1.1 Space-charge limitations in a collider

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1.1.1 Introduction

Design of several projects which envision hadron colliders operating at low energies such as NICA at JINR [1] and Electron-Nucleon Collider at FAIR [2] is under way. In Brookhaven National Laboratory (BNL), a new physics program requires operation of Relativistic Heavy Ion Collider (RHIC) with heavy ions at low energies at γ=2.7-10 [3].

In a collider, maximum achievable luminosity is typically limited by beam-beam effects. For heavy ions significant luminosity degradation, driving bunch length and transverse emittance growth, comes from Intrabeam Scattering (IBS). At these low energies, IBS growth can be effectively counteracted, for example, with cooling techniques. If IBS were the only limitation, one could achieve small hadron beam emittance and bunch length with the help of cooling, resulting in a dramatic luminosity increase. However, as a result of low energies, direct space-charge force from the beam itself is expected to become the dominant limitation. Also, the interplay of both beam-beam and space-charge effects may impose an additional limitation on achievable maximum luminosity. Thus, understanding at what values of space-charge tune shift one can operate in the presence of beam-beam effects in a collider is of great interest for all of the above projects.

Operation of RHIC for Low-Energy physics program started in 2010 which allowed us to have a look at combined impact of beam-beam and space-charge effects on beam lifetime experimentally. Here we briefly discuss expected limitation due to these effects with reference to recent RHIC experience.

1.1.2 Space-charge effects

The Coulomb force between the charged particles in a beam in an accelerator creates a self-field which acts on the particles inside the beam. The Lorentz force experienced by the particle in the radial direction consists of repulsive electric force and attractive magnetic forces which almost cancel each other yielding the factor of γ^2 in the denominator for the force expression. As a result, a direct space-charge effect is nonrelativistic in nature. In general, such space-charge force can change frequencies of oscillations of individual particles (incoherent effect) as well as frequencies of collective beam oscillations. This can lead to rather complex phenomena of space-charge driven resonances, as well as complicates response to the resonances driven by other effects. These effects are mostly of a concern for space-charge dominated beam transport and high-intensity storage rings operated close to the space-charge limit associated with low-order machine resonances. For discussion of these effects see for example Refs. [4-6] and references therein.

A convenient figure of merit for direct space charge effects in circular accelerator is change of the number of betatron oscillations of a single particle per machine turn (tune), which is called incoherent direct space-charge tune shift. In addition, beam surroundings in accelerator result in "indirect" space-charge effects due to image charges, which are omitted from present discussions.

For a beam with non-uniform transverse distribution, the radial force is non-linear which results in amplitude-dependent tune shift. The force can be linearized for small amplitudes near the beam center which provides expression for maximum tune shift. The tune shifts for particles having larger oscillation amplitudes are smaller than those for particles at the center of the beam. As a result, one has to consider the full tune spread introduced by the space charge. In bunched beam, transverse space-charge force is proportional to line charge density, which results in a tune shift being dependent on the longitudinal position within the beam. This causes a tune spread along the bunch as well. For a Gaussian transverse distribution, the maximum incoherent space-charge tune shift can be estimated as:

$$\Delta Q_{sc} = -\frac{Z^2 r_p}{A}\frac{N_i}{4\pi\beta^2\gamma^3\varepsilon}\frac{C_r}{\sqrt{2\pi}\,\sigma_s}, \qquad (1)$$

where r_p is the proton classical radius, N_i is the number of ions per bunch, A and Z are the ion atomic and charge numbers, γ, β are relativistic factors, ε is the unnormalized RMS emittance, σ_s is the rms bunch length, and C_r is ring circumference. Here we assumed round beam transversely and Gaussian longitudinal profile.

1.1.3 Beam-beam effects

Similar space-charge effect occurs in colliding beams. Each time the beams cross each other, the particles in one beam feel the electric and magnetic forces due to the particles of the other beam. The Lorentz force on the test particle linearized for small particle amplitudes gives linear incoherent beam-beam tune shift, which is also referred to as beam-beam parameter. For a round beam, linear incoherent beam-beam tune shift for hadrons is:

$$\xi = -\frac{Z^2 r_p}{A}\frac{N_i}{4\pi\beta^2\gamma\varepsilon}\frac{1+\beta^2}{2}, \qquad (2)$$

where ε is the unnormalized RMS emittance. Here we assumed colliding beams moving with the same velocity. The positive sign of β^2 corresponds to the case of the test particles and the bunch moving in opposite directions. When test particles and bunch travel in the same direction the sign of β^2 is negative, resulting in the same space-charge cancellation $1/\gamma^2$ as for direct space-charge tune shift. The sign of the incoherent beam-beam force depends on the signs of the charges in the two colliding beams. If the two beams contain particles of the same charge, the force is repulsive which results in negative tune shift similar to direct space-charge tune shift.

The periodicity of beam-beam interaction and the fact that beam-beam force has nonlinear dependence on particle amplitude cause two important effects: an excitation of the nonlinear resonances and tune dependence on particles amplitude. As a result, one has to consider the full tune spread within the beam similar to the tune spread due to direct space-charge. The beam-beam interactions are very complex phenomena and involve both incoherent and coherent effects. It has been the topic of many dedicated workshops and conferences and many excellent reviews on this subject are available.

