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SUMMARY/ABSTRACT

One of the greatest challenges in evaluating reliability of digital I&C systems is how to obtain better failure rate estimates of digital components. A common practice of the digital component failure rate estimation is attempting to use empirical formulae to capture the impacts of various factors on the failure rates. The applicability of an empirical formula is questionable because it is not based on laws of physics and requires good data, which is scarce in general. In this study, the concept of population variability of the Hierarchical Bayesian Method (HBM) is applied to estimating the failure rate of a digital component using available data. Markov Chain Monte Carlo (MCMC) simulation is used to implement the HBM. Results are analyzed and compared by selecting different distribution types and priors distributions. Inspired by the sensitivity calculations and based on review of analytic derivations, it seems reasonable to suggest avoiding the use of gamma distribution in two-stage Bayesian analysis and HBM analysis. [1]

[1] This paper was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, expressed or implied, or assumes any legal liability or responsibility for any third party's use, or the results of such use, of any information, apparatus, product, or process disclosed in this paper, or represents that its use by such third party would not infringe privately owned rights. The views expressed in this paper are not necessarily those of the U.S. Nuclear Regulatory Commission.

1. INTRODUCTION

Many reliability prediction methods have been proposed. The methods of Military Handbook 217 [1] are the most widely used by the defense industry. The prediction method of Telcordia SR-332 [2] is similar to that of the military handbook, and has been used by the telecommunication industry. In these methods, at the component level, generic failure rates are modified by π factors representing the effects of quality, electrical stress, and operating temperature, etc. Tables of the π factors are provided without indicating how they were derived. If test data or field data is available, Bayesian analysis is performed. Reliability prediction methods based on the military handbook have been criticized for lack of accuracy and failure to address important factors that affect failure rates of digital components [3]. The Reliability Analysis Center (RAC) developed a new method [4] which addresses some of the criticisms and implemented it in a software tool called PRISM [5]. However, the RAC method still depends on use of empirical formulae. All these reliability prediction methods attempt to use empirical formulae to capture the variability of component failure rates due to many factors, e.g., operating temperature, temperature cycling, electrical stress, relative humidity, vibration level, duty cycle, and cycle frequency. A basic problem with the use of empirical formulae is that the formulae are not laws of physics and their applicability is limited. It requires extensive applicable data to estimate the parameters of the models. With good data, an empirical formula may be demonstrated to make good predictions, i.e., consistent with the data. On the other hand, its range of applicability is still limited to the cases with good data and its extrapolation to other situations may not be valid. If good data is available, then estimating failure rates using the data directly seems a reasonable approach. This paper describes such an approach based on the Hierarchical Bayesian Method (HBM) [6].

The concept of population variability has been used in two-stage Bayesian analyses of probabilistic risk assessments (PRAs) to estimate initiating event frequencies [7,8]. It has also been used in development of component failure databases [9,10]. The two-stage Bayesian analysis method is a special case of the more general HBM [8]. In this study, the HBM was applied to estimating generic failure rates of digital components using available raw failure data extracted from the PRISM software. The hierarchical Bayesian method captures the variability in failure rates due to the factors considered in the reliability prediction methods of the defense industry. It generates generic component failure rates that can be further Bayesian updated if component specific failure data is available. The analysis consists of the procedures that follow: (1) data collection and grouping of the components to be studied; (2) Chi-square test of grouped data; (3) type and parameter selection of priors and hyperpriors used in HBM; and (4) population variability calculation and sensitivity analysis. The hierarchical method was implemented using Markov Chain Monte Carlo (MCMC) simulation.

The remainder of this paper is organized as follows: the hierarchical Bayesian method is briefly presented in Section 2. Section 3 gives a detailed description of the application of the hierarchical Bayesian model to prediction of the failure rate of a digital component using the data extracted from the PRISM database. A software tool WinBUGS [11], which is capable of calculating population variation using MCMC simulation, is used to perform the computation. Sensitivity analyses are presented and discussed in Section 4. Conclusive remarks are given in Section 5.

