BNL-75353-2006-CP

QUARK ANTIQUARK ENERGIES AND THE SCREENING MASS IN A QUARK-GLUON PLASMA AT LOW AND HIGH TEMPERATURES

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Presented at the Workshop on Extreme QCD
Swansea, Wales
2-5 August 2005

February 2006

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Abstract
We discuss quark antiquark energies and the screening mass in hot QCD using the non-perturbative lattice approach. For this purpose we analyze properties of quark antiquark energies and entropies at infinitely large separation of the quark antiquark pair at low and high temperatures. In the limit of high temperatures these energies and entropies can be related perturbatively to the temperature dependence of the Debye mass and the coupling. On the one hand our analysis thus suggests that the quark antiquark energies at (infinite) large distances are rather related to the Debye screening mass and the coupling than to the temperature dependence of heavy-light meson masses. On the other hand we find no or only little differences in all mass scales introduced by us when changing from quenched to 2-flavor QCD at temperatures which are only moderately above the phase transition.

1 The Debye screening mass

Color screening has been considered as an important mechanism for deconfinement, and in this context, quarkonium suppression as signal for quark-gluon plasma production in heavy ion collision experiments [1, 2]. The temperature dependence of the Debye screening mass, $m_D(T)$, provides an important measure for the strength of the screening property of the QCD medium. Due to difficulties when calculating the Debye mass in perturbation theory at temperatures which are only moderately above the phase transition temperature (T_c), the calculation of the screening mass has been to large extent considered using the non-perturbative lattice approach. In this approach one usually discusses either the infrared limit of the gluon propagator and/or the large distance color screened Coulomb behavior of quark antiquark free energies [3, 4, 5, 6, 7, 8]. We will discuss here the property that the screening mass can also be related at asymptotic high temperatures to the large distance behavior of quark antiquark energies and entropies [9, 10, 11, 12]. In particular, we conclude that the finite values of the finite temperature energies which are approached at (infinitely) large distances are rather related to the temperature dependence of the Debye mass and the coupling than to the temperature dependence of masses of corresponding heavy-light meson systems.

1.1 $Q\bar{Q}$ Free Energy, Entropy and the Screening mass

The presence of static color charges will polarize the medium and the parton density will change compared to the density distribution of the heat bath without containing static charges. In particular, the parton density is considered to rapidly increase in the close vicinity of the static charges and the energy and entropy which is needed to neutralize the additional charges will in general depend on temperature (T) and the distance (r) between the static charges. This property is illustrated in Fig. 1 (see also our discussion in Ref. [13]): In the limit of large distances ($r \to \infty$) the parton clouds can be considered as well separated (a). In this case the change in energy (free or internal energy) and entropy due to the presence of static charges in the heat bath will only depend on temperature [1]. While the change in free energy at (asymptotic) large distances, $F_\infty(T) \equiv \lim_{r\to\infty} F_1(r,T)$, is a steadily decreasing function with

[1] Of course this property changes qualitatively in quenched QCD at temperatures below deconfinement where the confinement forces should be related to a *string* (which cannot break in quenched QCD) rather than to *clouds* (which can be separated).

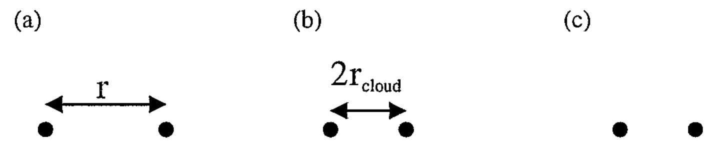

Fig. 1: Illustration of screening through the polarization of gluons in the medium (taken from Ref. [13]): (a) At large distances ($r \to \infty$) the gluon clouds which surround the static test charges are well separated. At smaller distances the clouds begin to overlap (b) and the geometric structure of the clouds will also depend on the separation between the test charges (c).

