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Stochastic Cooling

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Stochastic Cooling was invented by Simon van der Meer [1] and was demonstrated at the CERN ISR and ICE (Initial Cooling Experiment) [2]. Operational systems were developed at Fermilab [3] and CERN [4]. A complete theory of cooling of unbunched beams was developed [4–6], and was applied at CERN and Fermilab. Several new and existing rings employ coasting beam cooling [7].

Bunched beam cooling was demonstrated in ICE and has been observed in several rings designed for coasting beam cooling. High energy bunched beams have proven more difficult. Signal suppression was achieved in the Tevatron [8], though operational cooling was not pursued at Fermilab. Longitudinal cooling was achieved in the RHIC collider [9]. More recently a vertical cooling system in RHIC cooled both transverse dimensions via betatron coupling.

Cooling rates

Unbunched beam A simple, but useful, estimate of stochastic cooling rates is

$$\frac{1}{\epsilon}\frac{d\epsilon}{dt} = -\frac{W}{N}[2g - g^2(M+U)] \qquad (1)$$

where ϵ is the beam emittance, N is the number of particles in the ring, $W = f_{\max} - f_{\min}$ with $f_{\max,\min}$ the maximum and minimum frequency limits of the bandwidth W. The fastest cooling is obtained at the optimum system gain, i.e., $g = 1/(M+U)$. The mixing factor M may be defined to be the ratio of the peak Schottky power density to the average Schottky power density (averaged over all the Schottky bands in the cooling system bandwidth). U is the ratio of electronic noise power to the average Schottky power density.

Transverse cooling rate Applying kinetic theory and feedback theory, the betatron emittance cooling rate for particles with revolution frequency $f = \omega/2\pi$ is found to be[5]

$$S_\omega = \sum_{m,\pm Q}\left\{\frac{G[(m\pm Q)\omega]}{\epsilon_T[(m\pm Q)\omega]}\frac{\exp[im(\theta_p-\theta_k)\pm iQ\psi_\beta]}{\pm i} + \frac{\pi N f(\omega)}{|m\pm Q|}\left|\frac{G[(m\pm Q)\omega]}{\epsilon_T[(m\pm Q)\omega]}\right|^2\right\} \qquad (2)$$

where Q is the betatron tune, $f(\omega)$ is the normalized frequency distribution, G is the electronic gain, and ψ_β is the betatron phase between pickup and kicker. The dielectric response is

$$\epsilon_T[(m\pm Q)\omega] = 1 + NG((m\pm Q)\omega)\sum_{\ell,\pm\nu}\int d\omega_1 \frac{f(\omega_1)\exp(i\ell[\theta_p-\theta_k])}{i((m\pm Q)\omega) - i(\ell\pm\nu)\omega_1 + 0^+}, \qquad (3)$$

where 0^+ means take the limit as this term goes to zero though positive numbers. When pickup noise is included the equation for the transverse emittance of particles with revolution frequency ω is

$$\frac{d\epsilon_\omega}{dt} = S_\omega\epsilon_\omega + \sum_{m,\pm Q}\frac{\pi N}{\omega_0}\left|\frac{G[(m\pm Q)\omega]}{\epsilon[(m\pm Q)\omega]}\right|^2 \epsilon_{\text{noise,m}\pm\text{Q}} \qquad (4)$$

For low gain and resolution bandwidth larger than ω_0 the signal to noise ratio near frequency $\omega_0(m\pm Q)$ is $\epsilon_{rms}/\epsilon_{\text{noise,m}\pm\text{Q}}$ with $\epsilon_{rms} = \int f(\omega)\epsilon_\omega d\omega$ the root mean square emittance. Changing the gain downstream of the noise source does not change $\epsilon_{\text{noise,m}\pm\text{Q}}$. As cooling proceeds $\epsilon_{\text{noise,m}\pm\text{Q}}$ remains fixed and the signal to noise ratio drops.

Longitudinal Cooling Taking $x = E - E_0$ as the energy variable with no measurement noise the equation of motion for particle k is[5]

$$\dot{x}_k = \sum_{j=1}^{N}\sum_{n=-\infty}^{\infty} G(n\omega(x_j), x_j)\exp(in[\theta_k - \theta_j]), \qquad (5)$$

where N is the number of particles in the ring, $G(\Omega, x) = G_F(\Omega) + xG_P(\Omega)$ with $G_F(\Omega)$ the filter cooling gain and $G_P(\Omega)$ the electrical part of the Palmer cooling gain. A damped diffusion equation can be obtained

$$\frac{df(x,t)}{dt} = -\frac{\partial}{\partial x}F(x,t)f(x,t) + \frac{\partial}{\partial x}D(x,t)\frac{\partial}{\partial x}f(x,t) \qquad (6)$$

where

$$F(x,t) = \sum_m \frac{G(m\omega(x),x)}{\epsilon_L(m\omega(x))}\exp[im(\theta_p-\theta_k)]. \qquad (7)$$

