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AXIAL ANOMALY, DIRAC SEA, AND THE CHIRAL MAGNETIC EFFECT

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Gribov viewed the axial anomaly as a manifestation of the collective motion of Dirac fermions with arbitrarily high momenta in the vacuum. In the presence of an external magnetic field and a chirality imbalance, this collective motion becomes directly observable in the form of the electric current – this is the chiral magnetic effect (CME). I give an elementary introduction into the physics of CME, and discuss the experimental status and recent developments.

Keywords: Axial anomaly, QCD, chiral magnetic effect.

1. Anomalies, as a manifestation of the high momentum collective motion in the vacuum

In the article entitled "Anomalies, as a manifestation of the high momentum collective motion in the vacuum," V.N. Gribov offered a deep insight into "one of the most beautiful and non-trivial phenomena in modern field theory"[1] – the axial[2,3] and scale[4–8] anomalies. According to Gribov, the source of anomalies can be traced back to the collective motion of particles with arbitrarily large momenta in the vacuum. This collective motion defies any UV cutoff that we may try to impose and "transfers the axial charge and the energy-momentum from the world with infinitely large momenta to our world of finite momenta".[1]

Let us illustrate this statement for the case of axial anomaly by considering the Dirac sea of massless fermions. In the absence of external fields (or parity-odd interactions), the chirality is conserved and there are two disconnected Fermi surfaces of left- and right-handed fermions. Now let us turn on an external classical field capable of changing the chirality of

fermions – e.g. the parallel electric $\vec{E}$ and magnetic $\vec{B}$ fields. This field configuration will skew the balance between the Fermi surfaces of left- and right-handed fermions in the Dirac sea, transforming left-handed antiparticles into right-handed particles, or vice-versa, depending on the sign of the product $\vec{E} \cdot \vec{B}$.

The mechanism of the collective flow of chirality can be described as follows:[9,10] the presence of magnetic field B aligns the spins of the positive (negative) fermions in the direction parallel (anti-parallel) to $\vec{B}$. In the electric field E the positive fermions will experience the force eE and will move along $\vec{E}$; therefore their spin will have a positive projection on momentum, and we are dealing with the right fermions. Likewise, the negative fermions will be left-handed. After time t, the positive (right) fermions will increase their Fermi momentum to $p_R^F = eEt$, and the negative (left) will have their Fermi momentum decreased to $p_L^F = -p_R^F$. The one-dimensional density of states along the axis z that we choose parallel to the direction of fields $\vec{E}$ and $\vec{B}$ is given by $dN_R/dz = p_R^F/2\pi$. In the transverse direction, the motion of fermions is quantized as they populate Landau levels in the magnetic field. The transverse density of Landau levels is $d^2N_R/dxdy = eB/2\pi$. Therefore the density of right fermions increases per unit time as

$$\frac{d^4N_R}{dt\ dV} = \frac{e^2}{(2\pi)^2}\ \vec{E} \cdot \vec{B}. \tag{1}$$

The density of left fermions decreases with the same rate, $d^4N_L/dt\ dV = -d^4N_R/dt\ dV$. The rate of chirality $Q_5 = N_R - N_L$ generation is thus

$$\frac{d^4Q_5}{dt\ dV} = \frac{e^2}{2\pi^2}\ \vec{E} \cdot \vec{B}, \tag{2}$$

The quantity on the r.h.s. is the density of topological charge; its integral over four-dimensional space

$$q[A] = \frac{e^2}{8\pi^2} \int d^4x\ F^{\mu\nu}\tilde{F}_{\mu\nu}; \tag{3}$$

reveals the topological class to which the vector potential A belongs. It has to be integer, just as the difference between the numbers of right- and left-handed fermions. The relation (2) thus expresses the deep connection between the axial anomaly and the topology of classical gauge fields.

Having a classical field with an infinite number of quanta is important here since the picture described above involves changing the momenta of

an infinite number of particles, and "a finite number of photons is not able to change the momenta of an infinite number of particles".[1] This feature of the anomaly gives an intuitive explanation of the absence of perturbative quantum corrections to the axial anomaly that can be established formally through the renormalization group arguments.[11–13] As will be discussed below, this property of the (electromagnetic) axial anomaly persists even when the coupling constant that determines the strength of (non-electromagnetic) interactions among the fermions becomes infinitely large.

