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Strong Fields: From High Z Atoms to the Color Glass Condensate

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Strong Fields: From High Z Atoms to the Color Glass Condensate

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Abstract

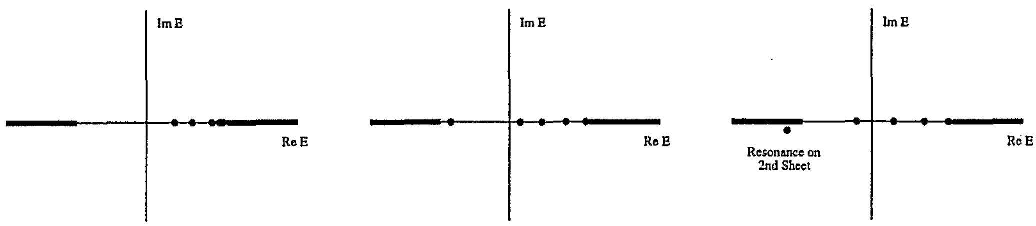
I review various strong field problems in field theory. I start with one of the earliest examples, a high Z Coulomb field. I discuss tunneling and thermally activated transitions in field theory. The latter problem may have applications to electroweak baryogenesis. Finally, I discuss the Color Glass Condensate, a form of high energy density gluonic matter which controls the high energy limit of QCD, and the Glasma which it makes in the collision of high energy nuclei.

1 High Z Atoms

Atoms with electric charge Z which is sufficiently large can induce spontaneous pair production. This happens when the potential is large enough so that

$$m \sim V(r)\mid_r = 1/m \tag{1.1}$$

For such large fields, the Coulomb field of the nucleus can be shielded by spontaneous pair production. When $m = V(1/m)$, $Z\alpha = 1$. there is indeed a singularity of the Dirac equation at this crtical value of the charge. Walter Greiner and colleagues were the first to understand that this singularity is an artifact of treating the charge in the Coulomb problem as pointlike. Indeed



a b c

Figure 1: (a) The singularities of the electron Green's function. (b) The singularities as Z increase. (c) The singularities after the field is strong enough to make pairs.

the true singularity corresponding to pair production may be found by treating the nuclear charge distribution as extended, and occurs at a larger value of $Z\alpha$ In figure 1 a, the singularity structure of the electron Greens function is shown. The cuts correspond to the continuum of positrons and electrons. The poles are bound states of electrons. As the field increases, Fig. 1 b, the poles move corresponding to more deeply bound states. Above some critical Z, the lowest bound state moves into the second Riemanian sheet through the positron cut. It corresponds to a resonance in positron atom scattering, where a positron scatters from a permanently bound electron and annihilates, followed by spontaneous pair production in the supercritical field, which then results in a continuum positron plus an electron permanently bound to the atom.

This supercritical field problem was to my knowledge first understood by Walter Greiner, and serves as the paradigm for many problems associated with the spontaneous decay of an unstable vacuum.[1]

The Frankfurt group suggested Coulomb barrier energy heavy ion collisions to test these ideas. The idea was that the ions would collide and stay close together long enough to produce pairs.[2] A program was established at GSI, and the results to date have in my opinion been inconclusive.

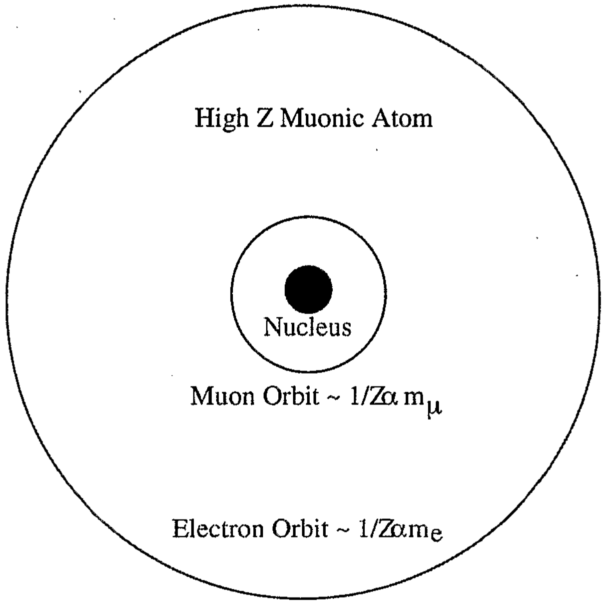


Figure 2: A high Z muonic atom surrounded by electrons with a captured muon.

