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Vibration Control in Accelerators

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1 General considerations

In the vast majority of accelerator applications, ground vibration amplitudes are well below tolerable magnet jitter amplitudes. In these cases, it is necessary and sufficient to design a rigid magnet support structure that does not amplify ground vibration. Since accelerator beam lines are typically installed at an elevation of 1 - 2 m above ground level, special care has to be taken in order to avoid designing a support structure that acts like an inverted pendulum with a low resonance frequency, resulting in intolerable lateral vibration amplitudes of the accelerator components when excited by either ambient ground motion or vibration sources within the accelerator itself, such as cooling water pumps [1] or helium flow in superconducting magnets [2]. In cases where ground motion amplitudes already exceed the required jitter tolerances, for instance in future linear colliders, passive vibration damping or active stabilization may be considered.

2 Passive damping

To passively suppress the transmission of high frequency ground vibration to the accelerator magnets, a support structure with a resonance frequency $f_r$ well below the unwanted vibration spectrum may be considered. For a single degree-of-freedom system with mass $m$ and support spring constant $D$, the equation of motion can be written as

$$m \ddot{y} + k(y - y_0) + D(y - y_0) = 0,$$

with $y(t)$ and $y_0(t)$ being the motion of the magnet and the ground base, respectively, while $k$ denotes the viscous damping constant. The corresponding complex transfer function denotes

$$H(s) = \frac{\omega_r^2 + 2\delta\omega_r s}{s^2 + 2\delta\omega_r s + \omega_r^2},$$

with $\omega_r = 2\pi f_r = \sqrt{D/m}$ and $2\delta\omega_r = k/m$ and $s = i\omega$. According to this transfer function, a passive vibration absorber attenuates ground vibration amplitudes by a factor proportional to $1/\omega^2$ at frequencies between the resonance frequency $\omega_r$ and approximately $\omega_r/\delta$, while for even higher frequencies the attenuation is proportional to $1/\omega$, as illustrated in Figure 1.

An overshoot occurs at the resonance frequency $f_r$, resulting in amplification of vibration amplitudes. This effect can be eliminated by increasing the damping coefficient $\delta$, at the expense of a degradation of the isolation performance for high frequencies, as illustrated in Figure 1.
Figure 1: Transfer function $|H|$ of a passive vibration absorber, according to Equation 2, for a resonance frequency $f_r = 1$ Hz and three different damping coefficients $\delta$. 
Figure 2: Transfer function $|H_m| = |Y/F|$, describing the displacement $Y$ of the magnet mass as a result of frequency dependent forces $F$ acting on the magnet itself, for a magnet mass $m = 100\,\text{kg}$ and a damping coefficient $\delta = 0.3$, for three different resonance frequencies $f_r$. See Equation 3.

Though such a passive system could be designed even with a low resonance frequency of only a few Hertz, it would be very sensitive to any type of force acting on the magnet directly. Denoting the Laplace transform of the resulting magnet vibration $y(t)$ as $Y(s)$,

$$Y(s) = \frac{\omega_r^2/D}{s^2 + 2\delta\omega_r s + \omega_r^2} F(s),$$

(3)

with $F(s)$ being the Laplace transform of the input force $f(t)$, describes the response in the frequency domain with $s = i\omega$.

In the limit of low frequencies, this reduces to

$$\lim_{s \to 0} Y(s) = \lim_{s \to 0} F(s)/D,$$

(4)

and with $D = \omega_r^2 \cdot m$ it becomes obvious that the sensitivity to static forces increases quadratically with decreasing resonance frequency $f_r$, as illustrated in Figure 2.
3 Active stabilization

Stabilization of low frequencies below several tens of Hertz requires active damping. A sensor on top of the accelerator structure provides a motion signal that is processed and sent to the active support structure to counteract the detected vibration with a closed feedback loop, as schematically depicted in Figure 3. The active support can be realized as either a soft or a stiff support with an appropriate actuator.

Assuming a motion sensor with a transfer function $H_s(s)$, a support structure described by the (passive) transfer function

$$H_r(s) = \frac{\omega_r^2 + 2\delta\omega_r s}{s^2 + 2\delta\omega_r s + \omega_r^2}, \quad (5)$$

and a feedback algorithm $R(s)$, the closed-loop transfer function of the entire feedback system is computed as

$$H_g(s) = \frac{H_r(s)}{1 + R(s)H_r(s)H_s(s)}, \quad (6)$$
which describes the transfer of ground motion to the magnet.

On the other hand, the transfer function for excitation forces acting on the magnet directly becomes

\[ H_m(s) = \frac{1}{1 + R(s)H_r(s)H_s(s)}. \] (7)

Now, since all motion sensors (e.g., piezoelectric accelerometers, geophones, or seismometers - see section on ground vibration) have a finite lower frequency limit, \( \lim_{s \to 0} H_s(s) = 0 \), it follows that in the limit of small frequencies the active support structure exhibits the same behavior as the corresponding passive support with the same spring constant \( D \), and therefore the same resonance frequency \( f_r \),

\[ \lim_{s \to 0} H_m(s) = \frac{1}{D}. \] (8)

Furthermore, since the transfer function of the support vanishes in the limit of high frequencies, \( \lim_{s \to \infty} H_r(s) = 0 \), the active support exhibits the same behavior as the passive support in the high frequency limit,

\[ \lim_{s \to \infty} H_m(s) = \frac{1}{D}. \] (9)

Therefore, while soft supports are efficient for vibration stabilization, static alignment of these structures is nearly impossible due to the inherent sensitivity even to small forces together with the finite lower frequency limit of the active feedback [3]. These limitations are avoided by stiff supports with resonance frequencies of hundreds of Hertz, which are typically based on piezoelectric actuators [4, 5]. For frequencies beyond 1 Hz, suppression of the rms vibration amplitude by a factor of 3 has been demonstrated.

References


