

*Violation of  $k$ -perpendicular factorization in quark production  
from the Color Glass Condensate*

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# Violation of $k_{\perp}$ factorization in quark production from the Color Glass Condensate

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We examine the violation of the  $k_{\perp}$  factorization approximation for quark production in high energy proton-nucleus collisions. We comment on its implications for the open charm and quarkonium production in collider experiments.

## 1. Introduction

Semi-hard processes, where  $\sqrt{s} \gg m_{q_{\perp}} \gg \Lambda_{\text{QCD}}$ , contribute significantly to particle production in high-energy collider experiments due to the large density of the small- $x$  gluons. The  $k_{\perp}$  factorization formalism[ 1] systematically resums corrections of  $(\alpha_s \ln(s/q_{\perp}^2))^n$  from gluon branchings in perturbative QCD. In this framework, the particle production cross-section is expressed as a convolution of a hard matrix element and *unintegrated* distributions of gluons in the hadrons with definite transverse momentum  $\mathbf{k}_{i\perp}$  and longitudinal fraction  $x_i$  in each projectile hadron ( $i=1, 2$ ).

Multiple-scattering (higher twist) effects become important at small  $x$  due to the large density of small- $x$  gluons. It is expected to be the origin of the Cronin enhancement and  $p_{\perp}$  broadening of hadrons observed in nuclear experiments. It is also relevant for the nuclear suppression of quarkonium production.

The simplest situation for studying the impact of higher twist effects on  $k_{\perp}$  factorization is in proton-nucleus (pA) collisions, wherein the proton is dilute and the nucleus is dense. The  $k_{\perp}$  factorization formalism was examined in the color glass condensate framework[ 2]. It is shown that the factorization is recovered when one keeps only the terms that are of the lowest order in the charge sources  $\rho_{p,A}$  of the projectiles[ 3]. The cross-sections at the leading order in  $\rho_p$ , but at all orders in the dense source  $\rho_A$  of the nucleus are obtained analytically. Gluon production by the “2-to-1” processes is shown to be  $k_{\perp}$ -factorizable [ 4, 5, 6] whereas the quark production is generally not[ 7, 8, 9].

Here we report the numerical estimates for the  $k_{\perp}$  factorization breaking in quark production within the McLerran-Venugopalan (MV) model[ 10]. We briefly discuss open charm production and quarkonium suppression in pA collisions in this framework.

## 2. Violation of $k_\perp$ factorization in quark pair production

The quark pair production cross-section is obtained as [ 7]:

$$\begin{aligned}
\frac{d\sigma}{d^2\mathbf{p}_\perp d^2\mathbf{q}_\perp dy_p dy_q} &= \frac{\alpha_s^2 N}{8\pi^4 (N^2 - 1)} \int_{\mathbf{k}_{1\perp}, \mathbf{k}_{2\perp}} \frac{\delta(\mathbf{p}_\perp + \mathbf{q}_\perp - \mathbf{k}_{1\perp} - \mathbf{k}_{2\perp})}{k_{1\perp}^2 k_{2\perp}^2} \\
&\times \left\{ \int_{\mathbf{k}_\perp, \mathbf{k}'_\perp} \text{tr}_d \left[ (\not{q} + m) T_{q\bar{q}} (\not{p} - m) \gamma^0 T_{q\bar{q}}^\dagger \gamma^0 \right] \phi_A^{q\bar{q}, q\bar{q}}(\mathbf{k}_{2\perp}; \mathbf{k}_\perp, \mathbf{k}'_\perp) \right. \\
&\quad + \int_{\mathbf{k}_\perp} \text{tr}_d \left[ (\not{q} + m) T_{q\bar{q}} (\not{p} - m) \gamma^0 T_g^\dagger \gamma^0 + \text{h.c.} \right] \phi_A^{q\bar{q}, g}(\mathbf{k}_{2\perp}; \mathbf{k}_\perp) \\
&\quad \left. + \text{tr}_d \left[ (\not{q} + m) T_g (\not{p} - m) \gamma^0 T_g^\dagger \gamma^0 \right] \phi_A^{g, g}(\mathbf{k}_{2\perp}) \right\} \varphi_p(\mathbf{k}_{1\perp}), \quad (1)
\end{aligned}$$

where the explicit forms for the Dirac matrices  $T_{q\bar{q}}(\mathbf{k}_{1\perp}, \mathbf{k}_\perp)$  and  $T_g(\mathbf{k}_{1\perp})$  are given in [ 7]. Here  $\varphi_p(l_\perp) \equiv (\pi^2 R_p^2 g^2 / l_\perp^2)$  F.T.  $\langle \rho_p^a(\mathbf{0}) \rho_p^a(\mathbf{x}_\perp) \rangle$  is the unintegrated gluon distribution for the proton, and F.T. denotes the Fourier transformation. One needs, however, *three* nuclear distributions defined as (see Eqs. (42), (43) and (45) in [ 7])

