

QCD Corrections to Dilepton Production near Partonic Threshold in $\bar{p}p$ Scattering*

H. Shimizu^a, G. Sterman^b, W. Vogelsang^c and H. Yokoya^d

^a Department of Physics, Hiroshima University, Higashi-Hiroshima 739-8526, Japan

^b C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794 – 3840, U.S.A.

^c Physics Department and RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, U.S.A.

^d Department of Physics, Niigata University, Niigata 950-2181, Japan

*Presented by H. Yokoya at the
7th International Symposium on Radiative Corrections
(RADCOR 2005)
Shonan Village, Japan
October 2-7, 2005*

January 2006

Physics Department/ RIKEN BNL Research Center

Brookhaven National Laboratory

P.O. Box 5000

Upton, NY 11973-5000

www.bnl.gov

Notice: This manuscript has been authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy. The publisher by accepting the manuscript for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this manuscript, or allow others to do so, for United States Government purposes.

This preprint is intended for publication in a journal or proceedings. Since changes may be made before publication, it may not be cited or reproduced without the author's permission.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

QCD Corrections to Dilepton Production near Partonic Threshold in $\bar{p}p$ Scattering*

H. Shimizu^a, G. Sterman^b, W. Vogelsang^c and H. Yokoya^d

^a Department of Physics, Hiroshima University, Higashi-Hiroshima 739-8526, Japan

^b C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794 – 3840, U.S.A.

^c Physics Department and RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, U.S.A.

^d Department of Physics, Niigata University, Niigata 950-2181, Japan

We present a recent study of the QCD corrections to dilepton production near partonic threshold in transversely polarized $\bar{p}p$ scattering. We analyze the role of the higher-order perturbative QCD corrections in terms of the available fixed-order contributions as well as of all-order soft-gluon resummations for the kinematical regime of proposed experiments at GSI-FAIR. We find that perturbative corrections are large for both unpolarized and polarized cross sections, but that the spin asymmetries are stable. The role of the far infrared region of the momentum integral in the resummed exponent and the effect of the NNLL resummation are briefly discussed.

1. INTRODUCTION

A polarized antiproton beam of energy $E_{\bar{p}} = 15 - 22$ GeV may be available in future experiments at the GSI-FAIR project. Measurements of dilepton production in transversely polarized $\bar{p}p$ collisions are the main motivation for the proposed GSI-PAX [1] and GSI-ASSIA [2] experiments. The measurements would be carried out using a transversely polarized fixed proton target, or a proton beam of moderate energy $E_p = 3.5$ GeV.

Measurements of the transverse double-spin asymmetry

$$A_{TT} \equiv \frac{\delta\sigma}{\sigma} = \frac{\sigma^{\uparrow\uparrow} - \sigma^{\uparrow\downarrow}}{\sigma^{\uparrow\uparrow} + \sigma^{\uparrow\downarrow}}, \quad (1)$$

defined as the ratio of transversely polarized and unpolarized cross sections, may provide information of the transversely polarized parton distribution functions of the proton, dubbed “transversity” δf [3,4]. Transversity will be probed by

measurements of A_{TT} in polarized pp collisions at the BNL-RHIC collider [5]. However, since the δf for sea quarks are expected to be small, the asymmetry is estimated to be at most a few percent [6]. In contrast, since for the Drell-Yan process in $\bar{p}p$ collisions the scattering of two valence quark densities contributes, and since in addition the kinematical regime of the planned GSI experiments is such that rather large parton momentum fractions $x \sim 0.5$ are relevant, a very large A_{TT} of order 40% or more is expected [7,8,9]. Therefore, unique information on transversity in the valence region may be obtained from the GSI measurements, and information from RHIC and the GSI would be complementary.

Here we give a brief report on a recent study of perturbative-QCD corrections to the cross sections and to A_{TT} for Drell-Yan dilepton production at GSI-FAIR [8]. We discuss the available fixed order corrections as well as all-order soft-gluon “threshold” resummation.