The flow of chirality, as the derivation above reveals, is accompanied by the collective motion of particles at all momenta, including the momenta around the UV cutoff scale Λ_{UV} that we may attempt to introduce. Therefore our world of particles with finite momenta $p < \Lambda_{UV}$ cannot be isolated from the world of particles with arbitrarily high momenta, and this according to Gribov is the essence of quantum anomalies.

2. The chiral magnetic effect and Landau levels of Dirac fermions

Consider now the situation in which there exists an external magnetic field, and an imbalance between the Fermi momenta of left- and right-handed fermions. In the absence of an external electric field, this imbalance cannot be caused by electromagnetic interactions, but we can imagine that the imbalance may originate from other sources – e.g. from strong interactions of the fermions (quarks) with a non-Abelian gauge field configuration with non-trivial topological contents.

The presence of magnetic field aligns the spins of positive and negative fermions in opposite directions – along or against the direction of $\vec{B}$, respectively. Therefore being, say, right-handed means for the positive fermion to move along the direction of magnetic field, and for a negative fermion - to move against the direction of $\vec{B}$. Therefore, if the densities and Fermi-momenta of left- and right-handed fermions are unequal in the presence of an external magnetic field, there should be an electric current and a separation of electric charge – this is the Chiral Magnetic Effect (CME).[14–18]

Let us introduce, in addition to magnetic field $\vec{B}$, an auxiliary electric field $\vec{E}$ and consider the energy balance of chirality generation. Changing chirality by one unit means transferring a massless fermion from the Fermi surface of left-handed particles to the one of the right-handed particles; this change costs an amount of energy equal to the difference of the correspond-

ing Fermi-momenta $\mu_R - \mu_L = 2\mu_5$. If we multiply this energy by the rate of chirality change (2), we get the energy spent per unit time:

$$P = (\mu_R - \mu_L)\ \frac{e^2}{(2\pi)^2}\ \int d^3x\ \vec{E}\cdot\vec{B}. \tag{4}$$

As we argued above this energy powers the electric current, the power of which is given by

$$P = \int d^3x\ \vec{J}\cdot\vec{E}. \tag{5}$$

We can take $\vec{E}$ in the direction of $\vec{B}$ in this expression, and then get rid of the auxiliary electric field by taking the limit $\vec{E}\to 0$. This allows us to find the following expression for the density of CME current:[17]

$$\vec{J} = \frac{e^2\mu_5}{2\pi^2}\,\vec{B}. \tag{6}$$

Note that this relation manifestly violates parity since magnetic field on the r.h.s. is a pseudo-vector whereas the electric current on the l.h.s. is a vector. Because of this, a static magnetic field with no curl cannot induce electric current in Maxwell electrodynamics (that is parity-even). In our case, the violation of parity is induced by the imbalance between the left- and right-handed fermions. Closely related phenomena have been discussed earlier in the physics of primordial electroweak plasma[19] and quantum wires.[20]

A more rigorous derivation[17] of (6) invokes the explicit sum over the contributions of all Landau levels of charged fermions. This sum is in general divergent, and one has to introduce a UV cutoff on the energy of Landau levels – this is the manifestation of the collective flow from the world of finite momenta to the world of infinite momenta discussed by Gribov. However, all excited Landau levels are degenerate in spin, and the opposite spin orientations give the contributions to the CME electric current that are opposite in sign and thus cancel each other. The lowest Landau level (LLL) of massless fermions is an exception since it is chiral, i.e. not degenerate in spin. Because of this, only the LLL contribution survives in the final expression (6) that does not contain any UV divergence.

3. The chiral magnetic effect and Maxwell-Chern-Simons electrodynamics

Let us now consider the CME in the effective theory of electromagnetism obtained by integrating the quarks out of the action.[18] Let us start from the

QCD coupled to electromagnetism; the resulting theory possesses $SU(3) \times U(1)$ gauge symmetry:

$$\mathcal{L}_{\rm QCD+QED} = -\frac{1}{4} G^{\mu\nu}_{\alpha} G_{\alpha\mu\nu} + \sum_f \bar{\psi}_f \left[i\gamma^\mu (\partial_\mu - igA_{\alpha\mu} t_\alpha - iq_f A_\mu) - m_f \right] \psi_f -$$

$$-\frac{\theta}{32\pi^2} g^2 G^{\mu\nu}_{\alpha} \tilde{G}_{\alpha\mu\nu} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \tag{7}$$

where A_μ and $F_{\mu\nu}$ are the electromagnetic vector potential and the corresponding field strength tensor, and q_f are the electric charges of the quarks.

