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Preface to the Series

The RIKEN BNL Research Center (RBRC) was established in April 1997 at Brookhaven National Laboratory. It is funded by the "Rikagaku Kenkyusho" (RIKEN, The Institute of Physical and Chemical Research) of Japan. The Memorandum of Understanding between RIKEN and BNL, initiated in 1997, has been renewed in 2002 and again in 2007. The Center is dedicated to the study of strong interactions, including spin physics, lattice QCD, and RHIC physics through the nurturing of a new generation of young physicists.

The RBRC has both a theory and experimental component. The RBRC Theory Group and the RBRC Experimental Group consists of a total of 25-30 researchers. Positions include the following: full time RBRC Fellow, half-time RHIC Physics Fellow, and full-time post-doctoral Research Associate. The RHIC Physics Fellows hold joint appointments with RBRC and other institutions and have tenure track positions at their respective universities or BNL. To date, RBRC has over 85 graduates (Fellows and Post- docs) of which 37 theorists and 19 experimenters have already attained tenure positions at major institutions worldwide.

Beginning in 2001 a new RIKEN Spin Program (RSP) category was implemented at RBRC. These appointments are joint positions of RBRC and RIKEN and include the following positions in theory and experiment: RSP Researchers, RSP Research Associates, and Young Researchers, who are mentored by senior RBRC Scientists. A number of RIKEN Jr. Research Associates and Visiting Scientists also contribute to the physics program at the Center.

RBRC has an active workshop program on strong interaction physics with each workshop focused on a specific physics problem. In most cases all the talks are made available on the RBRC website. In addition, highlights to each speaker's presentation are collected to form proceedings which can therefore be made available within a short time after the workshop. To date there are over one hundred proceeding volumes available.

A 10 teraflops RBRC QCDOC computer funded by RIKEN, Japan, was unveiled at a dedication ceremony at BNL on May 26, 2005. This supercomputer was designed and built by individuals from Columbia University, IBM, BNL, RBRC, and the University of Edinburgh, with the U.S. D.O.E. Office of Science providing infrastructure support at BNL. Physics results were reported at the RBRC QCDOC Symposium following the dedication. QCDSP, a 0.6 teraflops parallel processor, dedicated to lattice QCD, was begun at the Center on February 19, 1998, was completed on August 28, 1998, and was decommissioned in 2006. It was awarded the Gordon Bell Prize for price performance in 1998. QCDOC was decommissioned in May 2012. The next generation computer in this sequence, QCDCQ (600 Teraflops), is currently operational and is expected to produce many more interesting discoveries in the future.

N. P. Samios, Director
August 2012

***Work performed under the auspices of U.S.D.O.E. Contract No. DE-AC02-98CH10886.**

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P breaking effects in a quark (nuclear) medium with axial charge

Alexander A. Andrianov

With D. Espriu, V. Andrianov and X. Planells

Institut de Ciències del Cosmos, University of Barcelona
& Saint-Petersburg State University

CPODD 2012, Brookhaven, June 27, 2012

Outline

- **Maxwell-Chern-Simons Electrodynamics: possible manifestation**
  a. **Large-scale universe or galaxies**: Carroll-Field-Jackiw model (1990) and cosmic birefringence. Spontaneous generation of MCS Electrodynamics by axion condensation, A.A., R.Soldati (1995).

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  polarization splitting for photons and vector mesons in Chern-Simons constant background (axial chemical potential);
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  polarization splitting for photons and vector mesons in Chern-Simons constant background (axial chemical potential);
  A.A., D.Espriu, V.A.Andrianov,X.Planells
- **Finite volume effects**: passing through and reflecting from a boundary A.A., S.S.Kolevatov, R.Soldati
- **Quantization**: Bogoliubov transformation from vacuum to parity breaking medium and back.

Massive MCS electrodynamics (CFJ model)

$$\mathcal{L}_{MCS} = -\tfrac{1}{4}\, F^{\alpha\beta}(x) F_{\alpha\beta}(x) + \tfrac{1}{2}\, m^2\, A_\nu(x) A^\nu(x) + \tfrac{1}{2}\, \zeta_\mu A_\nu(x) \tilde{F}^{\,\mu\nu}(x) + \text{g.f.}$$

In momentum space wave Eqs.

$$\begin{cases} \left[g^{\,\lambda\nu} \left(k^2 - m^2 \right) - k^{\,\lambda} k^{\,\nu} + i\, \varepsilon^{\,\lambda\nu\alpha\beta}\, \zeta_\alpha\, k_\beta \right] \mathbf{a}_\lambda(k) = 0 \\ k^{\,\lambda}\, \mathbf{a}_\lambda(k) = 0 \end{cases}$$

Projection onto different polarizations with the help of

$$S^\nu_\lambda = \delta^{\,\nu}_{\,\lambda}\, \mathtt{D} + k^\nu\, k_\lambda\, \zeta^2 + \zeta^{\,\nu}\, \zeta_\lambda\, k^2 - \zeta \cdot k\, (\zeta_\lambda\, k^\nu + \zeta^\nu\, k_\lambda); \quad \mathtt{D} \equiv (\zeta \cdot k)^2 - \zeta^2\, k^2$$

Transversal polarizations,

$$\boldsymbol{\pi}^{\,\mu\nu}_{\pm} \equiv \frac{S^{\,\mu\nu}}{2\,\mathtt{D}} \pm \frac{i}{2}\, \varepsilon^{\mu\nu\alpha\beta}\, \zeta_\alpha\, k_\beta\, \mathtt{D}^{-\frac{1}{2}}; \quad \varepsilon^{\,\mu}_{\pm}(k) = \boldsymbol{\pi}^{\,\mu\lambda}_{\pm}\, \epsilon^{(0)}_\lambda$$

Scalar and longitudinal polarizations,

$$\varepsilon^{\,\mu}_{S}(k) \equiv \frac{k^{\,\mu}}{\sqrt{k^2}}, \qquad \varepsilon^{\,\mu}_{L}(k) \equiv \left(\mathtt{D}\, k^2 \right)^{-\frac{1}{2}} \left(k^2\, \zeta^{\,\mu} - k^{\,\mu}\, \zeta \cdot k \right)$$

Energy spectrum and birefringence

Transversal polarizations,

$$K_\nu^\mu \, \varepsilon_\pm^\nu(k) = \left(k^2 - m^2 \pm \sqrt{\mathtt{D}}\right) \varepsilon_\pm^\mu(k);$$

$$\omega_{\mathbf{k},\pm} = \begin{cases} \sqrt{\mathbf{k}^2 + m^2 \pm \zeta_0 |\,\mathbf{k}\,|}; & \zeta_\mu = (\zeta_0, 0, 0, 0) \\ \sqrt{\mathbf{k}^2 + m^2 + \frac{1}{2}\zeta_x^2 \pm \zeta_x \sqrt{k_x^2 + m^2 + \frac{1}{4}\zeta_x^2}} & \zeta_\mu = (0, -\zeta_x, 0, 0) \end{cases}$$

Polarizations of linearly polarized radio waves could be rotated with distance L!

$$\zeta_0 \ll |\,\mathbf{k} \quad k_\pm \simeq \omega_{\mathbf{k}} \mp \frac{1}{2}\zeta_0; \quad \Delta\phi_{rotation} = \frac{1}{2}(\phi_L - \phi_R) = \frac{1}{2}\zeta L.$$

Distances are of order the Hubble scale $\sim 10^{10}$ l.y. and from the analysis of 160 galaxies with linearly polarized radio waves $\Rightarrow |\zeta| < 10^{-33}$eV $\sim 1/R_{Universe}$

Large-scale Universe is not birefringent! (at low energies)

Motivation of local PB

Parity: well established global symmetry of strong interactions. Reasons to believe it may be broken in a finite volume?!

- **quantum fluctuations of θ parameter** (P-odd bubbles [T. D. Lee and G. C. Wick ...]: their manifestation in Chiral Magnetic Effect (CME))[D. E. Kharzeev, A.Zhitnitsky, L. D. McLerran, K.Fukushima, H. J. Warringa (an earlier proposal: A.Vilenkin, 1980)]

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- *Our special interest*:
  PB background inside a hot dense nuclear fireball in HIC !?

Topological charge

$$T_5(t) = \frac{1}{8\pi^2}\int_{\text{vol.}} d^3x \varepsilon_{jkl}\text{Tr}\left(G^j\partial^k G^l - i\frac{2}{3}G^j G^k G^l\right)$$

in a finite volume it may arise from quantum fluctuations in hot QCD medium
(due to sphaleron transitions!? [Manton, Klinkhamer, Rubakov, Shaposhnikov, McLerran])
(due to long-range P-odd correlations!? [Zhitnitsky's talk on this workshop])
and survive for a sizeable lifetime in a heavy-ion fireball (i.e. only a little dissipation due to gluon flux (jets))

$$\langle \Delta T_5\rangle \neq 0 \quad \text{for} \quad \Delta t \simeq \tau_{\text{fireball}} \simeq 5-10 \text{ fm},$$

For this period one can control the value of $\langle \Delta T_5\rangle$ introducing into the QCD Lagrangian a topological chemical potential

$$\Delta L = \mu_\theta \Delta T_5, \quad \Delta T_5 = T_5(t_f) - T_5(0) = \frac{1}{8\pi^2}\int_0^{t_f} dt \int_{\text{vol.}} d^3x \text{Tr}\left(G^{\mu\nu}\widetilde{G}_{\mu\nu}\right)$$

in a gauge invariant way.

Axial baryon charge

Partial conservation of isosinglet axial current broken by gluon anomaly (consider the light quarks only),

$$\partial_\mu J_5^\mu - 2im_q J_5 = \frac{N_f}{8\pi^2}\mathrm{Tr}\left(G^{\mu\nu}\widetilde{G}_{\mu\nu}\right)$$

predicts the induced **chiral (axial) charge**

$$\frac{d}{dt}\left(Q_5^q - 2N_f T_5\right) \simeq 0,\ m_q \simeq 0, \quad Q_5^q = \int_{\mathrm{vol.}} d^3x \bar{q}\gamma_0\gamma_5 q = \langle \underline{N_L - N_R} \rangle$$

to be conserved $\dot{Q}_5^q \simeq 0$ (in the chiral limit $m_q \simeq 0$) during $\tau_{fireball}$.

Axial chemical potential

Axial chemical potential can be associated with approximately conserved Q_5^q (for u, d quarks!)

$$\Delta L_q = \mu_5^q Q_5^q,$$

to reproduce a corresponding

$$\langle \Delta T_5 \rangle \simeq \frac{1}{2N_f} \langle Q_5^q \rangle, \Longleftrightarrow \mu_5^q \simeq \frac{1}{2N_f} \mu_\theta$$

LPB to be investigated in e.m. interactions of leptons and photons with hot/dense nuclear matter via heavy ion collisions.

- e.m. interaction implies

$$Q_5^q \to \tilde{Q}_5 = Q_5^q - T_5^{\text{em}}, \quad T_5^{\text{em}} = \frac{1}{16\pi^2} \int_{\text{vol.}} d^3x \varepsilon_{jkl} A^j \partial^k A^l$$

- μ_5 is conjugated to (nearly) conserved $\tilde{Q}_5$

Axial chemical potential in hadron Lagrangians

Bosonization of Q_5^q following VMD prescription

$$\mathcal{L}_{\text{int}} = \bar{q}\gamma_\mu \hat{V}^\mu q; \quad \hat{V}_\mu \equiv -eA_\mu Q + \frac{1}{2} g_\omega \omega_\mu \mathbb{I} + \frac{1}{2} g_\rho \rho_\mu^0 \lambda_3 + \frac{g_\phi}{\sqrt{2}} \phi_\mu \mathbb{I}_s,$$

$$(V_{\mu,a}) \equiv \left(A_\mu,\, \omega_\mu,\, \rho_\mu^0, \phi_\mu\right), \quad g_\omega \simeq g_\rho \equiv g \simeq 6; \quad g_\phi \sim 8$$

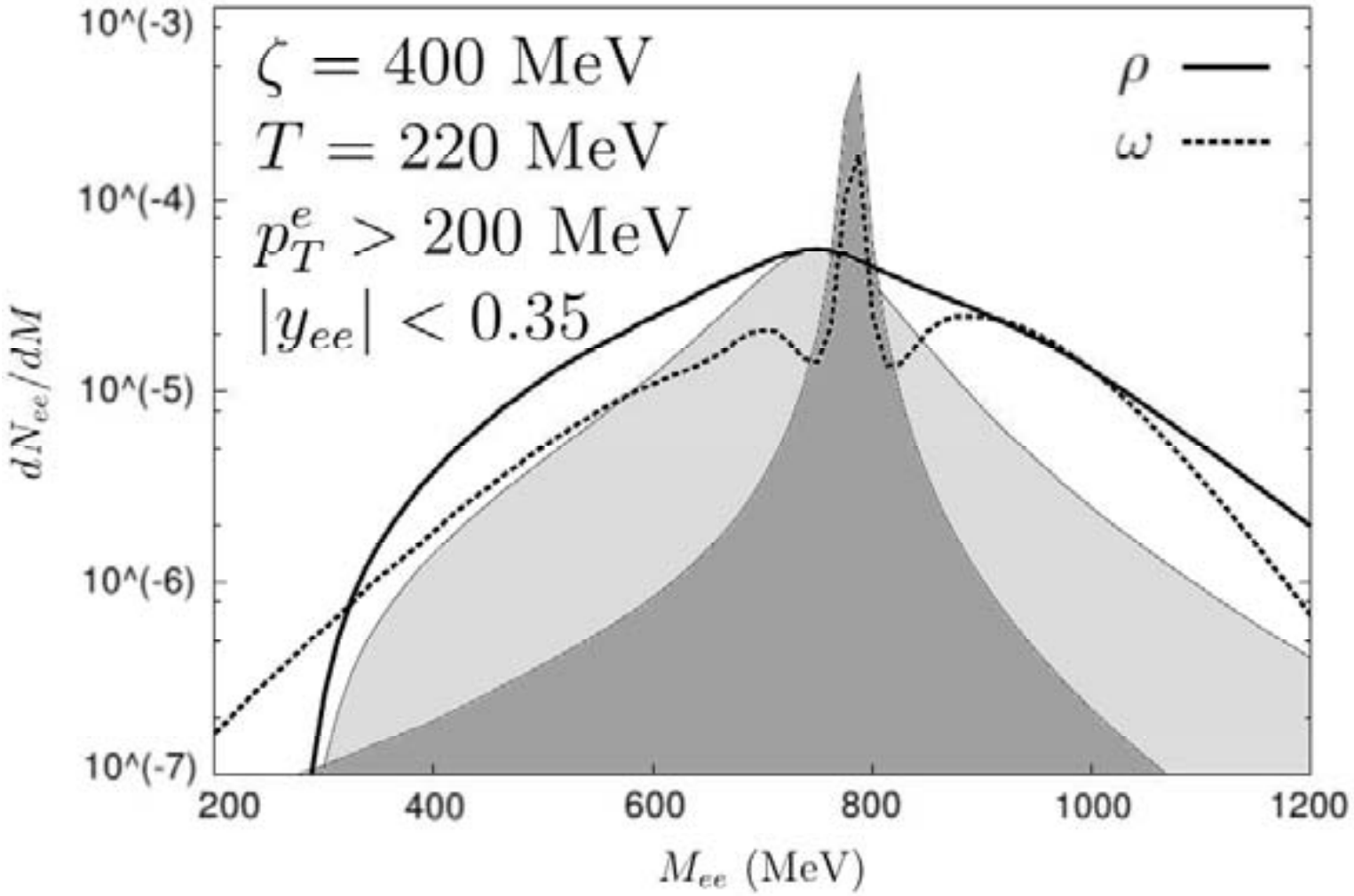
$$\mathcal{L}_{\text{kin}} = -\frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} + \rho_{\mu\nu}\rho^{\mu\nu} + \phi_{\mu\nu}\phi^{\mu\nu}\right) + \frac{1}{2} V_{\mu,a} (\hat{m}^2)_{a,b} V_b^\mu$$

***P*-odd interaction**

$$\mathcal{L}_{\text{mix}} \propto -\frac{1}{4}\varepsilon^{\mu\nu\rho\sigma} \text{Tr}\left[\hat{\zeta}_\mu V_\nu V_{\rho\sigma}\right] = \frac{1}{2}\text{Tr}\left(\hat{\zeta}\varepsilon_{jkl}\hat{V}_j \partial_k \hat{V}_l\right) = \frac{1}{2}\zeta\varepsilon_{jkl} V_{j,a} N_{ab} \partial_k V_{l,b}$$

with $\hat{\zeta}_\mu = \hat{\zeta}\delta_{\mu 0}$, spatially homogeneous and isotropic background.

Resonance splitting in polarizations



Finite volume: passing through boundary

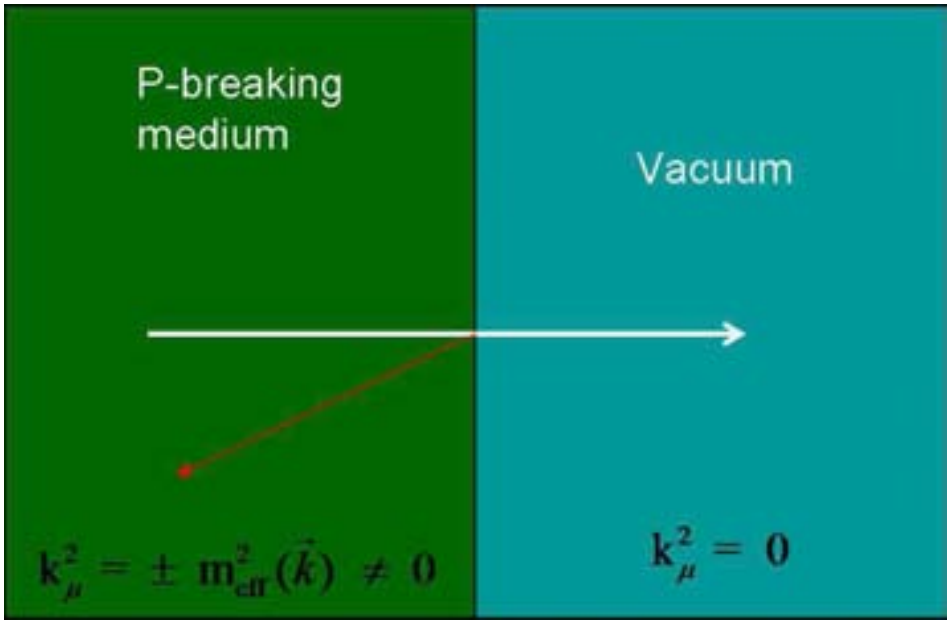
A.A., S.Kolevatov, R.Soldati, 2011

Mean free paths for vector mesons:

- $L_\rho \sim 0.8 fm$ $\ll L_{fireball} \sim 5 - 10 fm$
- $L_\omega \sim 16 fm \gg L_{fireball}$ Why it is relevant in medium?

LPB "vacuum"
$\neq$ empty vacuum
$=$ coherent state
of vacuum mesons

Matching on $\zeta \cdot x = 0$



Thus to save energy-momentum conservation transmission must be accompanied by reflection back. Enhancement of in-medium decays of ω mesons!

MCS electrodynamics in a half space

A possible gauge-invariant choice,

$$-\tfrac{1}{4}\, F^{\mu\nu}(x)\widetilde{F}_{\mu\nu}(x)\,\zeta_\lambda x^\lambda\,\theta(-\,\zeta\cdot x) \Rightarrow \tfrac{1}{2}\,\zeta_\mu A_\nu(x)\widetilde{F}^{\mu\nu}(x)\,\theta(-\,\zeta\cdot x),$$

which however associates a space-like boundary with a space-like CS vector $(\zeta_\mu)(x) = \zeta(0,\vec{a})\theta(-\,\vec{a}\cdot\vec{x}), \quad |\vec{a}| = 1$.

Compact dense stars filled by axions with density degrading to their surface?!

Another choice: time-like CS vector and space-like boundary $(\zeta_\mu)(x) = \zeta\big(\theta(-\,\vec{a}\cdot\vec{x}), -\vec{a}x_0\delta(-\,\vec{a}\cdot\vec{x})\big)$ gauge invariance condition $\partial_\nu\zeta_\mu = \partial_\mu\zeta_\nu$. Singular interaction on a space-like boundary!

Axial chemical potential for a fireball

⋆ ⋆ ⋆ ⋆ ⋆ ⋆

Matching on the boundary $\zeta\cdot x = 0$

$$\delta(\zeta\cdot x)\,[\,A^\mu_{\rm vacuum}(x) - A^\mu_{\rm MCS}(x)\,] = 0,$$

MCS dispersion laws

Axial chemical potential in a bounded volume

$\Rightarrow$ looking for boundary influence on meson propagation!

$$(\zeta_\mu)(x) = \zeta\big(\theta(-x_1), -x_0\delta(x_1), 0, 0\big); \qquad \zeta \sim \mu_5$$

The MCS dispersion laws for different polarizations (inside of fireball),

$$\begin{cases} k_{1L} = k_{10} = \sqrt{\omega^2 - m^2 - k_\perp^2} \\ k_{1-} = \sqrt{\omega^2 - m^2 - k_\perp^2 + \frac{\zeta^2}{2} + \zeta\sqrt{\omega^2 - m^2 + \frac{\zeta^2}{4}}} \\ k_{1+} = \sqrt{\omega^2 - m^2 - k_\perp^2 + \frac{\zeta^2}{2} - \zeta\sqrt{\omega^2 - m^2 + \frac{\zeta^2}{4}}} \end{cases}$$

$k_{1\star}$ are momentum components orthogonal to boundary;
k_{10} is outside the P-breaking medium;
$k_{1A}, \quad A = L, +, -$ are in the MCS vacuum;
$\vec{k}_\perp$ is parallel to the boundary;
ω is the energy.

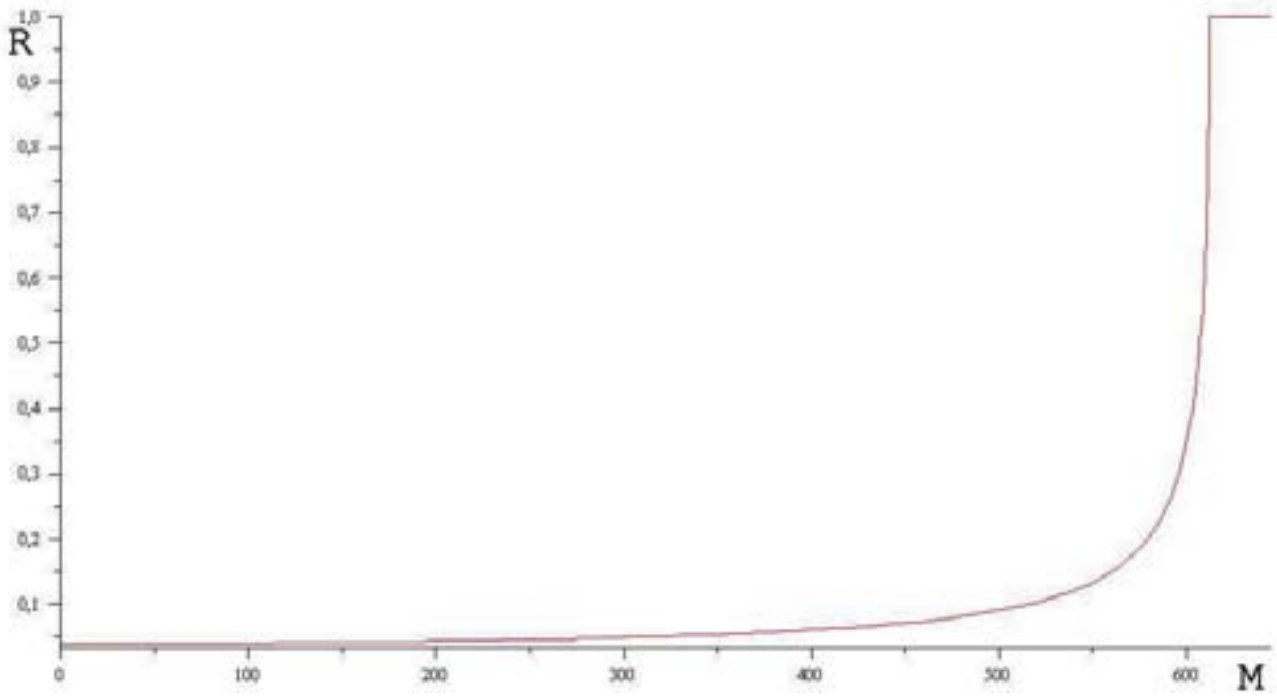
Reflection coefficient for transverse polarizations

In terms of effective mass $M^2 = k_\mu k^\mu$
adapted to describe dilepton decay channels!

$$k_{1\pm}^2 = \frac{(M^2 - m^2)^2}{\zeta^2} - k_\perp^2$$
$$k_{10}^2 = \frac{(M^2 - m^2)^2}{\zeta^2} + (M^2 - m^2) - k_\perp^2$$

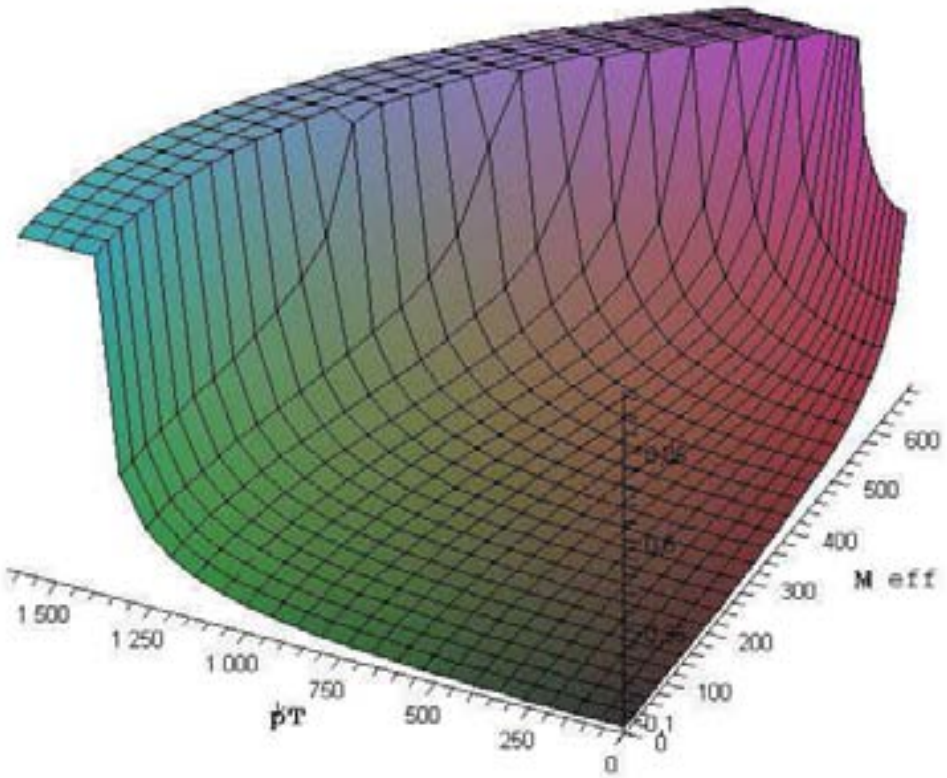
$$R = \frac{|\sqrt{\frac{(M^2-m^2)^2}{\zeta^2} - k_\perp^2} - \sqrt{\frac{(M^2-m^2)^2}{\zeta^2} + (M^2 - m^2) - k_\perp^2}|}{|\sqrt{\frac{(M^2-m^2)^2}{\zeta^2} - k_\perp^2} + \sqrt{\frac{(M^2-m^2)^2}{\zeta^2} + (M^2 - m^2) - k_\perp^2}|}$$

