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The Extraction of V(ud)

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The Extraction of V_{ud}

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The CKM 3-generation quark-mixing matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{1}$$

is a fundamental byproduct of electroweak symmetry breaking and mass generation. Determining its components can provide important insight regarding that phenomenon. In addition, testing the unitarity of that matrix with high-precision probes for "new physics," i.e., beyond the Standard Model effects. Toward that end, the unitarity condition for the first row,

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \tag{2}$$

is particularly important since it has been tested at nearly the $\pm 0.1\%$, and for many years seemed to suggest a 2-3 sigma deviation from unitarity. If true, that discrepancy could provide a window to "new physics." Alternatively, if eq.(2) is confirmed at $\pm 0.1\%$ or better, it can be used to constrain or rule out speculative ideas for appendages to the Standard Model, both at the tree and quantum loop level.

Critical to the unitarity test in eq.(2) is the fact that $|V_{ub}|^2 \simeq 2 \times 10^{-5}$ is negligibly small and can be, to good approximation, ignored. So, only V_{ud} and V_{us}, cornerstones of the CKM matrix, need be scrutinized with high precision.

In the case of V_{us}, there has been what amounts to an experimental revolution in the value obtained from $K_{\ell 3}$ decays, $K \to \pi \ell \nu$, the process traditionally used for its determination. As discussed elsewhere in this report, a series of new measurements now give [1]

$$|V_{us}| = 0.2264(12) \times (0.96/f_+(o)) \qquad (K_{\ell 3} \text{ decays}), \tag{3}$$

a value considerably larger than what prevailed (~ 0.220) in particle data tables for many years. Note, however, that we have chosen to normalize eq.(3) using the lattice form factor central value [2]

$$f_+(0) = 0.96(1) \qquad (\text{ Lattice}) \tag{4}$$

which is very similar to the classic value initially computed by Leutwyler and Roos [3], $f_+(0) = 0.961(8)$. More recent chiral perturbation theory studies [4] suggest values as large as $f_+(0) = 0.984(12)$. That discrepancy needs to be resolved.

Another new development is the lattice gauge theory determination of the pseudoscalar decay constants f_K and $f\pi$. Their ratio is already impressively precise [5]

$$f_K/f_\pi = 1.204(14) \tag{5}$$

due to the cancellation of correlated statistical and systematic uncertainties. In the long term, that determination may further significantly improve as new powerful computers allow more sophisticated treatments of chiral symmetry. Already, however, the result in eq.(5) can be combined with experimental measurements of $\Gamma(K \to \mu\nu(\gamma))$ and $\Gamma(\pi \to \mu\nu(\gamma))$ to give [6]

$$|V_{us}| = 0.2234(4)\ (1.204\ f\pi/f_K) \qquad \mathrm{K}_{\mu 2}/\pi_{\mu 2} \tag{6}$$

One can see that the f_K/f_π approach has the potential, as lattice calculations improve, to give the best determination of $|V_{us}|$ in the future.

Before ending this brief discussion of V_{us}, it should be mentioned that studies of Hyperon decays suggest [7] $|V_{us}| = 0.2250(27)$ (modulo SU(3) symmetry breaking corrections) while strangeness changing tau decays tend to give [8] $|V_{us}| = 0.2208(34)$, a lower value. Those cases will be discussed later in this report. Here, we merely use them to point out that although V_{us} has recently increased, its exact value is not without controversy and could still undergo some change.

To utilize the new V_{us} results in the unitarity relationship of eq. (2) requires a very precise determination of V_{ud}. That quantity has been extracted from 1) super-allowed, $0^+ \to 0^+$, nuclear beta decays, 2) neutron beta decays, $\eta \to pe\nu$, and 3) pion beta decay $\pi^+ \to \pi^0 e^+ \nu$. The latter two, subsequently discussed in this report, have smaller overall theoretical uncertainties and may in the long term be better ways to obtain V_{ud}; but currently, only super-allowed beta decays have the statistical power to determine V_{ud} to better than 0.05%; so, here we focus on the status of those decays and their implication for unitarity.