Let us discuss the electromagnetic sector of the theory (7). Electromagnetic fields will couple to the electromagnetic currents $J_\mu = \sum_f q_f \bar{\psi}_f \gamma_\mu \psi_f$. In addition, the θ-term in (7) will induce through the quark loop the coupling of $F\tilde{F}$ to the QCD topological charge. Let us introduce an effective pseudo-scalar field $\theta = \theta(\vec{x}, t)$ (playing the rôle of the axion[21–23] field, but without a kinetic term) and write down the resulting effective Lagrangian as

$$\mathcal{L}_{\rm MCS} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - A_\mu J^\mu - \frac{c}{4}\, \theta \tilde{F}^{\mu\nu} F_{\mu\nu}, \tag{8}$$

where

$$c = \sum_f q_f^2 e^2 / (2\pi^2). \tag{9}$$

This is the Lagrangian of Maxwell-Chern-Simons, or axion, electrodynamics that has been introduced previously in Refs.[24–26]

As we discussed above, the quantity $\tilde{F}^{\mu\nu} F_{\mu\nu}$ is the density of topological charge. Therefore the integral of this quantity over a four-dimensional volume should be an (integer) topological invariant sensitive only to the long distance, global properties of the gauge field. Such properties are determined by the asymptotic behavior of the field at the surface of the four-dimensional sphere, and thus the topological invariant has to be determined by the surface integral; Gauss theorem thus dictates that $\tilde{F^{\mu\nu}} F_{\mu\nu}$ has to be a full divergence:

$$\tilde{F}^{\mu\nu} F_{\mu\nu} = \partial_\mu J^\mu_{CS}; \tag{10}$$

the quantity J^μ_{CS} is the Chern-Simons current

$$J^\mu_{CS} = \epsilon^{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma}, \tag{11}$$

that is a three-dimensional Chern-Simons form[27] promoted in four dimensions to a current by adding an extra index to the antisymmetric tensor. The Abelian three-dimensional Chern-Simons form

$$CS[A] = \int d^3x \; \epsilon^{\nu\rho\sigma} A_\nu F_{\rho\sigma} \tag{12}$$

is so-called magnetic helicity $\int d^3x \; \vec{A} \cdot \vec{B}$ measuring the linkage of the lines of magnetic flux.

If θ is a constant, then the entire last term in (8) represents a full divergence – therefore it does not affect the equations of motion and thus does not have any effect on the electrodynamics of charges. The situation is different if the effective field $\theta = \theta(\vec{x}, t)$ varies in space-time. Indeed, in this case we have

$$\theta \tilde{F^{\mu\nu}} F_{\mu\nu} = \theta \partial_\mu J^\mu_{CS} = \partial_\mu \left[\theta J^\mu_{CS}\right] - \partial_\mu \theta J^\mu_{CS}. \tag{13}$$

The first term on r.h.s. is again a full derivative and can be omitted; introducing notation

$$P_\mu = \partial_\mu \theta = (M, \vec{P}) \tag{14}$$

we can re-write the Lagrangian (8) in the following form:

$$\mathcal{L}_{\rm MCS} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - A_\mu J^\mu + \frac{c}{4} \; P_\mu J^\mu_{CS}. \tag{15}$$

Since θ is a pseudo-scalar field, P_μ is a pseudo-vector; as is clear from (15), it plays a rôle of the potential coupling to the Chern-Simons current (11). However, unlike the vector potential A_μ, P_μ is not a dynamical variable and is a pseudo-vector that is fixed by the dynamics of chiral charge – in our case, determined by the fluctuations of topological charge in QCD.

Let us write down the Euler-Lagrange equations of motion that follow from the Lagrangian (15),(11) (Maxwell-Chern-Simons equations):

$$\partial_\mu F^{\mu\nu} = J^\nu - P_\mu \tilde{F}^{\mu\nu}. \tag{16}$$

The first pair of Maxwell equations (which is a consequence of the fact that the fields are expressed through the vector potential) is not modified:

$$\partial_\mu \tilde{F}^{\mu\nu} = J^\nu. \tag{17}$$

It is convenient to write down these equations also in terms of the electric $\vec{E}$ and magnetic $\vec{B}$ fields:

$$\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J} + c\left(M\vec{B} - \vec{P} \times \vec{E}\right), \tag{18}$$

$$\vec{\nabla} \cdot \vec{E} = \rho + c\vec{P} \cdot \vec{B}, \tag{19}$$

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0, \tag{20}$$

$$\vec{\nabla} \cdot \vec{B} = 0, \tag{21}$$

where $(\rho, \vec{J})$ are the electric charge and current densities. One can see that the presence of Chern-Simons term leads to essential modifications of the Maxwell theory and induces, as we will see, the chiral magnetic effect.