Reflection from boundary depending on effective mass: negative chirality

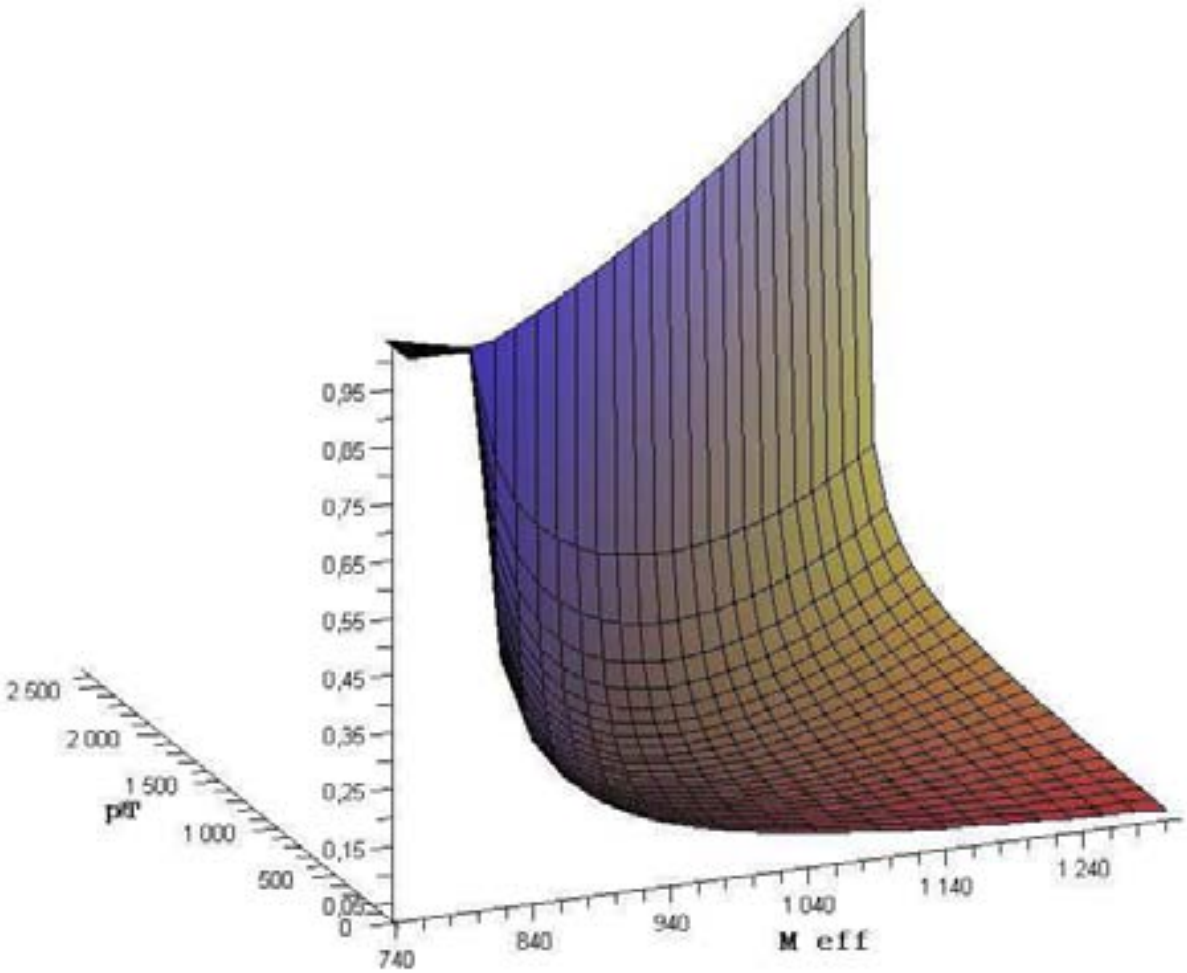


Dependence on effective mass for zero $p_T \equiv |\vec{k}_\perp|$**, CS vector** $\zeta = 300$ **MeV**

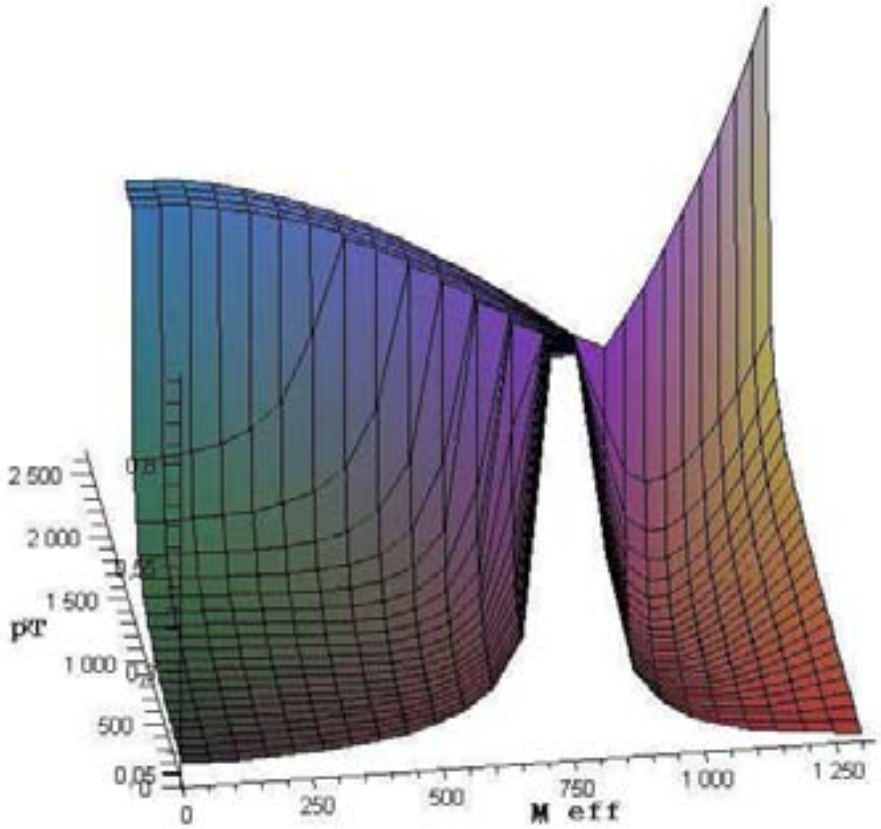
Reflection from boundary: negative chirality, $p_T \neq 0$



Reflection from boundary: positive chirality, $p_T \neq 0$



Reflection from boundary: both chiralities, $p_T \neq 0$



Quantization of MCS in a half-space: Bogolyubov transformation

In vacuum,

$$A^{\mu}_{\rm vacuum}(x) = \int \mathrm{d}^3\hat{k}\,\theta(\omega^2 - k_\perp^2 - m^2)\sum_{r=1}^{3}\left[\mathbf{a}_{\hat{k}\,,\,r}\,u^{\mu}_{\hat{k}\,,\,r}(x) + \mathbf{a}^{\dagger}_{\hat{k}\,,\,r}\,u^{\mu\,*}_{\hat{k}\,,\,r}(x)\right],$$

Canonical commutation relations $[\mathbf{a}_{\hat{k}\,,\,r}\,,\,\mathbf{a}^{\dagger}_{\hat{k}'\,,\,s}] = \delta(\hat{k} - \hat{k}')\,\delta_{rs}$

In P-breaking medium ($A \in \{L, +, -\}$),

$$A^{\nu}_{\rm MCS}(x) \;=\; \int \mathrm{d}^3\hat{k}\,\theta(\omega^2 - k_\perp^2 - m^2)\sum_{A}\left[c_{\hat{k},A}\,v^{\nu}_{\hat{k}\,A}(x) + c^{\dagger}_{\hat{k},A}\,v^{\nu *}_{\hat{k}\,A}(x)\right]$$

Canonical commutation relations, $\left[c_{\hat{k},A}\,,\,c^{\dagger}_{\hat{k}',A'}\right] \;=\; -\,g_{AA'}\,\delta(\hat{k} - \hat{k}')$

Matching

$$\delta(\zeta \cdot x)\,[\,A^{\mu}_{\rm vacuum}(x) - A^{\mu}_{\rm MCS}(x)\,] = 0,$$

Bogoliubov transformation

$$\mathbf{a}_{\hat{k},\,r} = \sum_{A=\pm,L} \left[\alpha_{\,rA}(\hat{k})\, c_{\hat{k},A} - \beta^{*}_{rA}(\hat{k})\, c^{\dagger}_{\hat{k},A}\right]$$

$$c_{\hat{k},A} = \sum_{r=1}^{3} \left[\alpha^{*}_{Ar}(\hat{k})\, \mathbf{a}_{\hat{k},\,r} + \beta^{*}_{Ar}(\hat{k})\, \mathbf{a}^{\dagger}_{\hat{k},\,r}\right]$$

Relations between coefficients

$$v^{\nu}_{\hat{k},A}(\hat{x}) = \sum_{s=1}^{3} \left[\alpha_{sA}(\hat{k})\, u^{\nu}_{\hat{k},s}(\hat{x}) - \beta_{sA}(\hat{k})\, u^{\nu *}_{\hat{k},s}(\hat{x})\right]$$

Two vacuums as coherent states

Two different Fock vacua

$$\mathbf{a}_{\hat{k},\,r}|0\rangle = 0 \qquad\qquad c_{\hat{k},A}\mid\Omega\rangle = 0$$

From Bogoliubov tranformation,

$$|0\rangle = \mathcal{N}\exp\left[\int \mathrm{d}^3\hat{k}\,\theta(\omega^2 - k_\perp^2 - m^2)\times\right.$$
$$\left.\times\left\{\frac{\beta^*_{r+}(\hat{k})}{2\alpha_{r+}(\hat{k})}(c^\dagger_{\hat{k},+})^2 + \frac{\beta^*_{r-}(\hat{k})}{2\alpha_{r-}(\hat{k})}(c^\dagger_{\hat{k},-})^2 + \frac{\beta^*_{rL}(\hat{k})}{2\alpha_{rL}(\hat{k})}(c^\dagger_{\hat{k},L})^2\right\}\right]\mid\Omega\rangle$$

and inversely

$$\mid\Omega\rangle = \tilde{\mathcal{N}}\exp\left[\int \mathrm{d}^3\hat{k}\,\theta(\omega^2 - k_\perp^2 - m^2)\times\right.$$
$$\left.\times\left\{\frac{-\beta^*_{A1}(\hat{k})}{2\alpha^*_{A1}(\hat{k})}(\mathbf{a}^\dagger_{\hat{k},\,1})^2 + \frac{-\beta^*_{A2}(\hat{k})}{2\alpha^*_{A2}(\hat{k})}(\mathbf{a}^\dagger_{\hat{k},\,2})^2 + \frac{-\beta^*_{A3}(\hat{k})}{2\alpha^*_{A3}(\hat{k})}(\mathbf{a}^\dagger_{\hat{k},\,3})^2]\right\}\right]|0\rangle$$

Conclusions

- Local (finite-volume) PB is not forbidden by any physical principle in QCD at finite temperature/density
- The PB leads to unexpected modifications of the in-medium properties of vector mesons and photons, in particular, resonance splitting in polarizations $\Rightarrow$ the detailed analysis is given in the next talk by Domenec Espriu .
- Boundary enhancement of in-medium ω decays + LPB $\rightarrow$ broadening of ω resonance in fireballs (observed on PHENIX!)
- Axion stars discovery from exotic photon spectra (boundary effects)??

Instantons and Sphalerons in a Magnetic Field

Gökçe Başar

Stony Brook University

06/27/2012

GB, G.Dunne & D. Kharzeev , arXiv:1112.0532, PRD **85** 045026

GB, D. Kharzeev, arXiv:1202.2161, PRD **85** 086012

Outline

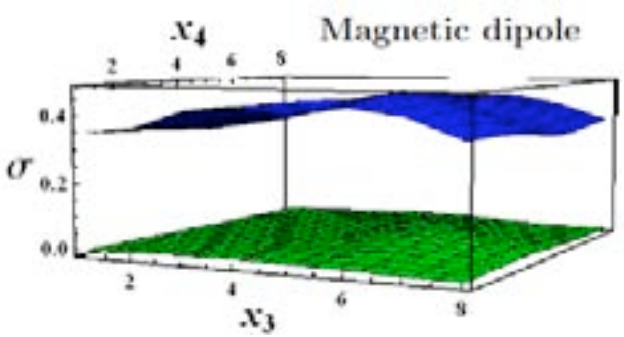
- Motivation & some lattice results
- General facts on Dirac operator
- Large instanton limit & dipole moments
- Sphaleron rate at strong coupling

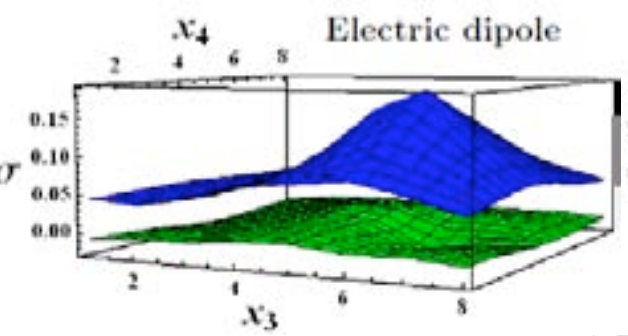
Motivation & some lattice results

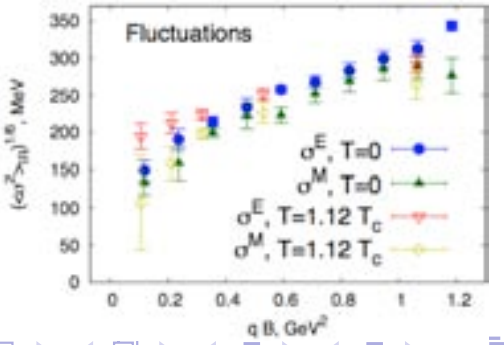
Interplay between topology & magnetic field

- Chiral magnetic effect $\vec{J} = \frac{q\vec{B}}{2\pi}\frac{\mu_5}{\pi}$
- What sources μ_5? sphalerons, η domains, etc..
- Instanton + magnetic field
- Lattice results
  - ITEP group (electric & dipole moments)
  - T. Blum et al. (zero modes $\propto$ B)
  - A. Yamamoto (C.M. conductivity)

(Polikarpov et al. '09)







Notation & conventions

work in: $\mathbb{R}^4(\mathbb{T}^4)$

chiral basis: $$\gamma_\mu = \begin{pmatrix} 0 & \alpha_\mu \\ \bar{\alpha}_\mu & 0 \end{pmatrix} \quad , \quad \gamma_5 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$$

$$\alpha_\mu = (\mathbb{1}, -i\vec{\sigma}) \quad , \quad \bar{\alpha}_\mu = (\mathbb{1}, i\vec{\sigma}) = \alpha_\mu^\dagger$$

Dirac operator: $$\not{\mathcal{D}} = \begin{pmatrix} 0 & \alpha_\mu \mathcal{D}_\mu \\ \bar{\alpha}_\mu \mathcal{D}_\mu & 0 \end{pmatrix} \equiv \begin{pmatrix} 0 & D \\ -D^\dagger & 0 \end{pmatrix}$$

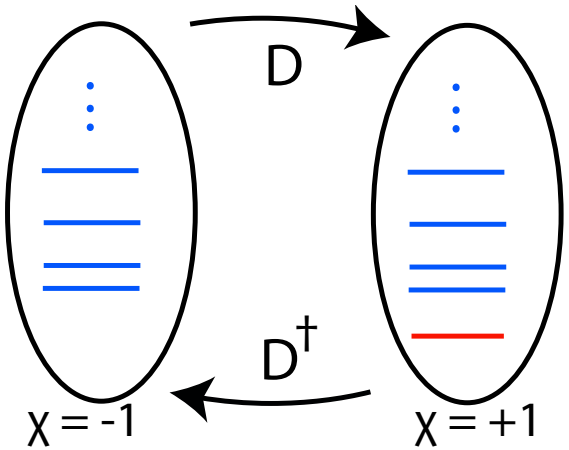
gauge field: $$\mathcal{A}_\mu = A_\mu + a_\mu$$

Notation & conventions

diagonal form: $\left(i\not{D}\right)^2 \psi_\lambda = \begin{pmatrix} DD^\dagger & 0 \\ 0 & D^\dagger D \end{pmatrix} \psi_\lambda = \lambda^2 \psi_\lambda$

$$\chi = +1: \qquad DD^\dagger = -\mathcal{D}_\mu^2 - \mathcal{F}_{\mu\nu}\bar{\sigma}_{\mu\nu}$$
$$\chi = -1: \qquad D^\dagger D = -\mathcal{D}_\mu^2 - \mathcal{F}_{\mu\nu}\sigma_{\mu\nu}$$

"supersymmetry:" for $\lambda \neq 0$, $DD^\dagger$ and $D^\dagger D$ has identical spectra



magnetic field:

$$DD^\dagger = D^\dagger D = -\mathcal{D}_\mu^2 - B\sigma_3$$

Zero modes: Positive spin, both chiralities

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BPST instanton:

$$DD^\dagger = -\mathcal{D}_\mu^2 \qquad , \qquad D^\dagger D = -\mathcal{D}_\mu^2 - F_{\mu\nu}\sigma_{\mu\nu}$$

Zero modes: Both spins, definite chirality

Index theorem: $N_+ - N_- = -N_- = -\frac{1}{32\pi^2}\int d^4x\, F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}$

Instanton & magnetic field

$$DD^\dagger = -\mathcal{D}_\mu^2 - B\sigma_3, \quad , \quad D^\dagger D = -\mathcal{D}_\mu^2 - F_{\mu\nu}\sigma_{\mu\nu} - B\sigma_3$$

Instanton & magnetic field

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Zero modes: Both spins, both chiralities

Index thm: $\mathrm{tr}\left(\mathcal{F}_{\mu\nu}\tilde{\mathcal{F}}_{\mu\nu}\right) = \mathrm{tr}\left(F_{\mu\nu}\tilde{F}_{\mu\nu}\right) + (\mathrm{dim})\, f_{\mu\nu}\tilde{f}_{\mu\nu}$

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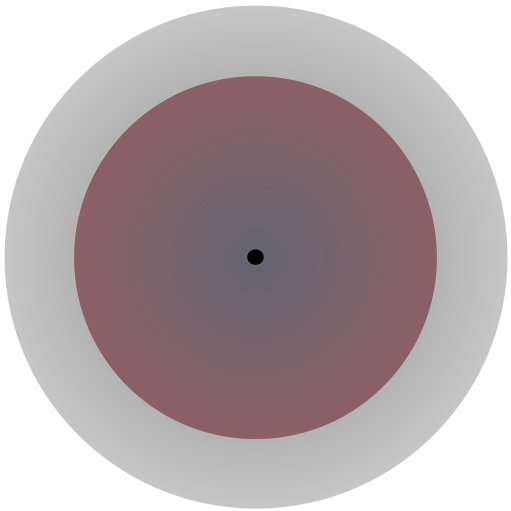
Competition between instanton and magnetic field

$\downarrow$ $\downarrow$

try to align chiralities align spins

Instanton zero mode: $|\psi_0|^2 = \frac{64\,\rho^2}{(x^2+\rho^2)^3}$

Topological charge: $q_5(x) = \frac{192\,\rho^4}{(x^2+\rho^2)^4}$



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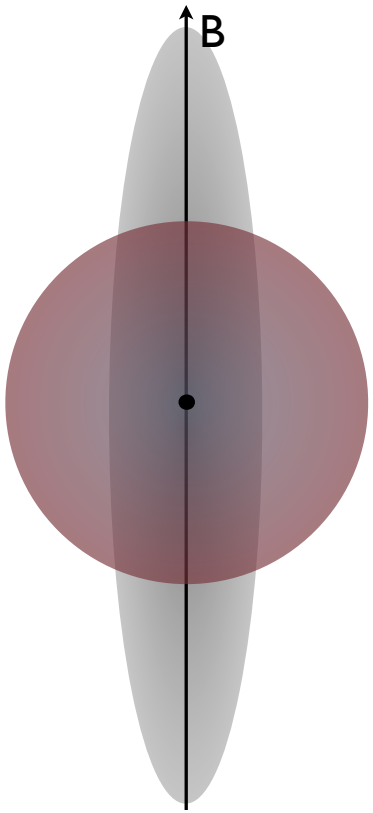

Large instanton limit

$$\text{suppose: } \frac{1}{\sqrt{B}} << \rho$$

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$$A^a_\mu = 2\frac{\eta^a_{\mu\nu}\, x_\nu}{x^2+\rho^2} \approx \frac{2}{\rho^2}\eta^a_{\mu\nu}x_\nu + \ldots$$

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$$A^a_\mu = 2\frac{\eta^a_{\mu\nu}\, x_\nu}{x^2+\rho^2} \approx \frac{2}{\rho^2}\eta^a_{\mu\nu}x_\nu + \ldots$$

after appropriate gauge rotation & Lorentz transformation:

$$\mathcal{A}_\mu = -\frac{F}{2}(-x_2, x_1, -x_4, x_3)\tau^3 + \frac{B}{2}(-x_2, x_1, 0, 0)\mathbb{1}_{2\times 2}$$

quasi-abelian, covariantly constant $\rightarrow$ soluble!

Large instanton limit

$$\mathcal{A}_\mu = -\tfrac{F}{2}(-x_2, x_1, -x_4, x_3)\tau^3 + \tfrac{B}{2}(-x_2, x_1, 0, 0)\mathbb{1}_{2\times 2}$$

$$\mathcal{F}_{12} = \begin{pmatrix} B - F & 0 \\ 0 & B + F \end{pmatrix}$$

$$\mathcal{F}_{34} = \begin{pmatrix} -F & 0 \\ 0 & F \end{pmatrix}$$

Landau problem with field strengths $\mathcal{F}_{12}$ & $\mathcal{F}_{34}$

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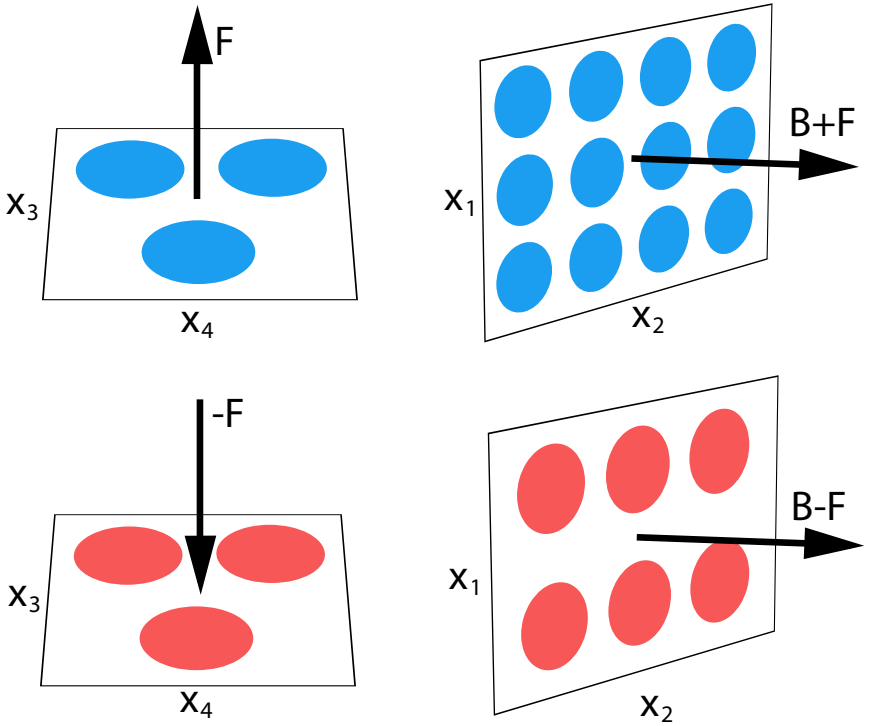
Topological charge: $\frac{1}{32\pi^2}\mathcal{F}^a_{\mu\nu}\tilde{\mathcal{F}}^a_{\mu\nu} = \frac{F^2}{2\pi^2}$

Zero modes

$$\tau = -1\,,\ \chi = -1\,,\ spin\uparrow,\ n_- = \frac{(B+F)}{2\pi}\,\frac{F}{2\pi}$$

$$\tau = +1\,,\ \chi = +1\,,\ spin\uparrow,\ n_+ = \frac{(B-F)}{2\pi}\,\frac{F}{2\pi}$$

$$n_+ + n_- = B\frac{F}{2\pi^2} \quad , \quad n_+ - n_- = -\frac{F^2}{2\pi^2}$$



Zero modes

$$B < F$$

$$\begin{aligned}
\chi = +1: \qquad n_+ &= 0 \\
\chi = -1: \qquad n_- &= \begin{cases} \frac{(B+F)}{2\pi}\frac{F}{2\pi} & , \quad (\tau_3 = -1 \quad , \quad spin \uparrow) \\ \frac{(-B+F)}{2\pi}\frac{F}{2\pi} & , \quad (\tau_3 = +1 \quad , \quad spin \downarrow) \end{cases}
\end{aligned}$$

$$n_+ + n_- = n_- = \frac{F^2}{2\pi^2} \quad , \quad n_+ - n_- = -n_- = -\frac{F^2}{2\pi^2}$$

$$n_\uparrow - n_\downarrow = \frac{BF}{2\pi^2}$$

Quantization on $\mathbb{T}^4$

- twisted boundary conditions $\rightarrow$ fractional Pontryagin index (' t Hooft '81)
- zero twist $\rightarrow \exists$ quasi-abelian, covariantly constant, SD solutions (van Baal '96)

M: Instanton flux N:Magnetic flux

Zero modes:

$$B > F$$

index: $N_+ - N_- = -2M^2 = -\frac{F^2\,L^4}{2\pi^2}$

total number: $N_+ + N_- = 2MN = \frac{B\,F\,L^4}{2\pi^2}$

$$B < F$$

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total number: $N_+ + N_- = 2M^2 = \frac{F^2\,L^4}{2\pi^2}$

Dipole moments

$$\sigma_i^M = \frac{1}{2}\epsilon_{ijk}\langle\bar{\psi}\Sigma_{jk}\psi\rangle \quad , \quad \sigma_i^E = \langle\bar{\psi}\Sigma_{i4}\psi\rangle$$

Dipole moments

$$\sigma_3^M = \tfrac{1}{2}\langle\bar{\psi}\Sigma_{12}\psi\rangle \quad , \quad \sigma_3^E = \langle\bar{\psi}\Sigma_{34}\psi\rangle$$

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$$\begin{aligned} m\langle\bar{\psi}\Sigma_{12}\psi\rangle &= \mathrm{tr}_{2\times 2}\left(\sigma_3\frac{m^2}{m^2+DD^\dagger}\right) + \mathrm{tr}_{2\times 2}\left(\sigma_3\frac{m^2}{m^2+D^\dagger D}\right) \\ m\langle\bar{\psi}\Sigma_{34}\psi\rangle &= -\mathrm{tr}_{2\times 2}\left(\sigma_3\frac{m^2}{m^2+DD^\dagger}\right) + \mathrm{tr}_{2\times 2}\left(\sigma_3\frac{m^2}{m^2+D^\dagger D}\right) \end{aligned}$$

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- $\sigma_3^M > \sigma_3^E$
- $\langle\bar{\psi}\Sigma_{34}\psi\,\bar{\psi}\Sigma_{34}\psi\rangle \approx \left(\frac{F}{2\pi^2 m^2 L^4}\right) B$

Sphaleron rate (basics)

$$\Gamma_{CS} = \frac{(\Delta Q_5)^2}{Vt} = \int d^4x \left\langle \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a(x) \frac{g^2}{32\pi^2} F^a_{\alpha\beta} \tilde{F}^{\alpha\beta}_a(0) \right\rangle$$

Diffusion of topological charge: $\frac{dN_5}{dt} = -c\, N_5\, \frac{\Gamma_{CS}}{T^3}$

- CP odd effects in QCD (CME)
- Baryon number (B+L) violation in E.W.

