Emittance Exchange Results

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EMITTANCE EXCHANGE RESULTS*

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Abstract
The promise of next-generation light sources depends on the availability of ultra-low emittance electron sources. One method of producing low transverse emittance beams is to generate a low longitudinal emittance beam and exchange it with a large transverse emittance. Experiments are underway at Fermilab’s A0 Photoinjector and ANL’s Argonne Wakefield Accelerator using the exchange scheme of Kim and Sessler. The experiment at the A0 Photoinjector exchanges a large longitudinal emittance with a small transverse emittance. AWA expects to exchange a large transverse emittance with a small longitudinal emittance. In this paper we discuss recent results at A0 and AWA and future plans for these experiments.

INTRODUCTION
The promise of next-generation light sources depends on the availability of ultra-low emittance electron sources. One method of producing low transverse emittance beams is to generate a low longitudinal emittance beam and exchange it with a large transverse emittance. There have been many schemes proposed to accomplish this, all of which are centered around a deflecting mode RF cavity. [1-4] Experiments are underway at Fermilab’s A0 Photoinjector and ANL’s Argonne Wakefield Accelerator using the scheme of Kim and Sessler.[1] In this paper we report on the theory of transverse to longitudinal emittance change, the progress of these experiments and future plans for them.

THEORY
The theory of transverse to longitudinal emittance exchange is discussed in References 1 and 2. We shall recap the main results following the treatment in Reference 2.

The 4x4 beam matrix is given by

\[
\sigma = \begin{pmatrix}
\sigma_x^2 & \sigma_{xy} & 0 & 0 \\
\sigma_{yx} & \sigma_y^2 & 0 & 0 \\
0 & 0 & \sigma_z^2 & \sigma_{z\delta} \\
0 & 0 & \sigma_{z\delta} & \sigma_{\delta}^2 \\
\end{pmatrix} = \begin{pmatrix}
\sigma_x & 0 \\
0 & \sigma_y \\
0 & 0 & \sigma_z \\
0 & 0 & 0 & \sigma_{\delta} \\
\end{pmatrix}
\] (1)

where the vertical plane is neglected and decoupling between the transverse and longitudinal planes is assumed. The determinant of the upper left block is the square of the transverse emittance, and the lower right block is the square of the longitudinal emittance.

The beam matrices at two points are related by the transport matrix \( R \)

\[
\sigma_2 = R \sigma_1 R^T.
\] (2)

We write the R matrix in 2x2 block form.

\[
R = \begin{pmatrix}
A & B \\
C & D \\
\end{pmatrix}
\] (3)

The beam matrix after traversing the beamline is

\[
\sigma_2 = \begin{pmatrix}
A \sigma_x A^T + B \sigma_y B^T & A \sigma_z C^T + B \sigma_{z\delta} D^T \\
C \sigma_x A^T + D \sigma_y B^T & C \sigma_z C^T + D \sigma_{z\delta} D^T \\
\end{pmatrix}
\] (4)

And the resulting emittances are [2]

\[
e_{x^2} = |A|^2 e_{x_1}^2 + (1 - |A|^2) e_{z_1}^2 + \lambda^2 e_{x_1} e_{z_1} \\
e_{z^2} = (1 - |A|^2) e_{x_1}^2 + |A|^2 e_{z_1}^2 + \lambda^2 e_{x_1} e_{z_1}.\] (5)

The goal of designing an emittance exchange beamline is to produce an \( R \) matrix with the A and D blocks identically zero. This will completely swap the emittances and leave them uncoupled after the exchange. If the A and D blocks of the \( R \) matrix are not identically zero, there will be residual coupling of the emittances. A common example of coupled emittances is any dispersive section of beamline. If the A and D blocks are not identically zero, but the determinant of these blocks is zero then the residual coupling resides in the \( \lambda^2 e_{x_1} e_{z_1} \) term. This contains terms from the input beam matrix and can be minimized by proper beam manipulation prior to the exchange.

All transverse to longitudinal emittance exchange beamlines proposed to date consist of three parts

1. A dispersive dogleg
2. A transverse mode deflecting RF cavity
3. A dispersive beamline section.[1-4]

The matrix for a thin deflecting cavity can be written as

\[
M_{\text{cav}} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & k & 0 \\
0 & 0 & 1 & 0 \\
k & 0 & 0 & 1 \\
\end{pmatrix}
\] (6)

