Understanding QGP through Spectral Functions and Euclidean Correlators

April 23-25, 2008

Organizers:

Organizers: Ágnes Mócsy and Péter Petreczky

RIKEN BNL Research Center

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Preface to the Series

The RIKEN BNL Research Center (RBRC) was established in April 1997 at Brookhaven National Laboratory. It is funded by the "Rikagaku Kenkyusho" (RIKEN, The Institute of Physical and Chemical Research) of Japan. The Center is dedicated to the study of strong interactions, including spin physics, lattice QCD, and RHIC physics through the nurturing of a new generation of young physicists.

The RBRC has both a theory and experimental component. The RBRC Theory Group and the RBRC Experimental Group consists of a total of 25-30 researchers. Positions include the following: full time RBRC Fellow, half-time RHIC Physics Fellow, and full-time, post-doctoral Research Associate. The RHIC Physics Fellows hold joint appointments with RBRC and other institutions and have tenure track positions at their respective universities or BNL. To date, RBRC has ~50 graduates of which 14 theorists and 6 experimenters have attained tenure positions at major institutions worldwide.

Beginning in 2001 a new RIKEN Spin Program (RSP) category was implemented at RBRC. These appointments are joint positions of RBRC and RIKEN and include the following positions in theory and experiment: RSP Researchers, RSP Research Associates, and Young Researchers, who are mentored by senior RBRC Scientists. A number of RIKEN Jr. Research Associates and Visiting Scientists also contribute to the physics program at the Center.

RBRC has an active workshop program on strong interaction physics with each workshop focused on a specific physics problem. In most cases all the talks are made available on the RBRC website. In addition, highlights to each speaker's presentation are collected to form proceedings which can therefore be made available within a short time after the workshop. To date there are eighty eight proceeding volumes available.

A 10 teraflops RBRC QCDOC computer funded by RIKEN, Japan, was unveiled at a dedication ceremony at BNL on May 26, 2005. This supercomputer was designed and built by individuals from Columbia University, IBM, BNL, RBRC, and the University of Edinburgh, with the U.S. D.O.E. Office of Science providing infrastructure support at BNL. Physics results were reported at the RBRC QCDOC Symposium following the dedication. QCDSP, a 0.6 teraflops parallel processor, dedicated to lattice QCD, was begun at the Center on February 19, 1998, was completed on August 28, 1998, and was decommissioned in 2006. It was awarded the Gordon Bell Prize for price performance in 1998.

N. P. Samios, Director
March 2007

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Understanding QGP through Spectral Functions and Euclidean Correlators

RBRC Workshop, April 23-25, 2008

Organizers:
Ágnes Mócsy, Péter Petreczky

In the past two decades, one of the most important goals of the nuclear physics community has been the production and characterization of the new state of matter – Quark-Gluon Plasma (QGP). Understanding how properties of hadrons change in medium, particularly, the bound state of a very heavy quark and its antiquark, known as quarkonium, as well as determining the transport coefficients is crucial for identifying the properties of QGP and for the understanding of the experimental data from RHIC.

On April 23rd, more than sixty physicists from twenty-seven institutions gathered for this three-day topical workshop held at BNL to discuss how to understand the properties of the new state of matter obtained in ultra-relativistic heavy ion collisions (particularly at RHIC-BNL) through spectral functions. In-medium properties of the different particle species and the transport properties of the medium are encoded in spectral functions. The former could yield important signatures of deconfinement and chiral symmetry restoration at high temperatures and densities, while the later are crucial for the understanding of the dynamics of ultra-relativistic heavy ion collisions.

Participants at the workshop are experts in various areas of spectral function studies. The workshop encouraged direct exchange of scientific information among experts, as well as between the younger and the more established scientists. The workshops success is evident from the coherent picture that developed of the current understanding of transport properties and in-medium particle properties, illustrated in the current proceedings. The following pages show calculations of meson spectral functions in lattice QCD, as well as implications of these for quarkonia melting/survival in the quark gluon plasma; Lattice calculations of the transport coefficients (shear and bulk viscosities, electric conductivity); Calculation of spectral functions and transport coefficients in field theories using weak coupling techniques; And certain spectral functions and also the heavy quark diffusion constant have been calculated in the strongly coupled limit of the N = 4 super-symmetric Yang Mills theory.

More information is available at http://www.bnl.gov/riken/qgp/
Heavy quark free energies and screening in lattice QCD

Olaf Kaczmarek

Universität Bielefeld

RBC-Bielefeld collaboration

We present results for heavy quark free energies, their relation to internal energy and entropy contributions and analyze screening properties of the medium. The results were obtained for 2+1 flavour QCD with almost realistic quark masses, i.e. a pion mass of 220 MeV and realistic strange quark mass using highly improved staggered quark action. Entropy contributions play an important role in the medium leading to a steeper slope of internal energy compared to free energy. In terms of potential model calculations this would lead to quarkonium states which are stronger bound and dissociate at higher temperatures.
\textit{Renormalized Polyakov loop}

Using short distance behaviour of free energies
Renormalization of \( F(r, T) \) at short distances
\[
e^{-F_1(r,T)/T} = (Z_r(g^2))^{2N_c} \langle \text{Tr} (L_x L_y) \rangle
\]
Renormalization of the Polyakov loop
\[
L_{\text{ren}} = (Z_R(g^2))^N L_{\text{lattice}}
\]
\( L_{\text{ren}} \) defined by long distance behaviour of \( F(r,T) \)
\[
L_{\text{ren}} = \exp \left( - \frac{F(r = \infty, T)}{2T} \right)
\]
Temperature depending running coupling

Non-perturbative confining part for $r \gtrsim 0.4$ fm

$$\alpha_{qq}(r) \simeq 3/4r^2\sigma$$

Present below and just above $T_c$

Remnants of confinement at $T \gtrsim T_c$

Temperature effects set in at smaller $r$ with increasing $T$

Maximum due to screening

Free energy in perturbation theory:

$$F_1(r, T) \equiv V(r) \simeq -\frac{4}{3} \frac{\alpha(r)}{r}$$ for $r\Lambda_{QCD} \ll 1$

$$F_1(r, T) \simeq -\frac{4}{3} \frac{\alpha(T)}{r} e^{-m_D(T)r}$$ for $rT \gg 1$

QCD running coupling in the $qq$-scheme

$$\alpha_{qq}(r, T) = \frac{3}{4r^2} \frac{dF_1(r, T)}{dr}$$

At which distance do $T$-effects set in?  

Definition of the screening radius/mass

Definition of the $T$-dependent coupling
Screening masses obtained from fits to:

\[ F_1(r, T) - F_1(r = \infty, T) = -\frac{4\alpha(T)}{3r} e^{-m_D(T)r} \]

at large distances \( rT \gg 1 \)

leading order perturbation theory:

\[ \frac{m_D(T)}{T} = \left( 1 + \frac{N_f}{6} \right)^{1/2} g(T) \]

\( T \) dependence qualitatively described by perturbation theory

But \( \lambda \approx 1.4 - 1.5 \Rightarrow \) non-perturbative effects

\( \lambda \to 1 \) in the (very) high temperature limit
Free energy vs. Entropy at large separations

Free energies not only determined by potential energy

\[ F_\infty = U_\infty - TS_\infty \]

Entropy contributions play a role at finite \( T \)

\[ S_\infty = -\frac{\partial F_\infty}{\partial T} \]

High temperatures:

\[ F_\infty(T) \approx -\frac{4}{3} m_D(T) \alpha(T) \approx -O(g^3 T) \]
\[ TS_\infty(T) \approx +\frac{4}{3} m_D(T) \alpha(T) \]
\[ U_\infty(T) \approx -4m_D(T) \alpha(T) \frac{\beta(g)}{g} \approx -O(g^5 T) \]
steeper slope of $V_{eff}(r,T) = U_1(r,T)$

$\Rightarrow J/\psi$ stronger bound using $V_{eff} = U_1(r,T)$

$\Rightarrow$ dissociation at higher temperatures compared to $V_{eff}(r,T) = F_1(r,T)$
Heavy Quark Potentials at Zero Temperature

- QQ systems are ideal for strong interactions studies
- Scales and Effective Field Theories: systematic approach
- pNRQCD: the QQbar and QQQ potentials
- Applications of pNRQCD: Potentials and Spectra, Decays, Transitions, SM parameters
- What at finite T?
**Q̅Q: A multiscale System**

<table>
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<th>MeV</th>
<th>Y(4S)</th>
<th>Y(3S)</th>
<th>ψ(3S)</th>
<th>ψ(3770)</th>
<th>X(2P)</th>
<th>X(1P)</th>
<th>X(1P)</th>
<th>h(1P)</th>
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</tr>
<tr>
<td>-500</td>
<td>Y(1S)</td>
<td>J_{PP}</td>
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<table>
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<tr>
<th>S states</th>
<th>P states</th>
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<td>Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$</td>
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The mass scale is perturbative:

$m_b \simeq 5$ GeV, $m_c \simeq 1.5$ GeV

The system is non-relativistic:

$\Delta m \sim m^2, \Delta E \sim m^4$

$v_b^2 \simeq 0.1, v_c^2 \simeq 0.3$

Non-relativistic bound states are characterized by at least three energy scales

$m \gg m \gg m^2, v \ll 1$

and $\Lambda_{QCD}$
EFTs for Quarkonium

The matching procedure enforces the EFT to be equivalent to QCD
EFTs for Quarkonium

In QCD another scale is relevant \( \Lambda_{QCD} \)

A potential picture arises at the level of pNRQCD:
- the potential is perturbative if \( mv \gg \Lambda_{QCD} \)
- the potential is non-perturbative if \( mv \sim \Lambda_{QCD} \)
pNRQCD for Quarkonium with small radius

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \text{Tr} \left\{ S^i \left( i\partial_0 - \frac{p^2}{m} - V_S \right) S \right\} + \mathcal{O} \left( iD_0 - \frac{p^2}{m} - V_o \right) \mathcal{O} \} \]

LO in \( r \)

\[ + V_A \text{Tr} \left\{ O^i \sigma \cdot gE S + S^i \sigma \cdot gE O \right\} \]
\[ + \frac{V_B}{2} \text{Tr} \left\{ O^i \sigma \cdot gE O + O^i O \sigma \cdot gE \right\} \]

NLO in \( r \)

\[ + \cdots \]

Static singlet potential: calculated in the matching

\[ e^{ig \oint ds \mu A_\mu} \]

NRQCD \quad \Rightarrow \quad + \quad \text{pNRQCD}

UP TO TWO LOOPS:

\[ \lim_{T \to \infty} \frac{i}{T} \ln \langle \Box \rangle = V_s(r, \mu) \]

The mu dependence cancels between the two terms

\[ \frac{g^2}{N_c} \int_0^\infty dt e^{-it(V_0 + V_s)} \langle \text{Tr}(r \cdot \hat{E}(t) \cdot r \cdot \hat{E}(0)) \rangle (\mu) + \ldots \]

ultrasoft contribution
In the talk I summarize the basic ideas of the Maximal Entropy Method, discuss in more detail some technical issues like minimization, treatment of $\alpha$, effect of noise, error estimation. I also show test results from QCD simulations.
Weight probability is especially appropriate tool to take into account prior knowledges. Now

\[ \sigma_A \text{ is real and } \sigma_A(\omega > 0) > 0. \]

\[ \Rightarrow \text{ weight probability should allow only positive values.} \]

Candidate: (continuous) Poisson distribution with pre-defined positive averages:

\[
\frac{\kappa^n e^{-\kappa}}{n!} \rightarrow \left\{ \begin{array}{l}
n \rightarrow \alpha \sigma_A(\omega) \\
dn \rightarrow \alpha d\sigma_A(\omega) \\
\kappa \rightarrow \alpha m(\omega) \end{array} \right\} \rightarrow \sqrt{\frac{\alpha}{2\pi \sigma_A}} e^{-\alpha S\sigma\omega(\sigma_A, h)},
\]

where \[ S\sigma\omega(\sigma_A, h) = \int d\omega \left[ \sigma_A(\omega) \ln \frac{\sigma_A(\omega)}{m(\omega)} - \sigma_A(\omega) + m(\omega) \right], \]

also known as the Shannon-Jaynes entropy.

\( m(\omega) \) is called default model.
Discretizing:

\[ \omega = \{0 \ldots N_\omega - 1\} \Delta \omega, \quad N_\omega \sim \mathcal{O}(1000) \]
\[ \tau = \{0 \ldots N_\tau / 2 - 1\} \beta / N_\tau \]

Minimum condition:

\[ \frac{\partial Q}{\partial \sigma_A(\omega)} = \alpha \ln \frac{\sigma_A(\omega)}{m(\omega)} + \sum_{\tau, \tau'} K(\tau, \omega) C_{\tau\tau'}^{-1} (D_A(\tau') - \bar{D}_A(\tau')) = 0. \]

where \( D_A(\tau') = \Delta \omega \sum_{\omega'} K(\tau', \omega') \sigma_A(\omega') \)

- \( N_\omega \) coupled nonlinear equations
- solution: recursive approximation with local linearization and finding approximate minimum
- number of parameters \( \sim \) number of independent vectors \( \sim \)
  rank of the second derivative matrix: number-of-data \( \sim (N_\tau / 2) \)
  \( \Rightarrow \) inversion problematic
Improvement

- $K(\tau, \omega)$ linear independent for different $\tau$s ($\sim \cosh(\frac{\omega}{2} - \tau)\omega$
- make a basis on $N_{\omega}$ dim. space: $\{ R_i(\omega) \}_{i=0 \ldots N_{\omega}-1}$
- where $R_i(\omega) = K(\tau_i, \omega)$ for $i = 0, \ldots, N_{\omega}/2 - 1$.
- Expand $\ln \frac{\sigma_A(\omega)}{m(\omega)}$ in this basis:

$$
\ln \frac{\sigma_A(\omega)}{m(\omega)} = \sum_{\tau} s(\tau) K(\tau, \omega) + \sum_{i \geq N_{\tau}/2} \tilde{s}_i R_i(\omega).
$$

- then we have

$$
\sum_{\tau, \tau'} K(\tau, \omega) \left[ \alpha s(\tau)\delta_{\tau\tau'} + C_{\tau\tau'}^{-1}(D_A(\tau') - \bar{D}_A(\tau')) \right] + \sum_{i \geq N_{\tau}/2} \tilde{s}_i R_i(\omega) = 0.
$$
linear independence requires $\bar{s}_i = 0$

first $N_r/2$ equation yields

$$\alpha \sum_{\tau'} C_{\tau\tau'} s(\tau') + \Delta \omega \sum_{\omega} K(\tau,\omega) \sigma_A(\omega) - \bar{D}_A(\tau) = 0,$$

where $\sigma_A(\omega) = m(\omega) \exp\{\sum K(\tau,\omega)s(\tau)\}$

This equation can be written as $\frac{\partial U}{\partial s} = 0$ with

$$U[s] = \frac{\alpha}{2} \sum_{\tau'\tau''} s(\tau) C_{\tau\tau'} s(\tau') + \int d\omega \sigma_A(\omega) - \sum_{\tau} s(\tau)\bar{D}_A(\tau).$$

$\Rightarrow$ potential minimization problem in $N_r/2$ dimensions
Error estimation:

- we want to see the particle peaks
- peak integral is pretty much insensitive to $\alpha$.

Example: Gaussian mock data, peak region $\omega a = [0.3 : 1.1]$

- we give the average peak height, and estimate the error by jackknife method.
Quarkonium Correlators and Spectral Functions from Anisotropic Lattice QCD

Alexander Velytsky
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and
HEP Division and Physics Division,
Argonne National Laboratory, 9700 Cass Ave., Argonne, IL 60439, USA

with A. Jakovac, P. Petreczky and K. Petrov
Phys.Rev.D75:014506,2007; hep-lat/0611017

Short summary

Using point heavy meson (cc and bb) operators we measure meson correlators on anisotropic lattices $\xi = a_s/a_T = 2$ and 4. Quenched approximation with standard Wilson action in the gauge sector and the anisotropic clover improved action for heavy fermions is used. Sommer scale is used to fix the physical units. Maximum Entropy Method (MEM) is used to extract spectral functions from euclidean correlators.

We find that:

- MEM can resolve finite width bound states.
- The $1S (\eta_c, J/\psi)$ charmonium correlators do not change in the deconfined phase up to $T \approx 1.5T_c$.
- The $1S$ spectral function at this temperature within precision of MEM corresponds to zero temperature spectral function.
- $1P (\chi_{c0}, \chi_{c1})$ charmonium correlators and spectral function show significant change at $1.1T_c$.
- Bottomonium states show similar behavior.
Test of MEM

Figure: The $\gamma = 0.01$ Breit-Wigner (left) and Dirac's delta (4 state) (right) spectral function reconstruction.
Charmonium: $T = 0$

Figure: Charmonium spectral function in the pseudo-scalar channel (left) and the scalar channel (right) at different lattice spacings and zero temperature.
The ratio \( G(\tau, T)/G_{\text{recon}}(\tau, T) \) of charmonium for the pseudo-scalar channel at \( a_s^{-2} = 8.18 \) and 14.11 GeV at different temperatures.
Figure: The ratio $G(\tau, T)/G_{\text{recon}}(\tau, T)$ of charmonium for the scalar channel at $a_t^{-2} = 8.18$ and $14.11 \text{GeV}$ at different temperatures.
Figure: Charmonium spectral functions in the pseudo-scalar channel at $a_t^{-2} = 14.11\text{GeV}$ at zero and above deconfinement temperatures.
I present results on quarkonia correlators in finite temperature lattice QCD. The behavior of the pseudoscalar correlators is consistent with no significant change in this channel on crossing deconfinement, while the vector channel shows a small modification which can be accounted for by the transport contribution in the $\omega \sim 0$ regime. Large modifications observed in the scalar and axial vector channels can also be accounted for by the small $\omega$ contributions. A quasiparticle description explains the small $\omega$ regime quite well. Based on works done with F.Karsch, A. Jakovác, P. Petreczky, I. Wetzorke.
Extracting the spectral function from the correlators involves an ill-defined inversion process, which can be regularized by a "maximum entropy" analysis. This involves supplying prior information about the high $\omega$ part of the spectral function. At high temperatures, low $\omega$ part is sensitive to the prior information about high $\omega$ part.

- Supply high $\omega$ information from low temperature studies.
- Compare high temperature correlators with correlators "reconstructed" from spectral function at low temperatures.

Reconstructed correlator

$$G_{\text{recon},T^*}(\tau, T) = \int d\omega \sigma(\omega, T^*) K(\omega, \tau, T)$$

Deviation of $G(\tau, T)$ from $G_{\text{recon}}(\tau, T)$ indicates medium modification.
1S charmonia survive till high temperatures

Datta et al., PRD 2004

Small modification in correlators does not necessarily imply small changes in the states. Mocsy and Petreczky, PRD 2008
Temperature modification of vector correlator: related to transport?

