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Recent progress in lattice QCD at finite temperature

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Recent progress in lattice QCD at finite temperature

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Abstract. I review recent progress in finite temperature lattice calculations, including the study of the nature of the deconfinement transition in QCD, equation of state, screening of static quarks and meson spectral functions.

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1. Introduction

One expects that at sufficiently high temperatures and densities the strongly interacting matter undergoes a transition to a new state, where quarks and gluons are no longer confined in hadrons, and which is therefore often referred to as a deconfined phase or quark-gluon plasma. The main goal of heavy ion experiments is to create such form of matter and study its properties. We would like to know at which temperature the transition takes place and what is the nature of the transition as well the properties of the deconfined phase, equation of state, static screening lengths, transport properties etc. Lattice QCD can provide first principle calculation of the transition temperature, equation of state and static screening lengths (see Ref. [1] for recent review). In this contribution I am going to review recent progress made in the study of QCD transition at finite temperature, calculations of equation of state, singlet free energy of static quarks and meson spectral functions.

2. The finite temperature transition and equation of state

One of the most interesting question for the lattice is the question about the nature of the finite temperature transition and the value of the temperature T_c where it takes place. For very heavy quarks we have a 1st order deconfining transition. In the case of QCD with three degenerate flavors of quarks we expect a 1st order chiral transition for sufficiently small quark masses. In other cases there is no true phase transition but just a rapid crossover. Lattice simulations of 3 flavor

QCD with improved staggered quarks (p4) using $N_\tau = 4$ lattices indicate that the transition is first order only for very small quark masses, corresponding to pseudo-scalar meson masses of about 60 MeV [6]. A recent study of the transition using effective models of QCD resulted in a similar estimate for the boundary in the quark mass plane, where the transition is 1st order [4]. This makes it unlikely that for the interesting case of one heavier strange quark and two light u, d quarks, corresponding to 140 MeV pion, the transition is 1st order. However, calculations with unimproved staggered quarks suggest that the transition is 1st order for pseudo-scalar meson mass of about 300 MeV [7]. Thus the effect of the improvement is significant and we may expect that the improvement of flavor symmetry, which is broken in the staggered formulation, is very important. But even when using improved staggered fermions it is necessary to do the calculations at several lattice spacings in order to establish the continuum limit. Recently, extensive calculations have been done to clarify the nature of the transition in the 2+1 flavor QCD for physical quark masses using $N_\tau = 4,\ 6,\ 8$ and 10 lattices. These calculations were done using the so-called *stout* improved staggered fermion formulations which improves the flavor symmetry of staggered fermions but not the rotational symmetry, The result of this study was that the transition is not a true phase transition but only a rapid crossover [8]. New calculations with stout action indicate that only for quark masses about ten times smaller than the physical quark mass the transition could be first order [9]. Even-though there is no true phase transition in QCD thermodynamic observables change rapidly in a small temperature interval and the value of the transition temperature plays an important role. The flavor and quark mass dependence of many thermodynamic quantities is largely determined by the flavor and quark mass dependence of T_c. For example, the pressure normalized by its ideal gas value for pure gauge theory, 2 flavor, 2+1 flavor and 3 flavor QCD shows almost universal behavior as function of T/T_c [5].

The chiral condensate $\langle\bar{\psi}\psi\rangle$ and the expectation value of the Polyakov loop $\langle L\rangle$ are order parameters in the limit of vanishing and infinite quark masses respectively. However, also for finite values of the quark masses they show a rapid change in vicinity of the transition temperature. Therefore they can be used to locate the transition point. The fluctuations of the chiral condensate and Polyakov loop have a peak at the transition temperature. The location of this peak has been used to define the transition temperature in the calculations with p4 action on lattices with temporal extent $N_\tau = 4$ and 6 for several values of the quark mass [3]. The combined continuum and chiral extrapolation then gives the value $T_c = 192(7)(4)$MeV. In this calculations the lattice spacing has been fixed by the Sommer parameter $r_0 = 0.469(7)$fm [10]. The last error in the above value of T_c corresponds to the estimated systematic error in the extrapolation. The transition temperature has been determined using the so-called *stout* staggered action and $N_\tau = 8,\ 10$ and 12 lattices. Here the lattice spacing has been fixed using the kaon decay constant f_K as an input [13]. The deconfinement temperature has been found to be 170(4)(3) MeV determined from the Polyakov loop [13] and 169(3)(3)MeV determined from the strangeness susceptibility. This value of the transition temperature is significantly

