. A very high performance system will be mounted in the decay region vacuum and at the margins of the downstream beam pipe. Behind the calorimeter will be a dipole magnet with field oriented to sweep charged particles traveling in the beam direction upwards or downwards into veto counters outside the beam profile.

KOPIO aims to collect about 150 $K_L \to \pi^0\nu\bar{\nu}$ events with varying signal to background. This will permit η to be determined to $\sim$ 14%, given expected progress in measuring m_t and V_{cb}. KOPIO will run during the ~20 hours/day the AGS is not needed for injection into RHIC. The experiment is presently awaiting funding ([6]).

3·3. $K_L \to \mu^+\mu^-$. – The short distance component of this decay, which arises out of the diagrams shown in Fig. 3, can be quite reliably calculated in the SM[32]. The most recent measurement of its branching ratio[80] based on $\sim$ 6200 events gave $B(K_L \to \mu^+\mu^-) = (7.18 \pm 0.17) \times 10^{-9}$. However the actual measurement was the ratio $R_{\mu\mu} \equiv B(K_L \to \mu^+\mu^-)/B(K_L \to \pi^+\pi^-)$, and the value of the denominator has recently changed significantly, so that the current value of $B(K_L \to \mu^+\mu^-)$ is $(6.87 \pm 0.12) \times 10^{-9}$[81][82] (this average also includes the results of older, less precise experiments). This is quite a precise determination for such a rare decay. However $K_L \to \mu^+\mu^-$ is dominated by long distance effects, the largest of which, the absorptive contribution mediated by $K_L \to \gamma\gamma$ shown in Fig. 16, accounts for $(6.64 \pm 0.07) \times 10^{-9}$.

Subtracting the two, yields $(0.23 \pm 0.14) \times 10^{-9}$ for dispersive part of $B(K_L \to \mu^+\mu^-)$.

([6]) On August 11, 2005, RSVP, of which this experiment is a component, was canceled by the US National Science Foundation

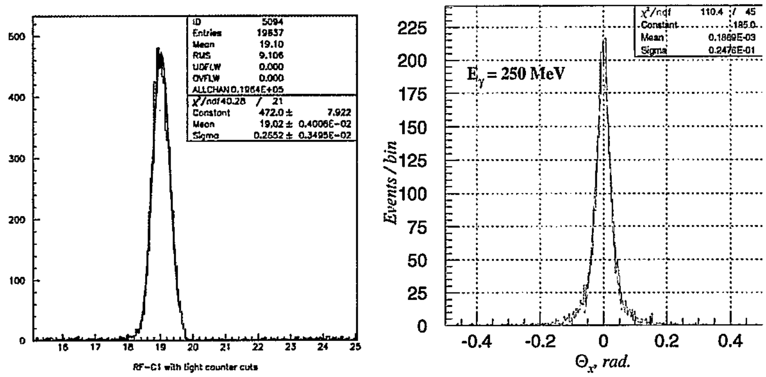

Fig. 15. – Tests of KOPIO components. **Left:** demonstration of microbunching of the AGS proton beam (93 MHz RF cavity at 22 KV). **Right:** angular resolution of prototype preradiator for 250 MeV photons (in a tagged photon beam).

One can do a little better than this in the following way. As mentioned above, the actual quantity measured in Ref [80] was $R_{\mu\mu} = (3.48 \pm 0.05) \times 10^{-6}$. It is necessary to subtract from this measured quantity the ratio $B^{abs}_{\gamma\gamma}(K_L \to \mu^+\mu^-)/B(K_L \to \pi^+\pi^-)$. Eq. 10 shows the components of this latter ratio, obtained from Ref. [83] augmented by a recent measurement $\Gamma(K_S \to \pi^+\pi^-)/\Gamma(K_S \to \pi^0\pi^0) = 2.236 \pm 0.003 \pm 0.015$ [84], whose product is $(3.344 \pm 0.053) \times 10^{-6}$.

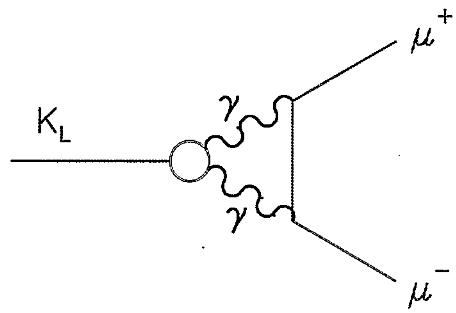

Fig. 16. – Long distance contribution to $K_L \to \mu^+\mu^-$.

