BNL-94123-2011-CP

Amplification of current density modulation in a FEL with an infinite electron beam

G. Wang, V.N. Litvinenko, S.D. Webb

Presented at the 2011 Particle Accelerator Conference (PAC'11)
New York, N.Y.
March 28 – April 1, 2011

Collider-Accelerator Department

Brookhaven National Laboratory

U.S. Department of Energy
Office of Science

Notice: This manuscript has been authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy. The publisher by accepting the manuscript for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this manuscript, or allow others to do so, for United States Government purposes.

This preprint is intended for publication in a journal or proceedings. Since changes may be made before publication, it may not be cited or reproduced without the author's permission.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

AMPLIFICATION OF CURRENT DENSITY MODULATION IN A FEL WITH AN INFINITE ELECTRON BEAM*

G. Wang#, V. N. Litvinenko, S. D. Webb, BNL, Upton, NY 11973, U.S.A.

Abstract

We show that the paraxial field equation for a free electron laser (FEL) in an infinitely wide electron beam with $\kappa-2$ energy distribution can be reduced to a fourth ordinary differential equation (ODE). Its solution for arbitrary initial phase space density modulation has been derived in the wave-vector domain. For initial current modulation with Gaussian profile, close form solutions are obtained in space-time domain.

INTRODUCTION

In developing an analytical model for a FEL-based coherent electron cooling system, an infinite electron beam has been assumed for the modulation and correction processes [1-3]. While the assumption has its limitation, it allows for an analytical close form solution to be obtained, which is essential for investigating the underlying scaling law, benchmarking the simulation codes and understanding the fundamental physics.

1D theory was previously applied to model a CeC FEL amplifier[4]. However, the theory ignores diffraction effects and does not provide the transverse profile of the amplified electron density modulation. On the other hand, 3D theories developed for a finite electron beam usually have solutions expanded over infinite number of modes determined by the specific transverse boundary conditions. Unless the mode with the largest growth rate substantially dominates other modes, both evaluation and extracting scaling laws can be complicated. Furthermore, it is also preferable to have an analytical FEL model with assumptions consistent with the other two sections of a CeC system.

Recently, we developed the FEL theory in an infinitely wide electron beam with $\kappa-1$ (Lorentzian) energy distribution[5]. Close form solutions have been obtained for the amplified current modulation initiated by an external electric field with various spatial-profiles. In this work, we extend the theory into $\kappa-2$ energy distribution and study the evolution of current density induced by an initial density modulation.

EQUATION OF MOTION

Assuming that the amplitude of the radiation field varies slowly with respect to the undulator period and that fast oscillation terms can be dropped, the amplitude of the radiation field is determined by the following paraxial field equation [6]

$$\begin{aligned}&\left(\nabla_\perp^2+2i\frac{\omega}{c}\frac{\partial}{\partial z}\right)\tilde{E}(\vec{r}_\perp,z)\\&=ij_0(\vec{r}_\perp)\int_0^z dz'\left[\frac{e\omega\theta_s^2}{2c^2\varepsilon_0}\tilde{E}(\vec{r}_\perp,z')+\frac{e}{\varepsilon_0\omega}\left(\nabla_\perp^2+2i\frac{\omega}{c}\frac{\partial}{\partial z}\right)\tilde{E}(\vec{r}_\perp,z')\right]\\&\times\int_{-\infty}^{\infty}e^{i\left(C+\frac{\omega}{c\gamma_z^2\mathrm{E}_0}P\right)(z'-z)}\frac{\partial}{\partial P}F(P)dP+\frac{i\theta_s\omega}{c\varepsilon_0}e\int_{-\infty}^{\infty}\tilde{f}_1(\vec{r}_\perp,P,0)e^{-i\left(C+\frac{\omega}{c\gamma_z^2\mathrm{E}_0}P\right)z}dP\end{aligned}, \tag{1}$$

where $\tilde{E}(\vec{r}_\perp,z)$ is the complex amplitude of the radiation field, ω is the radiation frequency, C is the detuning, E_0 is the nominal electron energy, P is the electron energy deviation, θ_s is the electron deflection angle, $F(P)$ is the energy distribution function, $j_0(\vec{r}_\perp)$ is the transverse spatial distribution of the unperturbed electron beam and $\tilde{f}_1(\vec{r}_\perp,P,0)$ is the initial phase space density perturbation. Assuming $j_0(\vec{r}_\perp)=j_0$ is independent of $\vec{r}_\perp$ and performing Fourier transformation to eq. (1) lead to [5]