*Presented by H. Yokoya at the “7th International Symposium on Radiative Corrections (RADCOR 2005)”, October 2-7, 2005, Shonan Village, Japan

2. DRELL-YAN CROSS SECTIONS

By virtue of the factorization theorem, the cross section for the Drell-Yan process at large lepton pair invariant mass M can be written in terms of a convolution of parton distribution functions and partonic scattering cross sections:

$$\frac{d(\delta)\sigma}{dM^2 d\phi} = \sum_{a,b} \int_{\tau}^1 dx_a (\delta) f_a(x_a, \mu^2) \quad (2)$$

$$\times \int_{\tau/x_a}^1 dx_b (\delta) f_b(x_b, \mu^2) \frac{d(\delta)\hat{\sigma}_{ab}}{dM^2 d\phi} + \mathcal{O}\left(\frac{\lambda}{M}\right)^p,$$

where $\tau = M^2/S$ with S the hadronic c.m. energy, and where ϕ is the azimuthal angle of one of the leptons. μ is the factorization scale. As indicated in Eq. (2), there are corrections suppressed with some power p and some hadronic scale λ . These corrections will become important for small M and in particular for lower-energy collisions.

2.1. Fixed-order perturbative calculation

The partonic cross section is calculated in QCD perturbation theory as a series in α_s ;

$$\frac{d(\delta)\hat{\sigma}_{ab}}{dM^2 d\phi} = (\delta)\hat{\sigma}_{ab}^{(0)} \left[\omega_{ab}^{(0)}(z) + \frac{\alpha_s}{\pi} (\delta)\omega_{ab}^{(1)}(z, r) \right. \\ \left. + \left(\frac{\alpha_s}{\pi}\right)^2 (\delta)\omega_{ab}^{(2)}(z, r) + \dots \right], \quad (3)$$

where $z = M^2/\hat{s}$, $\hat{s} = x_a x_b S$ and $r = M^2/\mu^2$. For the unpolarized cross section the calculation has been performed up to $\mathcal{O}(\alpha_s^2)$ [10], for the transversely polarized case to $\mathcal{O}(\alpha_s)$ [11]. The lowest order gives

$$\hat{\sigma}_{q\bar{q}}^{(0)} = \frac{2\alpha^2 e_q^2}{9M^2 \hat{s}}, \quad \delta\hat{\sigma}_{q\bar{q}}^{(0)} = \frac{\alpha^2 e_q^2}{9M^2 \hat{s}} \cos 2\phi \quad (4)$$

with $\omega_{q\bar{q}}^{(0)} = \delta(1-z)$. The higher-order functions may be found in the literature [10,11].

2.2. Threshold resummation

Threshold resummation addresses large logarithmic perturbative corrections to the partonic cross section that arise when the initial partons have just enough energy to produce the lepton pair. Only emission of relatively soft gluons is

allowed in this case. The large corrections exponentiate when Mellin moments of the partonic cross section, defined as

$$(\delta)\omega_{q\bar{q}}^{(k),N}(r) = \int_0^1 dz z^{N-1} (\delta)\omega_{q\bar{q}}^{(k)}(z, r), \quad (5)$$

are taken. To next-to-leading logarithmic (NLL) accuracy one then has for the resummed cross section [12,13]:

$$(\delta)\omega_{q\bar{q}}^{\text{res},N}(r, \alpha_s(\mu)) = \exp[C_q(r, \alpha_s(\mu))] \quad (6)$$

$$\times \exp\left\{ 2 \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \right. \\ \left. \times \int_{\mu^2}^{(1-z)^2 M^2} \frac{dk_T^2}{k_T^2} A_q(\alpha_s(k_T)) \right\},$$

where

$$A_q(\alpha_s) = \frac{\alpha_s}{\pi} A_q^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 A_q^{(2)} + \dots, \quad (7)$$

with $A_q^{(1)} = C_F$ and [14]:

$$A_q^{(2)} = \frac{C_F}{2} \left[C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} N_f \right], \quad (8)$$

where N_f is the number of flavors and $C_A = 3$. The coefficient $C_q(r, \alpha_s(\mu))$ collects mostly hard virtual corrections. It is given as

$$C_q(r, \alpha_s) = \frac{\alpha_s}{\pi} \left(-4 + \frac{2\pi^2}{3} + \frac{3}{2} \ln r \right) + \mathcal{O}(\alpha_s^2). \quad (9)$$

We note that it was shown in [15] that these coefficient functions also exponentiate.