$$\frac{B^{obs}_{\gamma\gamma}(K_L\to\mu\mu)}{B(K_L\to\pi^+\pi^-)} = \underset{\substack{\downarrow\\ 1.195\cdot 10^{-5}}}{\overset{\substack{\text{calculated}\\ \downarrow}}{\frac{B^{obs}_{\gamma\gamma}(K_L\to\mu\mu)}{B(K_L\to\gamma\gamma)}}}\;\underset{\substack{\downarrow\\ 0.632\pm 0.009}}{\overset{\substack{\text{last measured by}\\ \text{NA31 in 1987}\\ \downarrow}}{\frac{B(K_L\to\gamma\gamma)}{B(K_L\to\pi^0\pi^0)}}}\;\overbrace{\underset{\substack{\downarrow\\ (2.221\pm 0.014)^{-1}}}{\overset{\substack{\text{recently measured}\\ \text{by KLOE}\\ \downarrow}}{\frac{B(K_S\to\pi^0\pi^0)}{B(K_S\to\pi^+\pi^-)}}}\;\underset{\substack{\downarrow\\ 1-6(16.7\pm 2.6)\cdot 10^{-4}}}{\left(1-6Re\frac{\epsilon'}{\epsilon}\right)}}^{\frac{B(K_L\to\pi^0\pi^0)}{B(K_L\to\pi^+\pi^-)}}$$

(10)

This yields $(3.400\pm 0.053)\times 10^{-6}$. A similar calculation can be done using the results of recent determinations of the ratio $\Gamma(2\gamma)/\Gamma(3\pi^0)$ from KLOE [85] and NA48 [86]. For this, one also needs a value for the ratio $B(K_L \to 3\pi^0)/B(K_L \to \pi^+\pi^-)$ and I use the recent KTeV results on K_L branching ratios [87], since the correlations between the various measurements are given. This yields $(3.299 \pm 0.048) \times 10^{-6}$. Averaging the $2\pi^0$ and $3\pi^0$ determinations yields $(3.345 \pm 0.036) \times 10^{-6}$. The subtraction then yields $\frac{B^{disp}(K_L\to\mu^+\mu^-)}{B(K_L\to\pi^+\pi^-)} = (0.135 \pm 0.061) \times 10^{-6}$ (where B^{disp} refers to the dispersive part of $B(K_L \to \mu^+\mu^-)$). $\frac{B^{disp}(K_L\to\mu^+\mu^-)}{B(K_L\to\pi^+\pi^-)}$ can then be multiplied by $B(K_L \to \pi^+\pi^-) = (1.975 \pm 0.015) \times 10^{-3}$ [82] to obtain $B^{disp}(K_L \to \mu^+\mu^-) = (0.267 \pm 0.121) \times 10^{-9}$, or

$B^{disp}(K_L \to \mu^+\mu^-) < 0.42 \times 10^{-9}$ at 90% CL. (7)

Now if one inserts the result of even very conservative recent CKM fits into the formula for the short distance part of $B(K_L \to \mu^+\mu^-)$, one gets poor agreement with the limit of $B^{disp}(K_L \to \mu^+\mu^-)$ derived above. For example the 2σ fit of the CKM Fitter Group[88], $\bar{\rho} = 0.096 - 0.285$, gives $B^{SD}(K_L \to \mu^+\mu^-) = (0.64 - 0.93) \times 10^{-9}$. So why haven't we been hearing about this apparent violation of the SM? There are certainly viable candidates for BSM contributions to this decay [89, 90].

The answer is that unfortunately $K_L \to \gamma^*\gamma^*$ also generates a dispersive contribution, that is rather less tractable than the absorptive part and that interferes with the short-distance weak contribution one wants to extract. The problem in calculating this contribution is the necessity of including intermediate states with virtual photons of all effective masses. Such calculations can only be partially validated in kaon decays containing virtual photons in the final state. The degree to which this validation is possible is controversial with both optimistic [91, 92, 93] and pessimistic [94, 95] conclusions available. Recently there have been talks and publications on $K_L \to \gamma e^+e^-$ [96] (93,400 events), $K_L \to \gamma\mu^+\mu^-$[97] (9327 events), $K_L \to e^+e^-e^+e^-$[96] (1056 events), and $K_L \to \mu^+\mu^-e^+e^-$[10] (133 events). Figure 17-top shows the spectrum of $x = (m_{\mu\mu}/m_K)^2$ from Ref.[97]. The disagreement between the data (filled circles with error bars) and the prediction of pointlike behavior (histogram) clearly indicates the presence of a form factor. A candidate for this is provided by the DIP model[91] which depends on parameters α and β. The latter parameter applies only in decays in which both photons are virtual.

Fig. 17-bottom shows four determinations of this parameter from KTeV. These are mutually consistent, but there's a disagreement with a previous NA48 result for $K_L \to e^+e^-\gamma$ [98]. Both the DIP and the older BMS [99] parameterization predict a connection between the shape of the $\ell^+\ell^-$ spectra and the branching ratios, that can be exploited in the cases involving muon pairs. In those cases the parameters have been determined from both the spectra and the branching ratios. These determinations agree at the 1-2 σ level. However the available data is not yet sufficient to clearly favor either parameterization. In addition, it seems clear that a very large increase in the data of processes where both photons are virtual, such as $K_L \to e^+e^-\mu^+\mu^-$ would be needed to completely test the DIP parameterization [100, 101]. Additional effort, both experimental and theoretical, is required before the quite precise data on $B(K_L \to \mu^+\mu^-)$ can be fully exploited.