1.1.4 Space-charge and beam-beam limits in a collider

In a collider, when maximum luminosity is limited by beam-beam effects it can be expressed through beam-beam parameter as

$$L = \frac{A}{Z^2 r_p} \frac{N_i c}{\beta^* C_r} \frac{2\gamma\beta^2}{1+\beta^2} f\left(\frac{\sigma_s}{\beta^*}\right)\xi \ , \tag{3}$$

where β^* is the beta function at the interaction point (IP) and σ_s is the rms value of the longitudinal beam size. The factor $f(\sigma_s/\beta^*)$, which describes the "hourglass" effect, is close to unity when the longitudinal rms beam size is much less than the value of beta function at the IP, and decreases when σ_s is increased.

If the luminosity is limited by the space-charge tune shift value ΔQ_{sc}, then it can be expressed as:

$$L = \frac{A}{Z^2 r_p} \frac{N_i c}{\beta^*} \frac{\sqrt{2\pi}\,\sigma_s}{C_r^2} \gamma^3 \beta^2 f\left(\frac{\sigma_s}{\beta^*}\right) \Delta Q_{sc} \ , \tag{4}$$

where no transverse acceptance limitation was taken into account. For typical parameters of low-energy collider with bunch length much smaller than ring circumference and low values of relativistic parameter γ, strongest limitation comes from space charge, while for higher energies beam-beam limitation starts to dominate, as, for example, for low-energy RHIC operation [7].

Ultimate limits on maximum values of space-charge and beam-beam parameters are typically associated with beam instabilities. However, before such limits are reached beam lifetime is strongly influenced by beam response to high-order non-linear resonances.

In hadron colliders, the total achieved tune spread due to beam-beam interactions is much smaller than in electron machines, which is believed to be due to a negligible effect of strong damping mechanism through synchrotron radiation which counteracts beam-beam diffusions in electron machines. The largest total tune spread due to several beam-beam interactions per turn which was achieved in Tevatron is only about 0.03.

Similarly, when space-charge tune spread becomes significant, the beam overlaps many machine imperfection resonances, leading to large beam losses and poor lifetime. For machines where beam spends only tens of msec in high space-charge regime, and machines where the resonances are compensated, the tolerable space-charge tune shift can be as large as ΔQ_{sc}=0.2-0.5. However the acceptable tune shifts are smaller for

longer storage times, with beam lifetimes of several minutes achieved for space-charge tune shifts of about 0.1-0.2 [8]. For a collider when one is interested in even longer beam lifetimes, smaller acceptable space-charge tune spreads can be expected. In dedicated studies of IBS in RHIC, without beam-beam collisions beam lifetime of few hours was measured for space-charge tune spread of about 0.03. although no attempt was made to push space-charge tune shifts any further at that time.

A mostly unexplored effect at this moment is the interplay of direct space-charge and beam-beam effects, which is the case when one wants to collide beams with significant space-charge tune spread. In such a case, in its most simple manifestation beam-beam can excite resonances which will be crossed as a result of space-charge tune spread. We started to explore these effects in RHIC both in dedicated Accelerator Physics Experiments (APEX) and during 2010 physics Run at low energies which is briefly summarized in next section.

1.1.5 RHIC experience

An experimental investigation of the interplay of beam-beam and space-charge effects in RHIC started with APEX experiments in 2009 using protons beams at γ=25. In these experiments beam-beam parameter per single interaction was up to ξ=0.01 and space-charge tune spread up to ΔQ_{sc}=0.03. Strong excitation of beam-beam resonances was observed, which resulted in peeling of large amplitude particles after beams were put into collisions. This is shown in Fig. 1 for transverse beam emittances measured with the Ionization Profile Monitor (IPM). Resulting reduction of beam intensity is shown in Fig. 2. Fortunately, for such rather modest space-charge tune spread it was possible to find working point in the machine where effect of beam-beam resonances was minimized. An example of beam lifetime for such better working point is shown in Fig. 3.