Weak coupling: $\Gamma_{CS} = \kappa\, g^4 T \log(1/g)\, (g^2 T)^3$ (Bödeker '98)

Strong coupling: $\Gamma_{CS} = \frac{(g^2 N)^2}{256\pi^3} T^4$ (Son, Starinets '02)

Sphaleron rate (holography)

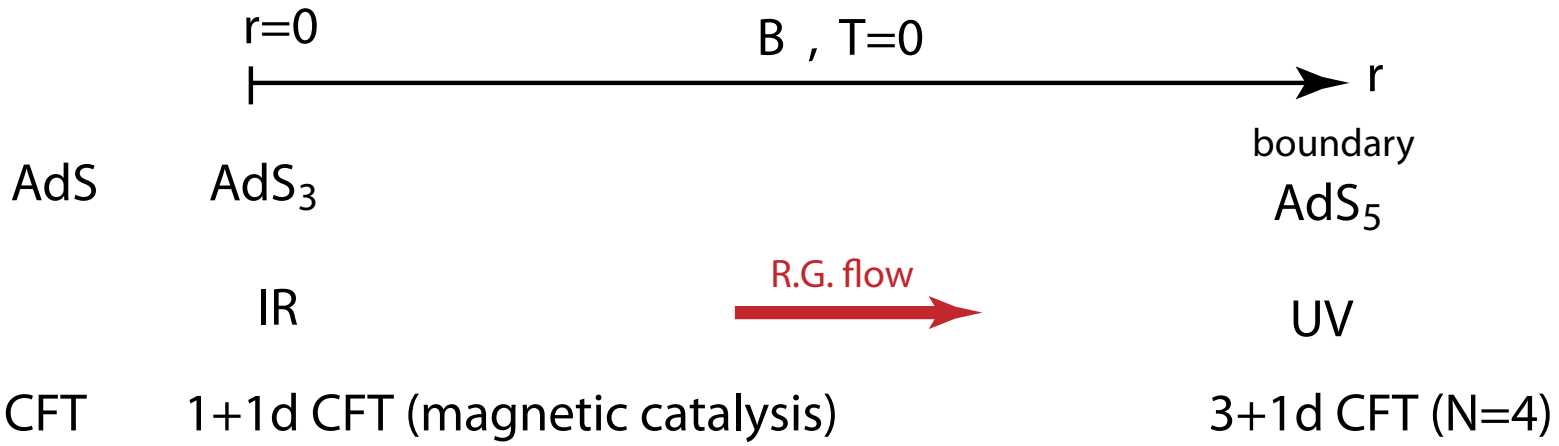
5-d Einstein-Maxwell-(Chern Simons):

$$S = -\frac{1}{16\pi G_5}\int d^5x\sqrt{-g}(R+F^{MN}F_{MN}-\frac{12}{l^2})+\frac{1}{6\sqrt{3}\pi G_5}\int A\wedge F\wedge F$$
$$F = Bdx_1\wedge dx_2$$

$\exists$ solutions of the form: (D'Hoker, Kraus '08-'11)

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + e^{2V(r)}(dx_1^2+dx_2^2) + e^{2W(r)}dx_3^2$$

with: $U(r_h) = 0 \quad , \quad e^{2V(r)}, e^{2W(r)}|_{r\to\infty} \to r^2 \ (AdS_5)$,



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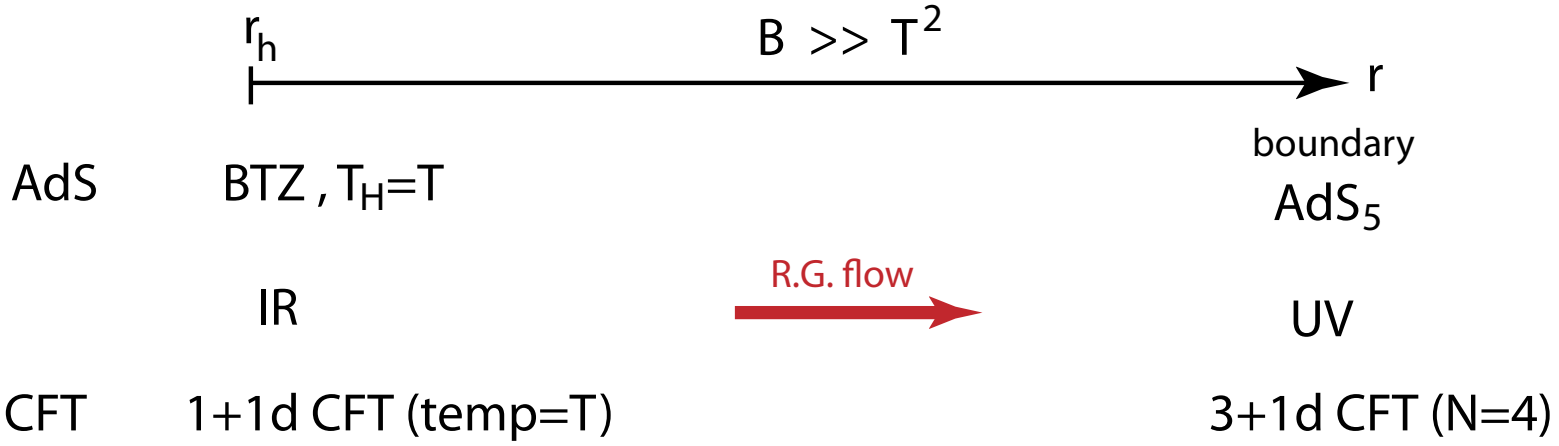

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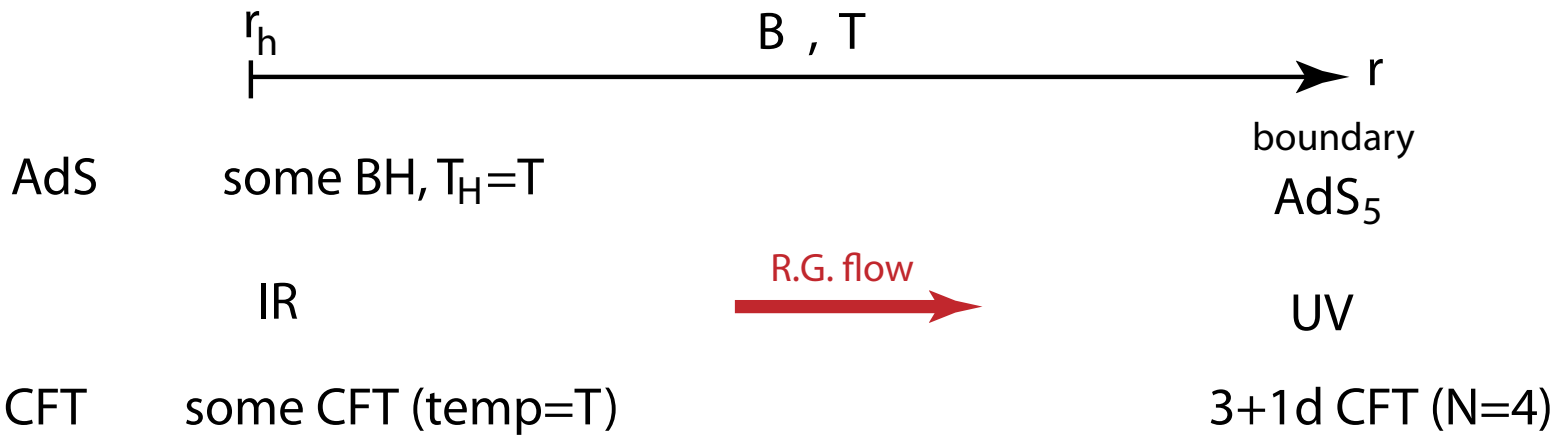
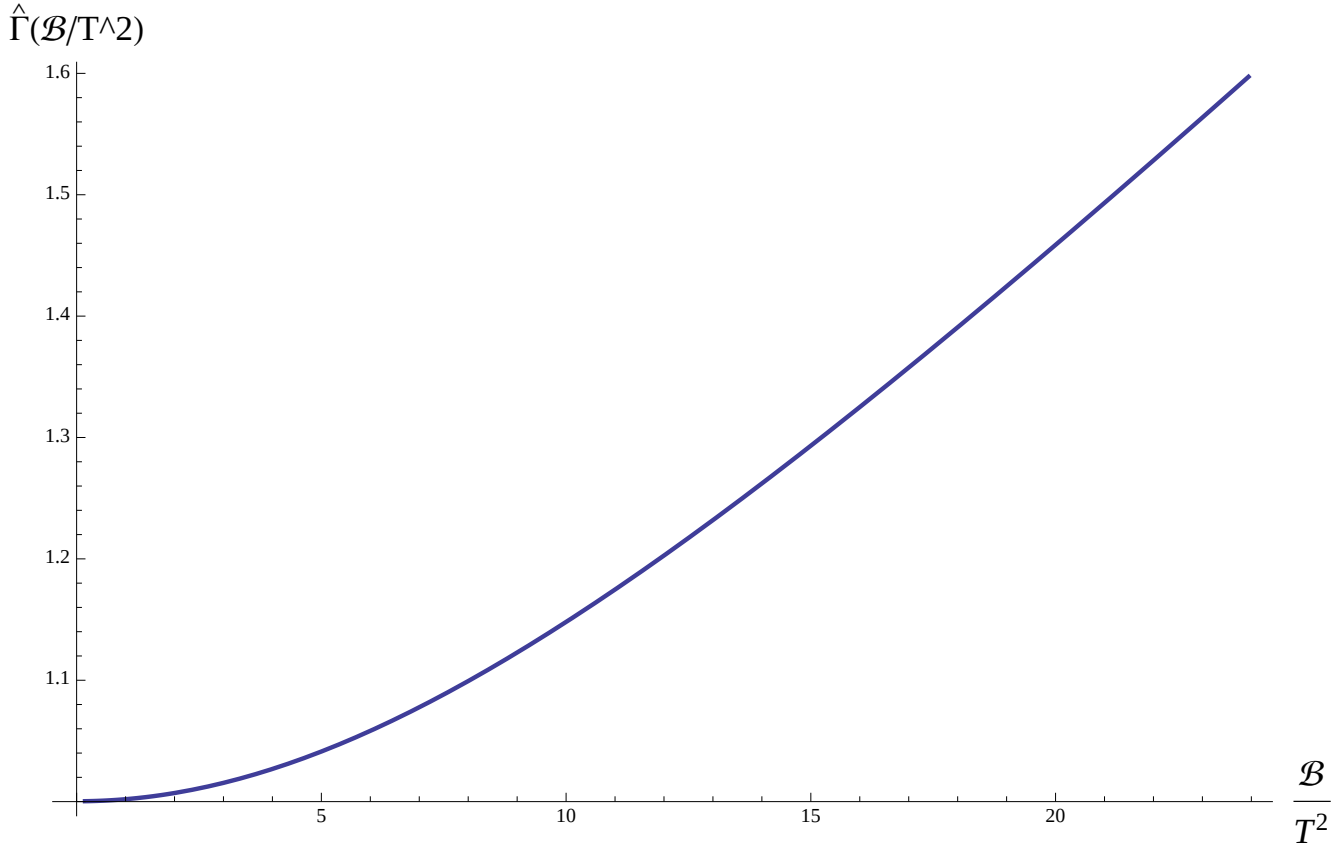


Sphaleron rate (holography)



$$\Gamma_{CS} = \begin{cases} \frac{(g^2 N)^2}{256\pi^3}\left(T^4 + \frac{1}{6\pi^4}\, B^2 + \mathcal{O}(\frac{B^4}{T^2})\right) & , \quad B << T^2 \\ \\ \frac{(g^2 N)^2}{384\sqrt{3}\pi^5}\left(B\, T^2 + 15.9\, T^4 + \mathcal{O}(\frac{T^6}{B})\right) & , \quad B >> T^2 \end{cases}$$

Sphaleron rate (holography)

$$B >> T$$

$$\Gamma_{CS} = \frac{(g^2 N)^2}{384\sqrt{3}\,\pi^4}\,\frac{B}{\pi} \times T^2$$

Sphaleron rate (holography)

$$B >> T$$

$$\Gamma_{CS} = \frac{(g^2 N)^2}{384\sqrt{3}\,\pi^4}\,\underbrace{\frac{B}{\pi}}_{\text{Landau level density}} \times T^2$$

- zero modes are magnetically confined in lowest Landau levels

Sphaleron rate (holography)

$$B >> T$$

$$\Gamma_{CS} = \frac{(g^2 N)^2}{384\sqrt{3}\,\pi^4}\,\underbrace{\frac{B}{\pi}}_{\text{Landau level density}} \times \underbrace{T^2}_{\text{diffusion scale in 1+1d}}$$

- zero modes are magnetically confined in lowest Landau levels
- strong interaction $\rightarrow$ back-reaction of B into the sphaleron
- spherical symmetry $\rightarrow$ axial symmetry

Conclusions & speculations

- Instanton + magnetic field has a rich structure
- Electric and magnetic dipole moments
- Zero modes play a crucial role
- 1st order derivative expansion captures some lattice results
- Confinement ? (instantons with nonzero holonomy)
- At strong coupling:
  - Magnetic field always increases the sphaleron rate
  - Back-reaction of magnetic field into non-abelian sector
  - Strong magnetic field leads to dimensional reduction
- Weak coupling? (Diamagnetic response of zero modes?)

Anomaly induced charge dependence of the elliptic flow in heavy ion collisions

Yannis Burnier

Bern University

Work done in collaboration with
D. E. Kharzeev, J. Liao, H. -U. Yee

Published in:
[YB, D. E. Kharzeev, J. Liao, H. -U. Yee, Phys. Rev. Lett. **107** (2011) 052303.]

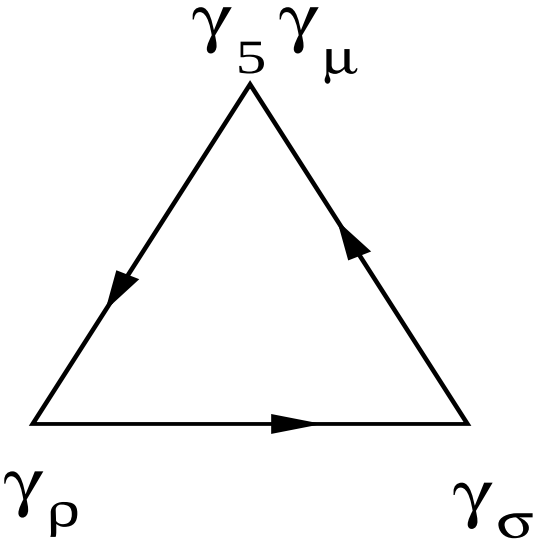
June 24, 2012

Outline

1. Introduction
   - Anomalies in the QCD plasma
   - Chiral magnetic effect
   - Chiral separation effect
2. Charge dependence of the elliptic flow
   - Chiral magnetic wave
   - Evolution of the wave
   - Estimate and comparison with experiment
3. Outlook

1. Introduction
Anomalies in the QCD plasma

Triangle diagrams ⇒
axial current non-conservation



For massless quarks:

- QED chiral anomaly:

$$\partial_\mu j_A^\mu = -2N_f \frac{e^2}{32\pi^2} F^{\mu\nu}\tilde{F}^{\rho\sigma} \tag{1}$$

- QCD chiral anomaly:

$$\partial_\mu j_A^\mu = -2N_f \underbrace{\frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a}_{Q} \tag{2}$$

Where $Q \neq 0$ only in topological transitions (sphalerons)

QED Axial anomaly in heavy ion collisions

In HIC, we have

- Large magnetic field perpendicular to the reaction plane.
- Charge and chirality fluctuations.

Anomalies lead to

- Chiral Separation Effect: At finite vector (electric, baryon number) chemical potential μ_V the chiral charge is separated along the axis of external magnetic field:

$$\vec{j}_A = \frac{N_c\ e}{2\pi^2}\mu_V \vec{B}, \tag{3}$$

- Chiral Magnetic Effect: At finite axial chemical potential μ_A, an external magnetic field induces the vector (electric) current:

$$\vec{j}_V = \frac{N_c\ e}{2\pi^2}\mu_A \vec{B}; \tag{4}$$

Chiral Magnetic Effect: Electric dipole in heavy ion collisions:

The chiral asymmetry from (QCD) sphalerons is transferred by the chiral (QED) anomaly in an electric charge separation along the magnetic field. [Kharzeev, McLerran, Warringa 0711.0950]

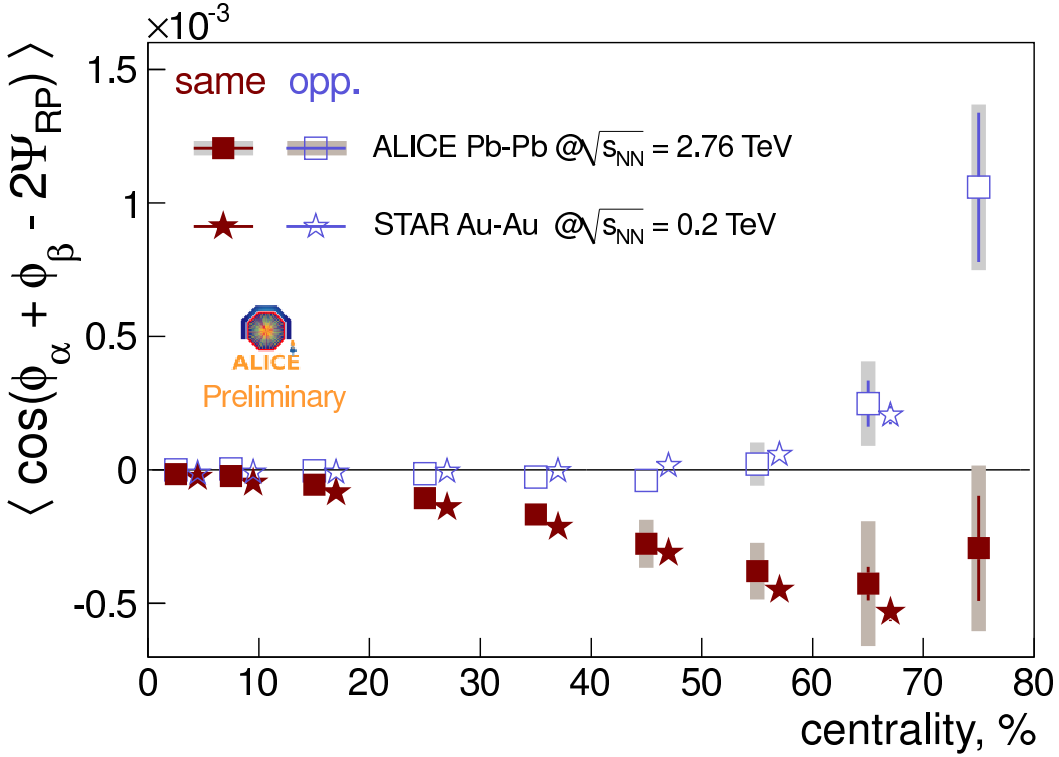
- Need both anomalies
- Formation of an electric dipole in the collision
- Event by event $\mathcal{P}$ violation

Can we see $\mathcal{P}$ violation in HIC?

- No $\mathcal{P}$ odd observable can be constructed
  - Sign of B not known.
  - Random sphaleron transitions
- Only charge correlations can be observed

Electric dipole: Experimental evidence?

Two particles correlations with respect to the reaction plane:



[Alice,1111.1875]

Correct order of magnitude and centrality dependence for CME

Chiral Separation Effect: Electric quadrupole in heavy ion collisions

Here the electric chemical potential induce an axial current

- Left-handed particles move along the magnetic field, Right-handed ones move opposite
- Additional + charges move towards the pole of the fireball
- Distribution of + charges is more elliptic

$\Rightarrow$ After hydro, elliptic flow of + charges is smaller

New physical effect!

- Need chiral anomaly in QED
- But no sphalerons
- No $\mathcal{P}$ violation

$\Rightarrow$ Definite sign

$\Rightarrow$ Present in all events

3. Charge dependence of the elliptic flow

The charges evolution can be represented by the Chiral Magnetic Wave:

Rewrite the anomaly formulas: [Kharzeev, Yee, 1012.6026]

$$\begin{pmatrix} \vec{j}_V \\ \vec{j}_A \end{pmatrix} = \frac{N_c\ e\vec{B}}{2\pi^2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \mu_V \\ \mu_A \end{pmatrix} \tag{5}$$

Dependencies of the chemical potential on the currents:

$$\begin{pmatrix} \mu_V \\ \mu_A \end{pmatrix} = \begin{pmatrix} \frac{\partial \mu_V}{\partial j_V^0} & \frac{\partial \mu_V}{\partial j_A^0} \\ \frac{\partial \mu_A}{\partial j_V^0} & \frac{\partial \mu_A}{\partial j_A^0} \end{pmatrix} \begin{pmatrix} j_V^0 \\ j_A^0 \end{pmatrix} + \mathcal{O}\left((j^0)^2\right) \equiv \begin{pmatrix} \alpha_{VV} & \alpha_{VA} \\ \alpha_{AV} & \alpha_{AA} \end{pmatrix} \begin{pmatrix} j_V^0 \\ j_A^0 \end{pmatrix} + \mathcal{O}\left((j^0)^2\right) \tag{6}$$

Remembering that: $\mu_i = \frac{\partial \mathcal{F}}{\partial j_i^0}$, $i = V, A$

where $\mathcal{F}$ is the Helmholtz free energy

α's: susceptibility matrices:

$$\alpha_{ij} = \frac{\partial^2 \mathcal{F}}{\partial j_i^0 \partial j_j^0} \tag{7}$$

By $\mathcal{P}$ transformation $V \to -V$ and $A \to A$, we have: $\alpha_{VA} = \alpha_{AV} = 0$.

Moreover one can show $\alpha_{VV} \sim \alpha_{AA} \equiv \alpha$.

Chiral magnetic wave

We get:

$$\begin{pmatrix} \vec{j}_V \\ \vec{j}_A \end{pmatrix} = \frac{N_c\ e\vec{B}\alpha}{2\pi^2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} j_V^0 \\ j_A^0 \end{pmatrix} \tag{8}$$

In the L,R basis:

$$\vec{j}_{L,R} = \mp \left(\frac{N_c\ e\vec{B}\alpha}{2\pi^2} \right) j_{L,R}^0 \tag{9}$$

Next: combine (9) with the conservation law $\partial_\mu j_{L,R}^\mu = 0$, add diffusion
For $\vec{B} = B\hat{x}^1$ we get:

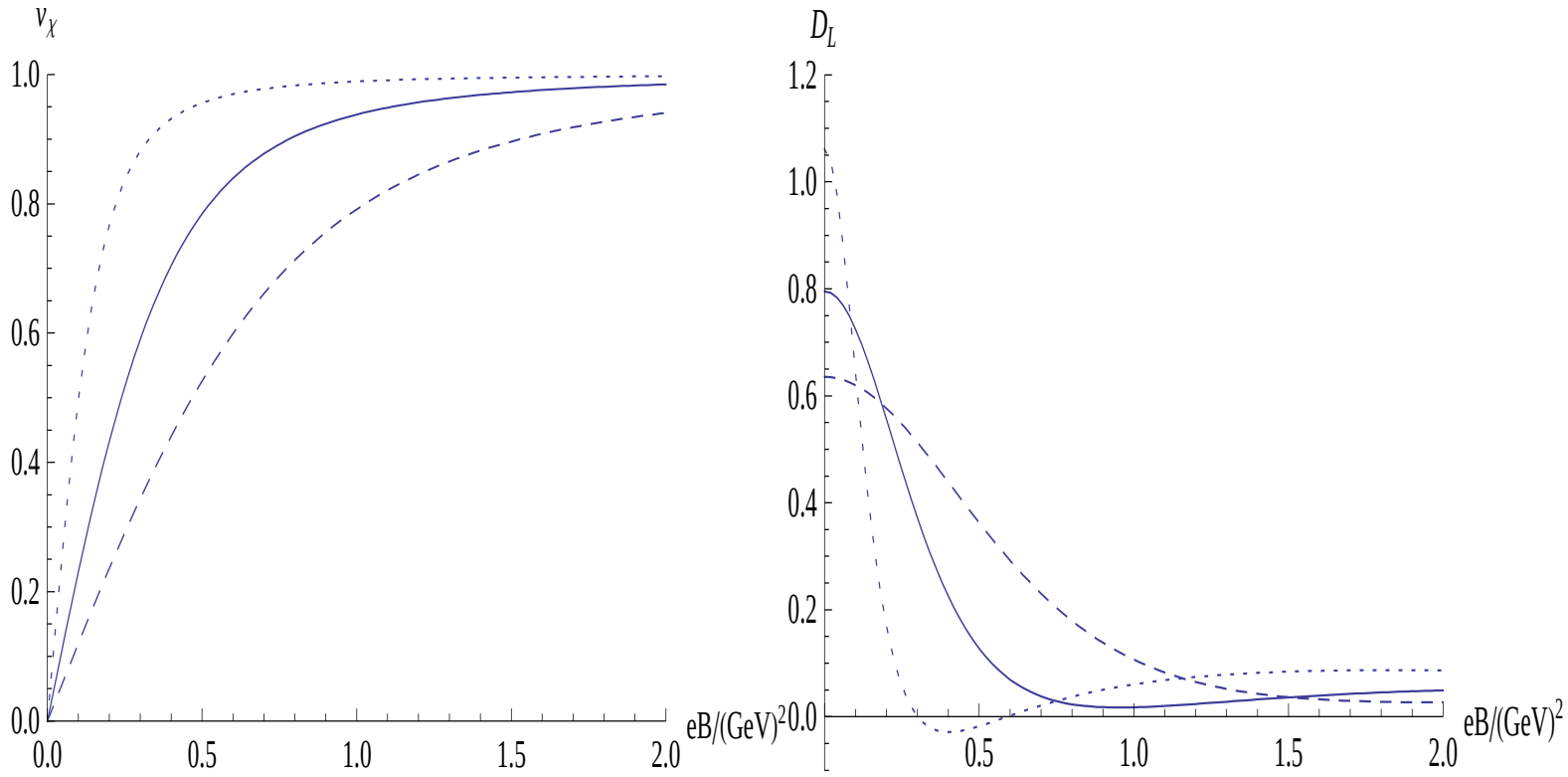
$$\left(\partial_0 \mp v\partial_1 - D_L\partial_1^2 - D_T\partial_T^2\right) j_{L,R}^0 = 0 \tag{10}$$

With $v = \frac{N_c e B\alpha}{2\pi^2}$ the velocity of the wave
Note $\alpha,\ D_L,\ D_T$ are B and T dependent and $v < c$

Parameters in the wave equation

We need now values for the functions $v(B,T), D_{L,T}(B,T)$

In [Kharzeev, Yee, 1012.6026] they were calculated in holographic QCD from the Sakai-Sugimoto model, which includes the triangle anomaly [Sakai, Sugimoto, 0412141] (T=150 (dots), 200 (full) 250 MeV (dashed)):



Evolution in the plasma: approximations

In principle: → The wave evolve in an expanding background

- Would require a full hydro simulation
- Require to write the CMW in an evolving metric

Approximation here: → The wave evolve in a fixed background, which is expanded afterwards

- Fine if the magnetic field has a short lifetime
- Hydro expansion moves the separated charges even further
- After part of the expansion, the path to separate the charges is longer

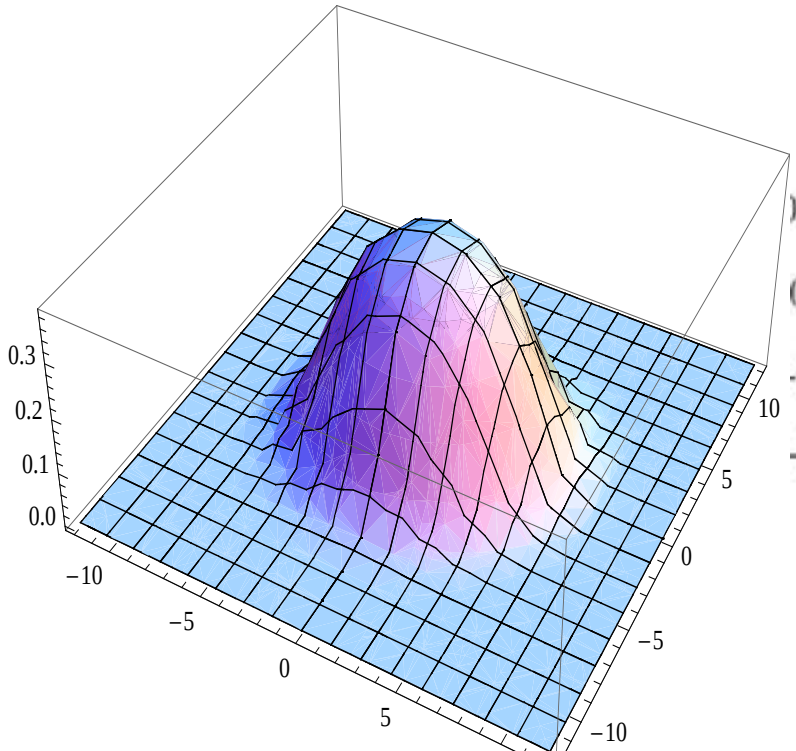
We solve the CMW starting from the almond shaped region with density given by the KLN model [Kharzeev, Levin, Nardi, 0111315].

