

*A multipurpose coherent instability simulation code*

**M. Blaskiewicz**

*Presented at the 22nd Particle Accelerator Conference (PAC)*  
Albuquerque, New Mexico  
June 25 – 29, 2007

**Collider-Accelerator Department**

**Brookhaven National Laboratory**

P.O. Box 5000  
Upton, NY 11973-5000  
[www.bnl.gov](http://www.bnl.gov)

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# A Multipurpose Coherent Instability Simulation Code\*

M. Blaskiewicz† BNL 911B, Upton, NY 11973, USA

## Abstract

A multipurpose coherent instability simulation code has been written, documented, and released for use. TRANFT (tran-eff-tee) uses fast Fourier transforms to model transverse wakefields, transverse detuning wakes and longitudinal wakefields in a computationally efficient way. Dual harmonic RF allows for the study of enhanced synchrotron frequency spread. When coupled with chromaticity, the theoretically challenging but highly practical post head-tail regime is open to study. Detuning wakes allow for transverse space charge forces in low energy hadron beams, and a switch allowing for radiation damping makes the code useful for electrons.

## INTRODUCTION AND THEORY

Coherent instabilities are of significant concern for a wide variety of planned and existing accelerators. The theory of these phenomena has been advancing steadily for decades [1, 2, 3, 7, 9, 8, 4, 5, 10, 11, 12, 13, 6, 14] and, quite recently, a crucial piece of the puzzle for transverse instabilities was found [15]. A theoretical treatment involving all the relevant pieces appears very difficult whereas simulation using particle tracking is conceptually straightforward [16].

The algorithm involves single particle evolution and multi-particle kicks. First consider the single particle motion. The single particle longitudinal update for one turn is given by

$$\bar{\epsilon} = \epsilon + \frac{q}{mc^2} [V(\tau) - V_s] + \delta\epsilon - T_0\epsilon/T_r \quad (1)$$

$$\bar{\tau} = \tau + \frac{T_0\eta}{\beta^2\gamma_0} \bar{\epsilon} \quad (2)$$

where  $\tau$  is the arrival time of the particle with respect to the synchronous phase,  $\epsilon = \gamma - \gamma_0$  is proportional to the energy deviation,  $\gamma_0$  is the reference Lorentz factor for a particle of mass  $m$  and charge  $q$ ,  $V(\tau)$  is the RF voltage,  $V_s$  is the synchronous voltage due to both acceleration and radiation,  $\beta = v/c$ ,  $T_0$  is the revolution period,  $\eta$  is the frequency slip factor,  $\delta\epsilon$  is a quantum excitation random kick,  $T_r$  is the longitudinal radiation damping time, and the updated variables are  $\bar{\tau}$  and  $\bar{\epsilon}$ .

Only one transverse variable is considered and it will be referred to as  $x$ . The single particle transverse update, without radiation, for one turn is

$$\bar{x} = x \cos \psi + p \sin \psi \quad (3)$$

$$\bar{p} = -x \sin \psi + p \cos \psi \quad (4)$$

$$\psi = \psi_0 + \frac{2\pi\xi}{\beta^2\gamma_0} \epsilon \quad (5)$$

where  $p$  is the transverse momentum variable,  $\psi_0$  is the on-momentum phase advance, and  $\xi$  is the chromaticity. Transverse radiation damping and quantum excitation are also included,

$$\bar{x} = \left(1 - \frac{T_0}{T_x}\right) x + \delta x \quad (6)$$

$$\bar{p} = \left(1 - \frac{T_0}{T_x}\right) p + \delta p, \quad (7)$$

where  $T_x$  is the transverse radiation damping time, and  $\delta x$  and  $\delta p$  are random variables. While equations (1) through (7) are written for one turn, TRANFT allows the user to choose the number of updates per turn.

The multiparticle forces are associated with three Green's functions that are referred to as wake potentials. The longitudinal voltage is

$$V_s(t) = - \int_{-\tau_b}^{\tau_b} W_s(\tau) I_b(t - \tau) d\tau, \quad (8)$$

where  $\tau_b$  is the bunch length,  $W_s(\tau)$  is the longitudinal wake potential, and  $I_b(t)$  is the instantaneous beam current. Note that  $I_b(t)$  is the linear superposition of the current impulses from each of the individual macro-particles. The transverse voltage is driven by two terms. The short range term is

$$V_x(x, t) = \int_{-\tau_b}^{\tau_b} [x W_d(\tau) I_b(t - \tau) + W_x(\tau) D_x(t - \tau)] d\tau, \quad (9)$$

where  $W_d(\tau)$  will be called the detuning wake [17, 18, 19],  $W_x(\tau)$  is the usual transverse wake potential, and  $D_x(t)$  is the instantaneous dipole density. Note that  $D_x(t)$  is the product of the instantaneous current and the instantaneous value of  $x$ .