2. HIERARCHICAL BAYESIAN MODELS FOR FAILURE RATES DETERMINATION

Bayesian estimation has been widely employed in PRAs to account for uncertainties. The uncertainties caused by a lack of knowledge can be expressed in terms of probability distributions using Bayesian estimation method. In a simple Bayesian analysis, Bayes's Theorem is applied to obtain a posterior distribution by updating the prior distribution of collected data. Often it is assumed that the data was collected from a single source. However, sometimes the data was collected from different sources, as is the case of the data of PRISM. The assumption of a single data source leads to a narrow posterior distribution because the source-to-source variation (population variation) is ignored. The two-stage Bayesian method [7] has been used to take into consideration the source–to-source variability [8, 9, 10]. In this study, this variability is addressed by using the HBM [6] which is a more general approach. In the HBM, the prior distribution is developed in multiple stages of a hierarchical structure, i.e., the parameters of the prior distribution are also considered uncertain and can be modeled as a probability distribution function with, again, uncertain parameters. This process can be repeated until the last stage, where the prior distribution is called hyperprior with corresponding constant hyperparameters. It can be demonstrated that a HBM model with 2 stages is the same as the two-stage Bayesian model. In two-stage analysis [7], a discretized probability distribution method is often used to solve the Bayesian equations. The HBM provides its own way of solving the model [6].

The population variation curve (PVC) is denoted as $g(\lambda)$, where λ is the parameter we are interested in, e.g., failure rate. Usually it is assumed to be lognormal or gamma distributed with $\underline{\theta}$ representing the parameter vector. Data is collected from m different sources/plants whose failure rates λ_i are random samples from $g(\lambda)$. Obviously, $\underline{\theta}$ might consist of different variables depending on the assumption of the population variability distribution. Different prior distribution can be selected for each element of parameter vector $\underline{\theta}$, e.g., $\underline{\theta}=[\alpha,\beta]$ for a gamma distribution. The prior distributions of the parameters are called hyper prior distributions and denoted as $\pi_0(\underline{\theta})$.

The posterior distribution of the uncertain parameter vector $\underline{\theta}$, i.e., the hyperposterior distribution, is required for PVC and can be calculated by applying the Bayes's Theorem in the multiple-dimensional form [8]:

$$\pi_1(\underline{\theta} \mid E)=\frac{L(E \mid \underline{\theta})\pi_0(\underline{\theta})}{\int_0^\infty \cdots \int_0^\infty L(E \mid \underline{\theta})\pi_0(\underline{\theta})d\underline{\theta}} \tag{1}$$

where $L(E \mid \underline{\theta})$ is the likelihood of the collected data. The likelihood function for a specific source/plant is given as $L(E_i \mid \underline{\theta})=\int_0^\infty P(x_i \mid t_i,\lambda_i)g(\lambda_i \mid \underline{\theta})d\lambda_i$, where λ_i is the failure rate of plant i, x_i is the number of failures that took place in time period t_i, and $P(x_i \mid t_i,\lambda_i)=\frac{(\lambda_i t_i)^{x_i} e^{-\lambda_i t_i}}{x_i!}$. The likelihood function for the entire set of the evidence is the product of the likelihood functions for the individual sources

$$L(E \mid \underline{\theta})=\prod_{i=1}^{m} L(E_i \mid \underline{\theta})=\int \cdots \int \prod_{i=1}^{m} P(x_i \mid t_i,\lambda_i)g(\lambda_i \mid \underline{\theta})d\lambda_1 \cdots d\lambda_m \tag{2}$$