Evolution of the wave

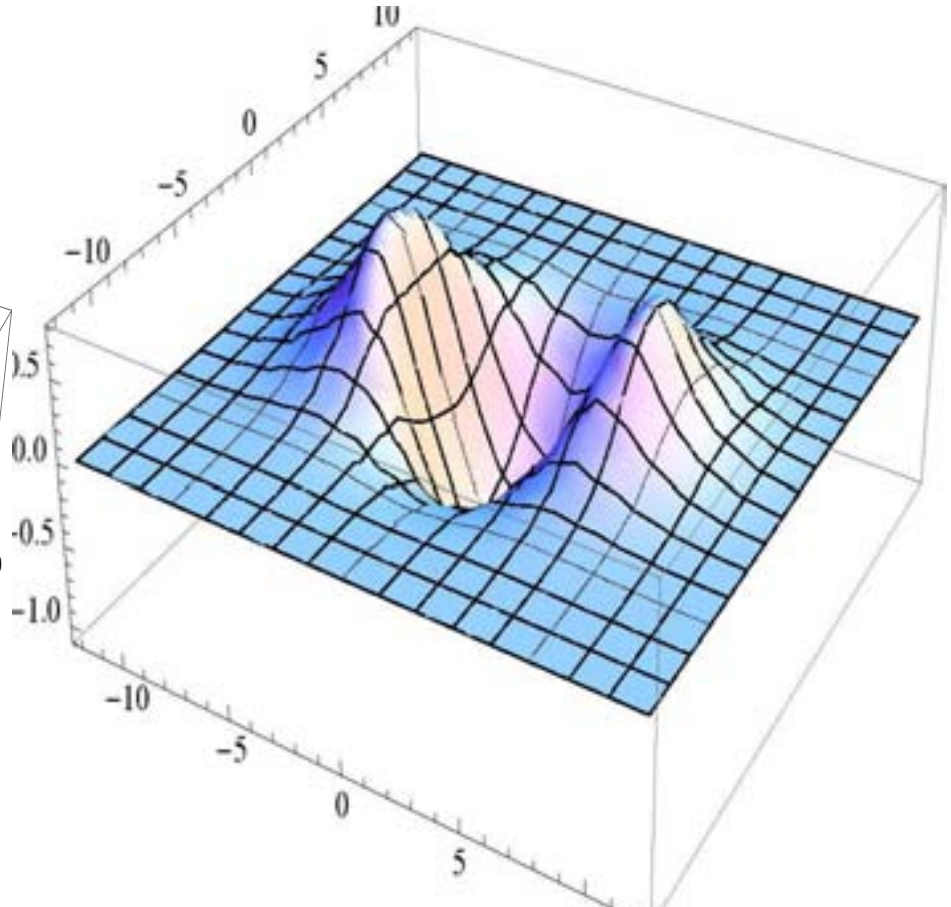
For all the different quark flavour/helicities we solve the corresponding CMW

The charges reaching the boundary of the plasma stops

Initial KLN density:



Evolution of the excess of charges:



Evolution after the plasma phase
How do the charges fly to the detector?

We consider a rapidity slice

Qualitatively:

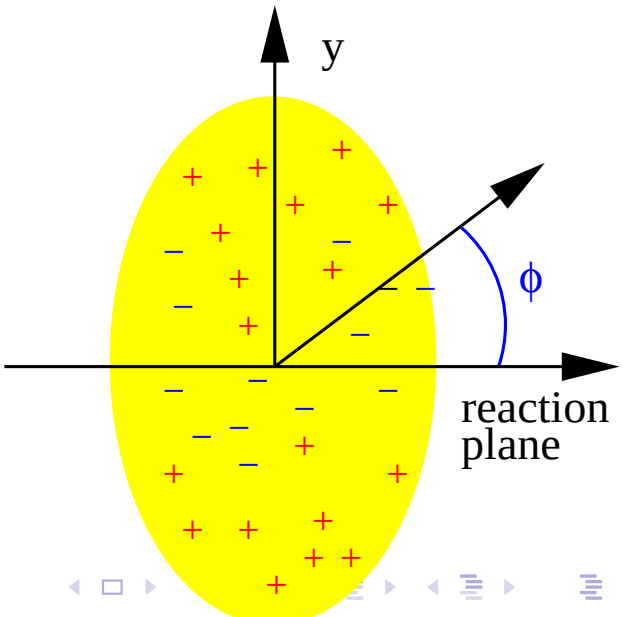
- The distribution of $+$ charges is more elliptic
- The relative - charges boosted more by the hydro flow

$\Rightarrow v_2^+ < v_2^-$.

For the charge asymmetry we approximate the flow to be radial

- Ellipticity is small except for large impact parameter
- We are not interested in v_2 but in the difference $v_2^- - v_2^+$

The strong flow aligns the particles momenta:
i.e. only their position matter



Charge dependent elliptic flow 1

The number of additional charges that will be measured at an angle ϕ are

$$\frac{d(N_+(\phi) - N_-(\phi))}{d\phi} = \int r\, dr\, j_e^0(r,\phi)$$

In the approximation of an elliptic distribution

$$\frac{d(N_+(\phi) - N_-(\phi))}{d\phi} = (\bar{N}_+ - \bar{N}_-)[1 + 2r_e \cos(2\phi)]$$

with

- $\bar{N}_\pm$ the total number of $\pm$ charged particles
- r_e is the ratio between the 2nd harmonic of the charge difference distribution to the total charge difference

$$r_e = \frac{q_e}{\rho_e}, \quad q_e = \int r\, dr\, d\phi\, \cos(2\phi)\left(j_e^0(r,\phi) - j_{e,B=0}^0(r,\phi)\right)$$

$$\rho_e = \int r\, dr\, d\phi\, j_e^0(r,\phi)$$

$\longleftarrow$ Obtained from the simulation of the CMW

Charge dependent elliptic flow 2

Supposing that the normal elliptic flow is not perturbed by the additional charges,

$$N_+ + N_- = (\bar{N}_+ + \bar{N}_-)[1 + 2v_2 \cos(2\phi)]$$

So that (to $\mathcal{O}(v_2 \times r\,A)$)

$$\frac{dN_\pm}{d\phi} = \bar{N}_\pm \langle [1 + 2(v_2 \mp r_e\,A)\cos(2\phi)\rangle]$$

Result: The elliptic flow is charge dependent! $v_2^\pm = v_2 \mp r_e\,A$

The difference is proportional to

- The second harmonic of the charge distribution r_e
- Charge difference $A = \frac{\bar{N}_+ - \bar{N}_-}{\bar{N}_+ + \bar{N}_-}$ in the rapidity slice

  $\longleftarrow$ At best: obtained from experiment, collision dependent
  - $\langle A \rangle$ small at high energy
  - A can be large in single events

What is the magnetic field in heavy ion collisions?

In a non-central collisions, the moving ions induce a magnetic field

- Perpendicular to the reaction plane
- Time dependent and impact parameter dependent

What happens to the magnetic field inside the plasma?

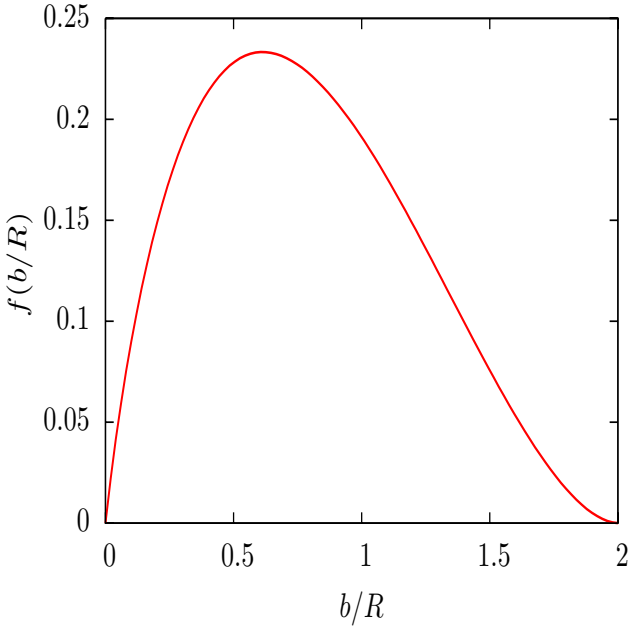
- It existed before the plasma was created
- Its evolution depends on the electrical properties of the plasma
- Electric conductivity σ not well known
- If $\sigma t,\ \sigma L \gg 1,\ \Rightarrow$ the system is described by magnetohydro:
  $\Rightarrow$ Magnetic flux conserved

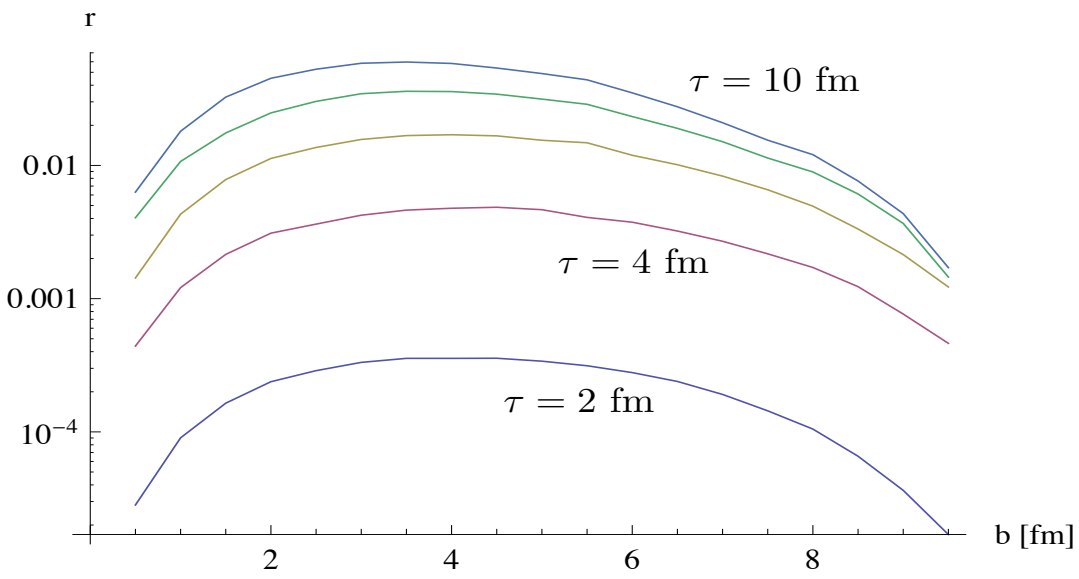
Done for this estimate:

- Use a constant magnetic field
- Evolution of CMW as function of its lifetime

Charge quadurpole as function of the magnetic field lifetime

- The duration of the magnetic field is considered as a parameter
- Originally we used $B_y(b)$ from [Kharzeev, McLerran, Warringa 0711.0950]
- Maximal magnitude $eB = m_\pi^2$ at $b \sim 3.5[fm]$
- At 200GeV collision energy $T = 165 MeV$ ($\sim$ freezout T)

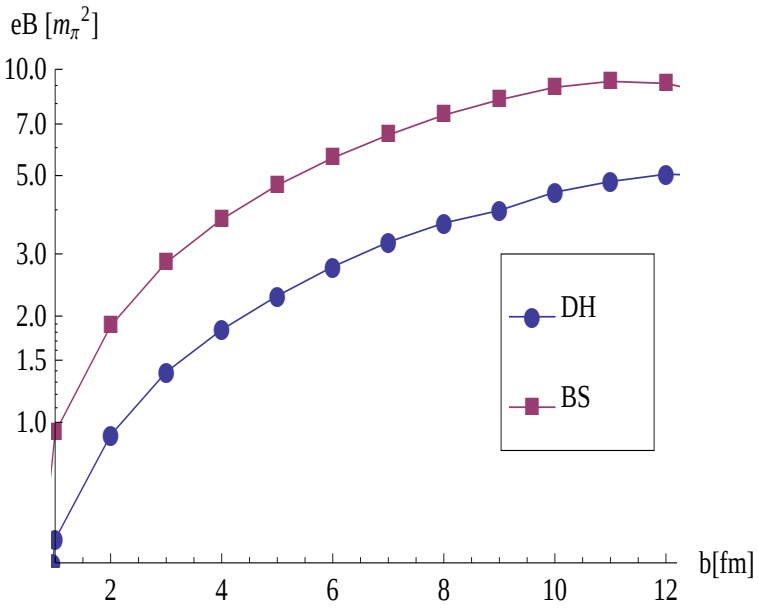


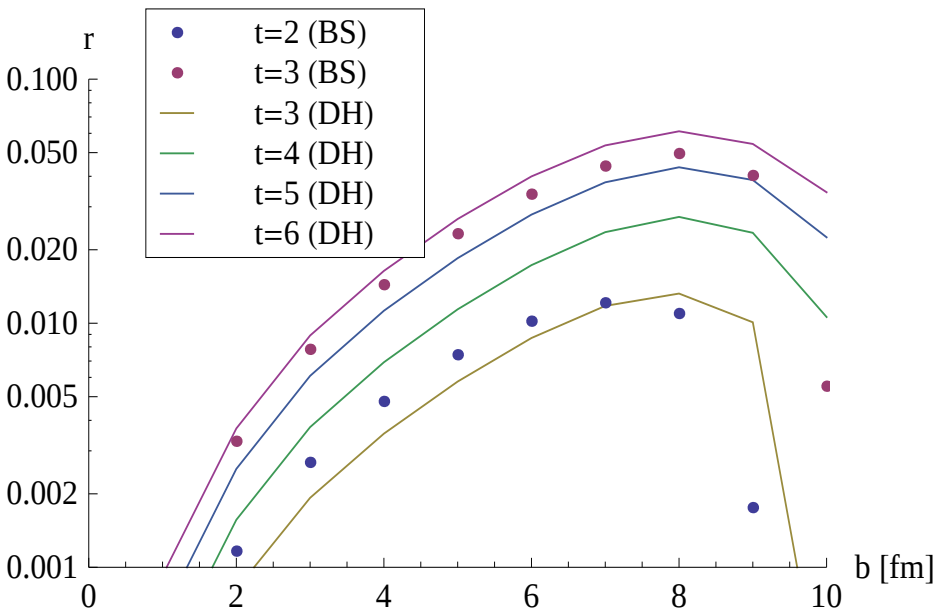


[YB, D. E. Kharzeev, J. Liao, H. -U. Yee 1103.1307]

Recently there were more precise calculation of the magnetic field

For instance [Bzdak, Skokov 1111.1949 (BS); Deng, Huang 1201.5108 (DH)]
They differ significantly from the previous estimate
$\Rightarrow$ New simulations for r_e:





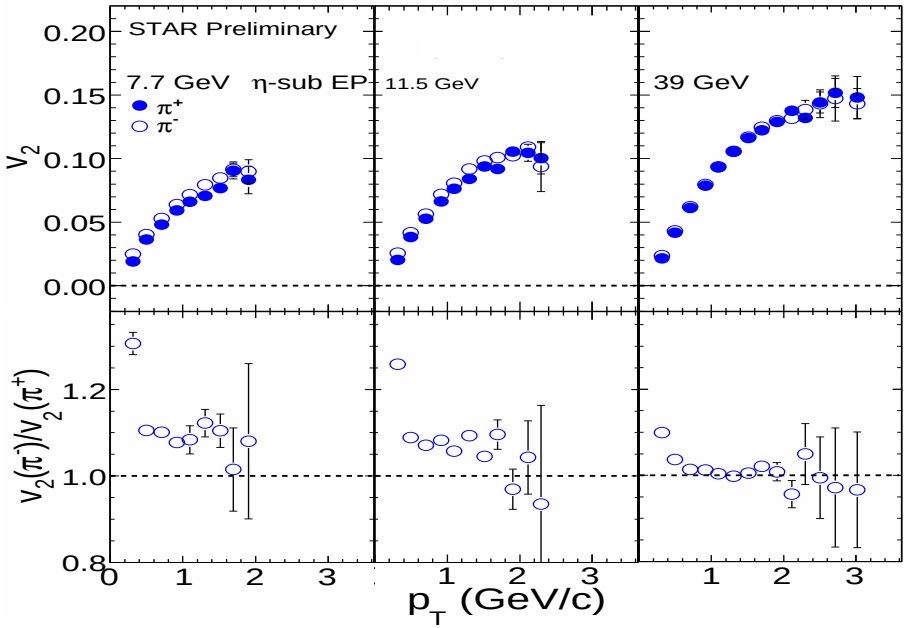
Comparison with experiment

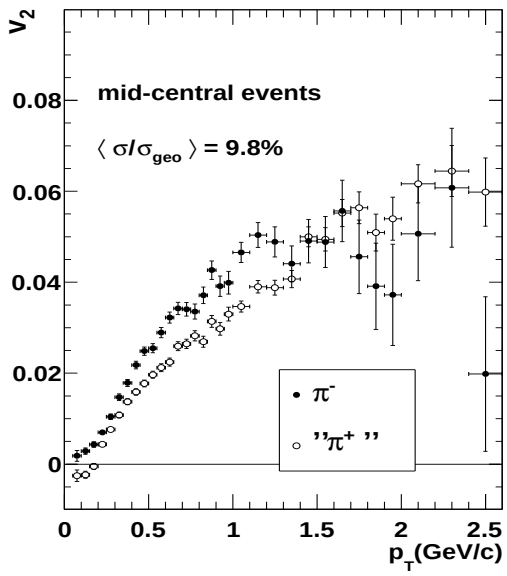
Positive and negative pions have similar absorption/scattering in the hadronic phase ⇒ so they represent the best channel

- Compare $v_2^- - v_2^+ = 2r_e A$

[STAR, 1106.5902] $v_2(p_T)$ for minimum bias (0-80% most central collisions)

Similar results at CERN NA-45 at 17.3GeV in [CERES 1205.3692]





- Compare r: See next talk!

4. Oulook: Is the effect seen due to the CSE?

Improve the estimate...

- Electric properties of the quark-gluon plasma
- Magnetic field during the collision
- Chiral magnetic wave in expanding medium
- Coefficient v_χ or α from QCD
- Diffusion coefficients from QCD (at least at $B = 0$)
- Full hydrodynamical simulation of the collision

Backgrounds and other effects

- Coulomb repulsion
- Decay of doubly charged particles
- Rescattering of the pions after the plasma phase
- What happens to Kaons, protons?

Spontaneous electromagnetic superconductivity of QCD×QED vacuum in (very) strong magnetic field

M. N. Chernodub

CNRS, University of Tours, France

Based on:

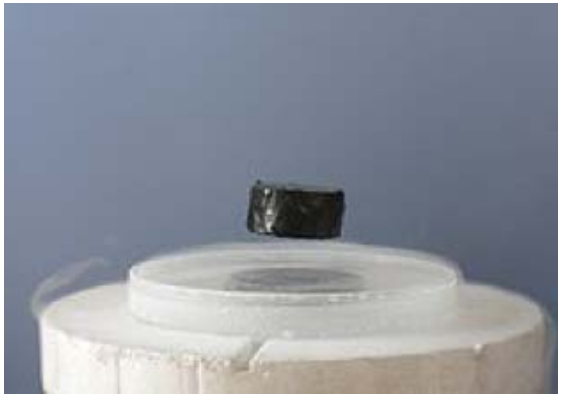
M.Ch., Phys. Rev. D 82, 085011 (2010) [arXiv:1008.1055]

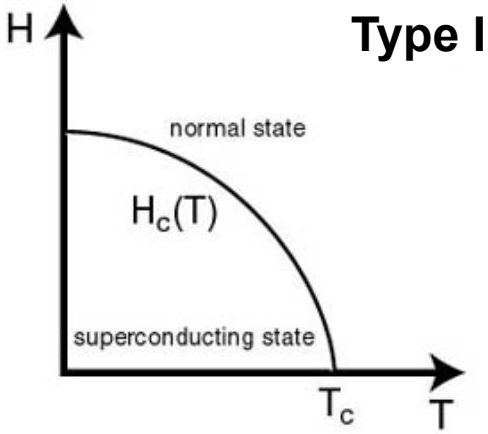
M.Ch., Phys. Rev. Lett. 106, 142003 (2011) [arXiv:1101.0117]

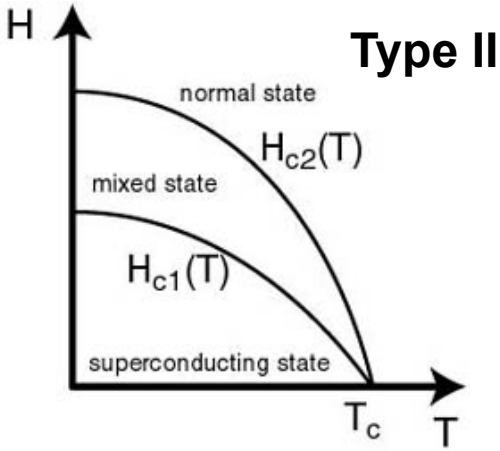
J. Van Doorsselaere, H. Verschelde, M.Ch., Phys. Rev. D 85, 045002 (2012) [arXiv:1111.4401]

+ arXiv:1104.3767 + arXiv:1104.4404 + ...

Superconductivity







I. Any superconductor has zero electrical DC resistance

II. Any superconductor is an enemy of the magnetic field:

1) weak magnetic fields are expelled by all superconductors (the Meissner effect)

2) strong enough magnetic field always kills superconductivity

Our claim:

In a background of strong enough magnetic field the <u>vacuum</u> becomes a superconductor.

The superconductivity emerges in empty space. Literally, "nothing becomes a superconductor".

The claim seemingly contradicts textbooks which state that:

1. Superconductor is a material (= a form of matter, not an empty space)
2. Weak magnetic fields are suppressed by superconductivity
3. Strong magnetic fields destroy superconductivity

General features

Some features of the superconducting state of vacuum:

1. spontaneously emerges <u>above</u> the critical magnetic field

$$B_c \simeq 10^{16}\ \text{Tesla} = 10^{20}\ \text{Gauss}$$

or

$$eB_c \simeq m_\rho^2 \simeq 31\, m_\pi^2 \simeq 0.6\ \text{GeV}^2$$

2...3 times stronger field ($eB \simeq$ 1...2 GeV^2) can be reached for short time in ultraperipheral Pb+Pb collisions at LHC at $\sqrt{s}$ =2.76TeV

A. Bzdak, V. Skokov, Phys.Lett.B (2012)

W. T. Deng, X. G. Huang, Phys. Rev. C (2012)

2. usual Meissner effect does not exist

3. perfect conductor (= zero DC resistance) in one spatial dimension (along the axis of the magnetic field)

4. insulator in perpendicular directions

1+4 approaches to the problem:

0. General arguments; (this talk)

1. Effective bosonic model for electrodynamics of ρ mesons based on vector meson dominance
   [M.Ch., PRD 2010; arXiv:1008.1055] (this talk)

2. Effective fermionic model (the Nambu-Jona-Lasinio model)
   [M.Ch., PRL 2011; arXiv:1101.0117] ~~(this talk)~~

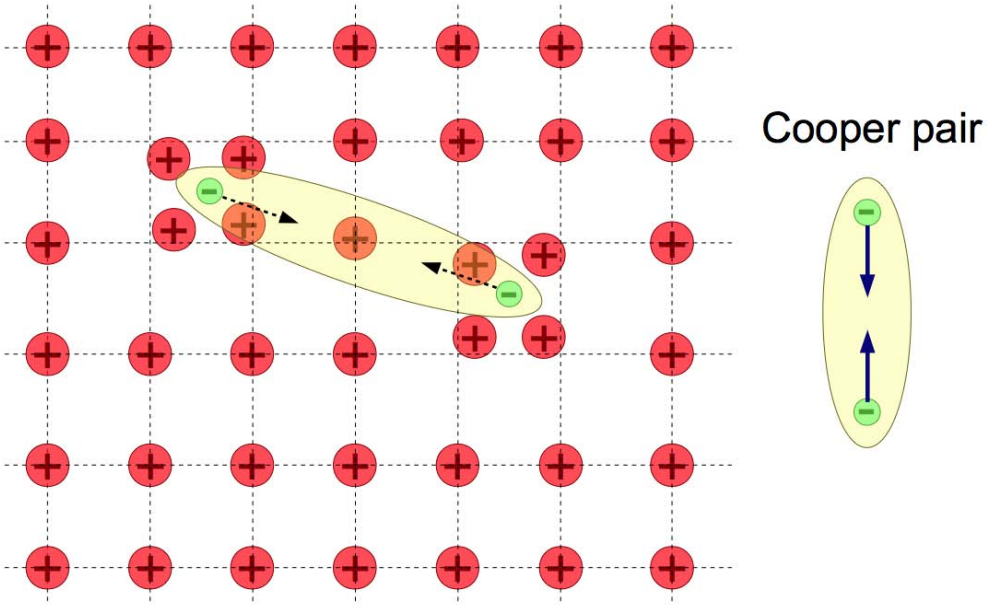
3. Nonperturbative effective models based on gauge/gravity duality (AdS/CFT)
   [Erdmenger, Kerner, Strydom (Munich, Germany), arXiv:1106.4551]
   [Callebaut, Dudal, Verschelde (Gent U., Belgium), arXiv:1105.2217];
   ~~(this talk)~~ → Talk by Johanna Erdmenger (24 hours 30 minutes ago)

5. First-principle numerical simulation of vacuum
   [ITEP Lattice Group, Moscow, Russia, arXiv:1104.3767] (this talk)

Conventional BCS superconductivity

1) The Cooper pair is the relevant degree of freedom!