increasing temperatures, the corresponding change in internal energy, $U_\infty(T) \equiv -T^2\partial(F_\infty/T)/\partial T$, and entropy, $S_\infty(T) \equiv -\partial F_\infty/\partial T$, behave more complicated as function of temperature [10, 15]. In particular, the energy and entropy which are needed to neutralize the static charges in the heat bath show a strong increase at temperatures close to the transition temperature and exhibit a sharp peak at the phase transition [10, 15]. At high temperatures, however, the internal energy is continuously decreasing with increasing temperatures while the entropy vanishes slowly in the limit of high temperatures. This behavior is also expected at asymptotic high temperatures from perturbation theory (following [10, 16]) where the temperature dependence of $F_\infty(T)$, $U_\infty(T)$ and $S_\infty(T)$ can be related to the Debye mass (m_D) and the coupling ($\alpha = g^2/(4\pi)$), *i.e.*

$$F_\infty(T) \simeq -\frac{4}{3}m_D(T)\alpha(T) , \quad (1)$$

$$U_\infty(T) \simeq 4m_D(T)\alpha(T)\frac{\beta(g)}{g(T)} \simeq -O(Tg^5) , \quad (2)$$

$$S_\infty(T) \simeq +\frac{4}{3}\frac{m_D(T)}{T}\alpha(T) , \quad (3)$$

where $\beta(g)$ denotes the QCD β-function. These relations already indicate that it could be misleading to relate the large distance values of the finite temperature energies to the temperature dependence of masses of corresponding heavy-light meson systems.

When going to smaller distances ($r < \infty$), however, the parton clouds will overlap (see Fig. 1: b, c) and the finite temperature energies and the entropy will also depend on distance, *i.e.* $F_1 \equiv F_1(r,T)$, $U_1 \equiv U_1(r,T)$ and $S_1 \equiv S_1(r,T)$. In this case high temperature perturbation theory suggests a color screened Coulombic behavior for the r-dependence of the free energy, *i.e.*

$$\delta F_1(r,T) \equiv F_1(r,T) - F_\infty(T) \simeq -\frac{4}{3}\frac{\alpha(T)}{r}e^{-m_D(T)r} . \quad (4)$$

In the limit of small distances ($r \to 0$), however, the quark antiquark free energy is given by the heavy quark potential, $V(r)$, at zero temperature, $F_1(r \ll 1/T, T) \simeq V(r)$. This property can be used to fix the finite temperature energies at small distances to the heavy quark potential. Once the free energy is fixed at small distances also the large distance behavior is properly determined, *i.e.* $F_\infty(T)$ is properly fixed, and it can no longer be assumed that the finite temperature energies and the entropy approach zero at large distances [9].

1.2 Lattice Results

We have calculated quark antiquark free energies as well as the corresponding internal energies and entropies at several temperatures below and above the deconfinement phase transition using the non-perturbative lattice approach in quenched [7, 9, 11] and 2-flavor QCD [8, 10, 14]. Above T_c, we extracted the Debye mass, $m_D(T)$, using a best fit analysis of the color singlet free energy with a color screened Coulomb form at large distances with respect to Eq. (4). Here, we may also define certain mass scales,

$\mu_{F_\infty}(T)$ and $\mu_{S_\infty}(T)$, by using Eqs. (1, 3), *i.e.* we use the temperature dependence of the large distance plateau values of the free energy (F_∞) and entropy (S_∞) and define

$$\mu_{F_\infty}(T) \equiv -\frac{3}{4\alpha(T)} \times F_\infty(T) , \quad \mu_{S_\infty}(T) \equiv +\frac{3}{4\alpha(T)} \times TS_\infty(T) . \tag{5}$$

In the limit of high temperatures both, $\mu_{F_\infty}(T)$ and $\mu_{S_\infty}(T)$, are expected to characterize the temperature dependence of the Debye mass, $m_D(T)$.

