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AtR transfer line***

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# TRANSFER OF POLARIZED $^3\text{He}$ IONS IN THE AtR TRANSFER LINE\*

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## Abstract

In addition to collisions of electrons with various unpolarized ion species as well as polarized protons, the proposed electron-hadron collider (eRHIC) will facilitate the collisions of electrons with polarized  $^3\text{He}$  ions. The AGS is the last acceleration stage, before injection into one of the RHIC's collider ring for final acceleration. The AtR (AGS to RHIC) transfer line will be utilized to transport the polarized  $^3\text{He}$  ions from AGS into one of the RHIC's collider rings. Some of the peculiarities of the AtR line's layout (simultaneous horizontal and vertical bends) may degrade the matching of the stable spin direction of the AtR line with that of RHIC's. In this paper we discuss possible simple modifications of the AtR line to accomplish a perfect matching of the stable spin direction of the injected  $^3\text{He}$  beam with the stable spin direction at the injection point of RHIC.

## INTRODUCTION

The AGS is currently the only synchrotron which accelerates polarized protons to an energy of  $\sim 24$  GeV. The extracted polarized protons from AGS are injected into RHIC for further acceleration to higher energies of 255 GeV to be used in physics experiments. The polarization of the extracted protons from AGS is between 65% and 70% and this is accomplished by the use of two partial helices which are installed in the AGS ring to eliminate the imperfection and the intrinsic spin resonances. The interest in polarization experiments using  $^3\text{He}$  ions [1,2] prompted the study of accelerating polarized  $^3\text{He}$  ions in AGS and injecting them into RHIC. Theoretical calculations show that for preserving the polarization of the  $^3\text{He}$  ions in AGS it is beneficial to extract the  $^3\text{He}$  ions from the AGS at an energy corresponding to  $G_{\text{He}}\gamma=65.5$  ( $G_{\text{He}}=(g-2)/2=-4.184$  where  $g$  is the gyromagnetic ratio of  $^3\text{He}$  nucleus and  $\gamma$  is the relativistic factor). In this paper we present results showing that it is possible to match the stable spin direction of the transferred polarized  $^3\text{He}$  ions at the injection point of RHIC for a wide range of extraction energies from AGS. The RHIC acceleration complex which includes the AGS is shown in figure 1.

## THEORETICAL BACKGROUND

One method to compute the trajectory and the spin motion of a charged particle with spin moving in an electric and magnetic field is by the numerical integration of the Lorentz and BMT equations shown below.

$$\vec{F} = \frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B}) + \nabla(\vec{\mu} \cdot \vec{B}) \quad \frac{d\vec{S}}{dt} = \vec{\Omega}_0 \times \vec{S}$$

where  $\vec{\mu} = g \frac{q}{2m} \vec{S}$ . In these equations the symbols ( $q, m, \vec{v}, \vec{\mu}, g, \vec{S}, \vec{E}, \vec{B}$ ) correspond to charge, mass, velocity, magnetic moment, gyromagnetic ratio, spin vector, and the electric and magnet fields in the laboratory frame.

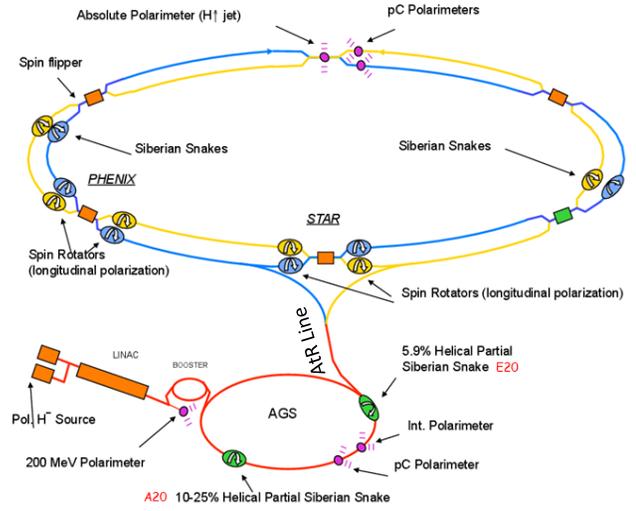


Figure 1. The RHIC accelerator Complex with the location of the A20 and E20 partial helices of the AGS.