Let us however start with a different phenomenon – the Witten effect:[28] magnetic monopoles at finite θ angle acquire electric charge and become "dyons". Consider, following Wilczek,[24] a magnetic monopole in the presence of a finite θ angle. In the core of the monopole $\theta = 0$, and away from the monopole θ acquires a finite non-zero value – therefore within a finite domain wall we have a non-zero $\vec{P} = \vec{\nabla}\theta$ pointing radially outwards from the monopole. According to (19), the domain wall thus acquires a non-zero charge density $c\vec{\nabla}\theta \cdot \vec{B}$. An integral along $\vec{P}$ (across the domain wall) yields $\int dl\ \partial\theta/\partial l = \theta$, and the integral over all directions of $\vec{P}$ yields the total magnetic flux Φ. By Gauss theorem, the flux is equal to the magnetic charge of the monopole g, and the total electric charge of the configuration is equal to

$$q = c\ \theta\ g = \frac{e^2}{2\pi^2}\ \theta\ g = \frac{e}{2\pi^2}\ \theta\ (eg) = e\ \frac{\theta}{2\pi}, \tag{22}$$

where we have used an explicit expression (9) for the coupling constant c, as well as the Dirac condition $ge = 4\pi \times$ integer.

Consider now a configuration where an external magnetic field $\vec{B}$ pierces a domain with $\theta \neq 0$ inside; outside $\theta = 0$. Let us assume first that the field θ is static, $\dot{\theta} = 0$. Assuming that the field $\vec{B}$ is perpendicular to the domain wall, we find from (19) that the upper domain wall acquires the charge density per unit area S of[15]

$$\left(\frac{Q}{S}\right)_{up} = +\ c\ \theta B \tag{23}$$

while the lower domain wall acquires the same in magnitude but opposite in sign charge density

$$\left(\frac{Q}{S}\right)_{down} = -\ c\ \theta B \tag{24}$$

Assuming that the domain walls are thin compared to the distance L between them, we find that the system possesses an electric dipole moment

$$d_e = c\ \theta\ (B \cdot S)\ L = \sum_f q_f^2\ \left(e\ \frac{\theta}{\pi}\right)\ \left(\frac{eB \cdot S}{2\pi}\right)\ L; \tag{25}$$

for brevity of notations we put $\sum_f q_f^2 = 1$; it is easy to restore this factor in front of e^2 when needed. Static electric dipole moment is a signature of $\mathcal{P}$, $\mathcal{T}$ and $\mathcal{CP}$ violation (we assume that $\mathcal{CPT}$ invariance holds). The spatial separation of charge will induce the corresponding electric field $\vec{E} = c\ \theta\ \vec{B}$. The mixing of pseudo-vector magnetic field $\vec{B}$ and the vector electric field $\vec{E}$ signals violation of $\mathcal{P}$, $\mathcal{T}$ and $\mathcal{CP}$ invariances.

The formula (25) allows a simple interpretation: since $eB/2\pi$ is the transverse density of Landau levels of charged fermions in magnetic field B, the floor of the quantity $eB \cdot S/2\pi$ (i.e. the largest integer that is smaller than $eB \cdot S/2\pi$) is an integer number of fermions localized on the domain wall. Each fermion species contributes independently to this number as reflected by the factor N_f. Again we see that the electric dipole moment (25) arises from the electric charge $q \sim e\theta/\pi$ that is induced on the domain walls due to the gradient of the pseudo-scalar field θ.

If the domain is due to the fluctuation of topological charge in QCD vacuum, its size is on the order of QCD scale, $L \sim \Lambda_{\rm QCD}^{-1}$, $S \sim \Lambda_{\rm QCD}^{-2}$. This means that to observe an electric dipole moment in experiment we need an extremely strong magnetic field $eB \sim \Lambda_{\rm QCD}^2$. Fortunately, such fields exist during the early moments of a relativistic heavy ion collision.[16,29] Here we have assumed that the domain is static; this approximation requires the characteristic time of topological charge fluctuation $\tau \sim 1/\dot{\theta}$ be large on the time scale at which the magnetic field B varies. This assumption is only marginally satisfied in heavy ion collisions, and so we now need to consider also the case of $\dot{\theta} \neq 0$. Note however that if the medium produced in heavy ion collisions conducts electricity, then the decaying with time magnetic field will induce the circular electric current which in accord with Lenz's law will in turn produce a magnetic field.[30] The estimates[30] indicate that this mechanism can extend the lifetime of magnetic field in a very significant way.

