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Design of Proton FFAG Accelerators

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When designing a FFAG accelerator for protons we found convenient to follow this procedure [1]. (1) We assume a *Non-Scaling Lattice* (NSL) because the aim is toward a compact layout, though we are aware of the issue of multiple resonance crossing. (2) We take a periodic sequence of FDF triplets as these have been proven to exhibit a very small dispersion function. (3) The reference trajectory is taken to be the injection orbit that corresponds to the lowest value of the acceleration momentum range. (4) Finally, the magnets in the triplet have all a linear field profile. We have indeed found recently [2, 3] that the *Adjusted Field Profile* (AFP) to cancel the horizontal chromaticity is exceedingly non-linear and it causes a too large betatron tune variation with the amplitude of motion.

A sequence of FDF triplets is shown in Figure 1. They are made of sector magnets having parallel entrance and exit planes facing each other. Only for the injection orbit the trajectory in the magnets is made of arcs of circle. The magnets have sharp edges and there is no entrance or exit angle only for the reference (injection) orbit [4]. This solution minimizes magnet width, has the most stable momentum range, and allows longer drifts between triplets.

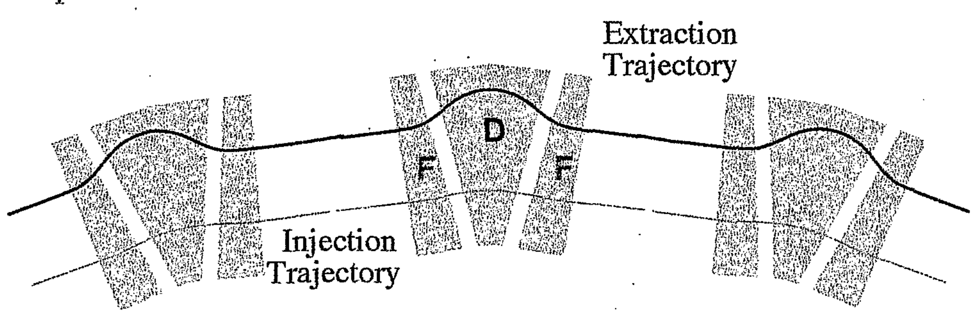

Figure 1. A Sequence of FDF Triplets making a FFAG Ring

Figure 2 shows a schematic of a FDF triplet with all the essential parameters. It is defined by 8 variables: the long drift S, the short drift g, the length L_F, the bending field B_F on the reference orbit, and the field gradient G_F of the focusing (F) sector magnets, and the same quantities L_D, B_D and G_D of the de-focusing sector magnet (D). To these we need to add the particle magnetic rigidity $B\rho$ (or equivalently the momentum p) and the full ring circumference C. We have thus a total of 10 variables that define uniquely the lattice of the FFAG ring for a given particle energy. Other useful parameters are the period length

$$P = S + g + 2L_F + L_D$$

the number of periods (periodicity)

$$N = C / P$$

the bending ratio

$$r = L_D B_D / 2 L_F B_F$$

and the packing factor

$$\alpha = (2 L_F + L_D) / P$$

By experience, after running several cases, we found that optimum values are given by

$$r = 2.34$$
$$\alpha = 0.472$$

that together determine 2 of the 10 quantities. Other quantities that are usually pre-assigned are the rigidity Br on the reference trajectory and the total circumference C. The D-sector magnet has a positive bending and the two F-sector magnets bend the beam in the opposite direction. The following relation imposes that the bending angle per period is $2\pi / N$ on the reference trajectory

$$B_D L_D - 2 B_F L_F = 2\pi B\rho / N$$

There are still a total of 5 variables to be determined.

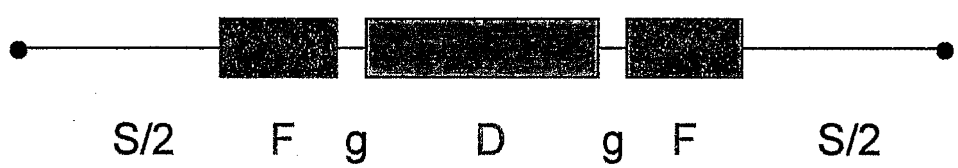

Figure 2. A FDF Triplet Period

A Reference Design. The AGS-FFAG

We have made the design of an FFAG accelerator [5, 6], following the procedure outlined above, for a new injector to the BNL AGS replacing the present Booster, with the goal of an average proton power of 1 MWatt at 28 GeV and the repetition rate of 2.5 Hz. The new injector would accelerate protons from 400 MeV to 1.5 GeV. The ring is to be located in the same AGS tunnel and has the same circumference. The main parameters of the accelerator are listed in Table 1. It is made of a sequence of periods similar to the one shown in Figure 2.