$$\begin{aligned}
\phi_A^{g, g}(l_\perp) &\equiv \frac{\pi^2 R_A^2 l_\perp^2}{g^2 N} \text{F.T. tr} \langle U(\mathbf{0}) U^\dagger(\mathbf{x}_\perp) \rangle, \\
\phi_A^{q\bar{q}, g}(l_\perp; \mathbf{k}_\perp) &\equiv \frac{2\pi^2 R_A^2 l_\perp^2}{g^2 N} \text{F.T. tr} \langle \tilde{U}(\mathbf{x}_\perp) t^a \tilde{U}^\dagger(\mathbf{y}_\perp) t^b U_{ba}(\mathbf{0}) \rangle, \\
\phi_A^{q\bar{q}, q\bar{q}}(l_\perp; \mathbf{k}_\perp; \mathbf{k}'_\perp) &\equiv \frac{2\pi^2 R_A^2 l_\perp^2}{g^2 N} \text{F.T. tr} \langle \tilde{U}(\mathbf{0}) t^a \tilde{U}^\dagger(\mathbf{y}_\perp) \tilde{U}(\mathbf{x}'_\perp) t^a \tilde{U}^\dagger(\mathbf{y}'_\perp) \rangle, \quad (2)
\end{aligned}$$

where  $U$  and  $\tilde{U}$  denote the path-ordered exponentials of the gauge fields in the nucleus in the adjoint and fundamental representations, respectively, and describe the multiple scatterings of the gluon and the quarks. The average  $\langle \dots \rangle$  is taken over the Gaussian distribution of the color charge sources characterized by the saturation scale  $Q_s^2$ .

$k_\perp$  factorization is violated by the transverse structure of the quark pair probed by the momentum  $\mathbf{k}_\perp^{(i)}$  from the nucleus since each quark from the pair can resolve and interact with several gluons from the nucleus. If any of the transverse masses  $m_{q_\perp}$  and  $m_{p_\perp}$  of the produced quarks is large compared with the typical rescattering scale,  $Q_s$ , we can neglect  $\mathbf{k}_\perp^{(i)}$  in  $T_{q\bar{q}}(\mathbf{k}_{1\perp}, \mathbf{k}_\perp^{(i)})$  and recover the  $k_\perp$  factorized formula thanks to the sum rule for  $\phi_A$ 's;  $\int_{\mathbf{k}_\perp, \mathbf{k}'_\perp} \phi_A^{q\bar{q}, q\bar{q}} = \int_{\mathbf{k}_\perp} \phi_A^{q\bar{q}, g} = \phi_A^{g, g}$ .

In Fig. 1 we compare the exact result with the  $k_\perp$  factorized approximation for single charm quark production. The breaking is relatively small for the saturation momentum  $Q_s^2=1$  GeV<sup>2</sup>, which may be the relevant scale for RHIC at central rapidity. At  $Q_s^2=15, 25$  GeV<sup>2</sup> (corresponding to very forward rapidities in the proton fragmentation region at RHIC and LHC) the correction can be as large as 40% at  $q_\perp \sim Q_s$ . For the bottom quark production the violation is smaller. To assess the model-dependence of our results, we compute them now, shown in Fig. 2, with a non-local Gaussian model known to be the

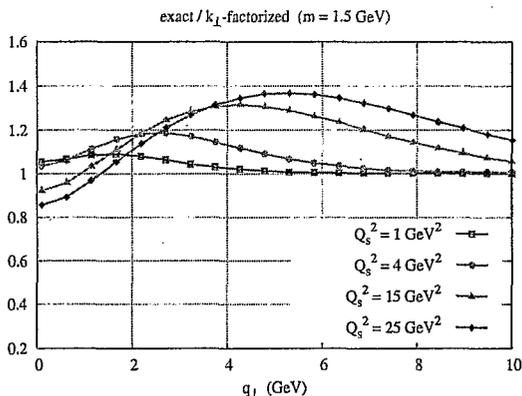


Figure 1. Breaking of  $k_{\perp}$  factorization in single charm quark production.

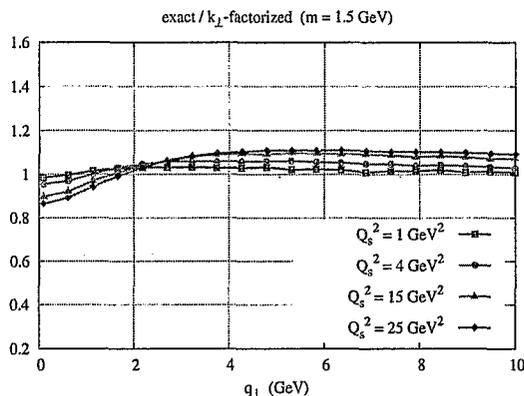


Figure 2. The same as in Fig. 1 but in the nonlocal Gauss model.

