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COMPARING EXTRACTIONS OF SIVERS FUNCTIONS

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A comparison is given of the various recently published extractions of the Sivers functions from the HERMES and COMPASS data on single-transverse spin asymmetries in semi-inclusive deeply inelastic scattering.

1. Introduction

Single-spin asymmetries (SSA) in semi-inclusive deeply inelastic scattering (SIDIS) off transversely polarized nucleon targets have been under intense experimental investigation over the past few years.¹⁻⁶ Substantial asymmetries have been reported in some cases, in particular, with best statistics, by the HERMES collaboration for scattering off a proton target.

The importance of SSA lies in the fact that they provide new insights into QCD and nucleon structure.⁷⁻¹⁴ For instance, the asymmetry in SIDIS may contain an angular dependence of the form $\sin(\phi - \phi_S)$, where ϕ and ϕ_S denote respectively the azimuthal angles of the produced hadron and the target polarization vector with respect to the axis defined by the hard virtual photon.¹¹ This angular dependence arises from the so-called Sivers effect⁷ tightly related to notions of an intrinsic asymmetry in the parton transverse momentum distribution and angular momenta. Factorization theorems¹⁵⁻¹⁷ proven to leading power in the photon virtuality Q provide the basis for a QCD description of the process, and allow to extract the Sivers function from SIDIS data¹⁻⁴ and to use it for predictions for the SSA in the Drell-Yan (DY) process, hopefully to be explored experimentally at RHIC, COMPASS and the GSI. Comparisons of SIDIS and the DY process will be particularly important for testing our understanding

of the underlying physics, since it has been predicted^{13,14} that the Siverson functions appear with opposite signs in these two processes. The approach just outlined has been followed recently in Refs. [18–23]. In this note we compare the results of these papers for the extracted Siverson functions

$$\Delta^N f_{q/p^\uparrow}(x, \mathbf{p}_T^2) \equiv -\frac{2|\mathbf{p}_T|}{M_N} f_{1T}^{\perp a}(x, \mathbf{p}_T^2) \equiv -\frac{2|\mathbf{p}_T|}{M_N} q_T(x, \mathbf{p}_T^2). \quad (1)$$

In the extractions of the Siverson functions from SIDIS several simplifying approximations were common between the groups, namely the neglect of the so-called “soft factor”^{16,17} and the Siverson antiquark functions. Different approaches were, however, followed in Refs. [19–23] concerning the treatment of the dependence of the distributions on transverse parton momenta.

The Siverson SSA is obtained^{2,24} by weighting the events entering the spin asymmetry with $\sin(\phi - \phi_S)$. When analyzed in this way, however, specific models for the dependence on parton transverse momenta need to be made in the theoretical expression. By assuming that the transverse momentum dependence of the Siverson function is of the form $f_{1T}^{\perp a}(x, \mathbf{p}_T^2) = f_{1T}^{\perp a}(x) G(\mathbf{p}_T^2)$ and/or similarly for other distribution or fragmentation functions, the Siverson SSA as defined at HERMES² can be written generically as

$$A_{UT}^{\sin(\phi - \phi_S)} = (-2) \frac{\sum_a e_a^2 x F_{\text{Siv}}^a(x) D_1^{a/\pi}(z)}{\sum_a e_a^2 x f_1^a(x) D_1^{a/\pi}(z)}. \quad (2)$$

The factor (-2) is due to conventions²⁴ and $F_{\text{Siv}}^a(x)$ is some functional depending on $f_{1T}^{\perp a}$ and the model used for parton transverse momenta.

Notice that by including in addition a factor of $P_{h\perp}/M_N$ into the weight in (2) the resulting SSA can be interpreted model-independently in terms of the transverse moment of the Siverson function¹¹

$$f_{1T}^{\perp(1)a}(x) \equiv \int d^2\mathbf{p}_T \frac{P_T^2}{2M_N^2} f_{1T}^{\perp a}(x, \mathbf{p}_T^2) = - \int d^2\mathbf{p}_T \frac{|\mathbf{p}_T|}{4M_N} \Delta^N f_{q/p^\uparrow}(x, \mathbf{p}_T^2). \quad (3)$$

Such weighted SSA were argued to be less sensitive to Sudakov suppression which can be important for predictions involving the Siverson function. Preliminary HERMES data for such SSA are available¹ and were studied in Ref. [18], where a first fit for the transverse moment of the Siverson function (3) was obtained. The result of [18] is in good agreement with the studies of SSA analyzed *without* a power of $P_{h\perp}$ in the weight^{2–4} reported in Refs. [19–23]. The next Sections review and compare the fit results for the Siverson functions extracted in the different approaches in Refs. [19–23].