2 Strong Constant Electric Fields

If there is a constant electric field of large enough extent, pairs will always be made in this and it will short itself out.[3] The criteria is that

$$m = Er \tag{2.2}$$

In QCD, this is taken as a model for how the color electric fields produce quark pairs, so that one can never see an unconfined color field. The electric field between source of color charge grows linearly in the absence of quarks which may short out the field. This has a consequence that the vacuum is a conductor of color electric charge (as is a quark gluon plasma). If one tries to separate a par of color charges to long distances, then they spontaneously generate a current between them which neutralizes their field.

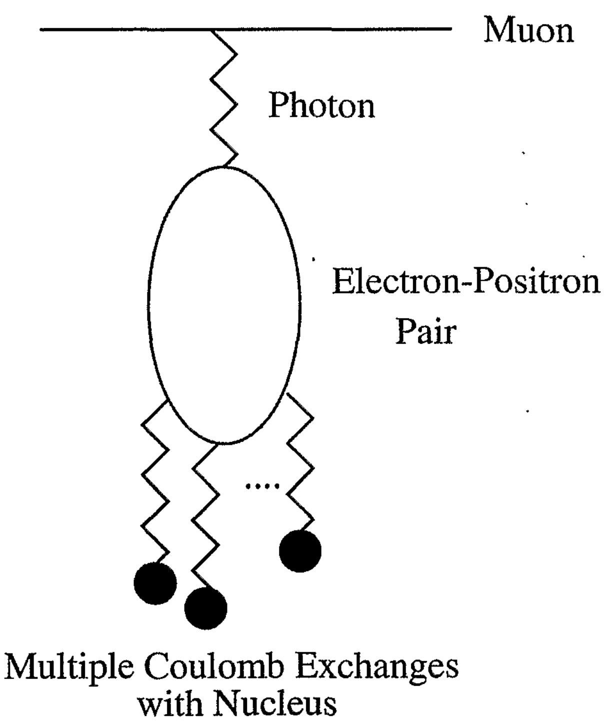


Figure 3: Vacuum polarization to all orders in $Z\alpha$.

3 Muonic Atoms

Muonic atoms provide clean tests of QED in high Z fields. This is because, as shown in Fig. 2, the muon can orbit far inside the first Bohr orbit of the electrons. This is because its mass requires that its deeply bound states be at distances of order $r \sim 1/Z\alpha m_\mu$, and $m_\mu >> m_e$ Miklos Gyulassy and I got to know one another as graduate students because we were both working on the problem of vacuum polarization in these strong fields. At the time, there was a discrepancy between theory and experiment. It later turned out that the experiments were wrong. In any case, we both carefully recomputed the vacuum polarization and accounted for finite size of the nucleus.[4]-[5] This was also how both of us got to know Walter, since he was the worlds expert on high Z atomic systems.

The computation was not easy, and involved computing a loop diagram to all orders in the Coulomb field, as shown in Fig. 3.

4 The Bounce

In field theory, one can imagine that for reasons of history, one is in the wrong vacuum. One can compute the rate of tunneling from the false vacuum into the correct one. If one uses the WKB approximation, the rate of tunneling is given by e^{-S}, where S is the Euclidian action for the classical solution which takes one between the false vacuum, bounces off of the upside down potential corresponding to entering the true set of states associated with the correct action and returning to the also vacuum again.[6]