In the notation of [5], section 4, $G_m(x) = G(m\omega(x),x)\exp(im(\theta_p-\theta_k)$. When summed over all revolution lines the dielectric response is purely a function of frequency,

$$\epsilon_L(\Omega) = 1 + \sum_m \int dx N \frac{\exp[im(\theta_p-\theta_k)]}{i\Omega - im\omega(x) + 0^+} G(\Omega,x)\frac{\partial}{\partial x}(\omega(x)f(x,t)) \qquad (8)$$

The diffusion coefficient is

$$D(x,t) = \sum_m \frac{N\pi}{|m|}\left|\frac{dx}{d\omega}\right|\left|\frac{G(m\omega(x),x)}{\epsilon_L(m\omega(x))}\right|^2 f(x,t) + \sum_m N\pi\left|\frac{G(m\omega(x),x)}{\epsilon_L(m\omega(x))}\right|^2 U_m \qquad (9)$$

where U_m is the ratio of noise to signal power measured with a large resolution bandwidth.

Longitudinal stacking Solutions of equation 6 with $\partial f/\partial t = 0$ approximate the central part of the solution during the accumulation of antiprotons. For no measurment noise and $\epsilon_L = 1$ one has

$$-f(x)\sum_m G_m(x) + f\frac{\partial f}{\partial x}\sum_m \frac{N\pi}{|m|}\left|\frac{dx}{d\omega}\right| |G_m(x)|^2 = \Psi_0. \tag{10}$$

Where Ψ_0 is the flux of cooling particles. Setting $G_m(x) = \alpha/f(x)$ for $m_- < |m| \leq m_+$ and maximizing the cooling with respect to α one finds

$$f(x) = f(x_1)\exp\left\{\frac{(m_+ - m_-)^2 d\omega/dx}{2\pi N\ln(m_+/m_-)\Psi_0}(x - x_1)\right\}, \tag{11}$$

where x_1 is an energy in the constant flux range.

Bunched beam cooling: For rough estimates one can use equation (1) with N corresponding to the number of particles in the ring that would create a current equal to the peak beam current. The major unknown is the mixing factor, M. With large synchrotron sideband overlap, as occurs in full rf buckets, M is the ratio of peak Schottky power to average Schottky power, as in coasting beams [10]. Mixing from intra-beam scattering (IBS) is important in the FNAL recycler [11]. For higher accuracy the damped diffusion equations for bunched beams are much more complicated than those for coasting beams [12] though calculations without synchrotron sideband overlap [12], or neglecting signal shielding [10], have been done. For application to RHIC it was found that multiparticle simulations proved both fast and reliable [9]. To simulate N_r real particles using N_s simulation particles one simply multiplies the real gain by N_r/N_s and tracks for N_s/N_r fewer turns [13]. The relevant algorithms are identical to those used for the simulation of coherent instabilities. As an example consider longitudinal cooling. The first update will take place at the RF cavity and, since synchrotron tunes are small compared to one, we may place the stochastic cooling kicker at the same spot.

$$\bar{x}_n = x_n + qV(\tau_n)$$
$$\bar{\tau}_n = \tau_n + \kappa_1\bar{x}_n$$

Where $x_n = E - E_0$ for particle n, τ_n is the arrival time of particle n with respect to the synchronous particle, and the bars denote updated variables. The parameter $\kappa_1 = f\eta T_{rev}/\beta^2 E_0$ accounts for particle slip when traversing a fraction of the ring f between kicker and pickup. At the pickup one accumulates the line density array for $k = 1, 2, \ldots, N_{grid}$

$$\lambda(t_k) = \sum_{n=1}^{N_s}\hat{\delta}(t_k - \bar{\tau}_n),$$

where $\hat{\delta(t)}$ is the triangle function for linear interpolation on the grid $t_k = k\Delta t$. Next one transports from the pickup to the kicker via $\bar{\bar{\tau}}_n = \bar{\tau}_n + (1 - f)\kappa_1\bar{x}_n$. The kick for simple filter cooling starts with $\Delta\lambda(t_k) = \lambda(t_k) - \lambda_{old}(t_k)$ where the array λ_{old} was accumulated on the previous turn. Then $\Delta\lambda(t)$ is Fourier transformed, multiplied by the frequency dependent gain, and inverse transformed. The voltage obtained is added to the rf voltage and the update sequence is repeated.