$$\begin{aligned}&\frac{\partial}{\partial\hat{z}}\tilde{R}(\hat{z},\hat{k}_\perp,\hat{C})\\&=\int_0^{\hat{z}}d\hat{z}'e^{i\hat{k}_\perp^2(\hat{z}-\hat{z}')}\left[\tilde{R}(\hat{z}',\hat{k}_\perp,\hat{C})+i\hat{\Lambda}_p^2\frac{\partial}{\partial\hat{z}'}\tilde{R}(\hat{z}',\hat{k}_\perp,\hat{C})\right]\int_{-\infty}^{\infty}d\hat{P}\frac{d\hat{F}}{d\hat{P}}e^{i(\hat{C}+\hat{P})(\hat{z}'-\hat{z})}\\&+\frac{e\theta_s}{2\varepsilon_0\Gamma}e^{i\hat{k}_\perp^2\hat{z}}\int_{-\infty}^{\infty}\tilde{f}_1(\vec{k}_\perp,\hat{P},0)e^{-i(\hat{C}+\hat{P})\hat{z}}d\hat{P}\end{aligned}, \tag{2}$$

where

$$\tilde{R}(z,k_\perp,C)\equiv e^{i\frac{k_\perp^2c}{2\omega}z}\tilde{E}(z,k_\perp,C). \tag{3}$$

In eq. (2), we used the normalized variables as defined in [6] and [5], i.e. $\hat{z}=\Gamma z$, $\hat{C}=C/\Gamma$, the 1D- gain parameter

$$\Gamma\equiv\left[\frac{\pi j_0\theta_s^2\omega}{c\gamma_z^2\gamma I_A}\right]^{1/3}, \tag{4}$$

the Alfven current $I_A\equiv m_ec^3/e$, the pierce parameter $\rho=\gamma_z^2\Gamma c/\omega$, the space charge parameter

$$\hat{\Lambda}_p\equiv\frac{1}{\Gamma^2}\left[\frac{4\pi j_0}{\gamma_z^2\gamma I_A}\right]^{1/2}, \tag{5}$$

the normalized transverse wave vector

$$\hat{\vec{k}}_\perp\equiv\sqrt{\frac{\rho}{2}}\frac{\vec{k}_\perp}{\gamma_z\Gamma}, \tag{6}$$

and the normalized energy distribution function $\hat{F}(\hat{P})$ satisfying

$$\int_{-\infty}^{\infty}\hat{F}(\hat{P})d\hat{P}=1\cdot$$

We assume the following $\kappa-2$ energy distribution,

* Work supported by Brookhaven Science Associates, LLC under Contract No.DE-AC02-98CH10886 with the U.S. Department of Energy.
#gawang@bnl.gov

$$F\left(\hat{P}\right)=\frac{2}{\pi\hat{q}}\frac{1}{\left(1+\frac{\hat{P}^2}{\hat{q}^2}\right)^2}=-\hat{q}^2\frac{\partial}{\partial\hat{q}}\left(\frac{1}{\hat{q}}F_1\left(\hat{P}\right)\right)\cdot \tag{7}$$

Applying the relation

$$\int_{-\infty}^{\infty}d\hat{P}e^{i\left(\hat{C}+\hat{P}\right)\left(\hat{z}'-\hat{z}\right)}\frac{\partial}{\partial\hat{P}}F\left(\hat{P}\right)=-i\left(\hat{z}'-\hat{z}\right)\left(1+\hat{q}\left|\hat{z}'-\hat{z}\right|\right)e^{i\hat{C}\left(\hat{z}'-\hat{z}\right)-\left|\hat{q}\left(\hat{z}'-\hat{z}\right)\right|} \tag{8}$$

to eq. (2) generates

$$\begin{aligned}&\frac{\partial}{\partial\hat{z}}\tilde{R}\left(\hat{z},\hat{k}_\perp,\hat{C}_{3d}\right)\\&=-i\int_0^{\hat{z}}d\hat{z}'\left[\tilde{R}\left(\hat{z}',\hat{k}_\perp,\hat{C}_{3d}\right)+i\hat{\Lambda}_p^2\frac{\partial}{\partial\hat{z}'}\tilde{R}\left(\hat{z}',\hat{k}_\perp,\hat{C}_{3d}\right)\right]\left(\hat{z}'-\hat{z}\right)\\&\times\left[1+\hat{q}\left(\hat{z}-\hat{z}'\right)\right]e^{-\left(\hat{q}+i\hat{C}_{3d}\right)\left(\hat{z}-\hat{z}'\right)}+\frac{e\theta_s}{2\varepsilon_0\Gamma}\int_{-\infty}^{\infty}\tilde{f}_1\left(\vec{k}_\perp,\hat{P},0\right)e^{-i\left(\hat{C}_{3d}+\hat{P}\right)\hat{z}}d\hat{P}\end{aligned}, \tag{9}$$

