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# *QCD Corrections in Transversely Polarized Scattering*

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# QCD Corrections in Transversely Polarized Scattering

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**Abstract.** We discuss two recent calculations of higher-order QCD corrections to scattering of transversely polarized hadrons.

**Keywords:** Nucleon spin structure, Soft-gluon resummation, Single-spin asymmetry
**PACS:** 12.38Bx, 13.88.+e, 13.60.-r, 13.85.-t

## INTRODUCTION

A basic concept underlying much of the theoretical description of high-energy hadronic scacttering is the factorization theorem, which states that large momentum-transfer reactions may be factorized into long-distance pieces that contain information on the structure of the nucleon in terms of its parton densities, and parts that are short-distance and describe the hard interactions of the partons. Two crucial points are that on the one hand the long-distance contributions are universal, i.e., they are the same in any inelastic reaction under consideration, and that on the other hand the short-distance pieces depend only on the large scales related to the large momentum transfer in the overall reaction and, therefore, may be evaluated using QCD perturbation theory.

The lowest order for the latter can generally only serve to give a rough description of the reaction under study. It merely captures the main features, but does not usually provide a quantitative understanding. The first-order ("next-to-leading order" (NLO)) corrections are generally indispensable in order to arrive at a firmer theoretical prediction for hadronic cross sections, and in some cases even an all-order resummation of large perturbative corrections is needed. In the present paper we will discuss two calculations [1, 2] of higher-order QCD corrections to transversely polarized scattering.

## THRESHOLD RESUMMATION FOR $p^\uparrow p^\uparrow \to \pi X$

In spite of extensive studies in recent years, the partonic structure of spin-1/2 nucleons is not yet completely known. Among the leading-twist collinear parton distributions, the unpolarized ($f$), longitudinally polarized ($\Delta f$), and transversely polarized ($\delta f$) densities, there is so far very little information about the latter. These "transversity" distributions $\delta f$ are defined as the differences of probabilities to find a parton of flavor $f$ at scale $\mu$ and light-cone momentum fraction $x$ with its spin aligned ($\uparrow\uparrow$) or anti-aligned ($\uparrow\downarrow$) with that of the transversely polarized nucleon:

$$\delta f(x,\mu) \equiv f_{\uparrow\uparrow}(x,\mu) - f_{\uparrow\downarrow}(x,\mu)\,. \tag{1}$$

Unlike the longitudinally polarized distribution functions which can be measured directly in deep-inelastic scattering (DIS), transversity is not accessible in inclusive DIS due to its chiral-odd nature. It will be highly desirable in the future to have direct probes of transversity. These are available in double-transverse spin asymmetries in hadronic collisions,

$$A_{TT} \equiv \frac{\frac{1}{2}[d\sigma(\uparrow\uparrow) - d\sigma(\uparrow\downarrow)]}{\frac{1}{2}[d\sigma(\uparrow\uparrow) + d\sigma(\uparrow\downarrow)]} \equiv \frac{d\delta\sigma}{d\sigma}, \tag{2}$$

where the arrows denote the transverse polarization of the scattering hadrons. A program of polarized $pp$ collisions is now well underway at the BNL Relativistic Heavy Ion Collider (RHIC), and measurements of $A_{TT}$ for various reactions should be feasible with sufficient beam-time for transverse polarization [3]. In the more distant future, there is also hope to have transversely polarized $\bar{p}p$ collisions at the GSI-FAIR facility, where there are plans to have an asymmetric polarized $\bar{p}p$ collider. Likewise, transversely polarized $pp$ collisions could become a possibility at the J-PARC facility.

We will consider single-inclusive pion production in transversely polarized hadronic collisions, $p^\uparrow p^\uparrow \to \pi X$, $\bar{p}^\uparrow p^\uparrow \to \pi X$, where the pion has large transverse momentum $p_T$. Compared to the Drell-Yan process, which is usually considered the "golden channel" for transversity measurements in hadronic scattering, the spin asymmetry for hadron production is typically considerably smaller, mostly due to the large contribution from gluonic scattering in the denominator of the asymmetry [4, 5, 6]. On the other hand, pions are very copiously produced in hadronic scattering, resulting in much smaller statistical uncertainties, and $A_{TT}$ in single-inclusive hadron production could thus conceivably present a viable alternative to Drell-Yan at RHIC as well as at GSI-FAIR and J-PARC [7].

