

*The powers of deconfinement*

**E. Megías<sup>1</sup>, E. Ruiz Arriola<sup>2</sup> and L.L. Salcedo<sup>2</sup>**

<sup>1</sup>Department of Physics, Brookhaven National Laboratory, Upton, NY, USA 11973, USA

<sup>2</sup>Departamento de Física Atómica, Molecular y Nuclear, Universidad de Granada, E-18071, Granada Spain

*Presented at the 14<sup>th</sup> International QCD Conference  
(QCD 08)*

Montpellier, France

July 7 to 12, 2008

**Physics Department  
Nuclear Theory Group**

**Brookhaven National Laboratory**

P.O. Box 5000

Upton, NY 11973-5000

[www.bnl.gov](http://www.bnl.gov)

Notice: This manuscript has been authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy. The publisher by accepting the manuscript for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this manuscript, or allow others to do so, for United States Government purposes.

This preprint is intended for publication in a journal or proceedings. Since changes may be made before publication, it may not be cited or reproduced without the author's permission.

## DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

# The powers of deconfinement \*

E. Megías<sup>a</sup>, E. Ruiz Arriola<sup>b</sup> and L.L. Salcedo<sup>b</sup>

<sup>a</sup>Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA

<sup>b</sup>Departamento de Física Atómica, Molecular y Nuclear, Universidad de Granada, E-18071 Granada, Spain

The trace anomaly of gluodynamics encodes the breakdown of classical scale invariance due to interactions around the deconfinement phase transition. While it is expected that at high temperatures perturbation theory becomes applicable we show that current lattice calculations are far from the perturbative regime and are dominated instead by inverse even power corrections in the temperature, while the total perturbative contribution is estimated to be extremely small and compatible with zero within error bars. We provide an interpretation in terms of dimension-two gluon condensate of the dimensionally reduced theory which value agrees with a similar analysis of power corrections from available lattice data for the renormalized Polyakov loop and the heavy quark-antiquark free energy in the deconfined phase of QCD [1,2].

**Introduction.** The Lagrangian of gluodynamics is conformal invariant, reflecting the absence of an explicit scale. The divergence of the dilatation current equals the trace of the improved energy-momentum tensor  $\Theta_\mu^\mu$  [3] and vanishes classically. Quantum-mechanically yields instead the so-called “trace anomaly” [4]. It reflects the breaking of scale invariance which introduces a single mass scale,  $\Lambda_{\text{QCD}}$ . The dimensionless “interaction measure”  $\Delta = T\partial_T(p/T^4) = (\epsilon - 3p)/T^4$  quantifies the departure from the conformal limit  $\epsilon = 3p$ , which corresponds to a gas of free massless particles. At finite temperature, the energy density  $\epsilon$  and the pressure  $p$  enter as [5,6,7,8],

$$T^4 \Delta \equiv \epsilon - 3p = \frac{\beta(g)}{2g} \langle (G_{\mu\nu}^a)^2 \rangle \equiv \langle \Theta_\mu^\mu \rangle, \quad (1)$$

where  $G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$  is the field strength tensor and  $\beta(g) = \mu \partial g / \partial \mu = -b_0 g^3 + \mathcal{O}(g^5)$  is the beta function, with  $b_0 =$

$11N_c/(48\pi^2)$ . A good knowledge of  $\Delta$  is crucial to understand the deconfinement process, where the non perturbative (NP) nature of low energy QCD seems to play a prominent role [9]. In this contribution we analyze the highly NP behaviour of the trace anomaly just above the phase transition and describe it in a way that is consistent with other thermal observables (see [10] for further details).

**Low and high temperatures.** At low temperatures  $\Delta$  is dominated by the lightest confined states in the spectrum. In gluodynamics the lightest glueball mass  $m_G \approx 1.3$  GeV is much heavier than  $T_c \approx 270$  MeV, and the pressure is  $p \sim e^{-m_G/T}$ , so  $\Delta \sim e^{-m_G/T}$ ,  $T \ll T_c$ , indicating an exponentially small violation of scale invariance.<sup>2</sup> At very high temperatures one also expects scale invariance to be restored while asymptotic freedom guarantees the applicability of perturbative QCD (pQCD). Actually, from the pressure to two loop one has [12]

$$\Delta = \frac{N_c(N_c^2 - 1)}{72} b_0 g^4(\mu) + \mathcal{O}(g^5), \quad T \gg T_c \quad (2)$$

where  $1/g^2(\mu) = b_0 \log(\mu^2/\Lambda_{\text{QCD}}^2)$ . It should be noted the ambiguity in this result, since generally

<sup>2</sup>In full QCD for massless quarks one has a gas of weakly interacting massless pions and  $\Delta \sim T^4/\Lambda_{\text{QCD}}^4$  as dictated by chiral symmetry [11].