Table 1. Parameters of AGS-FFAG Injector

Circumference, C	807.091 m
Periodicity, N	136
Period Length, P	5.9345 m
Magnetic Rigidity, Bρ	31.8308 kG-m (400 MeV)
Long Drift, S	2.5345 m
Short Drift, g	0.30 m
F-Sector Magnet	
Length, L_F	0.70 m
Bending Field, B_F	-0.7841 kG
Gradient, G_F	26.582 kG/m
D-Sector Magnet	
Length, L_D	1.40 m
Bending Field, B_D	1.8345 kG
Gradient, G_D	-23.296 kG/m

In this design the circumference C and the magnetic rigidity Bρ at injection were indeed pre-assigned. We intentionally wanted a large major drift S for locating RF cavities and magnet components for injection and extraction. The field gradients were adjusted to yield acceptable phase advances across a period on the injection orbit. The major lattice parameters of the AGS-FFAG are given in Table 2. Figure 3 plots the lattice functions across one period again on the injection orbit.

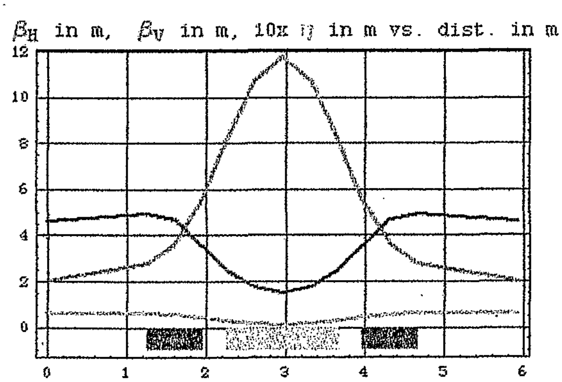

Figure 3. Lattice Functions across one AGS-FFAG Period

The linear field profile in each of the two types of sector magnets is plotted in Figure 4. For the AGS injector application the field value is modest, in the kGauss range. The magnets can be made as quadrupoles on which a dipole bending field is superimposed. Figure 5 shows the closed orbits along the length of half a period at different energies during acceleration. The physical aperture required to accommodate the momentum range is at most 17 cm to which the contribution from the betatron emittance is to be added as well some allowance for errors and clearance.

Table 2. Lattice Parameters on the Injection Orbit of the AGS-FFAG

Phase Advance / Period, H / V	105° / 100°
Betatron Tunes H / V	39.76 / 37.75
Transition Energy, γ_T	105.5
Max β value, H / V	4.6 m / 11.8 m
Max dispersion, η	6.0 cm
Chromaticity, H / V	-0.926 / -1.805

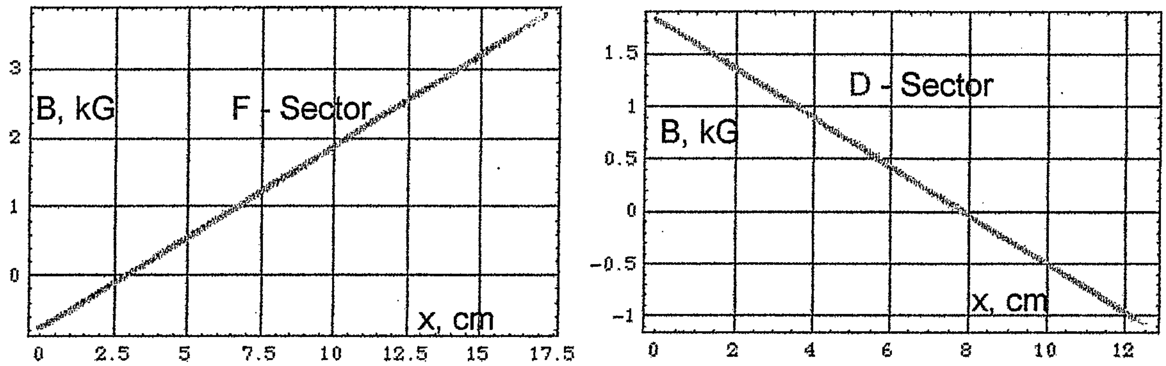

Figure 4. Linear Field Profile In the two Sector Magnets

Since we are dealing with a NSL, we expect large variation of the lattice functions across the momentum range of acceleration. Most dramatic is the variation of the betatron tunes shown in Figure 6. Several integral and half-integral resonances are swept through. The actual rate of tune change is expected to be large if assuming the acceleration cycle [5, 6] with the beam circulating a total of 2,200 revolutions during the period of 7 ms. It is not clear what would be the effect of the multiple resonance crossing on the beam stability. Figure 7 gives the change of the amplitude β-functions, that implies some modest increase of the betatron beam size as the beam is being accelerated. Figure 8 shows the variation of the dispersion η. Finally the variation of the transition energy γ_T is shown in Figure 9. Since $\gamma \ll \gamma_T$ and acceleration is done within the RF bucket, no consequences are expected to the beam longitudinal dynamics.