For energies typically available in fixed-target experiments, previous work [8] has shown that the NLO framework is no longer sufficient but that all-order resummations of large logarithmic corrections are needed. Here the value of $x_T \equiv 2p_T/\sqrt{S}$, with $\sqrt{S}$ the center-of-mass (c.m.) energy of the collision, is generally quite large. It turns out that the partonic hard-scattering cross sections relevant for $pp \to \pi X$ are then largely probed in the "threshold"-regime, where the initial partons have just enough energy to produce the high-transverse momentum parton that subsequently fragments into the hadron, and its recoiling counterpart. Relatively little phase space is then available for additional radiation of partons. In particular, gluon radiation is inhibited and mostly constrained to the emission of soft and/or collinear gluons. The cancellation of infrared singularities between real and virtual diagrams then leaves behind large double- and single-logarithmic corrections to the partonic cross sections. These logarithms appear for the first time at NLO, where they arise as terms of the form $\alpha_S \ln^2(1-\hat{x}_T^2)$ in the rapidity-integrated cross section, where $\hat{x}_T \equiv 2\hat{p}_T/\sqrt{\hat{s}}$ with $\hat{p}_T$ the transverse momentum of the produced parton and $\hat{s}$ the c.m. energy of the initial partons. At yet higher ($k$th) order of perturbation theory, the double-logarithms are of the form $\alpha_S^k \ln^{2k}(1-\hat{x}_T^2)$. When the threshold regime dominates, it is essential to take into account the large logarithms to all orders in the strong coupling $\alpha_S$, a technique known as "threshold resummation" [9, 10]. In our earlier work, we examined the effects of threshold resummation on the single-inclusive hadron cross section and on the double-longitudinal spin asymmetry $A_{LL}$ [8]. We found very significant enhancements of the cross section by resummation, which

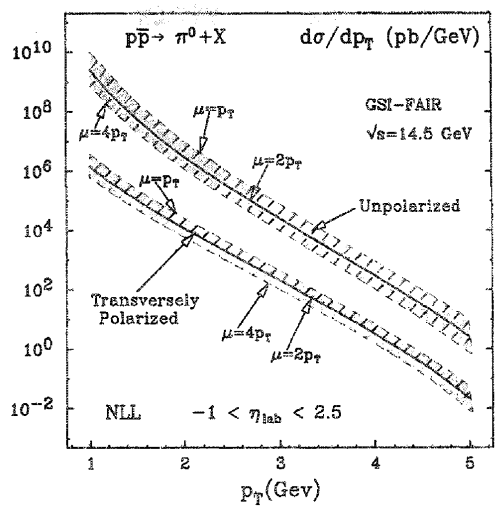


**FIGURE 1.** *NLL resummed cross section for unpolarized and transversely polarized $\bar{p}p \to \pi^0 X$ at $\sqrt{S} = 14.5$ GeV for $-1 < \eta_{lab} < 2.5$ [1]. The shaded bands represent the changes of the results if the factorization/renormalization scale is varied in the range $p_T \leq \mu \leq 4p_T$. The solid line corresponds to $\mu = 2p_T$.*

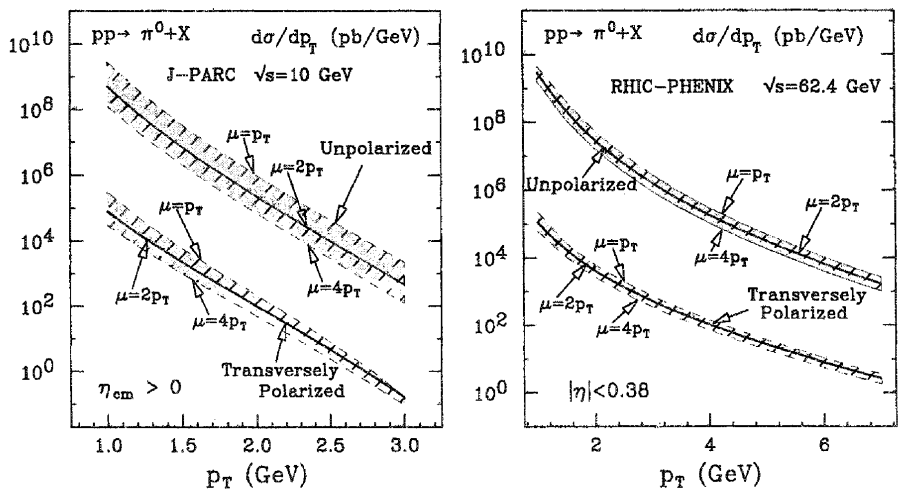


**FIGURE 2.** *NLL resummed cross section for unpolarized and transversely polarized $pp \to \pi^0 X$ at fixed-target $\sqrt{S} = 10$ GeV for $\eta_{cm} > 0$ (left) and at 62.4 GeV for $|\eta| < 0.38$ (right) [1]. The shaded bands represent the changes of the results if the factorization/renormalization scale is varied in the range $p_T \leq \mu \leq 4p_T$. The solid line corresponds to $\mu = 2p_T$.*