Of course, both mass scales defined in (5) will depend on the proper definition of the coupling which enters (5) and is used to fix $\mu_{F_\infty}(T)$ and $\mu_{S_\infty}(T)$. Assuming vanishing quark masses in QCD, we fix $\mu_{F_\infty}(T)$ and $\mu_{S_\infty}(T)$ using the temperature dependence of the 2-loop running coupling in the $\overline{MS}$-scheme,

$$g^{-2}_{2-loop}(T) = 2\beta_0 \ln\left(\frac{\mu T}{\Lambda_{\overline{MS}}}\right) + \frac{\beta_1}{\beta_0} \ln\left(2\ln\left(\frac{\mu T}{\Lambda_{\overline{MS}}}\right)\right) , \tag{6}$$

with

$$\beta_0 = \frac{1}{16\pi^2}\left(11 - \frac{2N_f}{3}\right) , \quad \beta_1 = \frac{1}{(16\pi^2)^2}\left(102 - \frac{38N_f}{3}\right) , \tag{7}$$

where we used $T_c/\Lambda_{\overline{MS}} = 1.14(4)$ [17, 18] in quenched ($N_f = 0$) and $T_c/\Lambda_{\overline{MS}} = 0.77(21)$ [19, 20] in 2-flavor ($N_f = 2$) QCD. In both cases, quenched and 2-flavor QCD, we fixed the renormalization scale μ to be 2π. Despite the dependence of the mass scales introduced in (5) from the definition of the coupling, $\mu_{F_\infty}(T)$ will also depend on the fixing of the heavy quark potential at zero temperature, $V(r)$, which is used for renormalization of the free energy and could add an overall constant contribution to $F_\infty(T)$. Here and in what follows we fixed the heavy quark potential as described in Refs. [9, 21]. Our results for the different mass scales $\mu_{F_\infty}(T)$ and $\mu_{S_\infty}(T)$ are shown in Fig. 2 as function of temperature, T/T_c, for quenched (filled symbols) and 2-flavor QCD (open symbols) at temperatures ranging from T_c up to temperatures about $5.5T_c$. We also compare in that figure $\mu_{F_\infty}(T)$ and $\mu_{S_\infty}(T)$ to the temperature dependence of the Debye mass, $m_D(T)$, recently obtained in quenched [7] and 2-flavor QCD [8].

Following the discussions of the Debye mass in Refs. [7, 8], $m_D(T)$ is about 500 MeV at temperatures close above the phase transition and continuously increases with increasing temperatures to about 2000 MeV at $4T_c$. In fact, the steadily increasing Debye mass can be well described with a perturbative inspired ansatz,

$$m_D(T) = A_{N_f}\sqrt{1 + \frac{N_f}{6}}\, g(T)\, T , \tag{8}$$

allowing for a non-perturbative overall multiplicative correction, A_{N_f}, and using for the temperature dependence of the coupling, $g(T)$, the perturbative 2-loop running coupling given in Eq. (6) and a renormalization scale $\mu = 2\pi$. For temperatures moderately above the transition one finds $A_{N_f=0} \simeq 1.51(2)$ in quenched [7] and $A_{N_f=2} \simeq 1.42(2)$ in 2-flavor QCD [8]. The latter estimate for $m_D(T)$ in 2-flavor QCD is shown in Fig. 2 as solid lines (including the error on $A_{N_f=2}$). We again stress here (see also our discussion of the screening mass and length in Refs. [8, 13, 15]) we see no or only little differences between $m_D(T)$ calculated in quenched and 2-flavor QCD in the entire temperature range shown in Fig. 2. A different discussion concerning this issue has recently been given in Refs. [22, 23].

We also show in Fig. 2 the temperature dependence of the mass scales $\mu_{F_\infty}(T)$ and $\mu_{S_\infty}(T)$ obtained from the infinite large distance values of the quark antiquark free energy and entropy. At low temperatures, *i.e.* temperatures below $3T_c$, $\mu_{F_\infty}(T)$ indicates negative values while it changes sign at about $3T_c$ and continuously increases with increasing temperature. In fact, below $3T_c$ the value which is approached by the free energy at large distances, $F_\infty(T)$, is still positive and thus may indicate a