smaller than the value obtained with p4 action. One reason for this discrepancy could be the fact that $N_\tau = 4$ and $N_\tau = 6$ lattices are too coarse for reliable continuum extrapolation. Calculations on $N_\tau = 8$ lattices indicate a relative shift of the transition temperature by 5MeV compared to the $N_\tau = 6$ results [14]. The transition temperature for the chiral transition was found to be $146 - 157$MeV depending on the observable, indicating that the chiral transition happens before the deconfiment transition contrary to the conclusion of Ref. [14]. However, it is possible that chiral transition was misidentified in Ref. [13] due to the effect of Goldstone modes below the transition temperature [15].

Lattice calculations of equation of state were started some twenty years ago. In the case of QCD without dynamical quarks the problem has been solved, i.e. the equation of state has been calculated in the continuum limit [16]. At temperatures of about $4T_c$ the deviation from the ideal gas value is only 15% suggesting that quark gluon plasma at this temperate is weakly interacting. Perturbative expansion of the pressure, however, showed very poor convergence at this temperature [17]. Only through the use of new re-summed perturbative techniques it was possible to get agreement with the lattice data [18, 19, 20]. To get a reliable calculation of the equation of state on the lattice, improved actions have to be used [21, 22]. Equation of state has been calculated using p4 and asqtad improved staggered fermion actions [23, 24] using $N_\tau = 4$ and 6 lattices. Very recently these calculations have been extended using $N_\tau = 8$ lattices by the HotQCD collaboration using both p4 and asqtad actions.

In lattice calculations the basic thermodynamic quantity is the trace of the energy momentum tensor, often referred to as the interaction measure $\epsilon - 3p$. This is because it can be expressed in terms of expectation values of gauge action and quark condensates (see discussion in Ref. [24]). All other thermodynamic quantities, pressure, energy density and entropy density $s = (\epsilon + p)$ can be obtained from it using integration

$$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \frac{\epsilon(T') - 3p(T')}{T'^5} \tag{1}$$

The value of T_0 is chosen to be sufficiently small so that it corresponds to vanishing pressure to a fairly good approximation. In Fig. 1 I show the interaction measure from the new calculations with two actions on $N_\tau = 6$ and 8 lattices [14]. At temperatures $T > 220$MeV the differences between calculations performed on $N_\tau = 6$ and $N_\tau = 8$ lattices are small, indicating that cutoff effects are under control in this region. Cutoff effects are seen in the peak region for the p4 action, but not for aqstad action. In this figure I also show the entropy density which raises rapidly in the temperature region $180 - 200$ MeV. At high temperatures it is only 10% or less below the ideal gas limit in agreement with expectations from improved perturbative calculations [20, 25]. We also compare the results for the entropy density with the weak coupling calculations of Ref. [26]. The entropy density of strongly coupled supersymmetric gauge theory is three quarters of the ideal gas value [27] and thus is significantly lower than the lattice result.

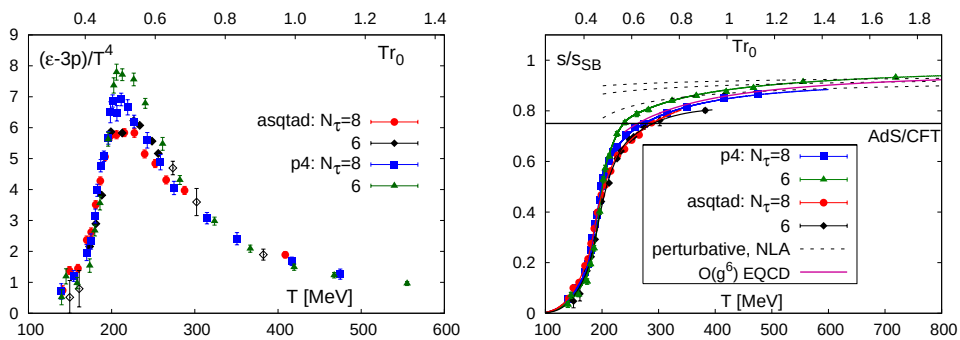

Fig. 1. The interaction measure calculated (left) and the entropy density divided by the corresponding ideal gas value (right) calculated with p4 and asqtad actions [14]. Comparison with resummed perturbation theory and the weak coupling expansion is also shown. The solid line is the prediction for strongly coupled gauge theory using AdS/CFT correspondence [27].