varying $\mu = \zeta p_T$, with $\zeta = 1, 2, 4$. It is worth noticing that after resummation the scale dependence is considerably reduced in the case of RHIC. As can be expected, the effects of resummation are much larger at $\sqrt{S} = 10$ GeV than at $\sqrt{S} = 62.4$ GeV. It is also interesting to notice that the $K$-factor is again much larger for the spin-averaged cross section than for the spin-dependent case, especially at $\sqrt{S} = 10$ GeV. This immediately implies that the spin asymmetry $A_{TT}^{\pi}$ will be strongly reduced when going from NLO to the NLL resummed case.

in fact lead to a relatively good agreement between resummed theory and experimental data. We also found a moderate decrease of the resummed $A_{LL}$, with respect to NLO. We concluded that threshold resummation is an essential part of the theoretical description in the fixed-target kinematic regime. Its effects at higher energies (such as at RHIC) are much smaller, even though it has to be said that one is typically much further away from the threshold regime here, so that the applicability of threshold resummation is not entirely clear.

Here we apply threshold resummation to the case of transverse polarization of the scattering partons. As we discussed in [8], the resummed partonic cross section for each subprocess can be written in Mellin moment space in the rather simple form

$$\frac{d\delta\hat{\sigma}^{(\mathrm{res})}_{ab\to cd}(N)}{d\phi} = \delta C_{ab\to cd}\,\Delta^a_N\,\Delta^b_N\,\Delta^c_N\,J^d_N\left[\sum_I \delta G^I_{ab\to cd}\,\Delta^{(\mathrm{int})ab\to cd}_{IN}\right]\frac{d\delta\hat{\sigma}^{(\mathrm{Born})}_{ab\to cd}(N)}{d\phi}\,, \quad (3)$$

where $\delta\hat{\sigma}^{(\mathrm{Born})}_{ab\to cd}(N)$ denotes the moments of the LO term in the perturbative expansion for a given partonic process. Each of the functions $J^d_N$, $\Delta^i_N$, $\Delta^{(\mathrm{int})ab\to cd}_{IN}$ is an exponential and embodies part of the resummation. $\Delta^{a,b}_N$ represent the effects of soft-gluon radiation collinear to initial partons $a,b$, and similarly for $\Delta^c_N$ for the final-state fragmenting parton $c$. The function $J^d_N$ embodies collinear, soft or hard, emission by the "non-observed" recoiling parton $d$. Large-angle soft-gluon emission is accounted for by the factors $\Delta^{(\mathrm{int})ab\to cd}_{IN}$, which depend on the color configuration $I$ of the participating partons. Finally, the coefficients $\delta C_{ab\to cd}$ contain $N-$independent hard contributions arising from one-loop virtual corrections. When performing the inverse Mellin transform, we use the *Minimal Prescription* proposed in Ref. [11] to treat the singularity of the perturbative strong coupling constant in the resummed exponent.

We first present our results [1] for $\bar{p}p$ collisions at $\sqrt{S} = 14.5$ GeV, corresponding to the GSI-FAIR project. At this relatively modest energy one expects the perturbative threshold corrections that we resum here to be particularly important. To match the experimental conditions realistically, we have to take into account the rapidity range that would be covered by the experiments. We assume this range to be $-1 < \eta_{\mathrm{lab}} < 2.5$, where $\eta_{\mathrm{lab}}$ is the pseudorapidity of the pion in the laboratory frame. We count positive rapidity in the forward direction of the antiproton. The results for the NLL resummed cross sections integrated over the range $-1 < \eta_{\mathrm{lab}} < 2.5$ are shown in Fig. 1. We also present in the figure the uncertainties in the prediction resulting from variation of the scale $\mu$ in the range $p_T \leq \mu \leq 4p_T$. One can see that the scale uncertainty remains rather large even after resummation.