We assume that the Stern Gerlach force  $\nabla(\vec{\mu} \cdot \vec{B}) \approx 0$ . The symbol  $\Omega_0$  is given by:

$$\Omega_0 = -\frac{e}{m\gamma} \left\{ \left[ 1 + G_{^3\text{He}}\gamma \right] \vec{B} - G_{^3\text{He}}(\gamma-1)(\vec{v} \cdot \vec{B})\vec{v} + \gamma \left[ G_{^3\text{He}} + \frac{1}{\gamma+1} \right] \frac{\vec{E} \times \vec{v}}{c^2} \right\}$$

The numerical integration of the Lorentz and BMT equations provide the trajectory and the spin direction of a particle moving in an electric and magnetic field. An alternative method to calculate the spin motion of a particle, is based on the Pauli matrices. In this method we form the  $2 \times 2$  spin rotation matrix  $M_{2 \times 2}$  of the beam line or of the synchrotron using,  $M_{2 \times 2} = M_n \dots M_i \dots M_3 M_2 M_1$  where  $M_i = \left\{ I \cos\left(\frac{\theta}{2}\right) - i(\vec{\sigma} \cdot \vec{P}) \sin\left(\frac{\theta}{2}\right) \right\}$  is the spin rotation matrix of each of the beam elements that the line is comprised of. The symbol  $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$  are the Pauli matrices,  $\vec{P} = (P_x, P_y, P_z)$  is the spin rotation axis, and  $\theta$  is the spin rotation angle around the spin rotation axis  $\vec{P}$ . For a synchrotron, the stable spin direction at the beginning of the element  $M_1$  is given by:

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$\vec{P} = i \left( \sin\left(\frac{\theta}{2}\right) \right)^{-1} Tr(\sigma \cdot M_{2 \times 2})$  and the spin rotation angle about the stable spin direction is:  $\cos(\theta/2) = \frac{1}{2} Trace(M_{2 \times 2})$ . Similarly

the stable spin direction  $\vec{S}_{end}$  at the end of a transfer line is given by:  $\vec{S}_{end} = \psi_{end}^* \vec{\sigma} \psi_{end} = \psi_{start}^* M_{2 \times 2}^T \vec{\sigma} M_{2 \times 2} \psi_{start}$ . In this

equation the symbol  $\psi$  is the spinor wavefunction which depends on the orientation of the spin direction. Alternatively, for a beam line, knowing the 2x2 spin rotation matrix we can generate the equivalent 3x3 spin rotation matrix  $M_{3 \times 3}$  which transforms the three components of the spin vector  $\vec{S}_{entr}$  at the entrance of a beam line. Subsequently the stable spin direction  $\vec{S}_{exit}$ , at the exit of the beam line can be calculated by using the relation  $\vec{S}_{exit} = M_{3 \times 3} \vec{S}_{entr}$ . Although the Pauli matrix method which transforms the spinors is equivalent to the raytrace method which transforms the three dimensional spin vector, the raytrace method is easier to apply when the magnetic field contains multipoles, or special magnets like helices.

### STABLE SPIN DIRECTION AT AGS'S INJECTION AND EXTRACTION POINTS

The beam-injection and extraction points of the AGS are located on the straight sections L20 and H10 respectively, and the stable spin direction at these points depends on the strength, and the location of the partial helices and also on the energy of the beam. Using the same strength of the partial helices as they are used for the polarized protons, the warm and cold helices rotate the stable spin direction of the  $^3\text{He}$  ions about the longitudinal axis, by  $25^\circ$  and  $44^\circ$  respectively, and this rotation is nearly constant from the injection energy to the extraction energy. The black trace in figure 2 shows the directional cosine of the stable spin direction with the vertical at the AGS ring, as a function of the beam energy ( $G\gamma$ ) at the L20 injection point. The black trace in figure 3 shows the directional cosine of the stable spin direction with the vertical at the AGS ring, as a function of the beam energy ( $G\gamma$ ) at the H20 extraction point. From the information of figures 2 one can deduce the optimum injection energy of the polarized  $^3\text{He}$  beam because the stable spin direction of the injection beam line into the AGS is along the vertical. In figures 2 and 3 one can observe a periodicity of the stable spin direction with the  $G\gamma$ . This periodicity is related to the location of the two partial helices on the AGS ring.