where we define

$$\hat{C}_{3d}\equiv\hat{C}-\hat{k}_\perp^2\cdot \tag{10}$$

The third derivative of eq. (9) with respects to $\hat{z}$ reads

$$\begin{aligned}&\tilde{R}^{(4)}+3\left(i\hat{C}_{3d}+\hat{q}\right)\tilde{R}^{(3)}+\left[\hat{\Lambda}_p^2+3\left(i\hat{C}_{3d}+\hat{q}\right)^2\right]\tilde{R}^{(2)}\\&+\left[\left(i\hat{C}_{3d}+3\hat{q}\right)\hat{\Lambda}_p^2+\left(i\hat{C}_{3d}+\hat{q}\right)^3-i\right]\tilde{R}^{(1)}\\&-i\left(i\hat{C}_{3d}+3\hat{q}\right)\tilde{R}=\frac{1}{n_0}\int_{-\infty}^{\infty}\left(\hat{q}-i\hat{P}\right)^3\tilde{f}_1\left(\hat{P},0\right)e^{-i\left(\hat{P}+\hat{C}_{3d}\right)\hat{z}}d\hat{P}\end{aligned}. \tag{11}$$

Eq. (11) is a fourth inhomogeneous ordinary differential equation (ODE) with constant coefficients and its Green function with initial condition

$$G(0)=G^{(1)}(0)=G^{(2)}(0)=G^{(3)}(0)=0 \tag{12}$$

is given by

$$G(\hat{z})=\sum_{(i,j,k,l)}\frac{e^{\lambda_i\hat{z}}}{\left(\lambda_i-\lambda_j\right)\left(\lambda_i-\lambda_k\right)\left(\lambda_i-\lambda_l\right)}, \tag{13}$$

where λ_i are solutions of the polynomial equation,

$$\begin{aligned}&\lambda^4+3\left(i\hat{C}_{3d}+\hat{q}\right)\lambda^3+\left[\hat{\Lambda}_p^2+3\left(i\hat{C}_{3d}+\hat{q}\right)^2\right]\lambda^2\\&+\left[\left(i\hat{C}_{3d}+3\hat{q}\right)\hat{\Lambda}_p^2+\left(i\hat{C}_{3d}+\hat{q}\right)^3-i\right]\lambda=i\left(i\hat{C}_{3d}+3\hat{q}\right)\end{aligned},$$

and the summation is over the cyclic permutation of the four indices. The particular solution of eq. (11) is

$$\begin{aligned}\tilde{R}_p(\hat{z})&=\int_0^{\hat{z}}G\left(\hat{z}-\hat{z}_1\right)f\left(\hat{z}_1\right)d\hat{z}_1\\&=\frac{e\theta_s}{2\varepsilon_0\Gamma}\sum_{(i,j,k,l)}\frac{1}{\left(\lambda_i-\lambda_j\right)\left(\lambda_i-\lambda_k\right)\left(\lambda_i-\lambda_l\right)}\\&\times\int_{-\infty}^{\infty}\frac{\left(\hat{q}-i\hat{P}\right)^3\cdot\left[e^{\lambda_i\hat{z}}-e^{-i\left(\hat{P}+\hat{C}_{3d}\right)\hat{z}}\right]\tilde{f}_1\left(\hat{k}_\perp,\hat{P},0\right)}{\lambda_i+i\left(\hat{P}+\hat{C}_{3d}\right)}d\hat{P}\end{aligned}. \tag{14}$$

Since eq. (14) satisfies

$$\tilde{R}(0)=\tilde{R}^{(1)}(0)=\tilde{R}^{(2)}(0)=\tilde{R}^{(3)}(0)=0, \tag{15}$$

the initial conditions are solely satisfied by the general solution, i.e.

$$\tilde{R}_g(\hat{z})=\sum_{i=1}^{4}A_i\left(\hat{C},\hat{k}_\perp\right)e^{\lambda_i\hat{z}}\cdot \tag{16}$$

