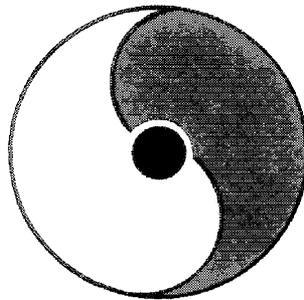


Hydrodynamics in Heavy Ion Collisions and QCD Equation of State

April 21-22, 2008



Organizers:

Frithjof Karsch, Dmitri Kharzeev, Dénes Molnár, Péter Petreczky and Derek Teaney

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Preface to the Series

The RIKEN BNL Research Center (RBRC) was established in April 1997 at Brookhaven National Laboratory. It is funded by the "Rikagaku Kenkyusho" (RIKEN, The Institute of Physical and Chemical Research) of Japan. The Center is dedicated to the study of strong interactions, including spin physics, lattice QCD, and RHIC physics through the nurturing of a new generation of young physicists.

The RBRC has both a theory and experimental component. The RBRC Theory Group and the RBRC Experimental Group consists of a total of 25-30 researchers. Positions include the following: full time RBRC Fellow, half-time RHIC Physics Fellow, and full-time, post-doctoral Research Associate. The RHIC Physics Fellows hold joint appointments with RBRC and other institutions and have tenure track positions at their respective universities or BNL. To date, RBRC has ~50 graduates of which 14 theorists and 6 experimenters have attained tenure positions at major institutions worldwide.

Beginning in 2001 a new RIKEN Spin Program (RSP) category was implemented at RBRC. These appointments are joint positions of RBRC and RIKEN and include the following positions in theory and experiment: RSP Researchers, RSP Research Associates, and Young Researchers, who are mentored by senior RBRC Scientists. A number of RIKEN Jr. Research Associates and Visiting Scientists also contribute to the physics program at the Center.

RBRC has an active workshop program on strong interaction physics with each workshop focused on a specific physics problem. In most cases all the talks are made available on the RBRC website. In addition, highlights to each speaker's presentation are collected to form proceedings which can therefore be made available within a short time after the workshop. To date there are eighty seven proceeding volumes available.

A 10 teraflops RBRC QCDOC computer funded by RIKEN, Japan, was unveiled at a dedication ceremony at BNL on May 26, 2005. This supercomputer was designed and built by individuals from Columbia University, IBM, BNL, RBRC, and the University of Edinburgh, with the U.S. D.O.E. Office of Science providing infrastructure support at BNL. Physics results were reported at the RBRC QCDOC Symposium following the dedication. QCDSF, a 0.6 teraflops parallel processor, dedicated to lattice QCD, was begun at the Center on February 19, 1998, was completed on August 28, 1998, and was decommissioned in 2006. It was awarded the Gordon Bell Prize for price performance in 1998.

N. P. Samios, Director
March 2007

CONTENTS

Preface to the Series	
Introduction	
<i>Frithjof Karsch, Dmitri Kharzeev, Dénes Molnár, Péter Petreczky and Derek Teaney</i>	i
Recent results from Hot QCD and benchmarks for hydrodynamic calculations	
<i>Ron Soltz</i>	1
Lattice calculation of EOS using asqtad action	
<i>Ludmila Levkova</i>	5
EoS with stout-link improved staggered fermions	
<i>Yasumichi Aoki</i>	11
QCD EoS from lattice calculations with p4-improved staggered fermions	
<i>Michael Cheng</i>	17
Sonic mach cones induced by fast partons in a perturbative quark-gluon plasma	
<i>Bryon Neufeld</i>	23
Comments on hydrodynamic calculations from an experimentalist	
<i>Huan Zhong Huang</i>	29
The effects of the order of phase transition, chemical equilibrium and freeze-out in ideal hydro	
<i>Pasi Huovinen</i>	35
Current status of a 3D hydro+cascade model	
<i>Tetsufumi Hirano</i>	41
The origin of thermal hadron production	
<i>Helmut Satz</i>	47
QCD critical point and its effects on physical observables – schematic consideration	
<i>Masayuki Asakawa</i>	53
Hydrodynamic expansion with QCD critical point	
<i>Chiho Nonaka</i>	61
New analytic results in hydrodynamics	
<i>Tamás Csörgő</i>	67
Viscous hydrodynamics and the QCD equation of state	
<i>Azwindini Muronga</i>	73

Non-Newtonian nature of causal hydrodynamics <i>Tomoi Koide</i>	79
Landau model and viscosity <i>Takeshi Kodama</i>	85
Viscosity and its effect on elliptic flow and thermal dileptons <i>Kevin Dusling</i>	91
Viscous hydrodynamics in different Israel-Stewart formalisms <i>Huichao Song</i>	97
Parton cascade with Yang-Mills fields <i>Adrian Dumitru</i>	103
Bulk viscosity in nuclear collisions (and other remarks) <i>Rainer Fries</i>	109
Comparing viscous hydrodynamics to a parton cascade <i>Pasi Huovinen</i>	115
A critical review of thermalization issue at RHIC -- results from STAR <i>Aihong Tang</i>	121
Hadronic transport coefficients from a microscopic transport model <i>Nasser Demir</i>	127
Calculating shear viscosity and relaxatime in a parton cascade <i>Andrej El</i>	133
Hydrodynamical behaviour in heavy ion collisions within parton cascade calculations <i>Zhe Xu</i>	139
List of Registered Participants	145
Agenda	147
Additional RIKEN BNL Research Center Proceeding Volumes	149
Contact Information	

INTRODUCTION

The interpretation of relativistic heavy-ion collisions at RHIC energies with thermal concepts is largely based on the relative success of ideal (nondissipative) hydrodynamics. This approach can describe basic observables at RHIC, such as particle spectra and momentum anisotropies, fairly well. On the other hand, recent theoretical efforts indicate that dissipation can play a significant role. Ideally viscous hydrodynamic simulations would extract, if not only the equation of state, but also transport coefficients from RHIC data. There has been a lot of progress with solving relativistic viscous hydrodynamics. There are already large uncertainties in ideal hydrodynamics calculations, e.g., uncertainties associated with initial conditions, freezeout, and the simplified equations of state typically utilized. One of the most sensitive observables to the equation of state is the baryon momentum anisotropy, which is also affected by freezeout assumptions. Up-to-date results from lattice quantum chromodynamics on the transition temperature and equation of state with realistic quark masses are currently available. However, these have not yet been incorporated into the hydrodynamic calculations.

Therefore, the RBRC workshop “Hydrodynamics in Heavy Ion Collisions and QCD Equation of State” aimed at getting a better understanding of the theoretical frameworks for dissipation and near-equilibrium dynamics in heavy-ion collisions. The topics discussed during the workshop included techniques to solve the dynamical equations and examine the role of initial conditions and decoupling, as well as the role of the equation of state and transport coefficients in current simulations.

We would like to thank Dr. Nicholas Samios and RIKEN BNL Research Center for providing the opportunity and support to organize this workshop. Our sincere thanks go to Pamela Esposito for her invaluable help in organizing and running the workshop.

The Organizers

June 20, 2008

Recent Results from HotQCD and Benchmarks for Hydrodynamic Calculations

R. Soltz and R. Gupta for the HotQCD Collaboration

June 20, 2008

Abstract

We present calculations for the finite temperature QCD equation of state and the chiral and deconfinement transition temperatures for $32^3 \times 8$ lattices using p4 and asqtad improved staggered actions. These calculations were performed by the HotQCD collaboration on BG/L supercomputing systems at LLNL, BNL, and SDSC. An initial motivation for using the lattice calculated equation of state is given, and comparisons are made to a typical resonance gas/bag model equation of state input to the Azhydro hydrodynamic code.

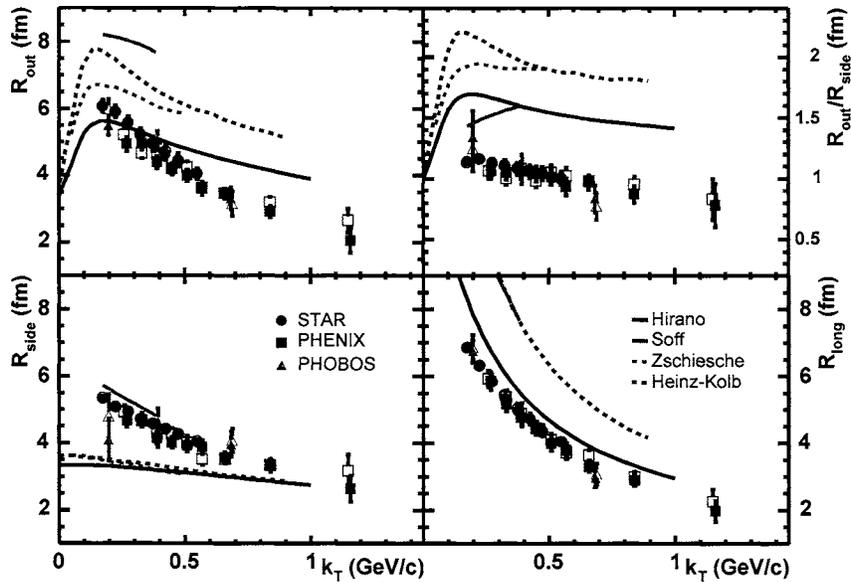


Figure 1: Gaussian radii and ratios from PHENIX, STAR, and PHOBOS compared to full hydrodynamic calculations [1].

The initial motivation for using the LLNL BG/L for equation of state calculations comes in part from the HBT Puzzle: the unexpected short emission duration as measured by the outwards Gaussian radius, R_{out} relative to the analogous sideways geometric component, R_{side} . The combined results from RHIC are shown in Fig. 1, taken from the 2005 review article by Lisa, Pratt, Soltz, and Wiedemann [1]. All of the models depicted in the figure lack at least one of two key components that are assumed necessary for bridging the gaps between experiment and theory: a hadronic cascade afterburner that can incorporate space-momentum correlations and a lattice generated equation of state. By reducing uncertainties in the equation of state with two separate LQCD codes using larger lattices, we seek to make the lattice generated equation of state more

attractive for inserting into hydrodynamics calculations, additional details of the hydrodynamics calculations and radii extraction are provided in [2, 3, 4, 5].

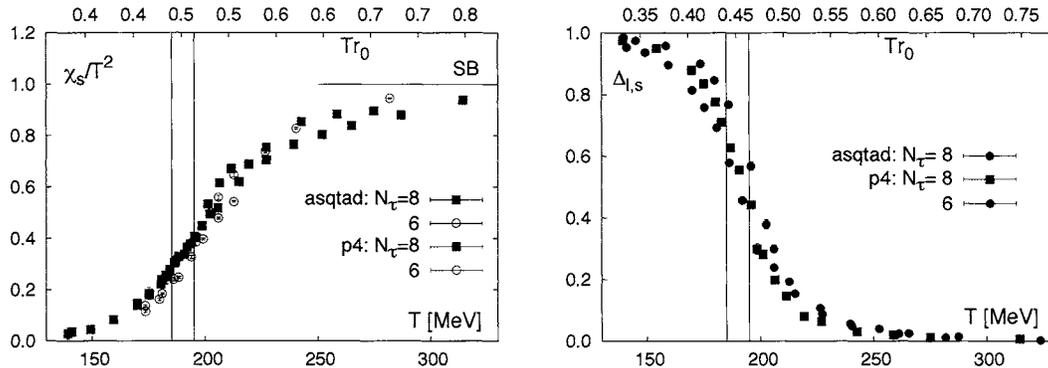


Figure 2: Preliminary HotQCD results for the transition temperature (left) and equation of state (right) using two lattice codes. Results from running the Columbia Physics System (CPS) code using a p4 improved staggered action are in blue and results from the MILC collaboration asqtad improved staggered fermion action are in red. Both codes ran on $N_\tau = 8$ finite temperature lattices along lines of constant physics with a pion mass of ~ 250 MeV ($m_q = 0.1m_s$).

In the fall of 2005 LLNL took delivery of a 64-rack BG/L computer, an architectural descendent of the QCDOC computer developed by a partnership between Columbia, BNL, Edinburgh and IBM. With a peak capacity of 360 TF, the LLNL supercomputer provided the opportunity to significantly extend calculations to lattice sizes of $N_s^3 \times N_\tau = 32^3 \times 8$, thus performing continuum corrections which restrict uncertainties in the transition temperature and equation of state to less than 10%. A workshop was held at LLNL, jointly hosted with LANL, in the spring of 2006 [6] and this led to a white paper which proposed a one-year research program to calculate the equation of state and transition temperature using a 10% allocation of the LLNL machine compared to 3–4 years using the existing resources available to the community. The HotQCD collaboration was formed in the summer of 2006, combining the RBC-Bielefeld and MILC collaborations together with Soltz, Vranas and Luu from LLNL and R. Gupta and T. Bhattacharya from LANL. First accounts on the machine were given in fall of 2006, followed by two short runs in the winter and summer of 2007 [7]. After 40 more racks were added the machine, the bulk of the calculation was performed on a 3-week allocation of 72 racks, with subsequent running on 8-racks for a period of several months. Equilibration runs for the asqtad action were performed the BG/L system at the San Diego Supercomputing Center (SDSC), and additional statistics for the p4 action were generated on the Stony Brook NYBlue BG/L system at BNL.

Both codes use improved staggered fermion actions, but differ in their treatment of discretization errors. In the p4 action employs a modified "fat" links, which lead to a four order correction in the momentum of the free-quark propagator [8], whereas the asqtad action uses tadpole improved links. Both actions have cut-off effects that are quadratic in a , the lattice spacing, which for finite temperature is equivalent to correctoins of order $\mathcal{O}(1/N_\tau^2)$. Although the magnitude cut-off effects can be estimated from ideal gas pressure calculations [10], calculating the equation of state on larger lattices is required to perform reliable extrapolations to the continuum limit. All calculations were performed using the RHMC algorithm and long the lines of constant physics for a pion mass of ~ 250 MeV ($m_q = 0.1m_s$). The overall scale is set by determining the shape of the static quark

potential and extracting the distance scales $r_0 = 0.469(7) \text{ fm}$ and $r_1 = 0.318(7) \text{ fm}$ at which the slope of the potential takes on the values $1.65/r_0^2$ and $1.0/r_1^2$, respectively. The former has been used in calculations with the p4 action, while the latter is used for the asqtad action.

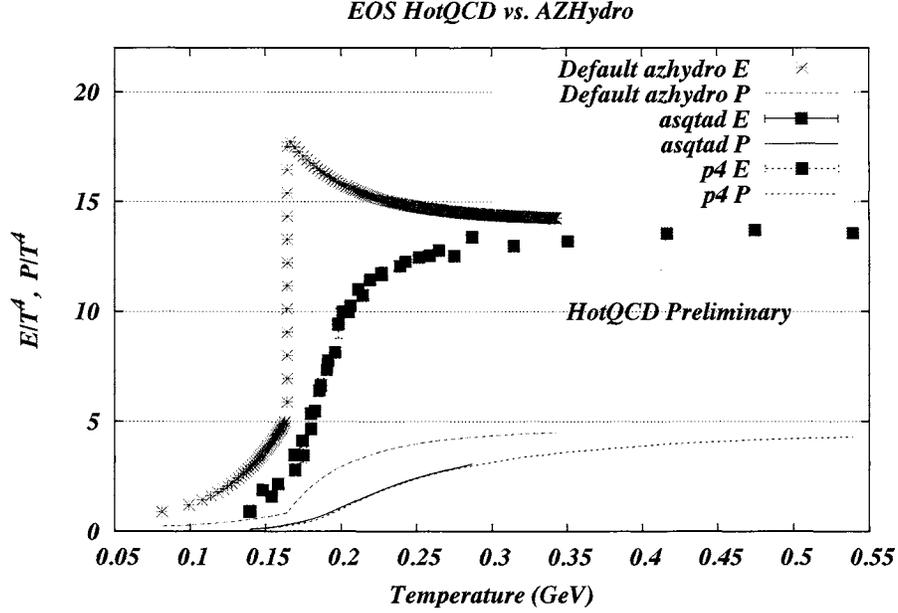


Figure 3: Preliminary HotQCD results for the pressure and energy density for both p4 and asqtad actions with $32^3 \times 8$ lattices compared to the input hadron resonance/bag model equation of state input to Azhydro.

The left panel of Fig. 2 shows the HotQCD preliminary results for the strange quark number susceptibility, given by Eq. 1,

$$\frac{\chi_s}{T^2} = \frac{1}{VT^3} \frac{\partial^2 \ln Z}{\partial (\mu_s/T)^2}, \quad (1)$$

for which the location of the inflection point is indicative of the deconfinement transition temperature. Vertical lines are drawn at 185 and 195 MeV to set limits for this transition for the $N_\tau = 8$ calculation. The right panel of Fig. 2 shows the subtracted chiral condensate defined by Eq. 2,

$$\Delta_{l,s}(T) = \frac{\langle \bar{\psi}\psi \rangle_{l,T} - \frac{m_l}{m_s} \langle \bar{\psi}\psi \rangle_{s,T}}{\langle \bar{\psi}\psi \rangle_{l,0} - \frac{m_l}{m_s} \langle \bar{\psi}\psi \rangle_{s,0}}. \quad (2)$$

The subtraction of normalized strange quark component and division by zero temperature values are used to remove additive and multiplicative divergences. As with the left panel, vertical lines are drawn to indicate the range of the chiral transition for this calculation. For both deconfinement and chiral quantities, the two actions give consistent results.

Fig. 3 shows the HotQCD preliminary equation of state results for the normalized energy density and pressure for two staggered fermion codes compared to the default input for the Azhydro 2+1 D hydrodynamic code [11]. As with the transition temperature measurements, the two codes calculations agree remarkably well for both the pressure and energy density. These results are

relevant because earlier hydro results by Bass *et al.* on the transition temperature and Huovinen on the equation of state have shown these differences can lead to 20% effects in the extracted radii, not large enough to close the gap with the models, but non-negligible and worth further investigation. In the future, and with the help of other participants at this workshop, we expect to investigate the impact of calculating particles spectra, flow, and space-time distributions in hydrodynamics using the HotQCD equation of state.

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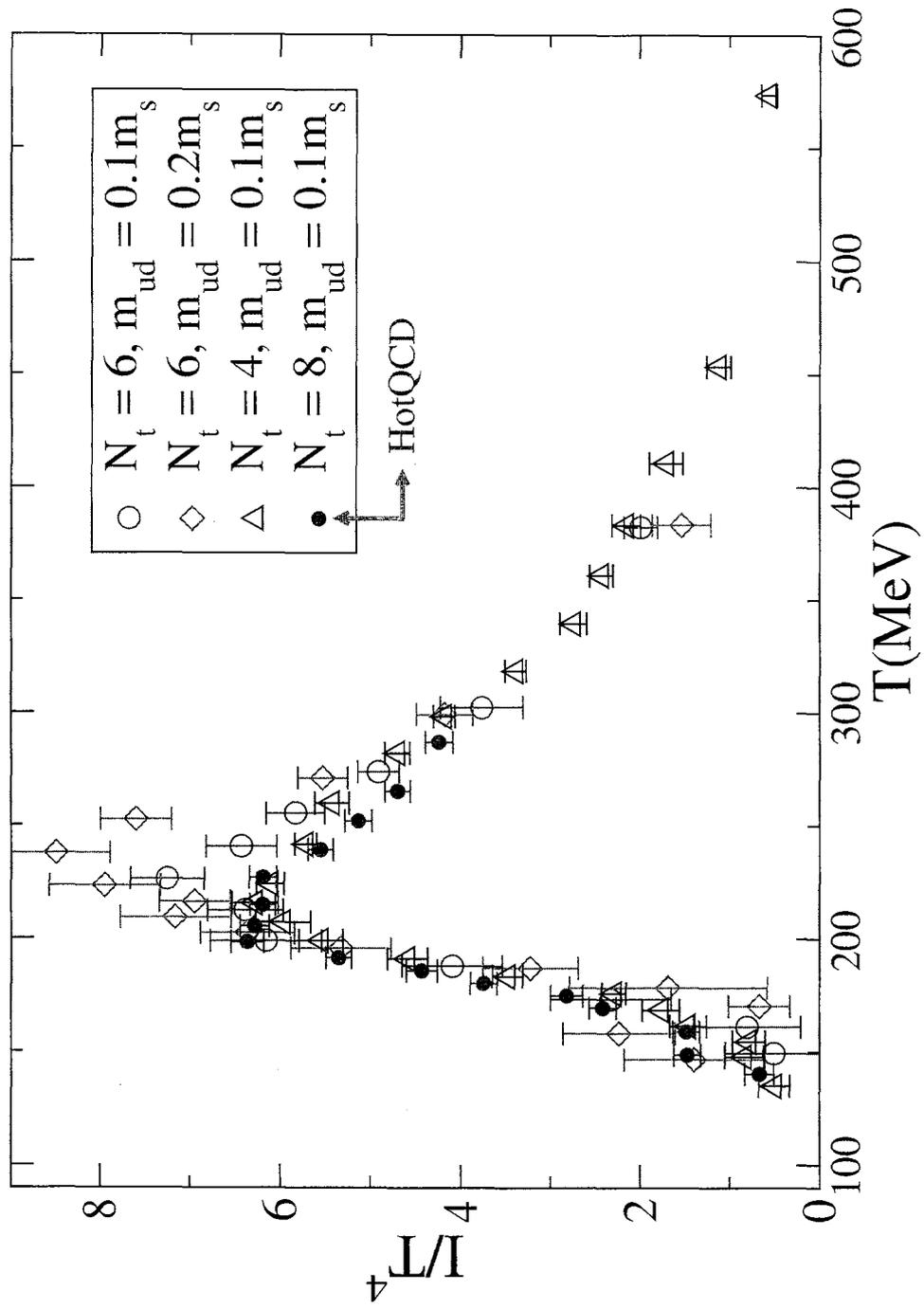
Lattice calculation of EOS using asqtad action

Ludmila Levkova
MILC Collaboration

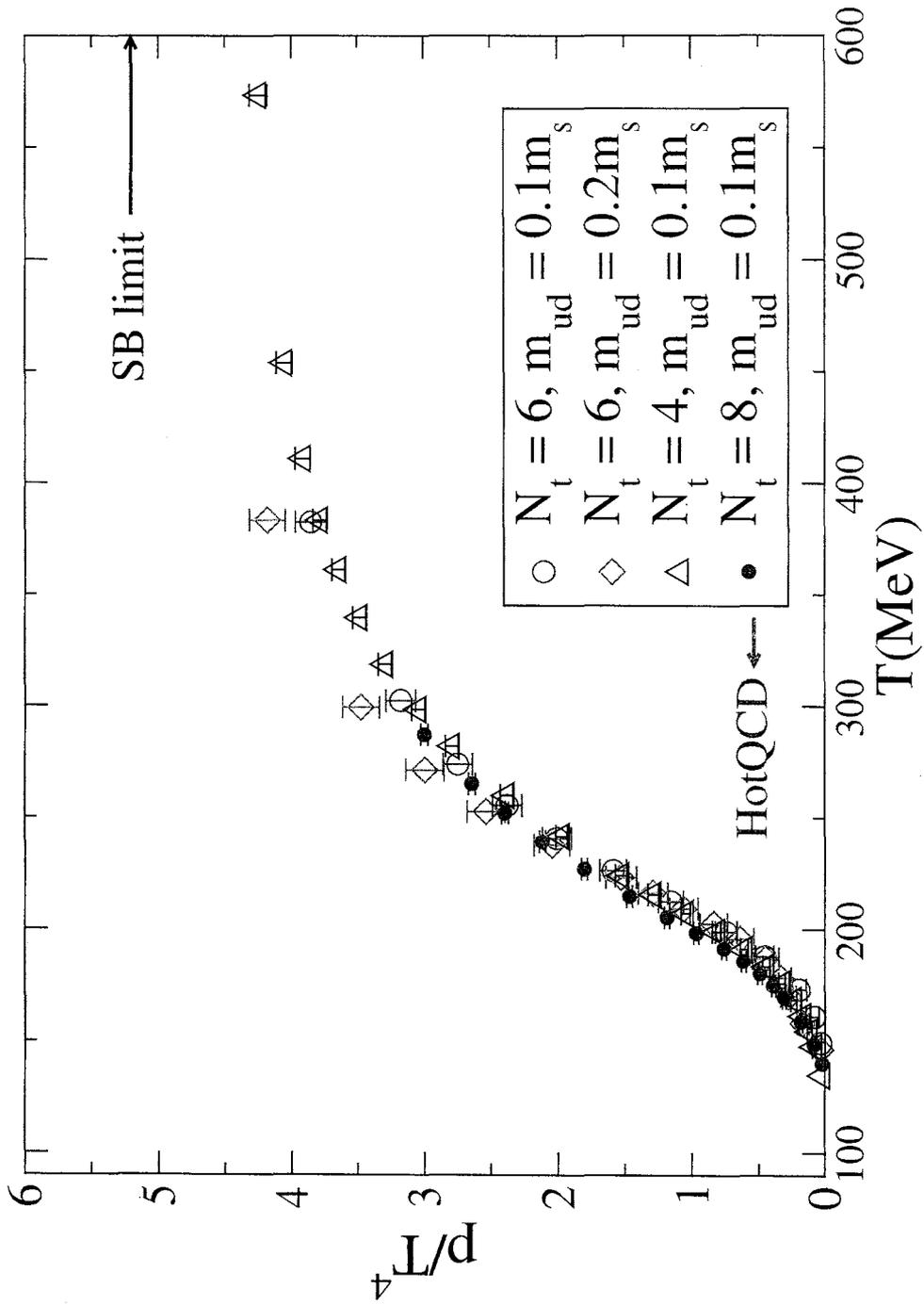
5

We report results for the interaction measure, pressure and energy density for nonzero temperature QCD with 2+1 flavors of improved staggered quarks. In our simulations we use a Symanzik improved gauge action and the Asqtad $O(a^2)$ improved staggered quark action for lattices with temporal extent $N_t = 4, 6$ and 8 . The heavy quark mass m_s is fixed at approximately the physical strange quark mass and the two degenerate light quarks have masses $m_{ud} \approx 0.1 m_s$ or $0.2 m_s$. We also present results for the QCD equation of state at non-zero chemical potential. The calculation is performed using the Taylor expansion method with terms up to sixth order.