Petreczky & Teany, PRD 73, 014508('06)

\[ \{ \rho_{00}^V(\omega), \rho_{ii}^V(l\omega l) \}_{\text{free}} = \frac{2\beta}{\pi} \omega \delta(\omega) \int dp p^2 \{-1, \frac{p^2}{p_0^2}\} e^{\beta p_0} n_F(p_0)^2 \]

Datta, Jakovác, Karsch, Petreczky, PANIC 2005

\[ G_{\text{sub}}(\tau) = G(\tau) - G_{\text{recon}}(\tau) \]

In interacting theory, \( \delta(\omega) \) in \( \rho_{ii}^V \) gets smeared to a Lorentzian

\[ \frac{1}{\pi} \frac{\eta}{\omega^2 + \eta^2} \]

where \( \eta \) is drag coefficient. Current data not good enough to distinguish diffusion from free streaming.
Much larger changes are seen in the scalar and axial vector correlators. The changes are dominated by contributions of zero modes.

\[ \rho_{\text{free}}^{\{SC,AX\}}(\omega \sim 0) = \frac{2\beta}{\pi} \omega \delta(\omega) \int dp \frac{2}{p_0^2} \left[ \frac{m^2}{p_0^2} + 1 + 2 \frac{m^2}{p_0^2} \right] e^{-\beta p_0} n_F(p_0)^2 \]
A quasiparticle description can describe the "zero mode" contributions reasonably well.

Petreczky, Quark Matter 2008

Mass of quasiparticles can be obtained from $G_{00}^V$

$$\frac{\chi_{00}}{T^3} \sim 12 \left( \frac{M_{\text{eff}}}{T} \right)^{3/2} e^{-M_{\text{eff}}/T}$$

$M_{\text{eff}}$ is very close to charm mass for $T > 1.5 T_c$ but becomes large as one approaches $T_c$. Zero modes in the other channels can be explained reasonably well using $M_{\text{eff}}$. 
Static quark-antiquark pairs at finite temperature

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In a framework that makes close contact with modern effective field theories for non-relativistic bound states at zero temperature, we have studied the real-time evolution of a static quark-antiquark pair in a medium of gluons and light quarks at finite temperature. For temperatures $T$ ranging from values larger to smaller than the inverse distance of the quark and antiquark, $1/r$, and at short distances, we have derived the potential between the two static sources, their energy and thermal decay width. Two mechanisms contribute to the thermal decay width: the imaginary part of the gluon self energy induced by the Landau damping phenomenon, and the quark-antiquark color singlet to color octet thermal break up. Parametrically, the first mechanism dominates for temperatures such that the Debye mass $m_D$ is larger than the binding energy, while the latter dominates for temperatures such that $m_D$ is smaller than the binding energy. If $m_D$ is of the same order as $1/r$, our results are in agreement with Laine et al. JHEP 0703(07)054. For temperatures smaller than $1/r$, we find new contributions to the potential, both real and imaginary, which may be relevant to understand the onset of heavy quarkonium dissociation in a thermal medium. Based on arXiv:0804.0993.
Framework: EFTs

\[ m \]
\[ \frac{1}{r} \sim mv \]
\[ V \sim mv^2 \]

\[ \text{QCD} \]
\[ \text{NRQCD} \]
\[ \text{pNRQCD} \]
\[ \text{pNRQCD}_{\text{HTL}} \]
\[ \text{NRQCD}_{\text{HTL}} \]
\[ T \]
\[ m_P \]

We assume that bound states exist for
- \( T < m \)
- \( \frac{1}{r} \sim mv > m_P \)

We neglect smaller thermodynamical scales.
Static quark antiquark at \( T \lesssim V \): energy and width

The real part of the diagram gives:

\[
\delta E = \frac{2}{3} N_c C_F \frac{\alpha_s^2}{\pi} r T^2 f(\frac{N_c \alpha_s}{(2rT)}) \quad , \quad f(z) \equiv \int_0^\infty dx \frac{x^3}{e^x - 1} P \frac{1}{x^2 - z^2}
\]

The imaginary part of the diagram gives

\[
\Gamma = \frac{N_c^3 C_F}{6} \frac{\alpha_s^4}{r} n_B(\frac{N_c \alpha_s}{(2r)})
\]

- Corrections coming from the scale \( m_D \) are suppressed by powers of \( m_D/T \).
- The width \( \Gamma \) originates from the fact that thermal fluctuations of the medium at short distances may destroy a color-singlet \( \bar{Q}Q \) into an octet plus gluons. This process is specific of QCD at finite \( T \); in QCD the relevant diagrams are of the type
Static quark antiquark at $1/r \gg T \gg m_D \gg V$

energy and width

$$\delta E = \frac{\pi}{9} N_c \frac{C_F}{3} \alpha_s^2 \frac{r^2}{T^2} \frac{\alpha_s}{\pi} \frac{r^2}{T} m_D^2 \frac{4}{3} \zeta(3) \frac{C_F}{3} \alpha_s^2 \frac{r^2}{T^3}$$

$$\Gamma = \frac{N_c^2 \frac{C_F}{3}}{3} \alpha_s^2 \frac{r^2}{T}$$

$$- \frac{C_F}{3} \alpha_s \frac{r^2}{T} \frac{m_D^2}{T} \left( 2 \gamma_E - \ln \frac{T^2}{m_D^2} - 1 - 4 \ln 2 - 2 \frac{\zeta(2)'}{\zeta(2)} \right) - \frac{8\pi}{9} \ln 2 N_c \frac{C_F}{3} \alpha_s^2 \frac{r^2}{T^3}$$

※ The non-thermal part of the static energy is the Coulomb potential $-C_F \alpha_s / r$.

※ The thermal width has two origins. The first term comes from the thermal break up of a quark-antiquark color singlet state into a color octet state. The other terms come from imaginary contributions to the gluon self energy that may be traced back to the Landau damping phenomenon. The first one is specific of QCD, the second one would also show up in QED. Having assumed $m_D \gg V$, the term due to the singlet to octet break up is parametrically suppressed by $(V/m_D)^2$ with respect to the imaginary gluon self-energy contributions.
Static quark antiquark at $T \gg 1/r \sim m_D$: real part

HTL propagators

potential contribution

mass contribution

$$\delta E = Re \left[ 2 e_m + \delta_1(r) \right] = -C_F e_m m_D - \frac{q_s^2}{r} e^{-m_D r}$$
Static quark antiquark at $T \gg 1/r \sim m_D$: imaginary part

Singlet to octet break up contribution

Damping rate of a static quark/antiquark

Landau damping contribution

$$\Gamma = -2\text{Im}\, \delta V_s(r) = \frac{N_c^2 C_F}{6} \alpha_s^3 T + 2 C_F \alpha_s T \left[ 1 - \frac{2}{r m_D} \int_0^\infty dx \frac{\sin(m_D r x)}{(x^2 + 1)^2} \right]$$

\begin{align*}
\text{a)} & \quad \sim \alpha_s m_D \times \left( \frac{V}{m_D} \right)^2 \times \frac{T}{m_D} \\
\text{b)} + \text{c)} & \quad \sim \alpha_s m_D \times \frac{T}{m_D} \gg \alpha_s m_D
\end{align*}

Pisarski PRD 47(93)5559

Laine Philipsen Romatschke Tassler JHEP 0703(07)054
Heavy quarkonium according to resummed perturbation theory

Mikko Laine (University of Bielefeld, Germany)

Miscellaneous remarks on:
1. Real-time static potential at finite temperature
2. Relation of static potential and quarkonium spectral function
3. Physics lessons for the dilepton production rate
4. Mystery with the scalar channel
At weak coupling:

What is the static limit of the time-ordered HTL-resummed gluon propagator in **real Minkowski time**?

\[
iD_{00}^T(0, q) = \frac{1}{\mathbf{q}^2 + m_D^2} - i\frac{\pi m_D^2 T}{|\mathbf{q}|(\mathbf{q}^2 + m_D^2)^2}.
\]

Beraudo et al, 0712.4394
So, there is a complex potential!

\[
\text{Re} \ V_>(\infty, r) = -\frac{g^2 C_F}{4\pi} \left[ m_D + \frac{\exp(-m_D r)}{r} \right],
\]

\[
\text{Im} \ V_>(\infty, r) = -\frac{g^2 T C'_F}{4\pi} \phi(m_D r),
\]

where

\[
\phi(x) = 2 \int_0^\infty \frac{dz \ z}{z^2 + 1} \left[ 1 - \frac{\sin(zx)}{zx} \right],
\]

is finite and strictly increasing, with the limiting values \( \phi(0) = 0, \phi(\infty) = 1. \)
What happens after the insertion of $V>(\infty, r)$?

(a) $T \sim g^2 M$
$\Rightarrow m_D r \sim g Tr \sim g^3 M r \sim g$
$\Rightarrow \text{Re} \, V_> \sim g^2 \exp(-m_D r)/r \sim g^4 M$
$\Rightarrow \text{Im} \, V_> \sim g^2 T (m_D r)^2 \sim g^6 M$
$\Rightarrow \text{width} \ll \text{binding energy} \Rightarrow \text{bound state exists.}$

(b) $T \sim g M$
$\Rightarrow m_D r \sim g Tr \sim g^2 M r \sim 1$
$\Rightarrow \text{Re} \, V_> \sim g^2 \exp(-m_D r)/r \sim g^4 M$
$\Rightarrow \text{Im} \, V_> \sim g^2 T \phi(m_D r) \sim g^3 M$
$\Rightarrow \text{width} \gg \text{binding energy} \Rightarrow \text{bound state has melted.}$

Burnier et al 0711.1743
Melting of the spectral fcn in the vector channel:

Basic structure as suggested by Matsui and Satz (1986) from phenomenological arguments. Melting temperature $\sim$ consistent with potential models and lattice QCD within $\pm 50$ MeV.
But tables turn for dilepton yield! \( \frac{dN_{\mu^+\mu^-}}{d^4x} \frac{d^4Q}{d^4Q} \propto \frac{\rho(\omega)}{\omega^2} e^{-\frac{\omega}{T}} \)

Burnier et al 0711.1743

No need to bind in order to produce a structure.
Quarkonia propagation in a hot-dense medium

Andrea Beraudo
ECT*-Trento and University of Torino

BNL, 23rd - 25th April 2008

Work done in collaboration with J.P. Blaizot and C. Ratti
arXiv:0712.4394 [nucl-th], to appear on NPA.

I will present a study of the in-medium propagator of a $Q\bar{Q}$ pair in the complex-time plane, focusing on some very general aspects of the problem.

The final goal is to give some solid basis to address the issue of propagation of quarkonia in the QGP.
The basic object of our study

\[ G^>(t,r_1; t,r_2|0,r'_1; 0,r'_2) \equiv \langle \chi(t,r_2)\psi(t,r_1)\psi^\dagger(0,r'_1)\chi^\dagger(0,r'_2) \rangle \]

\[ J_M(t) \quad J_M^\dagger(0) \]

QED toy-model

A \(Q\bar{Q}\) pair in a plasma of photons, electrons and positrons

\[ \mathcal{L}_{\text{QED}}^{M=\infty} = \mathcal{L}_{\text{em}} + \mathcal{L}_{\text{light}} + \psi^\dagger i(\partial_0 - igA_0)\psi + \chi^\dagger i(\partial_0 + igA_0)\chi \]

\[ \text{heavy } Q \quad \text{heavy } \bar{Q} \]
The strategy

- Consider the $Q\bar{Q}$ propagation in a given background configuration of the gauge-field $A_\mu$

$$G_A(t, r_1; t, r_2|0, r'_1; 0, r'_2) = \delta(r_1 - r'_1)\delta(r_2 - r'_2) \times$$

$$\times \exp \left( ig \int_0^t dt' A_0(r_1, t') \right) \exp \left( -ig \int_0^t dt' A_0(r_2, t') \right)$$

- Average over the gauge-field configuration with an action accounting for thermal effects

$$G^>(t, r_1; t, r_2|0, r'_1; 0, r'_2) = Z^{-1} \int [DA] G_A(t, r_1; t, r_2|0, r'_1; 0, r'_2) e^{iS[A]}$$

Which is the action to employ to weight the field configurations?
Real-time $Q\bar{Q}$ propagator

$\Rightarrow Q\bar{Q}$ current non-vanishing along $C_+$:

$$\bar{G}(t, r_1 - r_2) = \exp \left[ -\frac{i}{2} \int_{C_+} d^4 x \int_{C_+} d^4 y \, J^\mu(x) D_{\mu\nu}(x - y) J^\nu(y) \right]$$

with

$$J^\mu(z) = \delta^{\mu0} \theta(z^0) \theta(t - z^0) \left[ -g\delta(z - r_1) + g\delta(z - r_2) \right]$$

$\Rightarrow$ One gets:

$$\bar{G}(t, r_1 - r_2) = \exp \left[ -2ig^2 \int \frac{d\omega}{2\pi} \int \frac{dq}{(2\pi)^3} \frac{1 - \cos(\omega t)}{\omega^2} \left( 1 - e^{i\omega \cdot (r_1 - r_2)} \right) D_{00}(\omega, q) \right]$$
\[ \Rightarrow \text{Large time behavior} \]

- **$Q\bar{Q}$ propagator**

\[ \overline{G}(t, r_1 - r_2) \sim \exp[-i V_{\text{eff}}(r_1 - r_2)t] \]

- **Temporal evolution equation ($\sim$ Schrödinger!)**

\[ \lim_{t \to +\infty} [i \partial_t - V_{\text{eff}}(r_1 - r_2)]\overline{G}(t, r_1 - r_2) = 0 \]

where:

\[ V_{\text{eff}}(r_1 - r_2) \equiv g^2 \int \frac{dq}{(2\pi)^3} \left( 1 - e^{iq \cdot (r_1 - r_2)} \right) D_{00}(\omega = 0, q) \]

\[ = g^2 \int \frac{dq}{(2\pi)^3} \left( 1 - e^{iq \cdot (r_1 - r_2)} \right) \left[ \frac{1}{q^2 + m_D^2} - i \frac{\pi m_D^2 T}{|q|(q^2 + m_D^2)^2} \right] \]

\[ = -\frac{g^2}{4\pi} \left[ m_D + \frac{e^{-m_D r}}{r} \right] - i \frac{g^2 T}{4\pi} \phi(m_D r) \]
Imaginary-time $Q\bar{Q}$ propagator

⇒ Analyticity of $G^>(t) \rightarrow$ simply set $t = -i\tau$ with $\tau \in [0, \beta]$

$$G(-i\tau, r_1 - r_2) = \exp \left[ g^2 \int_0^\tau d\tau' \int_0^\tau d\tau'' \int \frac{dq}{(2\pi)^3} \left( 1 - e^{i\mathbf{q}(r_1 - r_2)} \right) \Delta_{00}(\tau' - \tau'', \mathbf{q}) \right]$$

⇒ Propagation till $\tau = \beta$:

$$G(-i\beta, r_1 - r_2) = \exp \left\{ -\beta g^2 \int \frac{dq}{(2\pi)^3} \left( 1 - e^{i\mathbf{q}(r_1 - r_2)} \right) \frac{1}{\mathbf{q}^2 + m_D^2} \right\}$$

Since:

$$G(-i\beta, r_1 - r_2) = \exp \left( -\beta \Delta F_{Q\bar{Q}}(r, T) \right)$$

One gets the $Q\bar{Q}$ free-energy:

$$\Delta F_{Q\bar{Q}}(r, T) = -\frac{g^2 m_D}{4\pi} - \frac{g^2}{4\pi} \frac{e^{-m_D r}}{r},$$

It coincides with the real part of the effective potential!
Motivation: expansion $\rightarrow$ anisotropy $\leftrightarrow$ viscosity

Covariant-gauge gluon propagator in aniso. QGP

Static limit: heavy-quark potential

Results (binding energy of small states)
Motivation: expanding plasma always (slightly) out of equilibrium

\[ f(p) = f_{iso}(\sqrt{p^2 + \xi p_z^2}) \]

anisotropy parameter

- Relation to viscosity: \( \xi = \frac{10}{T_T} \frac{\eta}{s} \)

Covariant-gauge gluon propagator in anisotrop. QGP

1) HTL (retarded) self-energy

\[ \Pi^{\mu\nu} = g^2 \int \frac{d^3 k}{(2\pi)^3} v^\mu \frac{\partial f(k)}{\partial k^\beta} \left( g^{\nu\beta} - \frac{v^\nu p^\beta}{p \cdot v + i\epsilon} \right) \]

\[ (\Delta^{-1})^{\mu\nu}(p, \xi) = -p^2 g^{\mu\nu} + p^\mu p^\nu - \Pi^{\mu\nu}(p, \xi) - \frac{1}{\lambda} p^\mu p^\nu \]

invert: \( (\Delta^{-1})^{\mu\sigma} \Delta^{\sigma\nu} = g^{\mu\nu} \)

static limit:

\[ \Delta^{00}(\omega = 0, p) = \frac{p^2 + m^2 + m^2}{(p^2 + m^2 + m^2)(p^2 + m^2) - m^4} \]
"Mass" scales:

\[
m^2_\alpha = - \frac{m_D^2}{2p_1^2 \sqrt{\xi}} \left( p_z^2 \arctan \sqrt{\xi} - \frac{p_z p^2}{\sqrt{p^2 + \xi p_1^2}} \arctan \frac{\sqrt{\xi} p_z}{\sqrt{p^2 + \xi p_1^2}} \right)
\]

\[
m^2_\beta = \frac{m_D^2}{2 \sqrt{\xi} (1 + \xi)(p^2 + \xi p_1^2)} \left[ (\sqrt{\xi} + (1 + \xi) \arctan \sqrt{\xi}) (p^2 + \xi p_1^2) + \xi p_z \left( p_z \sqrt{\xi} + \frac{p^2 (1 + \xi)}{\sqrt{p^2 + \xi p_1^2}} \arctan \frac{\sqrt{\xi} p_z}{\sqrt{p^2 + \xi p_1^2}} \right) \right]
\]

\[
m^2_\gamma = - \frac{m_D^2}{2} \left( \frac{p^2}{\xi p_1^2 + p^2} - \frac{1 + 2p_1^2}{\sqrt{\xi}} \arctan \sqrt{\xi} + p_z \frac{p^2 (2p^2 + 3 \xi p_1^2)}{\sqrt{\xi} (\xi p_1^2 + p^2) \frac{3}{2} p_1^2} \arctan \frac{\sqrt{\xi} p_z}{\sqrt{p^2 + \xi p_1^2}} \right)
\]

\[
m^2_5 = - \frac{\pi m_D^2 \xi p_z p_1 |p|}{4(\xi p_1^2 + p^2)^{3/2}}.
\]

**Potential: F.T. of static propagator**

\[
V(r, \xi) = -g^2 C_F \int \frac{d^3p}{(2\pi)^3} e^{i p \cdot r} \Delta^{00}(\omega = 0, p, \xi)
\]
\[ V(r \parallel \hat{z}, \xi \ll 1) = V_{iso}(r) \left[ 1 + \xi \left( \frac{2 e^\xi - 1}{\hat{r}^2} - \frac{2}{\hat{r}} - 1 - \frac{\hat{r}}{6} \right) \right] \]

valid for \( \hat{r} \equiv r m_D \ll 1 \)

\[ V(r \perp \hat{z}, \xi \ll 1) = V_{iso}(r) \left[ 1 + \xi \left( \frac{1 - e^\xi}{\hat{r}^2} + \frac{1}{\hat{r}} + \frac{1}{2} + \frac{\hat{r}}{3} \right) \right] \]

- weak anisotropy \( \xi \ll 1 \): angular dependence of screening

\[ V(\hat{r}) = V_{\text{vac}}(\hat{r}) \]

- screening weaker for \( r \parallel z \)

- \( r \parallel z \) versus \( r \perp z \):
  - angular dependence stronger at larger distance
  - as \( r \rightarrow 0 \), angular dependence disappears
• consider QQ state of size $\langle r^2 \rangle \sim r_{\text{med}}$ and $T$ sufficiently high so that $r_{\text{med}} < \sqrt{\alpha} / \sigma$: screened Coulomb dominates, potential and its angular dependence can be determined from pQCD

• anisotropy ($\xi > 0$) affects binding energy

$$\frac{\Delta E}{E_C} \approx \frac{4}{\alpha_s C_F} \frac{m_D}{m_Q} \left(-1 + \frac{\xi}{6} + \cdots\right)$$

ground state, $1 \gg \xi \gg m_D / (\alpha_s C_F m_Q)$
Quarkonia in the deconfined phase: Potential Models and Correlators

W.M. Alberico

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Abstract

Results are presented by employing a temperature dependent $Q\bar{Q}$ potential extracted from fits to lattice free energy calculations (with $N_f = 0, 2, 3$) and then plugged into Schroedinger equation. From the binding energies of the $Q\bar{Q}$ pair we obtain dissociation temperatures both for charmonium and bottomonium states, which agree with lattice estimates. We also evaluate mesonic spectral functions and correlators and find good qualitative agreement with the ratio of correlators (to the "reconstructed one") in all channels but for the pseudoscalar (including zero-mode contribution).