Finally, one might ask if it is possible to extract short distance information from the decay $K_L \to e^+e^-$. AGS E871 observed four events of this mode, establishing $B(K_L \to e^+e^-) = (8.7^{+5.7}_{-4.1}) \times 10^{-12}$ [102], the smallest branching ratio yet measured for an elementary particle. Unfortunately the SM short distance contribution is helicity suppressed with respect to $K_L \to \mu^+\mu^-$ by the ratio m_e^2/m_μ^2 while long distance contributions are relatively enhanced, making the extraction of SM short distance information almost impossible [94]. The theoretical prediction of the branching ratio agrees well

(7) Note that since $B(K_L \to \mu^+\mu^-)$ and $B^{abs}_{\gamma\gamma}(K_L \to \mu^+\mu^-)$ are so close, small shifts in the component values could have relatively large consequences for $B^{disp}(K_L \to \mu^+\mu^-)$.

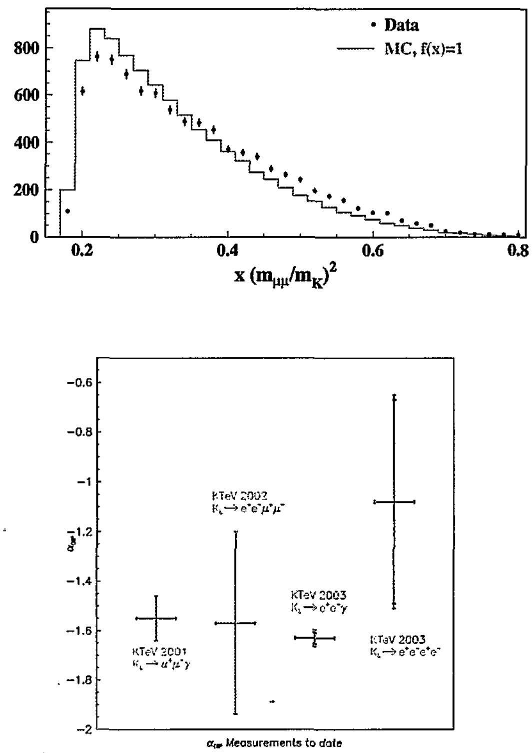

Fig. 17. – **Top:** spectrum of $x = (m_{\mu\mu}/m_K)^2$ in $K_L \to \mu^+\mu^-\gamma$ from Ref. [97]. **Bottom:** determinations of the parameter α_{DIP} from four K_L decays involving virtual photons.

with what is observed, which limits the presence of BSM pseudoscalar couplings in this decay.

3·4. $K_L \to \pi^0\ell^+\ell^-$. – Like $K_L \to \pi^0\nu\bar{\nu}$, the $K_L \to \pi^0\ell^+\ell^-$ are GIM-suppressed neutral current reactions sensitive to short-distance SM and BSM effects, but with far different experimental considerations. Like $K_L \to \pi^0\nu\bar{\nu}$, in the SM they are sensitive to $Im\lambda_t$, but in general they have different sensitivity to BSM effects [43]. Although their signatures are intrinsically superior to that of $K_L \to \pi^0\nu\bar{\nu}$, they are subject to a serious background that has no analogue in the neutral lepton case: $K_L \to \gamma\gamma\ell^+\ell^-$. This process, a radiative correction to $K_L \to \gamma\ell^+\ell^-$, occurs $10^3 - 10^4$ times more frequently than $K_L \to \pi^0\ell^+\ell^-$. Kinematic cuts are quite effective, but it is very difficult to improve the signal:background beyond about 1 : 1.5[103] and still maintain adequate acceptance. Both varieties of $K_L \to \gamma\gamma\ell^+\ell^-$ have been observed, $B(K_L \to \gamma\gamma e^+e^-)_{k_\gamma > 5MeV} = (5.84 \pm 0.15_{stat} \pm 0.32_{syst}) \times 10^{-7}$[104] and $B(K_L \to \gamma\gamma\mu^+\mu^-)_{m_{\gamma\gamma} > 1MeV/c^2} = (10.4^{+7.5}_{-5.9}{}_{stat} \pm 0.7_{syst}) \times 10^{-9}$[105]; both agree reasonably well with theoretical prediction. By comparison, the direct CP-violating part of $B(K_L \to \pi^0e^+e^-)$, is predicted to be $(4.4 \pm 0.9) \times 10^{-12}$ and that of $B(K_L \to \pi^0\mu^+\mu^-)$ to be

$(1.8 \pm 0.4) \times 10^{-12}$ [106].