From eq. (3), the solution for radiation field is

$$\tilde{E}\left(\hat{z},\hat{k}_\perp,C\right)=e^{-i\hat{k}_\perp^2\hat{z}}\left[\tilde{R}_g\left(\hat{z},\hat{k}_\perp,C\right)+\tilde{R}_s\left(\hat{z},\hat{k}_\perp,C\right)\right]. \tag{17}$$

The current density is related to the radiation field by [5, 6]

$$\tilde{j}_1\left(\hat{z},\hat{C},\hat{k}_\perp\right)=-\frac{c\Gamma}{2\pi\theta_s}\left[i\hat{k}_\perp^2+\frac{\partial}{\partial\hat{z}}\right]\tilde{E}\cdot \tag{18}$$

Inserting eq. (14), eq. (16), eq. (17) into eq. (18) leads to

$$\begin{aligned}&\tilde{j}_1\left(\hat{z},\hat{C},\hat{k}_\perp\right)\\&=-e^{-i\hat{k}_\perp^2\hat{z}}\left\{\frac{2c\varepsilon_0\Gamma}{\theta_s}\sum_{i=1}^{4}\lambda_iA_ie^{\lambda_i\hat{z}}+\sum_{(i,j,k,l)}\frac{ec}{\left(\lambda_i-\lambda_j\right)\left(\lambda_i-\lambda_k\right)\left(\lambda_i-\lambda_l\right)}\right.\\&\left.\times\int_{-\infty}^{\infty}\frac{\left(\hat{q}-i\hat{P}\right)^3\cdot\left[\lambda_ie^{\lambda_i\hat{z}}+i\left(\hat{P}+\hat{C}_{3d}\right)e^{-i\left(\hat{P}+\hat{C}_{3d}\right)\hat{z}}\right]\tilde{f}_1\left(\hat{k}_\perp,\hat{P},0\right)}{\lambda_i+i\left(\hat{P}+\hat{C}_{3d}\right)}d\hat{P}\right\}\end{aligned}, \tag{19}$$

where

$$\begin{pmatrix}A_1\\A_2\\A_3\\A_4\end{pmatrix}=-\frac{\theta_s}{2\varepsilon_0c\Gamma}M^{-1}\begin{pmatrix}-\frac{2\varepsilon_0c\Gamma}{\theta_s}\tilde{E}\\\tilde{j}_1\\\tilde{j}_1^{(1)}\\\tilde{j}_1^{(2)}\end{pmatrix}_{\hat{z}=0}, \tag{20}$$

with

$$M=\begin{pmatrix}1&1&1&1\\\lambda_1&\lambda_2&\lambda_3&\lambda_4\\\lambda_1\left(\lambda_1-i\hat{k}_\perp^2\right)&\lambda_2\left(\lambda_2-i\hat{k}_\perp^2\right)&\lambda_3\left(\lambda_3-i\hat{k}_\perp^2\right)&\lambda_4\left(\lambda_4-i\hat{k}_\perp^2\right)\\\lambda_1\left(\lambda_1-i\hat{k}_\perp^2\right)^2&\lambda_2\left(\lambda_2-i\hat{k}_\perp^2\right)^2&\lambda_3\left(\lambda_3-i\hat{k}_\perp^2\right)^2&\lambda_4\left(\lambda_4-i\hat{k}_\perp^2\right)^2\end{pmatrix}. \tag{21}$$

The linearized Vlasov equation for electrons in a straight field-free section reads[6]

$$\frac{\partial}{\partial\hat{z}}\tilde{f}_1+i\left(\hat{C}+\hat{P}\right)\tilde{f}_1=0\cdot \tag{22}$$

Assuming the electron beam goes through a field-free straight section before entering the undulator, the initial current modulation and its derivatives can be derived from eq. (22) as

$$\frac{\partial^{(n)}}{\partial\hat{z}^{(n)}}\tilde{j}\left(\hat{z},\hat{P}\right)\Big|_{\hat{z}=0}=\int_{-\infty}^{\infty}(-i)^n\left(\hat{C}+\hat{P}\right)^n\tilde{f}_1\left(0,\hat{C},\hat{P}\right)d\hat{P}\cdot \tag{23}$$

In order to proceed further, the initial phase space density modulation has to be specified.

