Melting Sequence of Quarkonia

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24th Winter Workshop on Nuclear Dynamics
South Padre, Texas, USA

August 7, 2008

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Introduction

One of the aims of relativistic heavy ion collisions is to produce a quark-gluon plasma (QGP), a state of matter in which the constituents of our hadronic world are deconfined. Deconfinement can happen when matter is heated up to high temperatures. High energy density matter has been already produced at CERN SPS and BNL RHIC, and is due later this year at CERN LHC. To have control over what temperatures are achieved and whether deconfined matter has been produced we need a thermometer. The sequential melting of quarkonium (bound state of heavy quark-antiquark) has been long considered to be exactly that: the QGP thermometer [1]. In a deconfined matter the force between a heavy quark and its antiquark is weakened by screening from light quarks and gluons, leading to dissociation of quarkonium. Different states are expected to melt at different temperatures. Measuring the dilepton spectrum can tell about the suppression of quarkonium yield.

$J/\psi$ suppression has been indeed measured, but the story turned out to be more complicated, and the observations are not yet understood. The reason is that the suppression pattern seen is not only due to screening, but might contain also effects of cold nuclear matter, as well as recombination. We need to disentangle these. One step toward is to know the properties of quarkonium in-medium and determine their dissociation temperatures.

In principle, everything about a given quarkonium channel is embedded in the spectral function. The position of a peak in the spectral function corresponds to the mass of a bound state, while its width determines its lifetime. Information
about the continuum and its threshold is also contained in the spectral function. Melting of a state corresponds to the disappearance of a peak. There are two main lines of studies to determine quarkonium spectral functions at finite temperature: lattice QCD and potential models (see figure 1). In the following I first discuss our current understanding of the lattice calculations; then I talk about potential models, highlighting when lattice data may be used (as input or as constrain) in context of these models.

![Fig. 1. Structural chart of lattice QCD and potential model calculations.](image)

**Understanding Lattice Data**

There are two lattice QCD calculations relevant for quarkonium studies: One is the measurement of the current-current correlation function of mesonic currents in different quarkonium channels, from which the extraction of spectral function is attempted, and the other is the measurement of the T-dependence of the free energy of a static quark-antiquark pair (see figure 1).

In lattice QCD quarkonium spectral functions are not calculated directly, but are extracted from the current-current correlation functions in Euclidean-time that are calculated directly [2] (see figure 1). In the pseudoscalar and vector channel the extracted spectral functions show no large T-dependence for up to about $1.5T_c$, while this is not the case in the scalar channel. The first peak has been commonly interpreted as the ground state. Based on this the melting temperature of the $J/\psi$ has been thought to be much greater than originally expected, while the $\chi_c$ melts near $T_c$. However, uncertainties in the spectral function are significant and details of this cannot be resolved! Their extraction using the Maximum Entropy Method is still difficult due to discretization effects and statistical errors. At this point
it is difficult to make any conclusive statement based on the shape of the spectral functions [3].

Contrary to the extracted spectral functions, the correlators are measured reliably. Based on their relation through an integral equation [3], it is instructive to look at the T-dependence of the correlator itself. The comparison of high T correlators to correlators reconstructed from spectral function at low T, the ratio \( G/G_{\text{recon}} \), shows no T-dependence in the pseudoscalar channel up to well above \( T_c \) while it strongly deviates from one near \( T_c \) for the scalar [3]. The initial interpretation of the deviation from one of correlator ratio as the dissociation temperature of the ground state was in seeming agreement with the spectral function interpretation. Recently it has been clarified though, that almost the entire T-dependence of the correlator ratio comes from the commonly overlooked zero-modes [4]. These are low-frequency contributions to the spectral functions at finite T describing the scattering states of single heavy quarks, in addition to the usual bound and unbound quark-antiquark pairs in a given channel. Zero-mode is understood in terms of quasi-free quarks with some effective mass [5], indicating the presence of free heavy quarks in the deconfined phase. Furthermore, the zero-mode gives a constant contribution to correlator. One can eliminate it by looking at ratio of the derivatives of correlators [4]. All the resulting correlators are flat in all channels up to \( 3T_c \)! One can conclude that the flatness is not related to survival, since it would also imply that the \( \chi_c \) survives until \( 3T_c \). The understanding is simple: the dramatic changes in spectral function are not reflected in the correlator.

The other, independent calculation from lattice is of the free energy of a static quark-antiquark pair [7]. More precisely, the change in the free energy of a medium at a given temperature when a static quark-antiquark pair is immersed into it. The results show that above deconfinement the range of interaction between the quark and antiquark is strongly reduced. This can be well described by exponential screening. Given this, it is even more difficult to imagine that the \( J/\psi \) survives in the QGP up to \( 1.5 - 2T_c \) even though strong screening is seen.