References: • W.M.A., A. Beraudo, A. De Pace, A. Molinari, Phys. Rev. D72, 114011 (2005); • Phys. Rev. D75, 074009 (2007); • arXive 0706.2846
• Extract the singlet internal energy (=potential energy for infinitely heavy sources). From $F = U - TS$:

$$U_1 = -T^2 \frac{\partial (F_1 / T)}{\partial T}.$$  

• Plug into Schrödinger equation

$$\left[ -\frac{\nabla^2}{2\mu} + V_1(r, T) \right] \psi(r, T) = \varepsilon(T) \psi(r, T),$$

• Quarkonium mass is given by:

$$M(T) = 2m_{c(b)} + \varepsilon(T) + U_1(r \to \infty, T);$$
\begin{table}
\centering
\begin{tabular}{|c|cc|cc|}
\hline
 & \multicolumn{2}{c|}{N_f = 0} & \multicolumn{2}{c|}{N_f = 2} \\
\hline
 & m_c = 1.4 GeV & m_c = 1.6 GeV & m_c = 1.4 GeV & m_c = 1.6 GeV \\
\hline
J/\psi, \eta_c & 1.40 & 1.52 & 1.45 & 1.59 \\
\chi_c & < 1 & < 1 & 1.00 & 1.00 \\
\psi' & < 1 & < 1 & 0.98 & 0.99 \\
\hline
 & m_b = 4.3 GeV & m_b = 4.7 GeV & m_b = 4.3 GeV & m_b = 4.7 GeV \\
\hline
Y, \eta_b & 2.96 [4.5] & 3.18 & 3.9(*) [6.7(*)] & 4.4(*) \\
\chi_b & 1.13 [1.55] & 1.15 & 1.15 [1.63] & 1.17 \\
Y' & 1.12 [1.40] & 1.14 & 1.13 [1.43] & 1.15 \\
\hline
\end{tabular}
\caption{Dissociation temperatures}
\end{table}
With eigenfunctions and eigenvalues of Schroedinger equation we evaluate the spectral function as the imaginary part of the \( Q\bar{Q} \) propagator:

\[
\sigma_M(\omega, T) = \frac{1}{\pi} \text{Im} G_M(\omega) = \sum_n |\langle 0 | j_M | n \rangle|^2 \delta(\omega - E_n)
\]

\[
= \sum_n F^2_{M,n} \delta(\omega - M_n) + \theta(\omega - s_0) F^2_{M,\omega-s_0}
\]

where \( E_n = s_0 + \epsilon_n \), \( s_0 \) is the continuum threshold:

\[
s_0(T) = 2m + U_{1Q\bar{Q}}(r \to \infty, T)
\]

NB Continuum is taken in two ways:
- from corresponding solution of Schroedinger equation [consistent with present approach]
- from QCD perturbative calculations

The resulting spectral functions are very sensitive to the continuum employed. The non-perturbative one produces sizable enhancement near
threshold.

**Correlators from potential model**

The Euclidean correlators follows from convolution of spectral functions and thermal Kernel:

\[
G_M(\tau, T) = \sum_n F_{M,n}^2 K(\tau, M_n, T) + \int_0^\infty d\epsilon F_{M,\epsilon}^2 K(\tau, \epsilon + s_0, T).
\]

On the lattice the following ratio is evaluated:

\[
R = \frac{G_M(\tau, T)}{G_M^{\text{rec}}(\tau, T, T_<)} = \frac{\int_0^\infty d\omega \sigma_M(\omega, T) K(\tau, \omega, T)}{\int_0^\infty d\omega \sigma_M(\omega, T_<) K(\tau, \omega, T)}
\]

The denominator is calculated using the kernel at \( T > T_c \) and the MEM spectral function at some reference temperature \( T_< \) (reconstructed correlator).
BUT... by including the effect of zero modes in spectral functions

P-wave ratio of correlators

(G.Aarts et al, arXiv: 0705.2198 [hep-lat])
The charmonium wave functions at finite temperature from lattice QCD calculations

T. Umeda, H. Ohno (Univ. of Tsukuba) for the WHOT-QCD Collaboration

We investigated $T_{\text{diss}}$ of charmonia from Lattice QCD using another approach to study charmonium at $T > 0$ without Bayesian analysis
- boundary condition dependence
- Wave function (Volume dependence)

No evidence for unbound $c\bar{c}$ quarks up to $T = 2.3 \, T_c$

RBRC Workshop, BNL, NY, USA, 23 April 2008
Current status on charmonium $T_{\text{dis}}$

- Lattice QCD studies (by MEM analysis) indicate
  - $J/\psi$ may survive up to $T=1.5T_c$ or higher
  - $\chi_c$ may dissolve just above $T_c$
    e.g. A.Jakovac et al. (2007)
  - no results on excited states, $\psi'$

- The 2nd statement may be misleading (1)
  small change even in P-wave state
  up to $1.4T_c$ w/o the constant mode

- On the other hand,
  the potential model studies suggest
  charmonium dissociation may also
  provide small change in the correlators
  e.g. A.Mocsy et al. (2007)

Therefore we would like to investigate $T_{\text{dis}}$
using new approaches with Lattice QCD
without Bayesian analyses

Fig: Temp. dependence of $M_{\text{eff}}(t)$
for $J/\psi$, $\chi_{c1}$ w/o constant mode.
T.Umeda (2007)
Bound state or scattering state?

We know three ways to identify the state in a finite volume:

- **Volume dependence**
  - $E$: energy
  - $V$: volume
  - $\Phi(r)$: wave function
  - $r$: $c - \bar{c}$ distance

- **Wave function**

- **Boundary Condition (B.C.) dep.**

---

RBRC Workshop

T. Umeda (Tsukuba)

H. Iida et al. ('06), N. Ishii et al. ('05)
Lattice setup

- Quenched approximation (no dynamical quark effect)
- Anisotropic lattices
  - lattice spacing: $a_s = 0.0970(5)$ fm
  - anisotropy: $a_s/a_t = 4$
- $r_s=1$ to suppress doubler effects
- Variational analysis with $4 \times 4$ correlation matrix

<table>
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<th>$N_t$</th>
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<th>26</th>
<th>20</th>
<th>16</th>
<th>12</th>
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<tbody>
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<td>$T/T_c$</td>
<td>0.88</td>
<td>1.08</td>
<td>1.40</td>
<td>1.75</td>
<td>2.33</td>
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<tr>
<td>V=32$^3$</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>100</td>
</tr>
</tbody>
</table>
Temperature dependence of charmonium spectra

\[ q(x_i + L_i) = b_i q(x_i) \]

\( b_i = 1 \) : periodic
\( b_i = -1 \) : anti-periodic

PBC : \( b = (1, 1, 1) \)
APBC : \( b = (-1, -1, -1) \)
HBC : \( b = (-1, 1, 1) \)

an expected gap in \( V = (2fm)^3 \)
(free quark case)
\~ 200\,MeV

\[ \square \text{ No significant differences in the different B.C.} \]
[\text{\square Analysis is difficult at higher temperature (} 2T_c\sim \)]
Volume dependence at $T=2.3T_c$

- Clear signals of bound states even at $T=2.3T_c$ (!)
- Large volume is necessary for $P$-wave states.

RBRC Workshop
T. Umeda (Tsukuba)
Spectral Functions of
One, Two, and Three Quark Operators
in the Quark-Gluon Plasma

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1. Importance of quark spectral functions (one, two, three,...quarks)
2. Review of previous results and the maximum entropy method
3. Formulation of analysis of the baryon spectral function
4. Baryon spectral functions below and above Tc
5. Summary
Microscopic Understanding of QGP

Importance of Microscopic Properties of matter, in addition to Bulk Properties

- In condensed matter physics, common to start from one particle states, then proceed to two, three, ... particle states (correlations)

Spectral Functions:

- One Quark
  - need to fix gauge  Kitazawa's talk
- Two Quarks
  - mesons
    - color singlet
    - octet  need to fix gauge
  - diquarks  need to fix gauge
- Three Quarks
  - baryons
- ......
Spectral Functions above $T_c$

$m_{ud} \ll m_s \sim T_c \ll m_c < m_b$

Asakawa, Nakahara & Hatsuda [hep-lat/0208059]
$J/\psi$ non-dissociation above $T_c$

Asakawa and Hatsuda, PRL 2004

$J/\psi (p = 0)$ disappears between $1.62T_c$ and $1.70T_c$.

Lattice Artifact
ccc baryon channel
Scattering Term (two body case)

\[
\begin{align*}
J_N & \\
\vec{p} & = 0
\end{align*}
\]

- This term is non-vanishing only for \(|\omega_2 - \omega_1| \leq |m_2 - m_1|\)

(Boson-Fermion case, e.g. Kitazawa et al., 2008)
Summary

- It is important to understand one, two, three...quark spectral functions for the understanding of QGP.

- Mesons and Baryons (new!) exist well above $T_c$.

- A sharp peak with negative parity is observed. This can be due to diquark-quark scattering term and imply the existence of diquark correlation above $T_c$.

- Direct measurement of SPF of one and two quark operators with MEM is desired.
Energy-momentum correlators and viscosity

Harvey Meyer

Center for Theoretical Physics, M.I.T.

Understanding QGP through Spectral Functions and Euclidean Correlators, Brookhaven 23-25 april 2008

I first discuss the problem of solving the equation
\[ C(x_0) = \int_0^{\infty} d\omega \; \rho(\omega) \frac{\cosh(\omega L/2 - x_0)}{\sinh(\omega L/2)} \]
for the spectral function \( \rho(\omega) \), knowing the Euclidean correlator \( C(x_0) \). I point out that a linear method allows one to quantify the systematic error through the resolution function prior to the acquisition of any lattice Monte-Carlo data.

I then show results for shear and bulk viscosity obtained from simulations of SU(3) gluodynamics. As an improvement of the method, I solve for a function that is smooth everywhere and goes to zero at high frequencies. This is achieved by taking appropriate linear combinations of spectral functions. Near \( T_c \), the observed overall increase of the \( T_{ii} \) two-point function suggests a corresponding increase in the bulk viscosity.
The functions $u_i(\omega) \quad (\hat{p}(\omega) = \sum_{i=1}^{N} c_i u_i(\omega))$
What is the error due to finite $N$? $x_{0}^{0} = L_{0}/2(1 - i/2(N - 1))$

$$\tilde{\rho}(\omega) = \int \tilde{\delta}(\omega, \omega')\rho(\omega')d\omega' \quad \tilde{\delta}(\omega, \omega') = \sum_{c=1}^{N} u_{c}(\omega)u_{c}(\omega').$$

$\tilde{\delta}(\omega, \omega')$ is called the resolution function.
Correlators in the confined phase

Tree level improved correlators at $T=T_c/2, (20^4, 28^4)$

- In the scalar channel, the two stable glueballs almost saturate the correlator beyond 0.5fm
- Calculation made possible by the multi-level algorithm (HM '03)
\[ \Delta \rho(T, \omega) = \rho_{ii, kk}(T, \omega) - \rho_{ii, kk}(0, \omega) + \rho_p(\omega) \]

<table>
<thead>
<tr>
<th>$\omega / T$</th>
<th>$\langle \omega^0 \rangle$</th>
<th>$\langle \omega^2 \rangle / 120 T^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.24 T_c$</td>
<td>5.08(72)</td>
<td>4.3(2)</td>
</tr>
<tr>
<td>$1.02 T_c$</td>
<td>33.6(6.3)</td>
<td>3.9(1)</td>
</tr>
</tbody>
</table>

Assuming for instance $\Delta \rho / (2T \tanh(\omega / 2T)) = \frac{9 \kappa}{\pi} \frac{r^2}{\omega^2 T^2}$ with $\Gamma = 2T \text{ or } 4T$, I get $\zeta / s \approx 1.3 \text{ or } 1.0$. 
understanding of transport coefficients beyond PQCD ⇒ lattice QCD

- linear methods are not restricted to positive integrands, and their performance is known before any data is available (resolution function $\delta(\omega, \omega')$)

- high statistics, control of cutoff effects, and making $\Delta \rho(\omega) \rightarrow \infty 0$ is more likely to help than increasing $N_T$

- you have to know in advance that you are dealing with a smooth $\omega$-integrand

- how large does the bulk viscosity really get close to $T_c$?

- are there quasiparticles responsible for transport properties in the QGP at $T_c < T < 3T_c$?

- elliptic flow at LHC?
Bulk Viscosity in SU(2) Gauge Theory

Kay Hübner

with Frithjof Karsch and Claudio Pica
Brookhaven National Lab

We study euclidean-time correlators of the energy-momentum tensor in 3+1 dim SU(2) gauge theory on lattices with $N_T = 4$. We find that the correlator connected to the bulk viscosity diverges like the specific heat and that the corresponding spectral function has a $\delta$-function singularity at $T_c$. We give an estimate for the bulk viscosity $\zeta$.

Understanding QGP through Spectral Functions and Euclidean Correlators

April 24th 2008
local energy-momentum tensor on the lattice (std Wilson action):

\[ \Theta_{\mu\nu}(x) = B(g) \sum_{\rho \neq \nu} \frac{1}{N_c} \text{Re} \text{ Tr} \mathcal{P}_{\mu\nu}(x), \quad B(g) \text{ is the } \beta\text{-function} \]

Correlation function connected to the bulk viscosity \( \zeta \):

\[ G_c(\tau, T) = \int d^3x \Theta(x, \tau) \Theta(0, 0) \]

spectral function at vanishing momentum:

\[ G_c(\tau, T) = \int_0^\infty d\omega \rho_c(\omega, T) K(\tau, \omega, T) \quad \text{with} \quad K(\tau, \omega, T) = \frac{\cosh(\omega(\tau - \frac{\tau}{2T}))}{\sinh(\frac{\tau}{2T})} \]

Bulk viscosity \( \zeta(T) \) through Kubo-formula:

\[ \zeta(T) = \frac{\pi}{9} \lim_{\omega \to 0} \frac{\rho_c(\omega, T)}{\omega} \]
Fit data with ansatz, parameters: $A_\alpha$, $B_\alpha$, $C_\alpha$, may depend on $\tau$:

$$G_c(\tau, T_c) / T_c^2 = A_\alpha N_{et}^{1/\nu} \left( 1 + B_\alpha N_{et}^{-\Delta/\nu} \right) + C_\alpha$$

- Critical behavior that of 3d Ising
- $G_c(\tau, T_c)$ independent of $\tau$
- $R(\delta t, T_c) = 1$, thus $\mu_c/\omega$ has $\delta$-fct-like singularity at $T_c$

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>$A_\alpha$</th>
<th>$B_\alpha$</th>
<th>$C_\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2$</td>
<td>9.3(1.2)</td>
<td>-2.4(0.3)</td>
<td>26.3(2.7)</td>
</tr>
<tr>
<td>$1/4$</td>
<td>9.1(1.2)</td>
<td>-2.4(0.3)</td>
<td>8.4(2.7)</td>
</tr>
<tr>
<td>combined</td>
<td>9.1(1.3)</td>
<td>-2.39(50)</td>
<td>29.4(1.5)  ($\nu = 1/4$)</td>
</tr>
</tbody>
</table>

$R(\delta t, T_c)$ is independent of $\nu$. $\mu_c/\omega$ has $\delta$-fct-like singularity at $T_c$. 

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K. Hüllbrecht (Bonn) [Bulk Viscosity in SU(2)] [Understanding QCD] 3/6
Scaling with \( T \)

- Fit \( T \)-dependence with ansatz, parameters \( A_\pm, B_\pm, C, D \), all may depend on \( \tau \):

\[
G_c(\tau, T) / T^5 = A_\pm \tau^{-\alpha} (1 + B_\pm \tau^\beta) + C + D t
\]

- fit range: \( T / T_c \in [0.94, 1.05] \)
- \( A_\pm, B_\pm \) agree for both \( \tau T \) within errors and with combined fit
**Bulk Viscosity III**

- estimate $\zeta/T^3 = 0.01 - 0.1$ at $2T_c$, rises by $O(10)$ when going to $1.2T_c$

- convert to $s/T^3 \simeq 0.3$ at $T_c$ and $s/T^3 \simeq 2$ at $2T_c$

References:
Conclusions and Outlook

- studied euclidean-time correlators of the energy momentum tensor in 3+1 dim SU(2) gauge theory on lattices with $N_T = 4$
- $G_\zeta$ diverges at $T_c$ with the 3d Ising critical exponent $\alpha$
- $\rho_\zeta$ has $\delta$-function-like singularity at $T_c$
- bulk viscosity $\zeta$ may diverge stronger than $\alpha$, $\omega_0$ may vanish at $T_0$
- want to use larger $N_T$-lattices to control $\tau$-dependence of $G_\zeta$ and to extract spectral functions, determine $\zeta$, $\omega_0$, and $\eta$ etc.
Bulk viscosity in QCD

D. Kharzeev
BNL
Physical picture:

Shear viscosity: how much entropy is produced by transformation of shape at constant volume

Bulk viscosity: how much entropy is produced by transformation of volume at constant shape
Kubo’s formula:

$$\eta(\omega) \left( \delta_{ii} \delta_{km} + \delta_{lm} \delta_{kl} - \frac{2}{3} \delta_{ik} \delta_{lm} \right) + \zeta(\omega) \delta_{ik} \delta_{lm}$$

$$= \frac{1}{\omega} \lim_{k \to 0} \int d^3 x \int_0^\infty dt \, e^{i(\omega t - kr)} \langle [\theta_{ik}(t, \mathbf{r}), \theta_{lm}(0)] \rangle$$

Bulk viscosity is defined as the static limit of the correlation function:

$$\zeta = \frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} \int_0^\infty dt \, \int d^3 r \, e^{i\omega t} \langle [\theta_{ii}(x), \theta_{kk}(0)] \rangle$$
In perturbation theory, shear viscosity is “large”:

\[ \frac{\eta}{s} \sim \frac{1}{\alpha_s^2} \]

and bulk viscosity is “small”:

\[ \frac{\zeta}{s} \sim \alpha_s^2 \]

At strong coupling, \( \eta \) is apparently small;

can \( \zeta \) get large?
Confinement as seen by the off-equilibrium thermodynamics
Full QCD with (almost) physical quark masses


\[
2 \int_0^\infty \frac{\rho(u, 0)}{u} du = T_s \left( \frac{1}{c_s^2} - 3 \right) - 4(E - 3P) + \left( T \frac{\partial}{\partial T} - 2 \right) \langle m\bar{q}q \rangle^* \\
+ 16|\epsilon_v| + 6(M_{\pi}^2 f_{\pi}^2 + M_K^2 f_K^2).
\]
Universality and the chiral critical point

Critical exponents of 3-d $O(4)$ [21] using $\beta$ and $\delta$ as input and $Z(2)$ [22] symmetric spin models

<table>
<thead>
<tr>
<th>Model</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O(4)$</td>
<td>$-0.21$</td>
<td>$0.38$</td>
<td>$1.47$</td>
<td>$4.82$</td>
</tr>
<tr>
<td>$Z(2)$</td>
<td>$0.11$</td>
<td>$0.33$</td>
<td>$1.24$</td>
<td>$4.79$</td>
</tr>
</tbody>
</table>

Specific heat $c_V = \partial \mathcal{E} / \partial T$

- 2nd order phase transition in $N_f=2$ QCD at zero baryon density - $O(4)$ universality class
  $c_V(t) = ct^{-\alpha} + \text{const}$
  No singularity at $T_c$
  (a spike) $c_V \sim h^{-\gamma/\beta \delta}$

- Chiral critical point - $Z(2)$ universality class
  $\gamma/\beta \delta \simeq 0.8$
  Singularity at CP!

