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DISCLAIMER

THE CORRECTION OF LINEAR LATTICE GRADIENT ERRORS USING AN AC DIPOLE*

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Abstract

Precise measurement of optics from coherent betatron oscillations driven by ac dipoles have been demonstrated at RHIC and the Tevatron. For RHIC, the observed rms beta-beat is about 10%. Reduction of beta-beating is an essential component of performance optimization at high energy colliders. A scheme of optics correction was developed and tested in the RHIC 2008 run, using ac dipole optics for measurement and a few adjustable trim quadruples for correction. In this scheme, we first calculate the phase response matrix from the measured phase advance, and then apply singular value decomposition (SVD) algorithm to the phase response matrix to find correction quadruple strengths. We present both simulation and some preliminary experimental results of this correction.

INTRODUCTION

It has been demonstrated in RHIC and Tevatron that ac dipole, as a non-destructive diagnostic tool, can be used to measure optics precisely [1, 2]. One important application of this technique is to correct the linear gradient errors. The general relation between quadruple strength and betatron phase variations under the linear approximation is given by

$$\begin{pmatrix} \Delta\phi_1 \\ \Delta\phi_2 \\ \cdots \\ \Delta\phi_{n_{bpm}} \end{pmatrix} = M \begin{pmatrix} \Delta kl_1 \\ \Delta kl_2 \\ \cdots \\ \Delta kl_{n_q} \end{pmatrix} \qquad (1)$$

, where $\Delta\phi_i$ is the phase variation at the location of the i th bpm, Δkl_i is the gradient variation of the i th quadruple, M is the phase response matrix defined as

$$M_{i,j} = \frac{\beta_j}{4\sin(2\pi\nu)}\{\sin(2\pi\nu)+\sin(2\phi_j-2\pi\nu) + sign(\phi_i-\psi_j)[\sin(2\pi\nu)+\sin(2|\phi_i-\psi_j|-2\pi\nu)]\} \qquad (2)$$

, β_j and ψ_j are the betatron function and phase at the position of the j th quadruple respectively, ϕ_i is the betatron phase at the position of the i th bpm and ν is the unperturbed betatron tune. Since the phase beat $\Delta\phi$ can be measured from the coherent oscillation excited by an ac dipole, the inversion of equation (1) can be used to find the proper strengths of a few adjustable quadruples such that the measured phase beat is reduced. The number of bpms is usually much larger than the number of adjustable quadruples, which makes equation (1) an over determined linear system. For such a system, the least χ^2 solution for the quadruples' strengths is given by singular value decomposition (SVD) algorithm[3].

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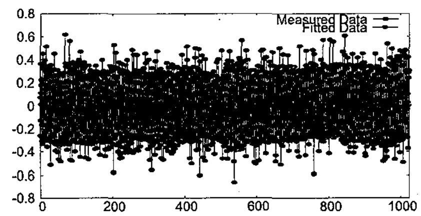


Figure 1: Example of excluded bpm data due to suspiciously large χ^2. The red dots are measured data and the green line is fitting result.

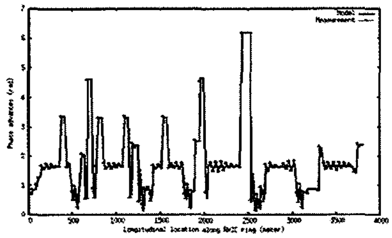
PHASE BEAT MEASUREMENT

For beam betatron oscillation driven by an ac dipole, the betatron phase at each bpm location is obtained by fitting the measured turn by turn data[1]. We used three filters to exclude the unreliable bpm data from our analysis. The first filter is the status bit which comes with the bpm data. The FFT of the turn by turn data provides the second filter, i.e. data with apparent driving tune error are screened out. After fitting all bpms, the fitting χ^2 serves as the third filter. Bpms with suspiciously larger fitting χ^2 compared with other bpms are excluded from further analysis. Figure 1 shows an example of bpm data with suspiciously large fitting χ^2.

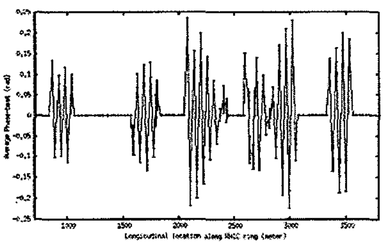
Figure 2 shows some preliminary results of phase beat measurement in RHIC 2008 run. As shown in Fig. 2 (b), the error bars are around 20%, which are calculated from the variance of the 5 measurements. Increasing the bpm data quality and the number of measurements are critical to reduce the statistical errors.