2) The electrons are bounded into the Cooper pairs by the (attractive) phonon exchange.



Three basic ingredients:

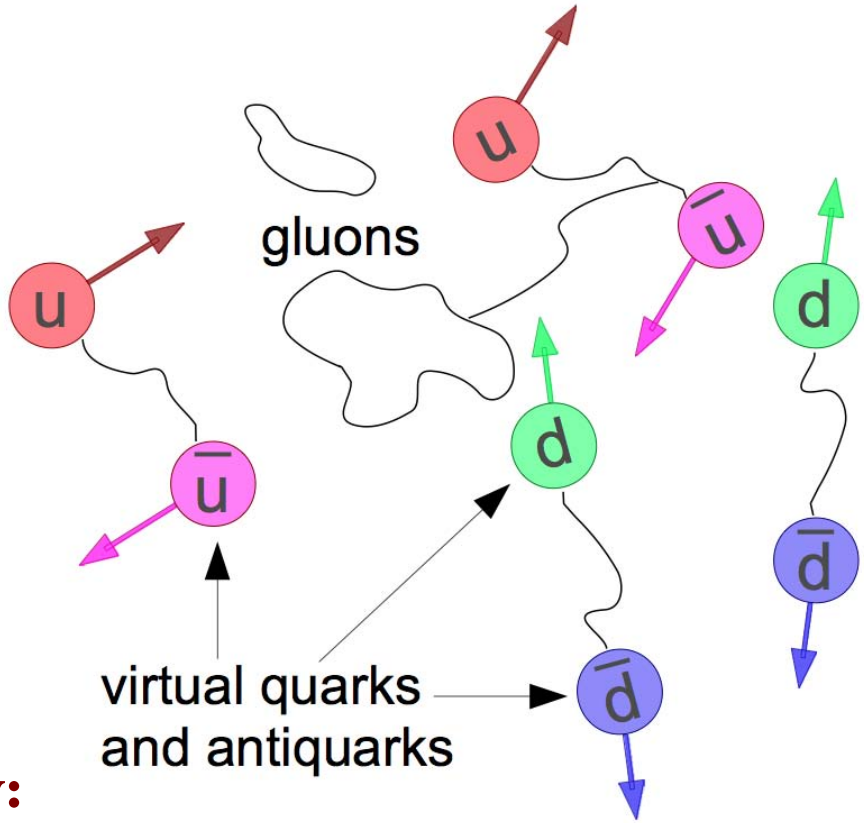
A) the presence of carriers of electric charge (of electric current);

B) the reduction of physics from (3+1) to (1+1) dimensions;

C) the attractive interaction between the like-charged particles.

Real vacuum, no magnetic field

1) Boiling soup of everything.
Virtual particles and antiparticles (electrons, positrons, photons, gluons, quarks, antiquarks ...) are created and annihilated every moment.



2) Net electric charge is zero.
An insulator, obviously.

3) We are interested in "strongly interacting" sector of the theory:

a) quarks and antiquarks,

i) *u* quark has electric charge $q_u = +2\,e/3$

ii) *d* quark has electric charge $q_d = -\,e/3$

b) gluons (an analogue of photons, no electric charge) "glue" quarks into bounds states, "hadrons" (neutrons, protons, etc).

The vacuum in strong magnetic field

Ingredients needed for possible superconductivity:

A. **Presence of electric charges?**

Yes, we have them: there are virtual particles which may potentially become "real" (= pop up from the vacuum) and make the vacuum (super)conducting.

B. **Reduction to 1+1 dimensions?**

Yes, we have this phenomenon: in a very strong magnetic field the dynamics of electrically charged particles (quarks, in our case) becomes effectively one-dimensional, because the particles tend to move along the magnetic field only.

C. **Attractive interaction between the like-charged particles?**

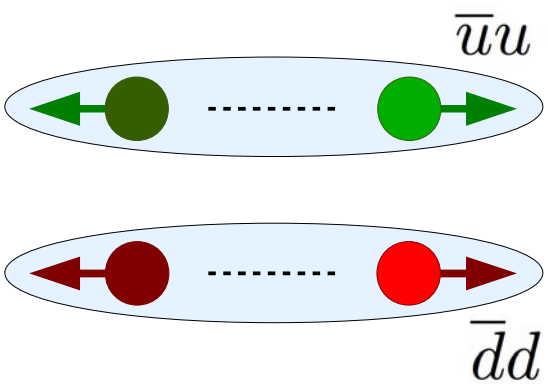
Yes, we have it: the gluons provide attractive interaction between the quarks and antiquarks (q_u=+2 $e/3$ and $q_{\bar{d}}$=+$e/3$)

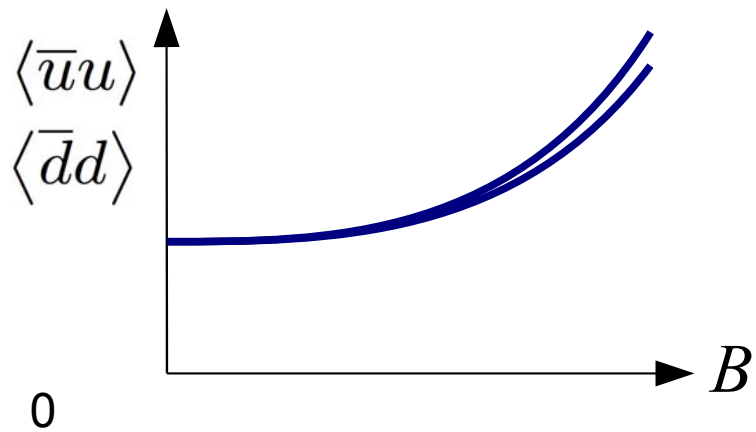
Pairing of quarks in strong magnetic field

Well-known "magnetic catalysis":

V. P. Gusynin, V. A. Miransky, I. A. Shovkovy ('94, '95, '96,...)

attractive channel: spin-0 flavor-diagonal states

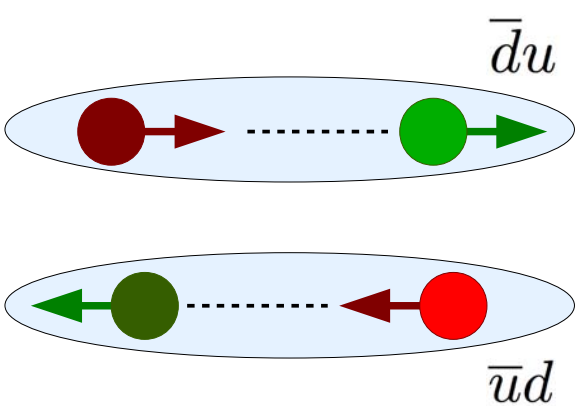


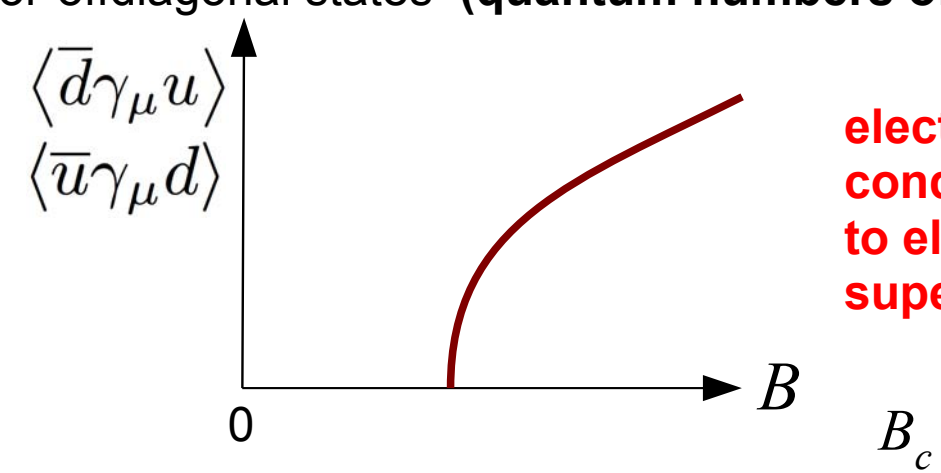


enhances chiral symmetry breaking

This talk:

attractive channel: spin-1 flavor-offdiagonal states **(quantum numbers of $\rho^{\pm}$ mesons)**





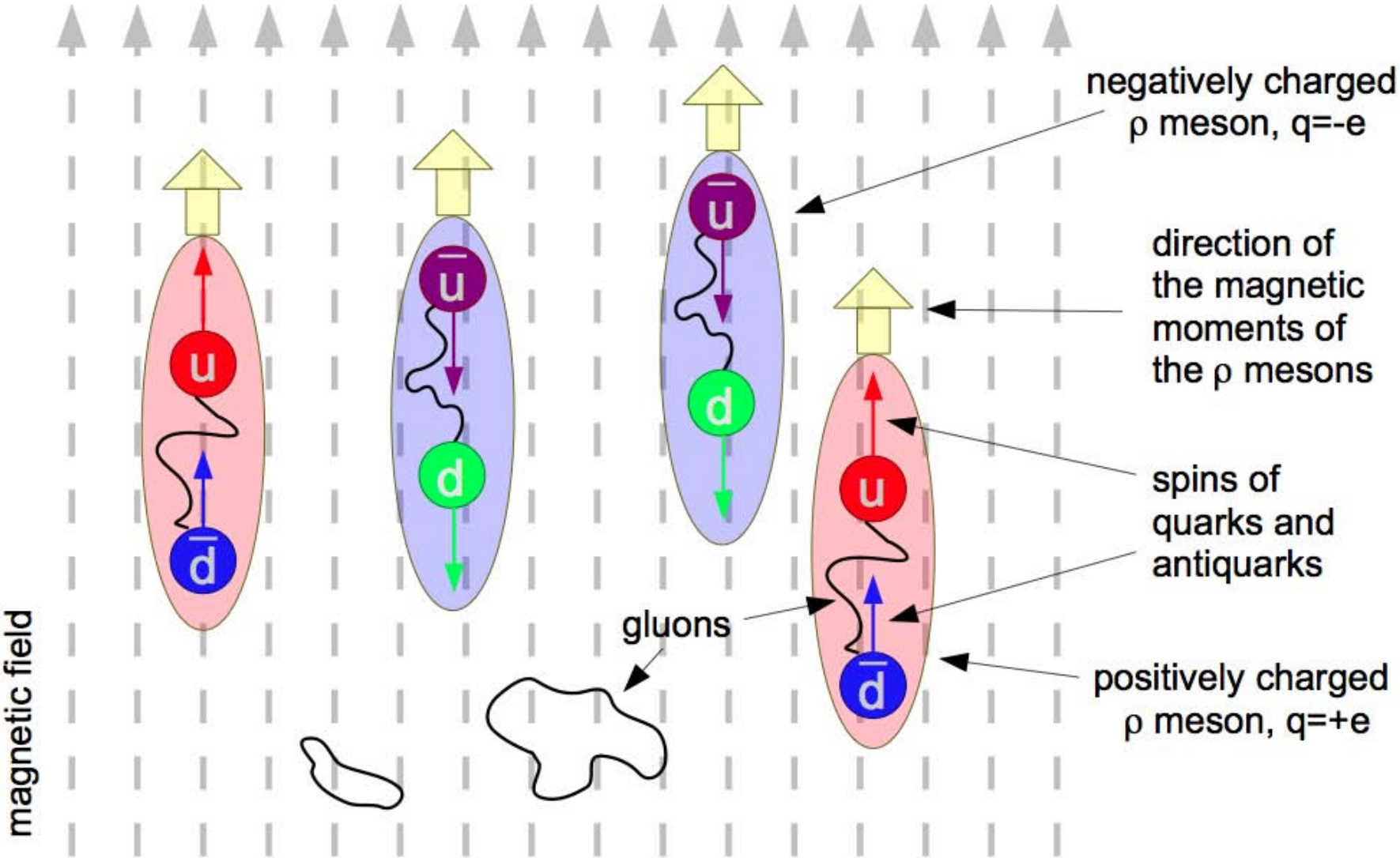
electrically charged condensates: leads to electromagnetic superconductivity

Key players: ρ mesons and vacuum

- ρ mesons:
  - electrically charged ($q = \pm e$) and neutral ($q=0$) particles
  - spin: s=1, vector particles
  - quark contents: $\rho^+ = u\bar{d}$, $\rho^- = d\bar{u}$, $\rho^0 = (u\bar{u}-d\bar{d})/2^{1/2}$
  - mass: m_ρ=775.5 MeV (approximately 1550 electron masses)
  - lifetime: τ_ρ=1.35 fm/c (very short: size of the ρ meson is 0.5 fm)

- vacuum: QED+QCD, zero tempertature and density

Strong magnetic field, picture



Naïve qualitative argument: charged relativistic particle in magnetic field

- Energy of a relativistic particle in the external magnetic field B_{ext}:

$$\varepsilon^2_{n,s_z}(p_z) = p_z^2 + (2n - 2s_z + 1)eB_{\text{ext}} + m^2$$

momentum along the magnetic field axis

nonnegative integer number

projection of spin on the magnetic field axis

(the external magnetic field is directed along the z-axis)

- Masses of ρ mesons and pions in the external magnetic field

$$m^2_{\pi^\pm}(B_{\text{ext}}) = m^2_{\pi^\pm} + eB_{\text{ext}}$$ becomes heavier

$$m^2_{\rho^\pm}(B_{\text{ext}}) = m^2_{\rho^\pm} - eB_{\text{ext}}$$ becomes lighter

$$\rho^\pm \to \pi^\pm \pi^0$$

- Masses of ρ mesons and pions:

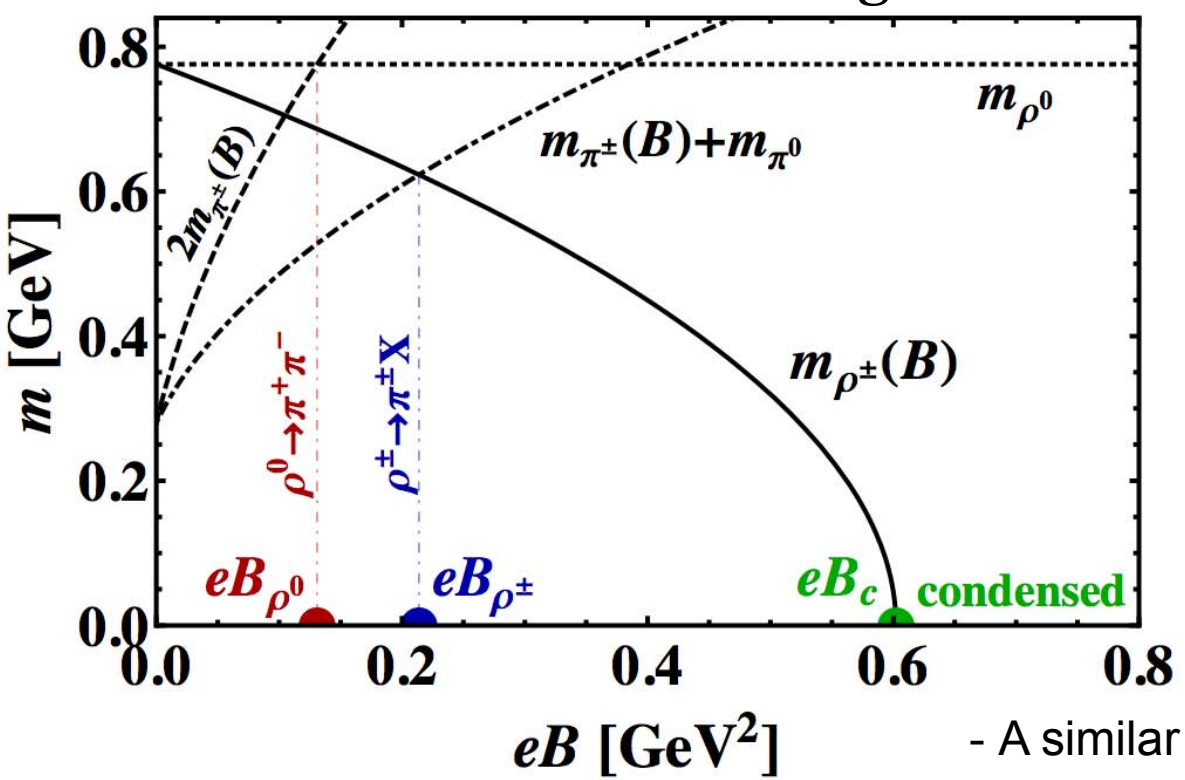
$$m_\pi = 139.6\,\text{MeV}\,, \qquad m_\rho = 775.5\,\text{MeV}$$

Condensation of ρ mesons

The $\rho^{\pm}$ mesons become massless and condense at the critical value of the external magnetic field

$$B_c = \frac{m_\rho^2}{e} \approx 10^{16}\,\text{Tesla}$$

masses in the external magnetic field



Kinematical impossibility of dominant decay modes

The pion becomes heavier while the rho meson becomes lighter

- The decay $\rho^\pm \to \pi^\pm \pi^0$ stops at certain value of the magnetic field

$$m_{\rho^\pm}(B_{\rho^\pm}) = m_{\pi^\pm}(B_{\rho^\pm}) + m_{\pi^0}$$

- A similar statement is true for $\rho^0 \to \pi^+\pi^-$

Quantitative approaches to the problem: bosonic or fermionic models

Superconductivity: conventional vs. vacuum (simplest models)

simplest models of superconductivity		
	conventional	vacuum
bosonic	Ginzburg–Landau model	ρ-meson electrodynamics vector dominance model
fermionic	Bardeen–Cooper– Schrieffer model	Nambu–Jona-Lasinio model with vector interactions

1. Effective bosonic model for electrodynamics of ρ mesons based on vector meson dominance
   [M.Ch., PRD 2010; arXiv:1008.1055]

2. Effective fermionic model (the Nambu-Jona-Lasinio model)
   [M.Ch., PRL 2011; arXiv:1101.0117]

Electrodynamics of ρ mesons

- Lagrangian (based on vector dominance models):

$$\mathcal{L} = -\frac{1}{4}\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\, \rho^\dagger_{\mu\nu}\rho^{\mu\nu} + m_\rho^2\, \rho^\dagger_\mu \rho^\mu$$
$$-\frac{1}{4}\, \rho^{(0)}_{\mu\nu}\rho^{(0)\mu\nu} + \frac{m_\rho^2}{2}\, \rho^{(0)}_\mu \rho^{(0)\mu} + \frac{e}{2g_s}\, F^{\mu\nu}\rho^{(0)}_{\mu\nu}$$

Nonminimal coupling leads to *g=2*

- Tensor quantities

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,,$$
$$f^{(0)}_{\mu\nu} = \partial_\mu \rho^{(0)}_\nu - \partial_\nu \rho^{(0)}_\mu\,,$$
$$\rho^{(0)}_{\mu\nu} = f^{(0)}_{\mu\nu} - i g_s(\rho^\dagger_\mu \rho_\nu - \rho_\mu \rho^\dagger_\nu)$$
$$\rho_{\mu\nu} = D_\mu \rho_\nu - D_\nu \rho_\mu\,,$$

- Gauge invariance

$$U(1): \quad \begin{cases} \rho^{(0)}_\mu(x) & \to \quad \rho^{(0)}_\mu(x)\,, \\ \rho_\mu(x) & \to \quad e^{i\omega(x)}\rho_\mu(x)\,, \\ A_\mu(x) & \to \quad A_\mu(x) + \partial_\mu \omega(x) \end{cases}$$

- Covariant derivative

$$D_\mu = \partial_\mu + i g_s \rho^{(0)}_\mu - ieA_\mu$$

- Kawarabayashi-Suzuki-Riadzuddin-Fayyazuddin relation

$$g_s \equiv g_{\rho\pi\pi} = \frac{m_\rho}{\sqrt{2} f_\pi} = 5.88$$
$$g_s \gg e \equiv \sqrt{4\pi\alpha_{\text{e.m.}}} \approx 0.303$$

[D. Djukanovic, M. R. Schindler, J. Gegelia, S. Scherer, PRL (2005)]

Homogeneous approximation

- Energy density:

$$\epsilon \equiv T_{00} = \frac{1}{2}F_{0i}^2 + \frac{1}{4}F_{ij}^2 + \frac{1}{2}\left(\rho_{0i}^{(0)}\right)^2 + \frac{1}{4}\left(\rho_{ij}^{(0)}\right)^2$$
$$+\frac{m_\rho^2}{2}\left[\left(\rho_0^{(0)}\right)^2 + \left(\rho_i^{(0)}\right)^2\right] + \rho_{0i}^\dagger\rho_{0i} + \frac{1}{2}\rho_{ij}^\dagger\rho_{ij}$$
$$+m_\rho^2\left(\rho_0^\dagger\rho_0 + \rho_i^\dagger\rho_i\right) - \frac{e}{g_s}F_{0i}\rho_{0i}^{(0)} - \frac{e}{2g_s}F_{ij}\rho_{ij}^{(0)}$$

- Disregard kinetic terms (for a moment) and apply B_{ext}:

$$\epsilon_0^{(2)}(\rho_\mu) = ieB_{\text{ext}}\left(\rho_1^\dagger\rho_2 - \rho_2^\dagger\rho_1\right) + m_\rho^2\rho_\mu^\dagger\rho_\mu$$
$$= \sum_{a,b=1}^{2}\rho_a^\dagger\mathcal{M}_{ab}\rho_b + m_\rho^2(\rho_0^\dagger\rho_0 + \rho_3^\dagger\rho_3)$$

mass matrix

$$\mathcal{M} = \begin{pmatrix} m_\rho^2 & ieB_{\text{ext}} \\ -ieB_{\text{ext}} & m_\rho^2 \end{pmatrix}$$

$$\vec{B} = (0, 0, B)$$

- Eigenvalues and eigenvectors of the mass matrix:

$$\mu_\pm^2 = m_\rho^2 \pm eB_{\text{ext}}\,, \qquad \rho_\pm = \frac{1}{\sqrt{2}}(\rho_1 \pm i\rho_2)$$

At the critical value of the magnetic field: imaginary mass (=condensation)!

Homogeneous approximation (II)

- The condensate of the rho mesons: $\rho_1 = -i\rho_2 = \rho$

- The energy of the condensed state:

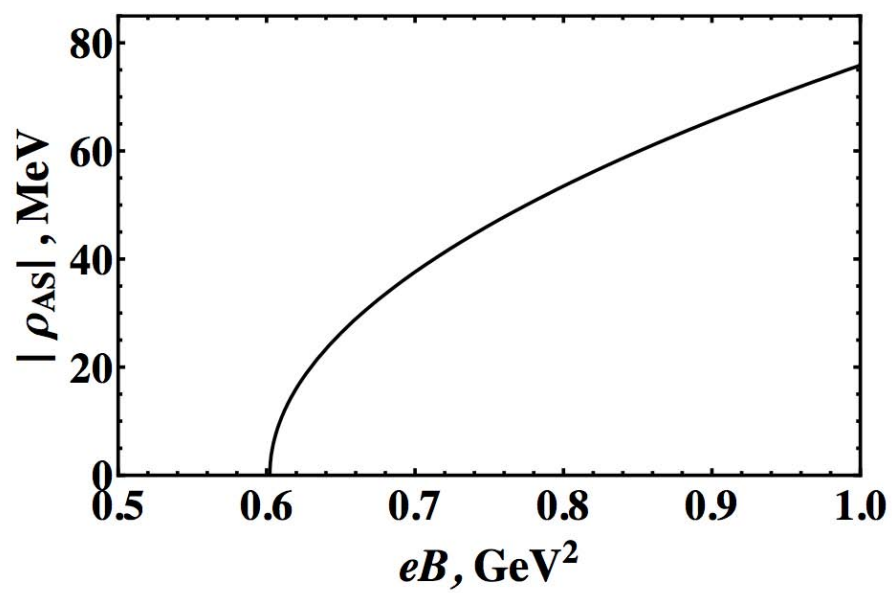
$$\epsilon_0(\rho) = \frac{1}{2}B_{\text{ext}}^2 + 2(m_\rho^2 - eB_{\text{ext}})\,|\rho|^2 + 2g_s^2\,|\rho|^4$$

(similar to a temperature-dependent Ginzburg-Landau potential for superconductivity!)

(qualitatively similar picture in the Nambu-Jona-Lasinio model)

- The absolute value of the condensate:

$$|\rho|_0 = \begin{cases} \sqrt{\frac{e(B_{\text{ext}}-B_c)}{2g_s^2}}, & B_{\text{ext}} \geqslant B_c \\ 0, & B_{\text{ext}} < B_c \end{cases}$$



Second order (quantum) phase transition, critical exponent = 1/2

Structure of the condensates

In terms of quarks, the state $\rho_1 = -i\rho_2 = \rho$ implies

$$\langle \bar{u}\gamma_1 d\rangle = \rho(x_\perp)\,, \qquad \langle \bar{u}\gamma_2 d\rangle = i\rho(x_\perp)$$

Depend on transverse coordinates only

(the same structure of the condensates in the Nambu-Jona-Lasinio model)

$$\vec{B} = (0, 0, B)$$

$U(1)_{\text{e.m.}}$: $\rho(x) \to e^{i\omega(x)}\rho(x)$ Abelian gauge symmetry

$O(2)_{\text{rot}}$: $\rho(x) \to e^{i\varphi}\rho(x)$ Rotations around B-axis

- The condensate "locks" rotations around field axis and gauge transformations:

$$U(1)_{\text{e.m.}} \times O(2)_{\text{rot}} \to U(1)_{\text{locked}}$$

(similar to "color-flavor locking" in color superconductors at high quark density)

Basic features of ρ meson condensation

- The condensate of the ρ mesons appears in a form of an inhomogeneous state, analogous to the Abrikosov lattice in the mixed state of type-II superconductors.