$h = A|T - T_c|/T_c + B|\mu_B - \mu_c|/\mu_c$
$T^\mu_\mu$ spectral weight and bulk viscosity

Where we can calculate it

Guy D. Moore (McGill University), with Omid Saremi

- Review of bulk viscosity, spectral weight
- Perturbative regime: kinetic theory
  * High frequency: rising cut
  * Low frequency: peak
- Near the critical point: universal scaling
  * Dynamical universality classes: QCD vs. liquid-gas
  * Critical slowing down and Bulk viscosity
- Summary and conclusions
Perturbative regime

Normalize so \( S = \int d^4x \frac{1}{2g^2} \text{Tr} \, G_{\mu\nu} G^{\mu\nu} \).

Do pure glue for simplicity. Conformal anomaly:

\[
T^\mu = \frac{\beta}{g^4} \text{Tr} \, G_{\mu\nu} G^{\mu\nu}, \quad \beta = \frac{\mu^2 d}{d\mu^2} g^2 \sim g^4.
\]

Evaluate Wightman correlator of \((\beta/g^4)G^2 - (1 - 3c_s^2)T_0^0\).

Leading diagram.

Note \((1 - 3c_s^2) \sim g^4\) is small;
\((1 - 3c_s^2)T_0^0\) is \(g^2\) suppressed.
Need to know width of peak

Bulk viscosity is \( G^>(\omega = 0)/T \). Need width of peak

Include imaginary parts on propagators: need ladders as well

Amounts to kinetic treatment. \( T^\mu_\mu \) in terms of \( f \):

\[
(T^i_i + 3c_s^2T^0_0) = \sum \int \frac{d^3p}{(2\pi)^3} \left[ (1 - 3c_s^2) p^2 + \frac{3\beta m^2_\infty}{g^2} \right] (f_0 + \delta f)
\]

Boltzmann equation

\[
\mathbf{v} \cdot \nabla f_0 + \partial_t f = -C[f]
\]

becomes

\[
\frac{f_0(1+f_0)}{ET} \left( \left[ \frac{1}{3} - c_s^2 \right] p^2 - \frac{\beta m^2_\infty}{g^2} \right) = -i\omega \delta f - C[f].
\]
Summary:

Shear viscosity

Bulk viscosity

\[ g^4 T^4 \]

\[ g^5 T^4 \]

\[ g^2 T^4 \]

\[ g T^4 \]

\[ T^4 \]

\[ \omega^4 \]

\[ g^4 \omega^4 \]

\[ g^5 \omega^4 \]

\[ g^2 \omega^4 \]
Another analytically tractable case

Critical region near second-order transition point:

Possible to compute parametric behaviors analytically
Slow dynamics: another low $\omega$ peak!
Transport Coefficients and nPI Methods
M.E. Carrington
Brandon University

In this talk I will discuss the calculation of the electrical conductivity in QED using the resummed 3PI effective action. I work to 3-loop order in the resummed effective action. I show that the formalism produces the integral equations that resum the pinching and collinear contributions, and thus that the full leading order contribution to the qed electrical conductivity can be obtained directly from the 3PI formalism. All leading order terms are produced naturally by the formalism, without the need for any kind of power counting arguments.

The result agrees with that obtained previously using kinetic theory. The calculation therefore provides a field theoretic connection to the kinetic theory approach. The method should be generalizable to the calculation of other transport coefficients, like the shear viscosity. In addition, it seems likely that quantum field theory provides a better framework than kinetic theory for calculations beyond leading order. The result of this calculation provides strong support for the use of nPI effective theories as a method to study the equilibration of quantum fields.
TRANSPORT COEFFICIENTS
AND $n$PI METHODS

tc measure efficiency with which a conserved quantity is transported over ‘long’ distances
(long compared to microscopic relaxation scales)

effective kinetic theory:
small deviations from thermal equilibrium
weak coupling
→ using equilibrium FT tools

will show:
3PI effective theory → same result ($\sigma_{\text{qed}}$)

motivation:
[1] in principle can use $n$PI far from equib
[2] possibility to go beyond leading order (?)
METHOD: 3PI EFFECTIVE ACTION

motivation:
need 3-loop diagram to get $t$- and $u$- channels in ME heirarchy: $\lim_{n \to \infty} PI|_{3\text{-loop}} = 3PI|_{3\text{-loop}}$


result:
3PI $\Gamma \to 2$ int eqns: pinch and collinear singularities
3PI $\Gamma$:

$$\Gamma[\psi, \bar{\psi}, A, S, D, V, U] = S_{cl}[\psi, \bar{\psi}, A] + \frac{i}{2} \text{Tr} \ln D_{12}^{-1} + \frac{i}{2} \text{Tr} \left[ (D_{12}^0)^{-1} \left( D_{21} - D_{21}^0 \right) \right]$$

$$-i \text{Tr} \ln S_{12}^{-1} - i \text{Tr} \left[ (S_{12}^0)^{-1} (S_{21} - S_{21}^0) \right]$$

$$+ \Gamma_2^0[S, D, V, U] + \Gamma_2^{\text{int}}[S, D, V, U]$$

$\Gamma_2^0 = \begin{array}{c}
\includegraphics[width=0.1\textwidth]{gamma_2_0.png}
\end{array}$

$\Gamma_2^{\text{int}} = \begin{array}{c}
\includegraphics[width=0.75\textwidth]{gamma_2_int.png}
\end{array}$
INTEGRAL EQUATIONS:

2 EoM from fcn derivs of $\Gamma$ wrt S and D (SD eqns)
differentiate wrt $A$
$\rightarrow$ BS eqn for $A$ and $\Omega$ (many cancellations)

sub $\Omega$ eqn into $A$ eqn and keep up to 2-loop order:

also: 2 EoM from fcn derivs of $\Gamma$ wrt V and U
→ substitute again and keep up to 2-loop order:

\[ \Lambda = + \]

\[ a + b + c + d \]

\[ e + f + g \]

\[ v = + + \]

1st is int eqn for \( \Lambda \) \( \sim \) same as from 2PI
2nd is sc int eqn for \( V \)

same eqns found using kinetic theory (AMY)
also found using a diagramatic approach

RESUMMATION IN EQUILIBRIUM AND NON-EQUILIBRIUM GAUGE THEORIES

Emil Mottola
Los Alamos National Lab

- Infrared Divergences in Perturbation Theory
- Gauge Invariance Issues
- Ward Identities
- Generating Functional for HTL and Higher Order Effects (Egri)
  - NEW RESUMMATION METHOD
- Linear Response, Spectral Fms.
  - Transport Coefficients (Non-Eqiv.)
- Real Time Evolution in Background Fields (Abelian, non-Abelian)

Gauge Dependence
Subleading Terms in $\frac{q^2}{\mu}$:

$\Sigma_1(w, p=0) = \frac{e^2 T}{8\pi} + (1-\frac{1}{3}) e^2 \omega \ln(4\mu) - \frac{1}{8\pi} \frac{e^2 \omega}{\mu} + O(\omega^2)$

- Imag. Part vanishes at LO
- Both Real and Imag. Parts are $\beta$ (Gauge Parameter) dependent at NLO
- Damping Rate requires Resummation (HTL)
- NLO Contributions to One-Loop $\Sigma_1$ come from "soft" internal momenta
  Re: $0 < k \mu < T \rightarrow \text{Re} \\
  \text{Im}: k \mu \sim eT \rightarrow \text{Im}$

Note: On-Shell Condition $\omega = \frac{eT}{\mu}$

gives NLO $\text{Re}\Sigma_1 - \frac{1}{\pi^2} \ln(4\mu)$
NLO $\text{Im}\Sigma_1 \sim e^2 T$ but gauge-dep.
on-shell
Large $N$ Method - Background Field

For QED

\[ S = S_{\text{cl}}[\bar{\Psi}, \Psi; A] = -\frac{1}{2} A \cdot \frac{\partial^2}{\partial x^2} A - \bar{\Psi} \cdot G^{-1}[A] \cdot \Psi \]

Maxwell

\[ (d^{-1})^\mu_{\nu} = -\frac{i}{\varepsilon}(\eta^\mu A - \eta^\nu A) \]

Dirac

\[ G^\mu[A] = -i (\gamma^\mu - i \gamma^\nu A) \quad (\text{as } m \to 0) \]

- Add sources \( J \cdot A + \bar{\Psi} \cdot \gamma^\mu \gamma^\nu \Psi \)
- Perform Gaussian integral over \( \bar{\Psi}, \Psi \) (exact)
- Perform remaining functional integral over \( A \) by steepest descent - valid if \( e^2 \to e^2/N, \bar{\Psi}, \Psi \to \sqrt{N} \) in large \( N \gg 1 \)
- Introduce Legendre transform variables

\[ \bar{\Psi} = \frac{\delta W}{\delta \eta}, \quad \bar{\Psi} = \frac{\delta W}{\delta \bar{\eta}} \]

\[ \bar{\Psi} = \frac{\delta W}{\delta \eta}, \quad \bar{\Psi} = \frac{\delta W}{\delta \bar{\eta}} \]

- Compute transform to classical fields (dotted)

\[ \frac{\delta S}{\delta A^2} \bigg|_{\bar{\Psi}, \bar{\bar{\Psi}}} = 0 \quad \text{Maxwell Eq. } \quad \bar{\bar{\Psi}} = \bar{\Psi} \]

RESULT Effective Action \( S, [\bar{\Psi}, \Psi; A] \)

\[ S_1[\bar{\Psi}, \Psi; A] = S_0[\bar{\Psi}, \Psi; A] - i \text{ Tr} \ln G^\mu[A] \]

\[ \sigma(N, \frac{1}{N}) = \frac{i}{2} \text{ Tr} \ln D^{-1}[\bar{\Psi}, \Psi; A] \]

\[ (D^{-1})^\mu_{\nu} [\bar{\Psi}, \Psi; A] = (d^{-1})^\mu_{\nu} - 2 \bar{\Psi} G^\mu[A] \gamma^\nu \Psi \]

\[ \Pi^{\mu\nu}[A] = -i \text{ Tr} \{ \gamma^\mu G[A] \gamma^\nu G[A] \} \]

One-Loop Photon Polarization (Debye Screening)

Self-Energy

\[ \text{propagator } \quad \text{so } D_{\mu\nu} = \frac{1}{d-1+i\pi} + \text{gauge terms} \]

is resummed (ringing) \( \beta_\pi \)

Automatically gives Free Energy/Pressure to order \( e^2 N^{1/2} \) with no extra work

\[ \frac{\delta S}{\delta A^2} \bigg|_{\bar{\Psi}, \bar{\bar{\Psi}}} = 0 \quad \text{Maxwell Eq. } \quad \bar{\bar{\Psi}} = \bar{\Psi} \]
How to resum $\Sigma_1$?

i.e. bring **external** self-energy
defined w.r.t. background "classical" fields
into internal lines.

Ans. Go back to original path integral
over quantum fields $\phi$, $\psi$ and
shift the saddle point by adding
and subtracting $\Sigma_1$:

$$\bar{\mathcal{F}} \cdot G^{-1}[A] \cdot \mathcal{F} = \bar{\mathcal{F}} \cdot \Lambda^\dagger[A] \cdot \mathcal{F} = \bar{\mathcal{F}} \cdot \mathcal{Z}[A] \cdot \mathcal{F}$$

$$\Lambda^\dagger[A] = G^{-1}[A] + Z[A]$$

then treat the last term as a
perturbation $\frac{1}{N}$ down from first term

The difference of this vs. previous method
are terms formally down by $\frac{1}{N}$
so allowed, BUT the new propagator
for the fermions will be $\Lambda^\dagger$ (note $\epsilon$)
which resums $\Sigma_1$ - OPT (used in $\phi^4$ theory)

Repeat procedure w. New Saddle Point
Obtain corrected effective action

$$S_2[\bar{\mathcal{F}}, \mathcal{F}; A] = S_0[\bar{\mathcal{F}}, \mathcal{F}; A]$$

$$- i \text{ Tr } \ln \Lambda^\dagger[A] + \frac{i}{2} \text{ Tr } \ln \mathcal{F} \cdot \mathcal{F}^\dagger[\bar{\mathcal{F}}; A]$$

$$\mathcal{Z}[A] = G[A] + \Sigma[A]$$

resums one-loop self-energy $\Sigma$,

$$\mathcal{F} \cdot \mathcal{F}^\dagger[\bar{\mathcal{F}}, \mathcal{F}; A] = (\mathcal{F} \cdot \mathcal{F}^\dagger)^{\mu} - 2 \mathcal{F} \Gamma^\mu \mathcal{F}^\dagger \Gamma^\nu \mathcal{F}$$

$$- i \text{ Tr } \{ \mathcal{Z}, \Gamma^\mu \mathcal{F} \}$$

contains dressed propagator $\bar{\mathcal{F}}$, and
vertices $\Gamma^\mu[A], \Gamma^\nu[A]$

with all LO HTL effective interactions
now part of internal lines

Taking $\bar{\mathcal{F}}, \mathcal{F}, A = 0$ and evaluating
in Equilibrium gives Pressure/Free Energy
connect to order $e^4$ (at least)

Corrected 2-loop topologies

Moreover effective action $\tilde{S}(A)$ gives

Gauge Invariant Formulation (to this order)

of Non-Equilibrium Eqs. of Motion

\[
\frac{\delta \tilde{S}(A)}{\delta A^\mu} = 0 \quad (\psi, \bar{\psi} = 0)
\]

\[\mathcal{L} \propto F_{\mu\nu} = \frac{i}{2} \tilde{S}(A)\]

Resums all effective 1-loop HTL propagator/vertices

without changing UV (counterterm) structure of underlying theory

(Unlike $\phi^4$-2PI or nPI formulations)

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**SUMMARY**

- Resummation/Re-Ordering of Perturbation theory necessary even for $e^2 \ll 1$

- Non-Trivial Problem in Gauge Theories (even QED) because of Gauge Dependence,
  vertices connected together with
  Self-Energy - Ward Identities

- Variation of Background Field
  Large $N_c$ Method proposed to
  Incorporate Resummed $Z$, $\Gamma$
  in Gauge Invariant Way

- Effective Action generates gauge-invariant
  n-point vertices which automatically satisfy Ward Identities and do
  not require modified UV counterterms
  (as in $\phi^4$, $\phi^3$,... formulations)
chain of propagators/vertices

sum of lower order

/vertices

sum, recovers all known

to free energy/predual

at order of

Action in Real Time (CTP)

of Motion Which can

out of equilibrium time evolution

cally Extendable to

orders

metric Generating Functional

cnear Response (Transport Coeff).

it Difficulty in Extending to QCD
Suppression of the Shear Viscosity as QCD Cools Into a Confining Phase

Yoshimasa Hidaka (RIKEN BNL Research Center)
Based on arXiv:0803.0453[hep-ph]

Abstract
We consider QCD near but above $T_c$. In this region, the pressure, susceptibilities and renormalized Polyakov loop, which is the order parameter of the confined deconfined phase transition, dramatically change. We show that the shear viscosity is suppressed by the power of the Polyakov loop. This suggests that $\eta/T^3$ decreases markedly as QCD cools down to temperatures near $T_c$. 
"Semi"-QGP system

\[ \langle \text{tr}L_r \rangle \sim e^{-f_r/T} \]

one particle free energy 

\[ f_r = \begin{cases} 
0 & \text{(deconfined)} \\
\text{finite} & \text{(semi-QGP)} \\
\infty & \text{(confined)} 
\end{cases} \]

quark anti-quark pair (singlet) 

no cost, color singlet 

\[ f_r = 0 \]

\( f_r \) depends upon the color representation, like chemical potential.

"Semi-QGP" is qualitatively different from the perturbative vacuum.
Semi-QGP Window

\[ \left\langle \frac{1}{N_c} \text{tr} L \right\rangle \approx 0 \]
\[ \left\langle \frac{1}{N_c} \text{tr} L \right\rangle < 1 \]
\[ \left\langle \frac{1}{N_c} \text{tr} L \right\rangle \approx 1 \]

Semi-QGP Window
\[ 0.8T_c - 2T_c \]

Maybe RHIC probes the Semi-QGP!!

Pressure, susceptibilities change dramatically in Semi-QGP.

How about transport coefficients?
Viscosity in the semi-QGP: pure glue

Y.H., Pisarski ('08)

Viscosity \( \eta = \frac{c_{\eta} T^3}{g^4 \log(1/g)} f(L) \quad f(L = 1) = 1 \)

\[ f(L) \sim \ell^2 \]

Suppression at small \( \ell \)

\( \star \) Semi-QGP

\[ \eta \sim \ell^2 T/\sigma \]

\( \sigma \) : cross section

Unlike classical dilute gas

\[ \eta \sim T/\sigma \]
Viscosity in semi-QGP with quarks (contd.)

Y.H., Pisarski ('08)

\[ \frac{N_f}{N_c} = \frac{1}{3} \]

\[ \frac{N_f}{N_c} = \frac{2}{3} \]

\[ \frac{N_f}{N_c} = \frac{3}{3} \]

\[ f(L) \text{ with quarks pure glue.} \]

\[ \text{Little change between two eig. dist.'s.} \]

\[ \text{Quark contribution dominates.} \]
Summary

- Shear viscosity suppressed, near \(T_c\), \(\sim \ell^2\). Quarks dominates.

- RHIC - probes semi-QGP? If so, not only \(\eta\), but \(R_{AA}\), real photons, dileptons, also suppressed by powers of \(\ell\).

- LHC - into complete QGP? If so, LHC \(\neq\) RHIC.
Transport Coefficients In Chiral Theories

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(work in collaboration with Ángel Gómez Nicola)

We present our recent work on the determination of transport coefficients in chiral perturbation theory. Our general perturbative scheme will be discussed and leading order results will be presented for the electrical and thermal conductivities and shear and bulk viscosities. We provide a consistent low-$T$ description and we will show that unitarity plays a crucial role in order to reproduce the expected behaviour as $T$ approaches the chiral transition. We will pay special attention to phenomenological applications in Heavy Ion physics, as elliptic flow or photon spectra, as well as to more formal issues such as the KSS bound or the chiral restoration behaviour.