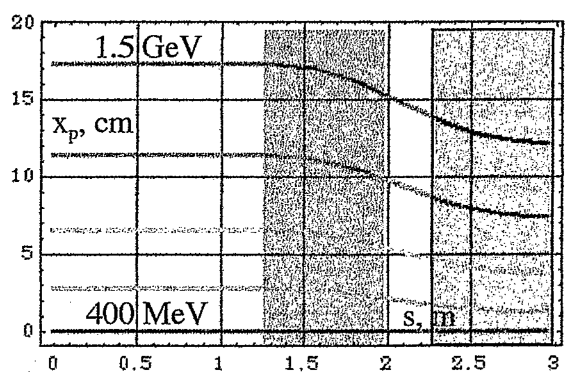

Figure 5. Closed Orbit along Half-Period at different Energies

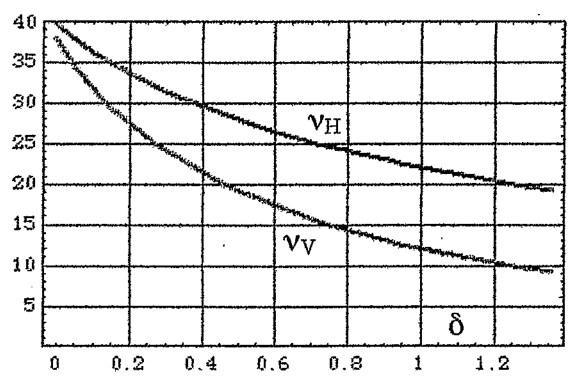

Figure 6. Tune change during Acceleration

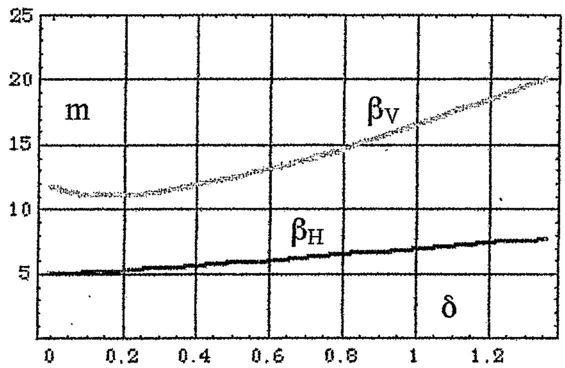

Figure 7. β_H (in front of F) and β_V (in the middle of D) during Acceleration

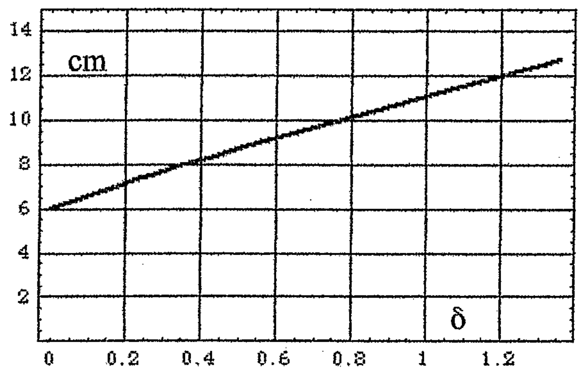

Figure 8. Dispersion η (in front of F) during Acceleration

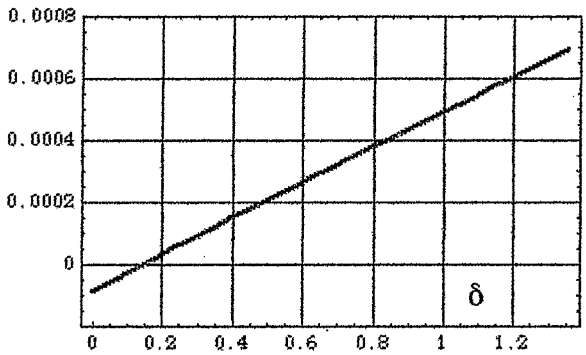

Figure 9. Momentum Compaction $\alpha_P = 1/\gamma_T^2$ during Acceleration

In Figures 6 to 9 the variable on the horizontal scale is the relative momentum deviation $\delta = (p - p_0) / p_0$ where p_0 is the momentum value on the injection orbit where $\delta = 0$ corresponding to a kinetic energy of 400 MeV. At the end of acceleration $\delta = 1.36$ that corresponds to 1.5 GeV.