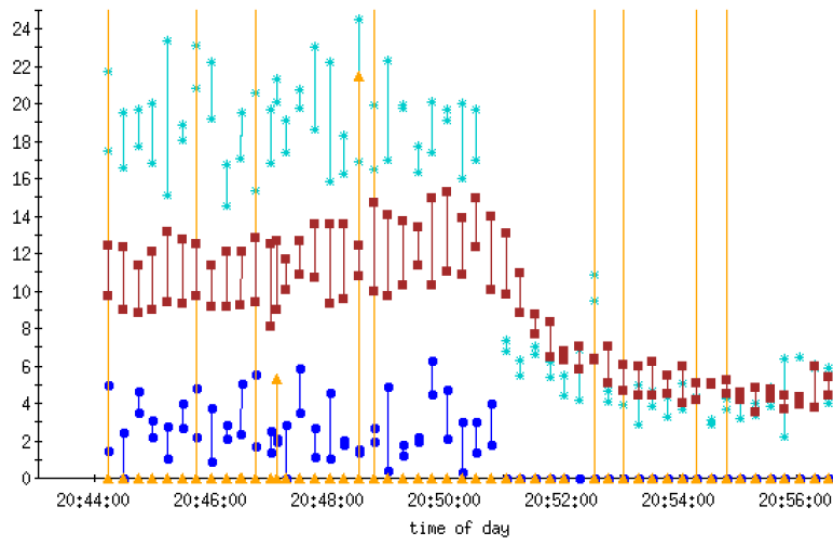

Figure 1: Measurements of transverse beam emittances (95%, normalized, [mm mrad]) during May 2009 RHIC APEX with proton beam at γ=25 for the working point where strong beam-beam resonances were excited. Brown rectangles show transverse emittance of one colliding beam while light-blue color stars show emittance of another colliding beam. After beams were put into collisions at about 20:50, dramatic loss of large amplitude particles was observed.

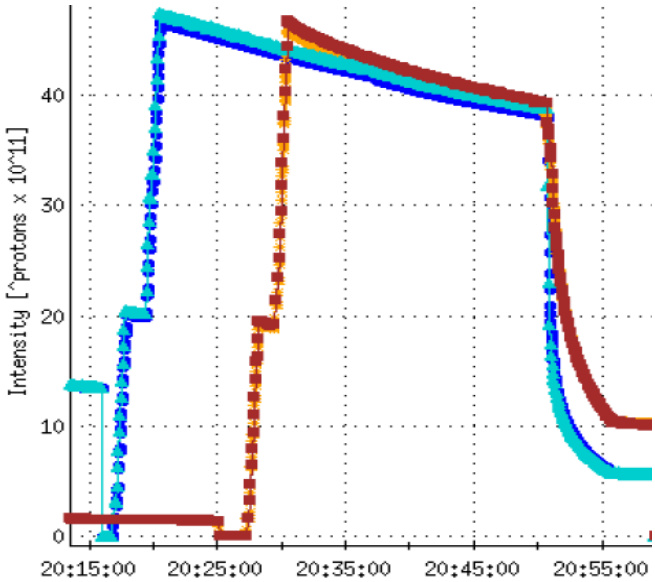

Figure 2: Measurements of total beam intensity loss (corresponding to Fig.1) during May 2009 RHIC APEX for proton beams at γ=25 before and after beams were put into collisions for the working point where strong beam-beam resonances were excited.

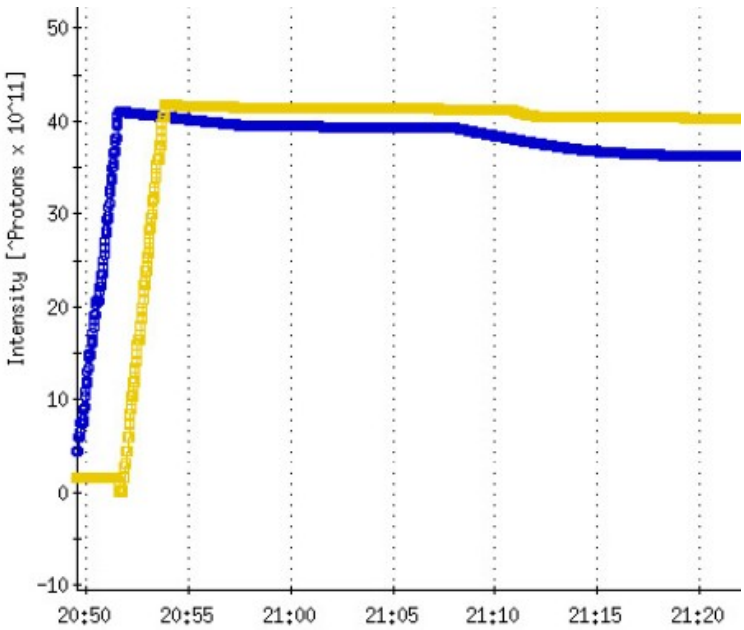

Figure 3: Measurements of total beam intensity loss during June 2009 RHIC APEX for proton beams at γ=25 before and after beams were put into collisions for a different working point where strong beam-beam resonances were avoided.

Experimental studies continued during RHIC operation with Au ion beams at low energies (γ=4-10) in 2010. Due to very low energies in this regime of parameters space-charge tune spread was much larger than beam-beam parameter. Thus, it was expected that beam-beam effects should be relatively small and one should be able to accommodate relatively large space-charge tune spreads. In March 2010 APEX experiments, for modest space-charge tune shift of ΔQ_{sc}=0.03, long beam lifetime was measured with almost no effect from beam-beam. For very large space-charge tune spread it appeared difficult to find sufficient space free from dangerous resonances on the tune diagram to achieve long beam lifetime. Although effects of beam-beam on beam lifetime were clearly observed we were able to provide collisions for physics with space-charge tune shifts up to 0.1. Detailed analysis of measured beam lifetime for various values of space-charge and beam-beam tune spreads are presently under way and will be presented elsewhere [9].

1.1.6 References

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