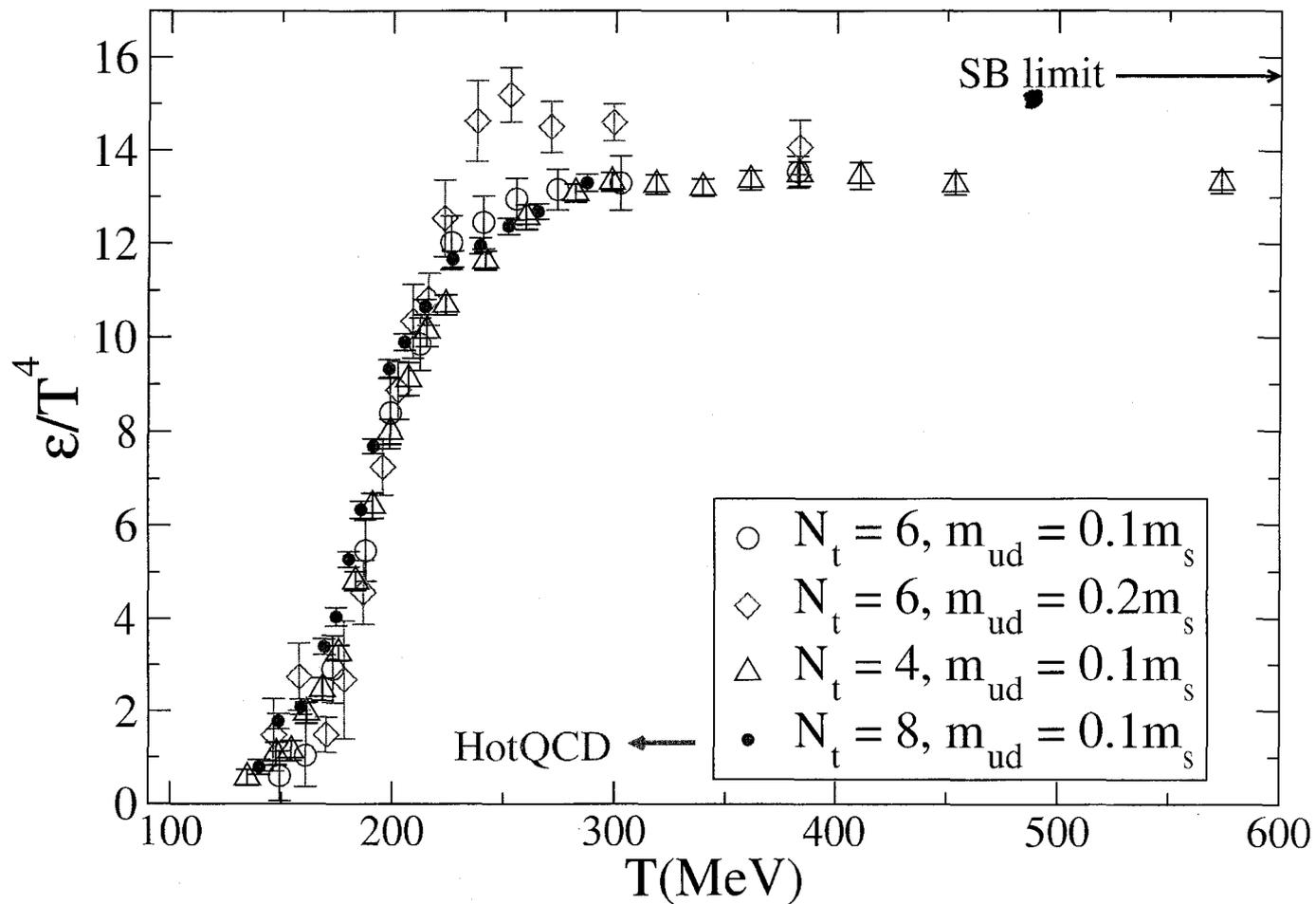
EOS at $\mu = 0$ – Interaction measure

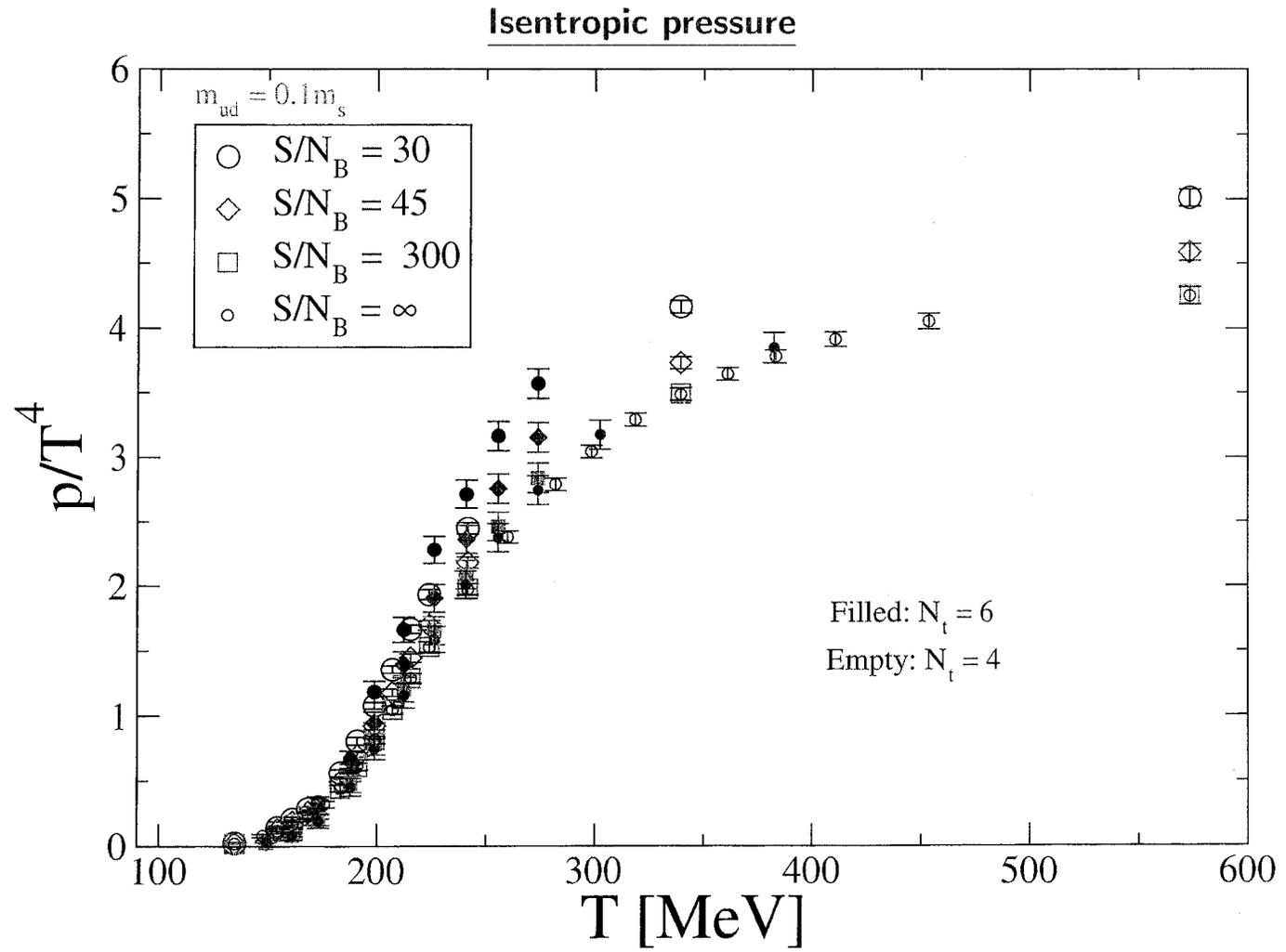


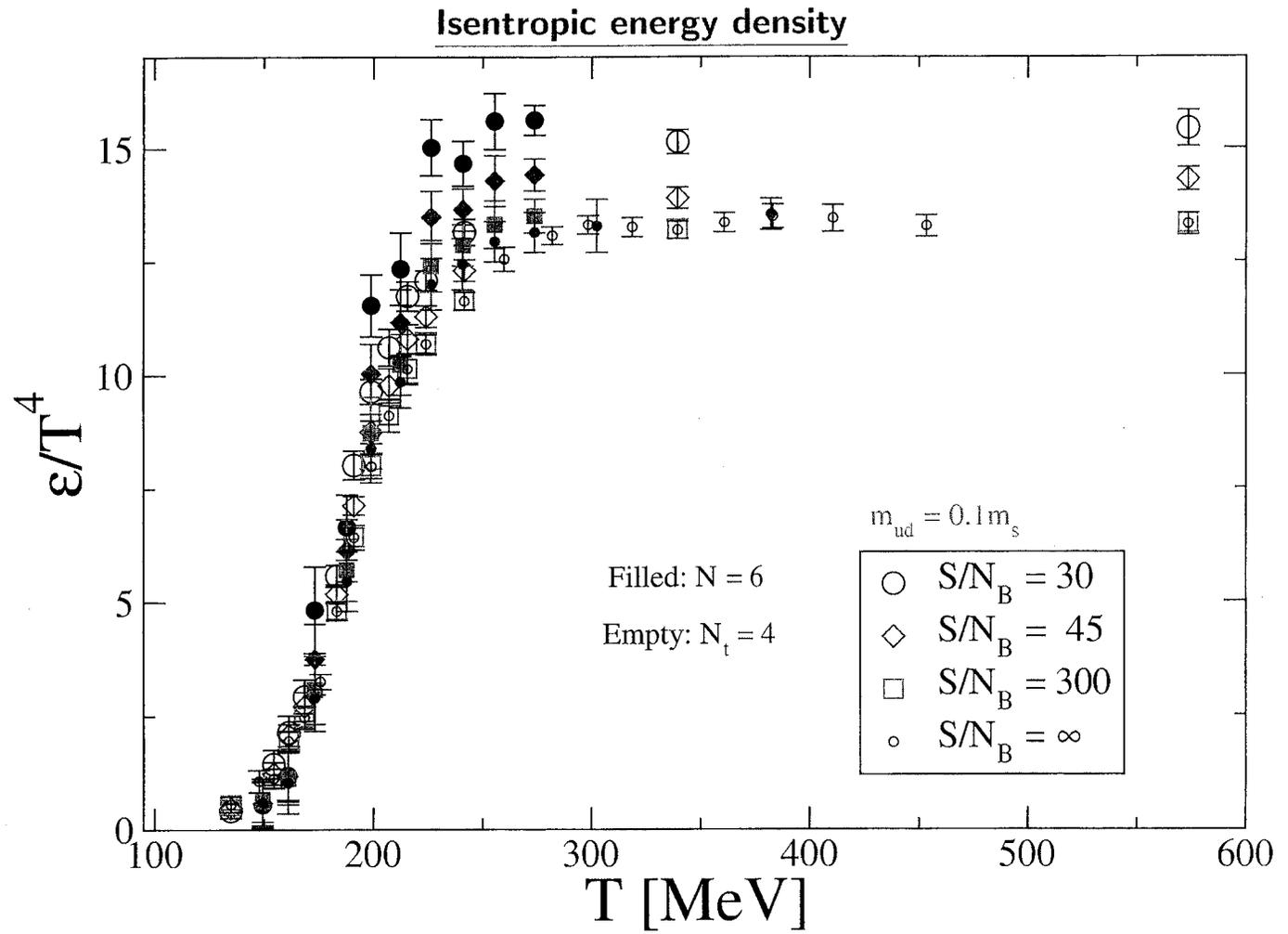
EOS at $\mu = 0$ - Pressure



EOS at $\mu = 0$ – Energy density







EoS with stout-link improved staggered fermions

Y. Aoki

RIKEN BNL Research Center

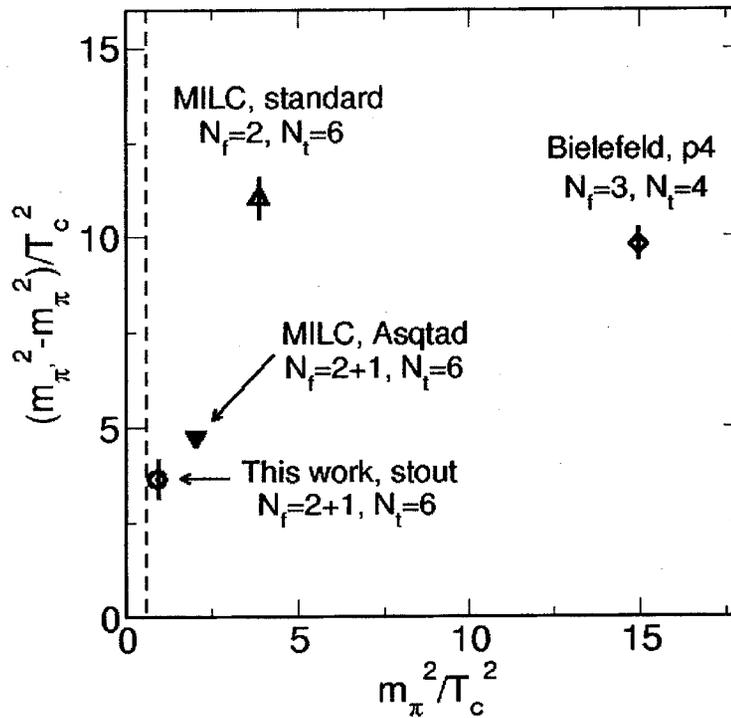
11

QCD equation of state is calculated with 2+1 flavor stout-link improved staggered fermions on $N_t=4$ and 6 lattices for the “physical quark masses”. We discuss systematic errors from the finite lattice spacing and the way the physical quark mass is determined. The method is promising though, further calculation with $N_t \geq 8$ are necessary for the controlled continuum extrapolation.

Thermodynamics with 2+1 flavor staggered fermions with stout link

- EoS (Equation of State)
 - Y.A., Z. Fodor, S. Katz, K. Szabo, JHEP 01(2006)089 [hep-lat/0510084].
 - › $N_t=4, 6$. LCP1.
- Order of the transition
 - Y.A., G. Endrodi, Z. Fodor, S. Katz, K. Szabo, Nature 443 (2006) 675 [hep-lat/0611014].
 - › $N_t=4, 6, 8, 10$. LCP2.
- T_c
 - Y.A., Z. Fodor, S. Katz, K. Szabo, PLB643 (2006) 46 [hep-lat/0609068].
 - › $N_t=4, 6, 8, 10$. LCP2.

a little outdated propaganda figure...



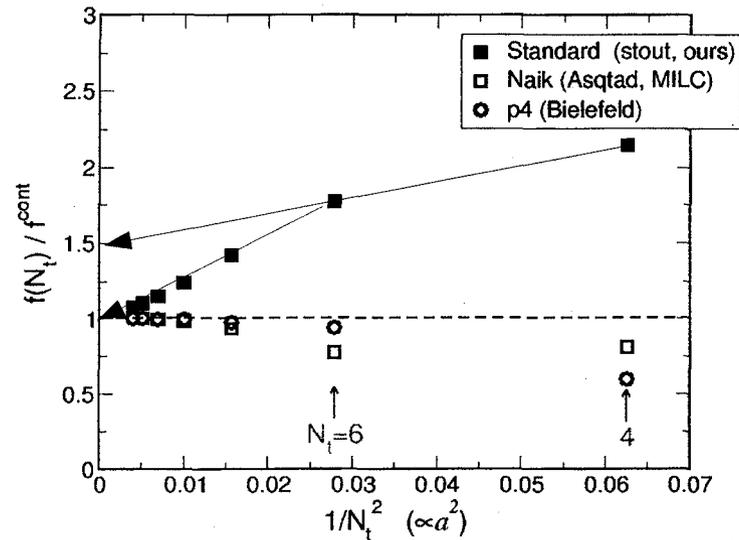
$$\Delta'_\pi = \frac{m_{\pi'}^2 - m_\pi^2}{T_c^2} = (m_{\pi'}^2 - m_\pi^2) N_t^2$$

($T = 0$ masses, measured at $\beta_c(N_t)$)

- Now other groups have $N_t=8$.

rotational symmetry

- our Symanzik improved gauge action \rightarrow improves gauge sector
- stout link fermion \rightarrow does not improve free energy at $T=\infty$, but

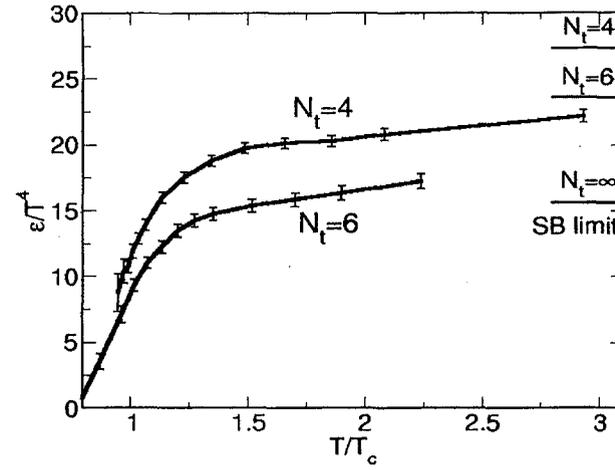
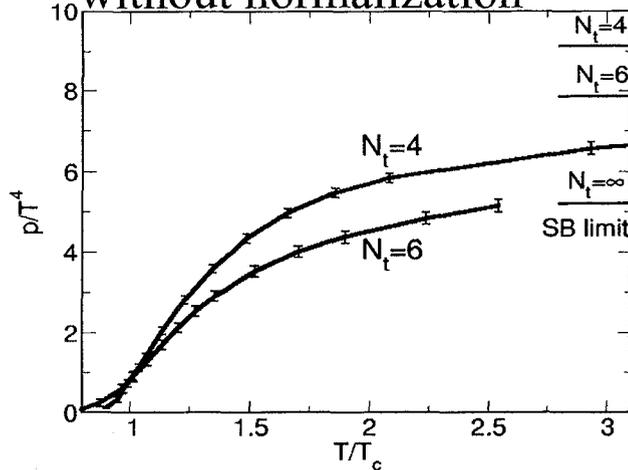


Heller, Karsch, Sturm

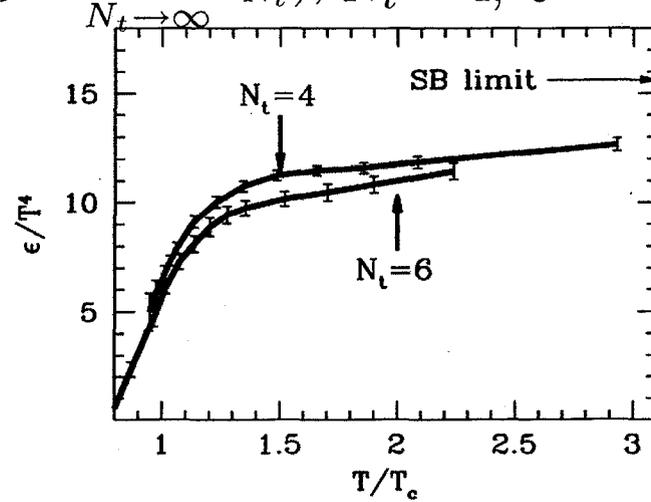
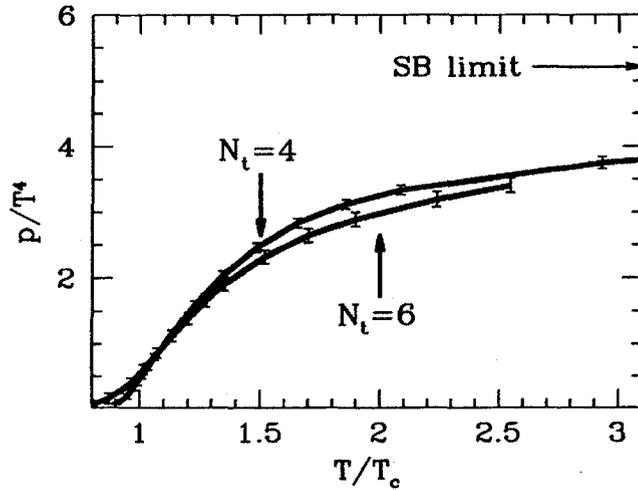
- $N_t=4 \rightarrow 6$, some higher order effect, but $N_t=6 \rightarrow 8$, $1/N_t^2$ very good
- a^2 extrapolation good for $N_t=6 \rightarrow 8$.

Equation of State

- without normalization

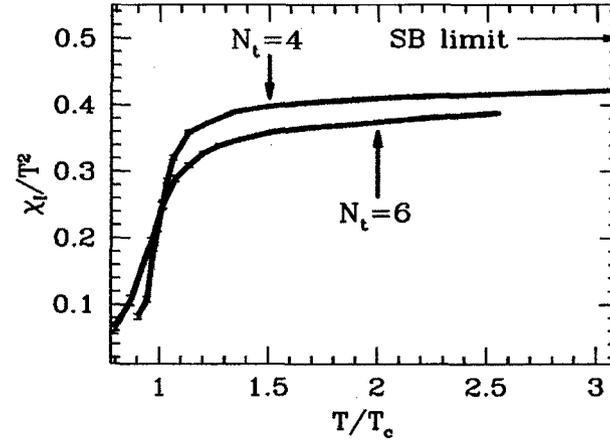
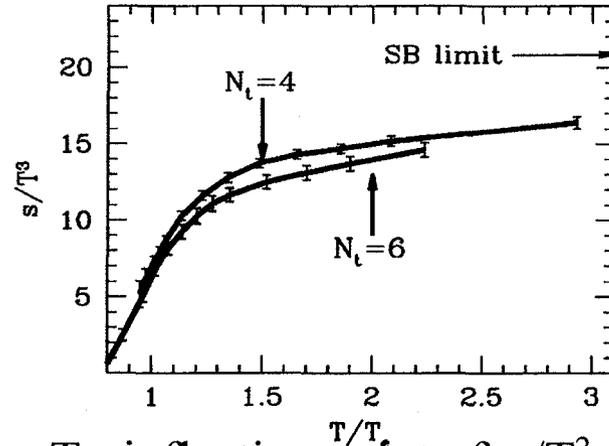


- multiplied with c_{SB}/c_{N_t} ($c_{SB} = \lim_{N_t \rightarrow \infty} c_{N_t}$), $N_t = 4, 6$

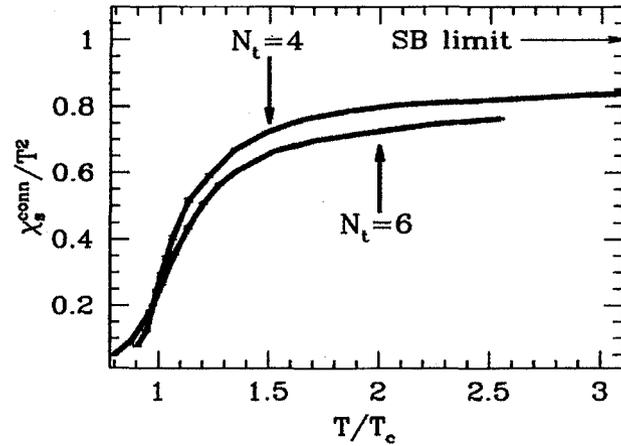
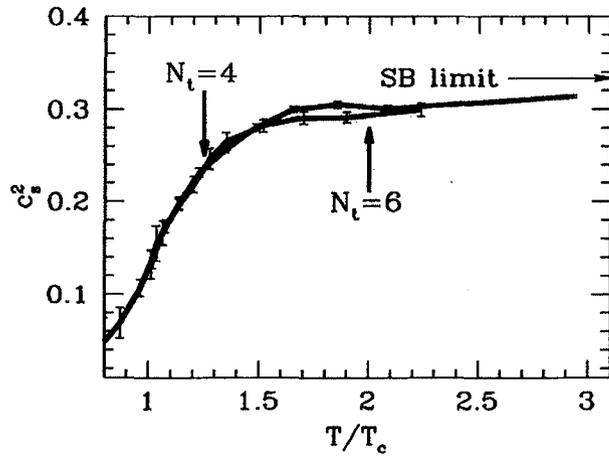


Equation of State

- multiplied with c_{SB}/c_{N_t} ($c_{SB} = \lim_{N_t \rightarrow \infty} c_{N_t}$), $N_t = 4, 6$



- T_c : inflection point of χ_l/T^2



QCD EoS from lattice calculations with p4-improved staggered fermions

Michael Cheng

Columbia University

Hydrodynamics in Heavy Ion Collisions and
QCD Equation of State
BNL: April 21-22, 2008

EoS calculations with p4 fermions

- Calculations at $N_t = 4, 6$ with RBC-Bielefeld Collaboration
- Use Rational Hybrid Monte Carlo (**RHMC**) to simulate two light dynamical flavors and 1 strange flavor (**2+1 flavor**)
- Light quark mass 2x heavier than physical mass ($m_l = 0.1 m_s$)
- $m_\pi \approx 220\text{MeV}$ Kaon mass tuned to physical value $m_K \approx 490\text{MeV}$

- Large temperature range:
 $140\text{ MeV} < T < 800\text{ MeV} \rightarrow 0.7 T_c < T < 4 T_c$

- Calculations at $N_t = 8$ with **HotQCD**
- Compare with **Asqtad** (talk by **L. Levkova**)

18

Calculating EoS on the lattice

The pressure:

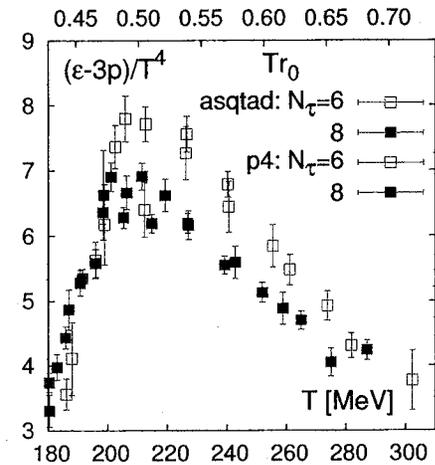
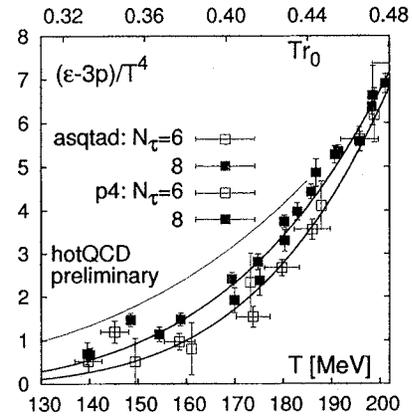
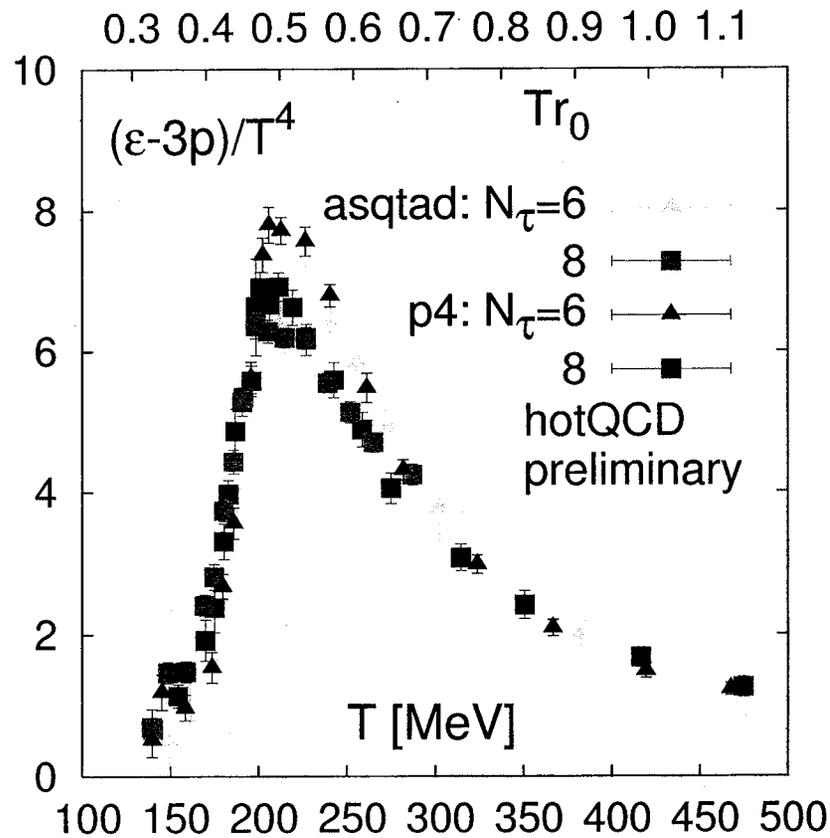
$$\frac{p}{T^4}(\beta) = N_t^4 \int_{\beta_0}^{\beta} d\beta \left[\frac{1}{N_s^3 N_t} (\langle S_g \rangle_0 - \langle S_g \rangle_T) - \sum_f \left(\frac{\partial m_f}{\partial \beta} \right) (\langle \bar{\psi} \psi_q \rangle_0 - \langle \bar{\psi} \psi_q \rangle_T) \right]$$

The interaction measure:

$$\frac{\epsilon - 3p}{T^4} = T \frac{d}{dT} \left(\frac{p}{T^4} \right) = a \frac{d\beta}{da} \frac{\partial p/T^4}{\partial \beta} = \left(\frac{\epsilon - 3p}{T^4} \right)_{gauge} + \left(\frac{\epsilon - 3p}{T^4} \right)_{fermion}$$

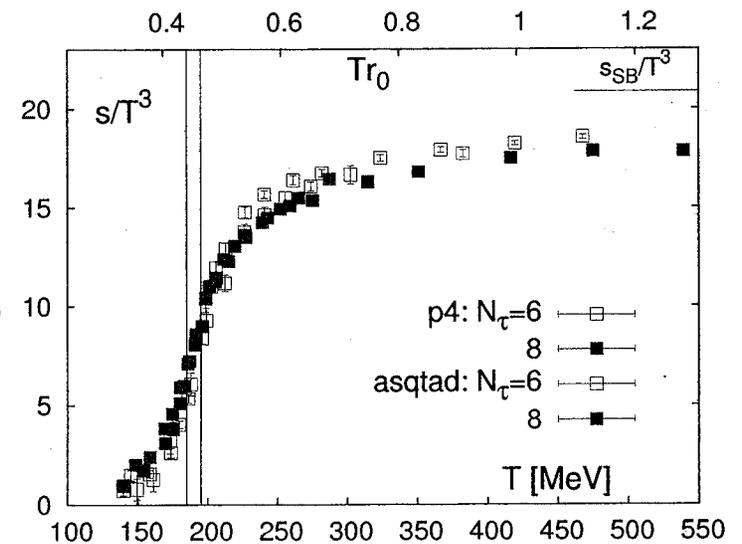
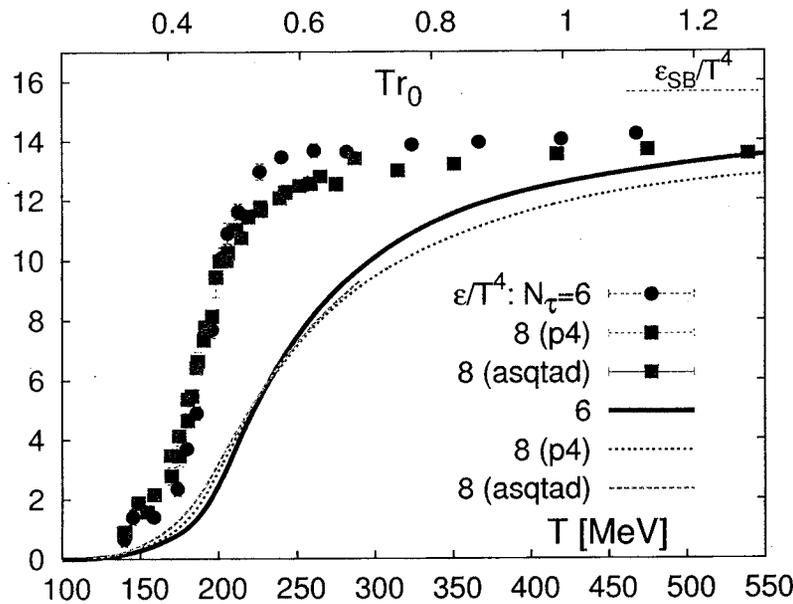
Interaction measure is the basic quantity measured on lattice.
 Pressure, energy density, entropy can be reconstructed via
 thermodynamic relations from $\epsilon - 3p$.

Interaction Measure



Pressure, Energy, Entropy

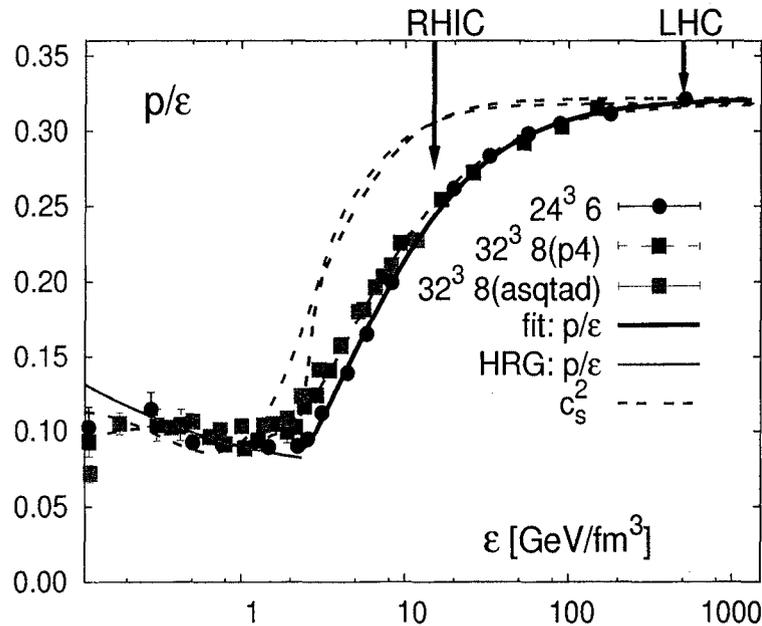
21



April 21-22, 2008

Hydrodynamics in Heavy Ion
Collisions and the QCD EoS

p/ϵ and Speed of Sound



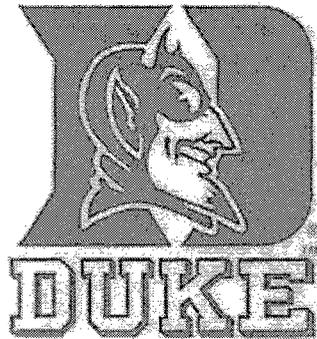
- Convenient to eliminate T in favor of ϵ for p/ϵ .
- Good agreement between **Nt = 6, 8 p4** and **Asqtad** calculations.
- Deviations at lowest temperatures when compared with HRG.
- SB limit:

$$c_s^2 = \frac{dp}{d\epsilon} = \epsilon \frac{dp/\epsilon}{d\epsilon} + \frac{p}{\epsilon}$$

$$c_s^2 \approx \frac{p}{\epsilon} \approx \frac{1}{3}$$

**Sonic Mach Cones Induced by Fast Partons in a
Perturbative Quark-Gluon Plasma [1]:**

We derive the hydrodynamic source of energy/momentum deposited by a fast parton in a perturbative QGP. We couple our result to a linearized hydrodynamical evolution and find a propagating Mach cone.



Presented by Bryon Neufeld (of Duke University) on March 20th 2008 in collaboration with:

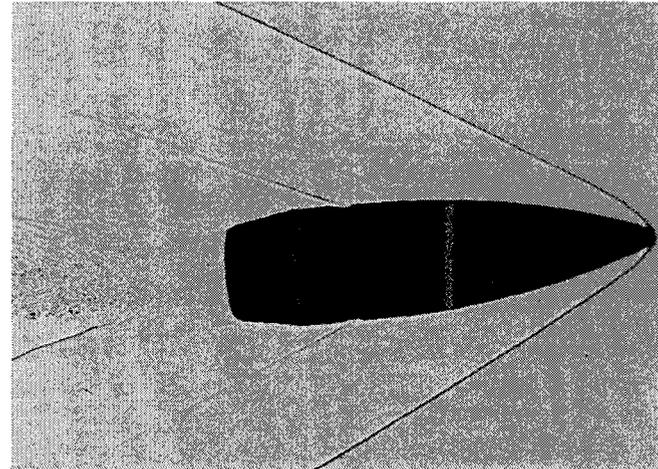
Berndt Mueller, J. Ruppert, M. Asakawa, C. Nonaka

[1] arXiv:0802.2254

Interesting Questions:

- **What is the energy and momentum perturbation of a QGP due to a fast parton?**
- **Similarly, Is a Mach cone created by a supersonic parton propagating through the quark gluon plasma?**

- A Mach cone is formed when an object moves faster than the speed of sound relative to its medium.



Back to the Question: **What is the energy and momentum perturbation of a QGP due to a fast parton?**

The answer:

$$\int \frac{d^3p}{(2\pi)^3} p^\nu (\nabla_{p_i} D_{ij} \nabla_{p_j} f) \equiv J^\nu$$

$$D_{ij} = \frac{g^2 C_2}{N_c^2 - 1} \int_{-\infty}^t dt' F_i^a(x) U_{ab}(x, x') F_j^b(x')$$

- J gives the energy/momentum deposited per unit time, it is a source term
- Assumptions: the medium is perturbative in coupling g, hydrodynamics

Linearized hydro

$$\begin{aligned}\delta e(\mathbf{k}, \omega) &= \frac{ikJ_L(\mathbf{k}, \omega) + J^0(\mathbf{k}, \omega)(i\omega - \Gamma_s k^2)}{\omega^2 - c_s^2 k^2 + i\Gamma_s \omega k^2} \\ \vec{g}_D(\mathbf{k}, \omega) &= \frac{i\omega \hbar J_T(\mathbf{k}, \omega) + ic_s^2 k J^0(\mathbf{k}, \omega)}{\omega^2 - c_s^2 k^2 + i\Gamma_s \omega k^2} \\ \vec{g}_T(\mathbf{k}, \omega) &= \frac{iJ_T(\mathbf{k}, \omega)}{\omega + \frac{3}{4}i\Gamma_s k^2}\end{aligned}$$

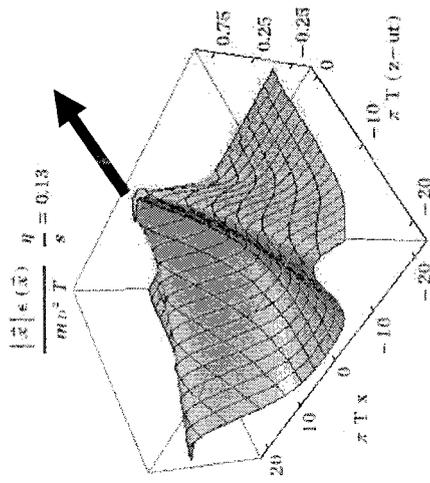
- These equations are valid in the limit of a weak source
- Solve for deposited energy density, sound momentum, and diffusion momentum
- We use: $u = 0.99955$ (gamma about 33), $c_s = \text{Sqrt}[1/3]$, $\Gamma_s = 4/(3 T) * (\eta/s)$ and $T = 350 \text{ MeV}$

See: Casalderrey-Solana et al. **Nucl.Phys.A774:577-580,2006.**

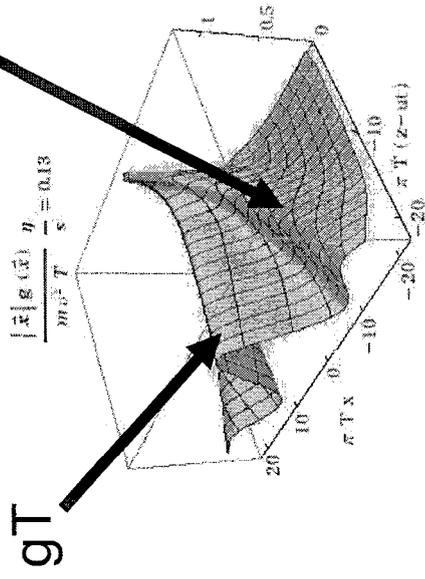
The Mach cone!

$$\frac{v}{s} = 0.13$$

Unscreened source with $\rho_{\text{min/max}}$ cutoff



Energy density

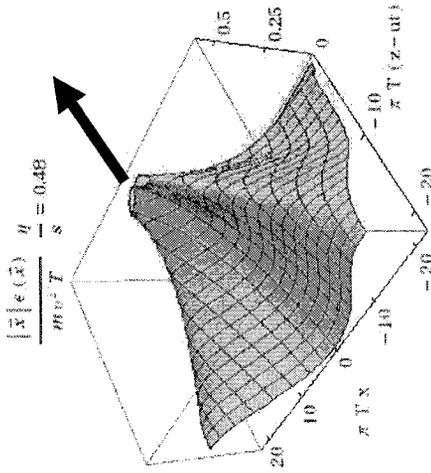


Momentum density

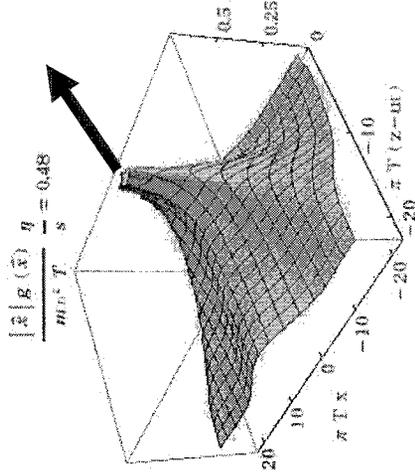
Still a Mach cone!

$$\frac{\eta}{s} = 0.18$$

Unscreened source with $\rho_{\min/\max}$ cutoff



Energy density



Momentum density

Comments on Hydrodynamic Calculations from an Experimentalist

Huan Zhong Huang
Department of Physics and Astronomy
University of California, Los Angeles

April 21-23, 2008 @RBRC, BNL

STAR White-Paper NPA 757, 102 (2005)

The indirect evidence of a phase transition of some sort in the elliptic flow results comes primarily from the sensitivity of hydrodynamics calculations of the magnitude and hadron mass-dependence of v_2 to the EOS. How does the level of this EOS sensitivity compare quantitatively to that of uncertainties in the calculations, gleaned from the range of parameter adjustments, from the observed deviations from the combination of elliptic flow, spectra and HBT correlations, and from the sensitivity to the freezeout treatment and to such normally neglected effects as viscosity and boost non-invariance?

