

***$Q_T$ -Resummation for Polarized Semi-Inclusive  
Deep Inelastic Scattering***

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# $Q_T$ -RESUMMATION FOR POLARIZED SEMI-INCLUSIVE DEEP INELASTIC SCATTERING

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We study the transverse-momentum distribution of hadrons produced in semi-inclusive deep-inelastic scattering. We consider cross sections for various combinations of the polarizations of the initial lepton and nucleon or the produced hadron, for which we perform the resummation of large double-logarithmic perturbative corrections arising at small transverse momentum. We present phenomenological results for the process  $ep \rightarrow e\pi X$  for the typical kinematics in the COMPASS experiment. We discuss the impact of the perturbative resummation and of estimated non-perturbative contributions on the corresponding cross sections and their spin asymmetry.

Semi-inclusive deep inelastic scattering (SIDIS) with polarized beams and target,  $ep \rightarrow ehX$ , for which a hadron  $h$  is detected in the final state, has been a powerful tool for investigating the spin structure of the nucleon. It also challenges our understanding of the reaction mechanisms in QCD. The bulk of the SIDIS events provided by experiments are in a kinematic regime of large virtuality  $Q^2$  of the exchanged virtual photon and relatively small transverse momentum  $q_T$ . In our recent paper<sup>1</sup>, we have studied the transverse-momentum dependence of SIDIS observables in this region, applying the resummation technique of<sup>2</sup>. The processes we considered were the leading-twist double-spin reactions:

$$\begin{aligned} \text{(i)} \quad e + p \rightarrow e + \pi + X, & \quad \text{(iv)} \quad \vec{e} + \vec{p} \rightarrow e + \pi + X, \\ \text{(ii)} \quad e + \vec{p} \rightarrow e + \vec{\Lambda} + X, & \quad \text{(v)} \quad \vec{e} + p \rightarrow e + \vec{\Lambda} + X. \\ \text{(iii)} \quad e + p^\uparrow \rightarrow e + \Lambda^\uparrow + X, & \end{aligned} \tag{1}$$

Here arrows to the right (upward arrows) denote longitudinal (transverse)

polarization. Needless to say, the final-state pion could be replaced by any hadron. The same is true for the  $\Lambda$ , as long as the observed hadron is spin-1/2 and its polarization can be detected experimentally. Here we present a brief summary of the main results of <sup>1</sup>.

There are five Lorentz invariants for SIDIS,  $\vec{e}(k) + A(p_A, S_A) \rightarrow e(k') + B(p_B, S_B) + X$ : the center-of-mass energy squared for the initial electron and the proton,  $S_{ep} = (p_A + k)^2$ , the conventional DIS variables,  $x_{bj} = \frac{Q^2}{2p_A \cdot q}$  and  $Q^2 = -q^2 = -(k - k')^2$ , the scaling variable  $z_f = \frac{p_A \cdot p_B}{p_A \cdot q}$ , and the magnitude of the “transverse” momentum  $q_T = \sqrt{-q_t^2}$  where the space-like vector  $q_t^\mu$  is defined as  $q_t^\mu = q^\mu - \frac{p_B \cdot q}{p_A \cdot p_B} p_A^\mu - \frac{p_A \cdot q}{p_A \cdot p_B} p_B^\mu$  which is orthogonal to both  $p_A$  and  $p_B$ . To write down the cross section, we use a frame where  $\vec{p}_A$  and  $\vec{q}$  are collinear, and we call the azimuthal angle between the lepton plane and the hadron plane  $\phi$ . In this frame, the transverse momentum of the final-state hadron  $B$  with respect to  $\vec{p}_A$  and  $\vec{q}$  is given by  $p_T = z_f q_T$ .

The lowest-order (LO) cross section differential in  $q_T$  (or  $p_T$ ) is of  $O(\alpha_s)$  and has been derived in <sup>3</sup>. It can be decomposed into several pieces with different dependences on  $\phi$ :

$$\frac{d^5 \sigma}{dQ^2 dx_{bj} dz_f dq_T^2 d\phi} = \sigma_0 + \cos(\phi) \sigma_1 + \cos(2\phi) \sigma_2, \quad (2)$$

for processes (i) and (ii) in (1),

$$\frac{d^5 \sigma}{dQ^2 dx_{bj} dz_f dq_T^2 d\phi} = \sigma_0 + \cos(\phi) \sigma_1, \quad (3)$$

for (iv) and (v), and

$$\begin{aligned} \frac{d^5 \sigma_T}{dQ^2 dx_{bj} dz_f dq_T^2 d\phi} &= \cos(\Phi_A - \Phi_B - 2\phi) \sigma_0^T + \cos(\Phi_A - \Phi_B - \phi) \sigma_1^T \\ &+ \cos(\Phi_A - \Phi_B) \sigma_2^T, \end{aligned} \quad (4)$$

for (iii). Here  $\Phi_A$  ( $\Phi_B$ ) is the azimuthal angle of the transverse spin vector of  $A$  ( $B$ ) as measured from the hadron plane around  $\vec{p}_A$  ( $\vec{p}_B$ ) in the so-called *hadron frame* for which  $q = (0, 0, 0, -Q)$ . At small  $q_T$ ,  $\sigma_0$  and  $\sigma_0^T$  develop the large logarithmic contribution  $\alpha_s \ln(Q^2/q_T^2)/q_T^2$ . At yet higher orders, corrections as large as  $\alpha_s^k \ln^{2k}(Q^2/q_T^2)/q_T^2$  arise in the cross section. We have worked out the NLL resummation of these large logarithmic corrections in  $\sigma_0$  and  $\sigma_0^T$  for all the processes in (1) within the  $b$ -space resummation formalism of <sup>2</sup>, extending the previous studies on the resummation for unpolarized SIDIS <sup>4</sup>. The  $\phi$ -dependent contributions to the cross sections in general also develop large logarithms <sup>5</sup>; their resummation would require an extension of the formalism.