2. The approach of Refs. [19,20]

In Ref. [19] the azimuthal angular dependence (Cahn effect) of the SIDIS unpolarized cross section was used to extract the widths of the Gaussian p_T -dependent parton distribution (pdf) and fragmentation (ff) functions respectively as $\langle p_T^2 \rangle = 0.25 \text{ (GeV}/c)^2$ and $\langle K_T^2 \rangle = 0.2 \text{ (GeV}/c)^2$. A first estimate of the Siverson functions was then obtained by fitting the data on $A_{UT}^{\sin(\phi-\phi_S)}$ observed by HERMES collaboration.^{1,2} In Ref. [20] a novel fit on the new HERMES data⁴ together with data from the COMPASS collaboration³ was performed. In both fits the full exact kinematics was always adopted. The Siverson function (u, d quarks) was parameterized as:

$$\Delta^N f_{q/p^\uparrow}(x, \mathbf{p}_T^2) = 2 N_q(x) f_{q/p}(x) g(\mathbf{p}_T^2) h(\mathbf{p}_T^2), \quad (4)$$

$$N_q(x) = N_q x^{a_q} (1-x)^{b_q} \frac{(a_q + b_q)^{(a_q + b_q)}}{a_q^{a_q} b_q^{b_q}}, \quad g(\mathbf{p}_T^2) = \frac{e^{-p_T^2/\langle p_T^2 \rangle}}{\pi \langle p_T^2 \rangle}. \quad (5)$$

Two options for the $h(\mathbf{p}_T^2)$ function were considered, namely:

$$(a) \quad h(\mathbf{p}_T^2) = \frac{2p_T M_0}{p_T^2 + M_0^2}, \quad (b) \quad h(\mathbf{p}_T^2) = \sqrt{2} e \frac{p_T}{M'} e^{-p_T^2/M'^2}, \quad (6)$$

the latter allowing, at leading order in p_T/Q , to give for F_{Siv}^a in Eq. (2):

$$F_{\text{Siv}}^a(x) = \frac{\sqrt{\pi}}{2} \frac{M_N}{\sqrt{\langle p_T^2 \rangle + \langle K_T^2 \rangle/z^2}} f_{1T}^{\perp(1)a}(x) \quad \text{with} \quad \langle \widehat{p_T^2} \rangle = \frac{\langle p_T^2 \rangle}{1 + \langle p_T^2 \rangle/M'^2}. \quad (7)$$

In the fits, $f_{q/p}(x)$ was taken from the LO MRST01 set²⁵, whereas Kretzer's set²⁶ for the LO ff was used. The 7 parameters were then extracted as²⁰:

$$\begin{aligned} N_u &= 0.32 \pm 0.11 & a_u &= 0.29 \pm 0.35 & b_u &= 0.53 \pm 3.58 \\ N_d &= -1.0 \pm 0.12 & a_d &= 1.16 \pm 0.47 & b_d &= 3.77 \pm 2.59 \\ M'^2 &= 0.55 \pm 0.38 & (M_0^2 &= 0.32 \pm 0.25) & & \text{(GeV}/c)^2, \end{aligned} \quad (8)$$

with a χ^2 per degree of freedom (χ_{dof}^2) of 1.06. The one-sigma band shown in Fig. 1 (Eq. (6b)) takes into account the errors with their correlations.

These results were then used to give predictions for SSA measurable in SIDIS and DY processes for various kinematical configurations.

These effects were also invoked^{9,10,27,28} to generate SSA for other processes in hadron-hadron-collisions^{29,30} although the status of factorization is less clear in this case. Here we only point out that the SIDIS data are sensitive to much smaller x values than the E704 (STAR) ones.