The expected PVC can be calculated using the hyperposterior distribution of $\underline{\theta}$:

$$\begin{aligned} g(\lambda \mid E) &= \int_0^\infty \cdots \int_0^\infty g(\lambda,\underline{\theta} \mid E)d\underline{\theta} \\ &= \int_0^\infty \cdots \int_0^\infty g(\lambda \mid \underline{\theta})\pi_1(\underline{\theta} \mid E)d\underline{\theta} \\ &= \int_0^\infty \cdots \int_0^\infty g(\lambda \mid \underline{\theta})\frac{\prod_{i=1}^{m} L(E_i \mid \underline{\theta})\pi_0(\underline{\theta})}{\int_0^\infty \cdots \int_0^\infty \prod_{i=1}^{m} L(E_i \mid \underline{\theta})\pi_0(\underline{\theta})d\underline{\theta}}d\underline{\theta} \end{aligned} \tag{3}$$

which can be used as a generic informative prior distribution for a Bayesian analysis of the data collected for the same component from a specific source/plant. Due to the unclear identification of the sources of data of the PRISM software discussed in Section 3, it is not likely that the specific source/plant can be associated with the sources of data of the PRISM software. Therefore, the issue of double counting the same plant specific data does not apply.

Usually it is impossible to evaluate equation (3) analytically. The solution using Markov chain Monte Carlo (MCMC) simulation considers the posterior distributions of all the parameters of interest, i.e., $\underline{\theta}$ and λ_i 's, and generates samples from the joint posterior distribution by constructing a Markov chain that has the parameters of interest as its state space and taking samples from the conditional distributions of the parameters. More specifically, Gibbs sampling or the Metreopolis-Hastings algorithm can be used in MCMC implementation [6].

3. FAILURE RATES ESTIMATION OF DIGITAL COMPONENTS USING HBM

3.1 DATA COLLECTION AND GROUPING

PRISM [5] is a software tool developed by the Reliability Analysis Center (RAC) for assessing system reliability. It includes a failure rate database for both electronic and non-electronic components while a separate database contains the failure mode and mechanism distributions, which allows partitioning of failure rates into failure modes and mechanisms. The RAC databases share some of the data sources. The PRISM database can be considered an update of military handbook 217 [1], which is no longer actively maintained, with more recent data up to year 2000 and improvements in the reliability prediction method.

The RAC database denoted as RACdata in PRISM contains failure data records in the form of the number of failures in a number of operating/calendar hours. The RAC database sources are not completely specified and only identified in format such as "warranty repair data from a manufacturer." The failure records of a specific type of component, e.g., memory, are further categorized according to sub-level component types, e.g., random access memory (RAM) or programmable read only memory (PROM); quality, e.g., commercial-grade or military-grade; environment, e.g., ground or airborne; hermeticity, e.g., plastic and ceramic; and time period that the data is collected, etc. In this study, the failure data of various digital components were extracted from the RACdata database. It was decided, for each sub-level component, to group the failure records of different qualities, environments, hermeticities, and time periods. In general, the data is not strong, i.e., many failure records do not have any failures, and those that do often scatter widely.

Table 1 lists the grouped failure records of a digital component. The definitions of sub-level component type, quality, and environment can be found in [1, 4, 5]. The failure records were used in estimating the population variability curve of the generic component. The last column of the table lists the point estimates of failure rates of those failure records with at least one failure. The point estimate is simply the number of failures divided by the number of hours. It provides information on the possible range of the population variability curve. The point estimate information was used in estimation of hyperprior parameters.

3.2 CHI-SQUARE TEST

A Chi-square (χ^2) test was performed for the data of each sub-level component type to determine whether the failure records can be pooled and the population variability should be used to model the failure rates of the components. It has been a common practice in statistics and the definition of the test can be found in many references, e.g., [6]. A Chi-square test was performed on the component failure data in Table 1 and a χ^2 value of 14481 was obtained. This indicates that for the data, the confidence is high that the failure records cannot be pooled, i.e., the failure records are samples from different failure rate sources and a population variability distribution should be used to model the variability in the failure rates.