In order to quantify these statements, and since the spectral functions from lattice are inconclusive, one needs to resort to potential models. As I discuss in the next section, in lack of knowledge of the finite T potential, the screening seen in the lattice free energy may serve as basis for input in potential models (see figure 1). The model calculations of the correlators have to been then constrained by the lattice data on correlators (see figure 1).

**Current Status of Potential Models**

Potential models are based on the assumption that heavy quark-antiquark interactions can be described by a potential. Due to the largeness of the heavy quark mass, \( m_{c,b} \gg \Lambda_{\text{QCD}} \) and the smallness of the heavy quark velocity, \( v \ll 1 \), one can solve the nonrelativistic Schrödinger equation to obtain the properties of the bound states. At zero temperature the Cornell potential has experienced great success:
It describes well the experimentally observed quarkonium spectroscopy [8]; it has been verified on the lattice [9]; it can be derived directly from QCD [10]. The latest is possible due to the hierarchy of energy-scales \( m \gg mv \gg mv^2 \), which allows to systematically integrate out the different scales and obtain non-relativistic QCD (NRQCD) [11] and potential NRQCD (pNRQCD), in which the Cornell potential shows up as matching coefficient [10].

Inspired by its success at zero temperature the potential model has been applied at finite temperature, with the assumption that medium effects can be accounted for as a screened temperature-dependent potential. Matsui and Satz argued for if the range of screening becomes smaller than the radius of a bound state the binding of this would be significantly reduced and the bound state would melt [1]. In order to quantify this statement knowing the potential at finite \( T \) is required. In principle, one should derive this directly from QCD, just as this has been done at \( T = 0 \) (see figure 1). But the existence of temperature-driven scales, \( T, gT, g^2T \), makes such derivation complicated and this has not been addressed until recently [12]. In lack of knowledge of the potential different phenomenological versions of these have been used, in particular, lattice-based potentials, i.e. potentials constrained by lattice data on the free energy of a static quark-antiquark pair (see figure 1).

Few years ago it has become clear [17] that instead of studying simply the discrete bound states by solving the Schrödinger equation (procedure good at \( T = 0 \) where quarkonium is well defined), we should rather obtain a unified treatment of bound states, threshold and continuum by determining the spectral function. This can be done, for instance, by solving the Schrödinger equation for the nonrelativistic Green’s function, and use the optical theorem to determine the spectral function.

One of the most debated questions is which lattice-based potential should be used in the Schrödinger equation. First it was the free energy \( F_1 \) [13]. It is now understood that this serves merely as a lower limit, due to an entropy contribution \( TS \). Removing the entropy the internal energy \( U_1 \) is obtained [7] which has also been used as potential [14]. \( U_1 \) is much deeper than \( F_1 \) leading to stronger binding and higher quarkonium dissociation temperatures. But the interpretation of \( U_1 \) as potential is questionable (includes medium polarization effects; its large increase near \( T_c \); increased strength at short distances compared to \( T = 0 \), serving as a sort of upper limit). Other lattice-based potentials on the market include the one proposed by Wong as a combination of \( F_1 \) and \( U_1 \) [15, 16], and a set of potentials shown in the left panel of figure 2 constructed using the general features of the lattice free energy: no deviation form the vacuum potential at short distances, while exponential screening at large distances [6].

Following the suggestion to compare correlators from the models to the ones from lattice QCD [17], the idea is to constrain different models using data from lattice [6]. The approach is the following: Assume a lattice-based potential and determine the corresponding spectral function in a given quarkonium channel. From the spectral function determine the ratio of correlators \( G/G_{\text{vac}} \) and compare it to lattice data. The “correct” potential and spectral function would be the one in best...
agreement with the data.

The surprising result is shown in the right panel of figure 2. This figure displays the correlators at $1.2T_c$ obtained using the set of potentials exploring the uncertainty within the allowed ranges [17] as shown in the left panel of figure 2 and compared to lattice data from [17]. The surprise is that the complete set of potentials all provide agreement of $1-2\%$ accuracy with lattice correlators, yielding indistinguishable results. Spectral functions that show peak structure for the ground state to higher temperatures, obtained with more binding potentials, and spectral functions with no resonance-like structure seen already near $T_c$ (see spectral functions in [17]), from less binding potential, yield flat correlator ratio. So possible dramatic changes in spectral function are not reflected in the correlator. Thus we cannot identify the "correct" potential, cannot determine the exact spectral function and quarkonium properties from such comparisons.