Eq. (6) is ill-defined because of the divergence in the perturbative running coupling $\alpha_s(k_T)$ at $k_T = \Lambda_{\text{QCD}}$. The perturbative expansion of the expression shows factorial divergence, which in QCD corresponds to a power-like ambiguity of the series. It turns out, however, that the factorial divergence appears only at nonleading powers of momentum transfer. The large logarithms we are resumming arise in the region [13] $z \leq 1 - 1/\bar{N}$ in the integrand in Eq. (6). Therefore to NLL they are contained in the simpler expression

$$2 \int_{M^2/\bar{N}^2}^{M^2} \frac{dk_T^2}{k_T^2} A_q(\alpha_s(k_T)) \ln \frac{\bar{N} k_T}{M} \quad (10)$$

for the second exponent in (6). Here we have chosen $\mu = M$. This form, to which we will return below, is used for “minimal” expansions [16] of the resummed exponent.

For the NLL expansion of the resummed exponent one finds from Eqs. (6),(10) [16]:

$$\ln \delta\omega_{q\bar{q}}^{\text{res},N}(r, \alpha_s(\mu)) = C_q(r, \alpha_s(\mu)) + 2 \ln \bar{N} h^{(1)}(\lambda) + 2h^{(2)}(\lambda, r), \quad (11)$$

where

$$\lambda = b_0\alpha_s(\mu) \ln \bar{N}. \quad (12)$$

The explicit expressions for the functions $h^{(1)}$ and $h^{(2)}$ can be found e.g. in Refs. [16,8].

The hadronic cross section is obtained by performing an inverse Mellin transformation of the resummed partonic cross section, multiplied by the appropriate moments of two parton densities:

$$\frac{d(\delta)\sigma^{\text{res}}}{dM^2 d\phi} = \int_C \frac{dN}{2\pi i} \tau^{-N} \times \sum_{ab} (\delta) f_a^N(\delta) f_b^N \frac{d(\delta)\hat{\sigma}_{ab}^{\text{res},N}}{dM^2 d\phi}. \quad (13)$$

In order to perform the inverse Mellin integral, we need to specify a prescription for dealing with the singularity in the perturbative strong coupling constant in Eq. (6). We will use the minimal prescription developed in Ref. [16], which relies on use of the NLL expanded form involving the $h^i(\lambda)$, and on choosing a contour to the left of the Landau singularity at $\lambda = 1/2$ in the complex- N plane.

Figure 1 shows the effects of the higher orders generated by resummation for $S = 30 \text{ GeV}^2$ and $S = 210 \text{ GeV}^2$. We define a resummed “ K -factor” as the ratio of the resummed cross section to the leading order (LO) cross section,

$$K^{(\text{res})} = \frac{d\sigma^{(\text{res})}/dM d\phi}{d\sigma^{(\text{LO})}/dM d\phi}, \quad (14)$$

which is shown by the solid line in Fig. 1. As can be seen, $K^{(\text{res})}$ is very large, meaning that resummation results in a dramatic enhancement over LO, sometimes by over two orders of magnitude for the collisions at lower energy. It is then interesting to see how this enhancement builds

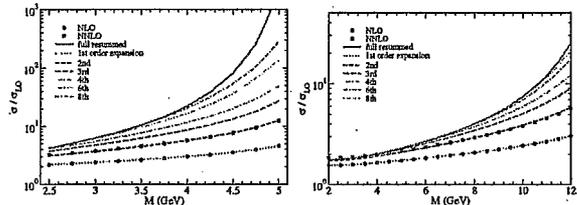


Figure 1. K -factors as defined in Eqs. (14), (15) for the Drell-Yan cross section as a function of lepton invariant mass M , in $\bar{p}p$ collision with $S = 30 \text{ GeV}^2$ (left), and $S = 210 \text{ GeV}^2$ (right).