\*Presented by E. Megías at the 14th International Conference In Quantum Chromodynamics (QCD 08) 7-12th Jul 2008, Montpellier, France. Work supported by the Fulbright Program of the U.S. Department of State and Spanish MEC, the Spanish DGI and FEDER funds with grant FIS2005-00810, Junta de Andalucía grant FQM-225-05, EU Integrated Infrastructure Initiative Hadron Physics Project contract RII3-CT-2004-506078, and U.S. Department of Energy contract DE-AC02-98CH10886.

one has both the temperature  $T$  and the  $(\overline{MS})$ -renormalization scale,  $\mu$ , for which one takes the reasonable but *arbitrary* choice  $\mu \sim 2\pi T - 4\pi T$ . Higher order corrections including up to  $g^6 \log g$  can be traced from [13]. The infrared problems of the perturbative expansion yield poor convergence at the lattice QCD available temperatures  $T < 6T_c$ . Perturbation theory contains only logarithms in the temperature, suggesting a mild temperature dependence. This feature is shared by hard thermal loops (HTL) and other resummation techniques of infrared divergencies (see e.g. [14,15]). Actually, the value they find  $\Delta_{\text{HTL}} = 0 \pm 0.5$  for  $T > T_c$  is compatible with zero within uncertainties. Furthermore, it is not clear at *what* temperatures is the pQCD result dominating  $\Delta$ .

**Thermal power corrections in gluodynamics.** The interaction measure on the lattice for gluodynamics [16] is shown in Fig. 1 and, as expected, is very small below  $T_c$ . It increases suddenly near and above  $T_c$  by latent heat of deconfinement, and raises a maximum at  $T \approx 1.1T_c$ . Then it has a gradual decrease reaching small values at  $T = 5T_c$ . The high value of  $\Delta$  for  $T_c < T < (2.5 - 3)T_c$  corresponds to a strongly interacting Quark-Gluon Plasma picture.

In previous works [1,2] we have detected the presence of inverse power corrections in other thermal observables. In Fig. 1 we plot  $(\epsilon - 3p)/T^4$  as a function of  $1/T^2$  (in units of  $T_c$ ) exhibiting an obvious straight line behaviour in the region slightly above the critical temperature,

$$\Delta_{\text{latt}} = a_{\text{tra}} + b_{\text{tra}} (T_c/T)^2, \quad (3)$$

and corresponding to a ‘‘power correction’’ in temperature. A fit of the lattice data ( $N_c^3 \times N_T = 32^3 \times 8$ ) for  $1.13T_c \leq T \leq 4.54T_c$  yields  $a_{\text{tra}} = -0.02(4)$ ,  $b_{\text{tra}} = 3.46(13)$ ,  $\chi^2/\text{DOF} = 0.35$ . Power corrections also appear in  $\epsilon$  and  $p$ , just by applying the standard thermodynamic relations. Note that if the power correction was absent the transition towards the free gas value would take over much faster indicating a weakly interacting quark-gluon plasma in the neighbourhood of  $T_c$ .

Pisarski [17] has suggested to interpolate between  $T = T_c$  where  $p = 0$  for non-interacting

and heavy glueballs and the free massless gluon gas value  $p \sim T^4$  at  $T \gg T_c$ . If we choose

$$p = \frac{(N_c^2 - 1)\pi^2}{45} T^4 \left[ 1 - \left( \frac{T_c}{T} \right)^n \right], \quad T \geq T_c, \quad (4)$$

with  $n$  arbitrary we get

$$\Delta = \frac{n(N_c^2 - 1)\pi^2}{45} \left( \frac{T_c}{T} \right)^n, \quad T \geq T_c, \quad (5)$$

which for  $n = 2$  corresponds to take  $a = 0$  and  $b = 3.51$  in Eq. (3), in excellent agreement with the fit to the lattice data. This thermodynamic consistency does not explain however *why* there is a power correction with  $n = 2$ . A fit of Eq. (5) to the data of Fig. 1 for the same range as in Eq. (3), yields  $n = 1.97(5)$  with  $\chi^2/\text{DOF} = 0.62$ , indicating the robustness of the power.