A similar state, the vortex state of W bosons, may appear in Electroweak model in the strong external magnetic field [Ambjorn, Olesen (1989)]

- The condensate forms a lattice, which is made of the new type of topological defects, the "ρ vortices".

- The emergence of the condensate of the charged $\rho^{\pm}$ mesons induces spontaneous condensation of the neutral ρ^0 mesons.

- The condensate of charged ρ mesons implies <u>superconductivity</u>.
- The condensate of neutral ρ mesons implies <u>superfluidity</u>.

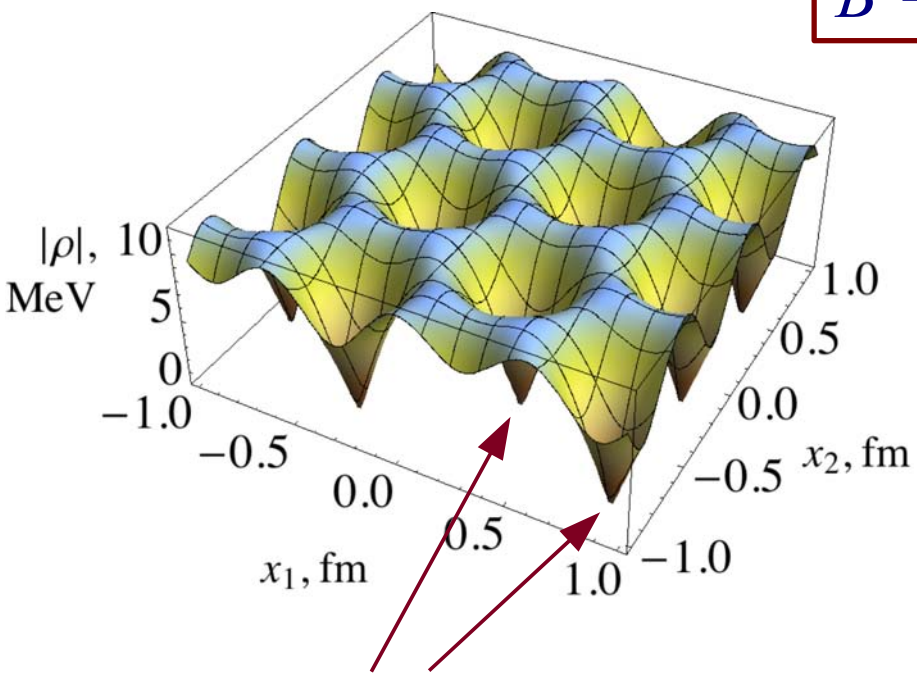
- Unusual optical properties of the superconducting state: It is a metamaterial ("perfect lens" optical phenomenon) with negative electrical permittivity (ε), negative magnetic permeability (μ), negative index of refraction (n) [Smolyaninov, PRL 107, 253903 (2011)].

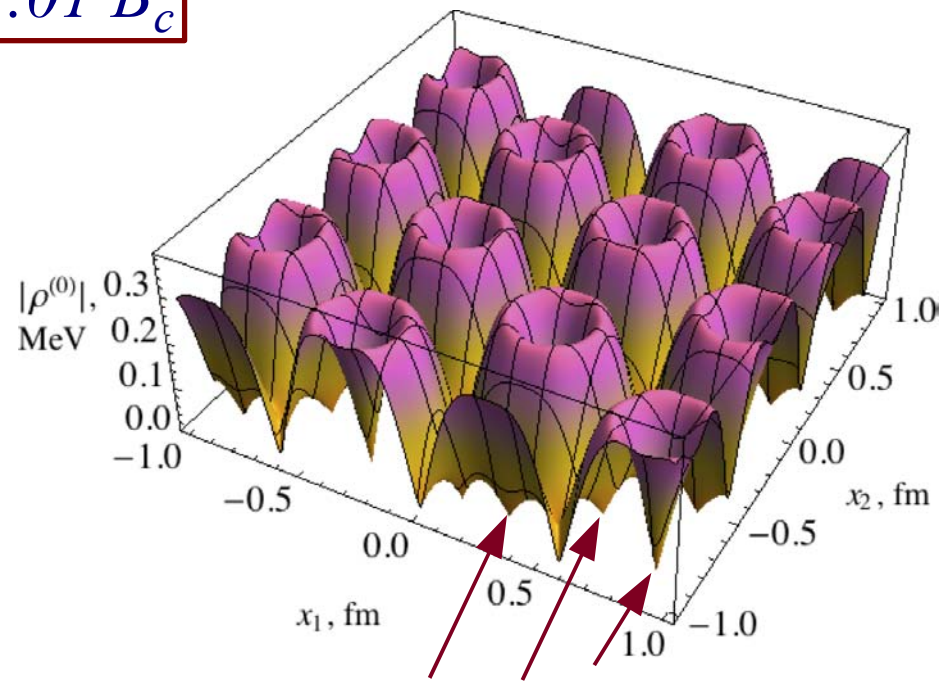
Condensates of ρ mesons, solutions

Superconducting condensate
(charged rho mesons)

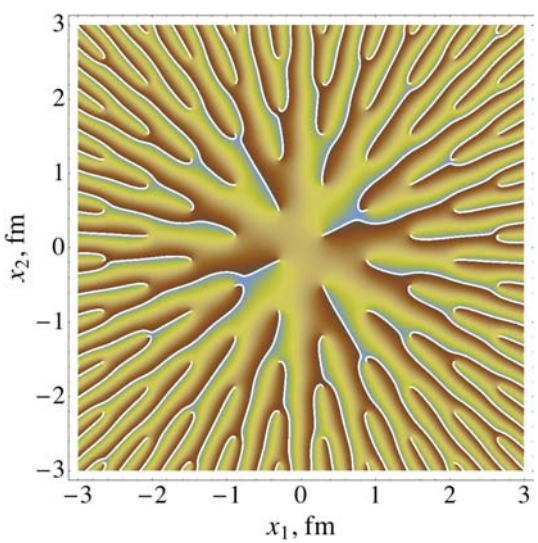
$B = 1.01\ B_c$

Superfluid condensate
(neutral rho mesons)



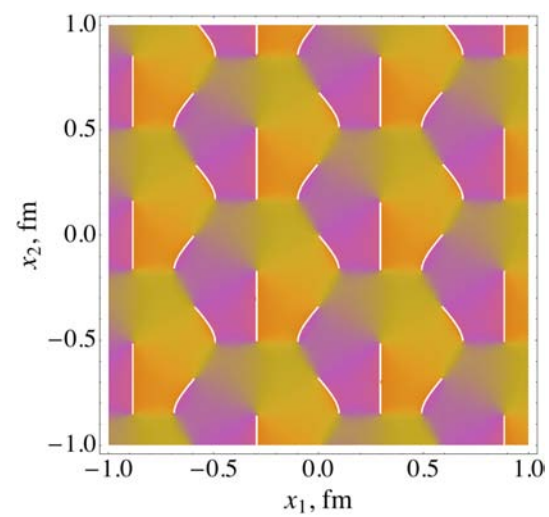


New objects, topological vortices, made of the rho-condensates

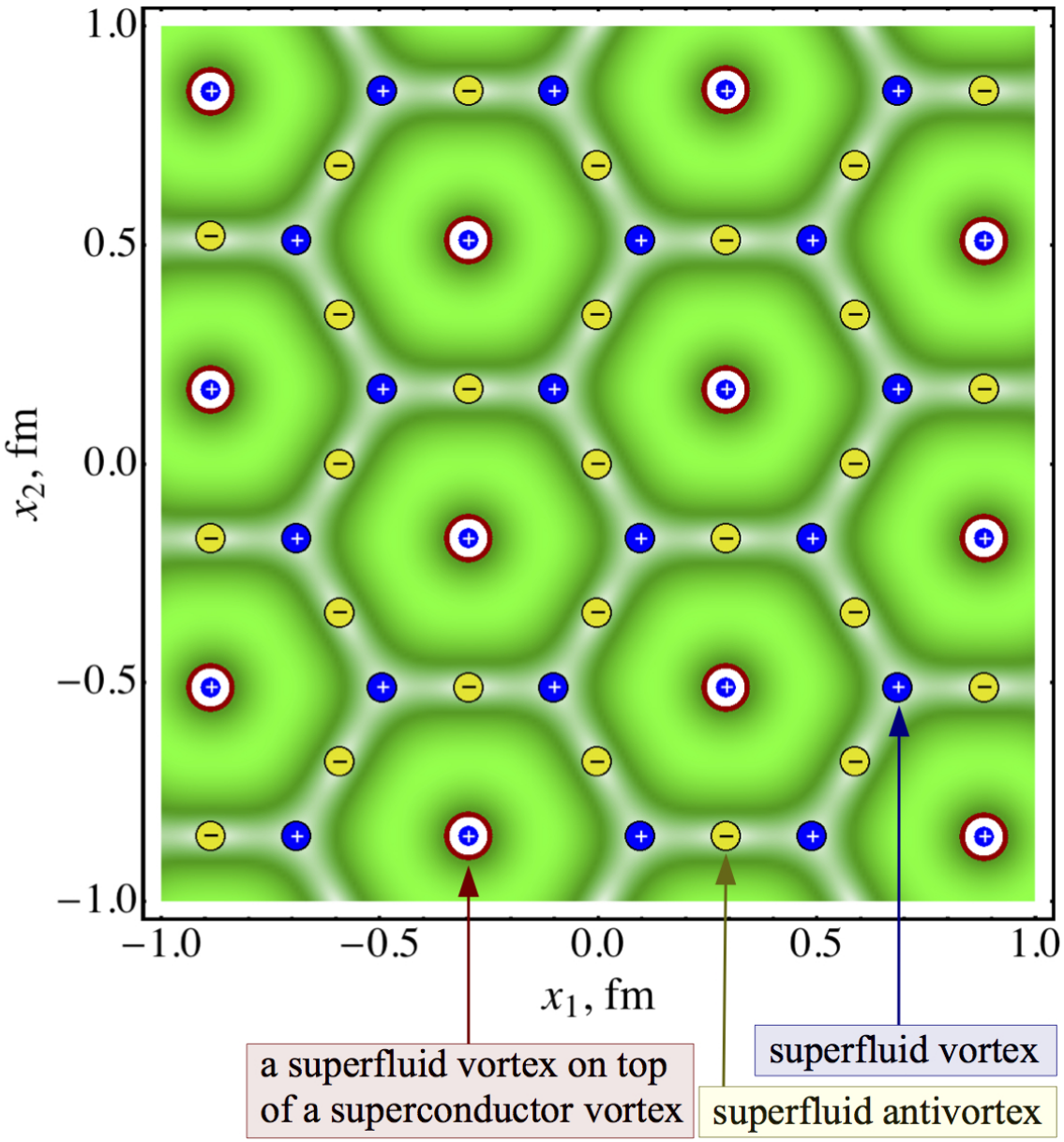


The phases of the rho-meson fields wind around vortex centers, at which the condensates vanish.

similar results in NJL and holographic approaches (J. Erdmenger)



Topological structure of the ρ mesons condensates



Anisotropic superconductivity
(via an analogue of the London equations)

- Apply a weak electric field E to an ordinary superconductor
- Then one gets accelerating electric current along the electric field:

$$\frac{\partial \vec{J}_{\rm GL}}{\partial t} = m_A^2 \vec{E} \qquad \text{[London equation]}$$

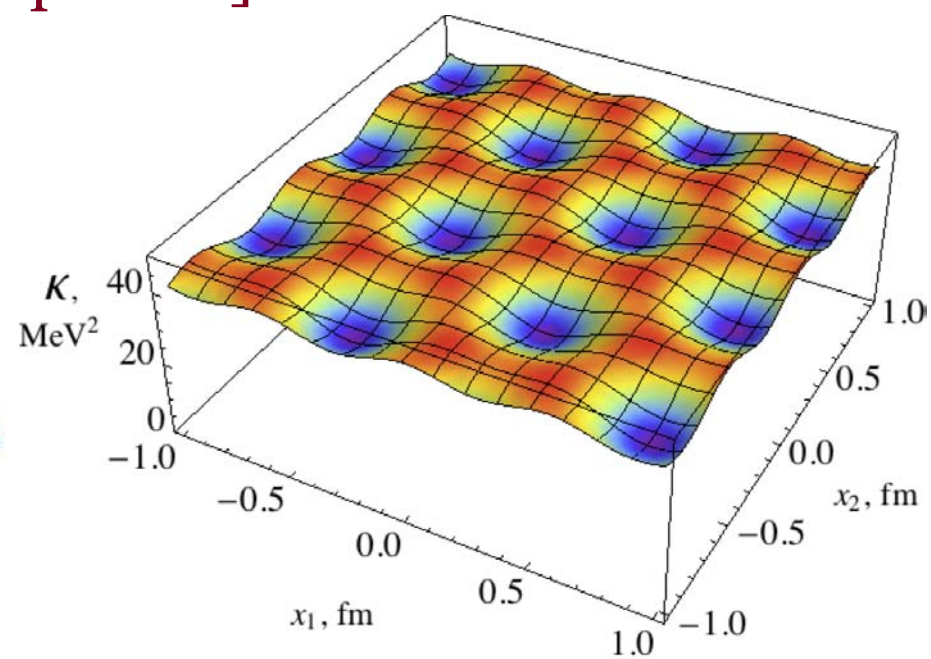
- In the QCDxQED vacuum, we get an accelerating electric current along the magnetic field $\boldsymbol{B}$:

$$\frac{\partial}{\partial t}\langle J_3 \rangle = -\frac{2e^3}{g_s^2}(B_{\rm ext} - B_c)E_3$$

$$\frac{\partial}{\partial t}\langle J_1 \rangle = \frac{\partial}{\partial t}\langle J_2 \rangle = 0$$

(for $B \geqslant B_c$)

Written for an electric current averaged over one elementary (unit) rho-vortex cell



(similar results in NJL)

Anisotropic superconductivity
(Lorentz-covariant form of the London equations)

We are working in the vacuum, thus the transport equations may be rewritten in a Lorentz-covariant form:

Electric current averaged over one elementary rho-vortex cell

$$\partial_{[\mu,} j_{\nu]} = \kappa \frac{(F \cdot \widetilde{F})}{(F \cdot F)} \widetilde{F}_{\mu\nu}$$

A scalar function of Lorentz invariants.
In this particular model:

$$\kappa = (e^3/g_s^2)(\sqrt{(F \cdot F)/2} - B_c)$$

(slightly different form of κ function in NJL)

Lorentz invariants:

$$(F \cdot \widetilde{F}) = F^{\mu\nu}\widetilde{F}_{\mu\nu} \equiv 4(\vec{B} \cdot \vec{E})$$

$$(F \cdot F) = F^{\mu\nu}F_{\mu\nu} \equiv 2(\vec{B}^2 - \vec{E}^2)$$

$$\widetilde{F}_{\mu\nu} = \tfrac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$$

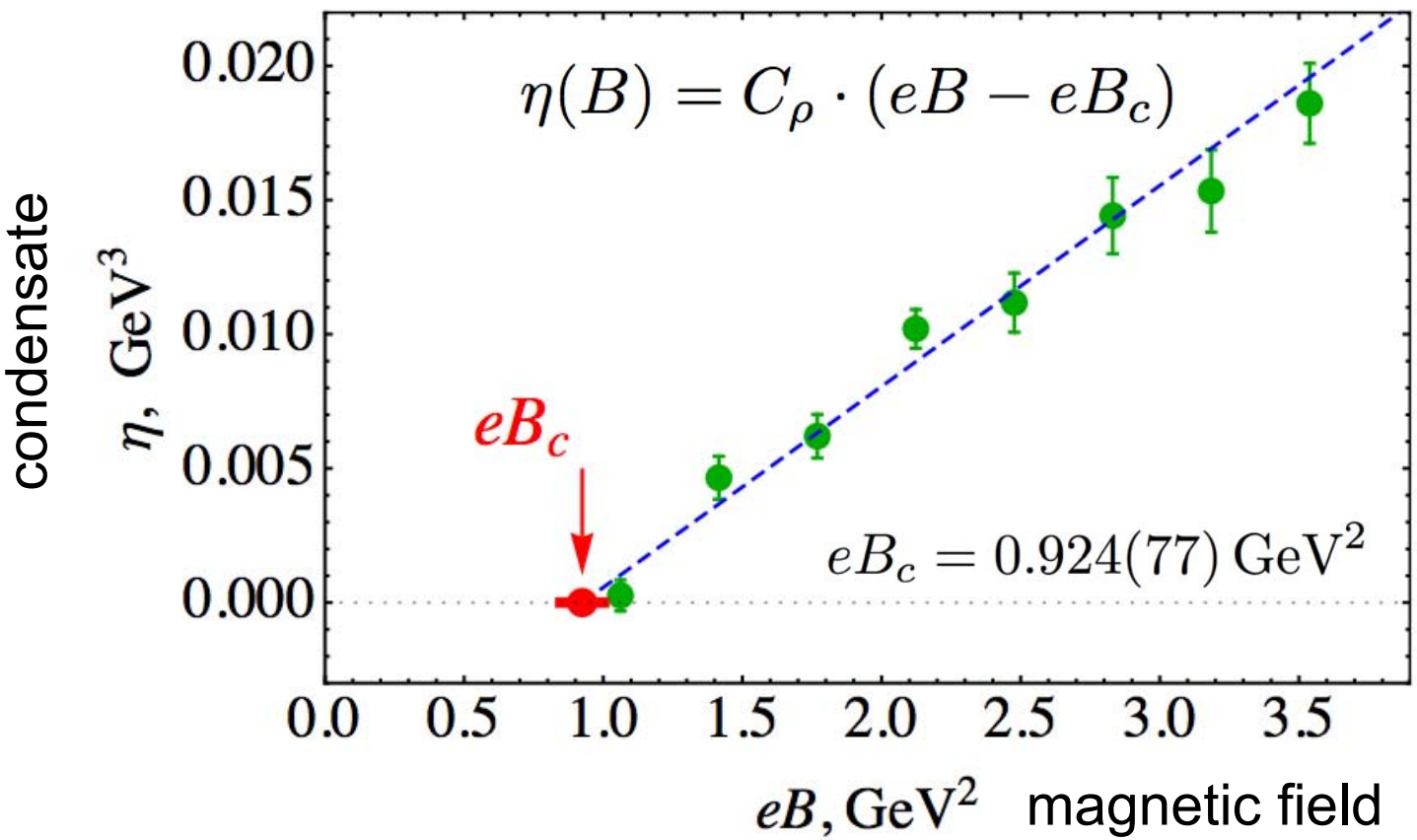
If B is along x_3 axis, then we come back to

$$\frac{\partial}{\partial t}\langle J_3 \rangle = -\frac{2e^3}{g_s^2}(B_{\text{ext}} - B_c)E_3 \qquad \text{and} \qquad \frac{\partial}{\partial t}\langle J_1 \rangle = \frac{\partial}{\partial t}\langle J_2 \rangle = 0$$

Numerical simulations of vacuum in the magnetic field background

V.Braguta, P. Buividovich, A. Kotov, M. Polikarpov, M.Ch., arXiv:1104.3767

Numerical simulation: $B_c = (1.56 \pm 0.13) \cdot 10^{16}\,\mathrm{Tesla}$



Theory:

$$B_c = \frac{m_\rho^2}{e} \approx 10^{16}\,\mathrm{Tesla}$$

$$\eta \sim \sqrt{B - B_c}$$

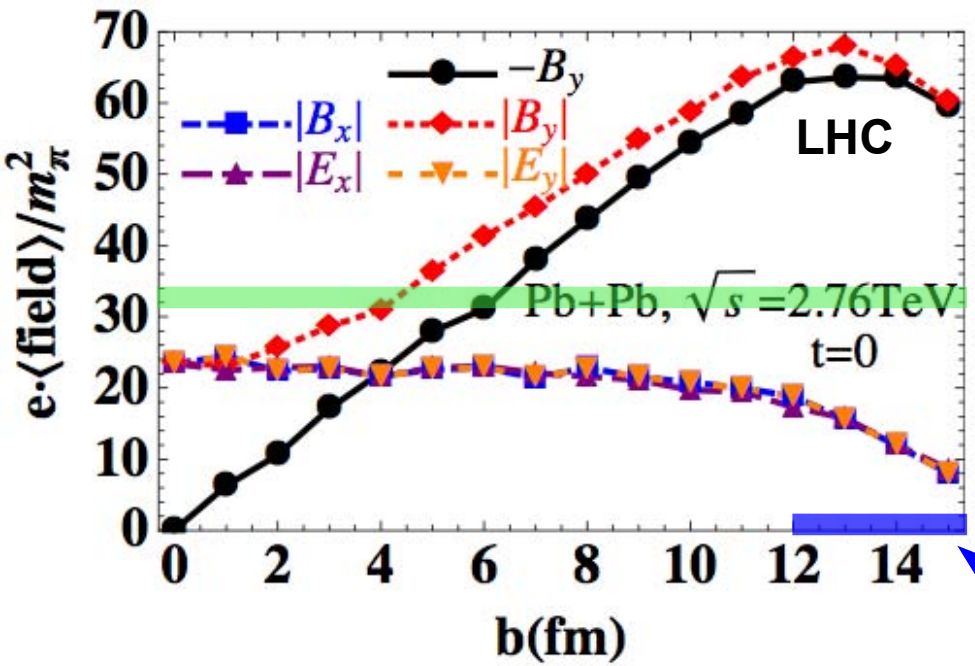
for $B \geqslant B_c$

[qualitatively realistic vacuum, quantitative results may receive corrections (20%-50% typically)]

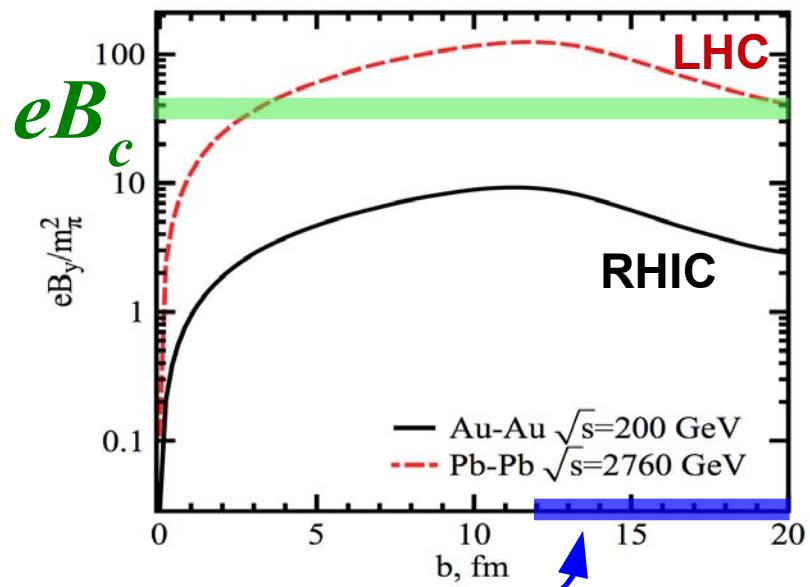
Too strong critical magnetic field?

$$eB_c \simeq m_\rho^2 \simeq 31\, m_\pi^2 \simeq 0.6\ \mathrm{GeV}^2$$

Over-critical magnetic fields (of the strength $B \sim 2...3\ B_c$) may be generated in ultraperipheral heavy-ion collisions
(duration is short, however – clarifications are needed)



eB_c



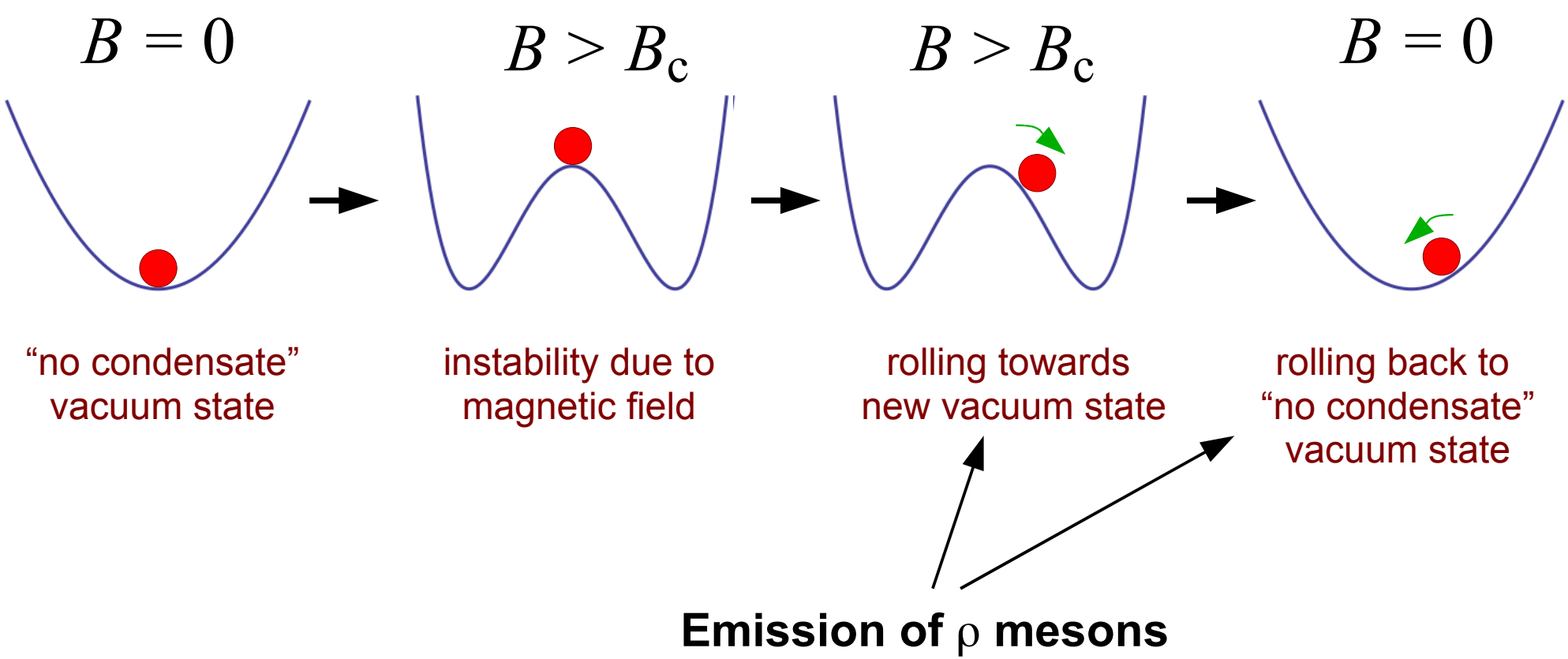
ultraperipheral

W. T. Deng and X. G. Huang,
Phys.Rev. C85 (2012) 044907

A. Bzdak and V. Skokov,
Phys.Lett. B710 (2012) 171
+ Vladimir Skokov,
private communication.

Too quick for the condensate to be developed, but signatures may be seen due to instability of the vacuum state

ρ-meson vacuum state between the ions in ultraperipheral collisions



Conclusions

- In a sufficiently strong magnetic field condensates with $\rho^{\pm}$ meson quantum numbers are formed spontaneously.

- The vacuum (= no matter present, = empty space, = nothing) becomes <u>electromagnetically</u> superconducting.

- The superfluidity of the neutral ρ^0 mesons emerges as well.