Lessons from p+A Collisions

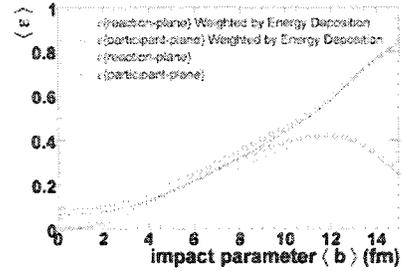
The energy deposition at mid-rapidity from incident proton in p+A collisions depends on the number of nucleon-nucleon collisions

---- not all participant nucleons are equal

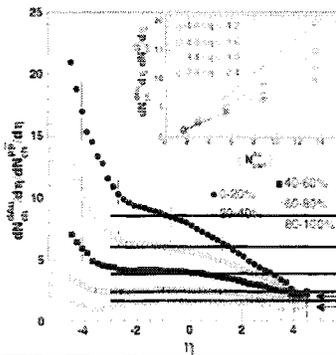
In p+A collisions the average number of particles produced at mid-rapidity $\sim (1+v)/2$
The energy deposition in rapidity is not uniform

---- the initial conditions – a dynamical issue
-- fluctuations

Noticeable Change in Eccentricity



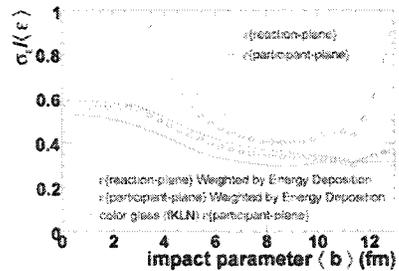
Lessons from p+A Collisions



The energy deposition at mid-rapidity and the rapidity distribution depends on dynamic

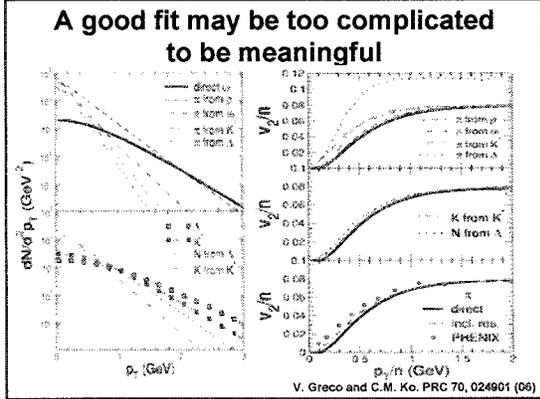
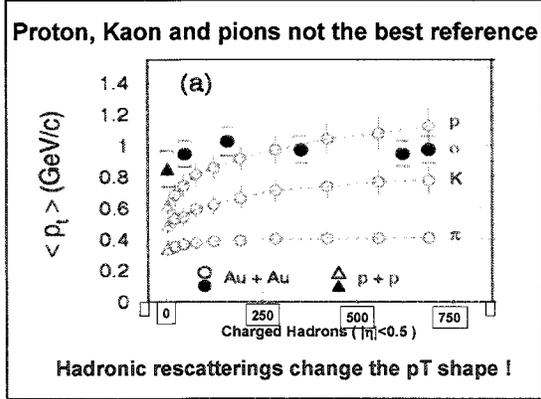
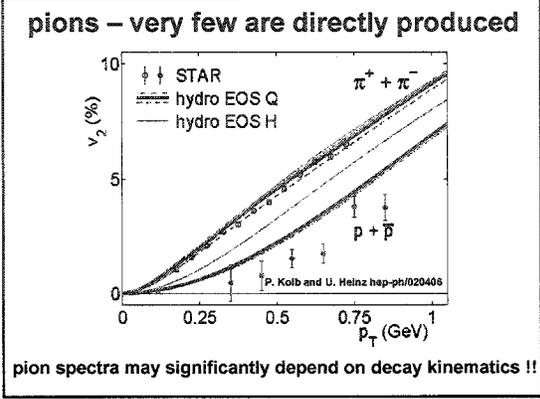
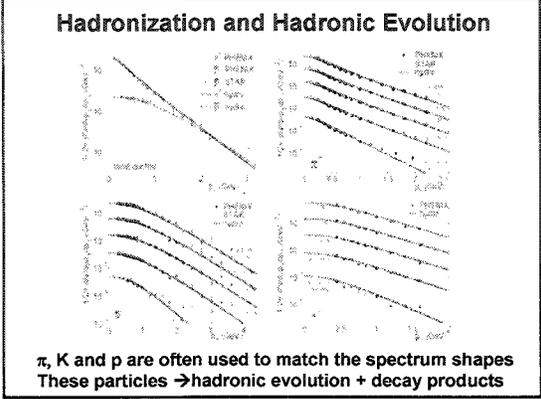
Expected ratios from previous p+A studies $(v+1)/2$

Fluctuations

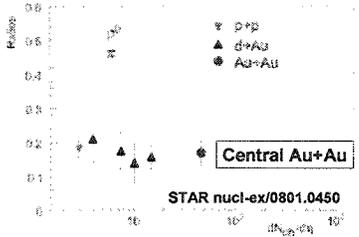


CGC Initial Condition – better approximation

Additional dynamical fluctuation due to e-by-e energy deposition

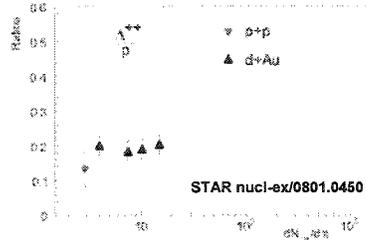


Resonances are difficult to measure in A+A



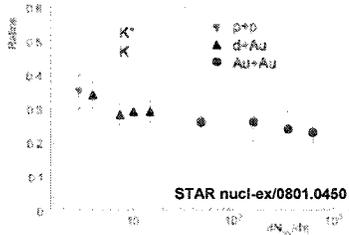
The directly produced number of pions is expected to be small ~ 10-15%; we do not know all resonance yields; is thermal statistical model good enough or not?

protons even more uncertain



~ half of the protons from hyperons decays
 - Δ resonances difficult to measure, pion wind can turn all protons to Δ s....

Kaons are affected too



Directly produced Kaons ~ 30-40% (?)

Maybe some particles are more suitable than others

- 1) Focus on particles with small hadronic rescattering cross sections and less resonance decay contributions ϕ , Ω and D
- 2) Hadronic afterburner --- resonance evolution

The effects of the order of phase transition, chemical equilibrium and freeze-out in ideal hydro

Pasi Huovinen

Physics Department, Purdue University, West Lafayette, IN 47907, USA,
phuovine@purdue.edu

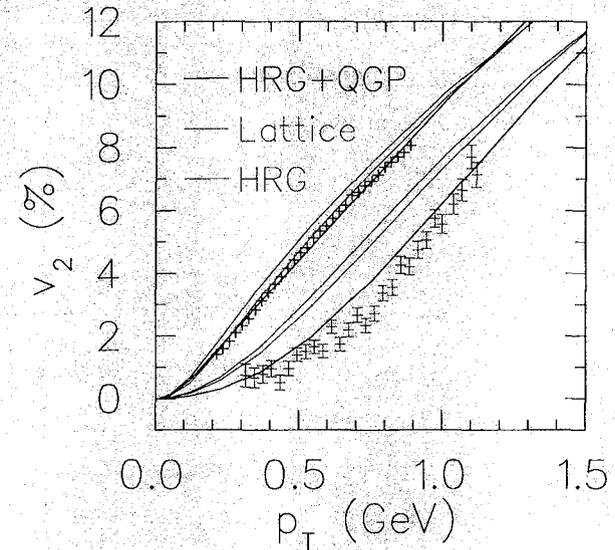
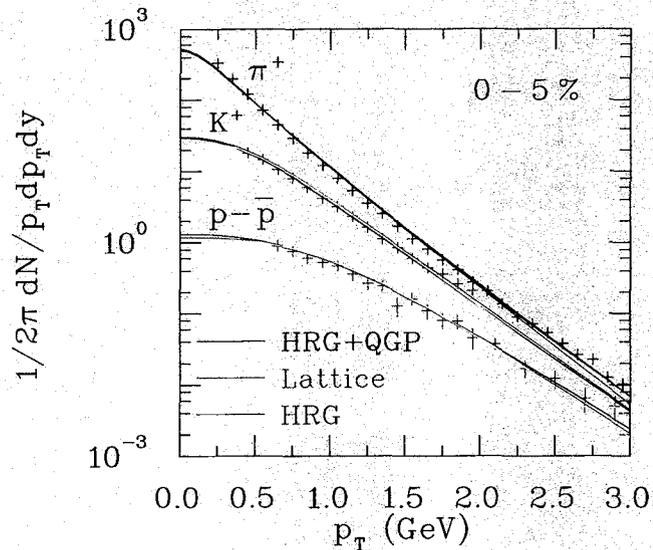
In ideal fluid description of heavy-ion collisions, the effects of equation of state to the particle distributions can be largely counteracted by the suitable choice of freeze-out temperature. Pion differential elliptic flow in minimum bias collisions is also almost independent of the equation of state, but the proton $v_2(p_T)$ shows observable sensitivity to it. Surprisingly the equation of state with a first order phase transition (HRG+QGP) leads to the best reproduction of data whereas lattice inspired equation of state leads to almost as bad result than pure hadron resonance gas equation of state (lattice and HRG in the figures, respectively). Solely changing the phase transition from first order transition to a cross-over does not solve the HBT puzzle either. However, it is worth noticing that the ratio $R_{\text{out}}/R_{\text{side}}$ is not largest for the first order transition equation of state, but for the pure hadron gas.

Unlike previously claimed in the literature, in principle it is possible to fit both particle yields and their p_T -distributions using ideal fluid hydrodynamics. This requires chemical freeze-out at $T_{\text{ch}} = 150$ MeV, kinetic freeze-out at $T_{\text{fo}} = 120$ MeV and very early thermalization at $\tau_0 = 0.2$ fm/c (the latter requirement can be relaxed if the equation of state is stiffer than the one with first order phase transition). Unfortunately the elliptic flow cannot be reproduced in this approach. (CE stands for chemical equilibrium and PCE partial chemical equilibrium, i.e. system not in chemical equilibrium between chemical and kinetic freeze-outs).

Lastly, requiring the freeze-out take place when expansion and scattering rates are equal leads to much smaller values of transverse flow velocity than in conventional freeze-out at constant temperature. This can again be counteracted by very short initial time when the system freezes-out soon after the phase transition in both when the temperature is chosen accordingly and when one requires the equality of the scattering and expansion rates.

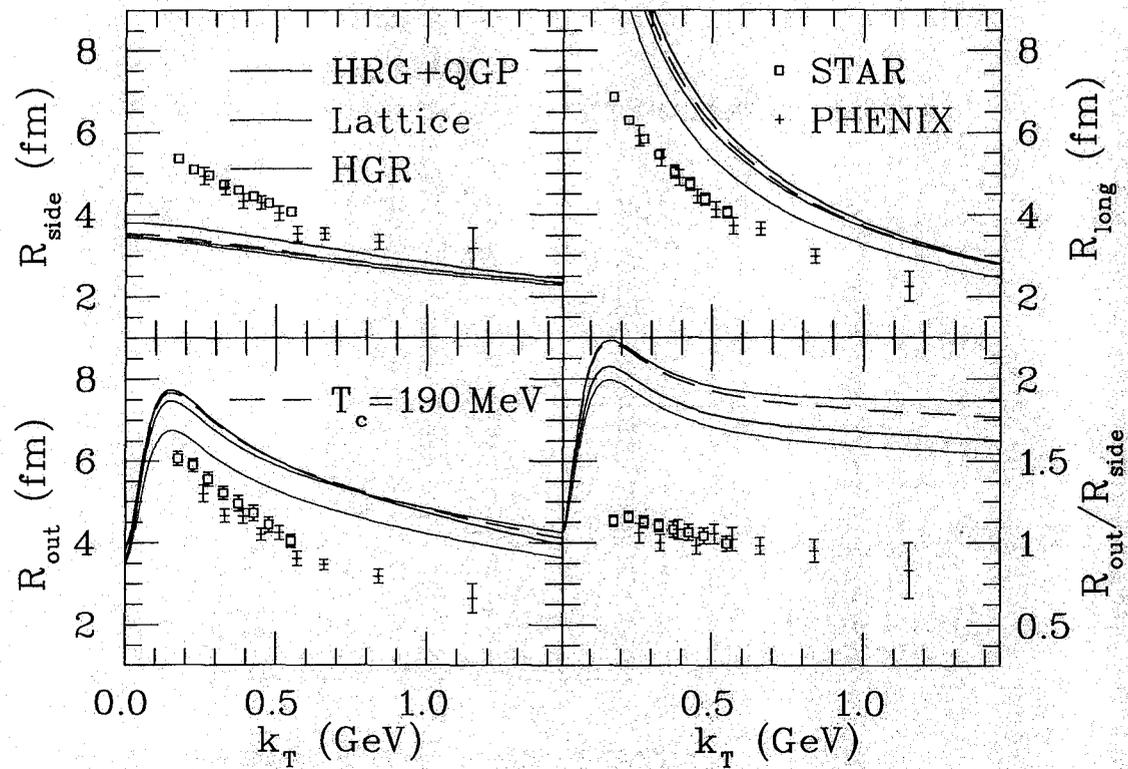
Anisotropy and p_T -spectra

Au+Au at RHIC ($\sqrt{s_{NN}} = 200$ GeV)



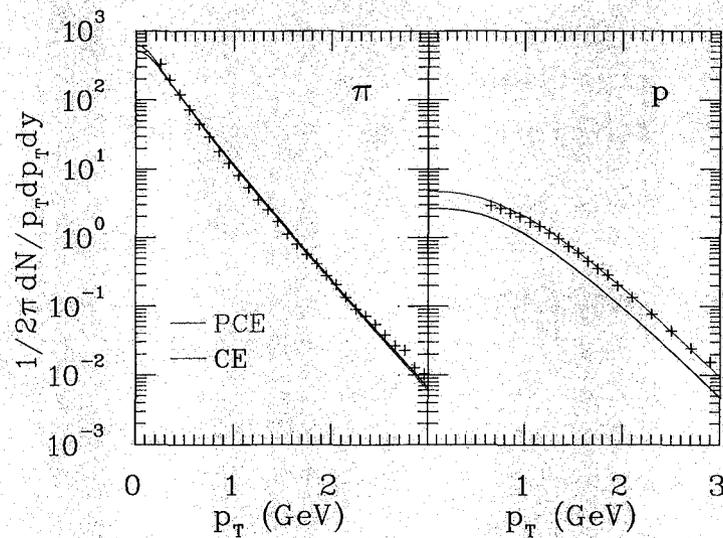
- Effect on spectra negated by freeze-out temperature
- All EoSs lead to similar pion $v_2(p_T)$
- Proton $v_2(p_T)$ is sensitive to EoS:
 - Lattice EoS gives as bad fit than EoS without any phase transition!
 - An EoS with a first order phase transition is closest to the data

Two particle correlations



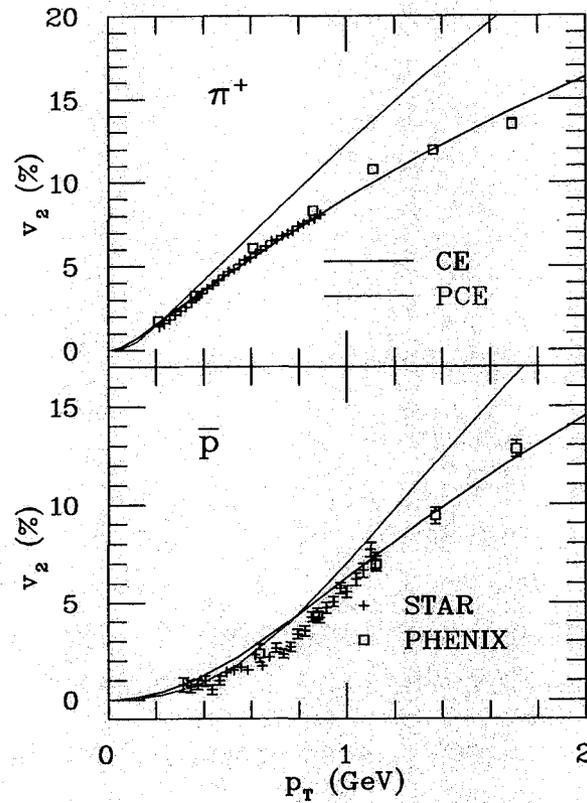
Initial state

- Need more transverse flow
 - Steeper initial density profile
 - Short initial time $\tau_0 = 0.2 \text{ fm}/c$ instead of $\tau_0 = 0.6\text{--}1.0 \text{ fm}/c$



- If momentum distribution is isotropic, $\epsilon = 3P$ holds
- No need for exact thermalization

Elliptic flow $v_2(p_T)$



- Dissipation required

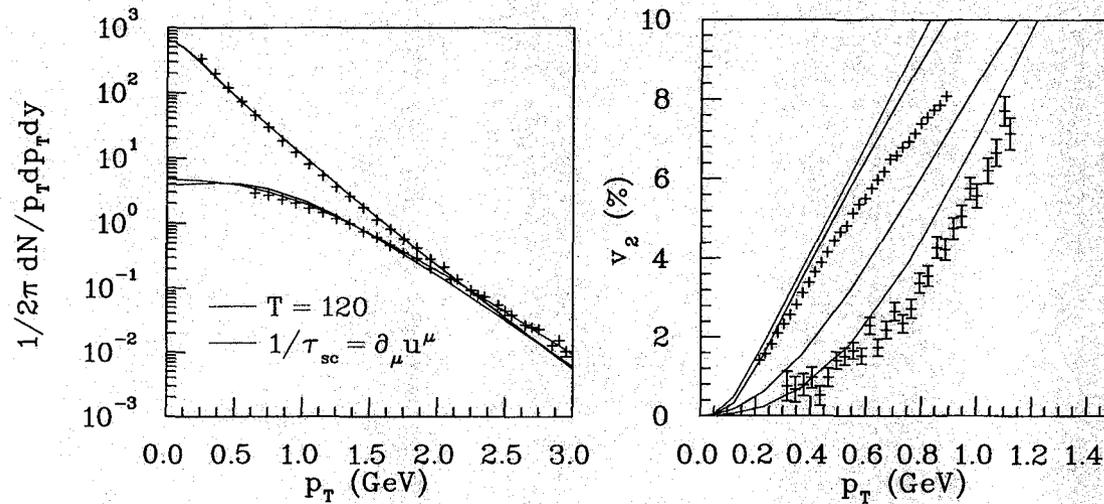
- But where?

- in hadronic phase?
- in plasma?
- or both?

Failed!

Early start?

- freeze-out has only a small effect
- system freezes out soon after phase transition anyway
- results for chemical non-equilibrium and $\tau_0 = 0.2 \text{ fm}/c$:



Current status of a 3D hydro+cascade model

Tetsufumi Hirano

*Department of Physics, the University of Tokyo, Tokyo
113-0033, Japan*

In this talk, we focus on the current status of dynamical modelling in relativistic heavy ion collisions at RHIC energies using a hybrid model in which ideal hydrodynamical description of the quark gluon plasma (QGP) is followed by a hadronic cascade.

Created fireballs are initialized first by the Glauber model which is suitably extended in the longitudinal direction. The hybrid model works quite well in description of collective flow.

By using the perfect fluid QGP picture, we reproduce a wide variety of v_2 data such as v_2 as functions of N_{part} , pseudorapidity η , transverse momentum p_T , particle species, system size, and collision energy. It turns out that mass ordering behavior of differential elliptic flow results mainly from hadronic rescatterings rather than perfect fluid evolution of the QGP. As a consequence, differential v_2 for ϕ meson does not follow the mass-ordering pattern due to its small cross section in the hadron gas.

We next investigate elliptic flow by changing initial conditions to the one from a color glass condensate model. Initial eccentricity in the CGC model is found to be larger than the one in the Glauber model. This, in turn, generates too much elliptic flow at a given centrality. This suggests that viscous effects and/or a soft equation of state are required even in the QGP phase.

References

- [1] T. Hirano, U. W. Heinz, D. Kharzeev, R. Lacey and Y. Nara, Phys. Lett. B **636**, 299 (2006) [arXiv:nucl-th/0511046]; J. Phys. G **34**, S879 (2007) [arXiv:nucl-th/0701075]; arXiv:0710.5795 [nucl-th] (Phys. Rev. C, in press).

Hydrodynamics in Heavy Ion Collisions and QCD Equation of State

Current status of a 3D hydro+cascade model

Tetsufumi Hirano
Department of Physics
The University of Tokyo

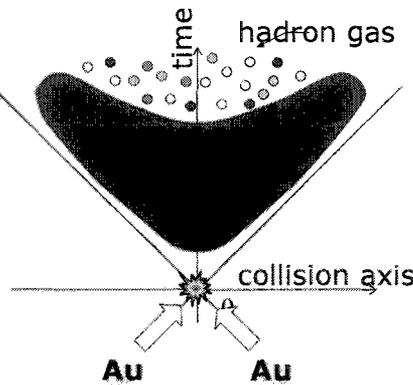


References:

T.Hirano, U.W.Heinz, D.Khaezeev, R.Lacey, Y.Nara
Phys.Lett.B636, 299 (2006);
J.Phys.G34, S879 (2007);
arXiv:0710.5795 [nucl-th] (Phys.Rev.C, in press.)

Our Strategy: QGP fluid + hadronic cascade in full 3D space

TH et al.('06)



Initial condition:

1. Glauber model
2. CGC model

QGP fluid:

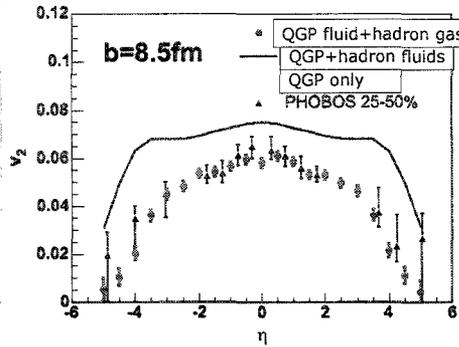
- 3D ideal hydrodynamics
- massless free u,d,s+g gas + bag const.
- $T_c = 170$ MeV

Hadron gas:

- Hadronic cascade, JAM1.13
- $T_{sw} = 169$ MeV

(1D) Bass, Dumitru (2D) Teaney, Lauret, Shuryak. (3D) Nonaka, Bass, Hirano et al.

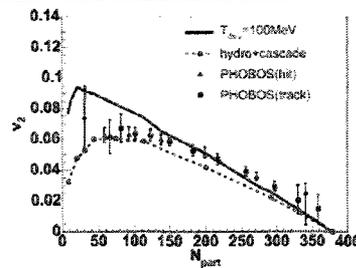
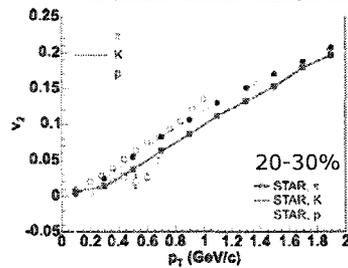
Importance of Hadronic "Corona"



- Boltzmann Eq. for hadrons instead of hydrodynamics
- Including effective viscosity through finite mean free path

T. Hirano et al., Phys. Lett. B636(2006)299.

Differential v_2 , centrality dependence

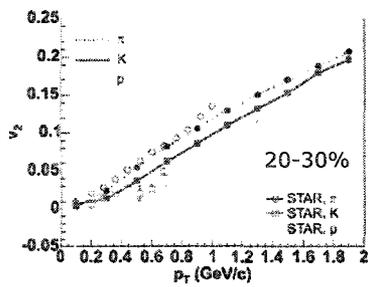


Mass dependence is o.k.
Note: First result was obtained by Teaney et al.

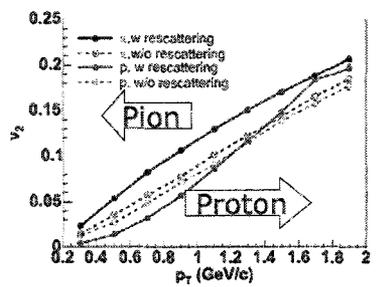
T. Hirano et al. ('07)

- Centrality dependence is ok
- Large reduction from pure hydro in small multiplicity events

Mass Ordering for $v_2(p_T)$

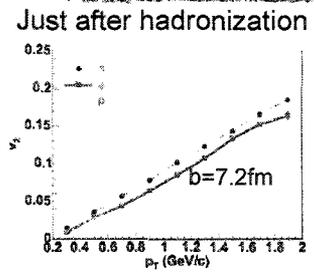


Mass dependence is o.k. from hydro+cascade.

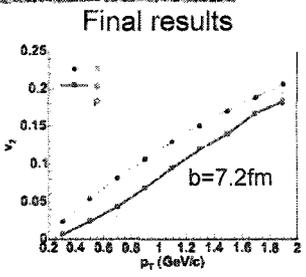
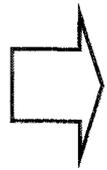


Mass ordering comes from hadronic rescattering effect. Interplay btw. radial and elliptic flows.

ϕ -meson case



$T = T_{sw} = 169 \text{ MeV}$



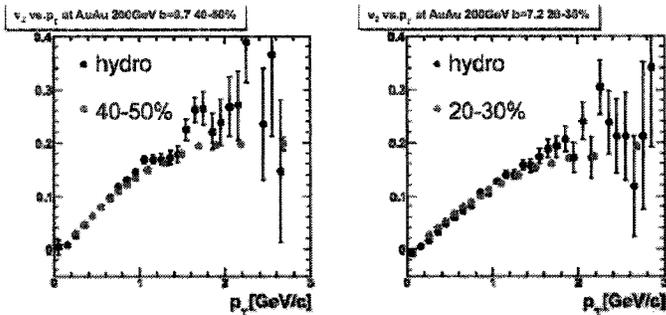
$v_{2,\phi} > v_{2,p}$
in $p_T < 1 \text{ GeV/c}$

Violation of mass ordering for phi mesons!
Clear signal of early decoupling!

Caveat: Published PHENIX data obtained in $p_{\sqrt{s}} > 1 \text{ GeV/c}$ for ϕ mesons

QGP fluid+hadron gas with Glauber I.C.

Centrality Dependence of Differential v_2

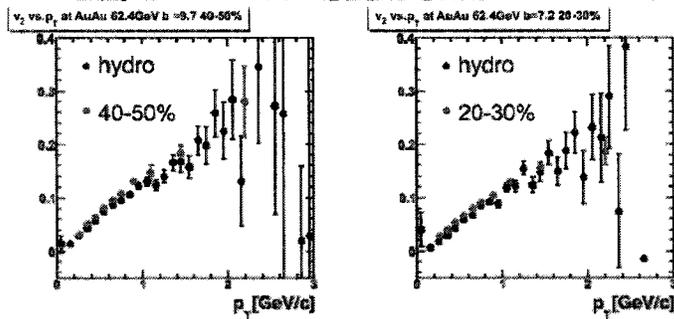


Pions, AuAu 200 GeV

Thanks to M. Shimomura

QGP fluid+hadron gas with Glauber I.C.

Hybrid Model at Work at $\sqrt{s_{NN}}=62.4$ GeV

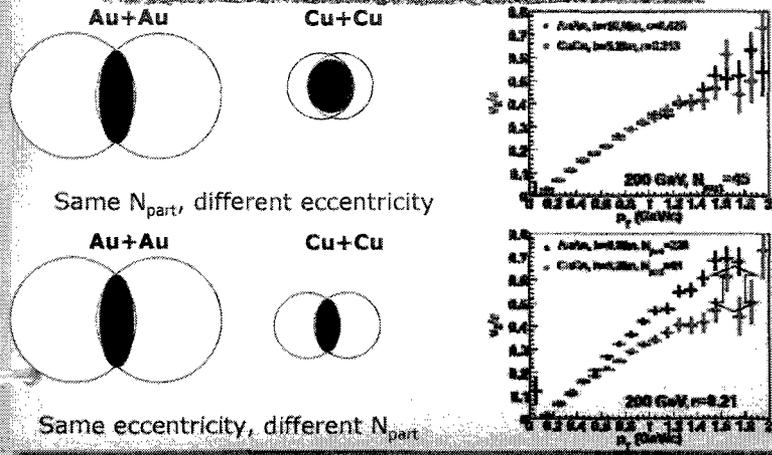


Pions, AuAu 62.4 GeV

Thanks to M. Shimomura

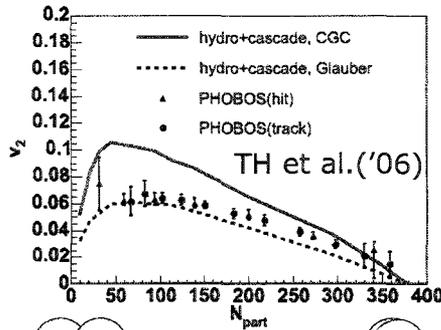
QGP fluid+hadron gas with Glauber I.C.

Differential v_2 in Au+Au and Cu+Cu Collisions



QGP fluid+hadron gas with CGC I.C.

$v_2(N_{part})$ Depends on Initialization



Glauber:

- ✓ Early thermalization
- ✓ Discovery of Perfect Fluid QGP

CGC:

- ✓ No perfect fluid?
- ✓ Additional viscosity required in QGP?

Important to understand initial conditions much better for making a conclusion Adil, Gyulassy, Hirano ('06)

April 21, 2008

The Origin of Thermal Hadron Production

Helmut Satz

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email: satz@physik.uni-bielefeld.de

Abstract:

The thermal multihadron production observed in different high energy interactions poses two basic problems: (1) why do even elementary interactions with comparatively few secondaries (such as e^+e^- annihilation) lead to thermal hadron abundances, and (2) why is there in such interactions a suppression of strange particle production, which is effectively removed for nuclear collisions? We show that the recently proposed mechanism of thermal hadron production through Hawking-Unruh radiation can naturally account for both. The event horizon of colour confinement leads to thermal behaviour, but the emission temperature depends on the strange quark content of the produced hadrons, causing a deviation from full equilibrium and hence a suppression of strange particle production. We show that the resulting formalism accounts well for multihadron production in e^+e^- annihilation over a wide energy range, providing a very good description of the observed abundances. It is fully determined in terms of the string tension and the bare strange quark mass, and contains no adjustable parameters.

- Why do elementary high energy collisions produce a thermal medium?
For nucleus-nucleus collisions possibly multiple parton interactions
→ kinetic thermalization; e^+e^- , $pp/pp\bar{p}$ not
- Is there another non-kinetic thermalization mechanism, providing a common origin of thermal production in all high energy collisions?
- Why is strangeness production universally suppressed in elementary collisions?
- Why no strangeness suppression in nuclear collisions?

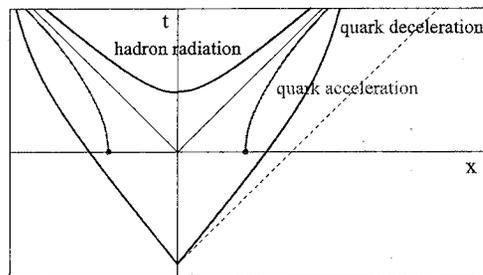
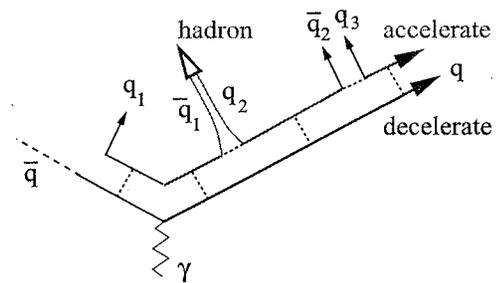
Conjecture:

physical vacuum \sim event horizon for colored constituents
thermal hadron production \sim Hawking-Unruh radiation of QCD

[Paolo Castorina, Dmitri Kharzeev, HS 2007]

self-similar pattern:

screening
string breaking
tunnelling
quark acceleration
/deceleration
Hawking radiation



Strangeness Production

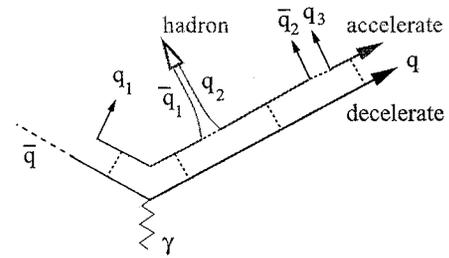
[Becattini, Castorina, Manninen, HS 2008]

Unruh temperature $\sim 1 / \text{mass of secondary}$.

we had for finite quark mass m_q

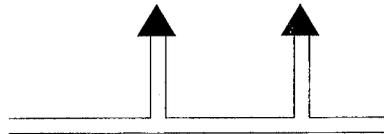
$$a_q \simeq \frac{\sigma}{\sqrt{m_q^2 + (\sigma/2\pi)}} \Rightarrow T_U = \frac{a_q}{2\pi}$$

produced meson consists
of quarks \bar{q}_1 and q_2

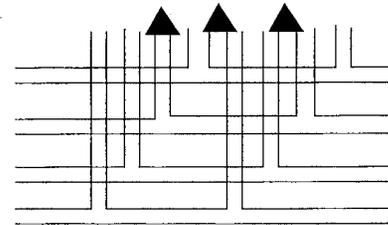


Heavy Ions

- elementary collisions
sequential $q\bar{q}$ pair production \Rightarrow independent hadron emission
- nuclear collisions
superposition of $q\bar{q}$ pair production, interference
exogamous pairing, not hadronic scattering



elementary



nuclear

Summary

- The physical vacuum is an event horizon for coloured quarks and gluons; thermal hadrons are Hawking-Unruh radiation produced by quark tunnelling through event horizon.
- The corresponding hadronization temperature T_H is determined by quark acceleration and deceleration in the colour field at the (quantum) horizon.
- Strangeness suppression arises through modified Unruh temperature for strange quark mass. It is effectively removed in nuclear collisions through exogamous pairing.
- Given string tension σ and strange quark mass m_s , the resulting scenario provides a parameter-free description of thermal hadron production in high energy interactions.