3. Color screening in the deconfined phase

To get further insight into properties of the quark gluon plasma one can study different spatial correlation functions. One of the most prominent feature of the quark gluon plasma is the presence of chromoelectric (Debye) screening. The easiest way to study chromoelectric screening is to calculate the singlet free energy of static quark anti-quark pair (for recent reviews on this see Ref. [28, 29]), which is expressed in terms of correlation function of temporal Wilson lines in Coulomb gauge

$$\exp(-F_1(r,T)/T) = \frac{1}{N}\mathrm{Tr}\langle W(r)W^\dagger(0)\rangle. \tag{2}$$

$L = \mathrm{Tr}W$ is the Polyakov loop. Instead of using the Coulomb gauge the singlet free energy can be defined in gauge invariant manner by inserting a spatial gauge connection between the two Wilson lines. Using such definition the singlet free energy has been calculated in SU(2) gauge theory [30]. It has been found that the singlet free energy calculated this way is close to the result obtained in Coulomb gauge [30]. The singlet free energy turned out to be useful to study quarkonia binding at high temperatures in potential models (see e.g. Refs. [31, 32, 33, 34, 35, 36]).

The singlet free energy was recently calculated in QCD with one strange quark and two light quarks with masses corresponding to pion mass of 220MeV on $16^3 \times 4$ lattices [37]. The numerical results are shown in Fig. 2. At short distances the singlet free energy is temperature independent and coincides with the zero temperature potential. In purely gluonic theory the free energy grows linearly with the separation between the heavy quark and anti-quark in the confined phase. In presence of dynamical quarks the free energy is saturated at some finite value at

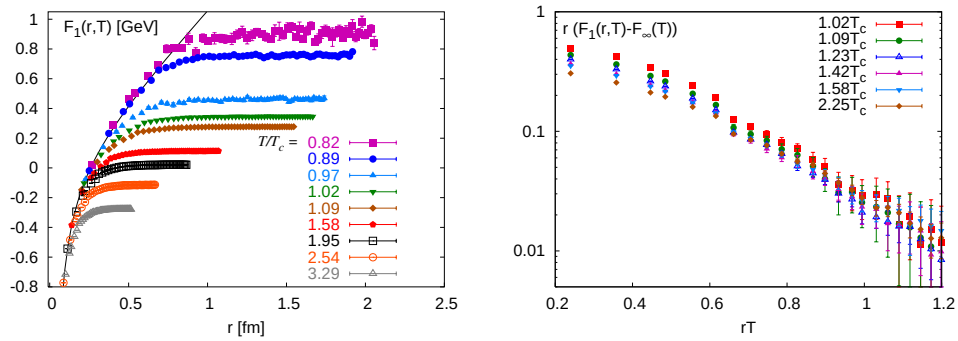

Fig. 2. The singlet free energy $F_1(r,T)$ calculated in Coulomb gauge on $16^3 \times 4$ lattices (left) and the combination $F_1(r,T) - F_\infty(T)$ as function of rT (right). The solid black line is the parametrization of the zero temperature potential.

distances of about 1 fm due to string breaking [28, 38, 40]. This is also seen in Fig. 2. Above the deconfinement temperature the singlet free energy is exponentially screened at sufficiently large distances [39] with the screening mass proportional to the temperature , i.e.

$$F_1(r,T) = F_\infty(T) - \frac{4}{3}\frac{g^2(T)}{4\pi r}\exp(-m_D(T)r), \;\; m_D \sim T. \tag{3}$$

Therefore in Fig. 2 we also show the combination $F_1(r,T) - F_\infty(T)$ as a function of rT. As one can see from the figure this function shows an exponential fall-off at distances $rT > 0.8$. The fact that the slope is the same for all temperatures means that $m_D \sim T$, as expected.