### STABLE SPIN DIRECTION AT THE INJECTION POINT OF RHIC

The directional cosine ( $S_y$ ) of the stable direction with the vertical at the extraction point of AGS is plotted in figure 3 (thin black line) as a function of  $G\gamma$ . The same quantity ( $S_y$ ), at the injection point of RHIC, is also plotted on figure 3 (thicker blue line). The blue line deviates

appreciably from the vertical for a wide range of the  $G\gamma$  values especially for  $G\gamma$  values greater than 54. This mismatch of the stable spin direction at the RHIC's injection point for a wide range of  $G\gamma$  values, as shown in figure 3 is due to the layout of the AtR line which serves as the transfer line between the AGS and RHIC. In particular two sections of the AtR line bend the transferred beam on the horizontal and vertical planes simultaneously to bring the transferred beam direction on the same horizontal plane as the RHIC's ring.

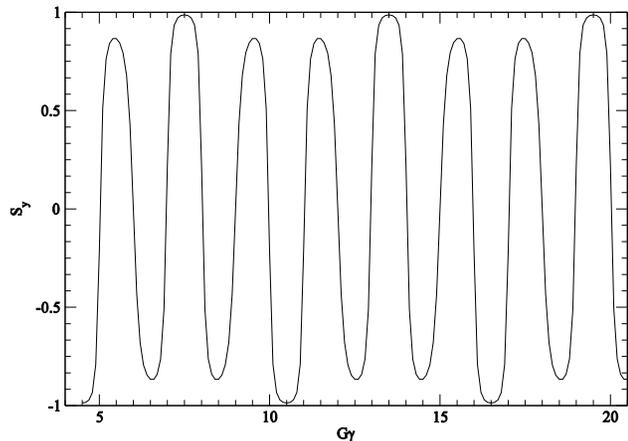


Figure 2. The  $S_y$  directional cosine of the stable spin direction at the injection point L20 of AGS.

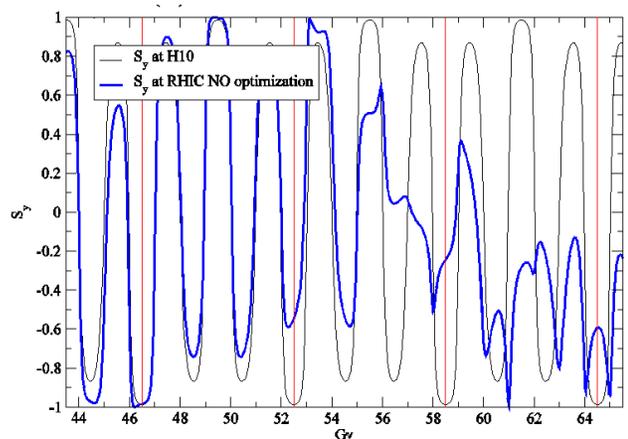


Figure 3. The  $S_y$  directional cosine of the stable spin direction at the extraction point H10 of AGS (thin black line). The  $S_y$  directional cosine of the stable spin direction at the Injection point of RHIC (thicker blue line).

This simultaneous bending of the beam in the horizontal and vertical planes, causes the mismatch of the stable spin direction at the injection point of RHIC. Figure 4 and 5 are schematic diagrams of these two sections of the AtR line where the beam bends simultaneously along the horizontal and vertical planes. Reference [3] provides a detailed description of the effect of the AtR line's layout on the stable spin direction and also describes a proposed method of using the three dipole magnets shown in figure 4, to match the stable spin direction at the injection point

of RHIC. In this paper we only show the results of the spin matching as applied to the polarized  $^3\text{He}$  beam transported along the AtR line when the three dipoles shown in figure 4 are off and when they are on. Figure 6 plots the stable spin direction of the  $^3\text{He}$  ions as they are transported by the AtR line, at the injection point of RHIC, as a function of  $G\gamma$ , for three different set ups of the AtR line discussed below.