“Understanding QGP through Spectral Functions and Euclidean Correlators”. BNL, April 2008.
**Transport coefficients in ChPT**

**Particle width in ChPT**

\[
\Gamma(k_1) = \frac{1}{2} \int \frac{d^3k_2}{(2\pi)^3} e^{-\beta E_2} \sigma_{\pi\pi} v_{\text{rel}} (1 - v_1 \cdot v_2) \sim \text{Im} \quad \text{Im}
\]

- Scattering cross section:
  \[
  \sigma_{\pi\pi}(s) \simeq \frac{32\pi}{3s} \left[ |t_{00}(s)|^2 + 9|t_{11}(s)|^2 + 5|t_{20}(s)|^2 \right] .
  \]

**Unitarity and the Inverse Amplitude Method (IAM)**

- ChPT violates the unitarity condition for high \(p\): \(S^\dagger S = 1 \Rightarrow \text{Im} \ t_{IJ}(s) = \sigma(s)|t_{IJ}(s)|^2\), with \(\sigma(s) \equiv \sqrt{1 - 4M^2 / s}\).

  Because partial waves are essentially polynomials in \(p\): \(t_{IJ}(s) = t^{(1)}_{IJ}(s) + t^{(2)}_{IJ}(s) + \mathcal{O}(s^3)\).

- IAM:
  \[
  t_{IJ}(s) \simeq \frac{t^{(1)}_{IJ}(s)}{1 - t^{(2)}_{IJ}(s; T) / t^{(1)}_{IJ}(s)} .
  \]

  It verifies the unitarity condition exactly.
Diagrammatic analysis

- **Ladder diagrams:**

  - Lines with $\Gamma = 0$. These loops (rungs) give a perturbative contribution $\sim X$.
  - $(q^0 \to 0^+, q = 0)$
  - $k^\mu$

  

  \[ O(X^n Y), \quad \text{for } T \ll M_\pi \]

  \[ O(X^n Y^{n+1}), \quad \text{for } T \simeq M_\pi \]

  If $T \gtrsim M_\pi$, $X \sim 1$, and derivative become important $\Rightarrow$ resummation may be relevant.

  Eg., if we only consider constant vertices, for the DC conductivity:

  - For $T \ll M_\pi$, $Y \sim \sqrt{\frac{M_\pi}{T}}$, $X \sim \frac{1}{Y} \left( \frac{M_\pi}{4\pi F_{\pi}} \right)^2$.
  - For $T \simeq M_\pi$, $Y \sim 1$, $X \lesssim \left( \frac{M_\pi}{4\pi F_{\pi}} \right)^2$.

  Each pair of lines with $\Gamma \neq 0$ and equal momentum give a pinching pole contribution $\sim Y$.

- **Bubble diagrams:**

  \[ O(0^n X^{n-1}) \]

  \[ \sim Y \]

  Contribution from a simple bubble $\sim Y$.

  Weinberg's theorem does not give the correct order for TC at low $T$: $O(p^{2n}) \gg O(p^{4n})$.

  This counting allows us to quickly obtain the functional form of TC at low $T$. 
**Electrical conductivity (pion gas)**

- **Kubo formula:**

\[
\sigma = -\frac{1}{6} \lim_{q_0 \to 0^+} \lim_{|q| \to 0^+} \frac{\partial \rho_\sigma(q_0, q)}{\partial q_0}, \quad \rho_\sigma(q_0, q) = 2 \text{Im} \int d^4x \, e^{iq \cdot x} \theta(t) \langle [J_i(x), J^i(0)] \rangle.
\]

- **Results:**

According to Kinetic Theory (KT):

\[
\sigma \sim \frac{e^2 n_{\text{ch}} T}{M_\pi}, \quad \text{but } \tau \sim 1/\Gamma, \quad \Gamma \sim n \sigma_{\pi\pi}.
\]

For \( T \ll M_\pi, \) \( n \sim (M_\pi T)^{3/2} e^{-M_\pi/T}, \)

\( v \sim \sqrt{T/M_\pi}, \) and \( \sigma_{\pi\pi} \) is a constant, \( \Rightarrow \)

\( \sigma \sim 1/\sqrt{T}. \) 🌈

---

[Graph showing electrical conductivity vs. temperature.]

---

[Note: The graph shows the variation of electrical conductivity with temperature, illustrating the behavior predicted by the Kubo formula and the results from Kinetic Theory. The graph highlights the unitarization of the conductivity and the contribution from a single hadron.]
Thermal conductivity (pion gas)

- **Kubo formula:**

\[ \kappa = -\frac{\beta}{6} \lim_{q^0 \to 0^+} \lim_{q \to q^0^+} \frac{\partial \rho_\kappa(q^0, q)}{\partial q^0} , \quad \rho_\kappa(q^0, q) = 2 \text{Im} \int d^4x \ e^{iq \cdot x} \theta(t) \langle [T_{0i}(x), T_{0i}(0)] \rangle . \]

- **Results:**

From KT: \( \kappa \sim c_p l \nu. \)

For \( T \ll M_\pi, \ c_p \sim T^{-1/2} e^{-M_\pi/T}, \Rightarrow \kappa \sim T^{-3/2}. \)
Shear and bulk viscosities (pion gas)

- Results:

For $\eta$ and $\zeta$ ladder diagrams could be important for $T < M_\pi$.

Good agreement with KT $\Rightarrow$ cancellation of ladders?

- AdS/CFT bound:

From KT: $\eta, \zeta \sim M_\pi v n_l$, but $l \sim \frac{1}{\sigma_{\pi\pi} n}$.

So for $T \ll M_\pi$, $\eta, \zeta \sim \sqrt{T}$. 🦢

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Quarkonia Measurements in PHENIX

Mike Leitch - PHENIX/LANL - leitch@bnl.gov
RBRC - 24 April 2008

- How are quarkonia produced?
- What CNM effects are important?
- How does the sQGP effect quarkonia?
- What are the CNM effects in AA collisions?
- Transverse Momentum Broadening
- Heavier Quarkonia
- Detector Upgrades & Luminosity for the future
Nuclear Dependence Nomenclature - Ratio ($R_{dAu}$, $R_{AA}$) and Alpha ($\alpha$)

$R_{dAu} = \alpha = 1$ if every N-N collision in a Nucleus contributes as if it were in a free nucleon

$$R_{dAu} = \frac{d\sigma^{dAu}/dy}{2\times197\cdot(d\sigma^{pp}/dy)}$$

$$= \frac{dN^{dAu}/dy}{\langle n_{coll}^{dAu}\rangle dN^{pp}/dy}$$

<{$n_{coll}$> from Glauber model calc. - can also be used for centrality bins

Where $dN^{dAu}/dy$ is an invariant yield w/o absolute normalization factors that would be needed for a cross section (lower systematical uncertainties)

Alternatively, a power law with $\alpha$ - especially useful when comparing expts that used different nuclear targets

$$\sigma_{pA} = \sigma_{pp}A^\alpha$$

$$\alpha = 1 + \ln(R_{pA})/\ln(A)$$

4/24/2008

Mike Leitch - PHENIX/LANL
What CNM effects are important?  
(CNM = Cold Nuclear Matter)

New Analysis of Run3 data consistent with EKS shadowing & absorption - clear need for new dAu data

Not universal vs $x_2$ as expected for shadowing, but closer to scaling with $x_F$, why?
- initial-state gluon energy loss?
- gluon saturation?
- Sudakov suppression (energy conservation)?
How does the QGP affect Quarkonia?

CNM Effects

CNM effects (EKS shadowing + dissociation) give large fraction of observed AuAu suppression, especially at mid-rapidity.

Normal CNM descriptions give similar AuAu suppression at mid vs forward rapidity.
• but if peaking in "anti-shadowing" region were flat instead then one would get larger suppression for forward rapidity as has been observed in AuAu data
• could come from gluon saturation or from a shadowing prescription that has no anti-shadowing

In any case more accurate dAu data is sorely needed.

4/24/2008
Mike Leitch - PHENIX/LANL
How does the QGP affect Quarkonia? 
Sequential Screening and Gluon Saturation

Some recent lattice calculations suggest \( J/\psi \) not screened after all
• suppression then comes only via feed-down from screened \( \chi_c \) & \( \psi' \)
• the situation could be the same at lower energies (SPS) as for RHIC mid-rapidity
• and the stronger suppression at forward rapidity at RHIC could come from gluon saturation (previous slide)

• Is suppression stronger than can come from \( \chi_c \) & \( \psi' \) alone?
• Can this picture explain saturation of the forward/mid-rapidity RAA super-ratio?
How does the QGP affect Quarkonia?
Regeneration - compensating for screening

- larger gluon density at RHIC expected to give stronger suppression than SPS
- but larger charm production at RHIC gives larger regeneration
- very sensitive to poorly known open-charm cross sections
- forward rapidity lower than mid due to smaller open-charm density there
- expect inherited flow from open charm
- regeneration much stronger at the LHC!

- need to know what happens to $\chi_c$ & $\psi'$ & measure $J/\psi$ flow?
- flat forward/mid-rapidity RAA super-ratio consistent with centrality trends of the two components (screening & regeneration)?
Quarkonia, Heavy Quarks and sQGP

Ralf Rapp
Cyclotron Institute
+ Physics Department
Texas A&M University
College Station, USA

With: H. van Hees, D. Cabrera (Madrid), X. Zhao,
V. Greco (Catania), M. Mannarelli (Barcelona)

RBRC Workshop on
“Understanding QGP Through Spectral Functions and
Euclidean Correlators”
Brookhaven National Laboratory (Upton, NY), 24.04.08
Outline

1.) Introduction

2.) Heavy Quarkonia in QGP
   - Charmonium Spectral + Correlation Functions
   - In-Medium T-Matrix with “lattice-QCD” potential
   - In-Medium Mass and Width Effects

3.) Open Heavy Flavor in QGP
   - Heavy-Light Quark T-Matrix
   - HQ Selfenergies + Transport
   - HQ and e± Spectra
   - Implications for sQGP

4.) Conclusions
2.2 Potential-Model Approaches for Spectral Fcts.

- Bound state + free continuum model
too schematic for broad / dissolving states

- Lippmann-Schwinger Equation

In-Medium $\bar{Q}$-$Q$ $T$-Matrix:

\[
T_L(E;q,q') = V_L(q,q') + \int k^2 dk \, V_L(q,k) \, G^0_{Q\bar{Q}}(E,k) \, T_L(E;k,q')
\]

- 2-quasi-particle propagator: $G^0_{Q\bar{Q}}(s) = 4\omega_k \sqrt{s - (2\omega_k + \Sigma_Q + \Sigma_{\bar{Q}})^2}$
- Bound + scatt. states, nonperturbative threshold effects (large)

- Correlator: $G_L(E) = \int G^0_{Q\bar{Q}} + \int G^0_{Q\bar{Q}} \int T_L \, G^0_{Q\bar{Q}} \quad L=S,P$
2.3 Charmonium Spectral Functions in QGP within T-Matrix Approach (lattice U$_1$ Potential)

Fixed $m_c=1.7\text{GeV}$

In-medium $m_c^*$ (U$_1$ subtraction)

- gradual decrease of binding, large rescatterting enhancement
- $\eta_c$, J/$\psi$ survive until $\sim 2.5T_c$, $\chi_c$ up to $\sim 1.2T_c$
3.2 Potential Scattering in sQGP

- $T$-matrix for $Q$-$q$ scatt. in QGP

\[ T^a_L = V^a_L + \int dk \, V^a_L \, G^0_{Qq} \, T^a_L \]

- Casimir scaling for color chan. $a$

- In-medium heavy-quark selfenergy:

\[ \Sigma_Q = \Sigma_g + \Sigma_T \]

Determination of potential

- Fit lattice $Q$-$Q$ free energy

\[ F_{Q\bar{Q}} = U_{Q\bar{Q}} - TS \quad V_{Q\bar{Q}}(r) = U_{Q\bar{Q}}(r) - U_{Q\bar{Q}}^\infty \]

- Currently significant uncertainty

![Graphs showing potential vs. temperature and distance](image1.png)

[Shuryak+ Zahed '04]

![Graphs showing potential vs. temperature and distance](image2.png)

[Wong '05]
3.4 Single-Electron Spectra at RHIC

- heavy-quark hadronization: coalescence at $T_c$ [Greco et al. ’04] + fragmentation
- **hadronic correlations at $T_c$** ↔ quark coalescence!
- charm bottom crossing at $p_T^e \sim 5\text{GeV}$ in d-Au ($\sim 3.5\text{GeV}$ in Au-Au)
- $\sim 30\%$ uncertainty due to lattice QCD potential
- suppression “early”, $v_2$ “late”
3.5 Maximal “Interaction Strength” in the sQGP

- potential-based description $\leftrightarrow$ strongest interactions close to $T_c$
  - consistent with minimum in $\eta/s$ at $\sim T_c$
  - strong hadronic correlations at $T_c$ $\leftrightarrow$ quark coalescence

- semi-quantitative estimate for diffusion constant:

  weak coupl. $\frac{\eta}{s} \approx \frac{4}{15} n <p>$ $\lambda_{tr} = \frac{1}{5} T D_s$

  strong coupl. $\frac{\eta}{s} \approx \frac{1}{4\pi} D_s(2\pi T) = \frac{1}{2} T D_s$

  $\Rightarrow \frac{\eta}{s} \approx \frac{2-4}{4\pi}$ close to $T_c$
4.) **Summary and Conclusions**

- **T-matrix approach with IQCD internal energy** \( U_{QQ} \): S-wave charmonia survive up to \( \sim 2.5T_c \), similar to IQCD correlators + spectral functions

- **T-matrix approach for (elastic) heavy-light scattering**: large c-quark width + small diffusion

- **“Hadronic” correlations dominant** (meson + diquark)
  - maximum strength close to \( T_c \leftrightarrow \) minimum in \( \eta/s \)?
  - naturally merge into quark coalescence at \( T_c \)

- **Open problems + challenges**:  
  - potential approach/definition, heavy-quark masses  
  - radiative processes, light-quark sector  
  - observables (open charm/bottom, quarkonia, dileptons,...)
Abstract:

We model the $Q\bar{Q}$ spectral function in terms of a finite width resonant state plus a continuum and compare its correlator behaviour relative to that at $T = 0$. 
Question: How to calculate quarkonium dissociation points?

Two possibilities:

- Schrödinger equation with temperature-dependent heavy quark potential $V(r, T)$
- quarkonium spectrum from finite $T$ lattice QCD

Both have intrinsic problems

- survival in potential theory for radii $r \geq 1/T$, binding energies $\Delta E \leq T$: what does that mean?
- spectral functions via MEM from correlator calculations, correlator ratios vs. reconstructed “vacuum” form: how much freedom?

report here on an attempt to address this problem

H.-T. Ding, O. Kaczmarek, F. Karsch, HS (in preparation)
idealized spectrum at $T = 0$ (no zero point mode)

$$\sigma(\omega, T = 0) = f \delta(\omega - M) + c \theta(\omega - s_0)\omega^2\sqrt{1 - (\omega/s_0)^2}$$

$f \sim$ strength of resonance

$c \sim$ strength of continuum

$s_0(T)$ continuum threshold

assume that at $T > 0$ resonance broadens (relativistic B-W), but retains same strength

$$\sigma_r(\omega, T) = N(\gamma) f \frac{M}{\pi} \left\{ \frac{2\omega\gamma}{\omega^2\gamma^2 + (\omega^2 - M^2)^2} \right\}$$

$N(\gamma)$ assures normalization for width $\gamma = \gamma(T)$
include $T$-independent continuum
reduces modifications due to finite width

remove resonance at finite $T$:
decreasing ratio
Basic Problem

- $G(\tau, T)/G_0(\tau, T) = 1 \forall \tau$ has unique solution: $\sigma(\omega, T) = \sigma(\omega, T = 0)$ [Baym & Mermin]

- given $G(\tau, T)/G_0(\tau, T) = 1$ for $N$ points $\tau_i, i = 1, \ldots, N$
  or more precisely,

- given $G(\tau, T)/G_0(\tau, T) = 1 \pm \epsilon$ for $N$ points $\tau_i, i = 1, \ldots, N$

how to define "the best solution"?
back to MEM....
Conclusions

• resonance width and continuum threshold have clearly visible effects on correlator ratios

• resonance broadening leads to increase at large $\tau$, due to low $\omega$ tail of RBW;
  shifting continuum threshold down enhances this

• resonance melting leads to decrease at large $\tau$, due to less contribution at low $\omega$;
  shifting continuum threshold down partially compensates this

• eventually compare to more precise correlator studies to model vector ($J/\psi$) and scalar ($\chi_c$) channels
Charmonium in strongly coupled Quark-Gluon Plasma

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(Dated: March 15, 2008)

The growing consensus that a strongly coupled quark-gluon plasma (sQGP) has been observed at the SPS and RHIC experiments suggests a different framework for examining heavy quark dynamics. We present both semi-analytical treatment of Fokker-Planck (FP) evolution in pedagogical examples and numerical Langevin simulations of evolving \(cc\) pairs on top of a hydrodynamically expanding fireball. In this way, we may conclude that the survival probability of bound charmonia states is greater than previously estimated, as the spatial equilibration of pairs proceeds through a “slowly dissolving lump” stage related to the pair interaction.

- The main parameter, fixed from \(R_{AA}\) and \(v_2\) for a single \(c \rightarrow \) charm diffusion constant \(D_c\)

Several theoretical groups have analyzed these data, in particular Moore and Teaney [23] provided information about the diffusion constant of a charm quark \(D_c\) by Langevin simulations. A conclusion following from this work is that both \(R_{AA}\) and \(v_2\) observed at RHIC can be described by one value for the charm diffusion constant in the range

\[
D_c(2\pi T) = (1.5 - 3). \quad (10)
\]

This can be compared with the perturbative (collisional) result at small \(\alpha_s\)

\[
D_c^{QCD}(2\pi T) = 1.5/\alpha_s^2. \quad (11)
\]
FP and Langevin eqns
with mutual interaction

\[ \frac{\partial P}{\partial t} = D \frac{\partial}{\partial r} f_0 \frac{\partial}{\partial r} (P/f_0) \]

\[ \frac{dp}{dt} = -\eta p + \xi - \nabla U \]

\[ \frac{dx}{dt} = \frac{p}{m_c} \]

**FIG. 3:** (Color online.) Numerical solution of the one-dimensional Fokker-Planck equation for an interacting \( \bar{c}c \) pair. The relaxation of the initial narrow Gaussian distribution is shown by curves (black, red, brown, green, blue, or top to bottom at \( r=0 \)) corresponding to times \( t = 0, 1, 5, 10 \) fm, respectively.

**FIG. 4:** (Color online.) Distribution over quark pair separation at fixed \( T = 1.5T_c \) after 9 fm/c, with (red squares) and without (green triangles) the \( \bar{c}c \) potential.
First p-equilibration and then quasiequilibrium

FIG. 7: (Color online.) Probability of $\bar{c}c$ pairs to be bound at RHIC Au+Au, $\sqrt{s} = 200$ GeV, mid-rapidity.