3.3 SELECTION OF HYPERPRIOR DISTRIBUTIONS

In the hierarchical Bayesian analysis, different distribution types can be assumed for the failure rates and the hyperpriors. The parameters for hyperpriors were chosen based on the range of the point estimates of failure rates in the data set and the properties of the type of the distribution. The criterion we used for selecting the mean values of the prior parameters is that the maximum and minimum values of the point estimate failure rates lie within the 95th and 5th percentile of the distribution defined by the selected mean values. The software tool WinBUGS was used to create the model and to calculate the population variability distributions. The details of the WinBUGS can be found in [11] and will not be discussed here.

Table 1: Failure Records of a Digital Component Extracted from PRISM RACdata Database

Quality	Environment	Number of Failures	Number of Hours (*1.0E6)	Point Estimate Failure Rate (per million hours)
Commercial	GB	12	633.8929	1.89e-02
Unknown	GB	0	0.2600	
Unknown	GB	0	0.0625	
Commercial	GB	16	2597.365	6.16e-03
Commercial	GM	4	701.1615	5.70e-03
Commercial	N/R	2	509.1335	3.93e-03
Commercial	GB	28	22751.18	1.23e-03
Commercial	GB	0	1105.13	
Unknown	GB	80	444.0000	1.80e-01
Unknown	GB	44	307.8874	1.43e-01
Unknown	GB	0	6.5937	
Commercial	GB	0	19.3613	
Commercial	GB	188	20069.9345	9.37e-03
Commercial	GM	1	692.6390	1.44e-03
Military	N/R	1	149.2384	6.70e-03
Military	AIF	0	0.0253	
Military	AIF	0	1.8755	
Military	AIF	0	11.3706	
Military	GB	0	0.7367	
Military	GF	0	53.6832	
Military	NS	0	29.2752	
Unknown	AIU	0	0.2376	
Unknown	AUF	0	1.5206	
Unknown	AUT	0	1.3585	
Unknown	GB	0	90.4280	
Unknown	GB	0	1.8878	
Unknown	GB	54	205.2583	2.63e-01
Unknown	GB	2	1.4060	1.42e+00
Unknown	GF	0	2.0275	
Unknown	GF	2	553.6315	3.61e-03
Unknown	GF	332	590.3949	5.62e-01
Unknown	GF	0	0.0080	
Unknown	GF	0	2.1948	
Unknown	NS	0	2.0799	
Unknown	NSB	0	0.0121	

There are no general rules about how to select the types of the priors and hyperpriors. Sensitivity calculations were performed by comparing different distributions and documented in Section 3. In the base case calculation described here, it was assumed that the population variability distribution is lognormally distributed with parameters μ and σ. The last column of Table 1 shows that the point estimate failure rates are approximately in the range of 1E-3 and 2E+0. Assuming that the range of 1E-3 and 2.0 is the 90% confidence interval of a lognormal distribution, the mean values of μ and σ can be calculated using formulae:

$$\bar{\sigma} = \frac{\ln(b/a)}{3.29}, \; \bar{\mu} = \ln b - 1.645\sigma \qquad (4)$$

where a and b are the lower bound and the upper bound of the point estimate, respectively, i.e., a=1E-3, b=2.0. We have $\bar{\sigma}$=2.31 and $\bar{\mu}$=-3.1073. According to the HBM, the parameters μ and σ are also associated with uncertainties. In the absence of any information concerning parameters μ and σ, the uncertainties can be addressed by further assuming that μ and σ are uniformly distributed with lower and upper bounds equal to -7 and -0.1, and 1 and 3.5, respectively.