The lattice behaviour of  $\Delta$  clearly contradicts perturbation theory [12,13] and resummations thereof [14,15] explaining why they have flagrantly failed to describe the data of the free energy below  $3T_c$ . Our discussion above shows that these approaches would yield a powerless contribution,  $\Delta_{\text{PT}}$ , which should ultimately be identified with the *almost* constant and vanishing  $a_{\text{tra}}$  of Eq. (3) rather than with the full result from the lattice [16]. Actually, the maximum lattice temperature  $T = 5T_c$  may still be far from the pQCD estimate since the power correction provides the bulk of the full result at this temperature. For  $N_c = 3$  the  $\mathcal{O}(g^5)$  correction to Eq. (2) corresponds to multiply it by  $(1 - 6g(\mu)/\pi)$  [13] which becomes small for  $g(\mu) \ll \pi/6$  or  $\mu \gg 10^{11} \Lambda_{\text{QCD}}$ . This delayed onset of pQCD is not new and happens in the study of exclusive processes at high energies (see e.g. Ref. [18]).

**Dimension two gluon condensate.** Power corrections are unmistakable high energy traces of NP low energy effects. Within QCD sum rules power corrections at high  $Q^2$  are usually related to *local* condensates as suggested by the OPE. The gluon condensate  $\langle G^2 \rangle \equiv g^2 \langle (G_{\mu\nu}^a)^2 \rangle$  describes the anomalous (and not spontaneous) breaking of scale invariance, and hence is not an order parameter of the phase transition. Actually, the order parameter is the vacuum expectation value of the Polyakov loop  $L(T)$  which signals the

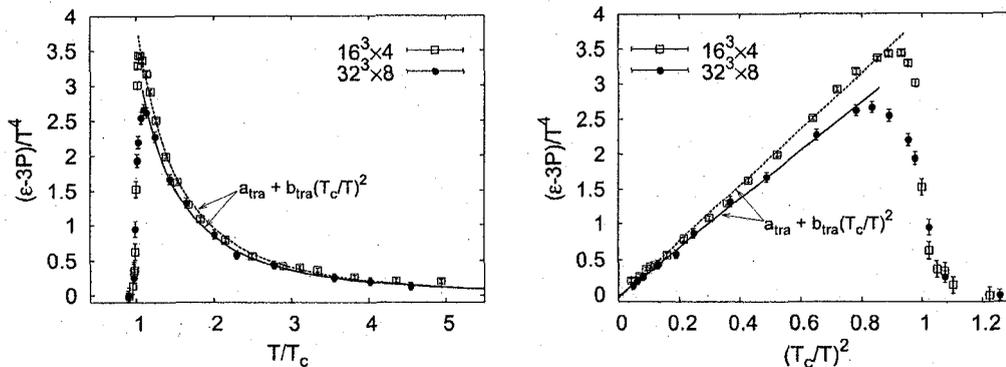


Figure 1. Trace anomaly  $(\epsilon - 3p)/T^4$  as a function of  $T$  (left) and  $1/T^2$  (right) (in units of  $T_c$ ). Lattice data are from [16] for  $N_\sigma^3 \times N_\tau = 16^3 \times 4$  and  $32^3 \times 8$ . The fits use Eq. (3) with  $a_{\text{tra}}$  and  $b_{\text{tra}}$  adjustable constants.

breaking of the  $\mathbb{Z}(N_c)$  discrete symmetry of gluodynamics as well as the deconfinement transition. A dimension two gluon condensate naturally appears from a computation of  $L(T)$  which in the static gauge,  $\partial_0 A_0(\mathbf{x}, x_0) = 0$ , in a Gaussian-like, large  $N_c$  motivated, approximation gives [1]

$$\left\langle \frac{1}{N_c} \text{tr}_c e^{igA_0(\mathbf{x})/T} \right\rangle = \exp \left[ -\frac{g^2 \langle A_{0,a}^2 \rangle}{4N_c T^2} \right], \quad (6)$$

valid up to  $\mathcal{O}(g^5)$  in pQCD.  $A_0$  is the gluon field in the (Euclidean) time direction.