In addition to this background, there are two other contributions that complicate the extraction of short-distance information from $K_L \to \pi^0\ell^+\ell^-$. First, there is an indirect CP-violating amplitude from the K_1 component of K_L that is proportional to $\epsilon A(K_S \to \pi^0\ell^+\ell^-)$. It is of the same order of magnitude as the direct CP-violating amplitude and can interfere with it. In the case of $K_L \to \pi^0 e^+e^-$, it yields [106]:

$$B(K_L \to \pi^0 ee)_{CPV} \approx \left[15.7a_S^2 \pm 6.2a_S\frac{Im\lambda_t}{10^{-4}} + 2.4\left(\frac{Im\lambda_t}{10^{-4}}\right)^2\right] \times 10^{-12} \tag{11}$$

where

$$B(K_S \to \pi^0 ee) \approx 5.2a_S^2 \times 10^{-9}d \tag{12}$$

For $K_L \to \pi^0\mu^+\mu^-$, the corresponding expressions are [106, 107]:

$$B(K_L \to \pi^0 \mu\mu)_{CPV} \approx \left[3.7a_S^2 \pm 1.6a_S\frac{Im\lambda_t}{10^{-4}} + 1.0\left(\frac{Im\lambda_t}{10^{-4}}\right)^2\right] \times 10^{-12} \tag{13}$$

and

$$B(K_S \to \pi^0 \mu\mu) \approx 1.2a_S^2 \times 10^{-9} \tag{14}$$

There are now measurements for both these processes. $B(K_S \to \pi^0e^+e^-)|_{m_{ee}>165} = (3.0^{+1.5}_{-1.2} \pm 0.2) \times 10^{-9}$ from NA48 [108], based on 7 observed events with an estimated background of 0.15 events (see Fig. 18). The branching ratio then needs to be corrected for a cut of $m_{ee} > 165\,\mathrm{MeV/c^2}$ that was imposed to eliminate $K_S \to \pi^0\pi^0$; $\pi^0 \to e^+e^-\gamma$ events. This yields: $B(K_S \to \pi^0e^+e^-) = (5.8^{+2.8}_{-2.3} \pm 0.8) \times 10^{-9}$. Eq. 12 then yields $|a_S| = 1.06^{+0.26}_{-0.21} \pm 0.07$.

NA48 has also observed six $K_S \to \pi^0\mu^+\mu^-$ candidates over a background of 0.22 events [109] (see Fig. 18). This yields $B(K_S \to \pi^0\mu^+\mu^-) = (2.9^{+1.5}_{-1.2} \pm 0.2) \times 10^{-9}$. Eq. 14 then gives $|a_S| = 1.54^{+0.40}_{-0.32} \pm 0.06$. Ref. [106] averages the electron and muon results to get a best estimate of $|a_S| = 1.2 \pm 0.2$. The ratio $\Gamma(K_S \to \pi^0\mu^+\mu^-)/\Gamma(K_S \to \pi^0e^+e^-) = 0.49^{+0.35}_{-0.29} \pm 0.07$ is consistent within 1 σ with the value ~ 0.2 predicted in the SM (and almost any other model).

Another contribution of similar order, but *CP-conserving*, is mediated by $K_L \to \pi^0\gamma\gamma$. In principle this contribution can be predicted from measurements of $K_L \to \pi^0\gamma\gamma$, of which thousands of events have been observed. The matrix element for this decay is given by [110]:

$$\begin{aligned}\mathcal{M}(K_L \to \pi^0\gamma\gamma) = \frac{G_8\alpha}{4\pi}\epsilon_\mu(k_1)\epsilon_\nu(k_2)\big[A(k_2^\mu k_1^\nu - k_1\cdot k_2 g^{\mu\nu}) + \\ B\frac{2}{m_K^2}(p_k\cdot k_1 k_2^\mu p_K^\nu + p_K\cdot k_2 k_1^\nu p_K^\mu - k_1\cdot k_2 p_K^\mu p_K^\nu - g^{\mu\nu}p_K\cdot k_1 p_K\cdot k_2)\big]\end{aligned} \tag{15}$$

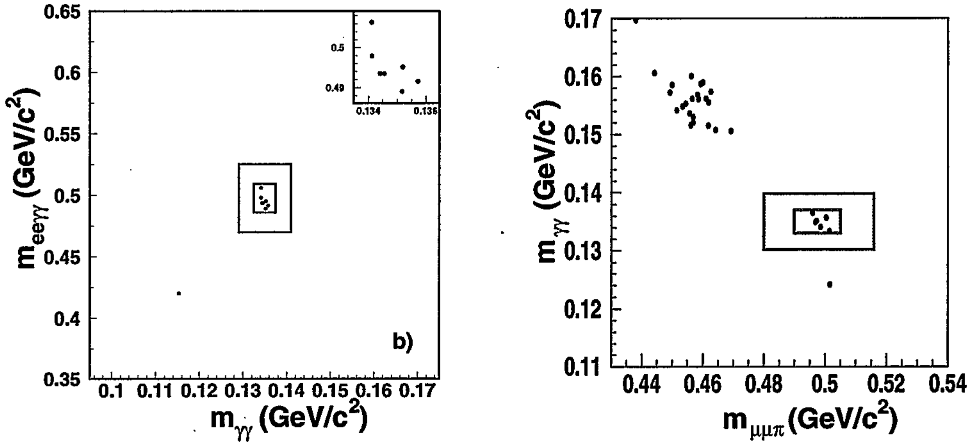

Fig. 18. – **Left:** NA48 $K_S \to \pi^0 e^+ e^-$ candidates (from Ref [108]). **Right:** NA48 $K_S \to \pi^0 \mu^+ \mu^-$ candidates (from Ref [109]).

