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QCD Thermodynamics With Almost Realistic Quark Masses

Christian Schmidt

Brookhaven National Laboratory
Upton, NY 11973-5000

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Physics Department

Brookhaven National Laboratory
P.O. Box 5000
Upton, NY 11973-5000
www.bnl.gov

QCD THERMODYNAMICS WITH ALMOST REALISTIC QUARK MASSES

C. Schmidt
(for the RBC-Bielefeld Collaboration)
Department of Physics, Brookhaven National Laboratory, Upton 11973 NY, USA

Ongoing calculations on the QCDOC supercomputer at Brookhaven National Laboratory and the APEnext installation at the University of Bielefeld aim to determine the critical temperature of the QCD phase transition as well as the equation of state with almost realistic quark masses. We will discuss preliminary results of the quark mass and cut-off dependence of order parameters, susceptibilities, static quark potentials and the critical temperature in (2+1)-flavor QCD. All these quantities are of immediate interest for heavy ion phenomenology.

1 Introduction and Lattice Setup

The calculations of the QCD phase diagram and bulk thermodynamic quantities from first principle give input for Heavy Ion Phenomenology, Cosmology and Astrophysics. In particular, it is mandatory to improve estimates on the critical temperature (T_c) to make contact with HIC Phenomenology. Since for the critical energy density (ε_c) we have $\varepsilon_c \sim T_c^4$, a small error on T_c is important. Moreover, an interesting question is whether or not the freeze-out temperature in HICs is connected to T_c.

The grand canonical QCD partition function on the lattice is given by the integral

$$Z_{GC}(am_q, am_s, N_s, N_t, \beta) = \int \mathcal{D}U \;\; [\det M(U, m_q)]^2 \, [\det M(U, m_s)] \exp\{-\beta S_G[U]\} \quad , \tag{1}$$

where a is the lattice spacing, am_q and am_s are the light and strange quark masses respectively (given in lattice units), N_s and N_t are the number of lattice points in spacial and temporal direction. M is the fermion matrix and U are the gauge fields, located on the links between the lattice points. S_G is the gauge part of the action and β is the coupling which controls the lattice spacing. The lattice action we use is especially designed for finite temperature QCD Simulations, where the lattice spacing is usually rather large. In the gauge sector we use a

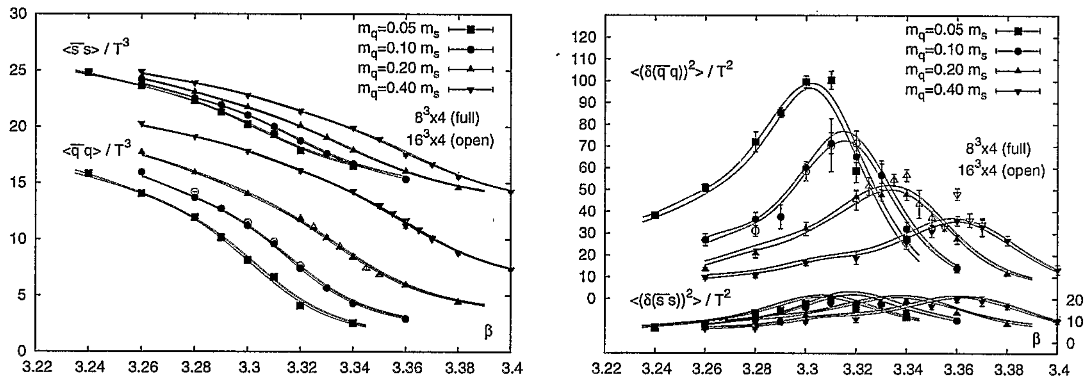

Figure 1: The light quark and strange quark chiral condensate at (left) and its susceptibility (right) as a function of the coupling. The strange quark chiral condensate was multiplied with a factor 2 for better visibility. Calculation have performed with various values of the light quark mass but fixed strange quark mass. Lattice sizes are $8^3 \times 4$ and $16^3 \times 4$.

(2×1)-Symanzik improvement scheme which eliminates all cut-off effects of order $\mathcal{O}(a^2)$. For the fermions we use the staggered fermion formulation. On top of that we add an improvement term which restores the rotational symmetry of the free quark propagator on the lattice up to order $\mathcal{O}(p^4)$ in the momentum [1]. In order to improve the flavor symmetry, which is violated in the staggered fermion formulation, we smear each link of the gauge field which is used in the standard part of the fermion action by its surrounding three link staples (p4fat3). The p4fat3 action was used for thermodynamical calculation earlier [2,3], reports of the ongoing project have been given by C. Jung [4] and M. Cheng [5]

We perform simulations with 2 light and one heavy quark flavor. The strange quark mass is always fixed to the physical value, whereas the light quark mass is varied in the range of $m_q = 0.05m_s - 0.4m_s$. The lattices have temporal extent $N_t = 4, 6$, which corresponds at the critical temperature (T_c) to a lattice spacing of $a \approx 0.13$ fm and 0.22 fm respectively. The lattice extent in spacial direction is $N_s = 8, 16, 32$. To determine the scale, we perform zero temperature simulations on $16^3 \times 32$ lattices. These calculations are being performed on the QCDOC computers at BNL and the APEnext installation at the University of Bielefeld.