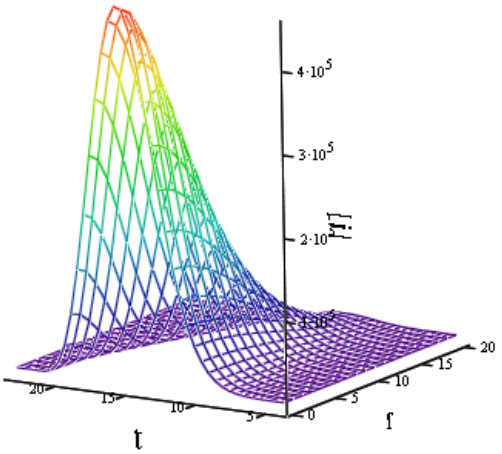
GAUSSIAN PROFILE

For simplicity, we ignore the energy modulation and assume the initial phase space density perturbation has the following form

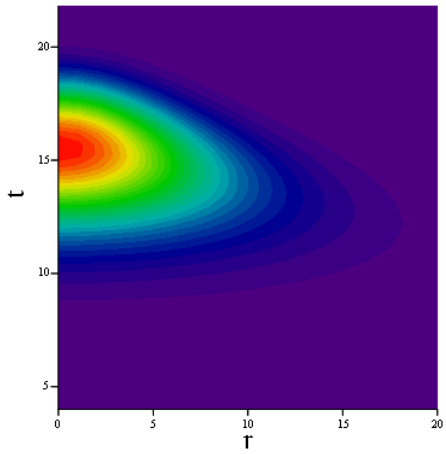
$$\tilde{f}_1\left(0,\hat{C},\hat{k}_\perp,\hat{P}\right)=\frac{1}{ec}\tilde{j}_1\left(0,\hat{C},\hat{k}_\perp\right)\delta\left(\hat{P}\right)\cdot \tag{24}$$

Inserting eq. (20), eq. (21), eq. (23) and eq. (24) into eq. (19) produces

$$\begin{aligned}\tilde{j}_1\left(\hat{z},\hat{C},\hat{k}_\perp\right)&=-e^{-i\hat{k}_\perp^2\hat{z}}\tilde{j}_1\left(0,\hat{C},\hat{k}_\perp\right)\sum_{(i,j,k,l)}\frac{1}{\left(\lambda_i-\lambda_j\right)\left(\lambda_i-\lambda_k\right)\left(\lambda_i-\lambda_l\right)}\\&\times\left[\lambda_i\left(B_{jkl}+\frac{\hat{q}^3}{\lambda_i+i\hat{C}_{3d}}\right)e^{\lambda_i\hat{z}}+\frac{i\hat{q}^3\hat{C}_{3d}}{\lambda_i+i\hat{C}_{3d}}e^{-i\hat{C}_{3d}\hat{z}}\right]\end{aligned} \tag{25}$$



(a)



(b)

Figure 2 The amplitude of amplified current density modulation as calculated from eq. (29). The snapshot is taken at $\hat{z}=21$ with $\hat{\sigma}_t=1.37\times10^{-5}$, $\hat{\Lambda}_p=0$, $\hat{\sigma}_\perp=2.32$ and $\hat{q}=0.1$. (a) surface plot; (b) contour plot.

where

$$B_{jkl}\equiv\lambda_j\lambda_k+\lambda_j\lambda_l+\lambda_k\lambda_l+i\hat{C}_{3d}\left(\lambda_j+\lambda_k+\lambda_l\right)-\hat{C}_{3d}^2\,. \quad (26)$$

Assuming the initial current density perturbation has Gaussian spatial profile, i.e.

$$j_1(z,t,x,y)=\frac{ec}{2\pi\sigma_\perp^2\sqrt{2\pi}\sigma_t}e^{-\frac{r^2}{2\sigma_\perp^2}}e^{-\frac{(z-\beta ct)^2}{2\sigma_z^2}} \quad (27)$$

for $z\le0$, the Fourier components of (27) at $z=0$ is

$$\tilde{j}_1\left(0,k_x,k_y,\hat{C}\right)=\frac{e}{\beta}e^{-\frac{k_\perp^2\sigma_\perp^2}{2}}e^{\frac{-\sigma_t^2}{2}\left(C-k-k_w\right)^2}\approx e\cdot e^{-\frac{\hat{k}_\perp^2\hat{\sigma}_\perp^2}{2}}e^{\frac{-\hat{\sigma}_z^2}{2}\left(\hat{C}-\hat{k}-\hat{k}_w\right)^2}. \quad (28)$$