where k_1 and k_2 refer to the photons. The A amplitude corresponds to $J_{\gamma\gamma} = 0$; B is a mixture of $J_{\gamma\gamma} = 0$ and $J_{\gamma\gamma} = 2$. G_8 is the octet coupling constant in χPT. Eq. 15 leads to

$$(16) \qquad \frac{\partial^2 \Gamma(K_L \to \pi^0 \gamma\gamma)}{\partial y \partial z} = \frac{m_K}{2^9 \pi^3} \left[z^2 |A + B|^2 + (y^2 - \frac{1}{4}\lambda(1, r_\pi^2, z))^2 |B|^2 \right]$$

where $z \equiv (k_1 + k_2)^2/m_K^2$, $y \equiv p_K \cdot (k_1 - k_2)/m_K^2$, $r_\pi \equiv m_\pi/m_K$ and $\lambda(a, b, c) \equiv a^2 + b^2 + c^2 - 2(ab + ac + bc)$. Since in $K_L \to \pi^0 e^+ e^-$ the effect of A is greatly suppressed by helicity conservation and $B = 0$ at leading order in χPT, it was initially thought that the CP-conserving contribution to $B(K_L \to \pi^0 e^+ e^-)$ would be very small. However the possibility of a substantial vector meson dominance (VDM) contribution to B was pointed out by Sehgal [111]. Such a contribution can arise at $\mathcal{O}(p^6)$ in χPT. Indeed, early measurements of $B(K_L \to \pi^0 \gamma\gamma)$ [112, 113] showed that although the simple $\mathcal{O}(p^4)$ calculation was in reasonable agreement with the $m_{\gamma\gamma}$ spectrum, it underestimated the decay rate by a factor > 2. Subsequent theoretical work has remedied this to a large extent [114, 115, 116, 117, 118]. Although a full $\mathcal{O}(p^6)$ calculation is not possible at present, in this work the $\mathcal{O}(p^4)$ calculation was improved by "unitarity corrections" and the addition of a VDM contribution characterized by a single parameter a_V. This produced satisfactory agreement with the observed branching ratio (at least until more precise measurements became available). A similar approach [119] was successful in predicting the characteristics of the closely related decay $K^+ \to \pi^+ \gamma\gamma$ that was measured by AGS E787 [120] and E949 [12]. The most recent data on $K_L \to \pi^0 \gamma\gamma$ is summarized in Table IV. Unfortunately the two results, from KTeV [121] and from NA48 [122] disagree by nearly 3σ in branching ratio. Their $m_{\gamma\gamma}$ spectra also differ rather significantly as shown in Fig. 19, leading to disagreement in their predictions for $B^{CP-cons}(K_L \to$

$\pi^0 e^+ e^-$). This is also reflected in differing extracted values of a_V seen in Table V. As discussed below, this disagreement complicates assessing the prospects of extracting $B^{direct}(K_L \to \pi^0 e^+ e^-)$ from data.

TABLE IV. – *Recent results on $K_L \to \pi^0\gamma\gamma$.*

Exp/Ref	$B(K_L \to \pi^0\gamma\gamma) \cdot 10^6$	a_V
KTeV [121]	$1.68 \pm 0.07_{stat} \pm 0.08_{syst}$	$-0.72 \pm 0.05 \pm 0.06$
NA48 [123]	$1.36 \pm 0.03_{stat} \pm 0.03_{syst} \pm 0.03_{norm}$	$-0.46 \pm 0.03 \pm 0.03 \pm 0.02_{theor}$

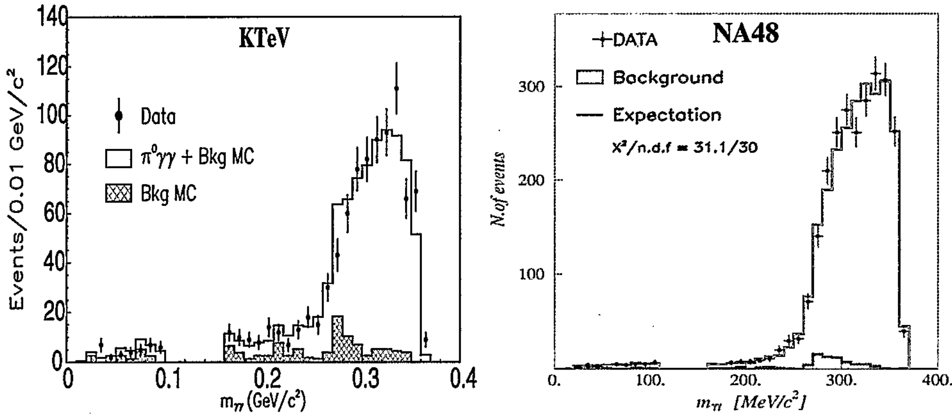

Fig. 19. – $m_{\gamma\gamma}$ spectrum of $K_L \to \pi^0\gamma\gamma$ candidates from (left) KTeV [121] and (right) NA48 [122]. The striking threshold at $m_{\gamma\gamma} \approx 2m_\pi$ is due to the amplitude $A(y,z)$ in χPT (or in pion loop models [124]).