A WinBUGS analysis of the data in Table 1 resulted in posterior distributions of μ and σ that are within the bounds of the uniform hyperprior distributions. This shows that the selected bounds of hyperprior distributions are reasonable. WinBUGS does not produce an output of the population variability distribution and only provides the characteristics of the posterior distributions of μ and σ separately. It may not be accurate to simply use the calculated mean values of μ and σ to define a population variability distribution, because μ and σ are correlated. Instead, a trick was used to generate information of the population variability distribution, by adding an artificial failure record with no failures and very small operating hours in the data set. Such a failure record is not expected to introduce any significant bias in the results of WinBUGS, and its posterior distribution is effectively the population variability distribution. The estimated population variability distribution has a mean value of 0.33, and 90% confidence interval of 8.8E-5 and 0.51. This is the base case shown in Table 2.

Table 2: Characteristics of Population Variability Distribution

Case	Mean	5th	Median	95th	Error Factor
Base	0.33	8.8E-5	7.2E-3	0.51	76
LNL1-1000000 samples	0.3	9.5E-5	7.5E-3	0.46	69
LNL2-3000000 samples	0.34	9.5E-5	7.4E-3	0.44	68
LNG	0.32	8.90E-05	7.80E-03	0.47	73
LUG	0.31	2.10E-04	1.10E-02	0.53	50
GEG-100000 Samples	0.09	3.4E-7	1.30E-02	0.41	1098
GEL	0.11	1.1E-7	1.30E-02	0.52	2142
GUU	0.15	2.0E-8	1.30E-02	0.77	1962

4. SENSITIVITY CALCULATIONS

A few sensitivity calculations were performed using different distribution types, and different hyperprior distributions. The results of the sensitivity calculations are shown in Table 2 using different models described below, where L represents lognormal distribution, N the normal distribution, G the gamma distribution, E the exponential distribution, and U the uniform distribution. The first capital letter indicates the type of the distribution of the population variability curve, and the second and the last letter indicate the distributions of its parameters.

LNL1- In this sensitivity calculation, the failure rate is assumed to be lognormal distributed with its parameters μ and σ distributed normally, and lognormally, respectively. That is, $\mu \sim$ Normal (μ_μ, σ_μ), and $\sigma \sim$ Lognormal ($\mu_\sigma, \sigma_\sigma$). The prior mean values of μ and σ were calculated such that the lognormal distribution based on the mean values has a confidence interval of 1E-3 and 2E+0. Using equation (4) we again have $\overline{\sigma}$=2.31 and $\overline{\mu}$=-3.1073, i.e., $\overline{\mu} = \mu_\mu$= -3.1073 and $\overline{\sigma} = 2.31$. The standard deviation of μ, i.e., σ_μ, was selected to be 15. According to $\overline{\sigma} = \exp(\mu_\sigma + \frac{\sigma_\sigma^2}{2})$, the parameters of σ, μ_σ and σ_σ, were selected to be -3.66 and 3, respectively.

The confidence intervals of the posterior distributions of μ and σ are well within the confidence intervals of the hyperpriors. The characteristics of the population variability distribution are very similar to those obtained with uniform hyperprior distributions. The mean values of the LNL1 model changed significantly with different sample size in WinBUGS. The mean value is 0.374 for 10,000 samples, 0.3025 for 100,000 samples, and 0.2946 for 1,000,000 samples. Thus, the calculation converged for 100,000 samples. Note that WinBUGS does not provide a tool to check convergence and the only way to assure the convergence is to compare the results using different sample sizes.

LNL2- This sensitivity calculation is the same as LNL1 except that the prior distribution of σ was changed to a narrower distribution (smaller variance) which still covers the confidence interval of the posterior distribution of σ obtained in case LNL1. The resulting confidence interval of σ is practically the same as that obtained from LNL1. The characteristics of the population variability distribution are close to those of the previous cases, except the mean value which deviates from that of the base by a larger factor. Sensitivity calculations were performed using this model by changing the number of samples. The results show that the mean value varies significantly, i.e., from 0.49 with 10,000 samples, to 0.22 with 100,000 samples (0.4 with 1,000,000 samples), while other characteristics do not change very much. It is easy to conclude that it has not converged. Using more samples was necessary for the simulation to converge. The mean value becomes 0.3361 for 3,000,000 samples and 0.3384 for more than 4,000,000 samples. It is shown in Table 2 that the mean value, median, and 5% and 95% percentiles of the population variability are very close to each other using LNL1 and LNL2 models once the convergence is achieved.