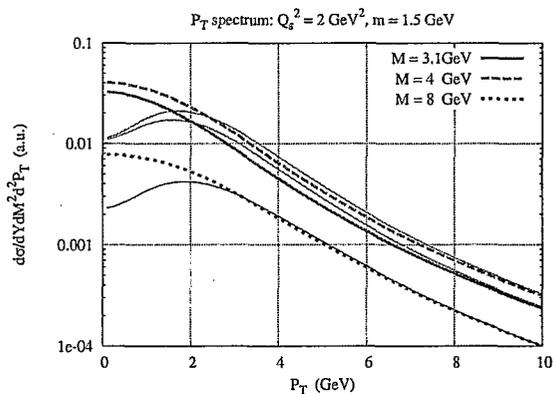


Figure 3.  $P_{\perp}$  spectrum of the quark pair with fixed invariant mass  $M$ .

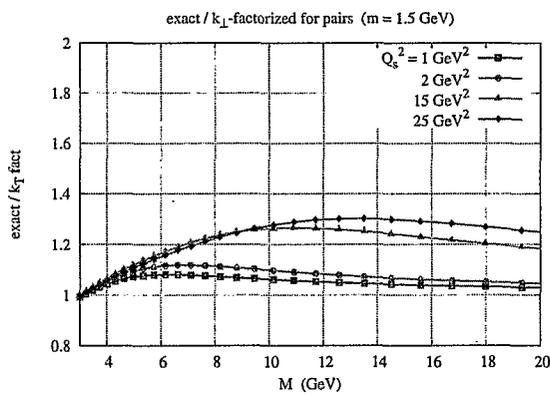


Figure 4. Breaking of  $k_{\perp}$  factorization in charm quark pair production.

asymptotic solution of renormalization equations for  $x$  evolution [11]; non-linear evolution effects reduce the magnitude of the violation of  $k_{\perp}$  factorization.

In Fig. 3 shown is the total  $P_{\perp}$  distribution of the charm quark pair with the fixed invariant masses  $M=3.1, 4, 8$  GeV. In the  $k_{\perp}$  factorized approximation (thin curves), either quark or antiquark exchanges all the momentum from the nucleus and we see the bump structure near  $Q_s$ , reflecting the gluon distribution of the nucleus. The bump is smeared out due to multiple scatterings of both the quark and antiquark in the full formula. Integrating over  $P_{\perp}$ , we show in Fig. 4, the magnitude of factorization breaking in the invariant mass spectrum of the pair.

### 3. Phenomenology

We study the importance of small- $x$  distributions in D meson production by convoluting the single quark spectrum with an appropriate fragmentation function [12]. We find, however, the production spectrum is determined not by the quark distribution with  $q_{\perp} \lesssim Q_s$ , but largely by the tail part  $\propto 1/q_{\perp}^4$  of the MV model. Moreover, in order to assess

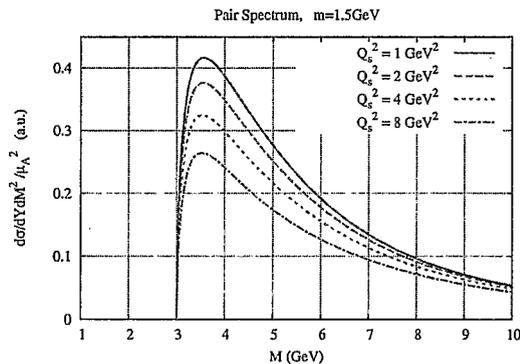


Figure 5.  $Q_s^2$  dependence of the charm pair production.

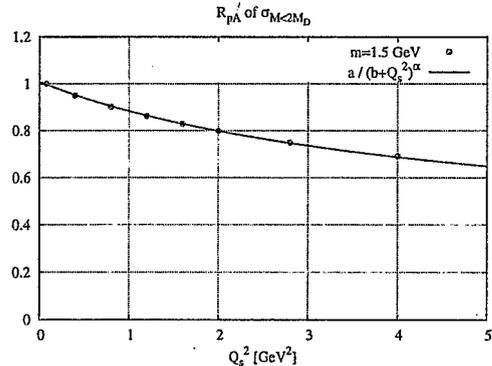


Figure 6. Suppression of low mass pairs in pA collisions.

the rapidity dependence of open charm production, the  $x$ -dependence of the unintegrated gluon distributions should be taken into account, which requires going beyond the MV model. Our results on open charm production will be reported elsewhere [13].

The  $Q_s^2$ -dependence of the pair spectrum (divided by the charge density  $\mu_A^2$ ) is displayed in Fig. 5. At larger  $M$ , where the high-density effects are diminished, all curves converge to a single one. The multiple scatterings of the pair quarks suppress the yield in the low  $M$  region. (The overall cross-section is of course enhanced with increasing  $Q_s^2$ .) One can get an idea about the normal suppression of the quarkonium production in the pA collisions, relying on the color evaporation picture. We show the nuclear modification ratio,  $R_{pA}$ , for the pairs with  $M$  less than the open charm threshold  $2M_D$ , as a function of  $Q_s^2$ . The suppression pattern fits the form  $1/(Q_s^2)^\alpha$  with  $\alpha \sim 0.42$ , and not the frequently assumed exponential form. One should note here that  $Q_s^2 \sim A^{1/3}$  in the MV model.

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