The dimension two gluon condensate  $g^2 \langle A_{0,a}^2 \rangle$  is obtained from the gluon propagator of the dimensionally reduced theory,  $D_{00}$ , by taking the coincidence limit. The perturbative propagator  $D_{00}^P(\mathbf{k}) = 1/(\mathbf{k}^2 + m_D^2) + \mathcal{O}(g^2)$ , being  $m_D \sim T$  the Debye mass, leads to the known perturbative result [19] and fails to reproduce lattice data below  $6T_c$  [20]. A NP model is proposed in Ref. [1] to describe the lattice data of the Polyakov loop, and it consists in a new piece in the gluon propagator driven by a positive mass dimension parameter:

$$D_{00}(\mathbf{k}) = D_{00}^P(\mathbf{k}) + D_{00}^{\text{NP}}(\mathbf{k}), \quad (7)$$

where  $D_{00}^{\text{NP}}(\mathbf{k}) = m_D^2/(\mathbf{k}^2 + m_D^2)^2$ . This ansatz parallels a zero temperature one [21], where the

dimension two condensate provides the short-distance NP physics of QCD and at zero temperature this contribution yields the well known NP linear term in the  $\bar{q}q$  potential. A justification of Eq. (7) based on Schwinger-Dyson methods has been given [22]. The Gaussian approximation has also been used in Ref. [2] to compute the singlet free energy of a heavy  $\bar{q}q$  pair [20,23], through the correlation function of Polyakov loops.

**Non perturbative contribution to the Trace Anomaly.** The model of Eq. (7) can easily be used to compute the trace anomaly Eq. (1) in gluodynamics. Assuming the leading NP contribution to be encoded in the  $A_{0,a}$  field and taking  $A_{i,a} = 0$  yields [24]

$$\langle (G_{\mu\nu}^a)^2 \rangle^{\text{NP}} = -6m_D^2 \langle A_{0,a}^2 \rangle^{\text{NP}}. \quad (8)$$

Note that the NP model is formulated in the dimensionally reduced theory, so the gluon fields are static. This formula produces the thermal power behaviour of Eq. (3) with

$$b_{\text{tra}} T_c^2 = -(3\beta(g)/g) \hat{m}_D^2 \langle A_{0,a}^2 \rangle^{\text{NP}}, \quad (9)$$

where  $\hat{m}_D \equiv m_D/T$ . If we consider the perturbative value of the beta function  $\beta(g) \sim g^3 + \mathcal{O}(g^5)$ , the r.h.s. of Eq. (9) shows a factor  $g^2$  in addition to the dimension two gluon condensate  $g^2 \langle A_{0,a}^2 \rangle^{\text{NP}}$ . So the fit of the trace anomaly

data is sensitive to the value of the smooth  $T$ -dependent  $g$ , without jeopardizing the power correction. When we consider the perturbative value  $g_P$  up to 2-loops, we get from the fit of the trace anomaly a value of  $g^2 \langle A_{0,a}^2 \rangle^{\text{NP}}$  which is a factor 1.5 smaller than from other observables. This disagreement could be partly explained on the basis of certain ambiguity of  $g$  in the NP regime. A better fit of the Polyakov loop and heavy quark free energy lattice data in the regime  $T_c < T < 4T_c$  is obtained for a slightly smaller  $g$  than  $g_P$ , i.e.  $g = 1.26 - 1.46$  [2]. Taking this value we get from Eq. (9)  $g^2 \langle A_{0,a}^2 \rangle^{\text{NP}} = (2.86 \pm 0.24 T_c)^2$ , a better overall agreement, see Table 1. From Renormalization Group requirements it is possible to observe that the consistency of the model is warranted if the coupling constant  $\alpha_s(\mu) \equiv g^2(\mu)/(4\pi)$  has a behaviour  $\sim 1/\mu^2$  at low enough temperature [10]. This behaviour is just what is obtained within the Analytic Perturbation Theory formalism, after extracting the Landau pole [25], with the corresponding decrease of the perturbative value of  $\alpha_s(\mu)$ . This decrease could approximately explain the best-fit value of  $g$ .