*QCD Critical Point and
Its Effects on Physical Observables
Schematic Consideration*

Masayuki ASAKAWA

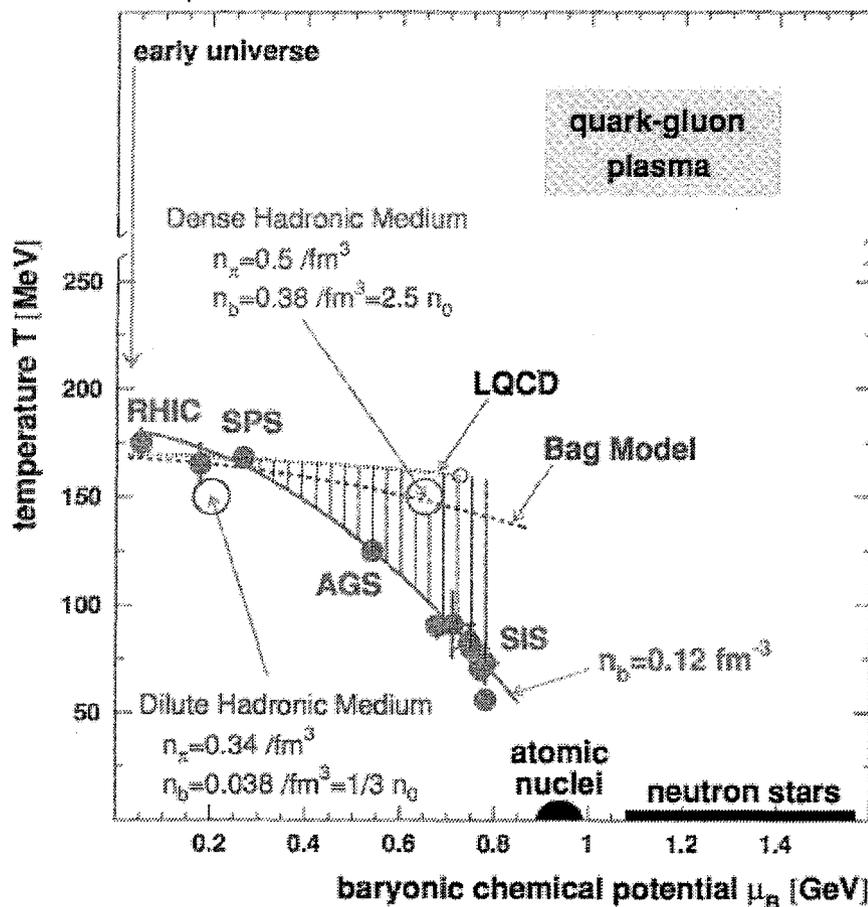
Department of Physics, Osaka University

1. QCD critical end point
2. Problems with usual hadronic observables
3. Universality and focusing of isotropic trajectories
4. Proposal of a new observable
5. Comparison with experimental data

Principles to Look for Other Observables

■ We are in need of observables that are not subject to final state interactions

➔ After Freezeout, no effect of final state interactions



Chemical Freezeout

- usually assumed momentum independent
- but this is not right

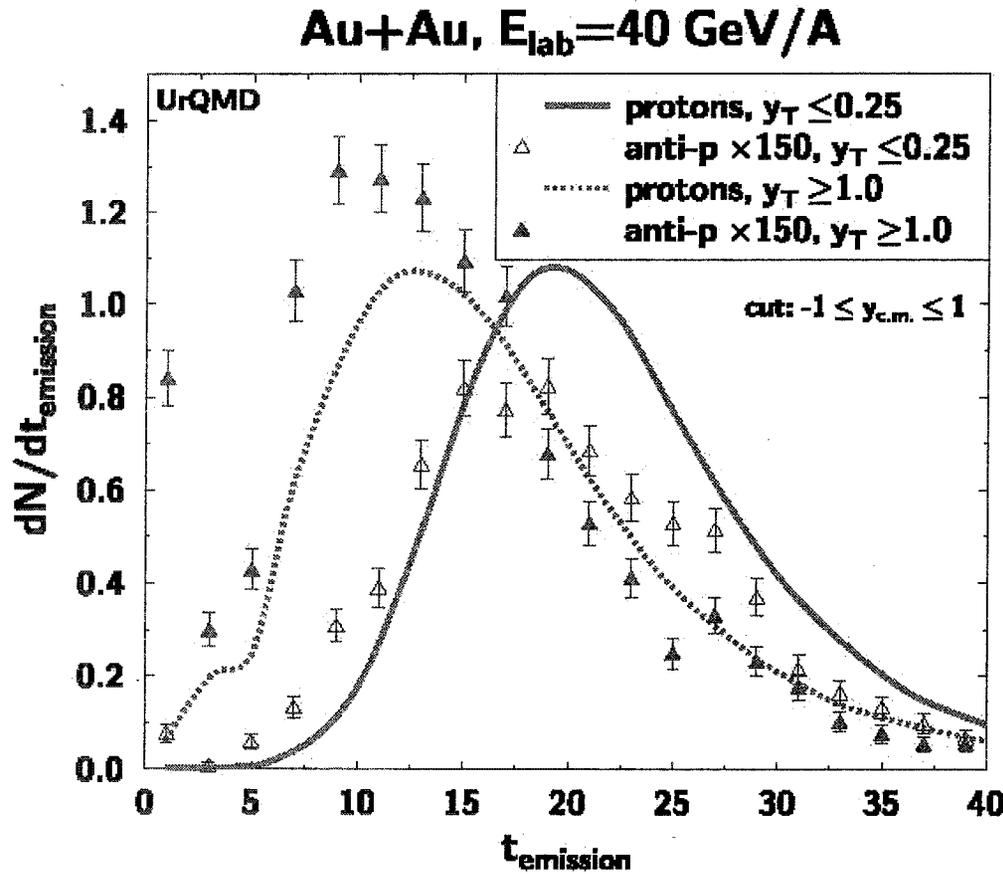
chemical freezeout time:
 p_T (or y_T) dependent

- ✓ Larger p_T (or y_T), earlier ch. freezeout

➔ **Principle I**

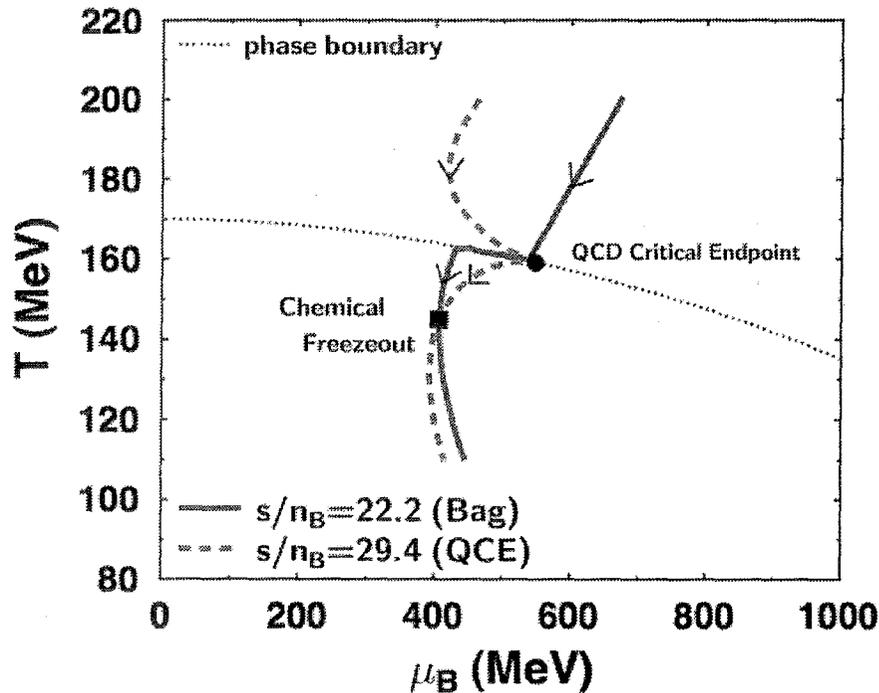
Emission Time Distribution

Emission Time



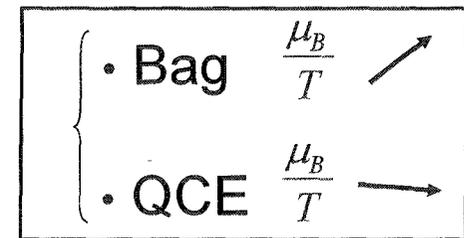
- Larger y_T , earlier emission
- To minimize resonance effect, y_T is used instead of p_T
- No CEP effect (UrQMD)

Consequence



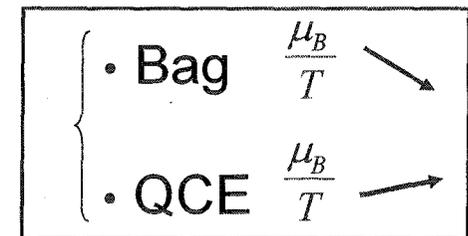
For a given chemical freezeout point, prepare two isentropic trajectories: w/ and w/o CEP

Along isentropic trajectory:



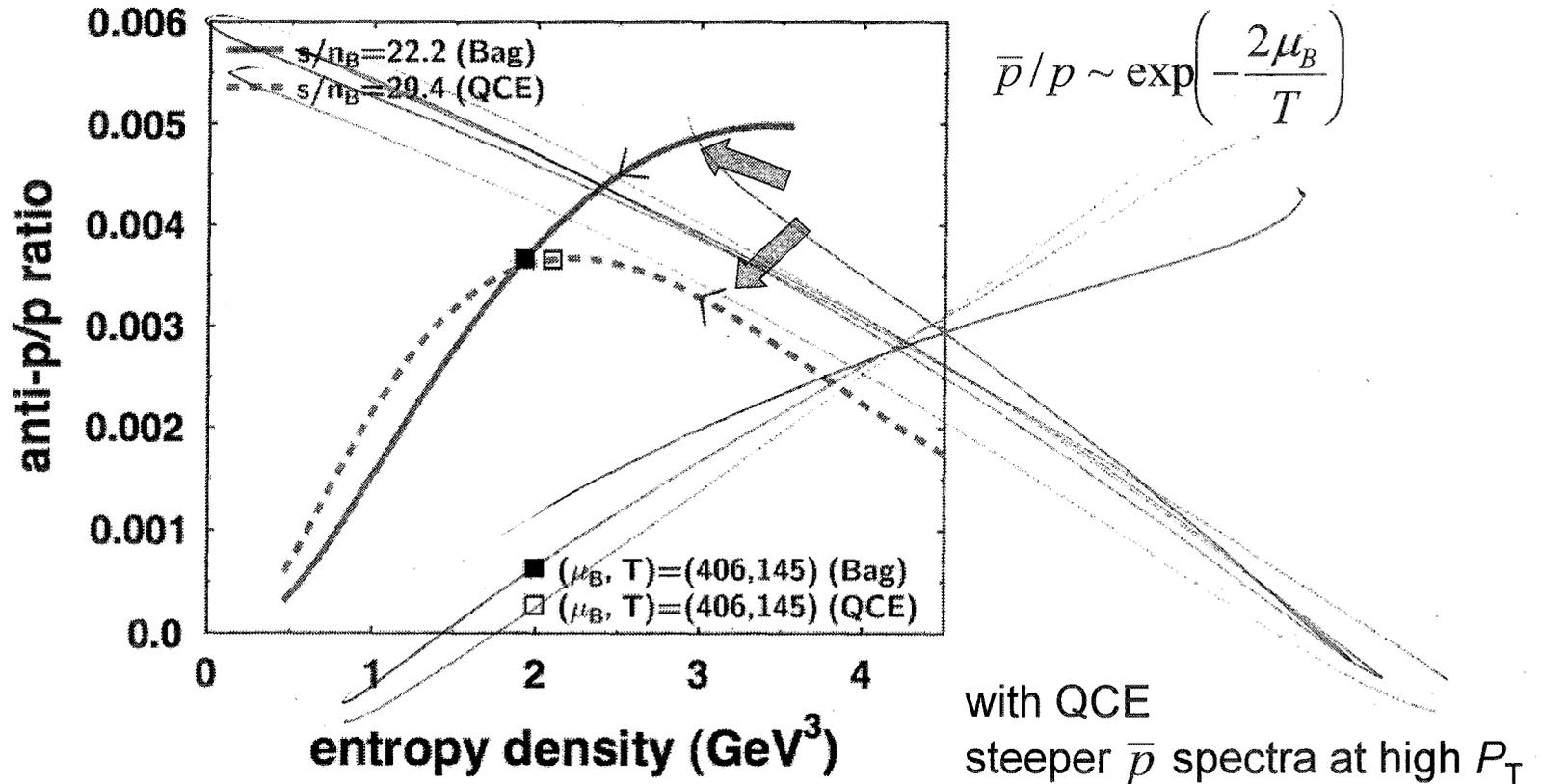
Principle I

As a function of $p_T(y_T)$:

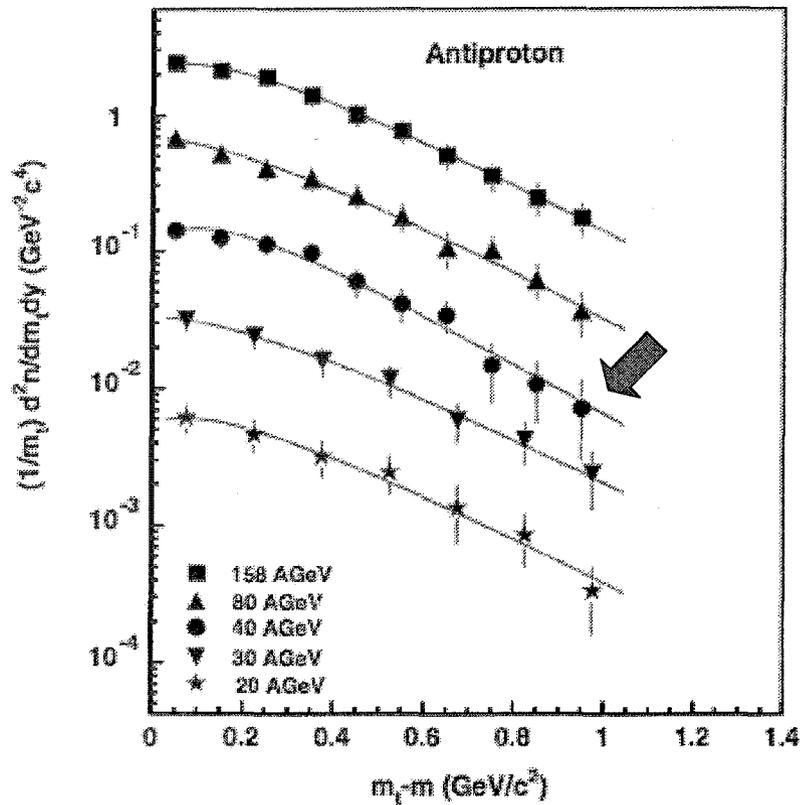


\bar{p}/p ratio : near CEP steeper

Evolution along Isentropic Trajectory



Result of One Temperature Fit



NA49, PRC73, 044910(2006)

	E_{beam} (A GeV)	dn/dy	T (MeV)	$\langle m_1 \rangle - m$ (MeV/c ²)
\bar{p}	158	1.66 ± 0.17	291 ± 15	384 ± 19
	80	0.87 ± 0.07	283 ± 30	385 ± 41
	40	0.32 ± 0.03	248 ± 35	385 ± 51
	30	0.16 ± 0.02	290 ± 45	395 ± 60
	20	0.06 ± 0.01	279 ± 64	394 ± 60
p	158	29.6 ± 0.9	308 ± 9	413 ± 13
	80	30.1 ± 1.0	260 ± 11	364 ± 16
	40	41.3 ± 1.1	257 ± 11	367 ± 16
	30	42.1 ± 2.0	265 ± 10	362 ± 14
	20	46.1 ± 2.1	249 ± 9	352 ± 13

- Only one experimental result for \bar{p} slope
- Still error bar is large

Summary

■ Two Principles:

- i) Chemical Freezeout is $p_T(y_T)$ dependent
- ii) Isentropic Trajectory behaves non-trivially near CEP (focusing)

⇒ \bar{p}/p ratio behaves non-monotonously near CEP

Information on the QCD critical point:
such as location, size of critical region,
existence...

■ We then made a data search

- turned out NA49 \bar{p} data shows non-trivial behavior around 40 GeV/A
- still error bar is large, *finer energy scans at SPS, FAIR, RHIC*: desirable

■ Effect on Flow ?

c_s changes differently from the case with EOS used in usual hydro cal.

(3D hydro cal. with CEP + UrQMD: C. Nonaka in progress)

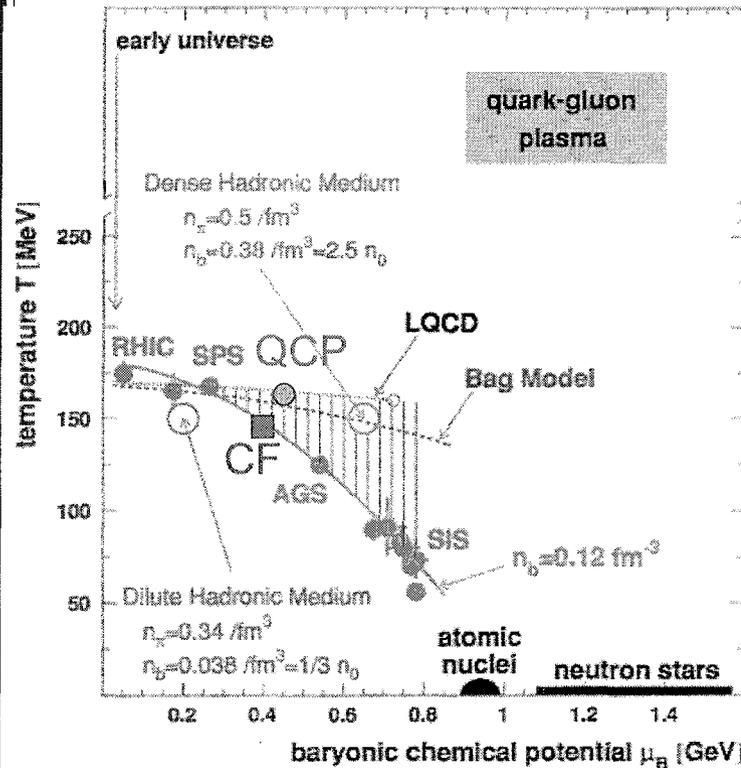
Hydrodynamic Expansion with QCD Critical Point

Nagoya University
Chiho NONAKA

Summary

- 3D Hydro + UrQMD Model with the QCD critical point
 - Isentropic trajectories, P_T spectra, hadron ratios
- Search of the QCD critical point from experiments
 - Anti-p/p ratio: promising and clear signature

Location of QCP?



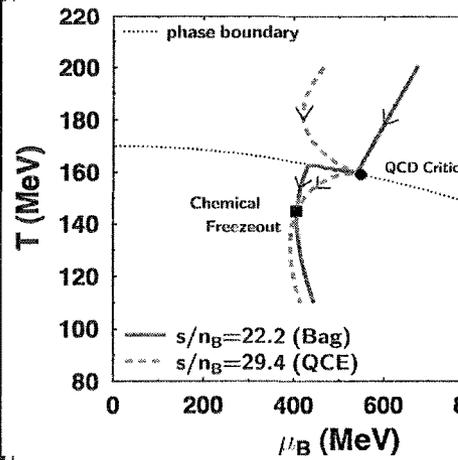
Search of the QCD critical point from experiments

SPS energy region

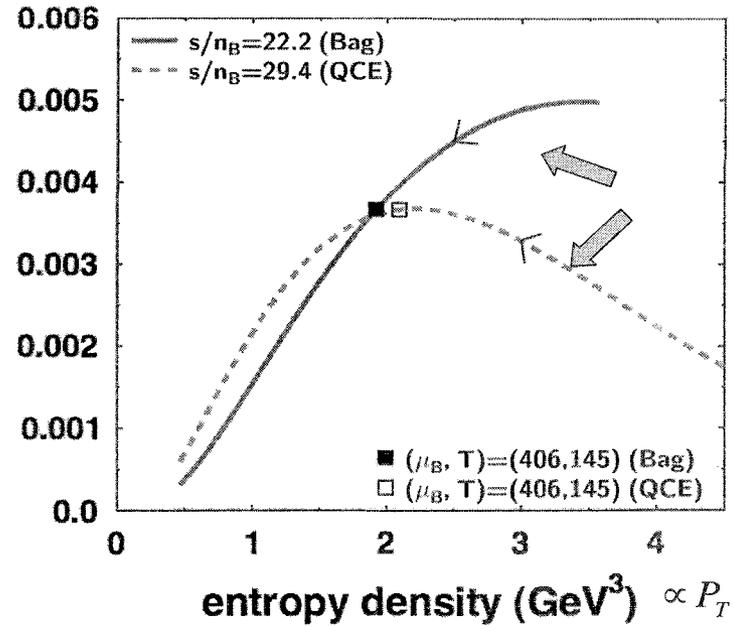
- Location of QCP
 $(\mu_B, T) = (550, 159)$
- Critical Region
- Chemical freezeout point
Hadron ratios are fixed.
 $(\mu_B, T) = (406, 145)$
from statistical model
- Hadronization
emission time distribution

Chiho NONAKA

Signature of QCP



anti-p/p ratio



$$\bar{p}/p \sim \exp\left(-\frac{2\mu_B}{T}\right)$$

- decreases (Bag)
- increases (QCE)

with QCE
steeper \bar{p} spectra at high P_T

3D Hydro+UrQMD with QCP

■ Initial Conditions

QCP: $T_E = 159$ MeV, $\mu_B = 550$ MeV

- Energy density

$$\varepsilon(x, y, \eta) = \varepsilon_{\max} W(x, y; b) H(\eta)$$

- Baryon number density

$$n_B(x, y, \eta) = n_{B\max} W(x, y; b) H(\eta)$$

- Parameters $\begin{cases} \tau_0 = 0.6 \text{ fm}/c \\ \eta_0 = 0.5 \quad \sigma_\eta = 1.5 \end{cases}$

- Flow

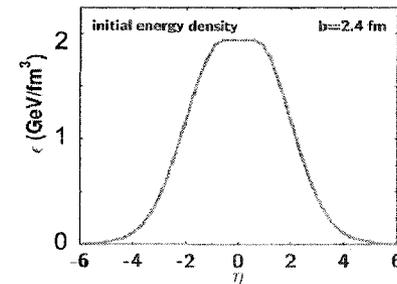
$$v_L = \eta \text{ (Bjorken's solution); } v_T = 0$$

- EOS: QCP, Bag Model

- Switching temperature

$$T_{\text{sw}} = 150 \text{ [MeV]}$$

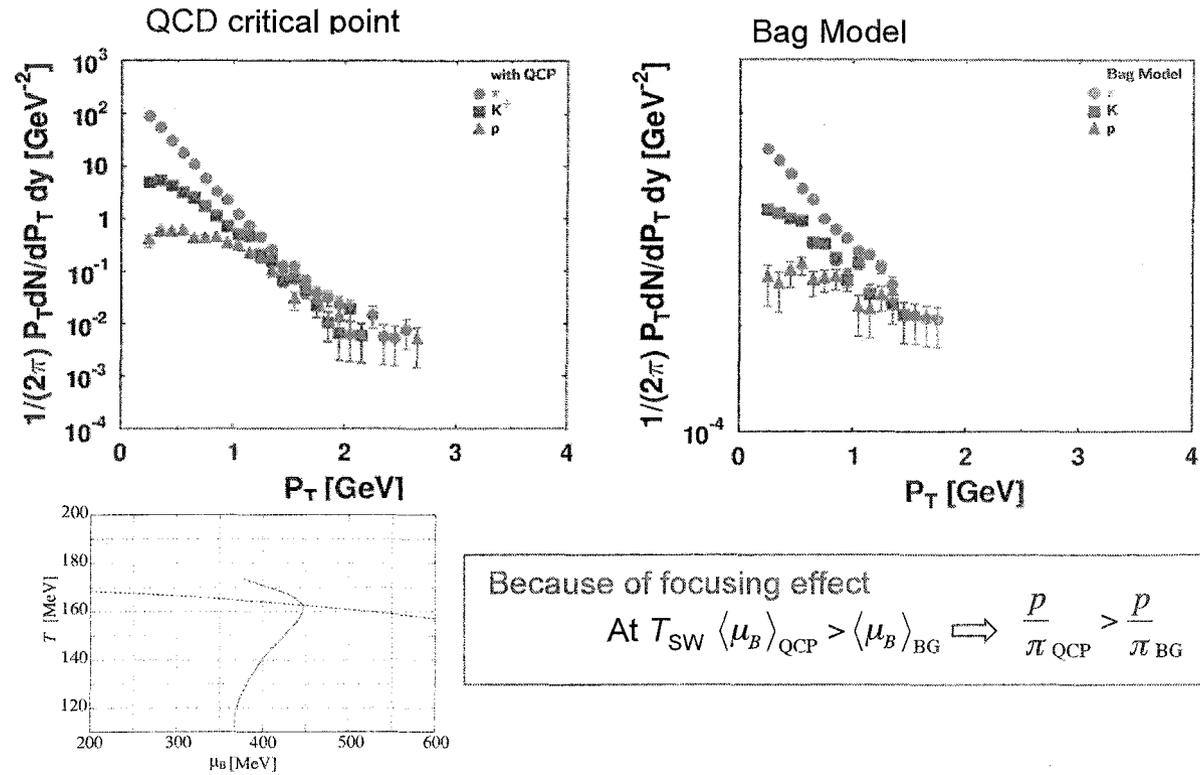
- longitudinal direction: $H(\eta)$



ε_{\max} GeV/fm ³	$n_{B\max}$ fm ⁻³
2.0	0.15

Chiho NONAKA

P_T Spectra



Chiho NONAKA

New analytic results in hydrodynamics

UTILIZING THE FLUID NATURE OF QGP

M. Csanád, T. Csörgő, M. I. Nagy



ELTE
MTA KFKI RMKI
Budapest, Hungary



Hydrodynamics at RHIC and QCD EOS Workshop,
BNL, USA
April 21, 2008

2008-04-21

M. Csanád, T. Csörgő, M.I. Nagy

Some general remarks

Hydrodynamics=

Initial conditions \otimes **dynamical equations** \otimes **freeze-out conditions**

Exact solution = formulas solve hydro without approximation

Parametric solution = shape parameters introduced,

time dependence given by ordinary coupled diff. eqs.

Hydro inspired parameterization

= shape parameters determined only at the freeze-out

their time dependence is not considered

Report on new class of exact, parametric solution of relativistic hydro

M.I. Nagy, T. Cs., M. Csanád, [arXiv:0709.3677v1](https://arxiv.org/abs/0709.3677v1), PRC77:024908 (2008)

T. Cs, M. I. Nagy, M. Csanád, [arXiv:nucl-th/0605070v4](https://arxiv.org/abs/nucl-th/0605070v4), PLB (2008)

M. Csanád, M. I. Nagy, T. Cs, [arXiv:0710.0327v3](https://arxiv.org/abs/0710.0327v3) [nucl-th] EPJ A (2008)

Initial conditions:

pressure and velocity on $\tau = \tau_0 = \text{const}$

EoS:

$\varepsilon - B = \kappa (p+B)$ $c_s^2 = 1/\kappa$

Freeze-out condition:

$T = T_f (\eta = 0)$, local simultaneity, $n^\nu = u^\nu$

New, simple, exact solutions

$$v = \tanh \lambda \eta,$$

$$p = p_0 \left(\frac{\tau_0}{\tau} \right)^{\lambda d \frac{\kappa+1}{\kappa}} \left(\cosh \frac{\eta}{2} \right)^{-(d-1)\phi_\lambda}$$

Possible cases (one row of the table is one solution):

Case	λ	d	κ	ϕ_λ
a.)	2	$\in \mathbb{R}$	d	0
b.)	$\frac{1}{2}$	$\in \mathbb{R}$	1	$\frac{\kappa+1}{\kappa}$
c.)	$\frac{3}{2}$	$\in \mathbb{R}$	$\frac{4d-1}{3}$	$\frac{\kappa+1}{\kappa}$
d.)	1	$\in \mathbb{R}$	$\in \mathbb{R}$	0
e.)	$\in \mathbb{R}$	1	1	0

Nagy,CsT, Csanád: arXiv:0709.3677v1

→ New, accelerating, d dimension

↘ d dimensional with $p=p(\tau,\eta)$

↗ (thanks T. S. Biró)

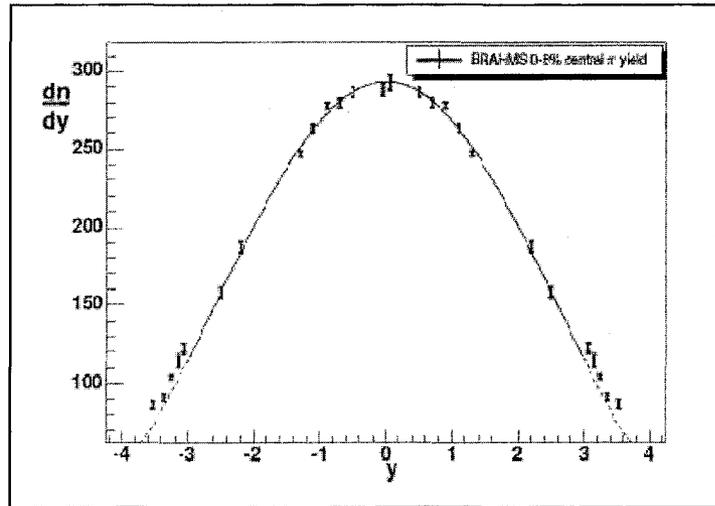
→ Hwa-Bjorken, Buda-Lund type

→ Special EoS, but general velocity

If $\kappa = d = 1$, general solution is obtained, for ARBITRARY initial conditions. It is STABLE !