4. Spectral functions

Information on hadron properties at finite temperature as well as the transport coefficients are encoded in different spectral functions. In particular the fate of different quarkonium states in the quark gluon plasma can studied by calculating the corresponding quarkonium spectral functions (for a recent review see Ref. [29]). On the lattice we can calculate correlation function in Euclidean time. This is related to the spectral function via integral relation

$$G(\tau,T) = \int_0^\infty d\omega \sigma(\omega,T) K(\tau,\omega,T), \quad K(\tau,\omega,T) = \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}. \tag{4}$$

Given the data on the Euclidean meson correlator $G(\tau,T)$ the meson spectral function can be calculated using the Maximum Entropy Method (MEM) [41]. For charmonium this was done by using correlators calculated on isotropic lattices [42, 43] as well as anisotropic lattices [44, 45, 46] in quenched approximation. It

has been found that quarkonium correlation function in Euclidean time show only very small temperature dependence [43, 46]. In other channels, namely the vector, scalar and axial-vector channels stronger temperature dependence was found [43, 46]. The spectral functions in the pseudo-scalar and vector channels reconstructed from MEM show peak structures which may be interpreted as a ground state peak [44, 45, 43]. Together with the weak temperature dependence of the correlation functions this was taken as strong indication that the 1S charmonia (η_c and J/ψ) survive in the deconfined phase to temperatures as high as $1.6T_c$ [44, 45, 43]. A detailed study of the systematic effects show, however, that the reconstruction of the charmonium spectral function is not reliable at high temperatures [46], in particular the presence of peaks corresponding to bound states cannot be reliably established. The only statement that can be made is that the spectral function does not show significant changes within the errors of the calculations. Recently quarkonium spectral functions have been studied using potential models and lattice data for the free energy of static quark anti-quark pair [36]. These calculations show that all charmonium states are dissolved at temperatures smaller than $1.2T_c$, but the Euclidean correlators do not show significant changes and are in fairly good agreement with available lattice data both for charmonium [43, 46] and bottomonium [46, 47]. This is due to the fact that even in absence of bound states quarkonium spectral functions show significant enhancement in the threshold region [36]. Therefore previous statements about quarkonia survival at high temperatures have to be revisited.

The large enhancement of the quarkonium correlators above deconfinement in the scalar and axial-vector channel can be understood in terms of the zero mode contribution [36, 48] and not due to the dissolution of the $1P$ states as previously thought. Similar, though smaller in magnitude, enhancement of quarkonium correlators due to zero mode is seen also in the vector channel [46]. Here it is related to heavy quark transport [49, 35]. Due to the heavy quark mass the Euclidean correlators for heavy quarkonium can be decomposed into a high and low energy part $G(\tau,T) = G_{\rm low}(\tau,T) + G_{\rm high}(\tau,T)$ The area under the peak in the spectral functions at zero energy $\omega \simeq 0$ giving the zero mode contribution to the Euclidean correlator is proportional to some susceptibility, $G^i_{low}(\tau,T) \simeq T\chi^i(T)$. Therefore it is natural to ask whether it can be described by a quasi-particle model. The generalized susceptibilities have been calculated in Ref. [52] in the free theory Replacing the bare quark mass entering in the expression of the generalized susceptibilities by an effective temperature dependent masses one can describe the zero mode contribution very well in all channels [53].

The spectral function for light mesons as well as the spectral function of the energy momentum tensor has been calculated on the lattice in quenched approximation [50, 51, 54, 55]. However, unlike in the quarkonia case the systematic errors in these calculations are not well understood.

5. Summary

Significant progress has been achieved in lattice calculations of thermodynamic quantities using improved staggered fermions. Pressure, energy density and entropy density can be reliably calculated at high temperatures when improved actions are used. Different lattice calculations show that for the physical quark masses the transition to the deconfined phase is not a true phase transition but a crossover. There is some controversy, however, concerning the location of the crossover. Lattice calculations provide detailed information about screening of static quarks which is important for the fate of heavy quarkonia in the quark gluon plasma. Some progress has been made in calculating spectral functions on the lattice, however, much more work is needed in this case.

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