- The superconductivity is anisotropic: the vacuum behaves as a perfect conductor only along the axis of the magnetic field.

- New type of tological defects,"ρ vortices", emerge.

- The ρ vortices form Abrikosov-type lattice in transverse directions.

- The Meissner effect is absent.

- Can be seen at LHC?

Charge dependent azimuthal correlations with the ALICE detector at the LHC

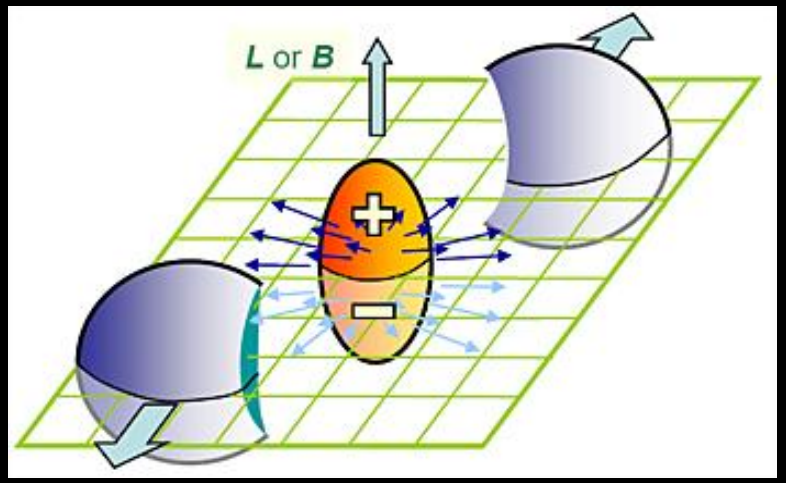
Panos Christakoglou[1], for the ALICE Collaboration

[1]Nikhef

Motivation

- Suggestions that heavy-ion collisions may form domains where the parity symmetry in strong interaction is locally violated
- In non-central collisions, these domains may manifest themselves by a separation of charge, above and below the reaction plane.
- The resulting charge separation is a consequence of two factors
  - the difference in numbers of quarks with positive and negative chiralities due to a non-zero topological charge of the region,
  - the interaction of these particles with the extremely strong and short lived magnetic field produced in such a collision (the Chiral Magnetic Effect-CME).
- The existence of the CME, is directly related to the Chiral Symmetry restoration and to extreme B field values
  - ~10^{18} Gaus, stronger than on the surface of a neutron star

• D. Kharzeev, Phys. Lett. **B633**, 260 (2006).
• D. Kharzeev and A. Zhitnitsky, Nucl. Phys. **A797**, 67 (2007).
• D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. **A803**, 227 (2008).
• K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. **D78**, 074033 (2008).



Proposed tools: azimuthal correlations

S. Voloshin, Phys. Rev. **C70**, 057901 (2004)

3–particle

$$\left\langle \cos\left(f_a + f_b - 2\Psi_{RP}\right)\right\rangle = \left\langle \cos\left(f_a + f_b - 2\psi_g\right)\right\rangle / v_{2g}$$

$$\left\langle \cos\left(f_a + f_b - 2\Psi_{RP}\right)\right\rangle = \left\langle \cos\left(\left(f_a - \Psi_{RP}\right) + \left(f_b - \Psi_{RP}\right)\right)\right\rangle =$$

$$\left\langle \cos\left(\Delta\psi_a + \Delta\psi_b\right)\right\rangle = \left\langle \cos\left(\Delta\psi_a\right)\cos\left(\Delta\psi_b\right)\right\rangle - \left\langle \sin\left(\Delta\psi_a\right)\sin\left(\Delta\psi_b\right)\right\rangle$$

2–particle

$$\left\langle \cos\left(f_a - f_b\right)\right\rangle = \left\langle \cos\left(\left(f_a - \Psi_{RP}\right) - \left(f_b - \Psi_{RP}\right)\right)\right\rangle =$$

$$\left\langle \cos\left(\Delta\psi_a - \Delta\psi_b\right)\right\rangle = \left\langle \cos\left(\Delta\psi_a\right)\cos\left(\Delta\psi_b\right)\right\rangle + \left\langle \sin\left(\Delta\psi_a\right)\sin\left(\Delta\psi_b\right)\right\rangle$$

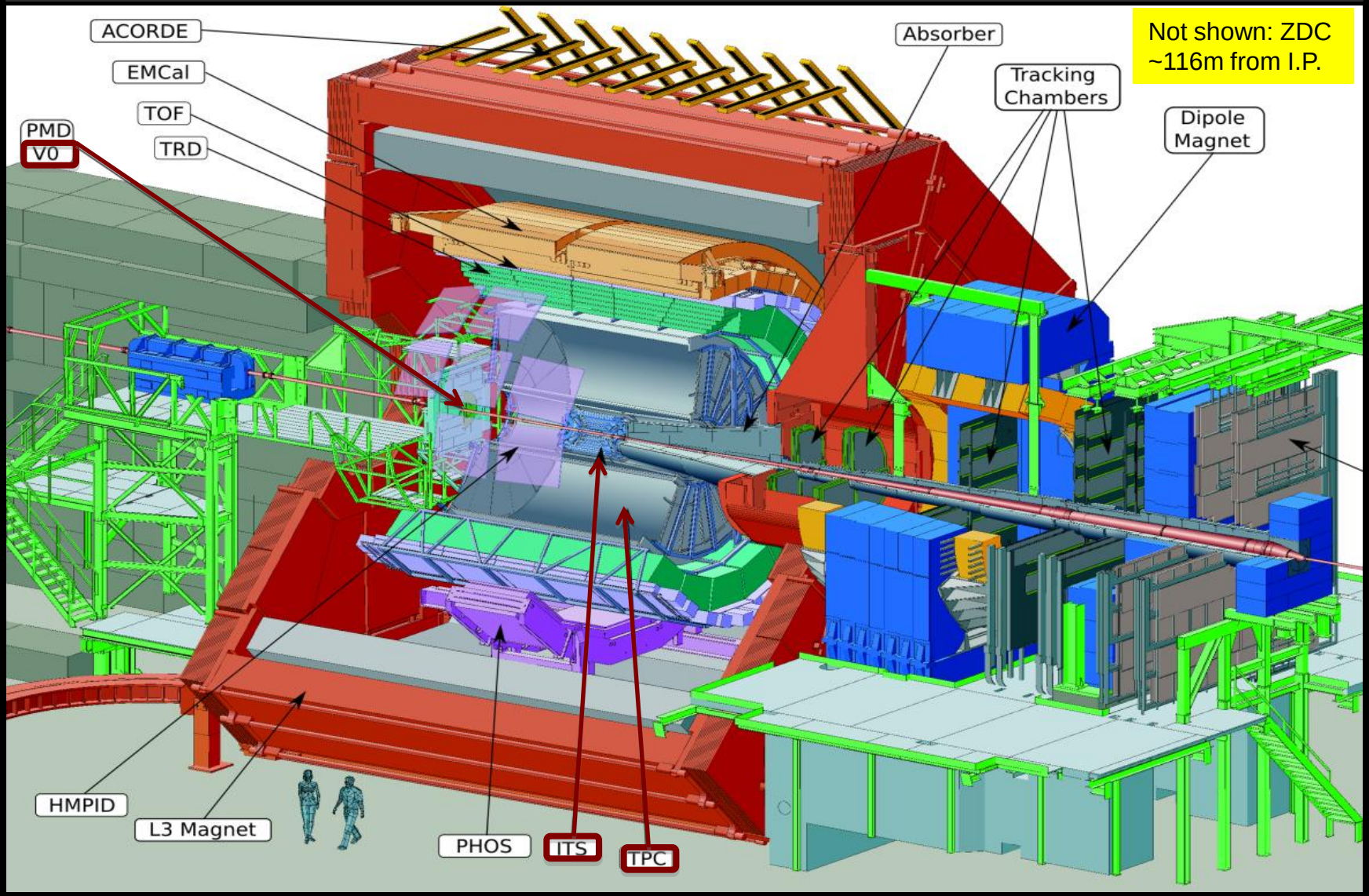
correlations in-plane

$$\left\langle \cos\left(\Delta\psi_a\right)\cos\left(\Delta\psi_b\right)\right\rangle = \frac{1}{2}\left[\left\langle \cos\left(\Delta\psi_a + \Delta\psi_b\right)\right\rangle + \left\langle \cos\left(\Delta\psi_a - \Delta\psi_b\right)\right\rangle\right]$$

correlations out-of-plane

$$\left\langle \sin\left(\Delta\psi_a\right)\sin\left(\Delta\psi_b\right)\right\rangle = \frac{1}{2}\left[\left\langle \cos\left(\Delta\psi_a - \Delta\psi_b\right)\right\rangle - \left\langle \cos\left(\Delta\psi_a + \Delta\psi_b\right)\right\rangle\right]$$

ALICE: Experimental setup

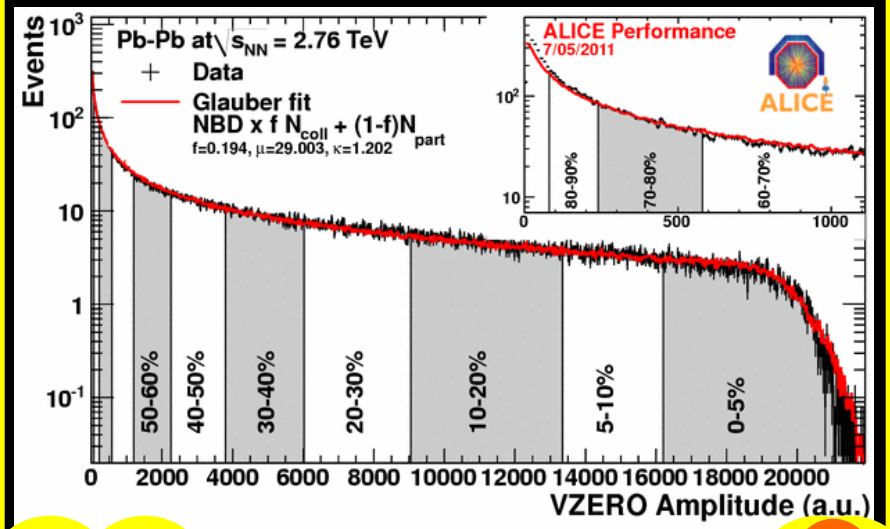


Studies in ALICE: Analysis details

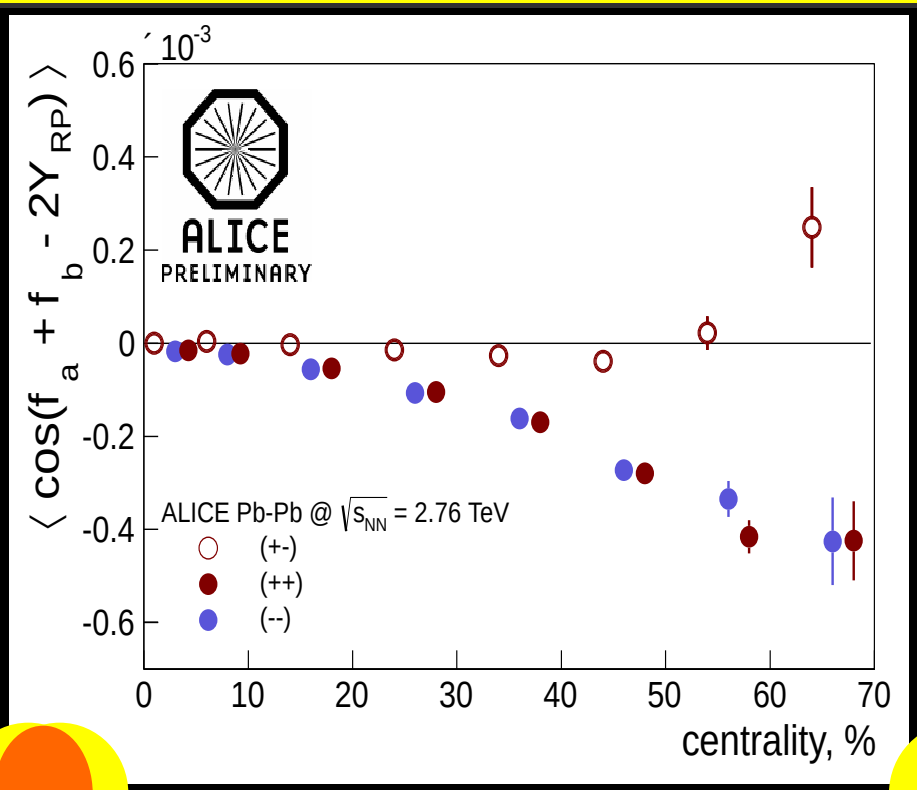
- Analysis of the Pb-Pb events recorded in November/December 2010 during the first LHC heavy-ion run
  - Event sample split in two sets having different magnetic field polarities (results used for the systematic uncertainties)
- Trigger conditions:
  - SPD, VZERO-A, VZERO-C (2 out of 3)
  - VZERO-A && VZERO-C

- Due to the small magnitude of the potential signal, we need to have the acceptance corrections under control:
  - The TPC tracks provide a uniform acceptance with minimal corrections
  - Disadvantage: contamination from secondaries
    - Investigated by varying the cut on the distance of closest approach (results used for the systematic uncertainty).

- The centrality is selected using the magnitude of the VZERO signal (~multiplicity) as the default estimator
  - Centrality bins: 0-5%, 5-10%, 10-20%,…,60-70%
  - Different centrality estimators (TPC tracks, SPD clusters) investigated
    - Results used for the systematic uncertainty



Centrality dependence: Charge combinations



Correlations measured with the cumulant technique

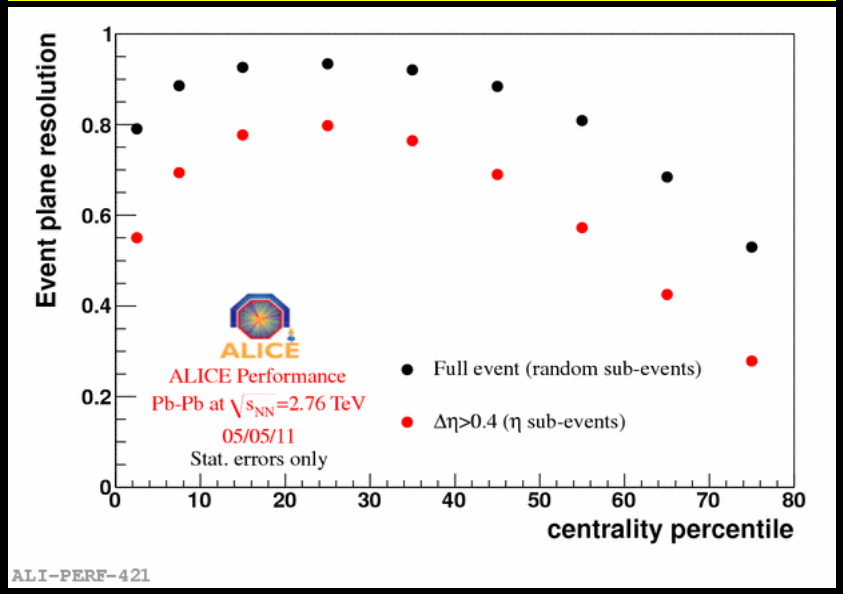
- Clear charge asymmetry observed
- Results for (++) and (--) consistent (combined later as “Same charge”)
- The magnitude of the correlations between the same charged pairs is larger than the one of the opposite charges (excluding the most peripheral collisions ➔ due to large non-flow?)

Different methods: event plane estimate from detectors

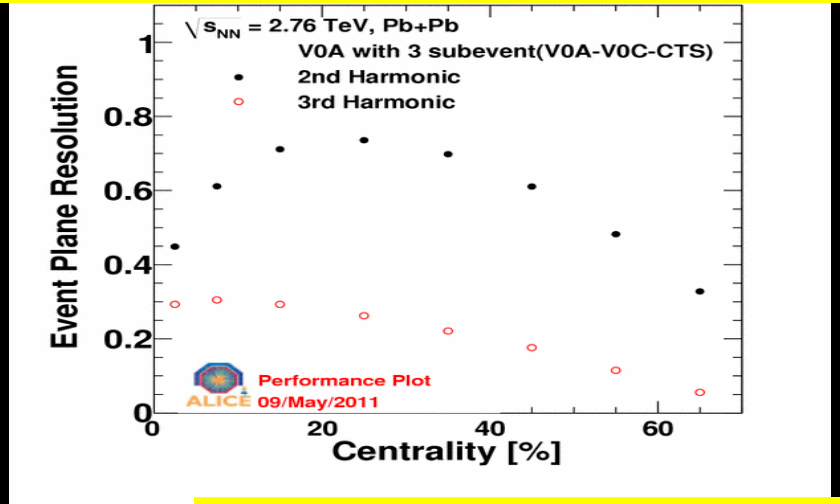
$$Q_{n,y} = \sum_i w_i \times \sin(n f_i) \qquad Q_{n,x} = \sum_i w_i \times \cos(n f_i)$$

$$\Psi_n = \operatorname{atan2}\left(\frac{Q_{n,y}}{Q_{n,x}}\right) / n$$

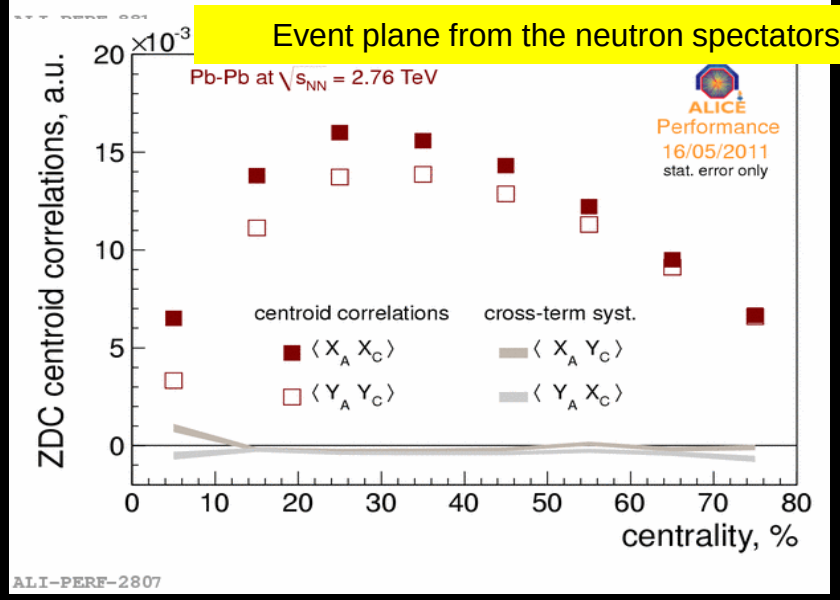
Event plane from charged particles at mid-rapidity



Event plane from charged particles at forward rapidity

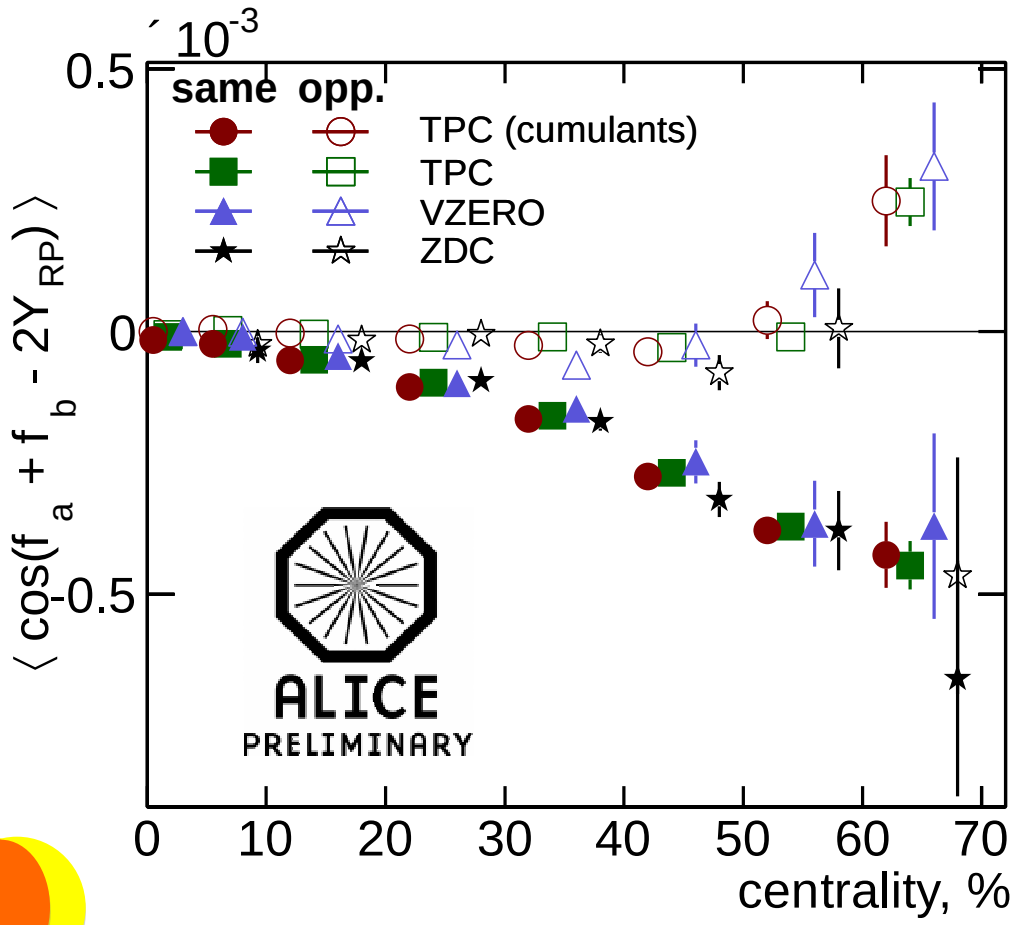


Event plane from the neutron spectators



Investigation with four independent methods

Centrality dependence: Comparison of methods

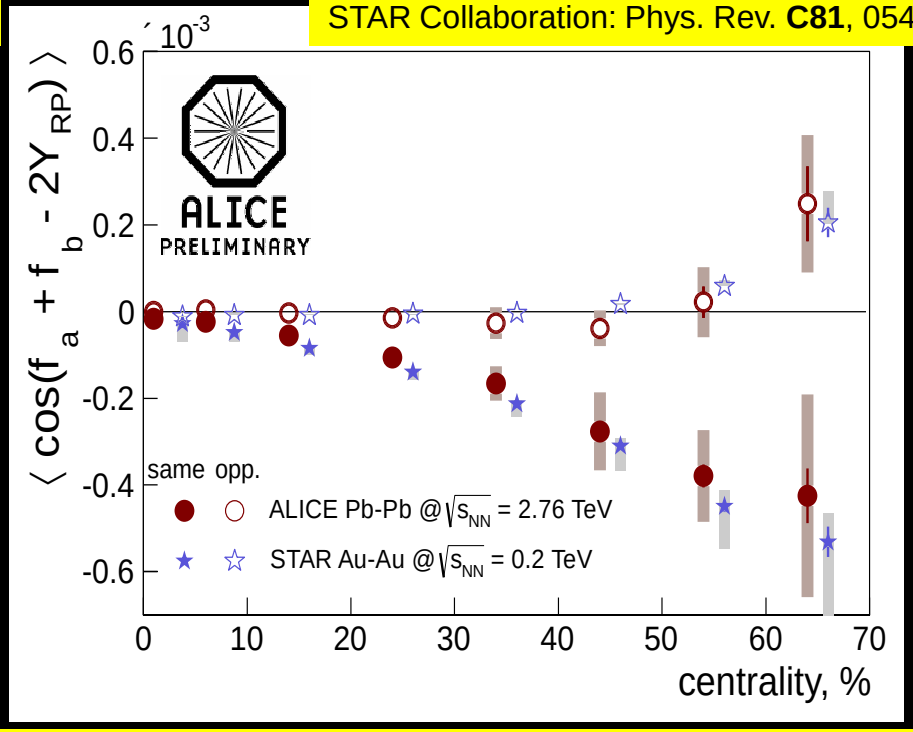


Charge asymmetry due to correlation wrt the reaction plane

3-particle correlator: LHC vs RHIC

Stat. error: error bars
Syst. error: shaded area

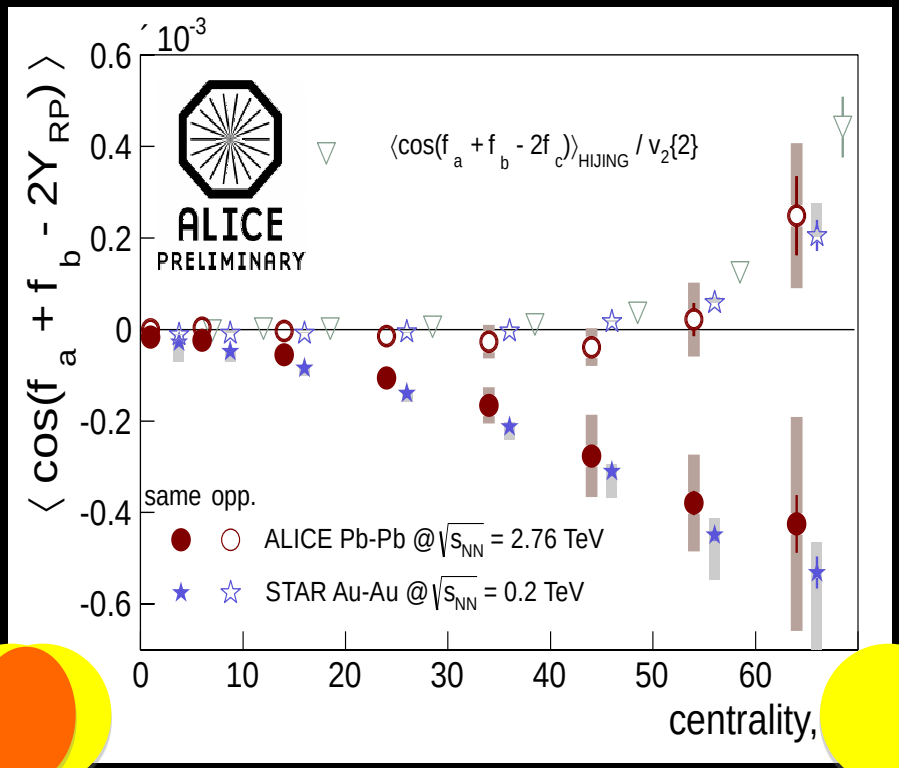
STAR Collaboration: Phys. Rev. Lett. **81**, 251601 (2009)
STAR Collaboration: Phys. Rev. **C81**, 054908 (2010)

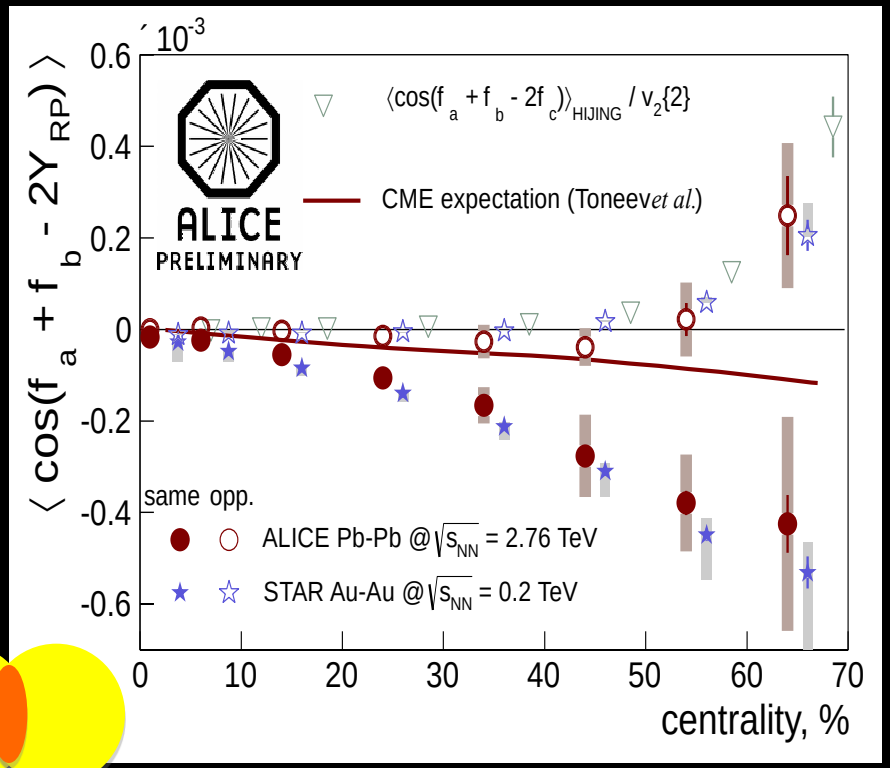


- Magnitude of the effect seems to be similar to what is reported by STAR.
- Some models predict a much lower effect at LHC energies (see next slide)
  - Signal and background should both scale with the inverse of the square of the multiplicity
- The effect can be similar depending on the t_0 of the magnetic field
  - D. Kharzeev et al., Nucl. Phys. **A803**, (227) 2008
  - A. R. Zhitnitsky, arXiv:1201.2665 [hep-ph].