The so-called super-allowed, $0^+ \to 0^+$, Fermi transitions between nuclei are very special [9]. Because they proceed (at the level) through pure weak vector current interactions, which are consumed in the $m_d = m_u$ limit; they are not renormalized by strong interactions at $q^2 = 0$. Hence, they are ideally suited for cleanly extracting V_{ud} with high precision. Corrections due to $q^2 \neq 0$ and $m_d \neq_u$ are negligibly small; so, one need only control uncertainties in the electroweak radiative corrections, isospin violating electromagnetic effects and nuclear structure dependence. How well that can be done is the subject of this section.

Last year, the prevailing value of V_{ud} obtained by averaging the nine best measured super-allowed β-decays was [10, 11]

$$V_{ud} = 0.9740(1)(3)(4) \rightarrow 0.9740(5) \qquad \text{(2004 value)} \tag{7}$$

where the errors are experimental, nuclear theory and radiative corrections. The very small experimental error illustrates the power of this averaging procedure. The largest uncertainty, associated with weak axial-vector induced loop effects [1, 12], primarily through γW box diagrams represents model dependent hadronic effects which until recently [13] were thought to be essentially irreducible or at least very difficult to reduce.

Two developments have led to a recent improvement in V_{ud} by nearly a factor of 2. First, a new global study of super-allowed β-decays by Hardy and Towner [14] has provided a more consistent treatment of Q values and lifetimes used in ft determinations, which in turn give V_{ud} via the master formula

$$|V_{ud}|^2 = \frac{2984.48(5)\text{sec}}{ft(1+RC)} \tag{8}$$

In that expression, RC designates the total effect of all radiative corrections from quantum loops as well as nuclear structure and isospin violating effects. RC is nucleus dependent, ranging from about +3.1% to +3.6% for the nine best measured super-allowed decays. That difference is of critical importance in bringing the values of V_{ud} obtained from separate decays into agreement with one another. The magnitude of the corrections is essential for establishing unitarity, as we shall see.

A second major advance in the determination of V_{ud} comes from a new study of the quantum loop corrections coming from the previously problematic γW box diagram due to weak axial-vector contributions. Previously, those effects, along with other smaller axial-vector current contributions, were found to shift the RC by about

$$\frac{\alpha}{2\pi}\left[ln\frac{m_Z}{m_A} + A_g + 2C_{\text{Born}}\right] \tag{9}$$

where $A_g = 0.34$ is a one-loop QCD correction to the short-distance logarithmic loop contribution and $C_{Born} \simeq 0.8 g_A(\mu_P + \mu_\eta) \simeq 0.9$ represents long-distance loop effects. The problematic intermediate loop momentum region was roughly estimated by employing $m_A \simeq 1.2$GeV in the log, while the crudely obtained error of $\pm 8 \times 10^{-4}$ in that quantity (which leads to $\pm 4 \times 10^{-4}$ in V_{ud}) was found [10, 12] by allowing the m_A cut-off scale to vary up or down by a factor of 2.

A new analysis [13] of the γW box diagram now divides the loop momentum into 3 integration regions:

$$(1.5GeV)^2 \leq Q_I^2 < \infty$$

$$(0.82GeV)^2 \leq Q_{II}^2 < (1.5\text{GeV})^2$$

$$0 \leq Q_{III}^2 < (0.82GeV)^2$$

The evaluation of region I has been supplemented by 3-loop QCD corrections to the leading term in the short-distance operator product expansion, rendering it effectively error free and, more important, allowing a smooth extrapolation to lower Q^2. Region II has been evaluated using interpolating vector and axial-vector resonances, a procedure motivated by large N_c QCD and vector meson dominance. That prescription has been well tested in other calculations; nevertheless, a conservative $\pm 100\%$ uncertainty has been assigned to that part of the calculation. Finally, region III was evaluated using well-measured nucleon dipole form factors and assigned a $\pm 10\%$ uncertainty. Those improvements have reduced the theoretical quantum loop uncertainty in V_{ud} from a crude $\pm 4 \times 10^{-4}$, about a factor of 2 improvement. Further error reduction may be possible if future lattice calculations can confirm the interpolating resonance approach, since the uncertainty from intermediate momenta is still dominant.