![Fig. 2. Set of potentials allowed by lattice data on free energies (left) and the corresponding correlator ratio compared to lattice data (right) at $1.2T_c$ (see [17] for details).](image)

So what can we learn then? There are a number of features common for all the spectral functions from potential models [6, 14, 16]:

1) There is a large threshold (rescattering) enhancement beyond what corresponds to free quark propagation. This is present even at high temperatures, indicating that there is a correlation persisting between the quark and antiquark, and it is true for all the channels. The threshold enhancement compensates for the melting of states keeping correlators flat.

2) The binding energy (distance between peak position and continuum threshold) determined from potential models decreases strongly with increasing temperature. So in a spectral function that exhibits a resonance-like peak the corresponding binding energy can be small. But what is the meaning of a $J/\psi$ with 0.2 MeV binding energy [15]? It has been customary to consider a state dissociated when its binding energy becomes zero. In principle, a state is dissociated when no peak structure is seen. But here I must warn that widths shown in spectral functions from current poten-
tial model calculations are not physical. Broadening of states as the temperature increases is not included in any of these models. At which \( T \) the peak structure disappears then? In [18] we argue that no need to reach \( E_{\text{bin}} = 0 \) to dissociate, but when \( E_{\text{bin}} < T \) a state is weakly bound and thermal fluctuations can destroy it. Let me quantify this statement.

As discussed above, we cannot determine the binding energy exactly, but we can nevertheless set an upper limit for it [18]. What I mean is that we can determine \( E_{\text{bin}} \) with the most confining potential that is still within the allowed ranges by lattice data on free energies. For the most confining potential the distance where deviation from \( T = 0 \) potential starts is pushed to large distances so it coincides with the distance where screening sets in [6]. From \( E_{\text{bin}} \) we can estimate the quarkonium dissociation rate due to thermal activation, obtaining this way the thermal width of a state \( \Gamma(T) \) [19]. At temperatures where the width (inverse of decay time) is greater than the binding energy (inverse of binding time) the state will likely to be dissociated. In other words, for example, already close to \( T_c \) the \( J/\psi \) would melt before it bounds. To quantify the dissociation condition we have set a more conservative condition for dissociation: \( 2E_{\text{bin}}(T) < \Gamma(T) \). The result for different charmonium and bottomonium states is shown in the thermometer of figure 3. Please recall, that all these numbers are to be though of as upper limits.

**Lessons**

From the temperature-dependence of quarkonium correlators from lattice QCD we learnt that small change in the ratio of correlators does not imply (un)modification of states. The dominant source of the T-dependence of this comes from zero-modes (low energy part of spectral function), which is understood in terms of free heavy quark gas. The high energy part which carries info about bound states shows almost no T-dependence until \( 3T_c \) in all channels. As for the spectral functions, although spectral functions obtained with MEM do not show much T-dependence, the details (like bound state peaks) are not resolved in the current lattice data.

Potential models utilizing a set of potentials between the lower and upper limit constrained by lattice free energy data yield agreement with lattice data on correla-
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Tors in all quarkonium channels. Therefore, precise quarkonium properties cannot be determined this way, only an upper limit can be estimated. The decrease in binding energies with increasing temperature can yield significant broadening, not accounted for in the currently shown spectral functions from potential models. The upper limit, most confining potential predicts that all bound states melt by 1.3T_c, except the upsilon, which survives until 2T_c. The large threshold enhancement above free propagation seen in the spectral functions even at high temperatures compensates for melting of states (flat correlators) and indicates that correlation between quark and antiquark persists. Lattice results are consistent with quarkonium melting.

Outlook

Implications of the new QGP thermometer of figure 3 for heavy ion collisions should be considered by phenomenological studies. This can have consequences for the understanding of the R_A^AA measurements, since now the J/ψ should melt at SPS and RHIC energies as well. Also, this results suggest that the T will be definitely suppressed at the LHC, but centrality dependence of this can reveal whether this happens already at RHIC. The correlations of heavy-quark pairs may lead to non-statistical recombination.

All of the above discussion refers to quarkonium at rest. Finite momentum calculations are to appear soon [?]. It is expected that a moving quarkonium dissociates faster. Also, all of the above discussion is for isotropic medium. Recently, the effect of anisotropic plasma has been considered [20]. Accordingly, quarkonium might be stronger bound in an anisotropic medium, especially if it is aligned along the anisotropy of the medium (beam direction). Qualitative consequences of these are further to be considered.

As for the exact determination of quarkonium properties the future is in the effective field theories from QCD at finite T. First works on this already appeared [12,21] and both real and imaginary parts of the potential have been derived in certain limits. There is indication though that medium effects cannot always be described as screened potential, but non-potential thermal effects on the static energy and decay width can show up [12].

Acknowledgment

I thank Péter Petreczky for collaboration on performing this work. I also thank the Organizers for inviting me to give this talk at the Winter Workshop. This work has been supported by U.S. Department of Energy under Contract No. DE-AC02-98CH10886.
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