up order by order in perturbation theory. We expand the resummed formula to next-to-leading order (NLO) and beyond and define the “soft-gluon K -factors”

$$K^n \equiv \frac{d\sigma^{(\text{res})}/dM d\phi|_{\mathcal{O}(\alpha_s^n)}}{d\sigma^{(\text{LO})}/dM d\phi}, \quad (15)$$

which for $n = 1, 2, \dots$ give the effects due to the $\mathcal{O}(\alpha_s^n)$ terms in the resummed formula. The results for K^{1-8} are also shown in Fig. 1. One can see that there are very large contributions even beyond NNLO, in particular at the higher M . Clearly, the full resummation given by the solid line receives contributions from high orders. We stress that the $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ expansions of the resummed result are in excellent agreement with the full NLO and NNLO ones, respectively (circle and square symbols in Figure 1). This shows that the higher-order corrections are really dominated by the threshold logarithms and that the resummation is accurately collecting the latter.

2.3. Far infrared cut-off

There is good reason to believe that the large enhancement from soft-gluon radiation seen above is only partly physical. The large corrections arise from a region where the integral in the exponent becomes sensitive to the behavior of the integrand at small values of k_T . As long as $\Lambda_{\text{QCD}} \ll M/\bar{N} \ll M$, the use of perturbation theory may be justified, but when $|N|$ becomes very large, k_T will reach down to nonperturbative scales. We seek a modification of the perturbative expression in Eq. (6) that excludes the region in which the absolute value of k_T is less than some nonperturbative scale μ_0 . To implement this idea, we will adopt a modified resummed

hard scattering, which reproduces NLL logarithmic behavior in the moment variable N so long as $M/\bar{N} > \mu_0$, but “freezes” once $M/\bar{N} < \mu_0$. If nothing else, this will test the importance of the region $k_T \leq \Lambda_{QCD}$ for the resummed cross section. If N were real and positive, we could simply replace the resummed exponent in (10) by

$$4 \int_{\rho(M/\bar{N}, \mu_0)}^M \frac{dk_T}{k_T} A_q(\alpha_s(k_T)) \ln \frac{\bar{N}k_T}{M}, \quad (16)$$

where $\rho(a, b) = \max(a, b)$, and where μ_0 then serves to cut off the lower logarithmic behavior. To provide an expression that can be analytically continued to complex N , we choose $\rho(a, b) = (a^p + b^p)^{1/p}$, with integer p . This simple form is consistent with the minimal expansion given above, and it also allows for a straightforward analysis of the ensuing branch cuts in the complex- N plane. For definiteness, we choose $p = 2$. We will continue to use the expansions in Eq. (11), but redefine λ in Eq. (12) by

$$\lambda = b_0 \alpha_s(\mu) \ln \bar{N} - \frac{1}{2} b_0 \alpha_s(\mu) \ln \left(1 + \frac{\bar{N}^2 \mu_0^2}{M^2} \right). \quad (17)$$

Of course, different choices of μ_0 give different results, but we should think of μ_0 as a kind of factorization scale, separating perturbative contributions from nonperturbative. Thus changes in μ_0 would be compensated by changes in a nonperturbative function. Our interest here, however, is simply to illustrate the modification of the perturbative sector, which we do by choosing $\mu_0 = 0.3$ GeV and $\mu = 0.4$ GeV.