In addition, the simple VDM-based formalism for predicting $B^{CP-cons}(K_L \to \pi^0 e^+ e^-)$ from $K_L \to \pi^0\gamma\gamma$ was criticized by Gabbiani and Valencia [125]. They pointed out that the use of a single parameter a_V can artificially correlate the A and B amplitudes. They showed that a three parameter expression inspired by $\mathcal{O}(p^6)$ χPT fits the KTeV data quite as well as the conventional one based on a_V and gives a significantly different prediction for $B^{CP-cons}(K_L \to \pi^0 e^+ e^-)$ as seen in Table V. In a subsequent paper [126], they find they can make a good simultaneous fit to the NA48 decay spectrum and rate using the same technique and show that this is not possible using just a_V. Not surprisingly, their predictions of their three parameter fits for $B^{CP-cons}(K_L \to \pi^0 e^+ e^-)$ also differ markedly between the two experiments as shown in Table V.

Finally there are significant uncertainties in the extraction of the dispersive contribution to $B^{CP-cons}(K_L \to \pi^0 e^+ e^-)$ [127, 128], which is similar in size to the absorptive contribution and so not at all negligible.

Subsequently Buchalla, D'Ambrosio and Isidori[129] took a different approach to analyzing the NA48 data. They used the same three parameters but applied a constraint given by the NA48 measurement of $B(K_S \to \gamma\gamma)$[86]. They obtained somewhat different values for the parameters but agreed that the CP-conserving component of $K_L \to \pi^0 e^+ e^-$ should be small ($< 3 \times 10^{-12}$). In fact for similar assumptions on the ratio of dispersive to absorptive contribution and using the data in the same way, the two approaches yield very similar results. For example, using the central value of the NA48 measurement of the low $m_{\gamma\gamma}$ part of the $K_L \to \pi^0\gamma\gamma$ spectrum and the recipe for the dispersive part of the contribution of Ref. [127], the method of Buchalla *et al.* yields the value listed in Table V. Based on the NA48 measurement, a reasonable estimate of $B^{CP-cons}(K_L \to \pi^0 e^+ e^-)$ is $(4 \pm 4) \times 10^{-13}$.

TABLE V. – *Predictions for* $B^{CP-cons}(K_L \to \pi^0 e^+ e^-)$.

Exp.	a_V fit from experimental paper	3 parameter fit by Gabbiani & Valencia	3 parameter fit by Buchalla *et al.*
KTeV	$(1.0 - 2.0) \cdot 10^{-12}$	$7.3 \cdot 10^{-12}$	
NA48	$(0.47^{+0.22}_{-0.18}) \cdot 10^{-12}$	$(0.46^{+0.22}_{-0.17}) \cdot 10^{-12}$	0.34×10^{-12}

To summarize, there is reasonable agreement on how to extract the absorptive contribution to $B^{CP-cons}(K_L \to \pi^0 e^+ e^-)$ from the observed low $m_{\gamma\gamma}$ part of the $K_L \to \pi^0\gamma\gamma$ branching ratio. There is some uncertainty over the dispersive part of the contribution, but this is immaterial if the NA48 result is correct. However, if the KTeV result is the correct one, $B^{CP-cons}(K_L \to \pi^0 e^+ e^-)$ could be a non-negligible part of the total and would complicate the extraction of short-distance information from results on this process. Thus further experimental work on $K_L \to \pi^0\gamma\gamma$ would be very welcome.

The situation in the muonic case is quite different. Here there is very little helicity suppression of the $J_{\gamma\gamma} = 0$ part of the CP-conserving contribution to $B(K_L \to \pi^0\mu^+\mu^-)$ and this is in fact expected to dominate the contribution. Recent work by Isidori *et al.* [106] has shown that although there are considerable uncertainties in the absolute calculation of the CP-conserving contribution, the ratio $B^{CP-cons}(K_L \to \pi^0\mu^+\mu^-)/B(K_L \to \pi^0\gamma\gamma)$ should be calculable to ~30%. Working from the average of the KTeV and NA48 results for $B(K_L \to \pi^0\gamma\gamma)$, they obtain $B^{CP-cons}(K_L \to \pi^0\mu^+\mu^-) = (5.2 \pm 1.6) \times 10^{-12}$.