3. The approach of Ref. [21]

In Ref. [21] it was assumed that the final hadron's transverse momentum is entirely due to the transverse-momentum dependence in the Siverson function. There is then no further assumption on the particular form of this dependence; rather it is integrated out in order to compare to the experimental data. The transverse momenta contributed by the other factors in the factorization formula will give some smearing effects which may be viewed as "sub-dominant". (However, we emphasize that this will not be true toward small z where the transverse momentum in the fragmentation functions will become important, likely resulting in a suppression of the asymmetry at small z .) The "1/2-moments" of the Siverson functions were then introduced in Ref. [21] in the fit to the experimental data:

$$q_T^{(1/2)}(x) \equiv \int d^2 \mathbf{p}_T \frac{|\mathbf{p}_T|}{M_N} f_{1T}^{1q}(x, \mathbf{p}_T^2). \quad (9)$$

These appear in an expression of the form (2) for the Siverson asymmetry, where

$$F_{\text{Siv}}^q(x) = \frac{1}{2} q_T^{(1/2)}(x). \quad (10)$$

In the actual fit to the HERMES data in²¹ the functions $q_T^{(1/2)}(x)$ were modeled in terms of the unpolarized u -quark distribution as

$$\frac{u_T^{(1/2)}(x)}{u(x)} = S_u x(1-x), \quad \frac{d_T^{(1/2)}(x)}{u(x)} = S_d x(1-x), \quad (11)$$

where $u(x)$ was taken from the GRV LO parameterizations for the unpolarized parton distributions.³¹ Furthermore, Kretzer's set for the LO fragmentation functions²⁶ was used. The fit to the new preliminary HERMES data gave

$$S_u = -0.81 \pm 0.07, \quad S_d = 1.86 \pm 0.28, \quad (12)$$

with $\chi_{\text{dof}}^2 \approx 1.2$. A fit to the old published HERMES data gave instead $S_u = -0.55 \pm 0.37$ and $S_d = 1.1 \pm 1.6$, with a similar size of χ_{dof}^2 . The COMPASS data were not included in the fit performed in²¹, but a comparison of the fit with the data was given, showing good agreement. The results of the fit to the HERMES data were furthermore used for making predictions for the SSAs in the Drell-Yan process and in di-jet and jet-photon correlations at RHIC.

4. The approach of Refs. [22,23]

In Ref. [22, 23] the distributions of transverse parton momenta in f_1^a , $f_{1T}^{\perp a}$ and D_1^a were assumed to be Gaussian with the respective widths $\langle p_T^2 \rangle$, $\langle p_T^2 \rangle_{\text{Siv}}$ and $\langle K_T^2 \rangle$ taken to be flavour- and x - or z -independent. In this model the F_{Siv}^a defined in (2) is given by the expression in Eq. (7) with $\langle \widehat{p_T^2} \rangle$ replaced by $\langle p_T^2 \rangle_{\text{Siv}}$.

The values $\langle K_T^2 \rangle = 0.16 (\text{GeV}/c)^2$, $\langle p_T^2 \rangle = 0.33 (\text{GeV}/c)^2$ were extracted²² from the HERMES data³² on $\langle P_{h\perp} \rangle$ and are similar to those discussed in Sec. 2, while $\langle p_T^2 \rangle_{\text{Siv}} \in [0.01; 0.32] (\text{GeV}/c)^2$ remained poorly constrained by positivity³³ – still allowing an extraction of the *transverse moment* of the Siverson function (3).

In order to reduce the number of fit parameters the prediction³⁴ from the limit of a large number of colours N_c was imposed:

$$f_{1T}^{\perp u}(x, \mathbf{p}_T^2) = -f_{1T}^{\perp d}(x, \mathbf{p}_T^2) \quad \text{modulo } 1/N_c \text{ corrections.} \quad (13)$$

The best fit²² (using parameterizations^{35,36}) to the *published data*² is

$$x f_{1T}^{\perp(1)u}(x) \stackrel{\text{ansatz}}{=} A x^b (1-x)^5 \stackrel{\text{fit}}{=} -0.17 x^{0.66} (1-x)^5 \quad (14)$$

with a $\chi_{\text{dof}}^2 \sim 0.3$, and a $1\text{-}\sigma$ uncertainty of roughly $\pm 30\%$. This result agrees well with the fit to the preliminary $P_{h\perp}$ -weighted HERMES data¹, which were analyzed in a (transverse parton momentum) model-independent way [18]. The good agreement of the results in Refs. [18, 22] is an important cross check for the applicability of the Gauss model to the description of SSA in SIDIS.

For sake of a better comparison to the results by the other groups^{19,20,21} the above fit procedure was applied²³ to the most recent and more precise *preliminary* HERMES data.⁴ The new fit has a $\chi_{\text{dof}}^2 \sim 2$ and is consistent²³ with that quoted in Eq. (14). One has to keep in mind that the large- N_c relation (13) is a useful constraint at the *present stage*, and will have to be relaxed when future more precise data will become available.