BRAHMS rapidity distribution

$$\frac{dn}{dy} \approx \left. \frac{dn}{dy} \right|_{y=0} \cosh^{\pm \frac{\alpha}{2} - 1} \left(\frac{y}{\alpha} \right) e^{-\frac{m}{T_f} \left[\cosh^{\alpha} \left(\frac{y}{\alpha} \right) - 1 \right]},$$

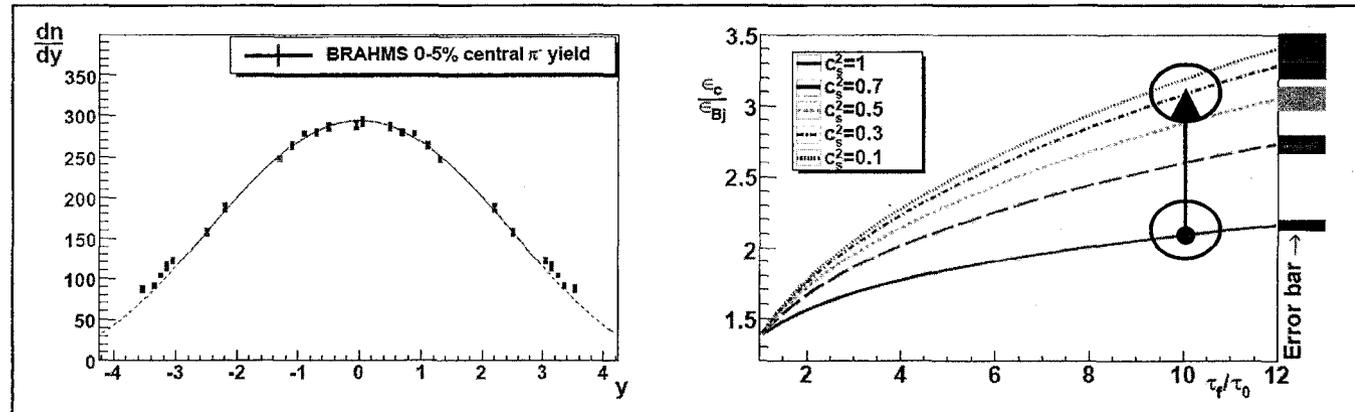


$$\lambda = \frac{\alpha - 1}{\alpha - 2}.$$

α	7.4 ± 0.13
$\left. \frac{dn}{dy} \right _{y=0}$	294 ± 1
χ^2/NDF	$30.6/14$
CL	0.6%
T_f (MeV)	200 (fixed)
m (MeV)	140 (fixed)
λ	1.18 ± 0.01 (derived)

BRAHMS dn/dy data fitted with the analytic formula

Conjectured EoS dependence of ε_0



71

$$\frac{\varepsilon_{cs}}{\varepsilon_{Bj}} = (2\lambda - 1) \left(\frac{\tau_f}{\tau_0} \right)^{\lambda-1} \left(\frac{\tau_f}{\tau_0} \right)^{(\lambda-1)(1-c_s^2)}$$

Using $\lambda = 1.18$, and $\tau_f/\tau_0 = 10$ as before

and $c_s = 0.35$, [PHENIX, [arXiv:nucl-ex/0608033v1](https://arxiv.org/abs/nucl-ex/0608033v1)] we get $\varepsilon_{cs}/\varepsilon_{Bj} = 2.9$

$\varepsilon_0 = 14.5 \text{ GeV/fm}^3$ in 200 GeV, 0-5 % Au+Au at RHIC

Conclusions

Explicit simple accelerating relativistic hydrodynamics

Analytic (approximate) calculation of observables

Realistic rapidity distributions; BRAHMS data well described

No go theorem: same final states, different initial states

New estimate of initial energy density:

$\varepsilon_c/\varepsilon_{Bj}$ at least 2 @ RHIC

dependence on c_s estimated, $\varepsilon_c/\varepsilon_{Bj} \sim 3$ for $c_s = 0.35$

Estimated work effects on lifetime:

at least 20% increase @ RHIC

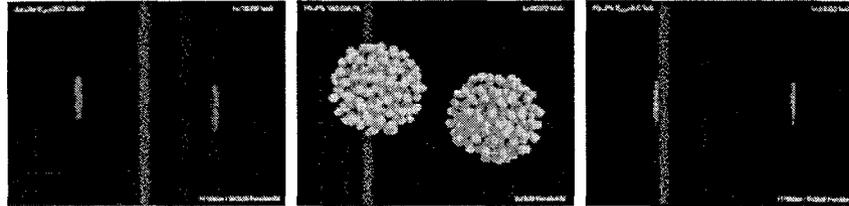
dependence on c_s estimated, $\tau_c/\tau_{Bj} \sim 1.4$ for $c_s = 0.35$

A lot to do ...

more general EoS

less symmetry, ellipsoidal solutions

asymptotically Hubble-like flows



Viscous hydrodynamics and the QCD equation of state

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Department of Physics, University of Cape Town,
South Africa

*Workshop on "Hydrodynamics in Heavy Ion Collisions
and QCD Equation of State"*

April 21-22, 2008 : BNL, Long Island, NY, USA

Entropy 4-current, density and flux

Entropy 4-current

$$S^\mu = su^\mu + \beta q^\mu - \frac{1}{2}\beta(\beta_0\Pi^2 - \beta_1 q^\nu q_\nu + \beta_2 \pi^{\nu\lambda} \pi_{\nu\lambda})u^\mu - \beta(\alpha_0\Pi q^\mu - \alpha_1 \pi^{\mu\nu} q_\nu)$$

Entropy density

$$s(\varepsilon, n, \Pi, q^\nu, \pi^{\nu\lambda}) = s_{eq}(\varepsilon, n) - \frac{1}{2}\beta(\beta_0\Pi^2 - \beta_1 q^\nu q_\nu + \beta_2 \pi^{\nu\lambda} \pi_{\nu\lambda})u^\mu$$

Entropy flux $\Phi^\nu = \beta(q^\nu - \alpha_0\Pi q^\nu - \alpha_1 q_\lambda \pi^{\nu\lambda})$

Second Law of thermodynamics

$$\beta^{-1}\partial_\mu S^\mu = \zeta^{-1}\Pi^2 - \lambda^{-1}q^\nu q_\nu + (2\eta)^{-1}\pi^{\nu\lambda} \pi_{\nu\lambda} \geq 0$$

The 3 new coefficients in the entropy density are related to the relaxation times and are responsible for causality while the 2 new coefficients in the entropy flux are related to the relaxation length and are responsible for the coupling between heat flow and viscous stresses

Entropy density and entropy production

$$s(\varepsilon, n, \Pi, q, \pi) = s_{eq}(\varepsilon, n) - \Psi(\varepsilon, n, \Pi, q, \pi)$$

with

$$\Psi(\varepsilon, n, \Pi, q, \pi) \equiv \frac{1}{2}\beta(\beta_0\Pi^2 - \beta_1q^2 + \beta_2\pi^2)$$

Entropy density

$$\text{Entropy production} \quad (Ds + s\Theta) = \Phi \geq 0$$

where

$$\Phi \equiv \beta(\zeta^{-1}\Pi^2 + \lambda^{-1}q^2 + (2\eta)^{-1}\pi^2)$$

Relevant ratios

$$\frac{s(\varepsilon, n, \Pi, q, \pi^{v\lambda})}{s_{eq}(\varepsilon, n)} = 1 - \frac{\Psi(\varepsilon, n, \Pi, q, \pi)}{s_{eq}(\varepsilon, n)}$$

$$Ds = -\left(1 - \frac{\Phi}{s\Theta}\right)s\Theta$$

Shear viscosity in the scaling solution

Energy equation $\frac{\partial s}{\partial \tau} = (R^{-1} - 1) \frac{s}{\tau}$ where $R^{-1} = \frac{\Phi}{T s}$

$$\Phi^{ideal} = 0$$

$$\Phi^{1st} = \frac{4}{3} \frac{1}{\eta T}$$

$$\frac{d\Phi^{2nd}}{d\tau} = -\frac{\Phi^{2nd}}{\tau} - \frac{1}{2} \Phi^{2nd} \left(\frac{1}{\tau} + \frac{T}{\beta_2} \frac{d}{d\tau} \left(\frac{\beta_2}{T} \right) \right) + \frac{2}{3} \frac{1}{\beta_2 \tau}$$

Reynold's number vs eta/s

$$s = 4aT^3, \quad \eta = bT^3, \quad \beta_2 = \frac{3}{4aT^4}, \quad \tau_\pi = \frac{3b}{2aT}$$

$$\frac{\eta}{s} = \frac{1}{4a} \frac{b}{T}$$

$$R^{-1} = \frac{1}{3aT} \frac{b}{T}$$

QCD EOS

Entropy equation of state

$$\frac{s}{s_c}(T) = \left[\frac{T}{T_c} \right]^3 \left(1 + \frac{d_Q - d_H}{d_Q + d_H} \tanh \left[\frac{T - T_c}{\Delta T} \right] \right)$$

where $s_c = \text{const.} \times \frac{1}{2} (d_Q + d_H) T_c^3$

For $\Delta T = 0$, MIT bag EOS

$$B = \frac{1}{2} \left[\frac{\frac{d_Q}{d_H} - 1}{\frac{d_Q}{d_H} + 1} \right] T_c s_c$$

$$\epsilon_Q - \epsilon_H = 4B$$

$$\epsilon_Q = \frac{1}{2} \left[\frac{4 \frac{d_Q}{d_H} - 1}{\frac{d_Q}{d_H} + 1} \right] T_c s_c, \quad \epsilon_H = \frac{3}{2} \left[\frac{1}{\frac{d_Q}{d_H} + 1} \right] T_c s_c$$

Beyond the idealistic view of space-time evolution of relativistic heavy ion collisions

This talk discussed the intriguing questions of the non-equilibrium fluid dynamical description of the space-time evolution of the relativistic heavy ion collisions. This description is quite different from the equilibrium description that we have learned to accept as the one that works.

It would be interesting to explore new non-equilibrium phenomena by combining the knowledge of non-equilibrium relativistic fluid dynamics and relativistic kinetic/transport theory in the description of the space-time evolution of relativistic heavy ion collisions.

Non-Newtonian nature of Causal Hydrodynamics

T. Koide

(Universidade Federal do Rio de Janeiro)

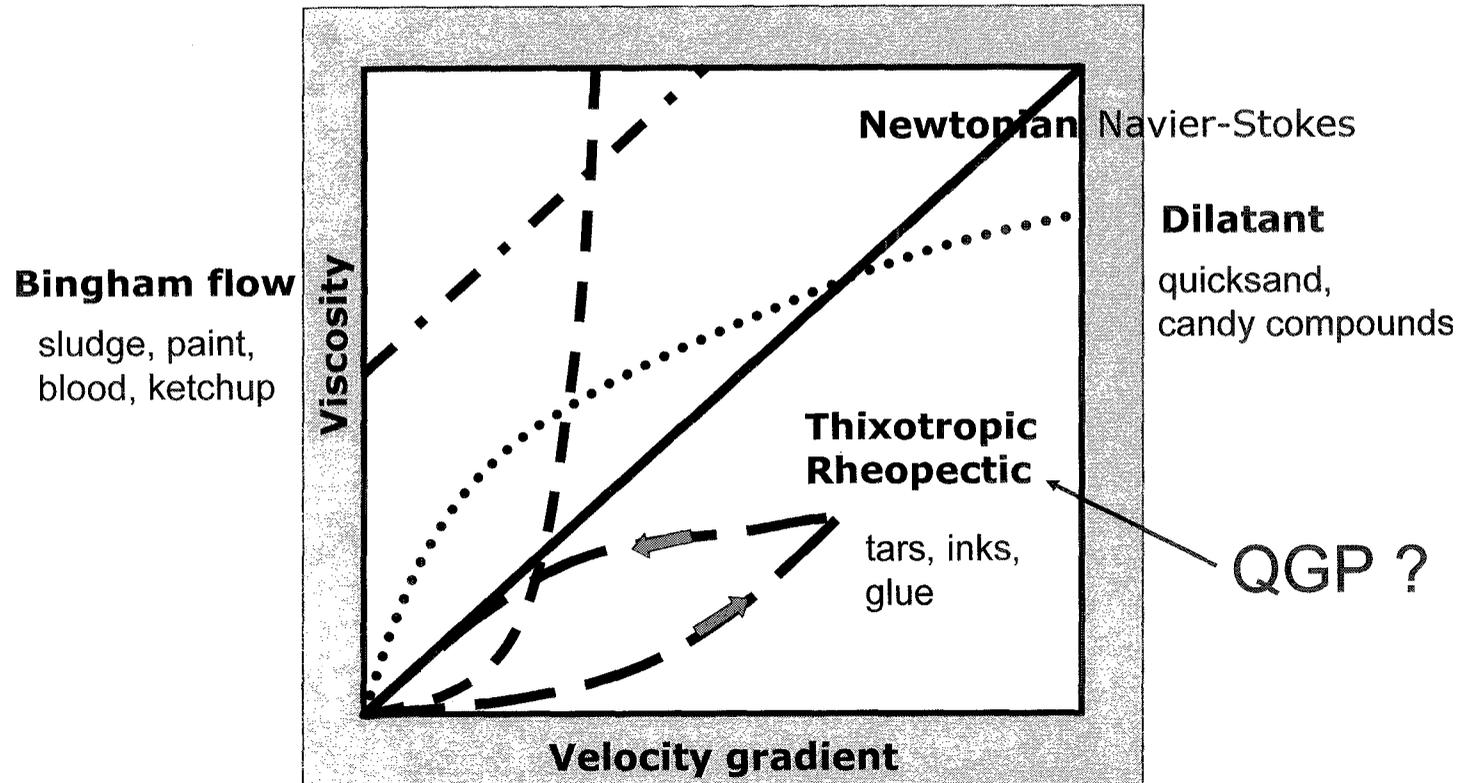
G.S. Denicol (UFRJ), T. Kodama (UFRJ), Ph. Mota (UFRJ)

Because of causality, the relativistic dissipative fluid will be a non-Newtonian fluid. Thus

- 1) The GKN formula should be modified.
- 2) $1/4\pi$ can be a lower bound of the shear of Newtonian fluids.
- 3) The fluid expands to vacuum by forming a stationary wave.
- 4) The additional viscosity is still necessary stabilize solutions.

Anomalous viscosity

Pseudoplastic latex, paper pulp, clay solns.



Generalization of GKN formula

T.K.&Maruyama (2004), T.K.(2005), T.K.(2007), T.K.&Kodama (2008)

GKN formula	$\chi_1 = \langle J(t); J \rangle$
New formula	$\chi_2 = \langle \rho(t); J \rangle$

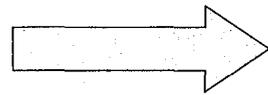
1. In the GKN formula, we need χ_1
In the generalized formula, we need χ_1 and χ_2
2. χ_2 characterizes the deviation from the GKN formula.
3. When χ_2 vanishes in the low momentum limit,
the new formula reproduces the GKN formula.
4. The result obtained in the new formula is consistent with
sum rules.

Universal relation between pressure and viscosity

We assume that the fluid forms a stationary wave at the boundary to vacuum. Then at the boundary,

$$\alpha(T^{00} + T^{11}) = vT^{01}(1 + v^2)$$

$$T^{11} = v^2 T^{00}$$



$$P = -\Pi$$

Additional causal viscosity

We need to introduce the artificial viscosity consistent with causality.

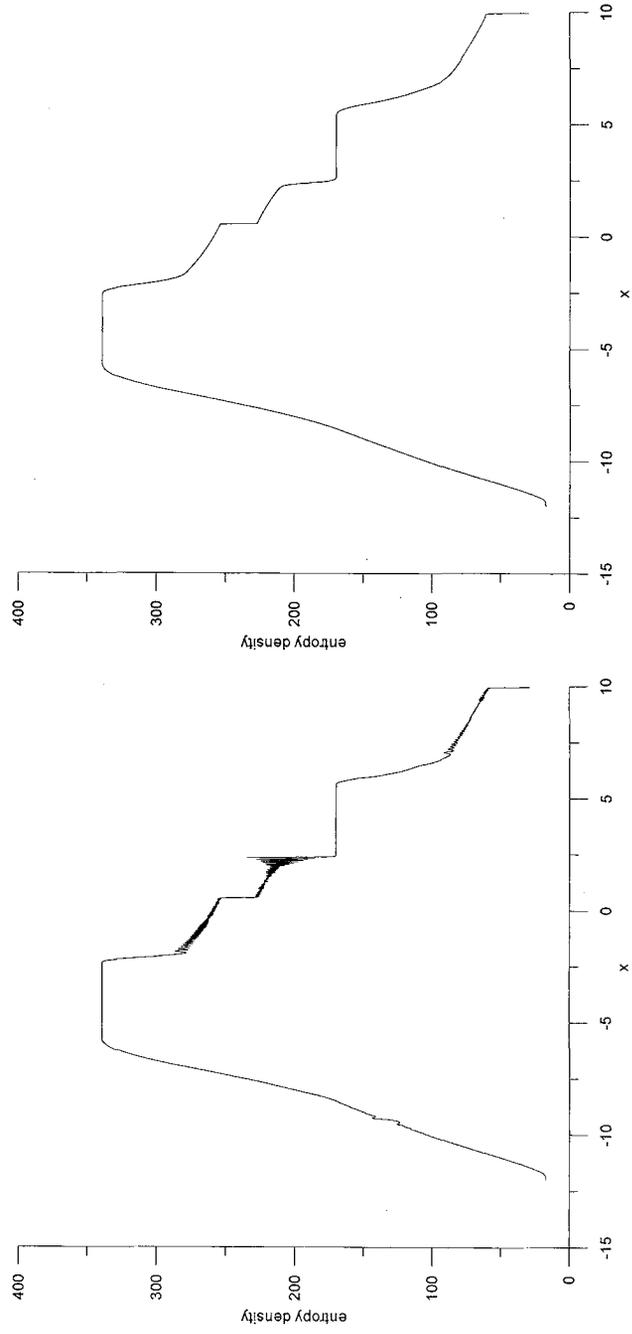
$$\Pi_{total} = \Pi + \Pi_{av}$$

$$\tau_{av} \frac{d}{d\tau} \Pi_{av} + \Pi_{av} = -\eta_{av} \partial_{\mu} u^{\mu}$$

$$\eta_{av} \propto hP$$

h :the size of the grid (0.01 fm)

Additional viscosity





Landau Model and Viscosity

G. Denicol, T. Koide, Ph. Mota and T. Kodama

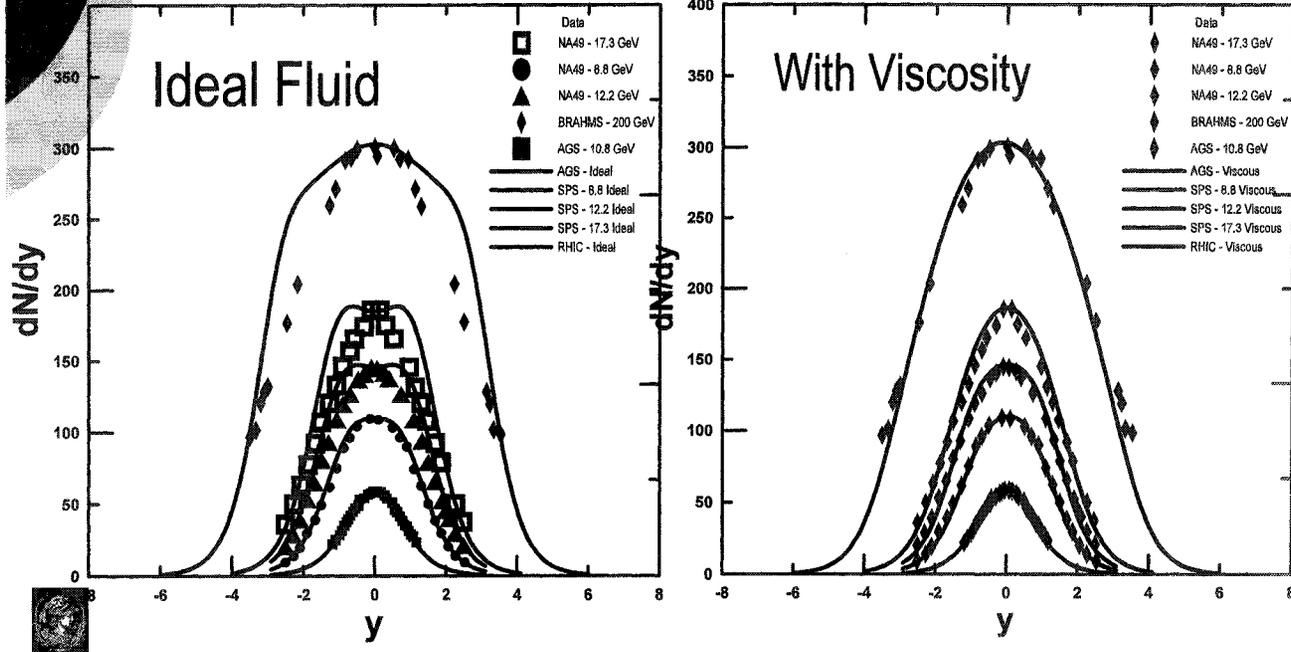
Instituto de Física - Universidade Federal do Rio de Janeiro

Abstract

Initial energy dependence of some global observables such as the total multiplicity and rapidity distribution in RHIC have been claimed to be consistent with the prediction from the Landau initial condition. Since the Landau model is essentially one dimensional, here we investigate if such initial condition can really be consistent with the observable data for more realistic 3D hydrodynamics. One possible interpretation why such a picture can be applicable just after the instant of the collision is discussed. It is pointed out that the effects of viscosity become much relevant for higher energies than RHIC.

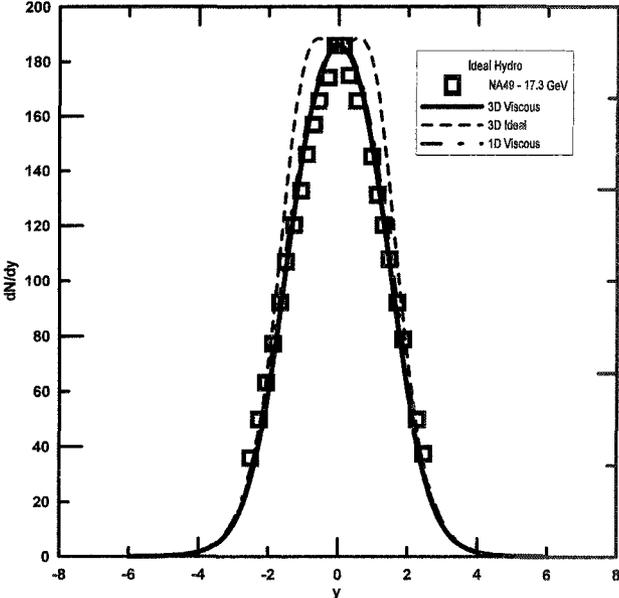


Landau Initial Condition (full stopping)
1D + thermal freezeout at $T=170$ MeV



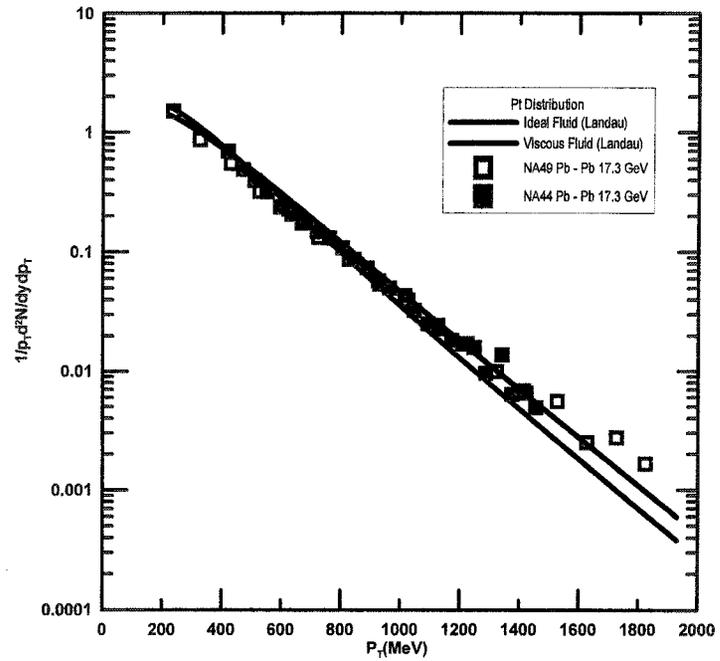
Rapidity with viscosity (3D)

SPS 17.3GeV





π -Transverse Momentum Distribution NA49 17.3GeV case



Conclusion:

For a system where the longitudinal dynamics is dominant, everything works as if a hydrodynamical system, but this may not have nothing to do with the *local thermal equilibrium*. Here, any "temperature" and "entropy",

$$"T" = K^{-1} \varepsilon^{1/4}, \quad "s" = \frac{4}{3} K \varepsilon^{3/4}$$

with any value of K. And also, the above argument valid for p+p if we substitute

$$T^{\mu\nu} \Rightarrow \langle T^{\mu\nu} \rangle$$



To be understood:

Does this reflect some 'glasma' dynamics from the vacuum?
 What is the conserved quantity corresponding to 'entropy', for
 $\text{Tr}(T)=0$,

$$T = K^{-1} \varepsilon^{1/4}, \quad s = \frac{4}{3} K \varepsilon^{3/4}$$

- Further studies such as v_2 and HBT observables should be done, changing IC and EoS (see L. M. Satarov, I. N. Mishustin, A. V. Merdeev and H. Stoecker, PHYS. REV. C **75**, 024903 (2007)). Also investigate the shear effect.

Interesting question: Study the Event-by-Event fluctuations of rapidity distribution varying the system size. See the role of τ (fluctuation-dissipation)

How to deal with the dynamics of baryon number?

How will be in LHC energies?



Viscosity and its effect on elliptic flow and thermal dileptons

Kevin Dusling

Department of Physics & Astronomy, State University of New York, Stony Brook, NY 11794-3800, U.S.A.

I present on the recent simulations of a viscous hydrodynamical model of non-central Au-Au collisions in 2+1 dimensions, assuming longitudinal boost invariance. The model fluid equations were proposed by Ottinger and Grmela. Freezeout is signaled when the viscous corrections become large relative to the ideal terms. Then viscous corrections to the transverse momentum and differential elliptic flow spectra are calculated. When viscous corrections to the thermal distribution function are not included, the effects of viscosity on elliptic flow are modest. However, when these corrections are included, the elliptic flow is strongly modified at large p_T . We also investigate the stability of the viscous results by comparing the non-ideal components of the stress tensor (π^{ij}) and their influence on the v_2 spectrum to the expectation of the Navier-Stokes equations ($\partial_j \pi^{ij} = -\eta \nabla^2 u_j$). We argue that when the stress tensor deviates from the Navier-Stokes form the dissipative corrections to spectra are too large for a hydrodynamic description to be reliable. For typical RHIC initial conditions this happens for $\eta/s \sim 0.3$.

In the second part of this presentation I discuss the first correction to the leading order $q=q$ dilepton production rates due to shear viscosity in an expanding gas. The modified rates are integrated over the space-time history of a viscous hydrodynamic simulation of RHIC collisions. The net result is a *hardening* of q_{\perp} spectrum with the magnitude of the correction increasing with invariant mass. We argue that a thermal description is reliable for invariant masses less than $M_{\max} \approx (2\tau_0 T_0^2)/(\eta/s)$. For reasonable values of the shear viscosity and thermalization time $M_{\max} \approx 4.5$ GeV. Finally, the early emission from a viscous medium is compared to emission from a longitudinally free streaming plasma. Qualitative differences in q_{\perp} spectrum are seen which could be used to extract information on the thermalization time, viscosity to entropy ratio and possibly the thermalization mechanism in heavy-ion collisions.

K. Dusling and D. Teaney, "Simulating elliptic flow with viscous hydrodynamics,"
Phys. Rev. C 77, 034905 (2008)
[arXiv:0710.5932 [nucl-th]].

K. Dusling and S. Lin, "Dilepton production from a viscous QGP,"
arXiv:0803.1262 [nucl-th].

April 21, 2008

Relativistic Navier-Stokes Equations (RNSE)

- RNSE difficult to solve
 - Unstable modes [Hiscock and Lindblom, PRD 31, 725 (1985).]
 - Violates Causality

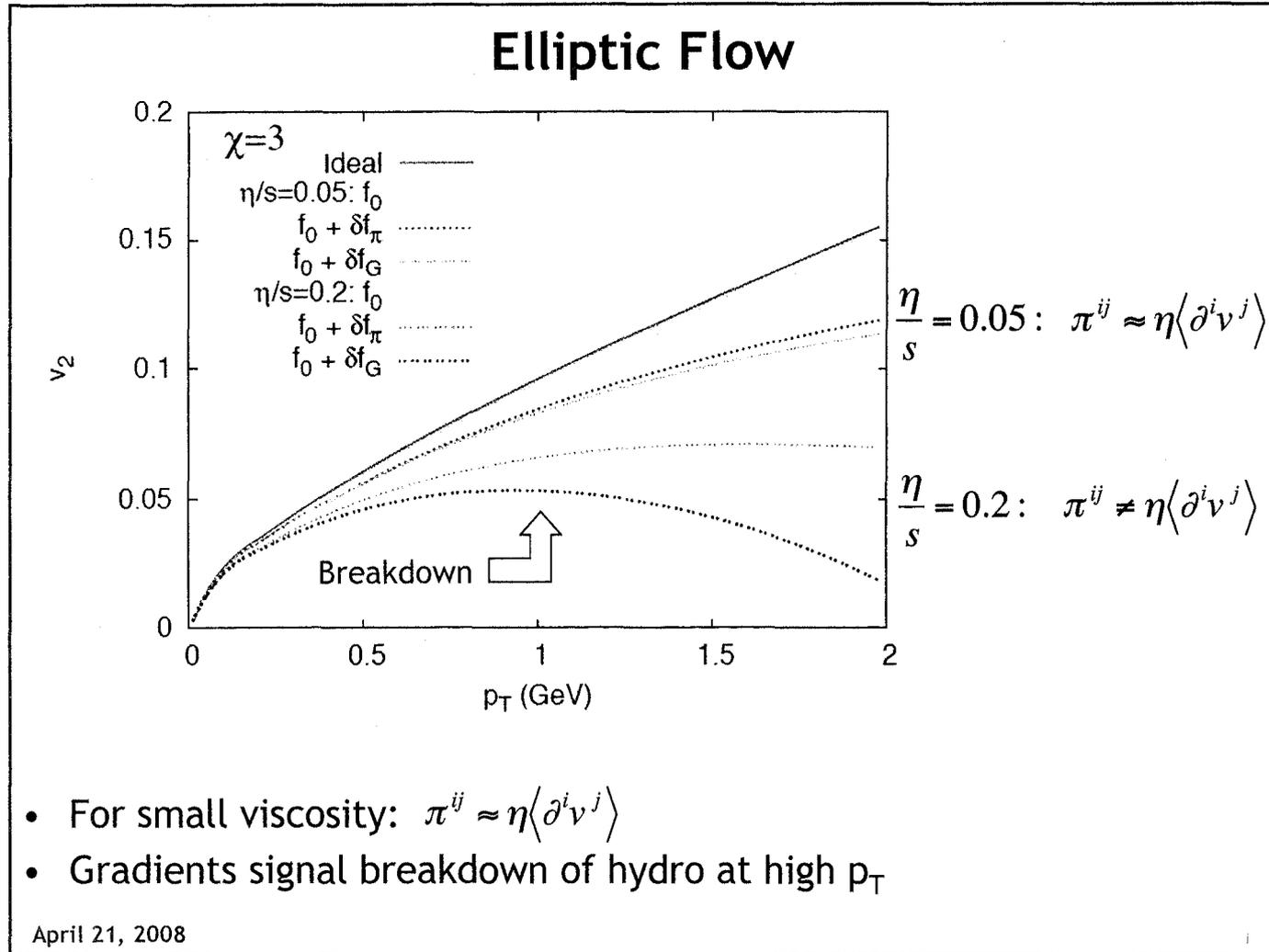
- RNSE stress tensor changes instantly

$$T_{vis}^{ij} \Big|_{\text{instantly}} = \eta \left(\partial^i v^j + \partial^j v^i - \frac{2}{3} \delta^{ij} \partial_i v^i \right)$$

- There are a number of models which relax to RNSE

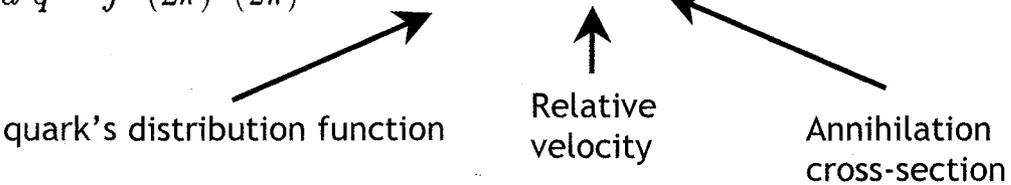
$$T_{vis}^{ij} \Big|_{\omega \rightarrow 0} \sim \eta \left(\partial^i v^j + \partial^j v^i - \frac{2}{3} \delta^{ij} \partial_i v^i \right)$$

- These models should agree with each other and with RNSE when hydrodynamics is applicable
 - Made Precise by Lindblom



Dilepton Production

- Look at qq annihilation

$$\frac{dN}{d^4q} = \int \frac{d^3\mathbf{k}_1}{(2\pi)^3} \frac{d^3\mathbf{k}_2}{(2\pi)^3} f(E_1, T) f(E_2, T) v_{12} \sigma(M^2) \delta^4(q - k_1 - k_2)$$


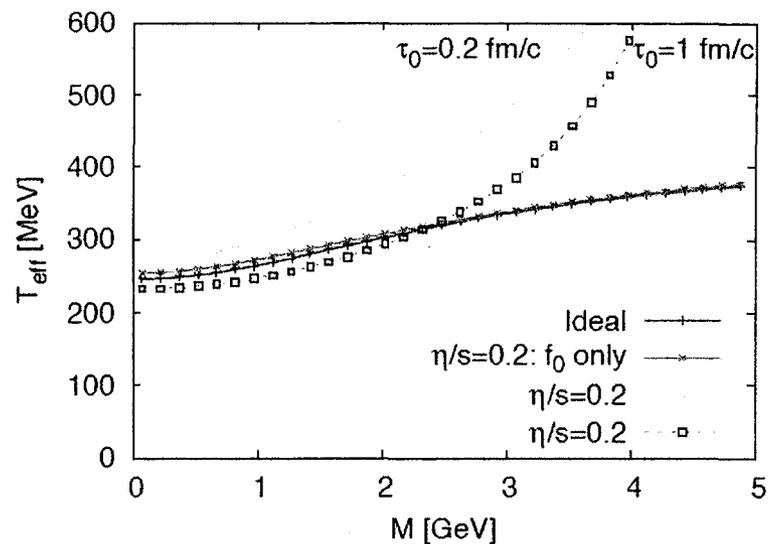
quark's distribution function
Relative velocity
Annihilation cross-section

- Replace quark distribution with viscosity modified:

$$f(p) \rightarrow f(p) + \frac{C_1}{2(\epsilon + p)T^2} f(p)[1 - f(p)] p^\alpha p^\beta \pi_{\alpha\beta}$$

Dilepton Effective Temperature

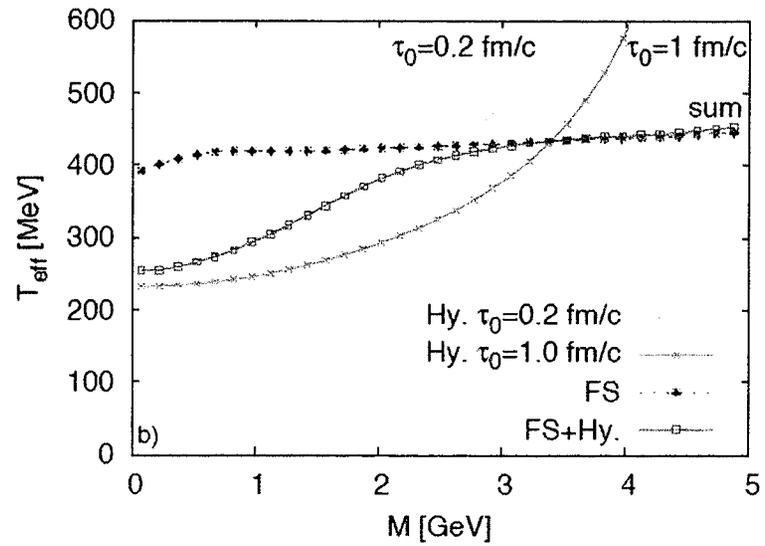
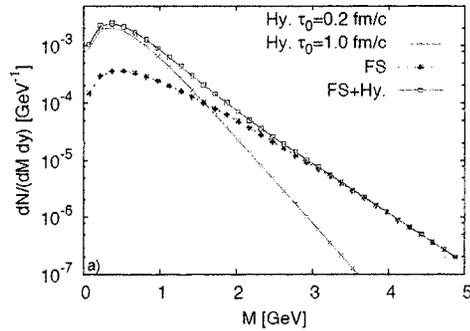
- Fit transverse mass spectrum to: $\frac{dN}{m_T dm_T} \propto \exp\left(-\frac{m_T}{T_{eff}}\right)$



- Without δf , viscous corrections are negligible
- Viscosity and thermalization time set mass limit on thermal dilepton production

$$M_{Max} \approx \frac{2\tau_0 T_0^2}{\left(\frac{\eta}{s}\right)}$$

T_{eff}: Free Streaming vs. Early Viscosity



$$M_{\text{Max}} \approx \frac{2\tau_0 T_0^2}{\left(\frac{\eta}{s}\right)} \approx \begin{cases} 4.5 \text{ GeV} & \text{for } \tau_0 = 1.0 \text{ fm/c} \\ 2.0 \text{ GeV} & \text{for } \tau_0 = 0.2 \text{ fm/c} \end{cases}$$

Viscous hydrodynamics in different Israel-Stewart formalisms

Huichao Song¹ and Ulrich Heinz^{1,2}

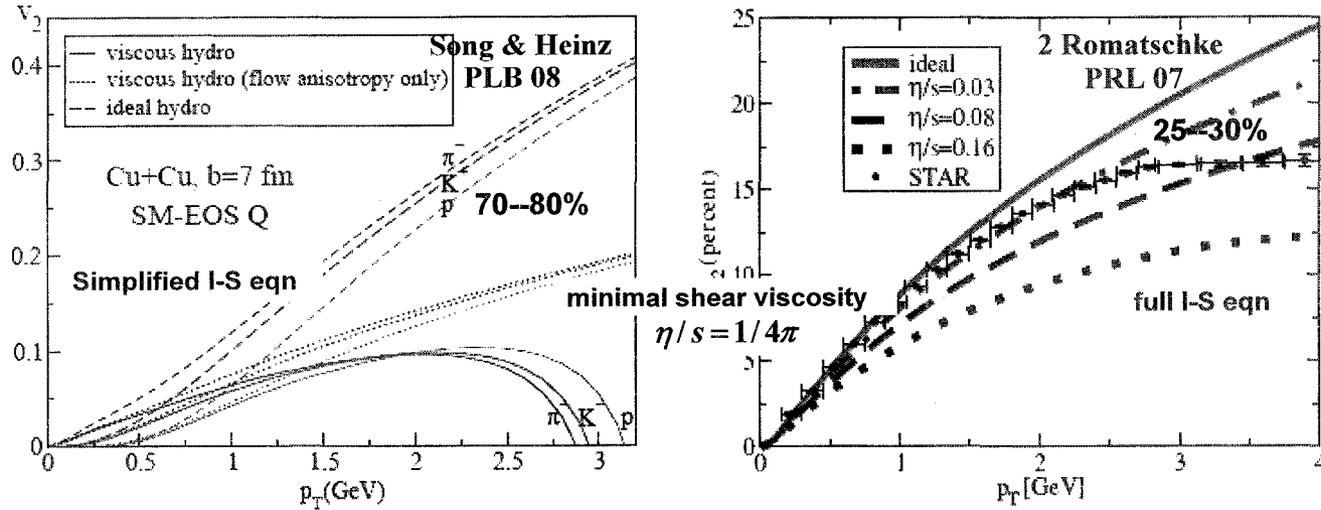
1) The Ohio State University 2) CERN

Abstract

With the efforts from different groups, the elliptic flow has now been widely accepted as the key observable to constrain the QGP shear viscosity. With the availability of several independently developed causal viscous hydrodynamic codes, we are at the threshold for a first attempt to extract the QGP shear viscosity from experiment. However, several issues that must be clarified before we do so. These include: 1) verification of the viscous hydro codes independently developed by different groups; 2) solving the ambiguities between different 2nd order formalisms: 2a) simplified Israel Stewart (I-S) formalism vs. full I-S formalism, 2b) I-S formalism vs. Öttinger-Grmela formalism; 3) the effects from 3a) system size, 3b) EoS, 3c) freeze-out procedures. Several of these issues (1, 2a, 3a,3b) have been investigated by us, and results are reported in this talk. The others require collaboration among the different groups in future studies.