A second term in the transverse force is included to account for multi-bunch effects. Each particle in the bunch receives the same transverse kick

$$V_x = Re \left\{ [X + iP] \sum_{k=1}^{\infty} W_x(kT_0/M) e^{i(\psi_0 - 2\pi s)k/M} \right\}, \quad (10)$$

Where  $X = \int D_x(t) dt$  and  $P = \int D_p dt$  are the total dipole moments in  $x$  and  $p$ . It is assumed that there are

\* Work performed under the auspices of the United States Department of Energy.

† blaskiewicz@bnl.gov

$M$  identical, equally spaced bunches interacting with coupled bunch mode number  $s$ . In actual accelerators there is usually a gap in the bunch train. For rigid modes the growth rate for a symmetric fill is never smaller than the growth rate for a partial fill[20]. For an uneven fill and arbitrary modes, one can prove that the largest magnitude tune shift for the symmetric fill is never smaller than the largest magnitude tune shift in an uneven fill[21]. With the typical error bars associated with accelerator impedances the error incurred by using (10) is probably benign.

## ALGORITHMS

All calculations, but wakefields, are done to machine precision using straightforward implementations of the equations already introduced. As an example of the wakefield calculations consider the longitudinal voltage, equation (8). Taking the instantaneous current to be a series of delta functions one obtains the first order approximation

$$V_{s,1}(t) = - \sum_{k=1}^N \hat{q} W_s(t - \tau_k), \quad (11)$$

where there are  $N$  macroparticles of charge  $\hat{q}$ . There are two problems with using (11) as it stands [22]. Firstly, since  $N$  is small compared to the actual number of particles within the bunch, there can be large statistical fluctuations in the applied voltage. This is especially worrisome since short range wake potentials tend to be very large. The net effect is that one can have a significant, unphysical, blow-up in the longitudinal emittance. The second problem, not fully unrelated to the first, is caused by the discrete time steps between updates. A typical particle makes a step  $2\pi Q_s \sigma_\tau$  each turn, where  $Q_s$  is the synchrotron tune and  $\sigma_\tau$  the rms bunch length. When length scales less than  $2\pi Q_s \sigma_\tau$  are important in the wake potential then it is possible for macro-particles to pass each other without interacting via the short range wake. Both of these problems can be alleviated by convolving (11) with a smoothing function of characteristic scale  $\Delta\tau \gtrsim 2\pi Q_s \sigma_\tau$  and, since convolution is commutative and associative, we may consider a smoothed wake potential  $\hat{W}_s(\tau)$ . This leads to a second approximation for the voltage that is physically reasonable

$$V_{s,2}(t) = - \sum_{k=1}^N \hat{q} \hat{W}_s(t - \tau_k). \quad (12)$$

To update the particles equation (12) needs to be evaluated for  $t = \tau_1, \dots, \tau_N$  and a naive algorithm requires  $O(N^2)$  operations. Instead of incurring this computational penalty, it was decided to use an approximate technique. First, a uniform grid of points spaced by  $\delta t \lesssim \Delta\tau/5$  is generated. Next, the macroparticles are placed on the grid via linear interpolation. A fast Fourier transform (FFT) is applied, multiplied by the FFT of  $\hat{W}_s$ , and an inverse FFT completes the calculation of  $V_{s,2}$ . The total number of grid points is a power of 2 and the total grid length is at least twice the total bunch length to eliminate ‘phantom’ or ‘ghost’ forces

[23]. There are two sources of error involved with this computation. The first is due to the application of linear interpolation in gridding the system and the second involves using numerical integration (via FFT) to evaluate the sums. The net effect is easily tested by cutting  $\delta t$  in half and rerunning the simulation until the answer converges.

## A SAMPLE CASE

Relevant parameters are given in Table 1. In addition to the table, the machine impedance was due to resistive wall and short range resonant impedances, both longitudinal and transverse. The total time required to do the simulations and make the plots was a couple of hours. Interested parties are encouraged to email the author for the code and user manual.