Design of a Generic other FFAG Accelerator

The other 5 variables that need to be determined for the design of a generic FFAG accelerator, with the procedure described above, are evaluated from the requirements:

- assign the ratio L_D / L_F of the two sector magnets,
- assign the ratio S / g of the long insertion to the short drift,
- assign the ratio B_D / B_F of the two bending fields on the injection orbit,
- chose the gradients G_D and G_F so that the phase advances per period ψ_H and ψ_V on the injection orbit are assigned for both horizontal and vertical planes.

From the design of the AGS-FFAG lattice we found convenient to use the following:

L_D / L_F	=	2
S / g	=	8.448
B_D / B_F	=	-2.43
ψ_H	=	105°
ψ_V	=	100°

The scaling with the number of periods N, the full circumference C (or the period length P), and the magnetic rigidity $B\rho$ on the reference (injection) orbit, gives the following relations uniquely determined:

L_D	=	(1.40 m)	(P / P_0)
S	=	(2.5345 m)	(P / P_0)
B_D	=	(1.8345 kG)	$(C_0 / C)\,(B\rho / B\rho_0)$
G_F	=	(26.582 kG/m)	$(P_0 / P)^2\,(B\rho / B\rho_0)$

where from the design of the AGS-FFAG

C_0	=	807.091m
N_0	=	136
P_0	=	5.9345 m
$B\rho_0$	=	31.8308 kG-m (400 MeV)

Using this approach we have designed other FFAG accelerators for other applications, namely one 1.0-GeV FFAG Proton Driver for high-power (up to 10 MW) [7], and a complex of FFAG accelerators for one 11.6 GeV (up to 20 MW) Proton Driver for Neutrino Factory [8]. As expected, we found that for all these other FFAG accelerators the behavior of the beam dynamics is similar to that of the AGS-FFAG, and that the results shown in Figures 3 to 9 are similar. In particular the following scaling also applies on the reference trajectory

ν_H	=	40	(N / N_0)
ν_V	=	38	(N / N_0)
γ_T	=	i 105	(N / N_0)
β_H	=	4.6 m	(P / P_0)
β_V	=	11.8 m	(P / P_0)
η	=	6.0 cm	$(P / P_0)\,(N_0 / N)$

where β_V is in the middle of the D-sector, and β_H and η at the entrance of the F-sector magnet. The variation of the betatron tunes across the momentum range is about the same for all accelerators and has the typical behavior shown in Figure 6, except that the range of change scales also as (N / N_0).

We wanted to explore the largest momentum range over which there is stability of motion versus the ring circumference, number of periods and initial beam energy. We found that, adopting the procedure outlined above, the result is the same for all cases and shown in Figure 10 where the two betatron tunes are plotted versus momentum deviation δ. In particular the vertical stability is lost earlier, at about $\delta = 2.176$. The horizontal motion remains stable over a longer range, the difference given by the extra focusing due the orbit curvature. When translating to the actual momentum range Δ centered to the average value, we have the relation $\Delta = \delta / (2 + \delta)$ that yields a limit of $\Delta = \pm\, 52\%$. Probably, in the design of an actual FFAG accelerator we should take safely, $\delta = 2$ and thus $\Delta = \pm\, 50\%$.

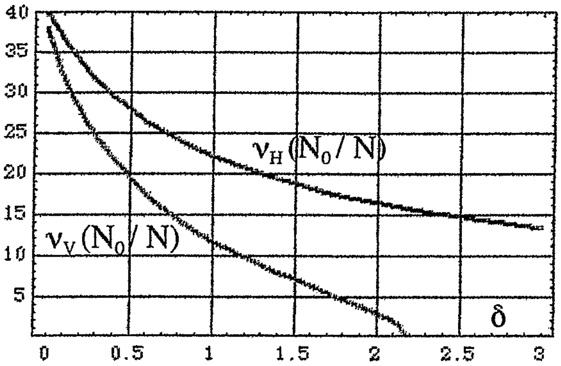

Figure 10. Universal FFAG Stability Limit

As an example, a Proton Medical FFAG accelerator operating between 0.5 and 250 MeV could be made of three rings with the following energy ranges:

0.5-4 MeV	$\delta = 1.83$	$\Delta = \pm 48\%$
4-32 MeV	$\delta = 1.85$	$\Delta = \pm 48\%$
32-250 MeV	$\delta = 1.95$	$\Delta = \pm 49\%$

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