The cavity strength \( k \) is given by

\[
k = \frac{eV\omega}{Ec} \] (7)

where \( eV \) is the deflecting field, \( \omega \) is the angular frequency, \( c \) is the speed of light, and \( E \) is the energy. If we assume that the dispersive dogleg produces a dispersion, \( \eta \), with some slope, \( \eta' \) then we can write the required cavity strength as
The post cavity dispersive section matrix needs to satisfy
\[
\begin{pmatrix}
M_{14}^\text{post-cav} \\
M_{24}^\text{post-cav}
\end{pmatrix}
= \begin{pmatrix}
M_{11}^\text{post-cav} & M_{12}^\text{post-cav} \\
M_{21}^\text{post-cav} & M_{22}^\text{post-cav}
\end{pmatrix}
\begin{pmatrix}
\eta \\
\eta'
\end{pmatrix}.
\] (9)

The \( M_{ab}^\text{post-cav} \) are the \( a,b \) components of the 4x4 transport matrix for the post cavity dispersive section. [5]. An uncoupled transverse to longitudinal emittance exchange will occur if these conditions are met. Two solutions for the post cavity section that meet these conditions have been proposed. The first is the dogleg bending in the same direction, which is the Kim and Sessler proposal.[1] The second, proposed by Helen Edwards is a quadrupole followed by a dipole. [4,6]

It is interesting to note that any solution that suppresses the dispersion in the absence of the cavity, such as a chicane, will not produce an uncoupled emittance exchange. This is because Equation 9 cannot be satisfied. [2, 5].

Until now, we have assumed that the deflecting cavity has zero length. In all of the current experiments the cavity’s length cannot be neglected. An \( n \) cell deflecting cavity has a matrix
\[
M_\text{cav} = \begin{pmatrix}
1 & L & kL/2 & 0 \\
0 & 1 & k & 0 \\
0 & 0 & 1 & 0 \\
k & kL/2 & 12n^2 & 12n^2
\end{pmatrix}
\] (10)

where \( L \) is the cavity length. If \( n>4 \) the coefficient on the 4,3 term differs from 1/6 by less than 2% and agrees with the result is Reference 2.

The 4,3 term of the long cavity matrix leads to a coupling that cannot be eliminated by beamline design. However, the residual coupling can be minimized if Equations 8 and 9 are satisfied. In this case, it may be possible to obtain an uncoupled emittance exchange by manipulating the input \( \sigma \) matrix prior to the exchange. However in the experiments to date, the effect of this residual coupling on the resulting emittances is negligible as compared to other effects.

**EMITTANCE DILUTATION EFFECTS**

There are a number of effects that can dilute the emittances in the ongoing experiments. If these effects are large enough they can completely swamp the exchange and the resulting emittances are merely blown up.

The coupling effects mentioned in the previous section are small, but we shall briefly consider them. For example, the A0 experiment has residual coupling due to the finite length of the cavity. The coupling term can be written in terms of the beam matrix at the deflecting cavity, the cavity length \( L \), and the dispersion at the cavity \( \eta \)
\[
\lambda^2 \varepsilon_x \varepsilon_z = \left( \frac{1+2n^2}{12n^2} \frac{L^2}{\eta} \right) \sigma_{x,\text{post-cav}}^2 \left( \sigma_{z,\text{post-cav}}^2 - \eta^2 \sigma_{x,\text{cav}}^2 \right)
\] (11)

Clearly this term can be eliminated if
\[
\sigma_{z,\text{post-cav}}^2 / \sigma_{x,\text{post-cav}}^2 = |\eta|
\] (12)

For the Fermilab experiment, this term cannot be eliminated. However its effect on the smaller final longitudinal emittance is only 6%. The effect on the final transverse emittance is negligible.[6]

Both the Fermilab and Argonne experiments operate at approximately 15 MeV, and can have bunch charges on the order of 1 nC or more. In this regime, space charge can blow up the emittance. Additionally Coherent Synchrotron Radiation (CSR) can also blow up the emittance. [7]

CSR occurs when a low energy, short bunch passes through a dipole magnet. If the dipole magnet is long enough, then the radiation emitted from the back of the bunch can interact with the front of the bunch. This can lead to large energy losses at the bunch tail, and some energy gain in the bunch head. The transverse emittance can also grow because of the effects of dispersion.[8] Most analytical treatments of CSR assume that the transverse dimensions of the bunch are much smaller than the longitudinal dimension. In the Fermilab experiment this is not the case, particularly in second dogleg after the deflecting cavity. Here the transverse size is much larger than the longitudinal. Start to end simulations using CSRtrack and ASTRA predict a factor of 11 increase in the smaller final longitudinal emittance from the effects of space charge and CSR for a 1nC beam.[7,9,10] Since CSR effects scale with the bunch charge squared and space charge goes as the bunch charge, these can be reduced if lower bunch charges are used.