Consider now the domain where $|\vec{P}| \ll \dot{\theta}$, i.e. the spatial dependence of $\theta(t, \vec{x})$ is much slower than the dependence on time.[16] Again, we will expose the domain to an external magnetic field $\vec{B}$ with $\vec{\nabla} \times \vec{B} = 0$, and assume

that no external electric field is present. In this case we immediately get from (18) that there is an induced CME current[17]

$$\vec{J} = -c\ M\ \vec{B} = -\frac{e^2}{2\pi^2}\ \dot{\theta}\vec{B}. \tag{26}$$

4. Chiral magnetic and chiral vortical effects at strong coupling; relativistic hydrodynamics

Many of our arguments were based on the weak coupling picture, e.g. on the existence of Landau levels obtained by solving Dirac equation in the external magnetic field. One may worry that once the strong interactions among the quarks are turned on, this simple picture will break down. Nevertheless this does not happen – the essentially topological nature of the phenomenon protects it from being modified by quantum corrections, even at strong coupling. In particular, in holographic models (at infinite 't Hooft coupling) the magnitude of the chiral magnetic effect[31–33] appears the same as at weak coupling.[31,34,35]

The CME has been studied in lattice QCD coupled to electromagnetism, both in the quenched[36–38] and dynamical (domain wall) fermion[39] formulations; these simulations fully take account of strong interactions among the (anti)quarks. This suggests that the CME exists even when the coupling among the quarks is strong.

Quark-gluon plasma at strong coupling has been argued to behave as a nearly perfect fluid (for review, see[40]), and an effective low-energy theory of strongly interacting fluids is well known – it is hydrodynamics. This invites a very interesting question about the role of axial anomaly in relativistic hydrodynamics that was addressed recently.[41] In a fluid, the role of magnetic field at finite baryon chemical potential μ_B can be played by vorticity $\vec{\omega}$ of the local fluid velocity $\vec{v}$:

$$\vec{\omega} = \vec{\nabla} \times \vec{v}. \tag{27}$$

This is quite natural since the rotating charged fluid generates an effective magnetic field $\mu_B\vec{\omega}$. As a result, the electric current can be induced by the rotation of the fluid with finite baryon and axial charge density even in the absence of an external magnetic field[15,41] – so-called "chiral vortical effect" (CVE). The topological origin of this phenomenon is manifest since the analog of Chern-Simons 3-form (12) in this case is the "kinetic helicity" $\int d^3x\ \vec{v}\cdot\vec{\omega}$. For a discussion of other vorticity-induced effects in heavy ion collisions, see e.g.[42–44]

A related effect—the emergence of a chiral current in a medium with finite baryon density, in an external magnetic field or in the presence of a vorticity the fluid—has been introduced in Refs.[41,45,46] The close connection between CME and the latter effect can be established for example by the method of dimensional reduction appropriate in the case of a strong magnetic field:[47] the simple relations $J_V^0 = J_A^1,\ J_A^0 = J_V^1$ between the vector J_V and axial J_A currents in the dimensionally reduced $(1+1)$ theory imply that the density of baryon charge must induce the axial current, and the density of axial charge must induce the charge current (CME).

5. Experimental status and a new test

Recently, STAR[48,49] and PHENIX[50,51] Collaborations at Relativistic Heavy Ion Collider reported experimental observation of charge asymmetry fluctuations. While the interpretation of the observed effect is still under intense discussion, the fluctuations in charge asymmetry have been predicted to occur in heavy ion collisions due to the Chiral Magnetic Effect (CME) in QCD coupled to electromagnetism.[14–18]

It is important to establish whether the CME explanation of charge asymmetry fluctuations is the correct one. First, it would be a direct observation of a topological effect in QCD. Second, the magnitude of this effect in the chirally broken phase is expected to be much smaller and hence the observation of the CME would manifest the restoration of chiral symmetry in the medium. The effort of quantifying the charge asymmetry fluctuations in QCD matter and of examining alternative explanations and backgrounds has already begun,[52–65] and there are plans to further study this effect at RHIC, LHC, NICA and FAIR.