FIG. 6: (Color online.) Evolving energy distribution for an ensemble of $\bar{c}c$ pairs at time moments $t = 2, 3, 10$ fm/c (circles, squares and triangles, respectively).
Langevin for c-barc pair on top of expanding fireball =>

\textbf{J/\psi survival probability}

- Pythia initiation
- Hydro => T(x)
- Langevin
- Projection to J/\psi after QGP, as T=T_c

The way we have chosen to display PHENIX data \[38\] is as follows: before we compare those with our results we “factor out” the cold nuclear matter effects, by defining (for any given rapidity \(y\)) the following ratio of Au+Au and \(d+Au\) data

\[
\frac{R^{\text{anomalous}}_{AA}(y)}{R_{PHENIX}(y)} = \frac{R^{\text{anomalous}}_{AA}(y)}{R_{Tc}(y)R_{Tc}(-y)} \quad (26)
\]

to be called the “anomalous suppression”. In principle...
Feeddown from higher states

we form the double ratios, at two relative energies corresponding to $\psi'$ and $J/\psi$ masses (minus $2m_{\text{charm}}$)

$$R_{\psi'}/J_\psi = \frac{f_0(0.8 \text{ GeV})/f_0(0.3 \text{ GeV})}{f(0.8 \text{ GeV})/f(0.3 \text{ GeV})}$$

(27)

particles produced from feeddown from higher charmonium states:

$$N_{J/\psi}^{\text{final}} = N_{J/\psi}^{\text{direct}} \left[ 1 + R_{\psi'}/J_\psi \sum_i (\frac{g_i}{3}) \exp(-\frac{\Delta M_i}{T})B_i \right]$$

(29)

where $i$ is summed over the $\chi_1, \chi_2$, and $\psi'$ particles which

- $R=>1$ means thermal population, as is indeed observed by NA50
- Sorge, Shuryak, Zahed, Andronic, PBM, Stachel

We don't do Np<100!

FIG. 9: (Color online.) The double ratio $R_{\psi'/J_\psi}$ defined in (27) versus centrality (number of participants). One point (green box) at $N_{\text{part}} = 2$ corresponds to experimental data for $\psi'$ and the direct $J/\psi$, for pp collisions.
J/psi survival, with and without feeddown

• Remaining issues:
• Is there a need/place for recombination? How large is it?
• What are pt, y distributions and $v_2 o^J$/psi from this simulation?

FIG. 10: (Color online.) The points are PHENIX data for $R_{AA}^{norm}(y = 0$), the same as used in Fig.III.D. Two curves are our model, with (solid, upper) and without (dashed, lower) feeddown.
Heavy Quark Diffusion at Next-To-Leading Order

Simon Caron-Huot
McGill University

April 26, 2008


Abstract

We present a calculation of the momentum diffusion coefficient of a nonrelativistic heavy quark moving through the quark-gluon plasma, at next-to-leading order in the weak coupling expansion. This transport coefficient characterizes the rate at which a heavy quark's momentum thermalizes with respect to the ambient medium. The next-to-leading order correction is $\mathcal{O}(g)$ relative to the leading result, and is calculable within the HTL effective theory. We find it to be large, being already significant at $\alpha_s \approx 0.03$, thus signaling convergence difficulties for the perturbative series at larger values of the couplings.
1.1. Introduction (II)

- Transport coefficients tend to be sensitive to $gT$-scale physics; this happens for: photon production rate, shear viscosity, jet quenching, heavy quark E loss...

- All of them could receive large $O(g)$ corrections!

  - The tools needed to calculate them already exist (HTL effective theory)
3.2. The HTL diagrams

- Must include all two-loop HTL diagrams.
- All propagators are soft ($p \sim gT$) and resummed.
4.1. The result

\[ \kappa^{\text{LO}} = \frac{g^4 C_H}{12\pi^3} \int_0^\infty k^2 dk \int_0^{2k} q^2 dq \left( \frac{m_B^2}{q^2 + m_B^2} \right)^2 \left\{ \frac{N_c n_B (1+n_B)(k)}{N_c} \left[ 2 - \frac{1}{k^2} + \frac{1}{4k^4} \right] + 2 N_f n_f (1-n_f)(k) \left[ 1 - \frac{1}{4k^2} \right] \right\} \]

\[ = \frac{g^2 T m_D^2 C_B}{6\pi} \left\{ \log \left( \frac{2T}{m_D} \right) + \xi + \frac{\frac{1}{2} N_f}{N_c + \frac{1}{2} N_f} \log 2 + \frac{7 g^2 N_c T}{8\pi m_D} \right\} \]

\[ \kappa^{\text{NLO}} = \frac{g^2 T m_D^2 C_B}{6\pi} \left\{ C \times \frac{N_c g^2 T}{3 m_D} \right\} \]

Where: \[ \xi = \frac{1}{2} - \gamma_E + \frac{\zeta(2)}{\zeta(3)} \approx -0.6472 \quad \text{(LO constant)} \]

\[ C \approx 1.4946 \quad \text{(NLO coefficient)} \]
4.1. The result (II)

\[ \kappa/g^4 L \]

\( \alpha_s \)

- Next-to-leading order
- Leading order
- Truncated leading order

\( \alpha_s \): Running coupling constant

\( \kappa/g^4 L \): Function of coupling constant and distance
4.2. Discussion

- The NLO correction is very large. Even for 3-flavors QCD at $T=100\text{GeV}$, $(\alpha_s \approx 0.1)$
  I wouldn't trust the perturbative series for $\kappa$.
- What happened? Half of the correction $C$ is from using the NLO Coulomb gauge potential. The remainder appears more complicated.
- Outlook:
  - Expansions in $(g_s^2N_c T/m_D)$ seem not to accurately describe the QCD gluons, even when $\alpha_s$ is small. Other approaches would be desirable.
Transport of Heavy Quarks in AdS/CFT

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(Dated: May 25, 2008)

The goal of the program was to review the current understanding of spectral functions in different channels. This talk computed the spectral density of the current-current channel for heavy quarks in \( \mathcal{N} = 4 \) Super Yang Mills (SYM) at strong coupling using the AdS/CFT correspondence. The work is based on the work done in collaboration with Jorge Casalderrey-Solana and was motivated by earlier work done in collaboration with Peter Petreczky [1-4]. It is impossible to summarize the literature, but the author would like to acknowledge an intellectual debt to two papers which also considered the drag and diffusion of heavy quarks [5, 6].

The first slide states the problem. To analyze the motion of a heavy quark which is placed in a strongly coupled SYM plasma we insert a string into the \( \text{AdS}_5 \times S^5 \) geometry. Generally, the statement of the correspondence is that it is sufficient to consider only the classical equations of motion. However it is clear that classical equations of motion will never reproduce the stochastic Brownian motion of a heavy particle in plasma. Indeed the equations of motion of a heavy quark in plasma can be written

\[
\frac{dx}{dt} = \frac{p}{M_Q} \quad \frac{dp}{dt} = -\eta p + \xi \quad \langle \xi(t)\xi(t') \rangle = \kappa \delta(t - t')
\]

(1)

where \( \eta \) is the drag coefficient and \( \xi \) is the noise. The drag and the noise are related by the Einstein relation \( \eta = \frac{k_B T}{2} \).

Restuffing this result we see that it can be written as

\[
M_Q \ddot{x} + \frac{\kappa}{2T} \dot{x} = \xi
\]

(2)

To see the stochastic equations of motion it is necessary study the quantum mechanical problem of a string in \( \text{AdS}_5 \). This is suggested in slide 2. Furthermore we need to examine the motion in the full Kruskal plane since the dynamics in Kruskal plane determines the Schwinger-Keylshirt contour. After considering small fluctuations of the string in \( \text{AdS}_5 \) we can determine an effective action for the dynamics of the endpoints of the string. Then the action of the endpoints of the string can be used to determine the dynamics of the endpoint.

The final result presented on slide three is that the endpoint of the string obeys a finite frequency generalization of the Langevin equations

\[
-M_Q^2 \omega^2 \chi(\omega) + G_R(\omega) \chi(\omega) = \xi(\omega) \quad \langle \xi(\omega)\xi(-\omega) \rangle = -(1 + 2\kappa_B(\omega)) \text{Im} G_R(\omega)
\]

(3)

In the low frequency limit we have

\[
G_R(\omega) = -i\omega \frac{\kappa}{2T} + \omega^2 \frac{\sqrt{\lambda} T}{2} \quad \text{with} \quad \kappa = \sqrt{\lambda} \pi T^3
\]

(4)

and the quark moves with a certain momentum diffusion coefficient \( \kappa \) and effective mass \( M_{\text{in}}(T) = M_Q^2 - \sqrt{\lambda} T^2/2 \) as found previously by different methods [5]. The final slide shows the full spectral density including the finite frequency generalization.

Acknowledgments. D. Teaney was supported in part by the Department of Energy under the Outstanding Junior Investigator (OJI) program and in part as a Sloan Fellow.


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The problem:

The quark doesn't move. There is no noise in "standard" AdS/CFT
Integrating out the Bulk

- The real time partition function of string for small fluctuations

\[ Z = \int \prod_{t_1} \! dX_1(t_1) \prod_{t_2} \! dX_2(t_2) \prod_{t,z} \! dx_1(t, z) \! dx_2(t, z) e^{iS_{NG}} \]

- The integrals over the internal coordinates can be done and yield

\[ Z = \int DX_1 DX_2 e^{iS_{\text{eff}}[X_{ct}(X_1(t_1), X_2(t_2))]} \]
Result

- Find the endpoint of the string obeys the expected Langevin equation

\[ M^0_Q \frac{d^2 X}{dt^2} + \int_0^t G_R(t - t')X(t') = \xi \]

- To quadratic order the retarded green function is

\[ G_R(\omega) = \frac{(\Delta M) \omega^2}{(\kappa \omega \frac{\kappa}{2T}) + \sqrt{\lambda T/2}} \]

- Then find the following effective equation of motion

\[ M_{\text{kin}}(T) \frac{d^2 X}{dt^2} + \frac{\kappa}{2T} \frac{dX}{dt} = \xi \]

with the correct kinetic mass (see HKKKY)

\[ M_{\text{kin}}(T) = M^0_Q - \frac{\sqrt{\lambda T}}{2} \]  \hspace{1cm} (1)
The full answer:

\[ \rho_{JJ}(\omega) = \chi_s \frac{\omega^2 \text{Im} G_R(\omega)}{(-M_Q \omega^2 + \text{Re} G_R(\omega))^2 + (\text{Im} G_R(\omega))^2} \]

Can compute high frequency corrections to Brownian motion.
Towards new relativistic hydrodynamics from AdS/CFT

Michael Lublinsky
Stony Brook

We propose to generalize viscous relativistic hydrodynamics by including all order derivative terms in the gradient expansion of the stress energy tensor. The gradient expansion is constructed to match hydro spectral functions with spectral functions computed at strong coupling via AdS/CFT correspondence.
Outline of the talk

- Quick tour to Relativistic hydro
- Brief visit into 5th dimension: Black Hole AdS/CFT
- All order hydro: momenta dependent viscosity

Motivation: Experiments (RHIC) probe systems with finite gradients.

Main Goal:

Introduce higher order dissipative terms in the gradient expansion of $T^{\mu\nu}$

Extract momenta dependent viscosities by matching two-point correlation functions of stress energy tensor with correlation functions computed from BH AdS/CFT.

(when applying to QCD we hope for some universality for transport coefficients)

We propose to use this hydro as a "nonlinear model" for real simulations at RHIC
\[ \Pi^{\mu\nu} = \Delta^{\mu m} \Delta^{\nu n} D_{mn,k}[\nabla] u^k \]

**Tracelessness condition:** \[ \Delta^{mn} D_{mn,k}[\nabla] u^k = 0 \]

\[ D_{mn,k} u^k = g_{mn} \left[ \frac{2}{3} \eta_1 - \frac{1}{3} \eta_2 \nabla^2 \right] (\nabla u) - \eta_1 \left[ g_{mk} \nabla^k + g_{nk} \nabla^k \nabla^m \right] u^k + \eta_2 \nabla_m \nabla_n (\nabla u) \]

\[ \eta_{1,2} = \eta_{1,2}[\sigma(u \nabla), \nabla^2] \rightarrow \eta_{1,2}[i \omega, \omega^2 - q^2] = \Re \eta_{1,2} + \Im \eta_{1,2} \]

\[ \eta_1[\omega \to 0, q \to 0] \to \eta \]

We keep the nonlinear dispersion to all orders, but

We neglect nonlinear interactions (though some terms could be recovered).
Shear (Diffusive) channel:

\[ G^T_R(\omega, q) = \frac{\eta_1 q^2/2}{-i \omega + \eta_1 q^2/2} \]
\[ \chi^R = \Im G^T_R \]

Sound channel:

\[ G^L_R(\omega, q) = \frac{2 i \omega c^2 q^2 \tilde{\eta} - c^2 q^2}{\omega^2 - c^2 q^2 + 2 i \omega c^2 q^2 \tilde{\eta}} \]
\[ \chi^L = \Im G^L_R \]

\[ \tilde{\eta} = \eta_1 + \eta_2 (\omega^2 - 2 q^2) / 4 \]

In order to extract \( \eta_{1,2} \) we have to invert these relations.
For that we need both imaginary and real parts of the correlators.

Poles of the correlators should reproduce the entire tower of quasi-normal modes + their dispersion relations.
\begin{align*}
\eta_1 &= 1 + i \tau_R \omega + \kappa q^2 + \lambda \omega^2 \ldots \\
\tau_R &= 2 - \ln[2] ; \quad \kappa \approx -1 , \quad \lambda \approx 1.7 \\
\tilde{\eta} &= 1 + i \tau_R \omega - \tau_R^2 \omega^2 \ldots
\end{align*}
Concluding Remarks

- Higher order terms in the gradient expansion seem to be important at early times. Taking them into account is likely to reduce the dependence on the initial time of the evolution.

- IS second order hydro does not agree with the all-order hydro from the AdS/CFT. This hints that this second order hydro is potentially less trustable tool than it could be previously thought.
Debye and Magnetic Masses in Full QCD Simulations and Comparison to AdS/CFT

Yu Maezawa

En'yo Radiation Laboratory, Nishina Accelerator Research Center, RIKEN, Wako, Saitama 351-0198, Japan

We study magnetic and electric screening masses of quark-gluon plasma in lattice QCD with two flavors of improved Wilson quarks at temperatures $T \sim T_{pc} - 4T_{pc}$, where $T_{pc}$ is the pseudocritical temperature. Imposing symmetries of the Euclidean-time reflection and charge conjugation on the Polyakov-loop correlator, we can decompose the Polyakov-loop correlation to various combinations of the Polyakov-loop operators with a gauge fixing, and extract the screening masses. The magnitude of the magnetic screening mass ($m_M$) turns out to be smaller than that of the electric screening mass ($m_E$), in accordance with an expectation of the thermal perturbation theory. We also find that the screening ratio $m_E/m_M$ shows a good agreement with a prediction from the AdS/CFT correspondence. At $T \geq 1.2T_{pc}$, our $m_E$ agrees well with a previous estimation of the screening mass from a heavy-quark free energy, implying dominance of electric contributions at our simulation parameters. The perturbation theory suggests that the higher-order magnetic contributions in heavy-quark free energies are suppressed at large distances when $m_E < 2m_M$. We confirm that our results for $m_M$ and $m_E$ satisfy this inequality.
Introduction

Screening properties in quark-gluon plasma

- Electric (Debye) screening mass ($m_E$)
  \[ \leftrightarrow \langle A_4(x)A_4(y) \rangle \]
  \[ \longrightarrow \text{Heavy-quark bound state (J/\Psi, } \Upsilon \text{) in QGP} \]

- Magnetic screening mass ($m_M$)
  \[ \leftrightarrow \langle A_i(x)A_i(y) \rangle \]
  \[ \longrightarrow \text{Spatial confinement in QGP, non-perturbative} \]

Attempts so far

- $\langle A_\mu A_\nu \rangle$ from lattice simulations in quenched approximation
  (Nakamura et al. PRD 69 (2004) 014506)

- Supergravity modes in AdS/CFT correspondence
  (Bak et al. JHEP 0708 (2007) 049)

Our approach

Polyakov-loop correlations in full lattice simulations ($N_f=2$)

Y. Maezawa @ BNL 4/25/2008
Decomposition of Polyakov-loop correlator

Extract electric and magnetic sector from Polyakov-loop correlator

\[
\begin{align*}
\text{Euclidean-time reflection (} T_E \text{)} & : \\
A(\tau, x) & \rightarrow \bar{A}(\tau, x) \\
A_4(\tau, x) & \rightarrow -A_4(\tau, x) \\
\text{Charge conjugation (} C \text{)} & : \\
A_\mu(\tau, x) & \rightarrow -A_\mu^*(\tau, x)
\end{align*}
\]

Arnold and Yaffe, PRD 52 (1995) 7208

Magnetic and electric gluons btw. Polyakov-loops

Intermediate states in z-direction

\[
\begin{pmatrix}
|\bar{A}\rangle \\
|A_4\rangle
\end{pmatrix}
\]

\[
\begin{array}{c}
T_E \\
C
\end{array}
\begin{array}{c}
+ \\
- \\
-
\end{array}
\]
Screening masses

- Mass inequality: $m_M < m_E$
- For $T > 2T_{pc}$, both $m_M$ and $m_E$ decreases as $T$ increases.
- For $T_{pc} < T < 2T_{pc}$, $m_M$ and $m_E$ behaves differently.
- $m_E$ well approximated by the NLO formula

$$m_E^{NLO}(T)/T = \sqrt{\frac{4}{3}} g(T, \kappa) \left[ 1 + g(T, \kappa) \frac{3}{2\pi} \sqrt{\frac{3}{4}} \left( \ln \frac{2m_E}{m_M} - \frac{1}{2} \right) + O(g^2) \right]$$

Rebhan, PRD 48 3967
Comparison with AdS/CFT

Screening masses in N=4 supersymmetric Yang-Mills matter

Spectra of supergravity modes

- Lightest $T_E$-odd mode (electric sector)
  $$m_D = 3.4041\pi T$$
- Lightest $T_E$-even mode (magnetic sector)
  $$m_{\text{gap}} = 2.3361\pi T$$

D.O.F btw. SYM and QCD different

Screening ratio

$$\frac{m_D(T_E= -)}{m_{\text{gap}}(T_E= +)} = 1.46$$

Good agreement at $T > 1.5T_{pc}$

Y. Maezawa @ BNL 4/25/2008
Comparison with quenched calculation

From \langle AA \rangle in Quenched QCD

From Polyakov-loops in \( N_f = 2 \) QCD
this work

\( m_E(T)/T \)
\( m_M(T)/T \)

\( m_E(T)/T \)
\( m_M(T)/T \)

\( m_\pi/m_\rho = 0.65 \)

\( T/T_c \)
\( T/T_{pc} \)

For \( T > 1.2T_{pc} \) qualitative behavior \((m_M < m_E)\) is the same.

For \( T < 1.2T_{pc} \)

\( N_f = 2 \) QCD

\( m_E \) decreases \( \iff \)
\( m_M \) increases

\( m_E \) increases
\( m_M \) decreases

as \( T \to T_{pc} \)

Order of the phase transition responsible?
STATIC QUARK CORRELATORS IN SU(2) GAUGE THEORY AT FINITE TEMPERATURE

Alexei Bazavov

Department of Physics
University of Arizona

April 25, 2008

in a collaboration with: Peter Petreczky, Alexander Velytsky
Static quark correlators

\[ G_1(r, T) = \frac{1}{2} \langle \text{Tr} L^\dagger(x) U(x, y; 0) L(y) U^\dagger(x, y, 1/T) \rangle, \]
\[ G_3(r, T) = \frac{1}{3} \langle \text{Tr} L^\dagger(x) \text{Tr} L(y) \rangle - \frac{1}{6} \langle \text{Tr} L^\dagger(x) U(x, y; 0) L(y) U^\dagger(x, y, 1/T) \rangle, \]

where \( U(x, y; t) \) are spatial transporters.

\[ \exp(-F_1(r, T)/T + C) = \frac{1}{2} \langle \text{Tr} L^\dagger(x) L(y) \rangle, \]
\[ \exp(-F_3(r, T)/T + C) = \frac{1}{3} \langle \text{Tr} L^\dagger(x) \text{Tr} L(y) \rangle - \frac{1}{6} \langle \text{Tr} L^\dagger(x) L(y) \rangle, \]
\[ r = |x - y|. \]
Static quark-antiquark free energies

Using the transfer matrix one can show that

\[ G_1(r, T) = \sum_{n=1}^{\infty} c_n^1(r) e^{-E_n(r, T)/T}, \quad (1) \]
\[ G(r, T) = \langle \text{Tr} L(r) \text{Tr} L(0) \rangle = \sum_{n=1}^{\infty} e^{-E_n(r, T)/T}, \quad (2) \]

where \( E_n \) are the energy levels of static quark and anti-quark pair. The coefficients \( c_n^1(r) \) depend on the choice of the transporters \( U \).