Comparison with Romatschke 07 results

-elliptic flow v_2 is sensitive to even minimal shear viscosity



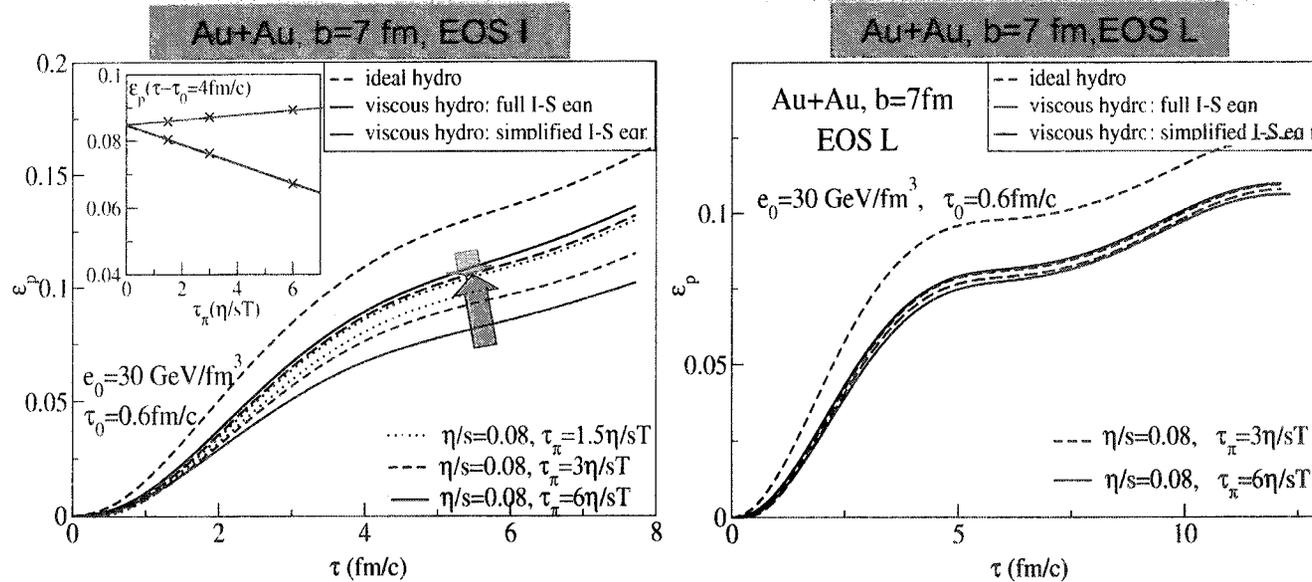
-different systems & EoS: Cu+Cu, b=7, SM-EOS Q vs. Au+Au, min bias, EOS Lattice

-different Israel-Stewart eqns. used: simplified I-S eqn. vs. full I-S eqn.

$$\Delta^{\mu\alpha} \Delta^{\nu\beta} D \pi_{\alpha\beta} = -\frac{1}{\tau_\pi} \left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu} \right] \quad \text{simplified I-S eqn}$$

$$\Delta^{\mu\alpha} \Delta^{\nu\beta} D \pi_{\alpha\beta} = -\frac{1}{\tau_\pi} \left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu} \right] + \frac{1}{2} \pi^{\mu\nu} [5D \ln T - \nabla_\alpha u^\alpha] - 2\pi^{\alpha(\mu} \omega_{\alpha}^{\nu)} \quad \text{full I-S eqn}$$

simplified I-S eqn. vs. full I-S eqn.

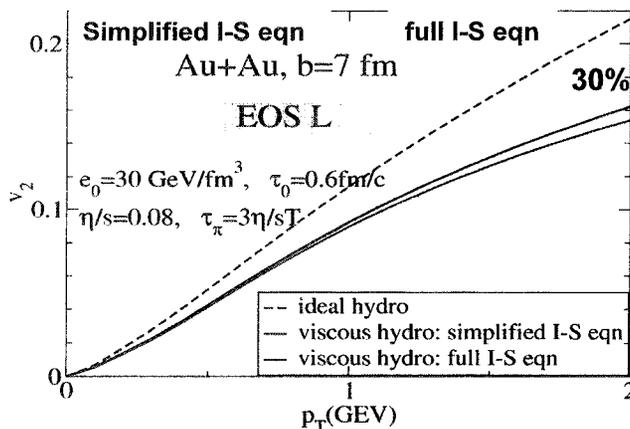
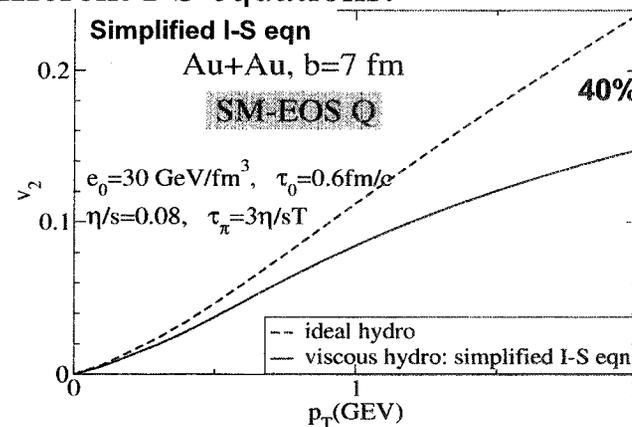
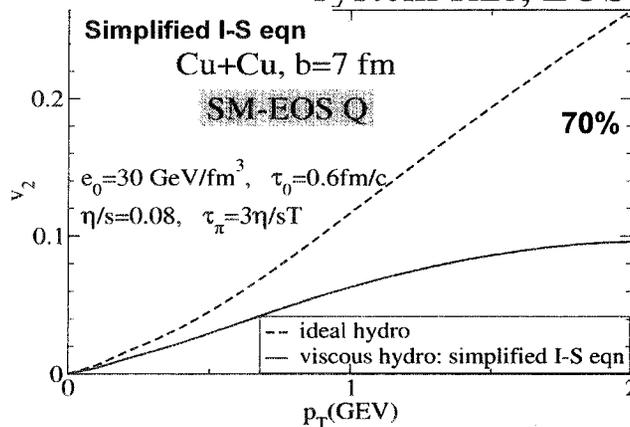


Momentum anisotropy evolution: simplified I-S eqn. vs. full I-S eqn. with different τ_π :

- for EOS I (conformal fluid) the effects from different I-S eqns are much larger (20-50% depending on initial energy, system size etc.), but will also vanish in the limit $\tau_\pi \rightarrow 0$
- for realistic EOS with a phase transition, the difference between simplified and full I-S eqns. for the viscous suppression of v_2 are small if the systems are not too small and the initial energy density is not too low

Different effects contributing to v_2 suppression

system size, EOS, different I-S equations:



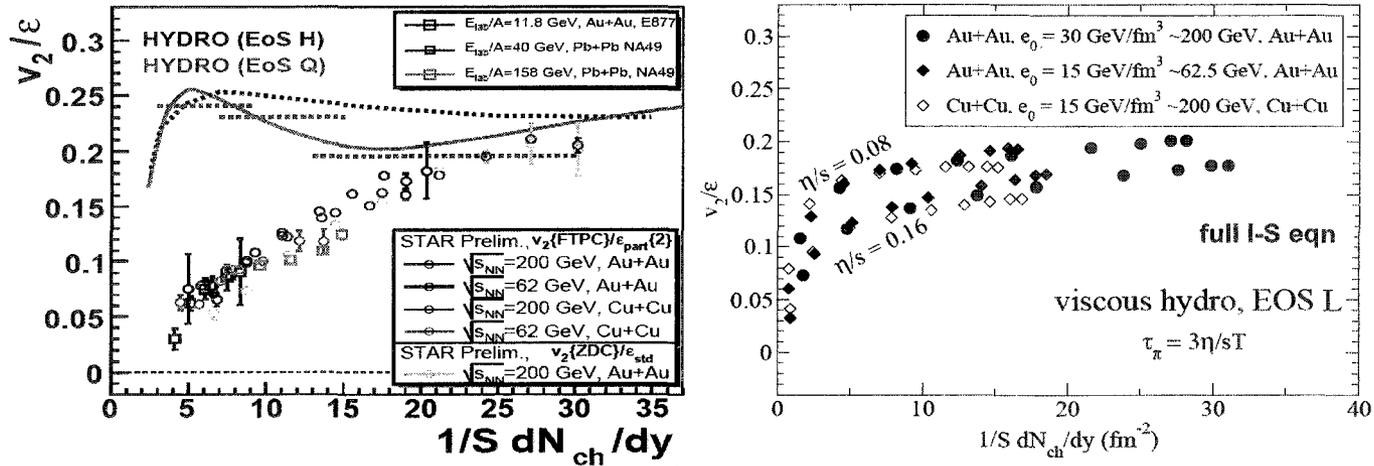
-system size: CuCu b=7fm vs. AuAu b=7fm:
20-30% effect

-EOS: SM-EOS Q vs. EOS L: ~10% effect

-different I-S eqns: simplified I-S eqn. vs. full I-S eqn.: ~5% effects (EOS Q and EOS L only)

Considering all of these effects, the final suppression of v_2 for Au+Au with EOS L and the full I-S eqn, for minimal shear viscosity $\eta/s=0.08$, is ~25%, approaching the results of P. & U. Romatschke (PRL'07).

Multiplicity scaling of v_2/ϵ *Preliminary*



101

- experimental data show qualitatively similar fine ordering as viscous hydro prediction (larger viscous effects in smaller systems and lower collision energies)
- to reproduce slope of v_2/ϵ vs. $(1/S)dN/dy$, a better description of the highly viscous hadronic stage is needed: viscous hydro + hadron cascade
- the experimental v_2/ϵ vs. $(1/S)dN/dy$ scaling (slope and fine structure) is another good candidate to constrain η/s (insensitive to Glauber-type vs. CGC initialization)
- this requires, however, experimental and theoretical improvements: reduced error bars, accounting for T -dependence of η/s , ζ/s near T_c , modeling hadronic phase with realistic cascade

Summary and discussion

- v_2 is sensitive to η/s
- multiplicity scaling of v_2/ε is a good candidate to extract the QGP viscosity:
 - larger viscous effects in smaller systems and at lower collision energies
 - multiplicity scaling of v_2/ε is insensitive to Glauber model vs. CGC initialization.

To extract QGP viscosity, one needs to consider at least the following aspects:

- resolve the ambiguities between different 2nd order formalisms used by different groups to simulate causal viscous hydrodynamics
 - a) simplified I-S eqn. (Song & Heinz 07-08) vs. full I-S eqn. (P.&U.Romatschke)
 - approach same Navier-Stokes limit as $\tau_\pi \rightarrow 0$
 - for non-conformal fluids (EOS Q & EOSL), both eqns. are OK (~5-10% diff.)
 - for conformal fluids (EOS I) we should use full I-S eqn. (which preserves conformal symmetry)
 - b) I-S formalism vs. Öttinger-Grmela (O-G) formalism (Dusling & Teaney) ~? %
- a realistic EOS: EOS L vs. SM-EOS Q ~10% (for v_2 and v_2/ε)
- initial conditions: CGC initialization vs. Glauber initialization ~15-30% (for v_2)
- bulk viscosity: with vs. without bulk viscosity ~?%

... ..

Parton Cascade with Yang-Mills fields

arXiv:0710.1223

Adrian Dumitru
Johann Wolfgang Goethe University
Frankfurt am Main

Collaborators: Y. Nara, B. Schenke, M. Strickland

- Collision term and (color-) Lorentz force, separation scale k^*
- Observable: jet transverse momentum broadening, \hat{q}
- Independent of lattice spacing and k^*

Boltzmann Eqn with self-consistent screening?

♦ screening mass:

$$m^2 = \frac{3\alpha_s}{\pi^2} \int d^3p \frac{f(p)}{p^0} \sim \alpha_s \frac{n_g}{p}$$

♦ elastic cross-section:

$$\frac{d\sigma}{dt} \sim \frac{\alpha_s^2}{(t - m^2)^2}$$

BUT:

- LL accuracy not good enough
- $(p^2 g^{\mu\nu} + p^\mu p^\nu + \Pi^{\mu\nu})$ exhibits unstable modes

• p_\perp broadening of hard particle in thermal medium:

elastic 2→2 collisions, LL approximation:

$$\hat{q} = \frac{d\langle p_\perp^2 \rangle}{dt} \sim g^4 n_g \log\left(C \frac{p}{m}\right)$$

take $p/m=10$: for $C = 2, 1, 0.5$: $\log() = 3.0, 2.3, 1.6$

→ expect strong sensitivity to cutoff !

Boltzmann-Vlasov-Yang-Mills Theory

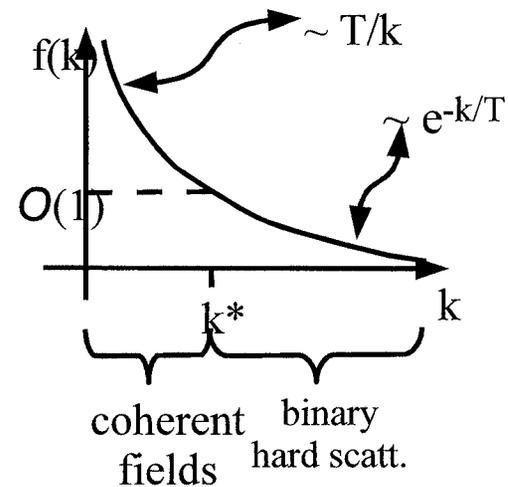
$$p^\mu \left[\partial_\mu - gQ^a F_{\mu\nu}^a \frac{\partial}{\partial p_\nu} + g f^{abc} A_\mu^b Q^c \frac{\partial}{\partial Q^a} \right] f = C[f]$$

$$(D_\mu F^{\mu\nu})^a = J^{a\nu}$$

- ◆ Soft exchange ($q < k^*$) via fields (Lorentz force)
- ◆ Hard exchange ($q > k^*$) via collision term
(2→2 elastic)
- ◆ NO CUTOFF

$$\rightarrow k^* = \pi/a \sim T$$

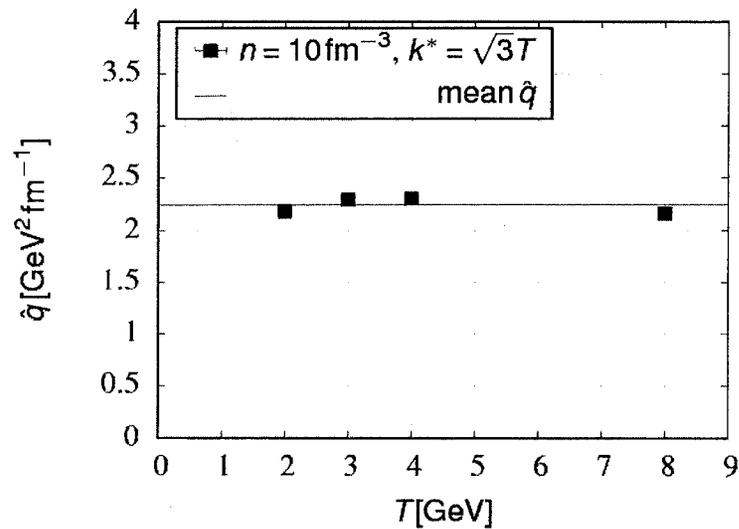
$$C[f] : \sigma(k^*) = \int_{k^*-2}^{s/2} dq^2 \frac{d\sigma}{dq^2}$$



• Continuum limit: $\frac{\pi}{a} \sim T$

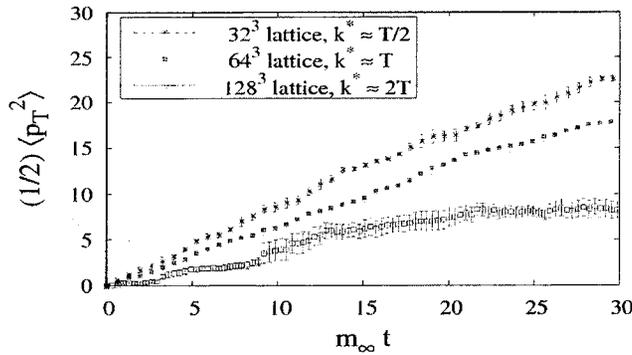
$$m_\infty a \ll 1 \quad \leftrightarrow \quad g^2 \frac{n_g}{T^3} \ll 1$$

for $g \sim 1$: $T \gg n_g^{1/3}$, should be ok for $\hat{q} \sim g^4 n_g \log \left(C \frac{p}{m} \right)$

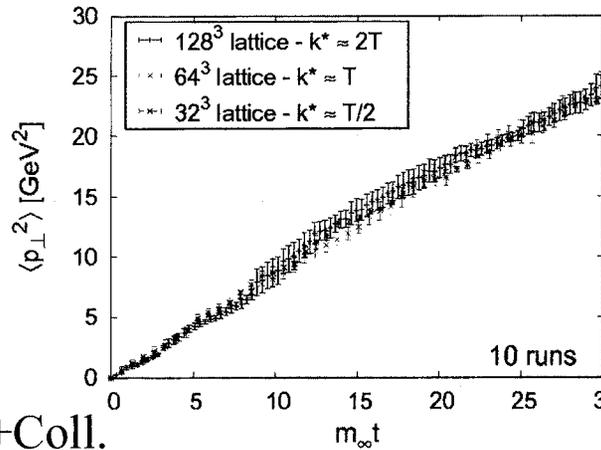
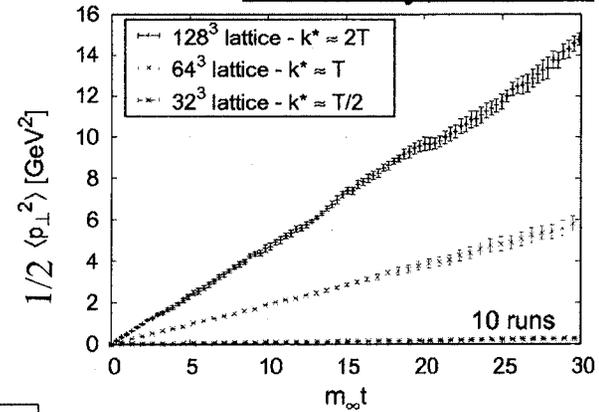


• p_{\perp} broadening in thermal SU(2) plasma: **arXiv:0710.1223**

Coll. only, no YM



Field only, no Coll.



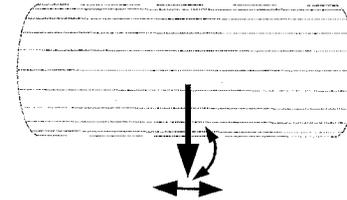
Field+Coll.

Independent of k^* !

$$\hat{q} = \frac{\langle p_{\perp}^2 \rangle}{t} = 2.2 \text{ GeV}^2/\text{fm}$$

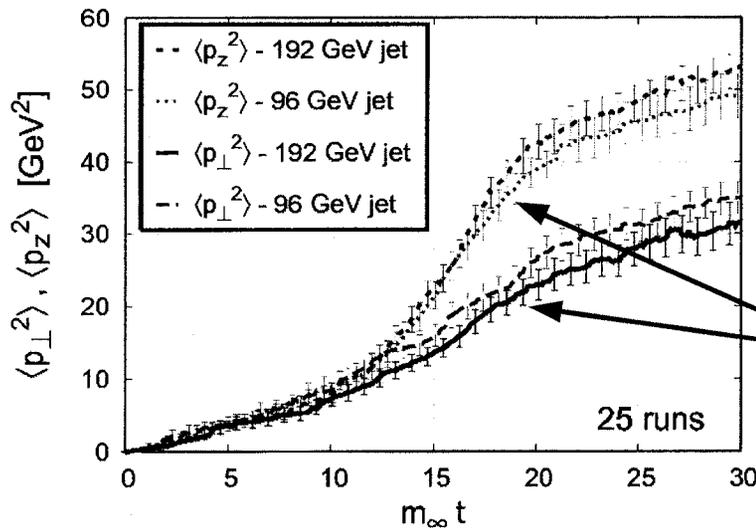
for $p_{\text{hard}}/(3T) = 5$, $n_g = 10/\text{fm}^3$

• anisotropic medium: instability...



medium: $f(\mathbf{p}) \sim \delta(p_z) e^{-p_\perp / p_{hard}}$

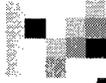
jet: $\mathbf{p} = (p_x, 0, 0)$, $p_x = 96, 192 \text{ GeV}$



$$\kappa_z \equiv \frac{d\langle p_z^2 \rangle}{dt}$$

$$\kappa_\perp \equiv \frac{d\langle p_x^2 + p_y^2 \rangle}{dt}$$

$\kappa_z / \kappa_\perp = 2.3$



Bulk Viscosity in Nuclear Collisions (And Other Remarks)

Rainer J. Fries (Texas A&M and RIKEN/BNL)

in collaboration with B. Müller, A. Schäfer

The initial conditions are an important ingredient of hydrodynamics calculations. Two important facts should be kept in mind. 1) the longitudinal pressure is negative at very early times due to the dominant longitudinal gluon field. 2) the transverse pressure in the early, field dominated phase of the collision is sizeable and leads to an expansion (and radial flow) of the system without equilibration.

The bulk viscosity over entropy ratio ζ/s has been calculated by several groups recently for quark and gluon matter [1,2]. These calculations predict a sharp peak around the critical temperature T_c with maximum values of order $O(1)$. This might imply a sizable contribution to dissipative entropy production around T_c . Furthermore, one can speculate that if dissipative effects are important around T_c , details of the equation of state, like the order of the phase transition, might be much more important than in ideal hydrodynamic calculations. We test this effect in a simple 0+1 dimensional hydrodynamic model. The longitudinal flow is fixed at Bjorken values and we use 2nd order Israel-Stewart equations with both shear viscosity ($\eta/s = 1/4\pi$ is kept fixed) and bulk viscosity. We use two scenarios. One utilizes a recent equation of state from lattice QCD exhibiting a smooth cross over [3], while the second one resembles a 1st order phase transition. The value of ζ/s as a function of temperature is the same in both scenarios and follows the work by Meyer [2].

Our preliminary results show a moderate contribution to the total entropy production from the bulk pressure, while the changes to the longitudinal pressure are quite dramatic. In the scenario with crossover the longitudinal pressure stays around 50% of the value of the equilibrium pressure during the entire QGP phase, while the same values of ζ/s even lead to negative values of the longitudinal pressure for a 1st order phase transition. This might have profound consequences for the applicability of viscous hydrodynamics around the phase transition and for the longitudinal expansion of the fireball.

[1] Kharzeev, Tuchin (2007); Karsch, Kharzeev, Tuchin (2007) [2] Meyer (2007) [3] Cheng et al. (2007)



Rainer Fries

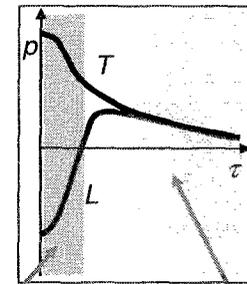
Bulk Viscosity

Initial Pressure

- n System starts with maximum pressure anisotropy.
- n Negative longitudinal pressure
- n Large transverse pressure
 - Leads to early transverse flow

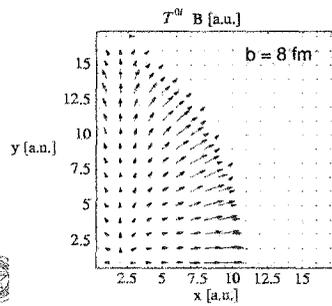
$$T^{0i} = -\frac{\tau}{2} \nabla^i \epsilon_0 \quad \longrightarrow \quad v_r = \frac{3}{2} \tau \frac{1}{\epsilon_0} \frac{\partial \epsilon_0}{\partial r}$$

- No equilibration necessary for flow



Longitudinal Field

(Ideal) plasma



$$T_{t(0)}^{\mu\nu} = \begin{pmatrix} \epsilon_0 & & & \\ & \epsilon_0 & & \\ & & \epsilon_0 & \\ & & & -\epsilon_0 \end{pmatrix}$$

$$T_{\text{equ}}^{\mu\nu} = \begin{pmatrix} \epsilon & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$$

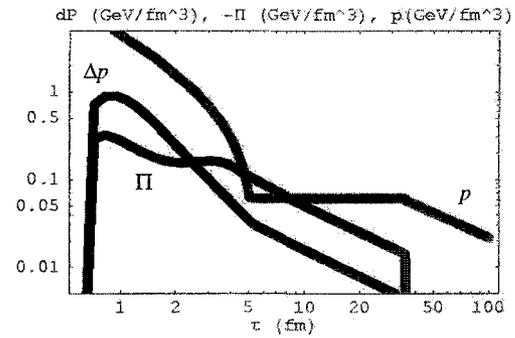
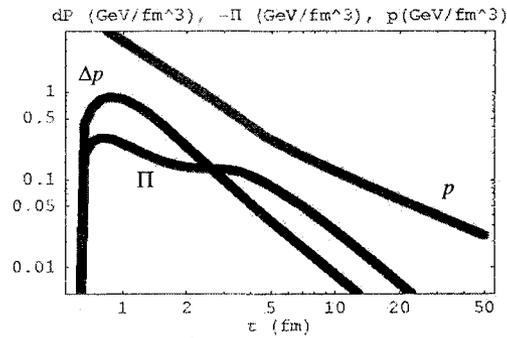
Results: Longitudinal Pressure

I: Lattice

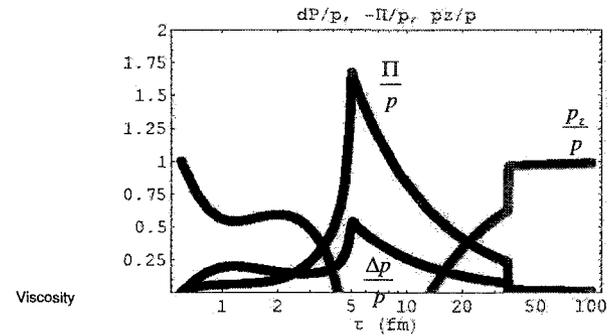
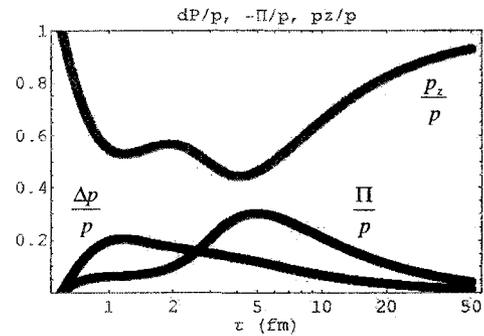
$$c_\zeta = 1$$

II: 1st order

n Pressure, bulk pressure and shear



n Relative long. pressure, bulk pressure and shear



III

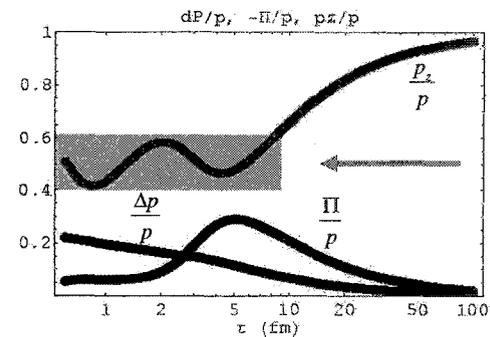
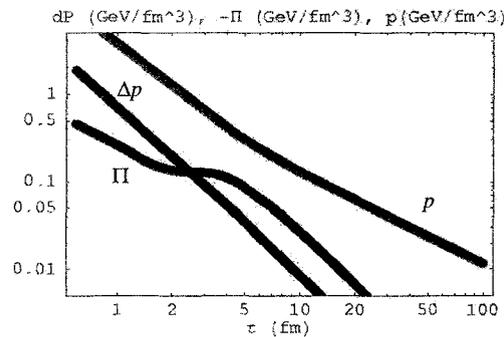


Viscosity

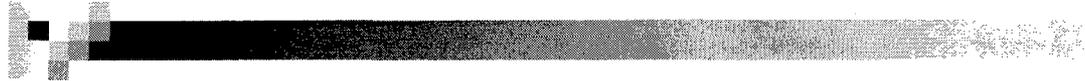


Longitudinal Pressure Revisited

- n Longitudinal pressure with “reasonable” initial conditions.
 - n I.e. $\Delta p(\tau)$ and $\Pi(\tau)$ are smooth functions around $\tau = \tau_0$:

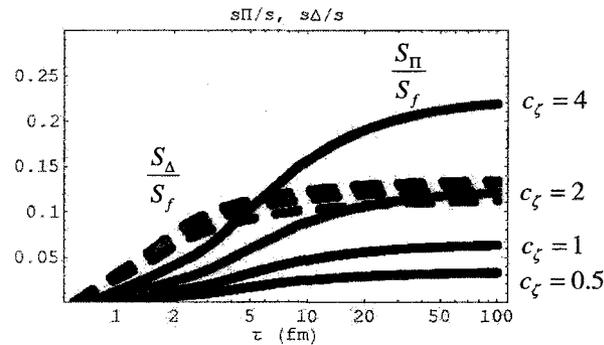


- n Even with $c_\zeta = 1$ longitudinal pressure $p_z \sim 1/2 p$ during entire QGP phase.
- n Observable consequences?