**FERMILAB EXPERIMENT**

The A0 Photoinjector has been described in numerous publications.[11] It consists of a 1.3 GHz, 1.5 cell normal conducting RF photoinjector and a TESLA technology booster cavity. The maximum energy is 16 MeV.

The experiment at the A0 Photoinjector utilized a horizontally bending double dogleg with a 3.9GHz deflecting mode RF cavity between them. The TM110 deflecting mode cavity is liquid nitrogen cooled, normal conducting, version of a superconducting version previously developed at Fermilab.[12] Such a beamline will produce a perfect exchange in the thin cavity limit. A vertical bending spectrometer is used after the exchanger to measure the momentum spread. Figure 1 shows a diagram of the beamline for the emittance exchange.

The input beam has a normalized rms transverse emittance of 5 mm-mrad, and a longitudinal emittance of 34 keV-ps or 20 mm-mrad with an energy of 14.3 MeV and charge of 250 pC. [13,14] Quadrupoles upstream of the emittance exchange beamline are used to manipulate the incoming transverse phase space. Adjusting the phase
Figure 1: Cartoon of the A0 Photoinjector. The emittance exchange beamline starts at the first blue dipole magnet. The yellow spectrometer on the emittance exchange beamline bends downward.

of the booster cavity can manipulate the incoming longitudinal phase space.

Diagnostics for the experiment include a number of strategically placed BPMs and flags, two sets of transverse emittance measuring slits, and a streak camera that is coupled to both lines to measure bunch length. A Martin Puplett Interferometer is also used in the emittance exchange line to measure the sub-picosecond bunch lengths. Two spectrometers are used for momentum spread measurement. The one after the exchanger bends in the vertical plane. [6,13]

The first set of data to emerge from this experiment was a measurement of the linear transport matrix through the beamline as a function of the cavity strength $k$. All of these measurements with the exception of the third row were done using difference orbits. The input parameters $x_{in}$, $x'_{in}$, or $\delta_{in}$, were adjusted and the changes in all of the beam output vector's elements, $x_{out}$, $x'_{out}$, $z_{out}$, and $\delta_{out}$ were measured. The third column is the change in the path difference, or equivalently the arrival time difference, $z_{in}$. This was measured by adjusting the phase of the deflecting cavity. The deflecting cavity is the only time dependant device in the beamline, and changing its phase is equivalent to changing the beam arrival time. The third row was measured using a streak camera. The elements of the matrix that couple to the vertical plane were also measured but are not discussed here.[13] The results of these measurements are shown in Figure 2.

Figure 2 shows the transport matrix elements as a function of the cavity strength. Also plotted are linear fits in green and comparisons to our optics model in red. There is good agreement between the model and the measurement for most matrix elements.

The emittance exchange beamline matrix when the cavity of at the proper strength ($k=100\%k_{ideal}$) is

$$
\begin{pmatrix}
-0.02 \pm 0.059 & -0.23 \pm 0.089 m & 4.75 \pm 0.350 m & 0.40 \pm 0.003 m \\
-0.02 \pm 0.167 m^{-1} & 0.11 \pm 0.041 & -0.02 \pm 0.540 m^{-1} & 0.21 \pm 0.002 \\
0.23 \pm 0.051 & 0.63 \pm 0.107 m & -0.21 \pm 0.312 & 0.00 \pm 0.148 m \\
-0.09 \pm 0.017 m^{-1} & 4.89 \pm 0.047 & 0.13 \pm 0.080 m^{-1} & 0.08 \pm 0.010 \\
\end{pmatrix}
$$


Figure 2: Measurement of the A0 emittance exchanger transport matrix as a function of the deflecting cavity strength $k$ in units of the ideal cavity strength ($k_{ideal}$). The green line is a fit to the data. The red line is the prediction.[13,14]
The measured emittance exchange transport matrix is in overall good agreement with the calculated transport matrix. However, from inspection of Figure 2, the measured $R_{21}$, $R_{23}$ and $R_{43}$ elements did not match expected values. In the case of $R_{23}$, the cryogenic supports of TM110 cavity are suspected to have abruptly changed the tune and phase during its measurement. The $R_{21}$ element difference is most likely due to cross talk between the dipoles in the doglegs. These three elements require a re-measurement.

The matrix measurement verified that a beamline could be constructed that could produce the appropriate transverse to longitudinal coupling. The next step is to measure the emittance to show that the emittances are actually exchanged.

The transverse emittance before and after the emittance exchange were measured using the multi-slit method.[13,14] The longitudinal emittance cannot be measured at the A0 beamline because there is no diagnostic to measure the energy-time correlation in the bunch. However, the energy spread and bunch length can be measured and therefore place an upper bound on the longitudinal emittance.