If \( c_1^1 = 1 \) the dominant contribution to \( G_3 \) would be the 1st excited state \( E_2 \), thus justifying the name singlet and triplet free energy.

In perturbation theory \( c_1^1 = 1 \) up to \( O(g^6) \) corrections \(^1\) and therefore at short distances, \( r \ll 1/T \) the color singlet and color averaged free energy are related \( F_a(r, T) = F_1(r, T) + T \ln 4 \).

Problems with extracting triplet

If $c_1^1 \neq 1$ then

$$e^{-\tilde{F}_a(r)/T} = G(r, t) = c_1^a e^{-E_1(r)/T},$$

$$e^{-\tilde{F}_1(r)/T} = G_1(r, T) = c_1^1(r) e^{-E_1(r)/T},$$

$$e^{-\tilde{F}_3(r)/T} = c_1^3(r) e^{-E_1(r)/T}$$

and

$$\tilde{F}_a(r, T) = E_1(r, T) - T \ln c_1^a,$$

$$\tilde{F}_1(r, T) = E_1(r, T) - T \ln c_1^1(r),$$

$$\tilde{F}_3(r, T) = E_1(r, T) - T \ln c_1^3(r),$$

$$c_1^a \sim 1, \quad c_1^3(r) \sim 1 - c_1^1(r).$$

$\tilde{F}_3(r, T)$ has a contribution from the singlet!

Coefficient $c_1^1$ at $\beta = 2.7$
Conclusion

- The singlet, triplet and color averaged static meson correlators calculated using different levels of APE smearing.
- APE smearing procedure allows to remove distance dependence from the matrix elements $c^1$ in the singlet channel thus providing the correct interpretation of the triplet correlator.
- Compared to fixing Coulomb gauge APE smearing procedure offers improvement in assessing triplet free energy contribution of static quark anti-quark pair.
- For higher temperatures dependence on levels of APE smearing vanishes.
- The screening function shows correct exponential fall-off behaviour.
A Hidden Local Field Theory Description of
Dileptons in Relativistic Heavy Ion Collisions

Gerald E. Brown
Stony Brook University

Summary

Using the notion of "hadronic freedom" inferred from the vector manifestation of hidden local symmetry, we argue that contrary to the commonly held belief, the dileptons measured in relativistic heavy ion collisions carry no appreciable information relevant to the spontaneous breaking of chiral symmetry and hence to the mechanism for mass generation of light-quark hadrons. The dileptons emitted from the vector mesons whose mass is shifted by the vacuum change in temperature and/or density are strongly suppressed in the hadronic free region between the chiral restoration point and the "flash point" at which the vector mesons recover ~ 90% of their free-space on-shell mass and their full strong coupling strength. The spectral function deduced from the dilepton yields reflects predominantly those vector mesons decaying at the "flash temperature" \( T_{\text{flash}} = 120 \text{ MeV} \). Relatively few of the dileptons that are off-shell (in vacuum) come out through the rho meson, so only the rho on-shell spectral function can be reconstructed.
Brown-Rho scaling in normal nuclei

J. Holt et al., PRL 100 (2008) 062501

![Graph showing experimental half-life versus nuclear density. The graph has a x-axis labeled 'Nuclear density n/n₀' and a y-axis labeled '1^4 C Half-life [yr]'. The graph includes a dashed line and a point representing 'Brown-Rho scaled NN interaction'.]
**Hadronic Freedom** (between $T_c$ and $T_{\text{flash}}$)


\[
\frac{\Gamma_{\rho}^*}{\Gamma_{\rho}} = \left( \frac{\langle \bar{q}q \rangle^*}{\langle q\bar{q} \rangle} \right)^5 \approx \left( \frac{m_{\rho}^*}{m_{\rho}} \right)^3 \left( \frac{g^*}{g} \right)^2
\]

<table>
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<th>$T$ (MeV)</th>
<th>$m_{\rho}^*/m_{\rho}$</th>
<th>$\Gamma^*/\Gamma$</th>
</tr>
</thead>
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<tr>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>164</td>
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<td>0</td>
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<tr>
<td>153</td>
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<tr>
<td>142</td>
<td>0.54</td>
<td>0.05</td>
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<tr>
<td>131</td>
<td>0.72</td>
<td>0.22</td>
</tr>
<tr>
<td>120</td>
<td>0.90</td>
<td>0.67</td>
</tr>
</tbody>
</table>

$T_{\text{flash}} = T_{\text{freezeout}}$

32 SU(4) Multiplet (massless at $T_c$)

$\rho(18), a_0(4), a_1(27), \pi(3), \sigma(2)$,

$\varepsilon = f_{1235}(12)$

66 pions result at flash point

\[
\frac{\rho^0}{\pi^-} = \frac{3}{(22-3)} = 0.16
\]
Blue shifted hadrons

\[ T_{\text{eff}} = \sqrt{\frac{1 + \beta_T^2}{1 - \beta_T^2}} \]

- \( \rho \) mass shifted most
- \( T_{\text{eff}}(\rho) \geq 300 \text{ MeV} \)

S. Damjanovic, NA60
NA60 reconstruction of on-shell $\rho$

- Most $\rho$'s seen are just the on-shell vacuum ones

$T_{\text{freezeout}} = 110$ MeV

S. Damjanovic, NA60
Low mass pion pairs

The $2\pi$ mass distribution $\eta_1(\mu^2)$ in the $2\pi$ exchange potential for the case of $I_{\pi\pi} = J_{\pi\pi} = 1$ exchange.\textsuperscript{[5]}
Quarkonia measurements at STAR

Zebo Tang
University of Science and Technology of China
Brookhaven National Laboratory

Summary:

We reported the STAR Preliminary results of J/ψ spectra from 200 GeV p+p collisions at p_T up to 14 GeV/c at mid-rapidity through the dielectron channel. The spectra can be described by color evaporation model and color singlet model with kt-factorization approach. The high p_T J/ψ production was found to follow the x_T scaling with a beam energy dependent factor ∼\sqrt{s_{NN}}^{(0.66±0.2)}. The J/ψ-hadron correlations were also discussed. We observed an absence of charged hadrons accompanying high p_T J/ψ on the near side in contrary to the strong correlation peak in the di-hadron correlations. The implications of J/ψ production mechanism and contribution from decay feed-down were discussed.

The J/ψ nuclear modification factor R_AA in Cu+Cu collisions at √s_{NN} = 200 GeV was also reported. The increase of R_AA from low p_T to high p_T challenges several model interpretations. The average of R_AA at p_T > 5 GeV/c is 0.9±0.2, consistent with no J/ψ suppression at high p_T. It implies that high p_T J/ψ may be produced from virtual photon or outside of the hot interaction region. We also reported the Upsilon results in p+p and Au+Au collisions at √s = 200 GeV. The Upsilon cross section times branch ratio is 91±28 (stat.) ± 22 (syst.) pb, which is consistent with pQCD calculation.
Low $p_T J/\psi$ in p+p at 200 GeV

$J/\psi$ trigger $0 < p_T < 5.5$ GeV/c

STAR has $J/\psi$ capabilities at low $p_T$
Mass and width consistent with MC simulation, low mass tail from electron bremsstrahlung

Integrated p+p luminosity at 200 GeV: 0.4 (pb)$^{-1}$

High $p_T J/\psi$ in p+p at 200 GeV

$J/\psi$ trigger

No background at $p_T > 5$ GeV/c
Reach higher $p_T$ ($\sim 14$ GeV/c)
**J/ψ spectra in p+p at 200 GeV**

- Significantly extend $p_T$ range of previous measurements in p+p at RHIC to 14 GeV/c
- Agreement of charm measurements between STAR and PHENIX
- Consistent with Color Evaporation calculations (R. Vogt, Private communication)

---

**CSM with $k_t$-factorization**

CSM can also describe the data with some improvement like the $k_t$-factorization approach

Are we done?
J/ψ-hadron correlation in p+p

- No significant near side J/ψ-hadron azimuthal angle correlation
- Constrain B meson's contribution to J/ψ yield
- Hints of CSM?

Yields in near/away side

- Away side: Consistent with leading charged hadron correlation measurement (η-h)
- Away side from gluon or light quark fragmentation
- Near side: Consistent with no associated hadron production
- B→J/ψ not a dominant contributor to inclusive J/ψ constrain J/ψ production mechanism
$x_T$ scaling:

- $\pi$ and proton: $n = 6.5 \pm 0.8$ (PLB 637, 161(2006))
- $J/\Psi$: $n = 5.6 \pm 0.2$
- $J/\Psi$ production: closer to $2 \rightarrow 2$ scattering

$n$ is related to the number of point-like constituents taking an active role in the interaction.

- $n=8$: diquark scattering
- $n=4$: QED-like scattering

Zebo Tang, USTC/BNL

STAR Preliminary
Nuclear modification factor $R_{AA}$

- Double the $p_T$ range to 10 GeV/c
- Consistent with no suppression at high $p_T$:
  $R_{AA}(p_T > 5 \text{ GeV/c}) = 0.9 \pm 0.2$
- Indicates $R_{AA}$ increase from low $p_T$ to high $p_T$
- Don't support AdS/CFT prediction
- Formed out of medium?
- Affect by heavy quark/gluon energy loss
- Produced from virtual photon?

2-component approach predicted slightly increase $R_{AA}$ after more consideration.

Compare to NA60

RHIC: $\sqrt{s_{NN}} = 200 \text{ GeV}$, consistent with no suppression at $p_T > 5 \text{ GeV/c}$
SPS: $\sqrt{s_{NN}} = 17.3 \text{ GeV}$, consistent with no suppression at $p_T > 1.8 \text{ GeV/c}$
PHENIX dileptons

This presentation focused upon results from the PHENIX measurements of dielectrons at central rapidity. The principal results included strong excess in the low mass region (below the rho). This region shows three distinct behaviors as a function of low transverse momentum. At intermediate \( p_T \), the spectra have slope constants that scale with mass similarly to charged single hadrons as one might expect for a radially flowing source. At low momentum a cold (T\(\sim\)120 MeV) component whose temperature is independent of exists. This component carries the bulk of the yield. Finally at \( p_T > 1 \) GeV/c, yield is similar to a direct virtual photon yield. The latter has been extracted as a virtual photon spectrum and compared to various models of the initial temperature and its evolution through the plasma expansion.

Thomas K. Hemmick
Stony Brook University
p+p Cocktail Comparison

- Data absolutely normalized
- Excellent agreement data-cocktail
- Extract charm and bottom cross section

Charm: integration after cocktail subtraction
- $\sigma_{cc} = 544 \pm 39 {\text{(stat)}} \pm 142 {\text{(sys)}} \pm 200 {\text{(model)}} \mu b$

Simultaneous fit of charm and bottom:
- $\sigma_{cc} = 518 \pm 47 {\text{(stat)}} \pm 135 {\text{(sys)}} \pm 190 {\text{(model)}} \mu b$
- $\sigma_{bb} = 3.9 \pm 2.4 {\text{(stat)}} +3/-2 {\text{(sys)}} \mu b$
Au+Au Cocktail Comparison

Low-Mass Continuum:
enhancement $150 \leq m_{ee} \leq 750$ MeV: $3.4 \pm 0.2 \text{(stat.)} \pm 1.3 \text{(syst.)} \pm 0.7 \text{(model)}$

Intermediate-Mass Continuum:
$\text{Single-}e \rightarrow p_T \text{ suppression & non-zero } v_2$: charm thermalized?
PYTHIA single- $e$ $p_T$ spectra softer than $p+p$ but coincide with Au+Au

Angular correlations unknown
Room for thermal contribution?

- Data absolutely normalized
- Cocktail filtered in PHENIX acceptance
- Charm from
  - PYTHIA
  - Single electron non photonic spectrum w/o angular correlations

$\sigma_{cc} = N_{\text{coll}} \times 567 \pm 57 \pm 193 \mu \text{b}$
p⁺p: follows the cocktail for all the mass bins
Au⁺Au: significantly deviate at low p_T
Yields and Slopes

- Intermediate $p_T$: inverse slope increase with mass, consistent with radial flow.
- Low $p_T$:
  - inverse slope of only $\sim 120$ MeV!!!
  - accounts for most of the yield!!!
- Cold and Bright
Direct $\gamma$ via $\gamma^*$ for p+p, Au+Au

- New p+p result with $\gamma^*$ method agrees with NLO pQCD predictions, and with the measurement by the calorimeter.
- For Au+Au there is a significant low $p_T$ excess above p+p expectations.
- The excess above TAA scaled p+p spectrum is characterized by the exponential fit explained in the previous slides. The inverse slope and the yield of the exponential is determined.
IN MEDIUM LIGHT MESON RESONANCES AND CHIRAL SYMMETRY RESTORATION

Angel Gómez Nicola
Daniel Fernández-Fraile
Universidad Complutense Madrid

ABSTRACT:

We review our recent work on scalar and vector resonances at finite temperature and density in the context of unitarized chiral perturbation theory and pion scattering poles, paying special attention to chiral symmetry restoration aspects. In particular, we describe the $\sigma f_0(600)$ pole behaviour and nature as the system is driven towards chiral restoration, in connection with related observed phenomena such as threshold enhancement. In the vector channel, our results for the $\rho$-resonance pole are compared with different scenarios aiming to describe dilepton data. We get consistency with broadening scenarios at finite temperature, although pure finitedensity chiral restoring effects could favor a scaling dropping-mass behaviour.

Understanding the QGP through Spectral Functions and Euclidean Correlators
BNL April 2008
MOTIVATION

$\rho \rightarrow$ dilepton spectrum (CERES, NA60) and nuclear matter

Broadening vs Mass shift (scaling?)

$\frac{M(\rho)}{M(0)} = 1 - \alpha \frac{\rho}{\rho_0}$

KEK-E325 (C, Fe-Ti): $\alpha = 0.092 \pm 0.002$

Jlab-CLAS (C, Cu): $\alpha = 0.02 \pm 0.02$

$f_0(600)/\sigma \rightarrow$ vacuum quantum numbers, chiral symmetry restoration

Observed (?) in nuclear matter experiments (CHAOS, ...)

through threshold enhancement (early suggested as a signal of chsr)

Any chance for Heavy Ions (finite $T$)?
OUR APPROACH: UNITARIZED CHIRAL PERTURBATION THEORY


CHIRAL SYMMETRY + UNITARITY

ππ scattering amplitude and πγ form factors in $T > 0$ SU(2) one-loop ChPT

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \cdots$$

Inverse Amplitude Method

$$\text{Im}[i^{\text{LM}}]^{-1} = -\sigma_T$$

$$\sigma_T(E) = \frac{1}{\sqrt{1 - \frac{4m^2}{E^2}}} \left[ 1 + \frac{2n_T(E/2)}{E} \right]$$

$$i^{\text{LM}}(E,T) = \frac{(\eta(E))^3}{\eta(E) + \eta(E)}$$

"Thermal" poles (dynamically generated, no explicit resonance fields)

Thermal phase space enhancement + Increase of effective ππ vertex, small mass reduction up to $T_c$. Form factor consistent with dilepton data.

Strong pole mass reduction (chiral restoration). Phase space squeezing overcomes low-$T$ thermal enhancement. However, remains wide even for $M - 2m_{\pi}$ (spectral function not peaked around the mass for broad resonances).

NO THRESHOLD ENHANCEMENT.
REAL AXIS POLES AND ADLER ZEROS


- Require extra terms in the IAM to account properly for Adler zeros → \( t(s_A) = 0 \).

- Otherwise, spurious real poles below threshold in 1st, 2nd Riemann sheets.

- Preserving chiral symmetry + unitarity:

\[
\frac{1}{i^{iAM}} \rightarrow \frac{1}{i^{iAM}} + g(s)
\]

\[
g(s) = \frac{t_A(s_A) - (s_A - s) [t_2(s_A) - t_1'(s_A)]}{[t_2(s)]^2}
\]

- No difference away from \( s_A \)

- Alternatively derived with dispersion relations.

- No problem for \( l=1 \rightarrow s_A = 4M^2 \)

No additional poles for \( T=0 \) with the redefined amplitudes.
THE NATURE OF THERMAL RESONANCES

Does not behave as a (thermal) $q\bar{q}$ state, not even near the chiral limit

$\rho$

- No BR-like scaling with condensate.
- Mass dropping only very near "critical" (too high) $T_\rho$, as in BR-HLS models
  Brown&Rho
  Harada&Sasaki
- Nature of our thermal $\rho$ dominated by non-restoring effects (broadening)
NUCLEAR CHIRAL RESTORING EFFECTS

\[ f_{\pi}^2(\rho) \left( \frac{\bar{q}q(\rho)}{\langle \bar{q}q \rangle(\rho)} \right) = 1 - \frac{\sigma_{\pi \pi}}{m_{\pi}^2 f_{\pi}^2(0)} \rho = 1 - 0.35 \frac{\rho}{\rho_0} \]

Justified by approximate validity of GOR \((\rho=0, T=0) \implies m_{\pi}^2 f_{\pi}^2 = -m_{\bar{q}q}(\langle \bar{q}q \rangle)\)

Non chiral-restoring many-body effects not included (relevant in the \(p\) channel)

\[ \text{Cabrera, Oset, Vicente-Vacas} \]

\[ \text{Threshold enhancement qualitatively compatible with experimental results of reactions } \pi \pi \rightarrow \pi \pi \text{ and } \pi \pi \rightarrow \pi \pi \]

The \(\rho\) pole approximately follows the condensate curve.

For high densities, a virtual \(\bar{q}q\)-like state exists with a \(\pi \pi\) bound state (compatible with other analyses).

\[ \text{No threshold enhancement for reasonably high densities in the } \rho \text{ channel.} \]

Mass linear fits: 
\[ \frac{M(\rho)}{M(0)} = 1 - \frac{\rho}{\rho_0} \quad \text{up to } \rho = \rho_0 \implies \alpha = 0.2 \]

Additional medium effects might lead to negligible mass shift
Quark Quasi-Particle Picture at Finite Temperature and Density in Effective Models

Teiji Kunihiro (Dep. of Physics, Kyoto)
In collaboration with M. Kitazawa, T. Koide, K. Mitsutani and Y. Nemoto

Abstract: If a QCD phase transition is of a second order or close to that, there should exist specific soft modes. We show that a coupling with such soft modes would generically modifies the quark spectral function significantly so that it gets to have a multiple-peak structure at low energies.