SVD

RHIC has 36 trim quadruples with separate power supplies. We plan to use them as knobs to correct the



(a)



(b)

Figure 2: Phase beat measurement of RHIC 2008 run. The abscissa is the location of the bpms in units of meter and the coordinate is the phase in units of radian. (a) shows the result of one measurement. The red line is calculated from model and the blue dots are measured data. (b) shows the averaged phase beat of 5 measurements.

linear gradient error. As the phase response matrix can be written as

$$M = UWV^T$$

with $U^TU = I$, $V^TV = I$ and W being diagonal matrix, the strengths of the trim quadruples are obtained by inversing equation (1), i.e.

$$\begin{pmatrix} \Delta kl_1 \\ \Delta kl_2 \\ \ldots \\ \Delta kl_{n_{tq}} \end{pmatrix} = V \begin{pmatrix} 1/w_1 & 0 & 0 & 0 \\ 0 & 1/w_2 & 0 & 0 \\ 0 & 0 & \ldots & \ldots \\ 0 & 0 & \ldots & 1/w_{n_{tq}} \end{pmatrix} U^T \begin{pmatrix} \Delta\phi_1 \\ \Delta\phi_2 \\ \ldots \\ \Delta\phi_{n_{bpm}} \end{pmatrix}. \quad (3)$$

Except for giving the least χ^2 solution for their strengths, the SVD algorithm also provides information about the correcting range of the trim quadruples and their vulnerabilities to noises in measured phase beat. In order to see it more clearly, we rewrite equation (5) as

$$\begin{pmatrix} \Delta kl_1' \\ \Delta kl_2' \\ \ldots \\ \Delta kl_{n_{tq}}' \end{pmatrix} = \begin{pmatrix} 1/w_1 & 0 & 0 & 0 \\ 0 & 1/w_2 & 0 & 0 \\ 0 & 0 & \ldots & \ldots \\ 0 & 0 & \ldots & 1/w_{n_{tq}} \end{pmatrix} \begin{pmatrix} \Delta\phi_1' \\ \Delta\phi_2' \\ \ldots \\ \Delta\phi_{n_{tq}}' \end{pmatrix} \quad (4)$$

, where

$$\Delta kl_i' = \sum_j V^T{}_{ij} \Delta kl_j \,,$$

$$\Delta\phi_i' = \sum_j U^T{}_{ij} \Delta\phi_j$$

and we arrange the system such that $w_1 > w_2 > \ldots > w_{tq}$. As shown in equation (4), the phase beat, as a n_{bpm} dimension vector, is mapped into a vector in a subspace with n_{tq} dimension and components normal to this subspace are out of the correcting range of the trim quadruples. Furthermore, equation (4) also grouped the phase beat and the correction trim quadruples into n_{tq} modes. Modes with larger eigenvalues are relatively easier to be correctted due to two reasons. Firstly and more obviously, for given phase beat amplitude, smaller correcting quadruple strengths are required for larger eigenvalue mode. Secondly, for given noises level in phase beat measurement, the resultant noises in the strengths of trim quadruples are much smaller for modes with larger eigenvalues than those with smaller eigenvalues.

PROOF OF PRINCIPAL

In RHIC 2007 and 2008 run, experiments were made to verify above correcting algorithm. In these experiments, gradient errors were intentionally assigned to trim quadruples and by applying the correcting algorithm, the resultant correcting strengths of the trim quadruples should reproduce the preset gradient errors with opposite signs. As shown in Figure 3, the SVD algorithm successfully reconstructed the preset gradient error for the experiment done in RHIC 2007 run. The data analysis for RHIC 2008 run is still in working progress. In the process of analyzing RHIC 2008 run data, we are facing two challenges, i.e. the noises in the measured phase beat and the effective range of the trim quadruples. As described in previous section, depending on its eigenvalue, each mode has different sensitivity to phase beat noises. Modes with

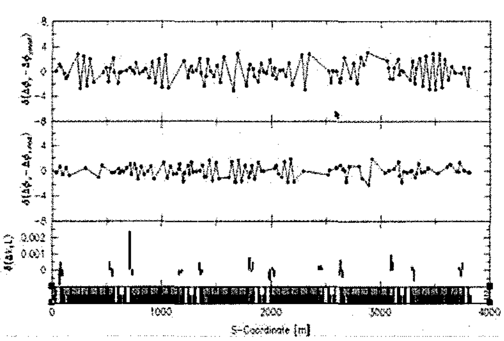


Figure 3: Experiment result in RHIC 2007 run. The top two graphs show the measured phase beat due to the preset trim quadruple error and the bottom graph show the strengths of trim quadruples required to correct the phase beat as calculated from the SVD algorithm.