LNG- This sensitivity calculation is the same as LNL1 except that σ is assumed to be gamma distributed with mean equal to 2.31. The two parameters μ_σ and σ_σ are assumed to be 2.31 and 1, respectively. The resulting population variability distribution is close to those of other cases.

LUG- It is assumed that the failure rate is of lognormal distribution with parameters μ and σ. The parameters μ and σ are uniformly and gamma distributed, respectively. That is, $\mu \sim$ Unif (a_1, b_1), and $\sigma \sim$ Gamma(α, β). The prior mean values of μ and σ are selected as -3.1073 and 2.3103 such that the lognormal distribution based on the mean values has a confidence interval of 1E-3 and 2. We choose a_1=-7, b_1=-0.1. The standard deviation of μ, i.e., σ_μ, is selected to be 15. The parameters of σ, μ_σ, and σ_σ, α=0.023103, and β=0.01. The calculation results are also close to previous results.

GEG- This sensitivity calculation assumes that the failure rate is gamma distributed. The mean values of its hyperpriors are selected as 0.44 and 0.87, such that the prior distribution of the failure rate has a confidence interval approximately between 1E-3 and 2 (between 0.001 and 2.0337). α is assumed to be exponentially distributed with mean of 0.44. β is assumed to be gamma distributed with parameters α_β and β_β equal to 0.01 and 0.0115 (such that the mean of β is 0.01/0.0115=0.87), respectively. With this choice of hyperpriors, the posterior distributions of the hyper parameters are covered by the hyperpriors. However, the population variability distribution is significantly different from the previous models.

GEL- It is assumed that the failure rate is gamma distributed with parameters of α and β, which are of exponential and lognormal distribution, respectively. The mean values of its hyperpriors are still 0.44 and 0.87. Lognormal distribution parameters are -2.1393 and 0.25 for β such that the mean of β is around 0.87. The mean value of the PVC is only slightly different from that of the GEG model but significantly different from those of other models.

GUU- It is again assumed that the failure rate is of gamma distribution with parameters of α and β. The parameters of α and β are both uniformly distributed, respectively. That is, $\alpha \sim$ Unif (a_1, b_1) and $\beta \sim$ Unif (a_2, b_2). We choose a_1=0.1, b_1=1, a_2=0.1, and b_2=2. The calculation shows that the mean value of PVC is slightly larger than those of GEG and GEL models but much smaller than those of other models.

An inspection of Table 2 shows that the mean values of failure rate of gamma distributions, i.e., using models GEG, GEL, and GUU, are close to each other and smaller than the mean values calculated using other models.

It is our experience in the HBM analysis of many different digital components that when assuming the failure rate is gamma distributed, the results tend to vary significantly with different values on the hyperparameters. This is consistent with the experience of the review of the SKI Data processing methodology [12], where a difference of a factor of 5 was observed using different hyperparameter values for gamma distribution. It was recommended that a lognormal distribution be used as a prior distribution instead in [10]. Bunea et al. [10] pointed out that the likelihood of α and β of gamma distribution has no maximum and is asymptotically maximal along a ridge, and improper hyperpriors do not always become proper when Bayesian updated [12]. As a result, a finite rectangle truncation of α and β can not be defined to contain most of the hyperposterior mass, and different choices could significantly shift the region in which the population variation is localized. This observation is very interesting and we would like to further illustrate it theoretically by elaborating on Hofer's derivation [13].