Plugging $a_S = 1.2 \pm 0.2$ and $\mathrm{Im}\lambda_t = (1.36 \pm 0.12) \times 10^{-4}$ into Eqs. 11 and 13.

$$B(K_L \to \pi^0 ee)_{CPV} \approx [(22.6 \pm 7.5)_{mix} \pm (10.1 \pm 2.0)_{int} + (4.44 \pm 0.87)_{dir}] \times 10^{-12} \tag{17}$$

and

$$B(K_L \to \pi^0 \mu\mu)_{CPV} \approx [(5.3 \pm 1.8)_{mix} \pm (2.6 \pm 0.5)_{int} + (1.8 \pm 0.4)_{dir} \times 10^{-12} \tag{18}$$

To obtain a prediction for the complete branching ratios, one needs to fix the sign of the interference between direct and indirect CP-violation, and decide what to take for the CP-conserving part. Positive interference has been preferred by theorists from the earliest days of this subject[130][131], and up-to-date arguments are given in Ref. [129]. If we accept this choice, Eqs. 17 and 18 give $B(K_L \to \pi^0 ee)_{CPV} \approx (3.7 \pm 0.8) \times 10^{-11}$ and $B(K_L \to \pi^0 \mu\mu)_{CPV} \approx (1.0 \pm 0.2) \times 10^{-11}$ respectively. The CP-conserving contribution is a problem for the electron mode. If we use the NA48 result, from the above discussion we estimate $(4 \pm 4) \times 10^{-13}$ for this. If instead, we accept the KTeV result, to be conservative we should use a much larger value, $(4.0 \pm 3.5) \times 10^{-12}$. These give:

$$B(K_L \to \pi^0 e^+ e^-)_{\mathrm{NA48}} \approx (3.8 \pm 0.8) \times 10^{-11} \tag{19}$$

$$B(K_L \to \pi^0 e^+ e^-)_{\mathrm{KTeV}} \approx (4.1 \pm 0.9) \times 10^{-11}. \tag{20}$$

In the muonic mode, the CP-conserving contribution is much less problematic, and using the value discussed above, one gets:

$$B(K_L \to \pi^0 \mu^+ \mu^-) \approx (1.5 \pm 0.3) \times 10^{-11} \tag{21}$$

The current experimental status of $K_L \to \pi^0 \ell^+ \ell^-$ is summarized in Table VI and Fig.20. A factor ~ 1.3 more $K_L \to \pi^0 \mu^+ \mu^-$ data is expected from the KTeV 1999 run.

TABLE VI. – *Results on* $K_L \to \pi^0 \ell^+ \ell^-$.

Mode	90% CL upper limit	Est. bkgnd.	Obs. evts.	Ref.
$K_L \to \pi^0 e^+ e^-$	2.8×10^{-10}	2.05 ± 0.54	3	[132]
$K_L \to \pi^0 \mu^+ \mu^-$	3.8×10^{-10}	0.87 ± 0.15	2	[133]

As can be seen from the table and figure, background in both modes is already starting to be observed at a sensitivity an order of magnitude short of the expected signal level. The problems of extracting a value of $\mathrm{Im}\lambda_t$ from these modes have been discussed in Ref. [135] among other places, and summaries of various schemes to deal with these problems are given in Refs. [129] and [106].

Although the situation appears difficult, it's worthwhile taking a closer look:

1. The observation of $K_S \to \pi^0 \ell^+ \ell^-$ with higher than expected rates has made it likely that the SM branching ratios of the K_L modes are also higher than previously believed.

2. If we compare the single event sensitivity of the recent data with that of the residual $K_L \to \gamma\gamma\ell^+\ell^-$ which is the least tractable background, the ratio is much less than an order of magnitude. In their 1999 $K_L \to \pi^0 ee$ run, KTeV had a single event sensitivity of 1.04×10^{-10} and an estimated residual $K_L \to \gamma\gamma ee$ background of 0.99 ± 0.35 events[132]. This implies a S:B (signal to background) of 1:2.5. In their 1997 $K_L \to \pi^0 \mu\mu$ data, KTeV had a single event sensitivity of 0.75×10^{-11}

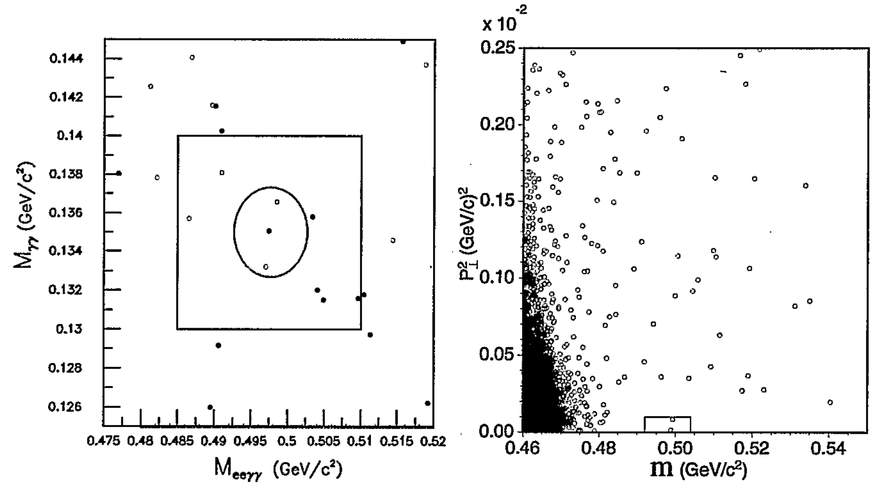

Fig. 20. – Signal planes showing candidates for $K_L \to \pi^0 e^+ e^-$ (left from Ref. [134] and [132]) and $K_L \to \pi^0 \mu^+ \mu^-$ (right from Ref. [133]).

and a calculated residual $K_L \to \gamma\gamma\mu\mu$ background of 0.37 ± 0.03 events [133]([8]). This implies S:B = 1:1.9.