While there might exist other explanations to the observed thermal power corrections our results of Table 1 suggest an unified and coherent description of observables in the non perturbative regime of the deconfined phase (sQGP) in terms of the dimension two gluon condensate [10].

Observable	$g^2 \langle A_{0,a}^2 \rangle^{\text{NP}}$
Polyakov loop [1]	$(3.22 \pm 0.07 T_c)^2$
Heavy $\bar{q}q$ free energy [2]	$(3.33 \pm 0.19 T_c)^2$
Trace Anomaly	$(2.86 \pm 0.24 T_c)^2$

Table 1

Values (in units of  $T_c$ ) of the dimension two gluon condensate from a fits in the deconfined phase of gluodynamics: Polyakov loop, singlet free energy of heavy quark-antiquark and trace anomaly. Lattice data are with  $N_\tau = 8$ . The error reflects an uncertainty in the coupling constant  $g = 1.26 - 1.46$ , being the highest value the perturbative  $g_P$  up to 2-loops at  $T = 2T_c$ . The critical temperature in gluodynamics is  $T_c = 270 \pm 2 \text{ MeV}$  [20].

## REFERENCES

1. E. Megias, E. Ruiz Arriola and L. L. Salcedo, *JHEP* **0601** (2006) 073.
2. E. Megias, E. Ruiz Arriola and L. L. Salcedo, *Phys. Rev.* **D75** (2007) 105019.
3. C. G. Callan, S. R. Coleman and R. Jackiw, *Annals Phys.* **59** (1970) 42.
4. J. C. Collins, A. Duncan and S. D. Joglekar, *Phys. Rev.* **D16** (1977) 438.
5. N. P. Landsman and C. G. van Weert, *Phys. Rept.* **145** (1987) 141.
6. P. J. Ellis, J. I. Kapusta and H. B. Tang, *Phys. Lett.* **B443** (1998) 63.
7. I. T. Drummond, R. R. Horgan, P. V. Landshoff and A. Rebhan, *Phys. Lett.* **B460** (1999) 197.
8. N. O. Agasian, *Phys. Lett.* **B519**(2001) 71.
9. K. Lichtenegger and D. Zwanziger, arXiv:0805.3804 [hep-ph].
10. E. Megias, E. Ruiz Arriola and L. L. Salcedo, (2008), in preparation.
11. P. Gerber and H. Leutwyler, *Nucl. Phys. B* **321** (1989) 387.
12. J. I. Kapusta, *Nucl. Phys. B* **148** (1979) 461.
13. K. Kajantie, M. Laine, K. Rummukainen and Y. Schroder, *Phys. Rev. D* **67** (2003) 105008
14. J. O. Andersen, E. Braaten and M. Strickland, *Phys. Rev.* **D61** (2000) 014017.
15. J. O. Andersen and M. Strickland, *Annals Phys.* **317** (2005) 281.
16. G. Boyd, *et al.*, *Nucl. Phys.* **B469** (1996) 419.
17. R. D. Pisarski, *Prog. Theor. Phys. Suppl.* **168** (2007) 276.
18. E. Ruiz Arriola and W. Broniowski, *Phys. Rev. D* **78** (2008) 034031.
19. E. Gava and R. Jengo, *Phys. Lett.* **B105** (1981) 285.
20. O. Kaczmarek, F. Karsch, P. Petreczky and F. Zantow, *Phys. Lett.* **B543** (2002) 41.
21. K. G. Chetyrkin, S. Narison and V. I. Zakharov, *Nucl. Phys.* **B550** (1999) 353.
22. V. Gogokhia, *Phys. Lett.* **B618** (2005) 103.
23. O. Kaczmarek, F. Karsch, F. Zantow and P. Petreczky, *Phys. Rev.* **D70** (2004) 074505.
24. E. Megias, E. Ruiz Arriola and L. L. Salcedo, arXiv:0805.4579 [hep-ph].
25. D. Shirkov, arXiv:0807.1404 [hep-ph].