In order to study the impact of resummation, we have carried out a numerical calculation for the process  $\bar{e}p \rightarrow e\pi X$ . The resummed cross section takes the form of an inverse Fourier transform into  $q_T$  space. To carry out the Fourier integral, one needs a recipe for treating the Landau pole present in the perturbatively calculated Sudakov form factor. We have followed the method of <sup>6</sup> which deforms the  $b$ -integral to a contour integral in the complex  $b$ -plane. This method introduces no new parameter and is identical to the original  $b$ -integral for any finite-order expansion of the Sudakov exponent. For comparison, we have also used the  $b^*$ -method proposed in <sup>2</sup>. In order to incorporate possible nonperturbative corrections, we introduce a Gaussian form factor by shifting the Sudakov exponent as  $e^{S(b,Q)} \rightarrow e^{S(b,Q)-gb^2}$ , where the coefficient  $g$  may be determined by comparison with data. In order to obtain an adequate description also at large  $q_T \sim Q$ , we “match” the resummed cross section to the fixed-order (LO,  $O(\alpha_s)$ ) one. This is achieved by subtracting from the resummed expression its  $O(\alpha_s)$  expansion and then adding the full  $O(\alpha_s)$  cross section <sup>6,7</sup>.

As an example, we show in Fig. 1 the  $z_f$ -integrated cross sections

$$\frac{1}{2\pi} \int_{0.2}^{z_f^{max}} dz_f d\phi \frac{d(\Delta)\sigma}{dx_{bj} dz_f dQ^2 dq_T d\phi}, \quad (5)$$

and their spin asymmetry for the typical kinematics of the COMPASS experiment,  $S_{ep} = 300 \text{ GeV}^2$ ,  $Q^2 = 10 \text{ GeV}^2$ ,  $x_{bj} = 0.04$ . As expected, the resummation tames the divergence of the LO cross section at  $q_T \rightarrow 0$  and enhances the cross section in the region of intermediate and large  $q_T$ . The nonperturbative Gaussian makes this tendency stronger. Although the cross sections vary slightly when different treatments of the  $b$ -integral and different values of  $g$  are chosen, the effects of resummation and the nonperturbative Gaussian are mostly common to both the unpolarized and the polarized cases. Accordingly, the spin asymmetry is relatively insensitive to these effects. It will be interesting to compare our results with forthcoming data from COMPASS and HERMES, and also to extend the analysis to the reaction  $\bar{e}p \rightarrow e\bar{\Lambda}X$  which is accessible at HERA.

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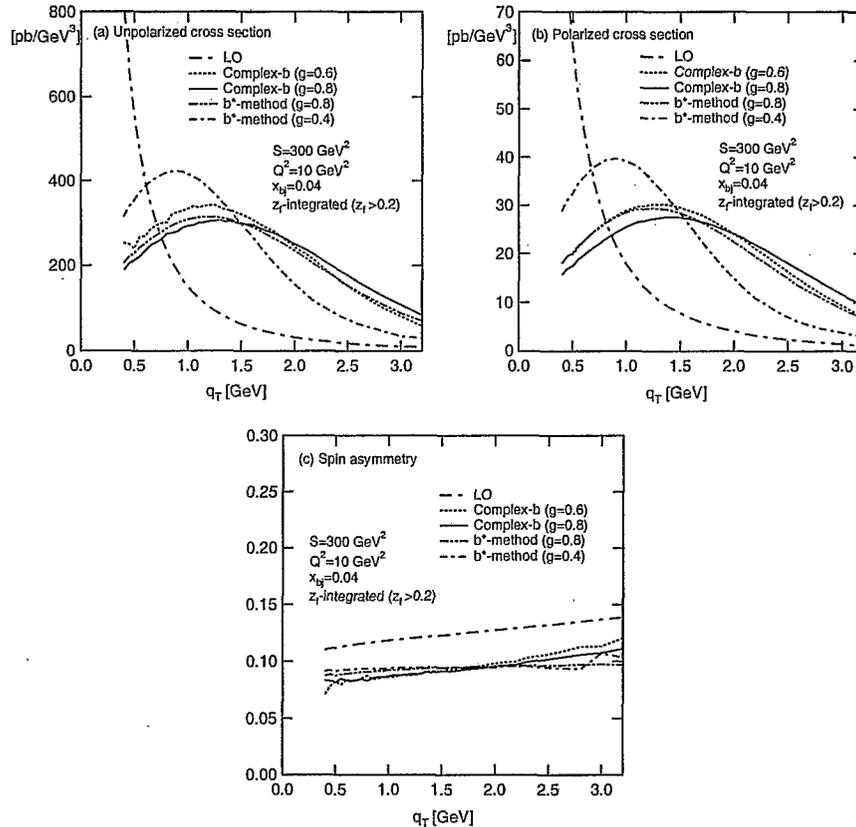


Figure 1. (a) Unpolarized SIDIS cross section for COMPASS kinematics. We show the fixed-order (LO) result, and resummed results for the complex- $b$  method with non-perturbative parameters  $g = 0.6, 0.8 \text{ GeV}^2$ , and for the  $b^*$  method with  $b_{\text{max}} = 1/(\sqrt{2} \text{ GeV})$  and  $g = 0.4, 0.8 \text{ GeV}^2$ . (b) Same for the longitudinally polarized case. (c) Spin asymmetries corresponding to the various cross sections shown in (a) and (b).

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