We now turn to $p^\uparrow p^\uparrow \to \pi X$ at $\sqrt{S} = 62.4$ GeV and at $\sqrt{S} = 10$ GeV, as relevant for experiments at RHIC (here, PHENIX) and J-PARC. In order to allow comparison of our theoretical predictions with future data we consider the regions of pseudo-rapidity as $\eta_{cm} > 0$ and $|\eta| < 0.38$ for J-PARC and RHIC (PHENIX), respectively, over which we again integrate. The first choice is based on the assumption of a forward spectrometer geometry with 200 mrad acceptance, similar to the one used by the COMPASS experiment at CERN. The results for the two cases are presented in Fig. 2 which shows the NLL spin-averaged and spin-dependent cross sections. As before we have taken into account the theoretical uncertainties in the factorization and renormalization scales by

# NLO CALCULATION OF THE SINGLE TRANSVERSE SPIN ASYMMETRY IN THE DRELL-YAN PROCESS

Single transverse spin asymmetries (SSAs) in high energy hadronic reactions continue to attract much theoretical and experimental interest. They are defined as differences of cross sections when one of the initial hadrons' transverse spin is flipped, divided by the sum: $A_N \sim (d\sigma(S_\perp) - d\sigma(-S_\perp))/(d\sigma(S_\perp) + d\sigma(-S_\perp))$. The theoretical description of SSAs has presented challenges, since the leading collinear partonic contribution to the asymmetries vanishes. Over the past few years, there have been a number of theoretical developments that have led to much progress in the exploration of the underlying physics for single spin asymmetry phenomena. These developments mainly follow two lines: the transverse momentum dependent (TMD) approach, which uses parton distributions that depend on partonic transverse momentum [12], and the twist-three quark-gluon correlation function approach [13, 14]. More recently, it has been found in some cases that the two approaches are closely related and describe the same physics [15].

So far, phenomenological applications of the approaches have been limited to the "bare" parton model, that is, to the zeroth order of perturbation theory without any QCD corrections, as the latter were generally not available. This situation was remedied very recently when the leading-order (LO) kernels for the scale evolution of the relevant twist-three correlation functions were derived [16]. Here we take a further step [2] by calculating next-to-leading order (NLO) QCD corrections for a particular observable, the SSA in Drell-Yan lepton pair production weighted by the transverse momentum of the virtual photon:

$$\langle q_\perp \Delta\sigma(S_\perp)\rangle \equiv \int d^2q_\perp |q_\perp| \sin\phi \frac{d\Delta\sigma(S_\perp)}{d^2q_\perp} \propto \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\frac{dx'}{x'} T_F^q(x_1,x_2)\bar{q}(x')H(x_1,x_2;x'), \tag{4}$$

where $T_F^q$ denotes a "Qiu-Sterman" matrix element [13], and $H$ is the hard part that we are calculating to NLO.

The NLO real-gluon radiation diagrams have already been studied in [15], and the results can be carried over to our present calculation with relatively little effort. Some of the virtual diagrams are shown in Fig. 3. Their result is in fact proportional to that for the NLO virtual corrections for the ordinary spin-averaged Drell-Yan cross section. We have verified that the soft divergence in real-gluon radiation is canceled by that in the virtual diagrams, so that we are left with only collinear divergences, which can be absorbed into the parton distribution and/or the twist-three correlation function, where they give rise to the scale evolution of the distributions. We find the following evolution equation for the "diagonal" twist-three quark-gluon correlation function at $x_1 = x_2 = x$:

$$\begin{aligned} \frac{\partial}{\partial \ln\mu^2} T_F(x,x;\mu^2) &= \frac{\alpha_s(\mu^2)}{2\pi}\int \frac{dx'}{x'}\left\{ C_F T_F(x',x';\mu^2)\left(\frac{1+z^2}{1-z}\right)_+ \right. \\ &\left. + \frac{N_c}{2}\left[T_F(x',x';\mu^2)z - (T_F(x',x';\mu^2) - T_F(x',x'z;\mu^2))\frac{1+z}{1-z}\right]\right\}. \end{aligned} \tag{5}$$

This evolution equation is consistent with the results derived recently by different methods [16]. The full NLO results may be found in [2].

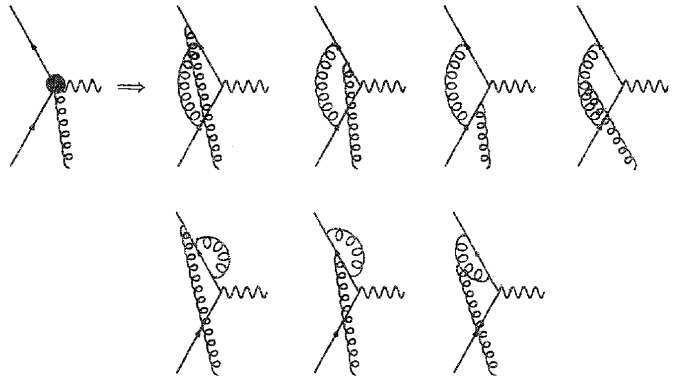

**FIGURE 3.** *Corrections to the quark-antiquark-photon vertex with gluon attachment.*

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