Note that for $\langle K_T^2 \rangle \rightarrow 0$ in (7) one obtains $F_{\text{Siv}}^a(x) \rightarrow \frac{1}{2} f_{1T}^{\perp(1/2)a}(x)$ within the Gaussian model. This limit means that the produced hadron acquires no additional transverse momentum from the fragmentation process, i.e. $D_1^a(z, \mathbf{K}_T^2) = D_1^a(z) \delta^{(2)}(\mathbf{K}_T)$. In this sense, the approach of Ref. [21] discussed in Sec. 3, c.f. Eq. (10), is contained as a limiting case in the Gauss ansatz.

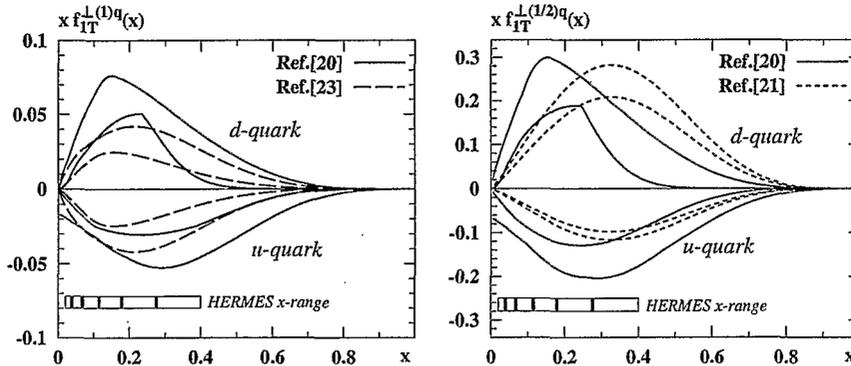


Figure 1. The first and 1/2-transverse moments of the Siverson quark distribution functions, defined in Eqs. (3, 9), as extracted in Refs. [20, 21, 23]. The fits were constrained mainly (or solely) by the preliminary HERMES data⁴ in the indicated x -range. The curves indicate the $1\text{-}\sigma$ regions of the various parameterizations.

5. Comparison of the results and Conclusions

It should be stressed that the various fit results, when used within the respective approaches, provide equally good descriptions of the HERMES and COMPASS data. Here we compare only those analyses^{20,21,23} in which the most recent and more precise *preliminary* HERMES data⁴ were used.

In Fig.1a we compare the fits for $f_{1T}^{\perp(1)q}$ from Refs. [20, 23], and in Fig.1b the fits for $f_{1T}^{\perp(1/2)q}$ from Refs. [20, 21]. (A direct comparison of [21] and [23] is not possible.) In view of the different models assumed for the transverse parton momenta and the varying fit Ansätze, we observe a satisfactory *qualitative* agreement — in the x -region constrained by the HERMES data. However, a closer look reveals differences between the results in Fig. 1, which indicate the size of the systematic uncertainties of the three Siverson function fits mainly due to the use of different models for the parton transverse momenta. These uncertainties were not estimated in Refs. [20, 21, 23].

We have presented a comparison of three extractions^{20,21,23} of Siverson functions from HERMES and COMPASS data on single-transverse spin asymmetries in SIDIS. The three approaches somewhat differ, but they describe the data with similar quality. The fits are in good qualitative agreement, though there are differences with regard to the size and shape

of the extracted Sivers functions. These differences reflect the model dependence of the fit results which gives rise to a certain theoretical systematic uncertainty of the fit results. The latter seems, however, less dominant than the statistical uncertainty of the fits at the present stage.

It is clear that further information from experiment will be vital. For now, one cannot really expect to obtain much more than a first qualitative picture of the Sivers functions. We also emphasize that it will be crucial for the future to experimentally confirm the leading-power nature of the observed spin asymmetries. For this, forthcoming COMPASS or JLab data for scattering off a proton target and studies of the Q^2 -dependence of the asymmetries will be important.

The good qualitative agreement between the different approaches observed here means that the predictions^{18–23} for the magnitude of the Sivers effect in DY are robust — in the kinematic region constrained by the HERMES data. This solidifies the conclusions^{18–23} that the predicted sign reversal of the Sivers function between SIDIS and DY, can be tested in running or future experiments at RHIC, COMPASS and PAX.

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