Results: Entropy

- n So far: $c_\zeta = 1$; now vary bulk viscosity using model I.
- n Entropy τ s: shear + bulk contributions relative to final entropy



- n In this hydro model $\zeta/s_{\max} \sim 0.4$ produces roughly as much entropy as $\eta/s = 1/(4\pi)$ over the lifetime of the fireball.
- n Caution: half of S_Π comes from $T < T_c$; need realistic ζ_{had} .





Summary

- n Importance of Initial Conditions
 - α Evolution of longitudinal pressure.
 - α Transverse pressure = early flow, also v_2 .
 - α No equilibration necessary for initial flow.

- n Interesting interplay bulk viscosity \leftrightarrow phase transition/equation of state.
 - α Sharp phase transition and/or large $\zeta(T_c)$ might lead to complete breakdown of p_z
 - α Applicability of (2nd order) hydrodynamics around the phase transition?

- n Our model I (“Lattice”): ζ important for p and s , but doesn’t overwhelm entropy production from η .

Comparing viscous hydrodynamics to a parton cascade

Pasi Huovinen

with Denes Molnar

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phuovine@purdue.edu

We gauge the validity and applicability of Israel-Stewart hydrodynamics to heavy-ion collisions by comparing the results calculated using viscous hydrodynamics and Boltzmann transport calculation. We use boost-invariant hydrodynamical model where the expansion in the two dimensions of the transverse plane is calculated numerically.

We find that the viscous hydrodynamics and transport lead to almost identical time-evolution of the components of the energy momentum tensor at the core of the system and the calculated elliptic anisotropy of the particles is very similar. This success requires that all the terms in the evolution equation, which result from the requirement of non-decreasing entropy, must be included in the calculation as previously advocated by Azwinndini Muronga and Paul Romatschke. These terms are also required to guarantee that entropy cannot decrease in any circumstances.

We conclude that the prospects of applying Israel-Stewart hydrodynamics to the description of heavy-ion collisions are promising. We expect to see 20-30% reduction in elliptic flow due to the postulated minimum shear viscosity $\eta = 1/(4\pi)$ compared to the ideal fluid calculation. However, the results depend somewhat on the freeze-out criterion at the end of the hydrodynamical evolution.

Viscous hydro vs transport

We solve the full Israel-Stewart-Muronga equations, including vorticity terms from kinetic theory, in a 2+1D boost-invariant scenario. Shear stress only.

Mimic a known reliable transport model:

- massless Boltzmann particles $\Rightarrow \epsilon = 3P$
- only $2 \leftrightarrow 2$ processes, i.e. conserved particle number
- $\eta = 4T/(5\sigma_{tr})$
- either $\sigma_{tr} = \text{const.} = 47 \text{ mb}$ ($\sigma_{tr} = 14 \text{ mb}$) \leftarrow the simplest in transport
or $\sigma_{tr} \propto \tau^{2/3}$ close to $\eta/s = 1/(4\pi)$

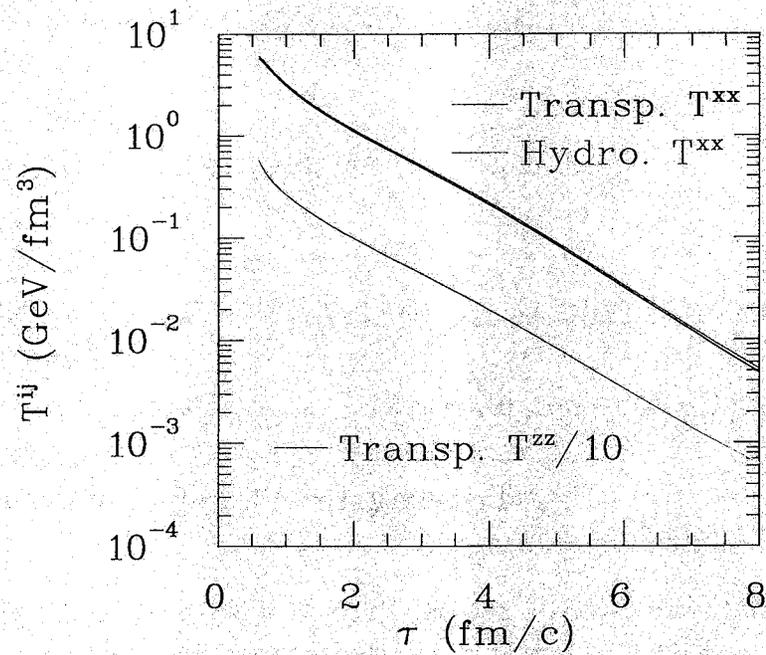
Our “RHIC-like” initialization:

- $\tau_0 = 0.6 \text{ fm}/c$
- $b = 8 \text{ fm}$
- $T_0 = 385 \text{ MeV}$ and $dN/d\eta|_{b=0} = 1000$
- freeze-out at constant $n = 0.365 \text{ fm}^{-3}$

Pressure evolution in the core

T^{xx} and T^{zz} averaged over the core of the system, $r < 1$ fm:

$$\eta/s \approx 1/(4\pi) \quad (\sigma_{\text{tr}} \propto \tau^{2/3})$$



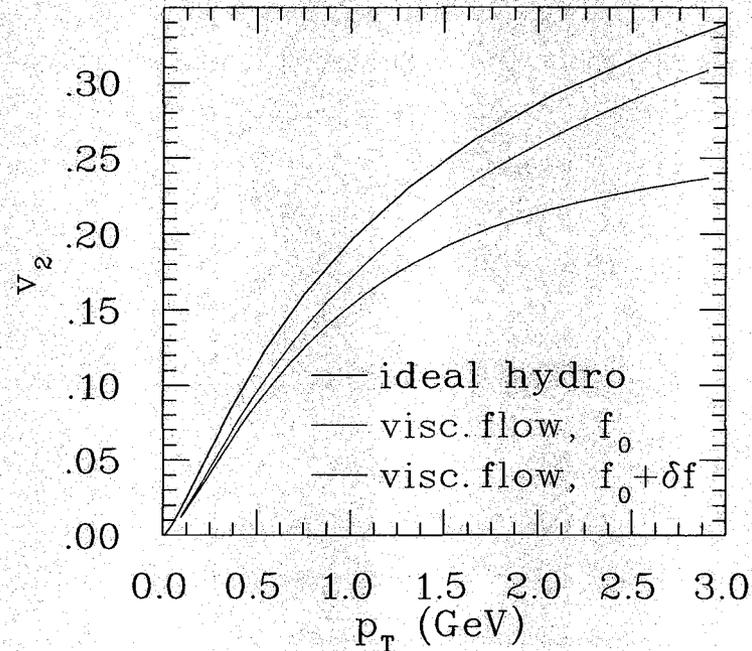
remarkable similarity!

Viscous hydro elliptic flow

- TWO effects: - dissipative corrections to hydro fields u^μ, T, n
 - dissipative corrections to thermal distributions $f \rightarrow f_0 + \delta f$

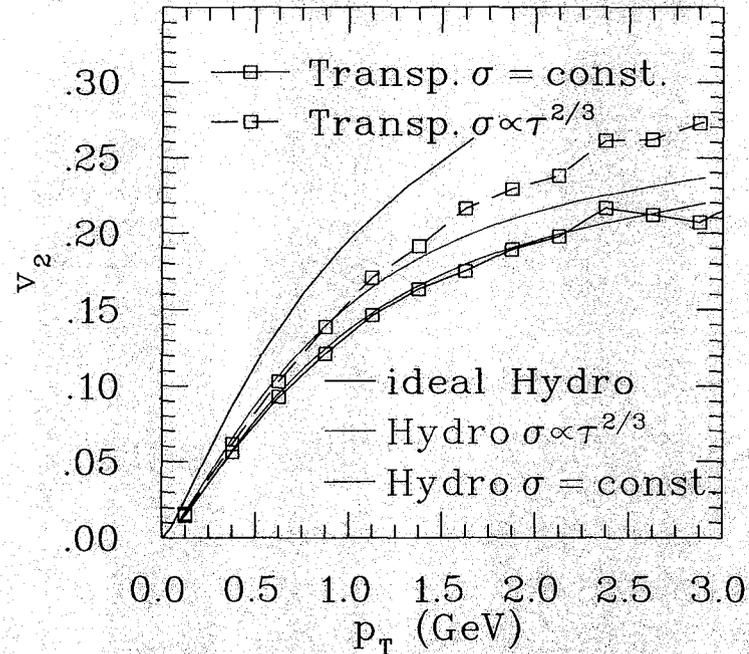
$$\eta/s \approx 1/(4\pi) (\sigma_{\text{tr}} \propto \tau^{2/3})$$

$$\delta f = f_0 \left[1 + \frac{p^\mu p^\nu \pi_{\mu\nu}}{8nT^6} \right]$$



Calculation for $\sigma_{\text{tr}} = \text{const.} \sim 15 \text{ mb}$ shows similar behaviour

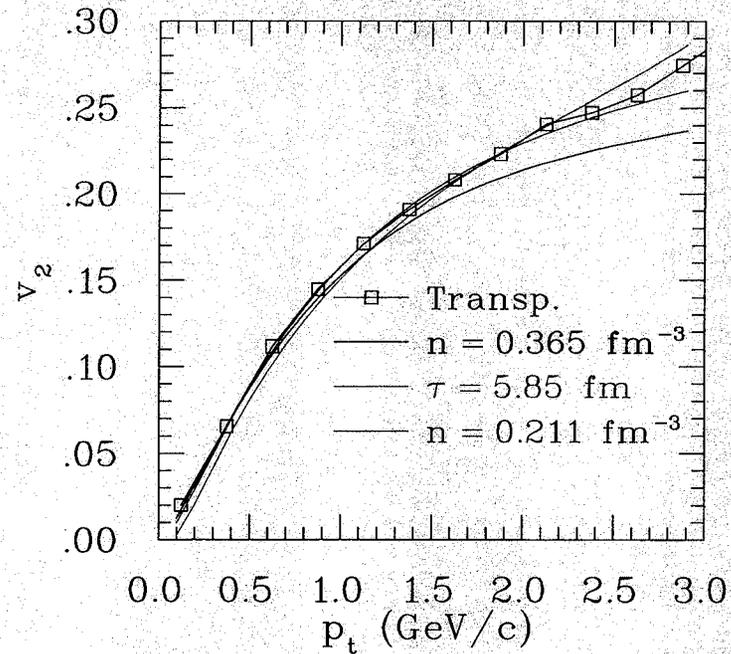
Viscous hydro vs transport v_2



- excellent agreement when $\sigma = \text{const} \sim 47 \text{ mb}$
- good agreement for $\eta/s \approx 1/(4\pi)$, i.e. $\sigma \propto \tau^{2/3}$
- BUT results sensitive to freeze-out criterion, especially at high p_T

Effect of freeze-out criterion

$$\eta/s \approx 1/(4\pi) (\sigma_{tr} \propto \tau^{2/3})$$



- some sensitivity to the freeze-out criterion
- not crucial for the results

A Critical Review of Thermalization Issue at RHIC

- Results from STAR

Aihong Tang for the STAR Collaboration

Aihong Tang
Hydro Workshop, BNL April 08



Collecting Evidences and Connecting Pieces

Scaling variables I have shown so far:

$$\frac{1}{S} \frac{dN}{dY}, \frac{dN}{d\eta}, \left(\frac{dN}{d\eta} \right)^{1/3}, N_{part}^{1/3}$$

Albeit in different formats, they are sensitive to the same quantity :

$$\frac{1}{K} \equiv \frac{R}{\lambda}$$

$$\lambda = \frac{1}{\sigma n}$$

$$n = \frac{1}{ct} \frac{1}{S} \frac{dN}{dy}$$

$$t \sim R / c_s$$

$$\frac{1}{K} = \sigma \left[\frac{1}{S} \frac{dN}{dy} \right] \frac{c_s}{c}$$

← **Number of collisions.**
Local thermal equilibrium
is achieved if $k^{-1} \gg 1$
(will come back to this in the
2nd part of this talk)

K: Knudsen number
R: system size
 λ : mean free path
n: particle density
 σ : parton cross section

Lots (but not all) of physics seem to be driven by the number of collisions per particle encountered on its way out!

In many cases a better linearity is seen if plotted against $x^{1/3}$. Proportional to $1/K$?



Choose the right $\{v_2, \varepsilon\}$ pairs

v_2 that are sensitive to anisotropy w.r.t. the Reaction Plane v_2 : $v_2\{4\}$, $v_2\{qDist\}$, $v_2\{qCumulant4\}$, $v_2\{ZDCSMD\}$	v_2 that are sensitive to anisotropy w.r.t. the Participant Plane : $v_2\{2\}$, $v_2\{EP\}$, $v_2\{uQ\}$ etc.
ε that are sensitive to anisotropy w.r.t. the Reaction Plane : $\varepsilon\{std\}$, $\varepsilon\{4\}$	ε That are sensitive to anisotropy w.r.t. the Participant Plane : $\varepsilon\{part\}$ $\varepsilon\{2\}$

In this slide, I assume that nonflow has been suppressed by external techniques (such as pseudorapidity gap etc.) in v_2 measurements that are based on two particle correlations ($v_2\{2\}$, $v_2\{EP\}$, $v_2\{uQ\}$).

R.Bhalerao and J-Y. Ollitrault, Phys. Lett. B 614 (2006) 260
S.Voloshin, A.Poskanzer, A.Tang and G.Wang, Phys. Lett. B 659 (2008) 537



How much deviation from ideal hydro ?

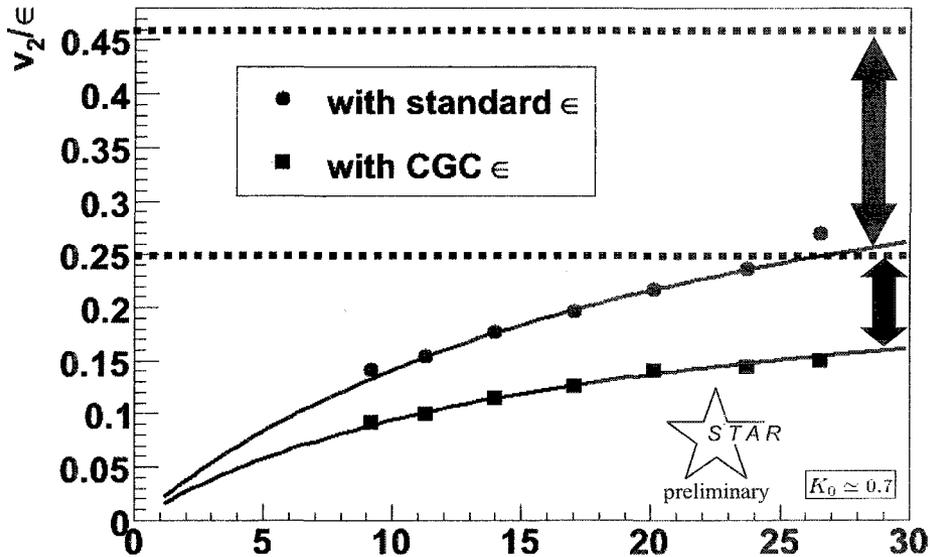
$$\frac{v_2}{\epsilon} = \left[\frac{v_2}{\epsilon} \right]_{\text{Perfect}} \frac{1}{1 + K / K_0}$$

$$K = \lambda / R$$

$$\frac{1}{K} = \frac{\sigma}{S} \frac{dN}{dy} C_s$$

For the case with
Standard ϵ :
 $\sigma=4.3\text{mb}$, $v_2/\epsilon=0.46$.
For 20-30% $K=0.85$

For the case with
CGC ϵ :
 $\sigma=5.7\text{mb}$, $v_2/\epsilon=0.25$.
For 20-30% $K=0.56$

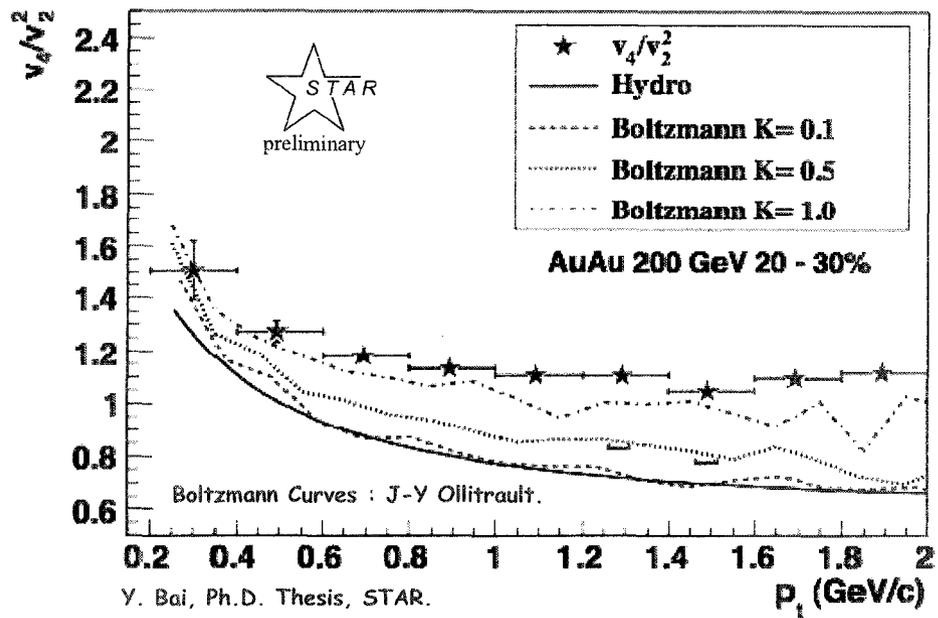


Fitting function from Drescher, Dumitru, Gombeaud, $1/S \, dN/dy$
J.Ollitrault, Phys. Rev. C76, 024905(2007)
CGC ϵ obtained from A.Adil, H-J Drescher, A.Dumitru,
A.Hayashigaki and Y.Nara, Phys. Rev. C 74 044905 (2006)

Dashed lines are hydro limit from fitting the data (as opposed to a pure theoretical calculation as adopted before)
~40% away from ideal hydro even in central collisions



How much deviation from ideal hydro ?



An improved analysis since QM06
Considerable deviation from ideal Hydro
Hints of incomplete thermalization ?

An Inconvenient Truth

(not really related to global warming)

- Many physics are driven by the Knudsen number, which when small, a thermal equilibrium is considered reached. While it is generally accepted that Hydrodynamics did a good job, for the first time, in describing RHIC's data, there are features that are not consistent with a complete thermalization, and they cannot be easily dismissed.

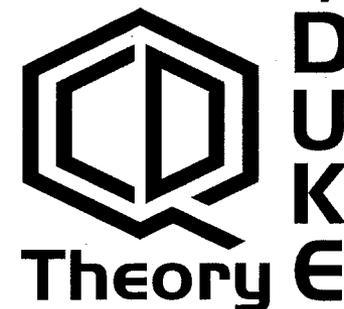


Hadronic Transport Coefficients from a Microscopic Transport Model

Nasser Demir

In collaboration with: Steffen A. Bass

Summary:



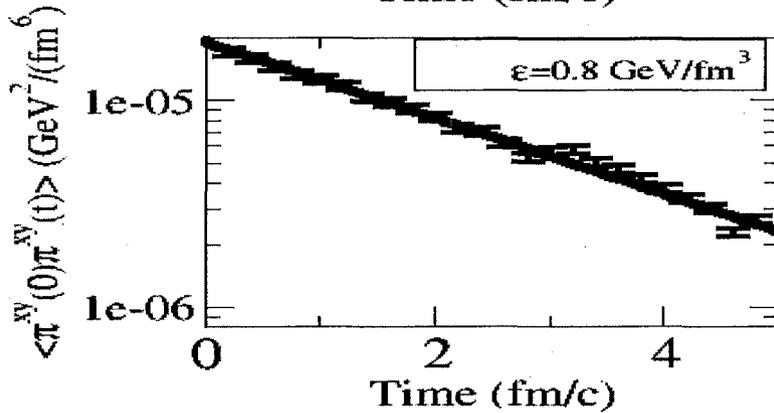
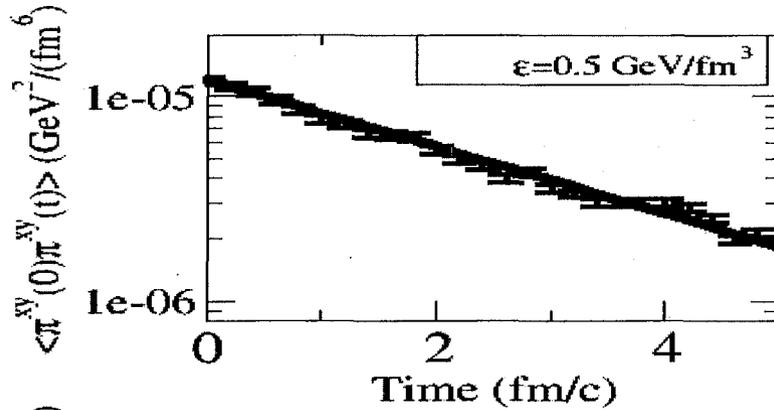
Ultrarelativistic heavy ion collisions at RHIC are thought to have created a Quark Gluon Plasma (QGP) with a low shear viscosity in the deconfined phase. However, as the QGP hadronizes it will evolve through a hadronic phase with rapidly increasing viscosity. In order to constrain the viscosity of the QGP state, one has to separately determine the viscosity of the hadronic phase. We present a calculation of the shear viscosity as a function of temperature and baryon number density for nuclear densities in the range $(0-2\rho_0)$. The hadronic medium is simulated using the Ultrarelativistic Quantum Molecular Dynamics (UrQMD) model in a box with periodic boundary conditions. The Kubo formalism is used to extract the shear viscosity by calculating correlation functions of the shear components of the system's energy momentum tensor near equilibrium. In addition, we present two schemes for computing the entropy of the system which are self-consistent. We find that finite baryon density notably reduces η/s .

Summarizing our technology

- Use UrQMD in box mode to describe infinite equilibrated hadronic matter.
 - Apply Green-Kubo formalism to extract shear viscosity.
 - Calculate entropy by weighting specific entropies particles (verified with Gibbs formula for entropy).
- Perform analysis of η , η/s as a function of T and baryon # density for a hadron gas **IN EQUILIBRIUM**.

Calculating Correlation Functions for Viscosity

$$\rho_B = \rho_0$$



$$\eta = \frac{V\tau_\pi}{T} \langle \pi^{xy}(0)^2 \rangle$$

$$\pi^{\mu\nu} = \int d^3p \frac{p^\mu p^\nu}{p^0} f(x, p)$$

$$\pi^{xy} = \frac{1}{V} \sum_{j=1}^{N_{part}} \frac{p^x(j)p^y(j)}{p^0(j)}$$

NOTE: correlation function found to empirically obey exponential decay.

Ansatz also used in Muronga, *PRC* **69**:044901, 2004

$$\langle \pi^{xy}(0)\pi^{xy}(t) \rangle \propto \exp\left(-\frac{t}{\tau_\pi}\right)$$

$$\eta = \frac{V}{T} \int_0^\infty dt \langle \pi^{xy}(0)\pi^{xy}(t) \rangle$$

Entropy Considerations

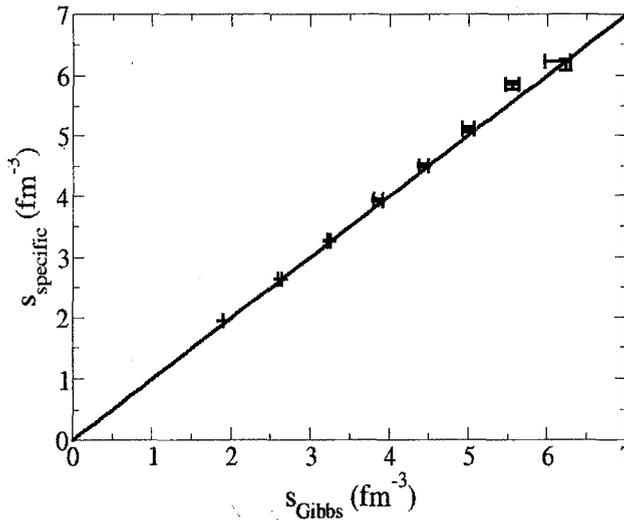
Method I: Gibbs formula for entropy:
 (extract μ_B for our system from SHAREv2,
 P and ϵ known from UrQMD.) Denote as
 s_{Gibbs} .

SHARE v2: Torrieri *et al.*, nucl-th/0603026
 -Tune particles/resonances to those in UrQMD.

**Method II: Weight over specific entropies
 of particles, where s/n is a function of
 m/T & μ_B/T ! Denote as s_{specific}**

$$s_{\text{Gibbs}} = \left(\frac{\epsilon + P - \mu_B \rho_B}{T} \right)$$

$$s_{\text{specific}} = \frac{1}{V} \sum_{i=1}^{N_{\text{part}}} \left(\frac{s}{n} \right)_i N_i$$

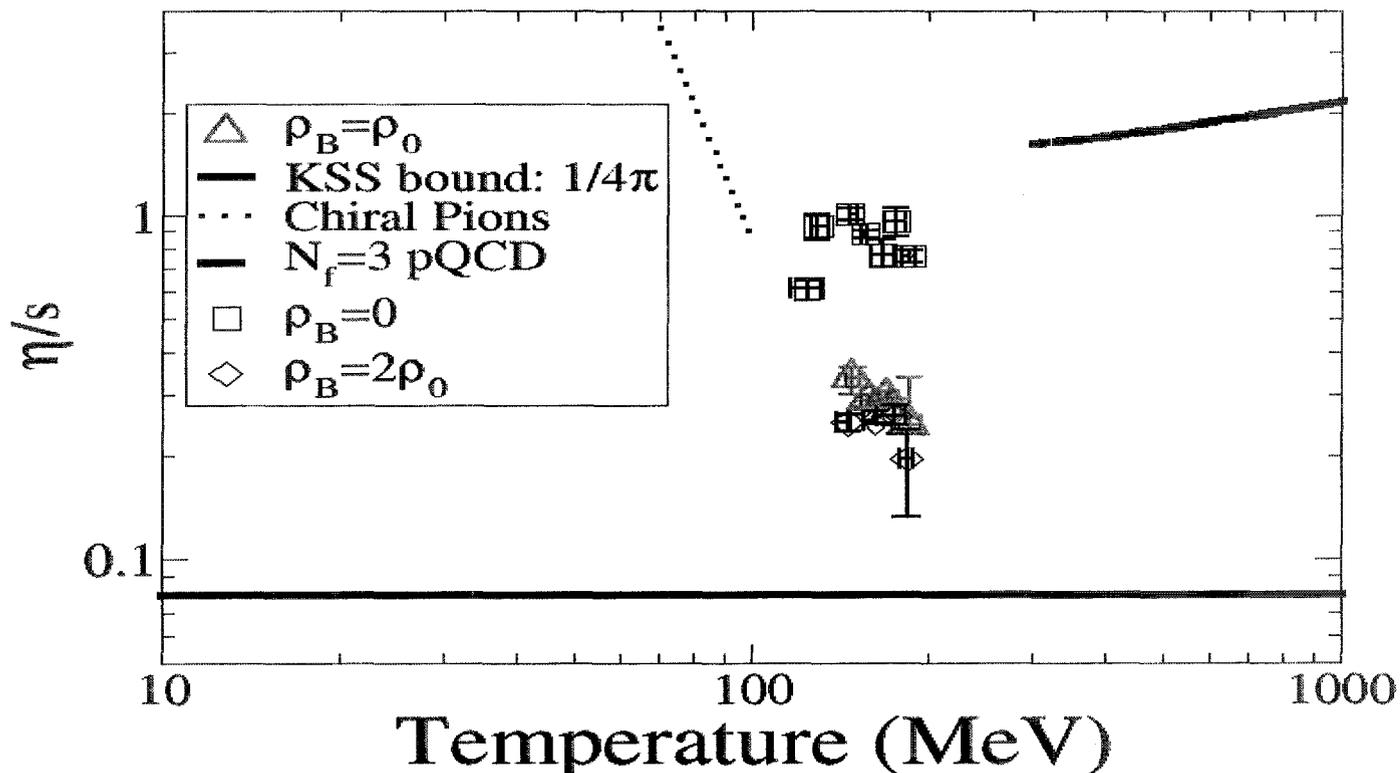


$$n = g \int_0^\infty \frac{d^3p}{(2\pi)^3} \frac{1}{\exp \left[\frac{\sqrt{p^2 + m^2} - \mu}{T} \right] \pm 1}$$

$$\epsilon = g \int_0^\infty \frac{d^3p}{(2\pi)^3} \frac{\sqrt{p^2 + m^2}}{\exp \left[\frac{\sqrt{p^2 + m^2} - \mu}{T} \right] \pm 1}$$

$$P = g \int_0^\infty \frac{d^3p}{(2\pi)^3} \frac{p^2}{3\sqrt{p^2 + m^2} \exp \left[\frac{\sqrt{p^2 + m^2} - \mu}{T} \right] \pm 1}$$

Where is the minimum viscosity?

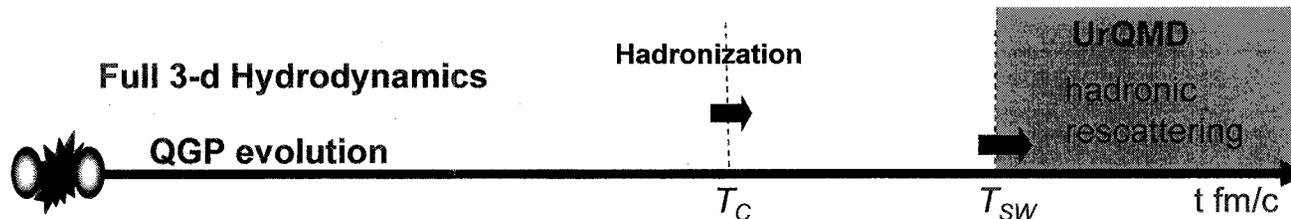


131

- η/s decreases w. finite μ_B .
- Minimum hadronic $\eta/s \approx 1.7/(4\pi)$
- Is minimum η/s near T_c ? Need $\mu=0$ results for $T < 100$ MeV to answer this question with certainty. (IN PROGRESS)

Summary/Outlook

- Can apply Green-Kubo formalism to hadronic matter in equilibrium:
 - Use UrQMD to model hadronic matter.
 - Use box mode to ensure equilibrium.Calculated entropy via 2 different methods (microscopic and macroscopic pictures self-consistent).
- Preliminary results:
 - Hadronic η/s satisfies viscosity bound from AdS/CFT (at least 1.7 times above bound).
 - η notably reduced at finite μ_B .In progress:
 - Analyzing $\mu=0$ mesonic matter for $T < 100$ MeV.
- Outlook:
 - Describe *time-evolution* of transport coefficient in relativistic heavy-ion reaction.



Calculating shear viscosity and relaxation time in a parton cascade.

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Zhe Xu ^{*2}
Carsten Greiner ^{*3}

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It is of great interest to investigate, under which conditions the parton cascade simulation and a dissipative hydrodynamic model yield equivalent results. In order to make a comparison between the two approaches, it is important to calculate the transport coefficients and the corresponding relaxation times in a parton cascade simulation.

In our work on thermalization of a Color Glass Condensate (CGC) in the parton cascade BAMPS[1] we have observed a quasi ideal hydrodynamic behavior of the gluonic system as it achieves kinetic equilibrium. The value of $\frac{\eta}{s}$ was found to be small, approximately

0.16. To evaluate the shear viscosity η in [1] we have used the Navier-Stokes equations:

$$\eta = \frac{t}{4}(T_{xx} + T_{yy} - T_{zz}) .$$
 However, the applicability of first order hydrodynamic equations in

calculations with the highly anisotropic CGC initial condition is questionable.

An alternative way to calculate the shear viscosity coefficient is using the Grad's method [2]. The derivation of the relativistic dissipative hydrodynamics from the Boltzmann Equation

$$p^\mu \partial_\mu f(x, p) = C[f(x, p)]$$

is discussed in [3,4]. In the present work we follow the discussion given by the authors in [3,4] and calculate the shear viscosity from the momenta of the collision term

$C[f(x, p)]$ in the RHS of the Boltzmann Equation, the underlying equation of our parton cascade. The presented results are obtained from simulations with a CGC initial condition in a 1-Dim expanding geometry. The shear relaxation times obtained from the calculated values of shear viscosity coefficient are consistent with the thermalization times obtained in [1]. The presented results are compared with the calculations using the Navier-Stokes equation. The values obtained from the two different approaches converge as the system is close to thermal equilibration. The presented formalism allows to calculate the heat conductivity, bulk and shear viscosities and the corresponding relaxation times, which enter second order hydrodynamic equations, in a parton cascade.