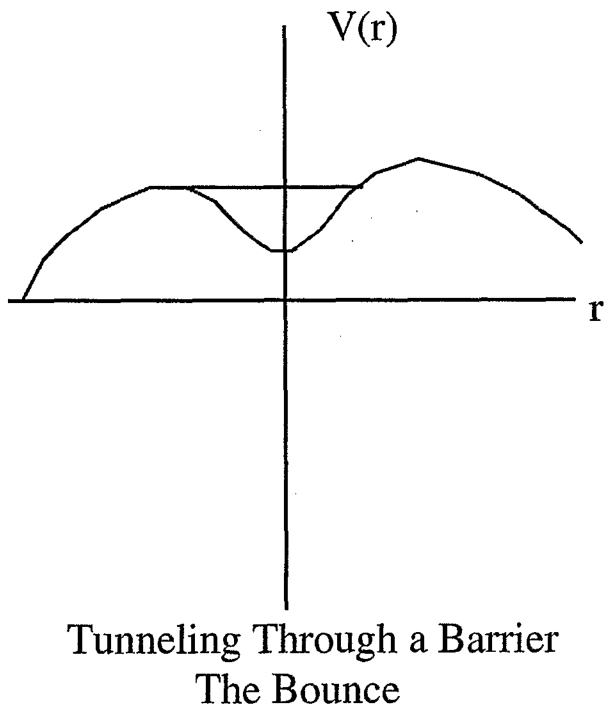


Figure 4: The bounce configuration.

This prescription is easy to understand: The Euclidean action arises because in tunneling, the Schrodinger wavefunction no longer oscillates, it exponentially decays. therefore $t \to it$. The equations of motion, $F = ma$, therefore correspond to an upside down potential, and one has motion in the forbidden region. The bounce solution is when inserted into the action gives precisely twice the WKB factor associated with tunneling, and therefore describes the rate of false vacuum decay.

This is a strong field problem, since for weakly coupled theories, a classical solution has a field which is of order $1/g$, where g is the interaction strength. The weak coupling limit corresponds, to the classical many quanta limit, and the typical number of quanta are of order $n \sim 1/\alpha$. False vacuum decay rates are therefore of order $e^{-\kappa/g^2}$.

5 Instantons and Sphalerons

In gauge theories such as QCD, there is multiple vacuum degeneracy. This degeneracy is labeled by the Chern-Simons charge number N_{CS}, as shown in Fig. 5. The various minima can all be mapped into one another by a gauge rotation. This gauge rotation has non-trivial topology. The internal space of the Yang-Mill field is mapped onto the coordinate space, and the winding number for this map is N_{CS}

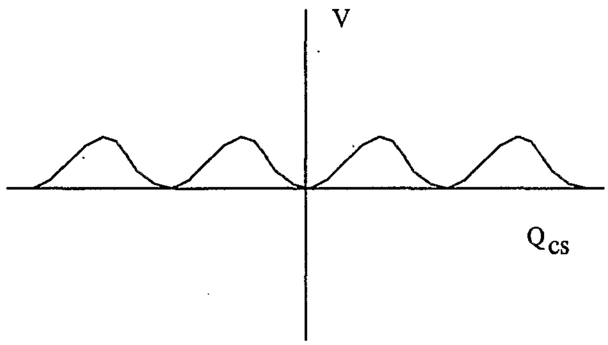
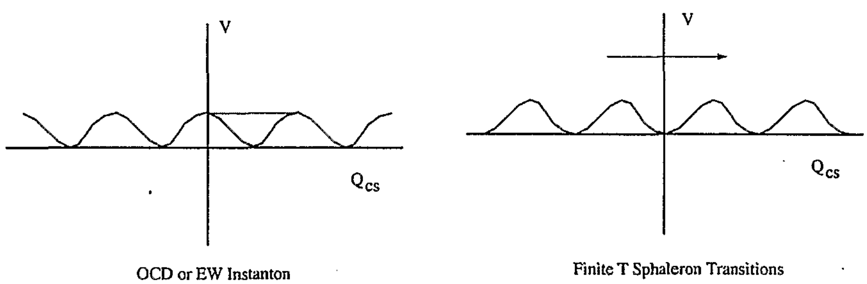


Figure 5: The energy of the as a function of N_{CS}.