If we assume that the failure rate is gamma distributed, the likelihood function of Equation (2) becomes [10,13]:

$$\prod_{i=1}^{m}\frac{\Gamma(x_i+\alpha)}{\Gamma(x_i+1)\Gamma(\alpha)}\left(\frac{\beta}{\beta+t_i}\right)^{\alpha}\left(\frac{t_i}{\beta+t_i}\right)^{x_i} \tag{4}$$

Hofer [13] also demonstrated that an expansion of the individual terms in equation (4) is:

$$\prod_{i=1}^{m}\frac{(\alpha+x_i-1)(\alpha+x_i-2)\cdots\alpha t_i^{x_i}}{(\beta+t_i)^{x_i}(1+\frac{t_i}{x_i})^{\alpha}x_i!}\approx\prod_{i=1}^{m}\frac{\alpha^{x_i}t_i^{x_i}}{\beta^{x_i}e^{\frac{\alpha}{\beta}t_i}x_i!}=\prod_{i=1}^{m}\frac{(\frac{\alpha}{\beta}t_i)^{x_i}e^{-\frac{\alpha}{\beta}t_i}}{x_i!}$$

for $\alpha >> x_i, \beta >> t_i$. The likelihood function in Equation (4) can thus be rewritten as:

$$\frac{(\frac{\alpha}{\beta}\sum t_i)^{\sum x_i}e^{-\frac{\alpha}{\beta}\sum t_i}}{(\sum x_i)!}\times\frac{\prod t_i^{x_i}(\sum x_i)!}{\prod(x_i)!(\sum t_i)^{\sum x_i}} \tag{5}$$

From equation (5), we can see that the likelihood function becomes the likelihood of a common incident rate model for large α and β. To find the maximum value of the likelihood function, we take the derivative of (5) with respect to $\frac{\alpha}{\beta}$. The maximum value of (5) occurs at $\frac{\alpha}{\beta}=\frac{\sum x_i}{\sum t_i}$ and converges to the asymptote that can be obtained by putting the value of $\frac{\alpha}{\beta}$ into (5) for larger α and β. Hofer used this derivation to show the common incident rate effect when α and β become large, but this also indicates that the likelihood function is improper and the result may not be different from the calculation performed by simply pooling all data together if α and β are allowed to take on large values in numerical evaluations.

Although it is possible that the hyperposterior might become proper by selecting the hyperpriors carefully [10], improper hyperposterior is clearly a major drawback of choosing a gamma prior distribution. Moreover, the fact that the likelihood function reduces to a common incident rate function shows the tendency of eliminating the population variability, which makes the whole Bayesian update meaningless. Therefore, it appears reasonable to suggest not to use gamma prior distributions in either HBM or two-stage Bayesian method.

On the other hand, using a lognormal distribution as the prior, the likelihood does peak. It is easy to select the truncation limits for the prior parameters μ and σ such that the peak of the likelihood is contained. Different choices of prior parameters might have only very minor effects on the results as long as the peak is contained. The mass of the population variability can be captured easily within the μ and σ rectangle. This explains the significant gaps of numerical results for gamma and lognormal priors although we carefully selected prior parameters for both models.

5. CONCLUSIONS

The hierarchical Bayesian method was used to estimate the failure rates of digital components using the data collected in the PRISM database. The hyperpriors and their parameters were carefully selected, often by performing sensitivity calculations. The population variability curves obtained using the MCMC method are very wide with very large error factors. This is mainly due to the large variability in the different sources of data. A close inspection of the grouped data, which can be found in [14], shows that (1) in all the tables of the failure data, the time periods of the data collected differ from each other significantly. It is always true that the number of hours of operation of the military grade components is very short compared to those of commercial grade components; and (2) most of the data of military grade components does not have any failures in the time period it was collected. This is possibly because of the short time period that data was collected for the higher quality military grade components. Furthermore, the failure rates could be very high if any failure occurs due to the extremely short time period. A real challenge for the failure rate prediction is the collection of the better data.

The analytic work by Hofer et al. provided an explanation of the difficulties with using the gamma distribution in two-stage Bayesian analysis [10], and our HBM analysis. The use of gamma distribution in such modeling should be re-considered.

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