The overall shift in V_{ud} due to the new evaluation of radiative corrections is relatively small, about ± 0.00007. However, the error reduction is more significant. Updating the most recent Hardy and Towner ft values [14] with the new RC results leads to the V_{ud} values given in Table 1. Combining all errors in quadrature now gives the weighted average

$$V_{ud} = 0.97390(27) \qquad \text{(2005 value)} \tag{10}$$

The central value has not shifted very much (see eq. (7)), but the error has been reduced by nearly a factor of 2.

Employing the values of V_{us} and V_{ud} in eqs. (3) and (10) leads to

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9998(5)(5)(11) \tag{11}$$

where the errors come from V_{ud}, V_{us} ($K_{\ell 3}$) and $f_+(0)$. The agreement with unitarity is outstanding. However, before a triumph for unitarity can be definitively declared, values of V_{us} determined in other ways must be reconciled with $K_{\ell 3}$ and $f_+(0)$ better clarified. Alternatively, if unitarity is to hold, the result in eq. (10) implies

$$|V_{us}| = 0.2269(12) \qquad \text{(implied by V}_{\text{ud}}\text{)} \tag{12}$$

in all processes.

The situation for V_{ud} looks very good; however, we caution that a recent remeasurement [15] of the Q value for ^{46}V increases its ft value and reduces the V_{ud} obtained from that nucleus $0.97363(50) \rightarrow 0.97280(43)$. Given that many of the errors are common, that movement makes ^{46}V differ from the others by about 3 sigma. Reconciliation would require significant change in the isospin corrections to ^{46}V. Alternatively, it could suggest a problem with the Z dependent radiative corrections or Q values of the other superallowed decays.

The latter possibility would effect all superallowed decays. We note that simply averaging in the new ^{46}V result leads to $V_{ud}^{ave} = 0.9738$ rather than 0.9739 not a significant shift.

The superallowed beta decays have now reached the very impressive $\pm 0.03\%$ level of precision in their determination of V_{ud}. Further studies of those reactions is clearly warranted, both to reduce the error and to clarify the new ^{46}V anomaly. In addition, future high statistics studies [10] of τ_π and g_A may be able to reach a level of precision for V_{ud} comparable to eq. (10), but without the nuclear physics uncertainties. Those measurements are difficult, but well worth the effort.

Table 1. Values of V_{ud} implied by various precisely measured superallowed nuclear beta decays. The ft values are taken from a recent update by Hardy and Towner [11]. Uncertainties in V_{ud} correspond to 1) nuclear structure and $z^2\alpha^3$ uncertainties added in quadrature with the ft error, 2) a common error assigned to nuclear conlomb distortion effects, and 3) a recently reduced (common) uncertainty in the radiative corrections from quantum loop

effects. Only the first error is used to obtain the weighted average.

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Nucleus	ft (sec)	V_{ud}
^{10}C	3039.5(47)	0.97381(77)(15)(19)
$^{14}0$	3043.3(19)	0.97368(39)(15)(19)
^{26}Al	3036.7(12)	0.97407(23)(15)(19)
^{34}Cl	3050.5(11)	0.97404(25)(15)(19)
^{38}K	3051.1(10)	0.97404(26)(15)(19)
^{42}Sc	3046.0(15)	0.97343(34)(15)(19)
^{46}V	3045.5(22)	0.97363(44)(15)(19)
^{50}Mm	3044.5(15)	0.97388(39)(15)(19)
^{54}Co	3047.4(15)	0.97389(42)(15)(19)
	Weighted Ave.	0.97390(11)(15)(19)