Inserting eq. (28) into eq. (25) and carrying out the inverse Fourier transformations yields[4]

$$\tilde{j}_1(\vec{x},t)=-\frac{ec\Gamma^3\gamma_z^4}{2\pi^2\rho}e^{-i\hat{k}_w\xi}\sum_{(i,j,k,l)}\int_{-\infty}^{\infty}d\hat{C}_{3d}e^{-i2\gamma_z^2(\hat{z}-c\hat{t})\hat{C}_{3d}}I_x\left(\hat{r},\hat{C}_{3d}\right)\times\frac{\lambda_i\left(B_{jkl}+\frac{\hat{q}^3}{\lambda_i+i\hat{C}_{3d}}\right)e^{\lambda_i\hat{z}}+\frac{i\hat{q}^3\hat{C}_{3d}}{\lambda_i+i\hat{C}_{3d}}e^{-i\hat{C}_{3d}\hat{z}}}{\left(\lambda_i-\lambda_j\right)\left(\lambda_i-\lambda_k\right)\left(\lambda_i-\lambda_l\right)}, \quad (29)$$

where

$$\xi=-\left[\hat{z}+2\gamma_z^2\left(\hat{z}-c\hat{t}\right)\right], \quad (30)$$

and

$$I_x\left(\hat{r},\hat{C}_{3d}\right)\equiv\int_0^\infty e^{-\left(\frac{\hat{\sigma}_\perp^2}{2}-i\xi\right)x}e^{\frac{-\hat{\sigma}_z^2}{2}\left(x+\hat{C}_{3d}-\hat{k}-\hat{k}_w\right)^2}J_0\left(\hat{r}_\perp\sqrt{x}\right)dx\,. \quad (31)$$

For $\hat{\sigma}_z=0$, the integration in eq. (31) can be carried out analytically and eq. (29) becomes

$$\tilde{j}_1(\vec{x},t)=\frac{-ec\Gamma^3\gamma_z^4e^{-i\hat{k}_w\xi}}{2\pi^2\rho}\frac{e^{-\frac{1}{4}\frac{\hat{r}^2}{\hat{\sigma}_\perp^2/2-i\xi}}}{\hat{\sigma}_\perp^2/2-i\xi}\sum_{(i,j,k,l)}\int_{-\infty}^{\infty}d\hat{C}_{3d}\frac{e^{-i2\gamma_z^2(\hat{z}-c\hat{t})\hat{C}_{3d}}}{\left(\lambda_i-\lambda_j\right)\left(\lambda_i-\lambda_k\right)\left(\lambda_i-\lambda_l\right)}\cdot\times\left[\lambda_i\cdot\left(B_{jkl}+\frac{\hat{q}^3}{\lambda_i+i\hat{C}_{3d}}\right)e^{\lambda_i\hat{z}}+\frac{i\hat{q}^3\hat{C}_{3d}}{\lambda_i+i\hat{C}_{3d}}e^{-i\hat{C}_{3d}\hat{z}}\right] \quad (32)$$

Fig. 1 shows the amplitude of the current density modulation as a function of the transverse spatial coordinate and arrival time as calculated from eq. (29). The transverse radius is in unit of $\sqrt{\rho}/\left(\sqrt{2}\gamma_z\Gamma\right)$, the longitudinal location is in unit of $1/\left(2\gamma_z^2\Gamma c\right)$, and the current density is in unit of $ec\Gamma^3\gamma_z^4/\left(2\pi^2\rho\right)$.

DISCUSSION

Eq. (29) has similar form as what have been previously obtained for current modulation due to external field excitation [5]. The contribution from the inhomogeneous driving term in eq. (11) is proportional to $\hat{q}^3$ and hence is negligible for $\hat{q}<<1$.

REFERENCES

[1] V. N. Litvinenko and Y. S. Derbenev, Physical Review Letters **102**, 114801 (2009).

[2] G. Wang and M. Blaskiewicz, Phys. Rev. E **78**, 026413 (2008).

[3] G. Wang, M.Blaskiewicz, and V. N. Litvinenko, in *International Particle Accelerator Conference 2010 (IPAC'10)* (Kyoto, Japan, 2010), p. 873.

[4] G. Wang, in *Department of Physics and Astronomy* (Stony Brook, 2008), Ph. D Dissertation.

[5] G. Wang, V. N.Litvinenko, and S. D. Webb, in *32th International Free Electron Laser Conference* (Malmo, Sweden, 2010), p. 60.

[6] E. L. Saldin, E. A. Schneidmiller, and M. V. Yurkov, *The Physics of Free Electron Lasers*(Springer, New York, 1999).