At zero temperature, transitions are made by quantum tunneling as shown in Fig. 6 a. The tunneling is calculated by computing the Euclidean action in configuration space. The classical solution which makes the transition between two vacuum configurations is called the instanton.[7] The rate associated with this transition is exponentially small in the inverse coupling, $R \sim e^{-\kappa/g^2}$

At finite temperature, thermal excitation can allow transitions over the top of the barrier which separates the various minima, as shown in Fig. 6

b.[8]-[10] The rate is $e^{-E_{barrier}/T}$ The energy of the barrier is determined by computing a stationary but unstable solution to the equations of motions which correspond to the classical fields localized at the top of the barrier. Such a solution is called the sphaleron. The energy of this barrier is of order the typical dimensional scale of the theory, divided by g^2. For example, in electroweak theory, this is of order $M_{weak}/\alpha_{weak} \sim 10\ TeV$. Note that there is always some high temperature where the rates of thermally activated transition become of order 1.



a b

Figure 6: (a) The instanton. (b) The spheleron.

In gauge theories, changes in topological charge result in anomalous violation of conserved quantum numbers. To see this, look at Figure 7. The spectra of the Dirac equation is the same in both gauge related minima. It deforms continuously as we slowly move from one minima to the other and one level in fact crosses zero as we go over the maximum which separates the vacua. When the theory is quantized, negative energy states are filled, and positive energy states are empty. Positive energy states are interpreted as physical particles. We see from the figure that the deformation creates a positive energy state. Thus a particle has been made, like in the case of strong Coulomb fields. Such a particle production typically violates some underlying symmetry of the theory. In electorweak theory, it is baryon plus lepton number conservation. In QCD, it is helicity conservation.

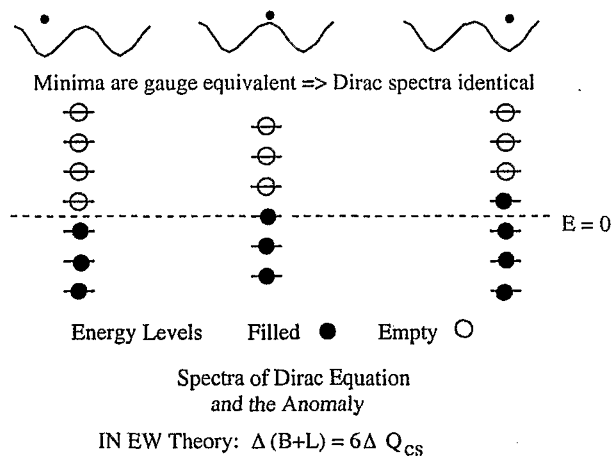


Figure 7: The generation of an anomaly.

6 The Color Glass Condensate and the Glasma

In ultrarelativistic heavy ion collisiions, the density of gluons is very high. This requires the interactions strength, $\alpha_S << 1$, becomes weak. The phase space density of gluons becomes large

$$\frac{dN}{dyd^2p_Td^2r_T} \sim \frac{1}{\alpha_S} \tag{6.3}$$

This has been described in much detail elsewhere.[11]-[12] I will not review this here, except to say that these gluons make a highly coherent condensate of strong color fields (characterized by a small interaction strength). The computation of properties of this condensate involves treating the Color Glass field as strong, and doing small fluctuations on top of this field. This lets one compute quark production in analogy to how Walter computer pair production long ago for strong Coulomb fields. [13]

It also turns out that in collisions, strong longitudinal electric and magnetic fields are produced. [14]-[16] The decay of these fields is the physics of the Glasma, a state intermediate between the Color Glass Condensate and the Quark Gluon Plasma. Such a Glasma might be responsible for early thermalization reported at RHIC. It also has a large density of Chern-Simons charge, which might manifest itself in CP or P odd observables.

It has also been suggested that the effects of rapid thermalization, such as strong elliptic flow patterns, may also be associated with the Glasma. This is because the Glasma contains a highly coherent strong gluon field. The interactions with this field are very rapid, and of the order of the time it takes light to travel the saturation distance scale $t \sim .1Fm/c$. This might take place by various instabilities of the classical equations which describe the evolution of the Glasma.[17]

7 Summary

The study of strong fields in quantum theory has become a rich and robust field. From a personal perspective, what is better is that it gave me a chance to get to know and become friends with Walter, Miklos, Horst and the young people Walter has produced at Frankfurt.

8 Acknowledgments

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