Additional effects such as betatron coupling and IBS are easily included, though one must be careful not to make N_s too small and introduce unphysical mixing due to the increased IBS rates. The results with several competing factors operating can be quite different from the estimates of individual effects. For instance, longitudinal action diffusion from IBS turns out to be very important for transverse cooling in RHIC. This is because particles with small synchrotron amplitude have poor cooling [12, 13] but longitudinal diffusion causes all particles to have signifcant synchroton amplitudes at least some of the time, leading to even cooling.

Hardware

The cooling system is a wide-band feedback loop with bandwidths of a few hundred MHz to a few GHz. Pickups are generally wide-band devices and employ multiple slots to extract sufficient beam power. The individual slot signals can be combined using a meander line[7]. Alternately, the coupling slots can modify the phase velocity of the waveguide mode leading to resonant buildup over a wide frequency range [14]. Both of these technologies require careful design but yield wide band high signal to noise devices. The very strong Schottky signals of the gold ions in the RHIC collider allowed the longitudinal pickup to be very simple. A pair of commercial waveguide launchers on either side of the beam pipe are combined in sum mode. A ceramic window keeps all the electronic components outside the vacuum, resulting in an inexpensive, robust design.

During design both the Panofsky-Wenzel theorem and the potential theorem [15] significantly augment the usefulness of electromagnetic simulation codes. Let $s = z + vt$ denote the longitudinal position of the kicked particle as a function of time. For an isolated kicker the Panofsky-Wenzel theorem states there exists a function $\Phi(x, y, z)$ such that the momentum kick is $\Delta(p_x, p_y, p_z) = \partial\Phi/\partial\mathbf{r}$ where $\mathbf{r} = (x, y, z)$. The potential theorem states that Φ obeys

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{1}{\gamma^2}\frac{\partial^2\Phi}{\partial z^2} = 0, \tag{12}$$

where γ is the Lorentz factor. For coasting beam cooling the front end noise in the pickup is a serious concern. Another difficulty is maintaining the appropriate gain and phase functions over very wide bandwidth. Various sorts of equalizers are often needed and feedback on delays is commonplace. For bunched beams there is the additional complication of beam driven coherent lines [16]. In RHIC, the signal power from coherent lines is often 100 times larger than the Schottky power. A very large dynamic range is needed to keep intermodulation products under control.

Longitudinal filter cooling usually employs one turn delays between pickup and kicker. The simplest and most common type is of the form $S_1(t) = S_0(t) - S_0(t - T_{rev})$ where $S_0(t)$ is the pickup signal, T_{rev} is a highly accurate one turn delay and $S_1(t)$ is the output signal. Single mode optical fibers allow for multi-GHz bandwidths and are inexpensive. Careful temperature control is required to keep length variations well under 1/4 wavelength and various sorts of feedback on the delay are standard.

Kickers for coasting beams look much like pickups, as expected from the Lorentz reciprocity theorem. One simply reverses the beam direction and uses the same solution to equation (12) [15]. For bunched gold beams in RHIC, longitudinal cooling requires a root mean square voltage of 3 kV. Fortunately, the bunch spacing of 107 ns is much larger than the 5 ns bunch length. This allowed for a Fourier series based approach using resonant cavities to generate the kick [9, 13].

New Techniques

Microwave stochastic cooling systems are limited to bandwidths of a few GHz. While this is adequate for condensing antiprotons and other rare particles or cooling the gold beams in RHIC it is not very interesting for cooling high density proton beams in colliders. For such beams bandwidths of several hundred GHz are required. Two technologies have been suggested for cooling such beams.

Optical stochastic cooling employs a wiggler pickup, optical amplifier, and wiggler kicker to close the cooling feedback loop [17, 18]. Operations at micron wavelengths with 10% bandwidth are envisioned. The bandwidth is of order 10^{13} Hz but there are significant challenges. The short wavelengths require linear but nearly achromatic optics and the laser amplifiers push the state of the art.

Coherent electron cooling [19] involves an electron bunch comoving with the hadron bunch. In the modulator the hadrons induce a density modulation on the electrons. The modulation is then amplified in a high gain free electron laser. In the kicker the electron bunch is again merged with the hadron bunch and the electrons kick the hadrons. Appropriate optics and timing result in cooling the hadrons. Central wavelengths of order 10 microns are envisioned and appropriate optics are being designed. High energy, high current energy recovery linacs appear capable of supplying the electron beams to cool proton colliders.

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