[1]hep-ph/0712.3734

[2]H.Grad, *Commun. Pure Appl. Math.* 2, 331 (1949)

[3]Azwinndini Muronga, *Phys. Rev C* 76, 014910 (2007)

[4]Dirk Rischke, *PhD Thesis*, (1993)

Thermalization of a CGC in BAMPS

Initial condition: simple form of CGC

$$f(x, p) = \frac{c}{\alpha_s N_c} \frac{1}{t_{ini}} \delta(p_z) \Theta(Q_s^2 - p_t^2)$$

$$\frac{dN}{d\eta} = c \pi R^2 \frac{N_c^2 - 1}{4\pi^2 \alpha_s N_c} Q_s^2$$

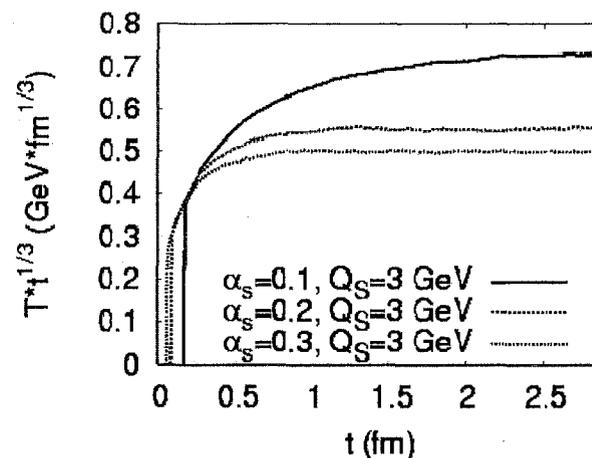
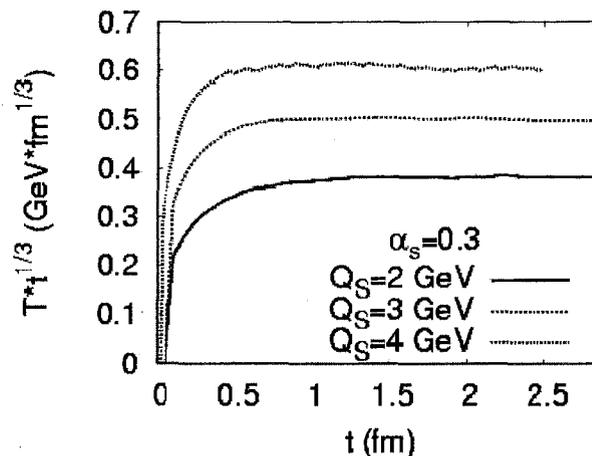
$Q_s = 2, 3, 4 \text{ GeV}, \alpha_s = 0.1, 0.2, 0.3$

RHIC

LHC

$$\alpha_s = 0.3 \quad \begin{aligned} t_{th}(Q_s = 2 \text{ GeV}) &= 1.2 \text{ fm}/c \\ t_{th}(Q_s = 3 \text{ GeV}) &= 0.75 \text{ fm}/c \\ t_{th}(Q_s = 4 \text{ GeV}) &= 0.55 \text{ fm}/c \end{aligned}$$

$$Q_s = 3 \text{ GeV} \quad \begin{aligned} t_{th}(\alpha_s = 0.1) &= 1.75 \text{ fm}/c \\ t_{th}(\alpha_s = 0.2) &= 1.0 \text{ fm}/c \\ t_{th}(\alpha_s = 0.3) &= 0.75 \text{ fm}/c \end{aligned}$$



From Boltzmann Equation to relativistic dissipative hydro.

$$\ln[f(x, p)] = y(x, p) = \phi(x, p) + y^{eq}(x, p)$$

↓ assuming ϕ is small

$$f(x, p) = e^y = e^{y^{eq}} \cdot e^\phi = f^{eq}(x, p)(1 + \phi(x, p))$$

↓ up to 2nd order in momentum

$$\phi(x, p) = \epsilon(x) - \epsilon_\mu(x) p^\mu + \epsilon_{\mu\nu}(x) p^\mu p^\nu$$

$$\partial_\mu S^\mu(x) = -\frac{g}{(2\pi)^3} \int dw p^\mu \partial_\mu f(x, p) \ln[f(x, p)] = -\frac{g}{(2\pi)^3} \int dw C[f] y(x, p) =$$

$$= -\frac{g}{(2\pi)^3} \int dw C[f] \cdot (\underbrace{\epsilon(x) - \epsilon_\mu(x) p^\mu + \epsilon_{\mu\nu}(x) p^\mu p^\nu}_{\text{vanishing momenta of collision term}} + y^{eq}(x, p))$$

vanishing momenta of collision term

because of

$$\partial_\mu N^\mu = 0 \wedge \partial_\mu T^{\mu\nu} = 0 \wedge \text{energy conservation}$$

$$\partial_\mu S^\mu = -\frac{g}{(2\pi)^3} \epsilon_{\mu\nu}(x) \int dw p^\mu p^\nu C[f] = -\epsilon_{\mu\nu} P^{\mu\nu} = \beta(\zeta^{-1} \Pi^2 - \lambda^{-1} q^\alpha q_\alpha + (2\eta)^{-1} \pi^{\alpha\beta} \pi_{\alpha\beta})$$

$$\epsilon_{\mu\nu} = A_2(3u_\mu u_\nu - \Delta_{\mu\nu})\Pi - B_1 u_{<\mu} q_{\nu>} + C_0 \pi_{\mu\nu}$$

To identify the transport coefficients, we need to decompose $P^{\mu\nu}$:

$$P^{\mu\nu} = \frac{4}{3} C_\Pi A_2 (3u^\mu u^\nu - \Delta^{\mu\nu})\Pi + 2C_q B_1 q^{(\mu} u^{\nu)} + \frac{1}{5} C_\pi C_0 \pi^{<\mu\nu>}$$

C_Π , C_q , C_π are unknown coefficients, involving integrals of the collision term

Taking projection of $P^{\mu\nu}$:

$$P^{<\mu\nu>} = \frac{1}{5} C_\pi C_0 \pi^{<\mu\nu>} = \frac{1}{5} C_\pi C_0 T^{<\mu\nu>} \quad \Rightarrow \quad C_\pi = \frac{5P^{<\mu\nu>}}{C_0 T^{<\mu\nu>}}$$

$$\eta = \frac{-5\beta}{2C_0^2 C_\pi} \quad \longrightarrow \quad \eta = \frac{-\beta T^{<\mu\nu>}}{2C_0 P^{<\mu\nu>}}$$

Calculating shear viscosity in BAMPS:

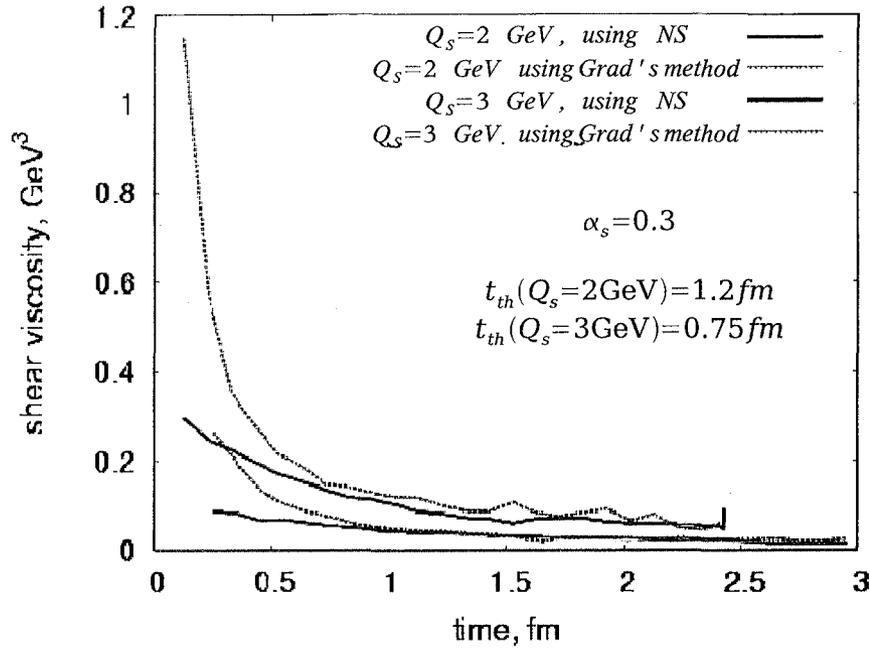
Navier-Stokes with Bjorken scaling

vs

Grad's method

$$\eta = \frac{\tau}{4} (T_{11} + T_{22} - 2T_{33})$$

$$\eta = \beta \frac{2T^{33} - T^{22} - T^{11}}{2C_0(P^{11} + P^{22} - 2P^{33})}$$

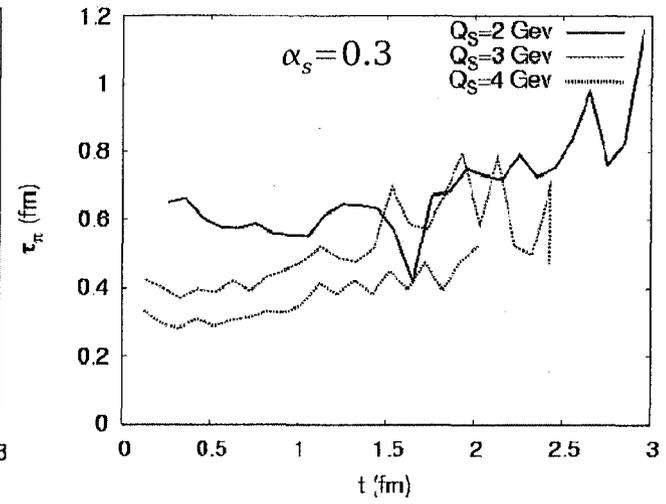
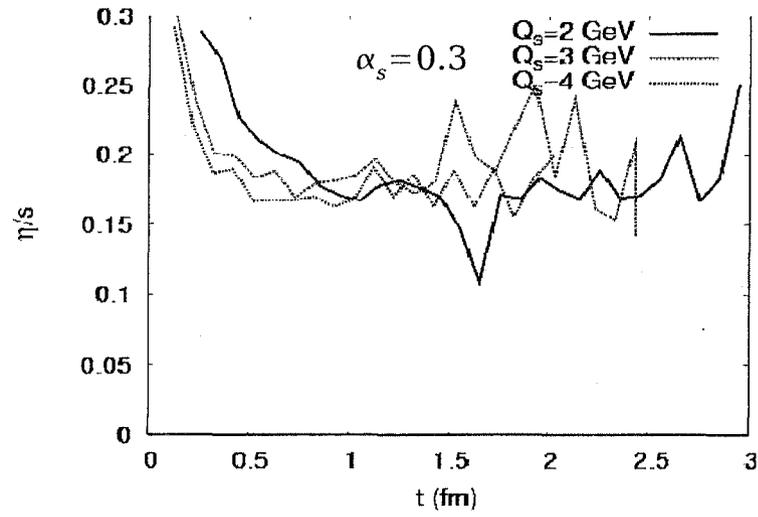


with

$$C_0 = \frac{1}{2J_{42}} \quad \beta = \frac{1}{T} = \left(\frac{\pi^2 e}{48}\right)^{-\frac{1}{4}}$$

$$J_{42} = \frac{64}{\pi^2} T^6$$

Shear to entropy density & relaxation times:



$$\begin{aligned} t_{th}(Q_s=2\text{GeV}) &= 1.2\text{ fm} \\ t_{th}(Q_s=3\text{GeV}) &= 0.75\text{ fm} \\ t_{th}(Q_s=4\text{GeV}) &= 0.55\text{ fm} \end{aligned}$$

Hydrodynamical behaviour in heavy ion collisions within parton cascade calculations

Zhe Xu

with A. El, O. Fochler, C. Greiner and H. Stöcker

Institut für Theoretische Physik, Johann Wolfgang Goethe-Universität Frankfurt, Germany

Abstract

Using the relativistic pQCD based parton cascade BAMPS we calculate the time scale of thermalization, the elliptic flow parameter v_2 , the shear viscosity and the nuclear modification factor R_{AA} of a gluon matter. The results show that the bremsstrahlung processes $gg \leftrightarrow ggg$ are essential for quick thermalization (1 fm/c), low shear viscosity over entropy ratio (0.08-0.13) and large v_2 as measured at RHIC. Also, the jet-quenching $R_{AA}=0.1$ is comparable with the experimental data. Hydrodynamical behavior (v_2) and energy loss (R_{AA}) are described quantitatively in a consistent manner within pQCD.

Presented on RBRC workshop, BNL, April 22, 2008

screened partonic interactions in leading order pQCD

$$|M_{gg \rightarrow gg}|^2 = \frac{9g^4}{2} \frac{s^2}{(q_{\perp}^2 + m_D^2)^2},$$

$$|M_{gg \rightarrow ggg}|^2 = \left(\frac{9g^4}{2} \frac{s^2}{(q_{\perp}^2 + m_D^2)^2} \right) \left(\frac{12g^2 q_{\perp}^2}{k_{\perp}^2 ((\vec{k}_{\perp} - \vec{q}_{\perp})^2 + m_D^2)} \right) \Theta_{LPM}(k_{\perp} \Lambda_g - \text{cosh } y)$$

J.F.Gunion, G.F.Bertsch, PRD 25, 746(1982)

T.S.Biro at el., PRC 48, 1275 (1993)

S.M.Wong, NPA 607, 442 (1996)

screening mass: $m_D^2 = 16\pi\alpha_s \int \frac{d^3p}{(2\pi)^3} \frac{1}{p} (3f_g + n_f f_q),$

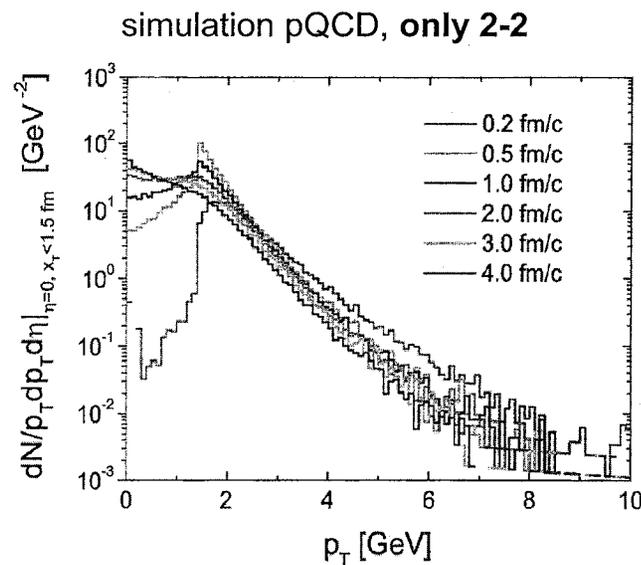
LPM suppression: the formation time $\Delta\tau \approx \frac{1}{k_{\perp}} \text{cosh } y < \Lambda_g$

Λ_g : mean free path

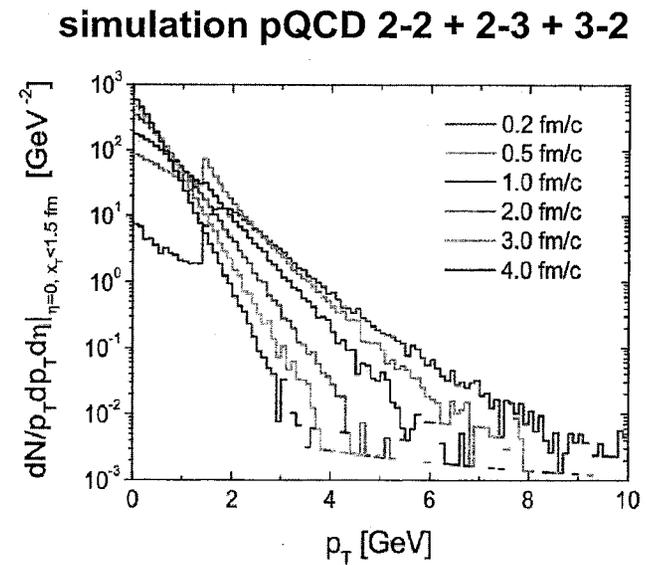
Zhe Xu

p_T spectra

at collision center: $x_T < 1.5$ fm, $\Delta z < 0.4$ t fm of a central Au+Au at $s^{1/2} = 200$ GeV
Initial conditions: **minijets** $p_T > 1.4$ GeV; coupling $\alpha_s = 0.3$



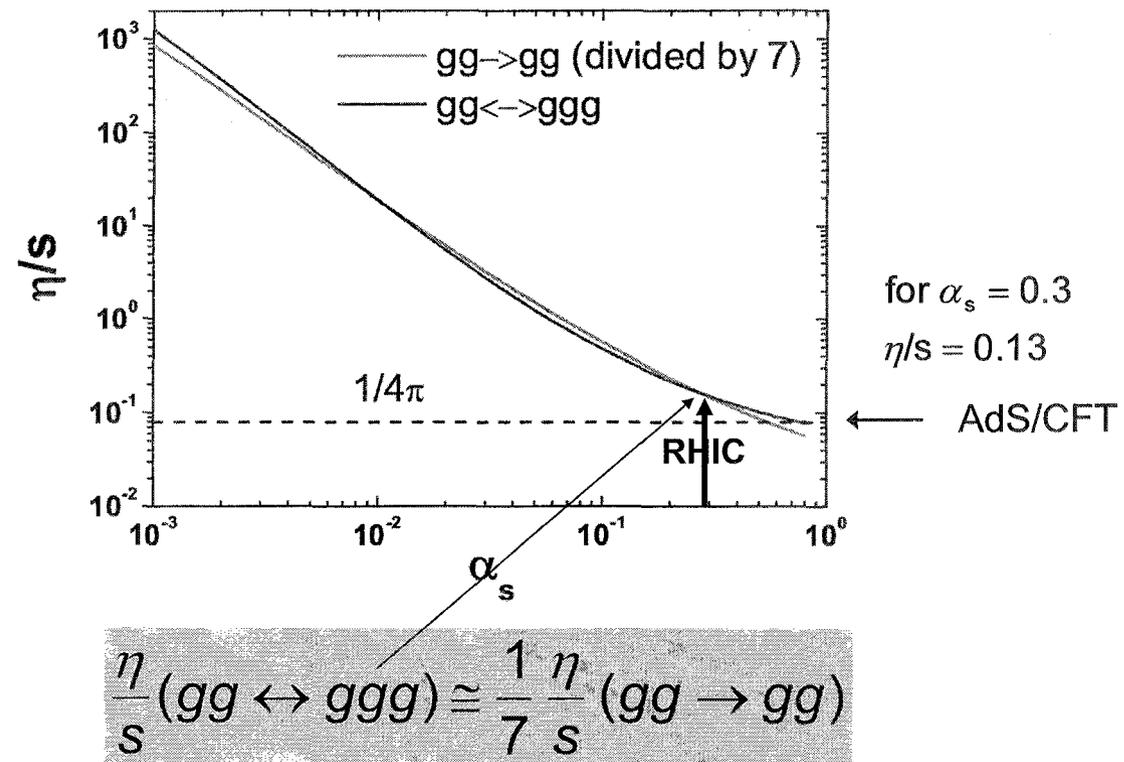
2-2: NO thermalization



**3-2 + 2-3: thermalization!
Hydrodynamic behavior!**

Zhe Xu

Ratio of shear viscosity to entropy density in 2-3



ZX and C.Greiner, arXiv: 0710.5719 [nucl-th], to be published in PRL.

Zhe Xu

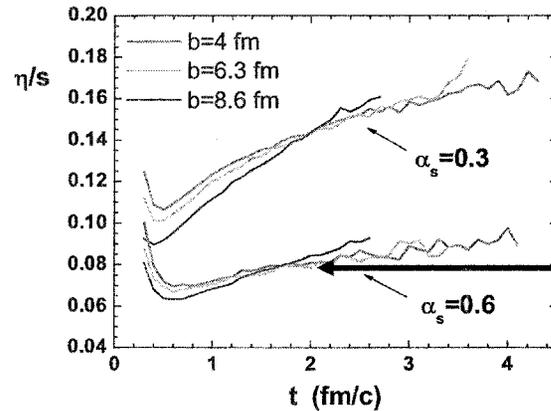
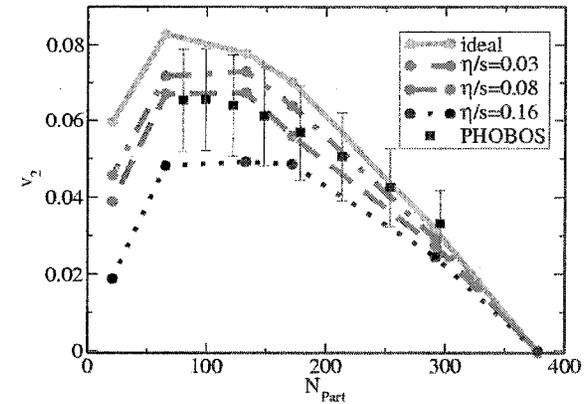
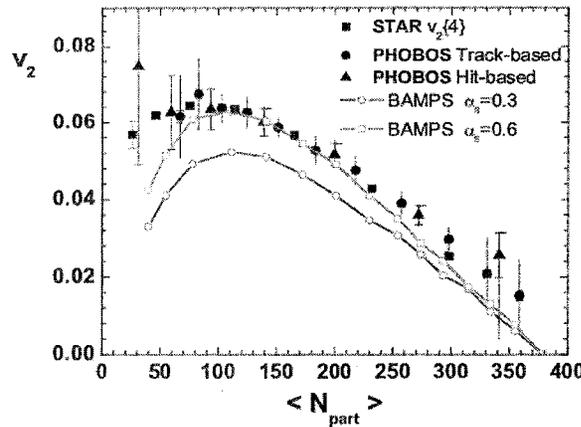
Elliptic Flow and Shear Viscosity in 2-3 at RHIC

2-3 Parton cascade BAMPs

viscous hydro.

ZX, Greiner, Stöcker, arXiv: 0711.0961 [nucl-th]

Romatschke, PRL 99, 172301, 2007



$$\eta = \frac{1}{5} n \frac{\left\langle E \left(\frac{1}{3} - \frac{p_z^2}{E^2} \right) \right\rangle}{\frac{1}{3} - \left\langle \frac{p_z^2}{E^2} \right\rangle} \frac{1}{\sum R^{tr} + \frac{3}{2} R_{23} - R_{32}}$$

$$s = 4n - n \ln \lambda$$

η/s at RHIC > 0.08

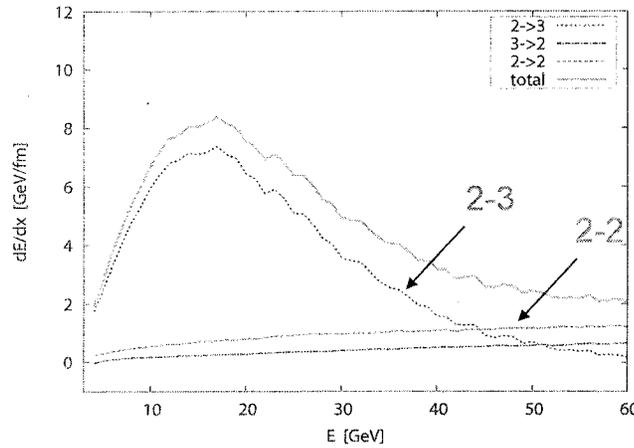
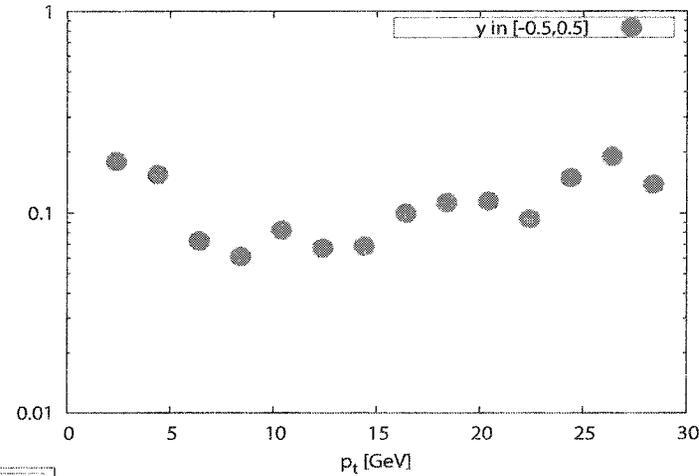
Zhe Xu

first realistic 3d results on jet-quenching with BAMPS

nuclear modification factor
 central ($b=0$ fm) Au-Au at 200 AGeV
 $\alpha_s = 0.3$

$$R_{AA} \sim 0.1$$

cf. S. Wicks et al.
 Nucl.Phys.A784, 426



O. Fochler

dE/dx, static medium (T = 400 MeV)

Zhe Xu

RIKEN BNL Research Center Workshop
Hydrodynamics in Heavy Ion Collisions and QCD Equation of State
April 21-22, 2008

List of Registered Participants

First Name	Last Name	Affiliation
Yasumichi	Aoki	RIKEN BNL Research Center
Masayuki	Asakawa	Osaka University
Michael	Cheng	Columbia University
Tamas	Csorgo	MTA KFKI RMKI
Nasser	Demir	Duke University
Adrian	Dumitru	ITP, Frankfurt Univ.
James	Dunlop	Brookhaven National Laboratory
Kevin	Dusling	Stony Brook University
Andrej	El	University of Frankfurt
Rainer	Fries	Texas A & M University & RIKEN BNL Research Center
Carsten	Greiner	Institut fuer Theoretische Physik, Universitaet Frankfurt
Urs	Heller	American Physical Society
Tetsufumi	Hirano	Department of Physics, the University of Tokyo
Huan	Huang	University of California, Los Angeles
Pasi	Huovinen	Purdue University
Olaf	Kaczmarek	University of Bielefeld
Frithjof	Karsch	Brookhaven National Laboratory
Dmitri	Kharzeev	Brookhaven National Laboratory
Takeshi	Kodama	Univ. Federal do Rio de Janeiro
Tomoi	Koide	Universidade Federal do Rio de Janeiro
Roy	Lacey	State University of NY, Stony Brook
J.H.	Lee	Brookhaven National Laboratory
Ludmila	Levkova	University of Utah
Jinfeng	Liao	Stony Brook University
Larry	McLerran	RIKEN BNL Research Center
Agnes	Mocsy	RIKEN BNL Research Center
Denes	Molnar	Purdue University & RIKEN BNL Research Center
Azwinndini	Muronga	University of Cape Town
Bryon	Neufeld	Duke University
Jason	Newby	Lawrence Livermore National Lab
Chiho	Nonaka	Nagoya University
Jorge	Noronha	Columbia University
Vitaly	Okorokov	Moscow Engineering Physics Institute
Peter	Petreczky	Brookhaven National Laboratory / RIKEN BNL Research Center
Helmut	Satz	University of Bielefeld
Ron	Soltz	Lawrence Livermore National Laboratory
Huichao	Song	The Ohio State University
Aihong	Tang	Brookhaven National Laboratory
Derek	Teaney	State University of NY, Stony Brook / RIKEN BNL Research Center
Raju	Venugopalan	Brookhaven National Laboratory
Zhe	Xu	ITP, Johann Wolfgang Goethe-Universität Frankfurt
Nu	Xu	Lawrence Berkley National Laboratory

RBRC Workshop

Hydrodynamics in Heavy Ion Collisions and QCD Equation of State (Physics Department Large Seminar Room)

Monday, April 21, 2008

8:20 **REGISTRATION**

8:50 Nicholas Samios : Welcome

9:00 Ron Soltz : EOS calculations on $N_t=8$ lattices and implications for experimental benchmarks for hydrodynamics

9:30 Ludmila Levkova : Lattice calculation of EOS using asqtad action

10:00 Yasumichi Aoki : Lattice calculations of EOS using stout staggered action

10:30-11:00 Coffee

11:00 Michael Cheng: EOS from lattice calculations with improved staggered action

11.30 Bryon Neufeld : Sounding out the QGP

12:00 Huan Z Huang : Comments on hydrodynamic calculations from an experimentalist

12:30-2:00 Lunch

2:00 Pasi Huovinen : The effects of the order of phase transition, chemical equilibrium and freezeout in ideal hydro

2:30 Tetsufumi Hirano : Current status of a 3D hydro+cascade model

3:00-3:30 Coffee

3:30 Helmut Satz : The origin of thermal hadron production

4:10 Masayuki Asakawa: QCD critical point and its effects on physical observables – schematic consideration

4:40 Chiho Nonaka : Hydrodynamic expansion with the QCD critical point

5:10 Tamás Csörgő : New analytic results in hydrodynamics

Tuesday, April 22, 2008

9:00 Azwindini Muronga : Viscous hydrodynamics and the QCD equation of state

9:30 Tomoi Koide : Non-Newtonian nature of relativistic dissipative fluid

10:00 Takeshi Kodama : Landau initial condition and effects of viscosity

10:30-11:00 Coffee

11:00 Kevin Dusling : Viscosity and its effect on elliptic flow and thermal dileptons

11:30 Huichao Song : Viscosity and its effect on elliptic flow

12:00-2:00 Lunch

2:00 Adrian Dumitru : Modelling the real-time dynamics of a non-Abelian plasma

2:25 Rainer Fries : Effects of bulk viscosity in hydrodynamic evolution

2:55 Pasi Huovinen : Comparing viscous hydrodynamics to a parton cascade

3:25-3:45 Coffee

3:45 Aihong Tang : Results from STAR

4:15 Roy Lacey : The implications of flow measurements at RHIC

4:45 Nasser Demir : Transport coefficients in the hadronic phase from transport models

5:10 Andrej El : Calculation transport coefficients in a parton cascade

5:35 Zhe Xu : Hydrodynamical behavior in heavy ion collisions within parton cascade calculations

Additional RIKEN BNL Research Center Proceedings:

- Volume 87 – RBRC Scientific Review Committee Meeting – BNL-79570-2007
- Volume 86 – Global Analysis of Polarized Parton Distributions in the RHIC Era, October 8, 2007, BNL-79457-2007
- Volume 85 – Parity-Violating Spin Asymmetries at RHIC-BNL, April 26-27, 2007 – BNL-79146-2007
- Volume 84 – Domain Wall Fermions at Ten Years, March 15-17, 2007 – BNL 77857-2007
- Volume 83 – QCD in Extreme Conditions, July 31-August 2, 2006– BNL-76933-2006
- Volume 82 – RHIC Physics in the Context of the Standard Model, June 18-23, 2006 – BNL-76863-2006
- Volume 81 – Parton Orbital Angular Momentum (Joint RBRC/University of New Mexico Workshop) February 24-26, 2006 – BNL-75937-2006
- Volume 80 – Can We Discover the QCD Critical Point at RHIC?, March 9-10, 2006 – BNL-75692-2006
- Volume 79 – Strangeness in Collisions, February 16-17, 2006 – BNL-79763-2008
- Volume 78 – Heavy Flavor Productions and Hot/Dense Quark Matter, December 12-14, 2005 – BNL-76915-2006
- Volume 77 – RBRC Scientific Review Committee Meeting – BNL-52649-2005
- Volume 76 – Odderon Searches at RHIC, September 27-29, 2005 – BNL-75092-2005
- Volume 75 – Single Spin Asymmetries, June 1-3, 2005 – BNL-74717-2005
- Volume 74 – RBRC QCDOC Computer Dedication and Symposium on RBRC QCDOC, May 26, 2005 – BNL-74813-2005
- Volume 73 – Jet Correlations at RHIC, March 10-11, 2005 – BNL-73910-2005
- Volume 72 – RHIC Spin Collaboration Meetings XXXI (January 14, 2005), XXXII (February 10, 2005), XXXIII (March 11, 2005) – BNL-73866-2005
- Volume 71 – Classical and Quantum Aspects of the Color Glass Condensate – BNL-73793-2005
- Volume 70 – Strongly Coupled Plasmas: Electromagnetic, Nuclear & Atomic – BNL-73867-2005
- Volume 69 – Review Committee – BNL-73546-2004
- Volume 68 – Workshop on the Physics Programme of the RBRC and UKQCD QCDOC Machines – BNL-73604-2004
- Volume 67 – High Performance Computing with BlueGene/L and QCDOC Architectures – BNL-
- Volume 66 – RHIC Spin Collaboration Meeting XXIX, October 8-9, 2004, Torino Italy – BNL-73534-2004
- Volume 65 – RHIC Spin Collaboration Meetings XXVII (July 22, 2004), XXVIII (September 2, 2004), XXX (December 6, 2004) - BNL-73506-2004
- Volume 64 – Theory Summer Program on RHIC Physics – BNL-73263-2004
- Volume 63 – RHIC Spin Collaboration Meetings XXIV (May 21, 2004), XXV (May 27, 2004), XXVI (June 1, 2004) – BNL-72397-2004
- Volume 62 – New Discoveries at RHIC, May 14-15, 2004 – BNL- 72391-2004
- Volume 61 – RIKEN-TODAI Mini Workshop on “Topics in Hadron Physics at RHIC”, March 23-24, 2004 – BNL-72336-2004
- Volume 60 – Lattice QCD at Finite Temperature and Density – BNL-72083-2004

Additional RIKEN BNL Research Center Proceedings:

- Volume 59 – RHIC Spin Collaboration Meeting XXI (January 22, 2004), XXII (February 27, 2004), XXIII (March 19, 2004)– BNL-72382-2004
- Volume 58 – RHIC Spin Collaboration Meeting XX – BNL-71900-2004
- Volume 57 – High pt Physics at RHIC, December 2-6, 2003 – BNL-72069-2004
- Volume 56 – RBRC Scientific Review Committee Meeting – BNL-71899-2003
- Volume 55 – Collective Flow and QGP Properties – BNL-71898-2003
- Volume 54 – RHIC Spin Collaboration Meetings XVII, XVIII, XIX – BNL-71751-2003
- Volume 53 – Theory Studies for Polarized pp Scattering – BNL-71747-2003
- Volume 52 – RIKEN School on QCD “Topics on the Proton” – BNL-71694-2003
- Volume 51 – RHIC Spin Collaboration Meetings XV, XVI – BNL-71539-2003
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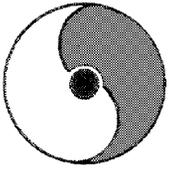
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Hydrodynamics in Heavy Ion Collisions and QCD Equation of State

April 21-22, 2008



Li Keran

*Nuclei as heavy as bulls
Through collision
Generate new states of matter.
T.D. Lee*

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