Recently, a new test of the chiral magnetic and chiral vortical effects (CME and CVE) has been proposed.[66] The test relies only on the general properties of triangle anomalies. Consider anomalous hydrodynamics,[41] and suppose that the system under consideration has a chemical potential μ, coupled to a charge $\bar{q}\gamma^0 Bq$, where B is a flavor matrix, and an axial chemical potential μ_5, coupled to the axial charge $\bar{q}\gamma^0\gamma^5 Aq$, where A is another flavor matrix. For simplicity, we shall assume that both μ and μ_5 are much smaller than the temperature T (this assumption usually holds in relativistic heavy ion collisions). We also assume that electromagnetism couples to the current $\bar{q}\gamma^\mu Qq$, with Q being the charge matrix. If one measures a vector current

$J^\mu = \bar{q}\gamma^\mu V q$, then the result is

$$\vec{J} = \frac{N_c \mu_5}{2\pi^2}\ [\mathrm{tr}(VAQ)\ \vec{B} + \mathrm{tr}(VAB)\ 2\mu\vec{\omega}] \tag{28}$$

where $\vec{B}$ and $\vec{\omega}$ are the external magnetic fields and the fluid vorticity respectively. The two parts of the current on the right hand side correspond to the CME and the CVE, respectively. The traces in the formula are related to the anomalous triangle diagram.

We shall consider two cases: $N_f = 3$, where u, d and s quarks are light, and $N_f = 2$ where only u and d quarks are light. In both cases, we assume A to be the unity matrix, $A = \mathbb{1}$ (which is expected if the chiral asymmetry is due to instanton events, which are flavor symmetric), and $B = (1/3)\mathbb{1}$. For $N_f = 3$, $Q = \mathrm{diag}(2/3, -1/3, -1/3)$, and for $N_f = 2$, $Q = \mathrm{diag}(2/3, -1/3)$. There are two currents that we will measure: the electromagnetic current J_E, corresponding to $V = Q$ and the baryon current J_B, corresponding to $V = B$. For CME, we get for the charge current (up to an overall factor of $N_c\ \mu_5\vec{B}/(2\pi^2)$ which is common for both charge and baryon currents)

$$J_E^{CME} \sim \frac{2}{3}\ \ (N_f = 3)\ \ \text{or}\ \ \frac{5}{9}\ \ (N_f = 2) \tag{29}$$

and for the baryon current

$$J_B^{CME} = 0\ \ (N_f = 3)\ \ \text{or}\ \ \sim \frac{1}{9}\ \ (N_f = 2). \tag{30}$$

For CVE, the results are (up to the overall factor $N_c\ \mu_5\mu\vec{\omega}/\pi^2$)

$$J_E^{CVE} = 0\ \ (N_f = 3)\ \ \text{or}\ \ \sim \frac{1}{3}\ \ (N_f = 2); \tag{31}$$

$$J_B^{CVE} \sim 1\ \ (N_f = 3)\ \ \text{or}\ \ \sim \frac{2}{3}\ \ (N_f = 2). \tag{32}$$

In the SU(3) case, the CME and CVE lead to completely different currents: the CME contributes only to the electromagnetic current and the CVE contributes only to the baryon current. In the SU(2) case, the separation is less clean, but the ratio of J_B/J_E still differs by a factor of ten.

Let us now discuss the implications for heavy ion collisions. It is known that the baryon chemical potential of the produced fireball depends on the collision energy: at smaller $\sqrt{s}$, μ is larger. Thus the CVE should be more important at lower energies. According to the computation above, J_B/J_E becomes larger as one lowers the energy of the collision. Moreover, since the symmetry arguments suggest that the magnetic field and the vorticity

of the fluid have to be aligned, our results show that the two vectors $\vec{J}_B$ and $\vec{J}_E$ should point in the same direction. Therefore, in addition to the charge separation, there must be a baryon number separation. The two effects are positively correlated on the event-by-event basis, and the relative importance of baryon number separation increases as one lowers the energy of the collision.

6. Summary

The picture proposed by Gribov identifies the high-momentum collective motion in the Dirac sea as a source of axial anomaly. In a strong magnetic field, and in the presence of a local chirality imbalance, this collective motion becomes directly observable in the form of electric CME current. In heavy ion collisions, the local chirality imbalance can be readily supplied by the topological gluon field configurations in hot QCD matter, and a sufficiently strong magnetic field is delivered by the colliding ions. The effect thus can become observable, and there is an intriguing evidence from RHIC experiments for the expected charge asymmetry fluctuations.[49,50] Much remains to be done to verify (or disprove) the anomaly-related origin of the observed effect; however this work has already begun.

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