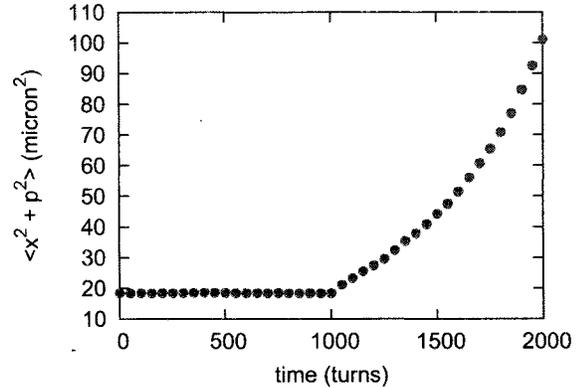


Figure 1: Evolution of the total transverse action for a marginally unstable beam.

parameter	value
h=1300 voltage	3.7 MV
h=3900 voltage	1.2 MV
rms bunch length	55 ps
particles/bunch	$14 \times 10^9$ and $15 \times 10^9$
betatron tunes	$Q_y = 16.3$
Lorentz factor	5871
circumference	780 m
transition gamma	52.1
transverse radiation damping time	13 ms
longitudinal radiation damping time	6.5 ms
chromaticity	5
rms transverse beam size	3 microns
rms spread in $\gamma$	5.4
coupled bunch mode number	17

Table 1: Parameters for the electron sample case

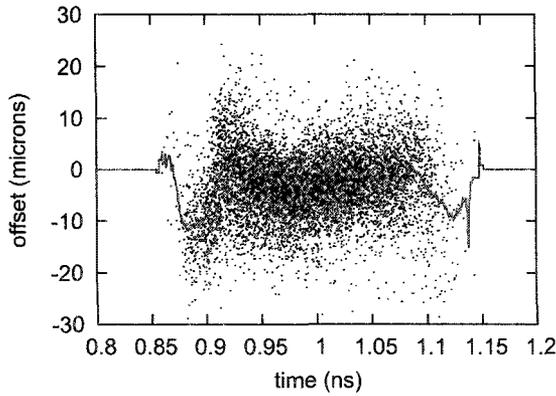


Figure 2: Transverse offset versus position along the bunch for a marginally unstable beam .

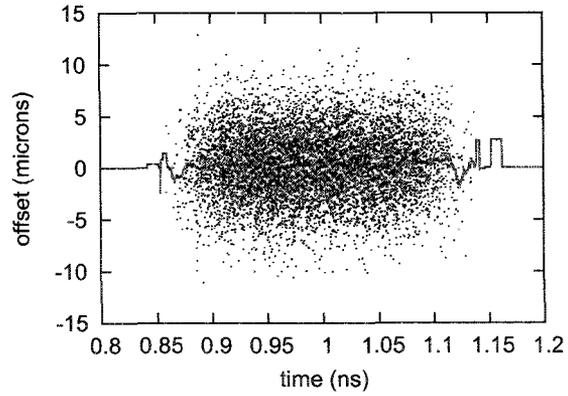


Figure 4: Transverse offset versus position along the bunch for a marginally stable beam .

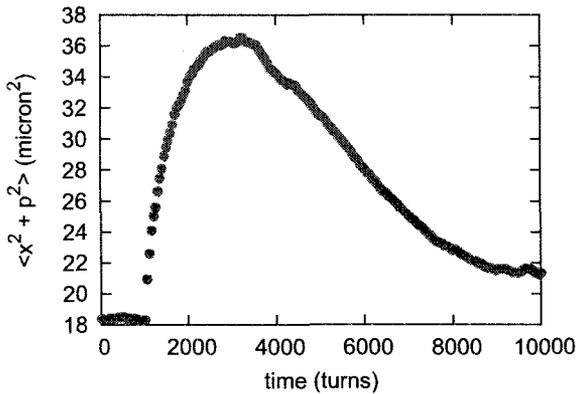


Figure 3: Evolution of the total transverse action for a marginally stable beam.

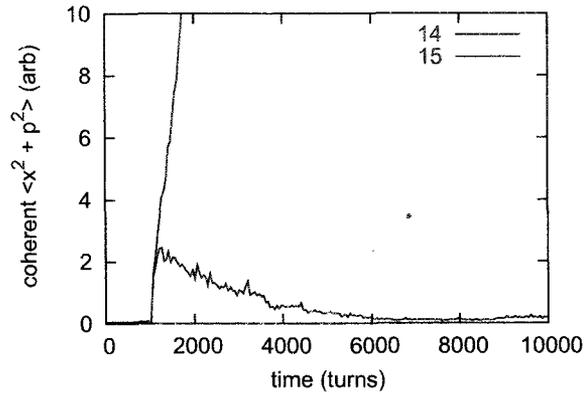


Figure 5: Comparison of coherent amplitude for stable and unstable beams.

## ACKNOWLEDGEMENTS

Thanks to Michael Borland, Yong-Chul Chae, Sam Krinsky, Ryutaro Nagaoka, Thomas Roser, Todd Satogata and Gang Wang for useful discussions and encouragement.

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