2 Order Parameters and Susceptibilities

Connected to chiral symmetry breaking is the chiral condensate. In the staggered formulation of lattice QCD it is given by

$$\langle \bar{q}q \rangle \equiv \frac{d}{dm} \ln Z_{GC}(am, am_s, N_s, N_t, \beta)\Big|_{m=m_q} = N_s^{-3} N_t^{-1} \frac{1}{2} \left\langle \mathrm{Tr} M_{KS}^{-1}(m_q) \right\rangle \quad , \qquad (2)$$

$$\langle \bar{s}s \rangle \equiv \frac{d}{dm} \ln Z_{GC}(am_q, am, N_s, N_t, \beta)\Big|_{m=m_s} = N_s^{-3} N_t^{-1} \frac{1}{4} \left\langle \mathrm{Tr} M_{KS}^{-1}(m_s) \right\rangle \quad , \qquad (3)$$

where M_{KS} is the staggered fermion matrix. In the limit of vanishing quark masses the chiral condensate is an exact order parameter of the spontaneous chiral symmetry breaking. Its expectation value is non-zero below the critical temperature (T_c) and zero above. At finite quark masses where the chiral symmetry is explicitly broken, the chiral condensate still signals the transition by a rapid change. In Fig. 1 we show the expectation value of the light quark chiral condensate ($\langle \bar{q}q \rangle$) and strange quark chiral condensate ($\langle \bar{s}s \rangle$) as function of the coupling β (left) and its susceptibility (right) on $N_t = 4$ lattices. Results on the chiral susceptibility on $N_t = 6$ lattices are shown in Fig. 2(left). The results have been interpolated in the coupling β by using

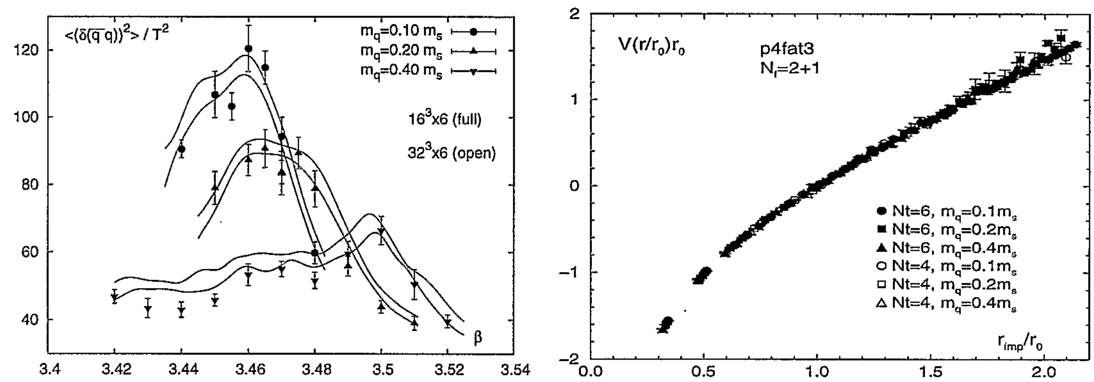

Figure 2: The light quark chiral susceptibility as a function of the coupling measured on $16^3 \times 6$ lattices with three different light quark masses (left). The zero temperature static quark potential in units of the Sommer scale. Results for different light quark masses and lattice spacings are plotted seem to fall on one universal curve.

the multi-histogram re-weighting technique[6]. Since the coupling controls the lattice spacing a it thus also controls the temperature, which is given by $T = 1/N_t a$. Large couplings β corresponds to large temperatures T and small couplings to small temperatures. At the coupling where the condensate shows the most drastic change, the corresponding susceptibility peaks. We define the critical coupling β_c by the peak position of the chiral susceptibility. In Fig. 1, we compare results for different quark masses. The calculation with $m_q = 0.05 m_s$ is roughly at the physical point. A clear dependence of the critical coupling (critical temperature) on the quark mass can be observed. The peak positions of the light quark and strange quark susceptibilities do however coincide within our statistical accuracy. Which indicates that chiral symmetry restoration for light and strange quark occurs at the same temperature. The strength of the transition decreases with increasing quark masses, this is reflected in a decreasing peak height of the susceptibilities.