Simulations showed that CSR and space charge may dilute the signature of the emittance exchange at a bunch charge of 1nC.[7] The transverse emittance after the exchange was expected to blow up 32% and the longitudinal emittance would increase a factor of 11. This would still show an exchange, but would only confuse the analysis. Since CSR effects scale with the bunch charge squared and space charge with the bunch charge, the Fermilab measurements were done using a 250 pC beam.

The emittance measurements are detailed in References 10 and 11. Table 1 shows some of their results.[14]. There is a clear emittance exchange between the horizontal and longitudinal planes. The vertical plane is unaffected.

Table 1: Emittance Exchange results from A0 Photoinjector, from Table 1 of Reference 14.

<table>
<thead>
<tr>
<th>Plane</th>
<th>$\varepsilon_{[\text{mm-mrad}]}$</th>
<th>$\varepsilon_{[\text{mm-mrad]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>4.67 ± 0.22</td>
<td>20.08 ± 2.00</td>
</tr>
<tr>
<td>Vertical</td>
<td>5.11 ± 0.21</td>
<td>6.00 ± 0.42</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>21.1 ± 1.50</td>
<td>7.06 ± 0.43</td>
</tr>
</tbody>
</table>

The Fermilab experiment is now measuring the emittance exchange for a variety of input phase space configurations. They will also increase the beam charge to measure the effects of space charge and CSR on the exchange, as this could have important implication for future applications. They also anticipate installing a second deflecting cavity downstream of the emittance exchange beamline that could be used in conjunction with a spectrometer magnet to measure the energy time correlation in the longitudinal emittance.

**ANL Experiment**

The Argonne Wakefield Accelerator (AWA) has been described in numerous publications.[15] It consists of a 1.3 GHz, 1.5 cell normal conducting RF photoinjector and a normal conducting booster cavities. The maximum energy is 15 MeV.

The emittance exchange experiment will also use the Kim and Sessler scheme. The TM110 deflecting mode cavity for the exchange was built by Tsinghua University.[16] As part of their diagnostics suite, they anticipate adding a second deflecting cavity to measure the longitudinal correlation. The anticipated layout of the AWA beamline is shown in Figure 3.

![Figure 3: Proposed layout of the AWA emittance exchange experiment.](image-url)

The AWA experiment plans to exchange a large transverse emittance with a small longitudinal emittance. The horizontal emittance is 18 mm-mrad. The longitudinal emittance is simulated to be 6.5 mm-mrad and momentum spread measurements are in agreement with this value.[17] The beam energy will be 12 MeV with a bunch charge of 100 pC. Installation of the first dogleg and deflecting cavity is planned for the summer of 2009. Simulations are being done to optimize the emittance exchange and explore the limitations of the exchange in this experiment. [18]

**Future Uses of Emittance Exchange**

It has already been proposed to couple a round to flat beam transformer with a transverse to longitudinal emittance exchange to produce a beam with a large longitudinal emittance and small transverse emittances.[3] The round to flat transformer would make a beam with a large ratio of the transverse emittances. The larger transverse emittance would then be exchanged with the smaller longitudinal emittance. Such a beam could be useful for driving an FEL.

One of the useful properties of an emittance exchange beamline is that it takes longitudinal properties and maps them into the transverse and vice versa. This opens the door to some potentially useful applications. Also proposed by Emma, et.al. is that an exchanger could be used as a jitter exchanger.[2] Timing or energy jitter in the input beam would be exchanged for transverse jitter in the output beam. This may be advantageous as the
transverse jitter may be more tolerable or more easily mitigated than the timing jitter.

Sun and Piot have also proposed producing a train of microbunches using a transverse to longitudinal exchange. The idea is that one would place a set of slits into the beam prior to the exchanger. This would create a set of beamlets in the transverse phase space. This beam is passed through the exchanger and comes out as a train of bunches. They have proposed to do this at the A0 Photoinjector.[19]

**CONCLUSIONS**

The benefits of a transverse to longitudinal emittance exchange have prompted several experimental programs. A proof of principle emittance exchange has been performed at the Fermilab A0 Photoinjector. The demonstration exchanged a smaller input transverse emittance with a larger input longitudinal emittance. Continuing studies are investigating effects of space charge and other coherent effects. Steps are being taken to incorporate a second deflecting mode cavity, in conjunction with a magnetic spectrometer, to measure the longitudinal energy correlation of the final emittance exchanged beam.

Another emittance exchange beamline is being implemented at the ANL AWA facility. The existing AWA beamline is also being reconfigured to the Kim-Sessler double dogleg configuration which will follow a round-to-flat beam transformer. ANL has taken delivery the dogleg magnets and the first TM110 deflecting mode cavity. Initial input beam parameters are being measured.

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