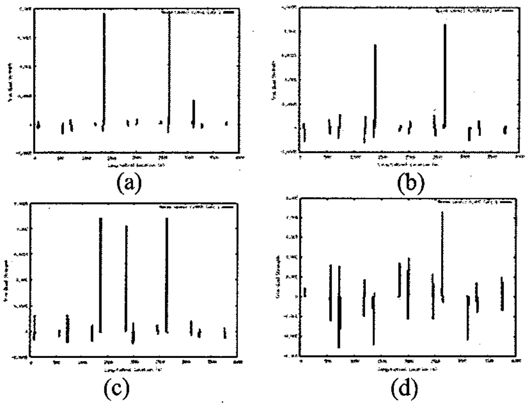


Figure 4: Simulation results of reconstructing gradient errors in presence of phase beat noises. The abscissa is the longitudinal location along the ring and the coordinate is the quadruple strength. (a) Reconstruction of gradient errors set to trim quadruples without large components in modes with small eigenvalues. The noise level is 2%; (b) Same as (a) with noise level of 20%; (c) Reconstruction of gradient errors set to trim quadruples with large components in modes with small eigenvalues. The noise level is 1%. (d) Same as (c) with noise level of 10%.

substantially small eigenvalues compared with other modes are most vulnerable to phase beat noises and are typically cut off in order to avoid the ill-conditioned equations. However, if the phase beat due to the preset gradient errors has large components in modes with very small eigenvalues, it becomes very difficult to reconstruct these errors by SVD as the cut off process results in major information losses. Figure 4 shows the simulation results for two set of preset gradient errors reconstructed by SVD algorithm. As shown in Figure 4 (d), if the trim quadruple with preset gradient errors has large components in the noises sensitive modes, it becomes very difficult to reconstruct those gradient errors.

In order to verify the correction algorithm and test the range of the 36 trim quadruples, we did a few simulations for RHIC 2009 run optics. In these simulations, random gradient errors were assigned to quadruples all along the ring and SVD algorithm was applied to find the proper correcting strengths for the 36 trim quadruples. The phase beats before and after the corrections are shown in Figure 5. Figure 5 (a) shows the effective correction of gradient errors randomly assigned to all 'QF' quadruples with peak relative amplitude of 2.5%. The rms phase beat reduced from 7.6% to 2.8% as a result of correction which indicates that the randomly generated phase beat has major components falling into the correction range of the trim quadruples. On the contrary, Figure 5 (b) shows the correction result for another set of gradient errors which fall out of the correction range. Figure 5 (c) and Figure (d) shows similar results with the gradient errors assigned to all 'QD' quadruples.

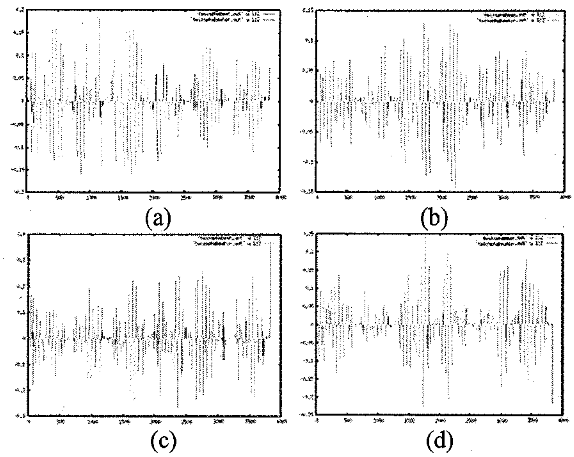


Figure 5: Simulations of gradient errors correction with RHIC 2009 run optics. The abscissa is the longitudinal location along the ring and the coordinate is the phase beat before (red) and after (green) the correction.

SUMMARY

The algorithm described here for linear gradient errors correction is verified by simulation and experiment in RHIC 2007 run. In the process of analyzing RHIC 2008 run data, the bpm noises presents to be a challenge for reconstructing preset gradient errors, especially when the gradient errors have large components in modes with small eigenvalues. Simulations to study the noise effects have been done and the results show that modes with smaller eigenvalue are more vulnerable to noises and more difficult to be reconstructed. Simulations also show the correction range of the 36 trim quadruples and within the range, the correction is very effective.

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