3. The interest in $K_L \to \pi^0 \ell^+ \ell^-$ has evolved from being a possible source of information on $\mathrm{Im}\lambda_t$, to its role as an arena for probing BSM effects.

With this in mind one can ask, for example, what single event sensitivity would be needed to establish a factor two effect at 3σ in these modes, given the background levels mentioned above. For the electronic case, the answer is 10^{-12}; for the muonic case, 0.4×10^{-12}. In fact this sensitivity for the latter mode could have been reached by the KaMI experiment at Fermilab [136], had it gone forward, in about 3 years of running. If instead, the SM were correct, a 30% measurement of the BR would have resulted. Thus a next-generation experiment could make very useful measurements. Another example, given by Buchalla *et al.*, is illustrated in Fig. 21. It shows $B(K_L \to \pi^0 \mu^+ \mu^-)$ vs $B(K_L \to \pi^0 e^+ e^-)$ for the SM, and that for a recent model with enhanced electroweak penguins [137] designed to explain apparent anomalies in hadronic B decays. In this BSM scenario, a $K_L \to \pi^0 \mu^+ \mu^-$ experiment of 0.4×10^{-12}/event statistical sensitivity would observe 180 ± 13.4 events of which 110 would be signal, to be compared to 37 signal events expected for the SM.

([8]) They observed a total of 2 (presumably background) events. I am assuming the backgrounds other than $K_L \to \gamma\gamma\mu\mu$ can be suppressed in a future experiment.

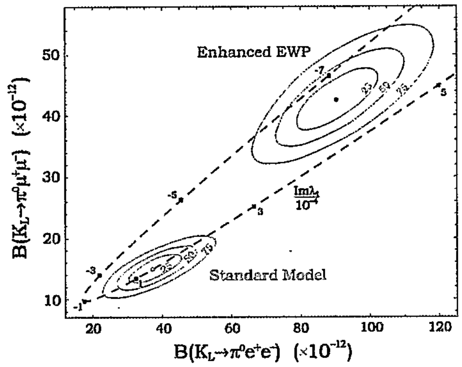

Fig. 21. – Relationship between $B(K_L \to \pi^0\mu^+\mu^-)$ and $B(K_L \to \pi^0 e^+e^-)$ for the SM and for a model that introduces new physics (see text for details). The 25%, 50%, and 75% CL regions (due to theoretical and parametric uncertainties) are shown for each case. From Ref. [106].

4. – Conclusions

Lepton flavor violation experiments have probed sensitivities corresponding to mass scales of well over 100 TeV, making life difficult for models predicting accessible LFV in kaon decay and discouraging new dedicated experiments of this type.

The existing precision measurement of $K_L \to \mu^+\mu^-$ will be very useful if theorists can make enough progress on calculating the dispersive long-distance amplitude, perhaps helped by experimental progress in $K_L \to \gamma\ell^+\ell^-$, $K_L \to 4$ leptons, etc. The exploitation of $K_L \to \mu^+\mu^-$ would also be aided by higher precision measurements of the branching ratios of some of the normalizing reactions, such as $K_L \to \gamma\gamma$.

$K^+ \to \pi^+\nu\bar{\nu}$ should clearly be further exploited. Two initiatives are devoted to reaching the 10^{-12}/event level: the P326 in-flight proposal to CERN, and an LOI for an advanced stopped-K^+ experiment at J-PARC. The first dedicated experiment to seek $K_L \to \pi^0\nu\bar{\nu}$ (E391a) is proceeding and there are plans to continue the pursuit of this reaction at J-PARC with the eventual goal of making a $\leq 10\%$ measurement of the branching ratio.

Measurements of $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ can determine an alternative unitarity triangle that will offer a critical comparison with results from the B system. If new physics is in play in the flavor sector, the two triangles will almost certainly disagree. Moreover these reactions can be calculated very precisely so that they can elucidate flavor couplings in almost any BSM theory.

There are no near-term plans to pursue $K_L \to \pi^0\ell^+\ell^-$ although the recent observation of $K_S \to \pi^0\ell^+\ell^-$ indicates that these reactions may be much more tractable than previously believed. They can probe BSM operators not accessible to $K \to \pi\nu\bar{\nu}$, so it is unfortunate that they will probably not be pursued unless a BSM signal is seen in the latter.

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