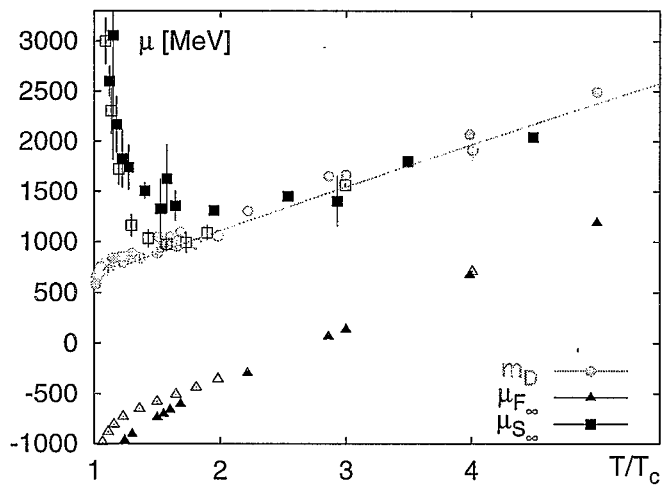

Fig. 2: The Debye screening mass, $m_D(T)$, obtained in quenched (filled circles) [7] and 2-flavor QCD (open circles) [8] as function of T/T_c. To convert $m_D(T)$ to physical units we used $T_c = 270$ MeV for quenched and $T_c = 202$ MeV for 2-flavor QCD. The lines indicate a best-fit analysis of the perturbatively inspired ansatz (8) for $m_D(T)$ in 2-flavor QCD (see Ref. [8] for further details). We also show here lattice results for the different mass scales $\mu_{F_\infty}(T)$ and $\mu_{S_\infty}(T)$ introduced in the text (see Sec. 1) obtained in quenched (filled symbols) and 2-flavor QCD (open symbols).

temperature regime which is not quite accessible with leading order perturbation theory, *i.e.* the leading order approximation given in Eq. (1) does indicate a misleading sign for $F_\infty(T)$ in this temperature regime (below $3T_c$). At low temperatures it thus makes no sense to identify $\mu_{F_\infty}(T)$ with a scale that characterizes the Debye mass. The mass scale, however, which is obtained from the finite values of the quark antiquark entropy, $\mu_{S_\infty}(T)$, is also shown in Fig. 2 for quenched (filled squares) and full QCD (open squares) and does not depend on the fixing of $V(r)$. At temperatures close above the deconfinement phase transition $\mu_{S_\infty}(T)$ approaches unexpected values about 3000 MeV. It, however, rapidly drops below 1500 MeV already at temperatures about twice as large than T_c and than starts to increase with increasing temperatures as expected in perturbation theory for a mass scale that might mimic the temperature dependence of the Debye mass. In particular, at temperatures larger than $2T_c$ the mass scales $\mu_{S_\infty}(T)$ and $m_D(T)$ become of similar magnitude in the entire temperature range analyzed here. Again, when comparing our results for $\mu_{F_\infty}(T)$ and $\mu_{S_\infty}(T)$ obtained in quenched and 2-flavor QCD we find no or only little differences at temperatures which are only moderately above the transition.

2 Summary

The present status of our analysis of the quark antiquark free energy, internal energy and the entropy at finite temperature concerning Debye screening effects suggest the following two statements (see also Refs. [8, 10, 13]): (i) It might be misleading to relate the large distance values of the finite temperature energies to the mass dependence of the corresponding heavy-light meson systems; in particular, the temperature dependence of $F_\infty(T)$ and $U_\infty(T)$ might be rather related to the temperature dependence of the Debye mass and the coupling. (ii) The large differences between Debye screening effects observed in Refs. [22, 23] when changing from quenched to 3-flavor QCD are not apparent in any of the scales analyzed here when changing from quenched to 2-flavor QCD at temperatures which are only moderately above the transition temperature.

Acknowledgments

We thank F. Karsch, E. Laermann and H. Satz for helpful discussions. This work has partly been supported by DFG under grant FOR 339/2-1 and by BMBF under Grant No.06BI102 and partly by Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy.

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