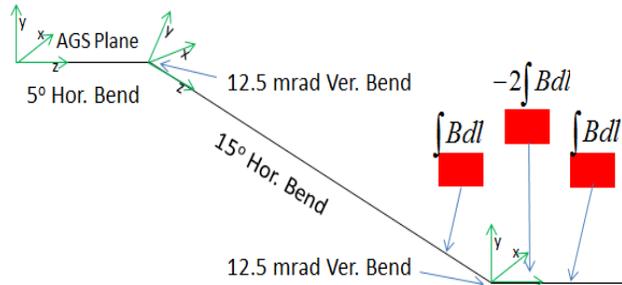


Figure 4. The first section of the AtR line where the beam is bent simultaneously along the horizontal and vertical planes. The red boxes represent dipole magnets which can be placed at specified locations along the AtR line to rotate the stable spin direction and match it at the RHIC's injection point.

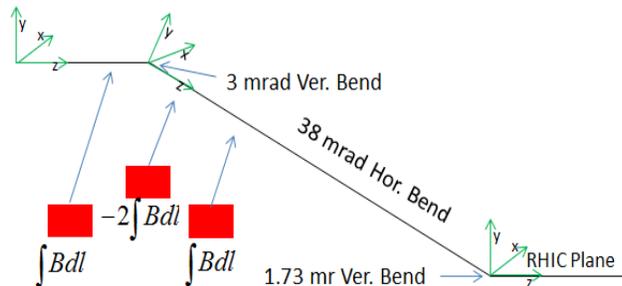


Figure 5. The second section of the AtR line where the beam is bent simultaneously along the horizontal and vertical planes. The red boxes represent dipole magnets which can be placed at specified locations along the AtR line to rotate the stable spin direction and match it at the RHIC's injection point.

The continuous blue line corresponds to the case when the AtR line does not use any special devices to alter the stable spin direction at the injection point of RHIC. In this case it is obvious that the stable spin direction of the polarized beam as it is delivered by the AtR line is not matched with that of the vertical stable spin direction of the circulating beam of RHIC. In order to optimize the stable spin direction of the AtR line we have devised a method [3] which uses the three dipole magnets shown in figure 4. By exciting these three magnets we can optimize the stable spin direction of the AtR line at the injection point of RHIC for any energy of the  $^3\text{He}$  ions extracted from the AGS, without affecting the beam optics of the

transferred beam. In figure 6 the dashed-point green line corresponds to the stable spin direction of the AtR line which has been optimized to deliver the optimum spin matching at  $G\gamma=65.5$ , and the brown dashed line corresponds to the stable spin direction of the AtR line which optimizes the matching of the stable spin direction at  $G\gamma=59.5$ . It is however possible, as we have shown in calculations that by employing an additional set of three dipole magnets to make the stable spin direction of the AtR line at the injection point of RHIC to match exactly the stable spin direction of RHIC ( $S_y=1$ ).

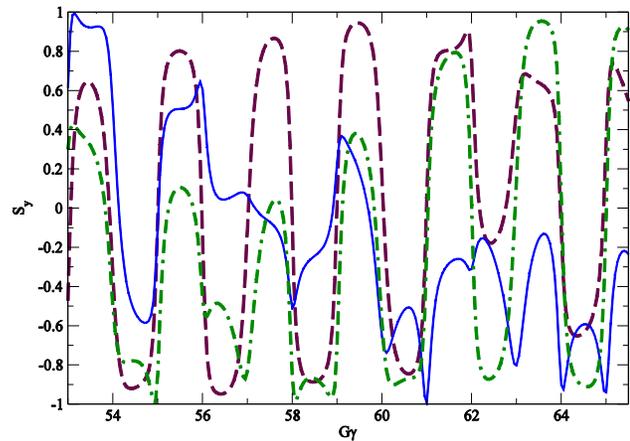


Figure 6. The stable spin direction at the RHIC's injection point as it is transported by the AtR line as a function of  $G\gamma$ , for three different cases which are discussed in the text.

## CONCLUSIONS

Calculations show that by inserting two sets of three dipole magnets in the AtR line we can transport polarized  $^3\text{He}$  ions extracted from the AGS synchrotron to the injection point of RHIC, with perfect "Spin matching" at the RHIC injection point and for a wide range of extraction energies.

## REFERENCES

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