Results for the “ K -factor” with these values of μ_0 are shown in Fig. 2, compared to the same NLO, NNLO and resummed cross sections as

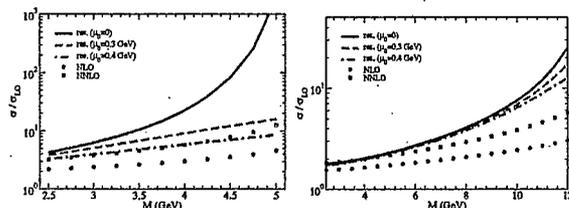


Figure 2. K -factors as in Fig. 1, at $S = 30$ GeV² (left) and $S = 210$ GeV² (right). The dashed (dot-dashed) lines show the effects of a lower cut-off $\mu_0 = 300$ MeV (400 MeV) for the k_T integral

presented before. The ratios of the infrared-regulated resummed cross sections to LO show a smoother increase than the “purely minimally” resummed ones. The difference is particularly marked at the lower center of mass energy in Fig. 2 (left), with only a modest enhancement over NNLO remaining. We interpret these results to indicate a strong sensitivity to nonperturbative dynamics at the lower energies, and much less at the higher.

3. SPIN ASYMMETRY A_{TT}

To perform numerical studies of the asymmetry A_{TT} we need to make a model for the transversity densities in the valence region. Here, guidance is provided by the Soffer inequality [17]

$$2 |\delta q(x, Q^2)| \leq q(x, Q^2) + \Delta q(x, Q^2), \quad (18)$$

which gives an upper bound for each δq . Following [6] we utilize this inequality by saturating the bound at some low input scale $Q_0 \simeq 0.6$ GeV using the NLO GRV [18] and GRSV (“standard scenario”) [19] densities $q(x, Q_0^2)$ and $\Delta q(x, Q_0^2)$, respectively. For $Q > Q_0$ the transversity densities $\delta q(x, Q^2)$ are then obtained using the NLO evolution equations [11].

Figure 3 shows that A_{TT} is very robust under the QCD corrections, including resummation with and without a cutoff. This is expected to some extent because the emission of soft-gluons does not change the spin of the parent parton.

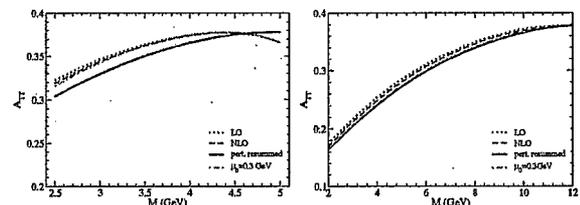


Figure 3. Spin asymmetry $A_{TT}(\phi = 0)$ at LO, NLO and for the NLL-resummed case at $S = 30$ GeV² (left) and $S = 210$ GeV² (right).

4. NNLL RESUMMATION

Thanks to the recent calculation of the three-loop splitting functions by Moch, Vermaseren and

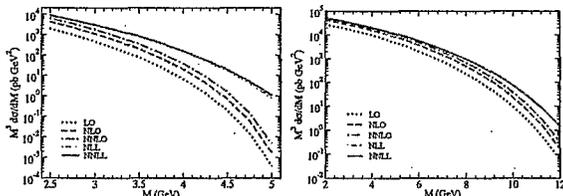


Figure 4. Unpolarized cross section $M^3 d\sigma/dM$ at $S = 30 \text{ GeV}^2$ (left) and $S = 210 \text{ GeV}^2$ (right) at LO, NLO, NNLO, and NLL-, NNLL-resummed, as function of lepton pair invariant mass M .

Vogt [20], we can now perform the threshold resummation for the Drell-Yan process to NNLL accuracy. This leads to a new term in the exponent in Eq. (11):

$$\begin{aligned} \ln \delta\omega_{q\bar{q}}^{\text{res},N}(r, \alpha_s(\mu)) = & C_q(r, \alpha_s(\mu)) \\ & + 2 \ln \bar{N} h^{(1)}(\lambda) + 2h^{(2)}(\lambda, r) \\ & + 2\alpha_s(\mu)h^{(3)}(\lambda, r), \end{aligned} \quad (19)$$

where $h^{(3)}$ includes the new $A_q^{(3)}$ and $D_{DY}^{(2)}$ coefficients [21,22], and where an additional $C_q^{(2)}$ term is included in the coefficient function, which may be extracted [23] from the known [10] NNLO results for the Drell-Yan process. The effects of NNLL resummation on the unpolarized cross section are displayed in Fig. 4. One finds that the resummed cross section has a fast convergence, even at the lower energy.