RBRC Workshop at Brookhaven National Laboratory
“Understanding QGP Through Spectral Functions and Euclidean Correlators”, at BNL, April 23-25, 2008
Possible pseudogap formation in heated quark matter

M. Kitazawa, T. Koide, T. K. and Y. Nemoto

\[ N(\omega) = \int \frac{d^3k}{(2\pi)^3} \rho(k, \omega) \]

\[ \varepsilon = \frac{T - T_c}{T_c} \]

- **Pseudogap structure manifests itself in \( N(\omega) \).**
- The pseudogap survives up to \( \varepsilon \approx 0.05 \) (5% above \( T_c \)).

Cherenkov-like emission of diquark-fluctuation mode around Fermi energy.

\[ \Sigma(\omega, k) = \ldots \]
Quarks coupled to chiral soft modes near $T_c$

We incorporate the fluctuation mode into a single particle Green function of a quark through a self-energy.

$$G(\omega_n, p) = \frac{1}{G_0(\omega_n, p) - \Sigma(\omega_n, p)} = + \left\{ \begin{array}{c} \Sigma \\ \Sigma \\ \Sigma \end{array} \right\} + \cdots$$

Non self-consistent $T$-approximation (1-loop of the fluctuation mode)

$$\Sigma(\omega_n, p) = \sum_{\mathbf{q}, i\omega_m} \frac{\Sigma}{D^{1/2}(\mathbf{k} + \mathbf{q}, i\omega_n + i\omega_m)} = T \sum_m \int \frac{d^3q}{(2\pi)^3} D(\omega_n + \omega_m, p + q) G_0(\omega_m, q)$$

N.B. This is a complicated multiple integral owing to the compositeness of the para-sigma and para-pion modes.
Spectral Function of Quark

\[ A(p, p^0) = \rho_+ (p, p^0) \Lambda_+ \gamma^0 + \rho_- (p, p^0) \Lambda_- \gamma^0 \]

\[ \varepsilon = \frac{T - T_c}{T_c} \]


- Three-peak structure emerges.
- The peak around the origin is the sharpest.

Quasi-dispersion relation

\[ \text{Re} \left[ S_+(\omega, p) \right]^{-1} = \omega - |p| - \text{Re} \Sigma_+(\omega, p) = 0 \]
Origin of the 3 peaks

The level crossing is shifted by the mass of the fluctuation modes.

\[ \rho_-(\omega, p) \quad \rho_+(\omega, p) \]

\[ q_h - m_b \quad q_l + m_b \]

\[ p \quad p \]

\[ \bar{q}_h + m_b \]

\[ \bar{q}_h - m_b \]

\[ \omega \quad \omega \quad \omega \quad \omega \quad \omega \]

\[ m_b \rightarrow 0 \]

: the HTL result only with the normal quark and plasmino.
4. Summary and concluding remarks

- If a QCD phase transition is of a second order or close to that, there should exist specific soft modes, which may be easily thermally excited.
- In the fermion-boson system with $m_F \ll m_B$, the fermion spectral function has a 3-peak structure at 1-loop approximation at $T \sim m_B$.

If the chiral transition is close to a second order, quarks may have a 3-peak structure in the QGP phase near $T_c$.

- The physical origin of the 3-peak structure is the Landau damping of quarks and anti-quarks owing to the thermally excited massive boson, which induces a mixing between quarks and anti-quark hole,
- The boson may be vector-type or glueballs.

Future problems:
Full self-consistent calculation
Confirmation in the lattice QCD
Experimental observables; e.g., Lepton-pair production (PHENIX?)
Transport coefficients
Soft mode (density-fluctuations) at the CEP and quark spectrum
Spectral Properties of Quarks
at Finite Temperature in Quenched Lattice QCD

Masakiyo Kitazawa
(Osaka Univ.)

with
Frithjof Karsch

We analyze the spectral properties of the quark propagator at finite temperature in quenched lattice QCD in Landau gauge. The bare quark mass, temperature, and momentum dependences of the quark spectral function is analyzed. We assume a two-pole structure for the quark spectral function, which is numerically found to work quite well. It is shown that in the chiral limit the quark spectral function above $T_c$ has two collective modes that correspond to the normal and plasmino excitations.

Correlation Function

\[ C(\tau,0) = C_+ (\tau) \Lambda_+ + C_- (\tau) \Lambda_- = C_0 (\tau) \gamma^0 + C_S (\tau) \]

64\times16, \beta = 7.459, \kappa = 0.1337, 51\text{confs.}

Fitting result

\[ C_+ (\tau) = z_1 e^{-E_1 \tau} + z_2 e^{-E_2 (\beta-\tau)} \]

We neglect 4 points near the source from the fit.

2-pole ansatz works quite well!! (\chi^2/\text{dof.} \sim 2\text{ in correlated fit})
- Limiting behaviors for $m_0 \to 0, m_0 \to \infty$ are as expected.
- Quark propagator approaches the chiral symmetric one near $m_0=0$.
- $E_2 > E_1$: qualitatively different from the 1-loop result.
Extrapolation of Thermal Mass

Extrapolation of thermal mass to infinite spatial volume limit:

\[ T = 1.25 T_c \]
\[ m_T / T = 0.816(20) \]
\[ m_T = 274(8) \text{MeV} \]

\[ T = 1.5 T_c \]
\[ m_T / T = 0.800(15) \]
\[ m_T = 322(6) \text{MeV} \]

\[ T = 3 T_c \]
\[ m_T / T = 0.771(18) \]
\[ m_T = 625(15) \text{MeV} \]

- Small \( T \) dependence of \( m_T / T \),
- while it decreases slightly with increasing \( T \).
- Simulation with much larger volume is desirable.
Pole Structure for $p>0$

- $E_2 < E_1$; consistent with the HTL result.
- $E_1$ approaches the light cone for large momentum.

2-pole approx. works well again.
Summary

We analyzed the quark spectral function at finite $T$ in lattice QCD.

- Above $T_c$,
  The quark degrees of freedom have a simple quasi-particle picture similar to that in the high $T$ limit even near $T_c$.
  - Light quarks have the plasmino and thermal mass.
  - The ratio $m_f/T$ is insensitive to $T$ near $T_c$.

- Below $T_c$,
  The pole approximation fails completely.

Future Work

gauge dependence / volume dependence
/ full QCD / gluon propagator / ...
RIKEN BNL Research Center Workshop
Understanding QGP through Spectral Functions and Euclidean Correlators
April 23-25, 2008

List of Registered Participants

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<tr>
<th>First Name</th>
<th>Last Name</th>
<th>Affiliation</th>
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<tr>
<td>Wanda</td>
<td>Alberico</td>
<td>University of Torino</td>
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<tr>
<td>Masayuki</td>
<td>Asakawa</td>
<td>Osaka University</td>
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<tr>
<td>Oleksiy</td>
<td>Bazaev</td>
<td>University of Arizona</td>
</tr>
<tr>
<td>Andrea</td>
<td>Beraudo</td>
<td>ECT*</td>
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<tr>
<td>Nora</td>
<td>Brambilla</td>
<td>Dipartimento di Fisica, U. Milano</td>
</tr>
<tr>
<td>Gerald</td>
<td>Brown</td>
<td>State University of NY, Stony Brook</td>
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<td>Simon</td>
<td>Caron-Huot</td>
<td>McGill University</td>
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<td>Constantinos</td>
<td>Constantinou</td>
<td>State University of NY, Stony Brook</td>
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<td>Margaret</td>
<td>Carrington</td>
<td>Brandon University</td>
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<td>Saumen</td>
<td>Datta</td>
<td>Tata Institute of Fundamental Research</td>
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<tr>
<td>Adrian</td>
<td>Dumitru</td>
<td>TTP, Frankfurt Univ.</td>
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<tr>
<td>Kevin</td>
<td>Dusling</td>
<td>State University of NY, Stony Brook</td>
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<td>Shinji</td>
<td>Ejiri</td>
<td>Brookhaven National Laboratory</td>
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<tr>
<td>Daniel</td>
<td>Fernandez-Fraile</td>
<td>University Complutense of Madrid</td>
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<td>Angel</td>
<td>Gomez Nicola</td>
<td>Universidad Complutense Madrid</td>
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<td>Urs</td>
<td>Heller</td>
<td>American Physical Society</td>
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<td>Thomas</td>
<td>Hemmick</td>
<td>State University of NY, Stony Brook</td>
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<td>Yoshimasa</td>
<td>Hidaka</td>
<td>RIKEN BNL Research Center</td>
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<td>Jeremy</td>
<td>Holt</td>
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<td>Kay</td>
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<td>Pasi</td>
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<td>University of Wuppertal</td>
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<td>Olaf</td>
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<td>University of Bielefeld</td>
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<td>Frithjof</td>
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<td>Dmitri</td>
<td>Kharzeev</td>
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<td>Masakiyo</td>
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<td>Teiji</td>
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<td>Mikko</td>
<td>Laine</td>
<td>University of Bielefeld</td>
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<td>Michael</td>
<td>Leitch</td>
<td>Los Alamos National Laboratory</td>
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<td>Jinfeng</td>
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<td>Shu</td>
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<td>Michael</td>
<td>Lublinsky</td>
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<td>Yu</td>
<td>Maezawa</td>
<td>RIKEN</td>
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<tr>
<td>Larry</td>
<td>McLerran</td>
<td>RIKEN BNL Research Center</td>
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<td>Eugenio</td>
<td>Megias Fernandez</td>
<td>Brookhaven National Laboratory</td>
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<tr>
<td>Harvey</td>
<td>Meyer</td>
<td>Massachusetts Institute of Technology</td>
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<tr>
<td>Agnes</td>
<td>Mocsy</td>
<td>RIKEN BNL Research Center</td>
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<tr>
<td>Guy</td>
<td>Moore</td>
<td>McGill University</td>
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<tr>
<td>Emil</td>
<td>Mottola</td>
<td>Los Alamos National Laboratory</td>
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<tr>
<td>Chiho</td>
<td>Nonaka</td>
<td>Nagoya University</td>
</tr>
</tbody>
</table>
Vitaly Okorokov  Moscow Engineering Physics Institute
Andrei Parnachev  State University of NY, Stony Brook
Peter Petreczky  Brookhaven National Laboratory
Rob Pisarski  Brookhaven National Laboratory
Krishna Rajagopal  MIT Physics
Ralf Rapp  Texas A&M University
Claudia Ratti  State University of NY, Stony Brook
Helmut Satz  University of Bielefeld
Edward Shuryak  State University of NY, Stony Brook
Wolfgang Soeldner  Brookhaven National Laboratory
Zebo Tang  USTC/ Brookhaven National Laboratory
Derek Teaney  State University of NY, Stony Brook / RIKEN BNL Research Center
Takashi Umeda  Univ. of Tsukuba
Antonio Vairo  University of Milano
Alexander Velytsky  University of Chicago
Raju Venugopalan  Brookhaven National Laboratory
Zhangbu Xu  Brookhaven National Laboratory
Bin Zhang  Arkansas State University
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<th>Time</th>
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<th>Thursday</th>
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<td>08:45-09:15</td>
<td>Registration Opening: Nicholas Samios RBRC Director</td>
<td>09:00-09:30</td>
<td>H. Meyer (MIT) Energy-momentum Tensor Correlators and Viscous Hydro</td>
<td>09:00-09:40</td>
<td>D. Teaney (Stony Brook) Spectral Densities and Viscous Hydro</td>
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<td>09:30-10:00</td>
<td>O. Kaczmarek (Bielefeld) Static Quarks at Finite Temperature in Lattice QCD</td>
<td>09:30-10:00</td>
<td>K. Huebner (BNL) Bulk Viscosity in SU(2) Gauge Theory</td>
<td>09:40-10:10</td>
<td>M. Lublinsky (Stony Brook) New Relativistic Hydro from AdS/CFT Spectral Functions</td>
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<td>10:00-10:30</td>
<td>N. Brambilla (Milano) Quark-Antiquark and Three-Quark Potentials at T=0</td>
<td>10:00-10:30</td>
<td>D. Kharzeev (BNL) Bulk Viscosity in QCD</td>
<td>10:10-10:40</td>
<td>A. Paramechew (Stony Brook) Progress in Holographic Model of QCD</td>
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<td>10:30-10:50</td>
<td>Coffee</td>
<td>10:30-10:50</td>
<td>Coffee</td>
<td>10:40-11:00</td>
<td>Coffee</td>
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<tr>
<td>11:20-11:50</td>
<td>A. Velytsky (Chicago) Quarkonium Correlators and Spectral Functions from Anisotropic Lattice QCD</td>
<td>11:30-12:00</td>
<td>M. Carrington (Brandon U) Transport Methods and nPI Methods</td>
<td>11:30-12:00</td>
<td>A. Bazavov (Arizona U) Static Quark Correlators in SU(2) Gauge Theory at Finite Temperature</td>
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<tr>
<td>11:50-12:20</td>
<td>S. Datta (TIFR) Quarkonia Correlators from Isotropic Lattice QCD</td>
<td>12:00-12:30</td>
<td>E. Mottola (Los Alamos) Resummation in Equilibrium and Non-Equilibrium Gauge Theories</td>
<td>12:30-13:00</td>
<td>Lunch</td>
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<td>12:20-13:40</td>
<td>Lunch</td>
<td>12:30-14:00</td>
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<td>12:00-12:40</td>
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<td>14:00-14:30</td>
<td>A. Valro (Milano) Static Quark-Antiquark Pairs at Finite Temperature</td>
<td>14:00-14:30</td>
<td>Y. Hidaka (RBRC) Suppression of the Shear Viscosity as QCD Cools into a Confining Phase</td>
<td>13:40-14:10</td>
<td>G. Brown (Stony Brook) A Field Theory for Relativistic Heavy Ion Collisions</td>
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<td>14:30-15:00</td>
<td>M. Laine (Bielefeld) Heavy Quarkonium According to Resummed Perturbation Theory</td>
<td>14:30-15:00</td>
<td>D. Fernandez-Fraile (Madrid) Transport Coefficients in Chiral Theories</td>
<td>14:10-14:40</td>
<td>Z. Tang (USTC/BNL) Quarkonium Measurements in STAR</td>
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<td>15:00-15:30</td>
<td>A. Beraudo (ECT) Real and Imaginary Time Correlators of a Q-Qbar Pair in a Thermal Medium</td>
<td>15:00-15:30</td>
<td>M. Leitch (Los Alamos) Quarkonium Measurements in PHENIX</td>
<td>14:40-15:10</td>
<td>T. Hemmick (Stony Brook) Light Meson Sector Measurements from PHENIX</td>
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<td>15:30-16:00</td>
<td>GROUP PHOTO</td>
<td>15:30-15:50</td>
<td>Coffee</td>
<td>15:10-15:30</td>
<td>Coffee</td>
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<tr>
<td>15:50-16:20</td>
<td>A. Dumitru (Frankfurt) The Heavy-Quark Potential in an Anisotropic QGP</td>
<td>15:50-16:20</td>
<td>R. Rapp (Texas A&amp;M) Quarkonia, Heavy Quarks, and sQGP</td>
<td>15:30-16:00</td>
<td>A. Nicola (Madrid) In-medium Light Meson Resonances and Chiral Symmetry Restoration</td>
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<td>16:20-16:50</td>
<td>W. Alberico (Torino) Quarkonia in the Deconfined Phase: Potential Models and Correlators</td>
<td>16:20-16:50</td>
<td>H. Satz (Bielefeld) On the In-Medium Behavior of Finite Width Charmonia</td>
<td>16:00-16:30</td>
<td>T. Kunihiro (Kyoto) Quark Quasi-Particle Picture at High Temperature in Effective Models</td>
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<td>16:50-17:20</td>
<td>T. Umeda (Tsukuba) Charmonium Wave Functions at Finite Temperature from Lattice QCD Calculations</td>
<td>16:50-17:20</td>
<td>E. Shuryak (Stony Brook) Charmonium in Strongly Coupled Plasma</td>
<td>16:30-17:00</td>
<td>M. Kitazawa (Osaka) Spectral Properties of Quarks at Finite Temperature in Quenched Lattice QCD</td>
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<td>17:20-17:50</td>
<td>M. Asakawa (Osaka) Spectral Functions of One, Two, and Three Quark Operators in the Quark-Gluon Plasma</td>
<td>17:20-17:50</td>
<td>S. Caron-Huot (McGill) Heavy Quark Momentum Diffusion at Next-to-leading Order</td>
<td>18:30-20:30</td>
<td>Dinner</td>
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Organizers: Ágnes Mócsy and Péter Petreczky

RBRC Workshop at Brookhaven National Laboratory

"Understanding QGP Through Spectral Functions and Euclidean Correlators"

Large Seminar Room, Physics Department, Bldg 510

April 22-25, 2008

AGENDA
Additional RIKEN BNL Research Center Proceedings:

Volume 88 – Hydrodynamics in Heavy Ion Collisions and QCD Equation of State, April 21-22, 2008, BNL-
Volume 87 – RBRC Scientific Review Committee Meeting – BNL-79570-2007
Volume 86 – Global Analysis of Polarized Parton Distributions in the RHIC Era, October 8, 2007, BNL-
Volume 79570-2007
Volume 84 – Domain Wall Fermions at Ten Years, March 15-17, 2007 – BNL 77857-2007
Volume 83 – QCD in Extreme Conditions, July 31-August 2, 2006– BNL-76933-2006
Volume 81 – Parton Orbital Angular Momentum (Joint RBRC/University of New Mexico Workshop) February
24-26, 2006 – BNL-75937-2006
Volume 80 – Can We Discover the QCD Critical Point at RHIC?, March 9-10, 2006 – BNL-75692-2006
Volume 79 – Strangeness in Collisions, February 16-17, 2006 – BNL-79763-2008
Volume 78 – Heavy Flavor Productions and Hot/Dense Quark Matter, December 12-14, 2005 – BNL-76915-
2006
Volume 77 – RBRC Scientific Review Committee Meeting – BNL-52649-2005
Volume 76 – Odderon Searches at RHIC, September 27-29, 2005 – BNL-75092-2005
Volume 75 – Single Spin Asymmetries, June 1-3, 2005 – BNL-74717-2005
Volume 74 – RBRC QCDOC Computer Dedication and Symposium on RBRC QCDOC, May 26, 2005 –
BNL-74813-2005
Volume 72 – RHIC Spin Collaboration Meetings XXXI(January 14, 2005), XXXII (February 10, 2005),
XXXIII (March 11, 2005) – BNL-73866-2005
Volume 71 – Classical and Quantum Aspects of the Color Glass Condensate – BNL-73793-2005
Volume 69 – Review Committee – BNL-73546-2004
Volume 68 – Workshop on the Physics Programme of the RBRC and UKQCD QCDOC Machines – BNL-
73604-2004
Volume 67 – High Performance Computing with BlueGene/L and QCDOC Architectures – BNL-
Volume 65 – RHIC Spin Collaboration Meetings XXVII (July 22, 2004), XXVIII (September 2, 2004), XXX
(December 6, 2004) - BNL-73506-2004
Volume 64 – Theory Summer Program on RHIC Physics – BNL-73263-2004
Volume 63 – RHIC Spin Collaboration Meetings XXIV (May 21, 2004), XXV (May 27, 2004), XXVI (June 1,
2004) – BNL-72397-2004
BNL-72336-2004
Volume 60 – Lattice QCD at Finite Temperature and Density – BNL-72083-2004

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Additional RIKEN BNL Research Center Proceedings:

Volume 58 – RHIC Spin Collaboration Meeting XX – BNL-71900-2004
Volume 57 – High pt Physics at RHIC, December 2-6, 2003 – BNL-72069-2004
Volume 56 – RBRC Scientific Review Committee Meeting – BNL-71899-2003
Volume 52 – RIKEN School on QCD “Topics on the Proton” – BNL-71694-2003
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