In Fig. 1 we also compare results form $8^3 \times 4$ and $16^3 \times 4$ lattices. Since we see almost no volume dependence the results suggest that the transition is in fact not a true phase transition in the thermodynamic sense but a rapid crossover.

3 Scale Setting and the Static Quark Potential

On $16^3 \times 32$ lattices we calculate the zero temperature static quark potential for all quark mass values at their corresponding critical couplings. The Sommer scale r_0 is defined as the distance where the derivative of the potential take a certain value, to be precise it is defined as

$$r^2 \frac{\mathrm{d}V}{\mathrm{d}r}\bigg|_{r=r_0} = 1.65 \quad . \tag{4}$$

The Sommer scale is often used to set the scale in lattice calculations, a phenomenological value is $r_0 = 0.5$ fm. In Fig. 2 (right), we plot the static quark potential for different quark masses and lattice spacing in units of the Sommer since all results seem to fall on a universal curve, the static quark potential shows almost no quark mass or cutoff dependence. In order to remove short range lattice artifacts, we have used improved distances r_{imp} in the Coulombic part of the potential. For our choice of the lattice action r_{imp} is given by

$$\frac{1}{4\pi r_{imp}} \equiv \int \frac{d^3k}{(2\pi)^3} \frac{e^{ikr}}{4\sum_i(\sin^2\frac{k_i}{2}) + \frac{1}{3}\sin^4\frac{k_i}{2}} \quad . \tag{5}$$

To estimate the systematic uncertainties of our potential fits, we perform various types of fits, e.g. different fit-ranges in r or different fit-forms (3 & 4 params. fits). In our future analysis, of

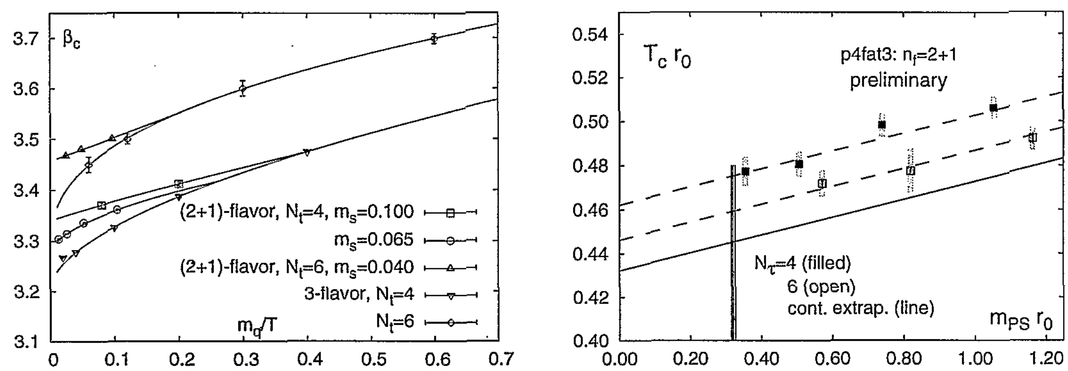

Figure 3: The critical coupling as a function of the light quark mass (left) and the critical temperature as a function of the pion mass (right). The critical temperature as well as the pion mass are given in units of the Sommer scale.

the critical temperature we will use the Sommer scale to convert our results form lattice units to physical units.

4 The Critical Temperature

In Fig. 3 (left) we plot the critical coupling as function of the light quark mass. We find that $\Delta\beta \equiv \beta_c(N_t = 6) - \beta_c(N_t = 4) = 0.13 - 0.14$ is almost quark mass independent. This independence if the cut-off effect from quark masses holds in leading order also for the critical Temperature as can be seen in Fig. 3 (right). Here we show T_c in units of the Sommer scale as function of the pion mass m_{PS} (also in units of the Sommer scale). We perform a combined chiral and continuum extrapolation of T_c by using the ansatz

$$T_c r_0 = [T_c r_0]_{\text{cont}}^{\text{chiral}} + b_1(m_\pi r_0) + b_2/N_t^2 \quad , \tag{6}$$

where b_1 and b_2 are free fit parameters. This extrapolation leads to critical temperature in the chiral and continuum limit of $T_c \approx 185$ MeV. The linear extrapolation in quark mass is on the one hand suggested by the data, on the other hand it is validated by the fact that one expects a critical point in the chiral limit which is in the $O(4)$-universality class. The leading order in the scaling behavior of the transition temperature is then given by $T_c \sim (m_{PS})^{1.1}$, which is sufficiently close to a linear scaling behavior.

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