5. SUMMARY

We have studied the perturbative QCD corrections to Drell-Yan dilepton production in transversely polarized $\bar{p}p$ collisions for the kinematical regime of proposed experiments at GSI-FAIR. We find that the K -factor for the available fixed-order corrections, and for the all-order NLL soft-gluon resummation, can be very large. In contrast, the spin asymmetry is quite stable. We have highlighted the importance of rather small momentum scales in the resummed exponent at the lower energies. We have also examined the resummation to NNLL and found it to give a rather modest correction.

Acknowledgments

The work of G.S. was supported in part by the National Science Foundation, grants PHY-0098527, PHY-0354776, and PHY-0354822. W.V. is grateful to RIKEN, BNL and the U.S. Department of Energy (contract number DE-AC02-98CH10886) for providing the facilities essential for the completion of his work. The work of H.Y. was supported in part by a Research Fellowship of the Japan Society for the Promotion of Science.

REFERENCES

1. V. Barone *et al.* [PAX Collaboration], arXiv:hep-ex/0505054.
2. M. Maggiora *et al.* [ASSIA Collaboration], arXiv:hep-ex/0504011.
3. J.P. Ralston and D.E. Soper, Nucl. Phys. B **152** (1979) 109.
4. R.L. Jaffe and X. Ji, Phys. Rev. Lett. **67** (1991) 552; Nucl. Phys. B **375** (1992) 527.
5. See, for example: G. Bunce *et al.*, Ann. Rev. Nucl. Part. Sci. **50** (2000) 525.
6. O. Martin *et al.*, Phys. Rev. D **57** (1998) 3084; Phys. Rev. D **60** (1999) 117502.
7. M. Anselmino *et al.*, Phys. Lett. B **594** (2004) 97; A. V. Efremov, K. Goeke and P. Schweitzer, Eur. Phys. J. C **35** (2004) 207.
8. H. Shimizu, G. Sterman, W. Vogelsang and H. Yokoya, Phys. Rev. D **71** (2005) 114007.
9. V. Barone *et al.*, arXiv:hep-ph/0512121; A. Bianconi and M. Radici, Phys. Rev. D **72** (2005) 074013.
10. R. Hamberg, W. L. van Neerven and T. Matsuura, Nucl. Phys. B **359** (1991) 343 [Erratum-ibid. B **644** (2002) 403]. See also: R. V. Harlander and W. B. Kilgore, Phys. Rev. Lett. **88** (2002) 201801.
11. W. Vogelsang, Phys. Rev. D **57** (1998) 1886.
12. G. Sterman, Nucl. Phys. B **281** (1987) 310.
13. S. Catani and L. Trentadue, Nucl. Phys. B **327** (1989) 323; *ibid.* **353** (1991) 183.
14. J. Kodaira and L. Trentadue, Phys. Lett. B **112** (1982) 66; Phys. Lett. B **123** (1983) 335.
15. T. O. Eynck, E. Laenen and L. Magnea, JHEP **0306** (2003) 057.
16. S. Catani, M. L. Mangano, P. Nason and

- L. Trentadue, Nucl. Phys. B **478** (1996) 273.
17. J. Soffer, Phys. Rev. Lett. **74** (1995) 1292.
18. M. Glück, E. Reya, and A. Vogt, Eur. Phys. J. **C5** (1998) 461.
19. M. Glück, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. D **63** (2001) 094005.
20. S. Moch, J. A. M. Vermaseren and A. Vogt, Nucl. Phys. B **688** (2004) 101; *ibid.* **691** (2004) 129.
21. A. Vogt, Phys. Lett. B **497** (2001) 228.
22. S. Catani, D. de Florian, M. Grazzini and P. Nason, JHEP **0307** (2003) 028.
23. S. Moch and A. Vogt, Phys. Lett. B **631** (2005) 48.