3-particle correlator: Comparison with models

V.D. Toneev and V. Voronyuk, arXiv:1012.1508v1 [nucl-th]

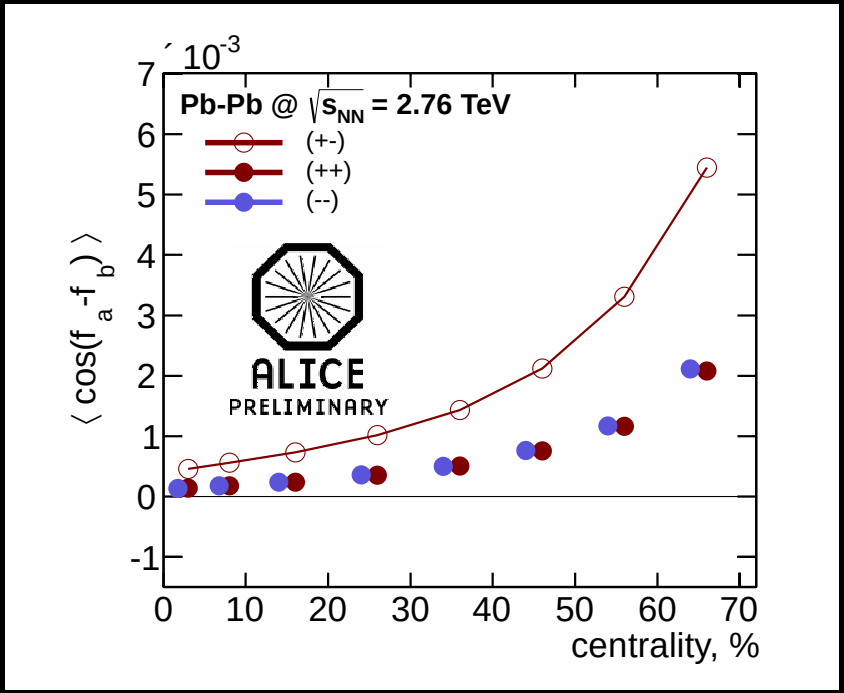




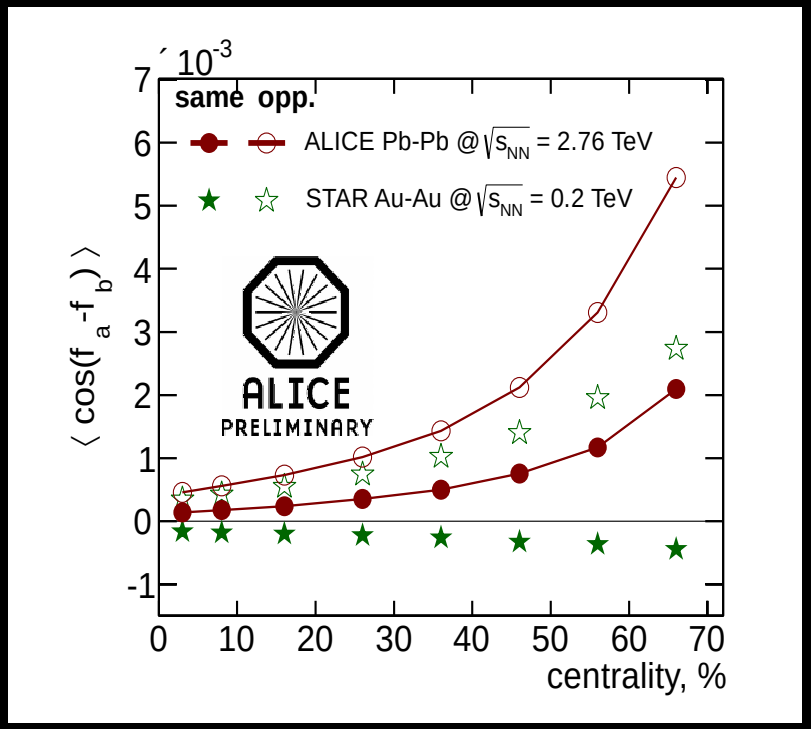
- HIJING results between pairs of same and opposite charge are consistent ➔ combined into one point
- HIJING points consistent with the (+-) data points
- HIJING points scaled with the square of the multiplicity, consistent with the idea of having the correlations originating from emerging clusters (jets, resonances)

- The only available quantitative prediction for LHC energies (@4.5 TeV)
- According to the authors the magnitude should roughly scale with 1/√s
  - Applied in the figure to convert the prediction to $\sqrt{s_{NN}}$ = 2.76 TeV

2–particle correlations: Centrality dependence

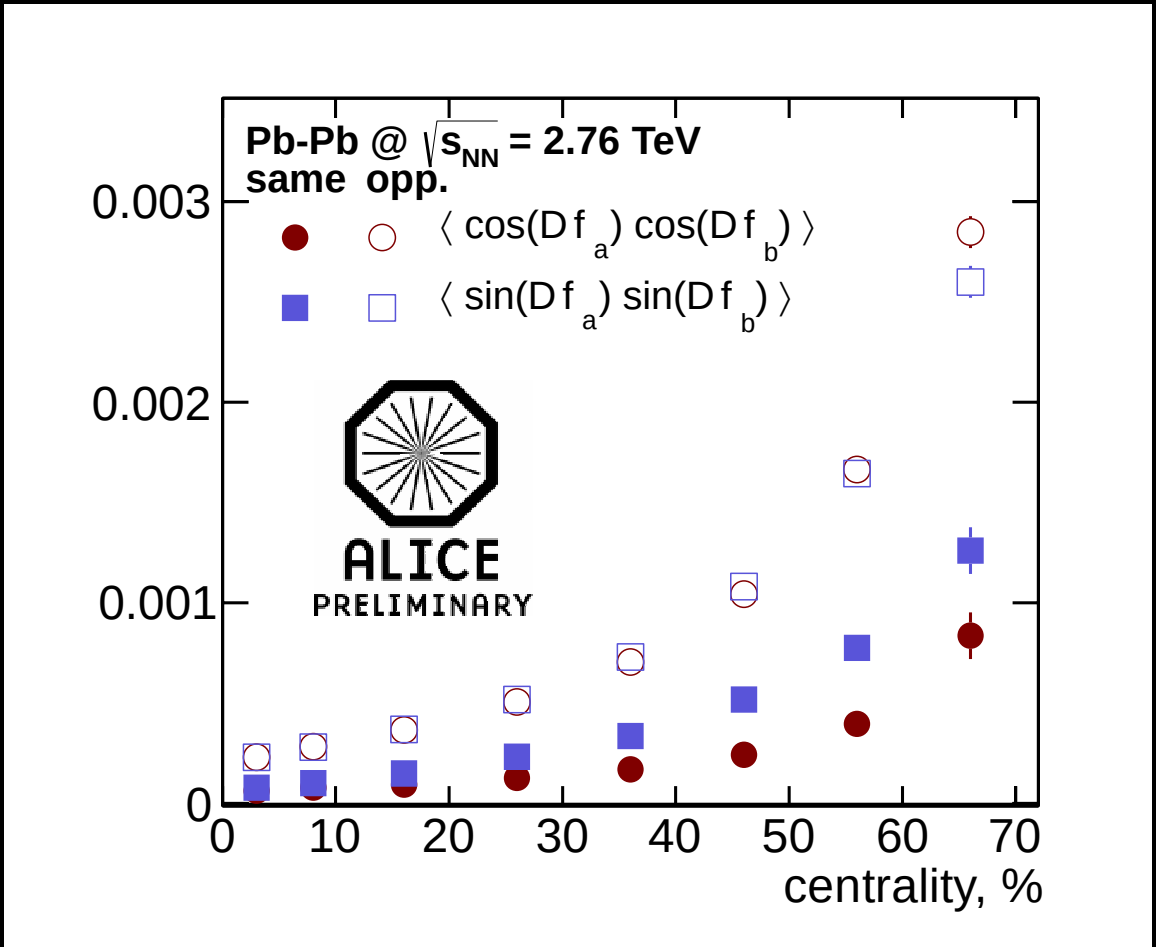


- (++) and (--) combined into one set of points ("Same charge").
- Similarity to STAR: the magnitude of the opposite charged pairs which is larger than the same charged ones.
- Difference with STAR:
  - Sign of the same charged correlations
  - Strength of the correlations



- Correlations between opposite charges are positive and large
- Correlations of same charged pairs are also positive and have a smaller magnitude
- Results between (++) and (--) are consistent

Decomposition

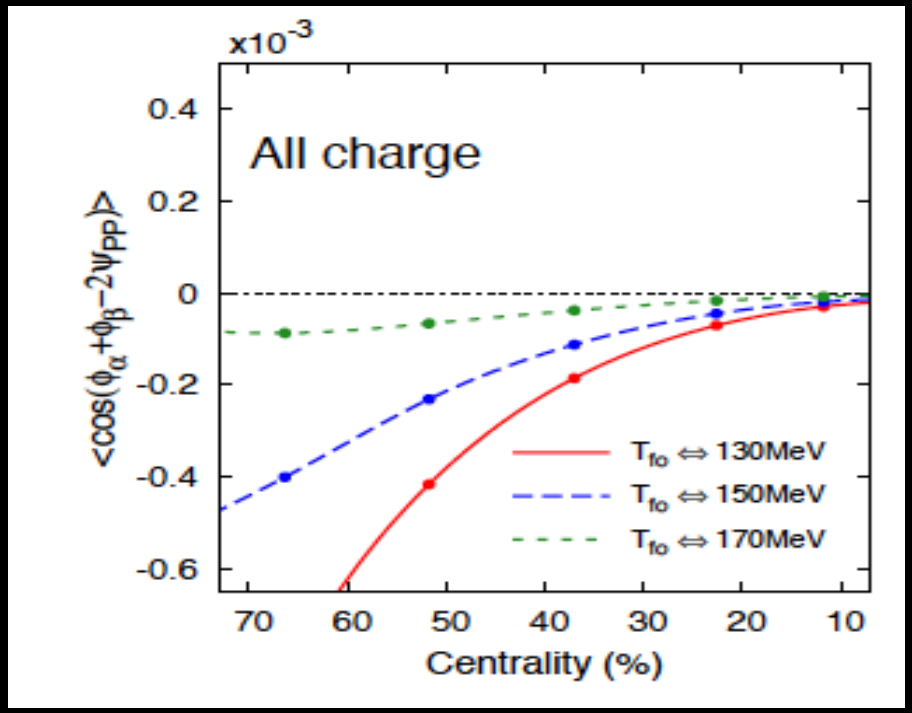


- Similar magnitude for the cos terms for same and opposite charged pairs
- Higher magnitude for the sin terms for same than opposite charged pairs

Background effects: flow fluctuations

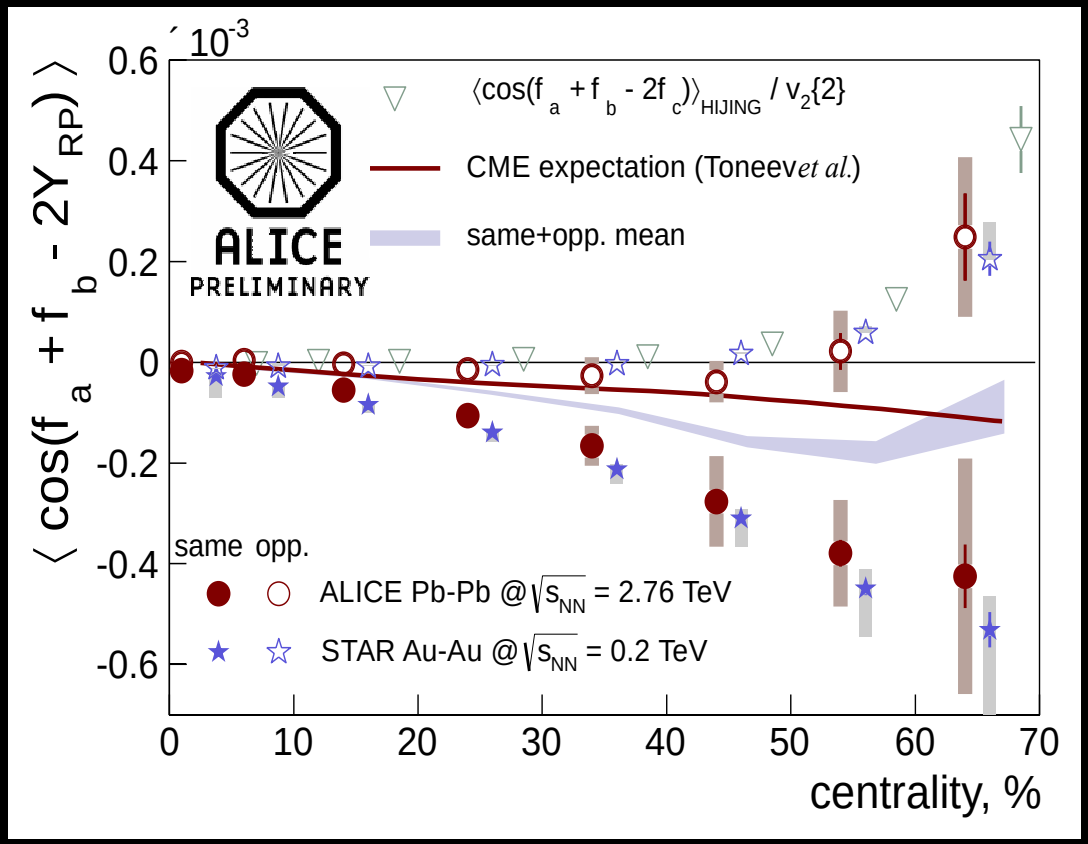
- The orientation angle of the dipole asymmetry shows a preference out-of-plane.
  - This results in a net v1 out of plave with a small magnitude
- The magnitude of the correlations depending on the freeze-out conditions can give a potentially significant contribution
  - The hydrodynamic calculation though does not describe the charge separation!
- Baseline shift in our measurement?

D. Teaney and L. Yan, arXiv:1010.1876v1 [nucl-th]



Background effects: initial state fluctuations (cont.)

Charge independent correlator



P. Christakoglou (for the ALICE Collaboration), Phys. G **G38**, (2011) 124165
Paper at the last stage of the Collaboration review (will released soon after the workshop):
2- and 3-particle integrated correlator + differential analysis (3-particle correlator)

Summary

- The possibility of observing parity odd domains was investigated by using both a 2-particle and a 3-particle P-even correlator.
- The results from the 2-particle correlator studies show that the sign of the correlations is the same regardless of the charge combination, contrary to what was observed in STAR
  - Need to take into account the different non-flow contributions
- The centrality dependence of the 3-particle correlator illustrates a remarkable agreement in both the magnitude and the behavior with the results reported by STAR in Au-Au collisions at $\sqrt{s_{NN}}$ = 0.2 TeV
  - Hydro calculations indicate that the dipole asymmetry's preferential out-of-plane orientation might result into a v_1 contribution out-of-plane, but the charge asymmetry is not explained.
  - Baseline shift from the fluctuations of the initial geometry?
- Theory was not clear about the possible energy dependence of the effect
  - Significant need for quantitative (realistic) calculations of the CME effects for both RHIC and LHC energies

Charge asymmetry is seen experimentally with a similar magnitude as at the highest RHIC energy ➔ Theory is challenged by the latest findings!

Outlook (towards QM…and beyond)

- Charge conservation coupled to elliptic flow seems to describe the difference of the 3-particle correlator for same and opposite charged pairs at RHIC
  - Look at the balance function wrt Ψ

S. Schlichting and S. Pratt, Phys. Rev. **C83**, 014913 (2011).
S. Pratt, S. Schlichting and S. Gavin, Phys. Rev. **C84**, 024909 (2011)

- Look at other correlators (e.g. double harmonics)

S. Voloshin, arXiv:1111.7241 [nucl-ex]

- Correlations between identified particles

- Chiral vortical effect studies

D. Kharzeev Phys. Rev. Lett. **106** (2011) 062301

BACKUP

Topological Effects and the Deconfinement Transition

Massimo D'Elia
University of Pisa & INFN

In collaboration with Francesco Negro (Genoa Univ.)

based on arXiv:1205.0538

CPODD 2012 - Brookhaven National Laboratory - June 25-27 2012

1 – OUTLINE

- **Dependence of the deconfinement temperature on θ:**

  – **lattice determination for $SU(3)$ pure gauge theory**

  – **large N_c estimate**

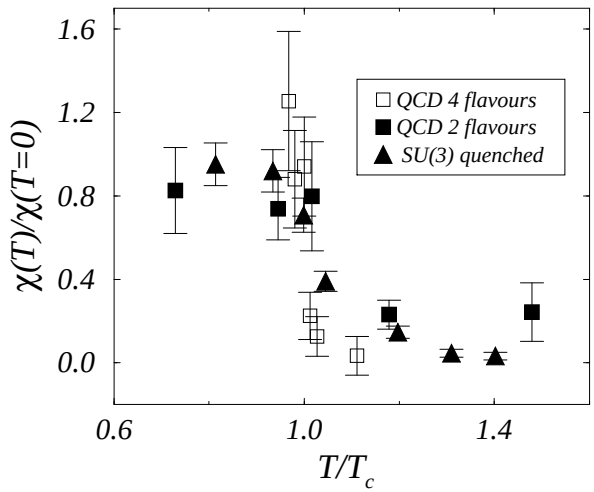
- **Speculations about the $T - \theta$ phase diagram**

2 – Introduction

- **Effects related to configurations with non-trivial topology are expected to play a significant role in strong interactions.**

  This is particularly true around the phase transition.

We know that the topological activity changes at the deconfinement transition, e.g. the topological susceptibility χ drops.



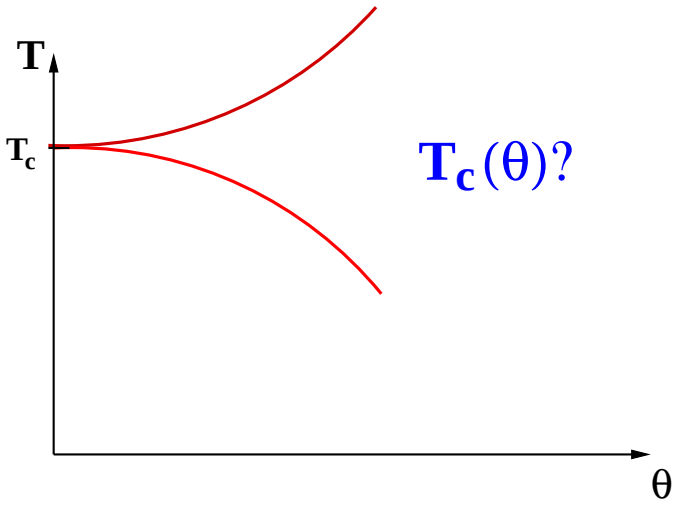
B. Allès, M. D'E., A. Di Giacomo, '96-'00

- **A different question: can topology affect the location and nature of the transition?**

  How does T_c changes if we switch a $\theta \neq 0$ parameter on?

  Lattice QCD simulations are a possible way to approach the problem

We are looking for yet another extension of the QCD phase diagram, the θ-axis



We consider $SU(3)$ pure gauge theory

1st **order transition for**

$T_c(\theta = 0) \simeq 270$ **MeV**

$$Z(T,\theta) = \int [dA]\, e^{-S_{QCD}[A] + i\theta Q[A]} = e^{-V_s f(\theta)/T} \qquad Q = \int d^4x \frac{g_0^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x)$$

General features

- **The theory is CP even at $\theta = 0$ $\Longrightarrow$ T_c must be an even function of θ**
- **The measure is complex for $\theta \neq 0$ $\Longrightarrow$ we have to face yet another sign problem**

We can borrow methods and strategies used to partially overcome the sign problem for QCD at finite baryon chemical potential μ_B

We exploit here analytic continuation: $\theta = i\,\theta_I \qquad Z(T,\theta_I) = \int [dA]\, e^{-S_{QCD}-\theta_I Q}$

Azcoiti *et al*, hep-lat/0203017; Allés - Papa 0711.1496; S. Aoki *et al*, 0808.1428; Panagopoulos - Vicari, 1109.6815

$$\frac{T_c(\theta_I)}{T_c(0)} = 1 + R_\theta\,\theta_I^2 + O(\theta_I^4) \quad \Longrightarrow \quad \frac{T_c(\theta)}{T_c(0)} = 1 - R_\theta\,\theta^2 + O(\theta^4)$$

We shall determine R_θ

- **we assume that the slope of $T_c(\theta^2)$ is well defined at $\theta = 0$ (consistent with data)**
- **determining higher order terms may be difficult, as it happens in the μ_B case**

  see e.g. P. Cea, L. Cosmai, M. D'E., A. Papa, F. Sanfilippo, arXiv:1202.5700

3 – Lattice implementation

$$Z_L(T,\theta) = \int [dU]\, e^{-S_L[U]-\theta_L Q_L[U]}$$

$S_L = \beta \sum_{x,\mu>\nu}(1 - \mathrm{ReTr}\,\Pi_{\mu\nu}(x)/N) \qquad \beta = 2N/g_0^2$ **(Wilson action)**

Which choice for $Q_L = \sum_x q_L(x)$?

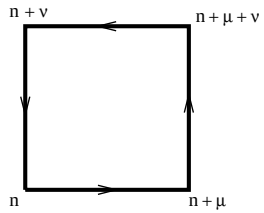
- **A gluonic definition tipically leads to renormalizations**

  $q_L(x) \overset{a\to 0}{\sim} a^4 Z(\beta) q(x) + O(a^6) \qquad \Longrightarrow \qquad \theta_I = Z(\beta)\,\theta_L + O(a^2)$

- **A fermionic, renormalization free definition (e.g. based on overlap operators) would lead to unreasonable computational requirements**

Optimal Strategy: **simplest gluonic definition (no smearing) so that heat-bath + over-relaxation works, then compute the multiplicative renormalization $Z(\beta)$**

$q_L(x) = \frac{-1}{2^9\pi^2} \sum_{\mu\nu\rho\sigma=\pm 1}^{\pm 4} \tilde{\epsilon}_{\mu\nu\rho\sigma}\, \mathbf{Tr}\,(\Pi_{\mu\nu}(x)\Pi_{\rho\sigma}(x))$ $\qquad \Pi_{\mu\nu} \rightarrow$



Simulation parameters

- **three different lattice spacings:**
  $a \sim 1/(N_t T_c) = 1/(4T_c), 1/(6T_c), 1/(8T_c) \quad (0.182 \rightarrow 0.091 \text{ fm})$ **in order to perform the continuum limit extrapolation**

- **three different lattices with equal physical spatial volume:**
  $16^3 \times 4, \quad 24^3 \times 6, \quad 32^3 \times 8$

- **we determine** $T_c(\theta_I)/T_c(0)$**:**
  most finite size effects are expected to cancel out in the ratio

- **Typical statistics:**
  $10^5 - 10^6$ **MC sweeps for each** θ_L , β
  Autocorrelation times up to 10^3 **at the transition.**

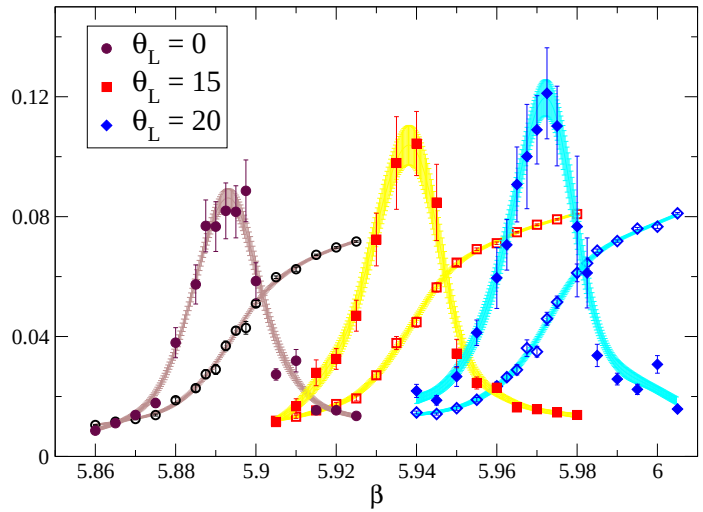
Locating the phase transition

$Z(N)$ **center symmetry, which is spontaneously broken at the deconfinement transition of pure** $SU(N)$ **gauge theories, is still exact in presence of a** θ **term.**

$\Longrightarrow$ **The Polyakov loop is still a good order parameter to locate deconfinement**

$$\langle L \rangle \equiv \frac{1}{V_s} \sum_{\vec{x}} \frac{1}{N} \langle \mathrm{Tr} \prod_{t=1}^{N_t} U_0(\vec{x}, t) \rangle \qquad \chi_L \equiv V_s \left(\langle L^2 \rangle - \langle L \rangle^2) \right) ,$$

Polyakov loop and its susceptibility as a function of β **for** $N_t = 6$ **and a few** θ_L



$\beta_c(\theta_L)$ **located at the peak of** χ_L

$$\beta_c(\theta_L) \quad \longrightarrow \quad T_c(\theta_L) = \frac{1}{N_t \, a(\beta_c(\theta_L))}$$

non perturbative $a(\beta)$ **from** **G. Boyd *et al.*, Nucl. Phys. B 469, 419 (1996).**

Renormalization: $T_c(\theta_I = Z\theta_L)$ $\quad Z = Z(\beta_c(\theta_L))$

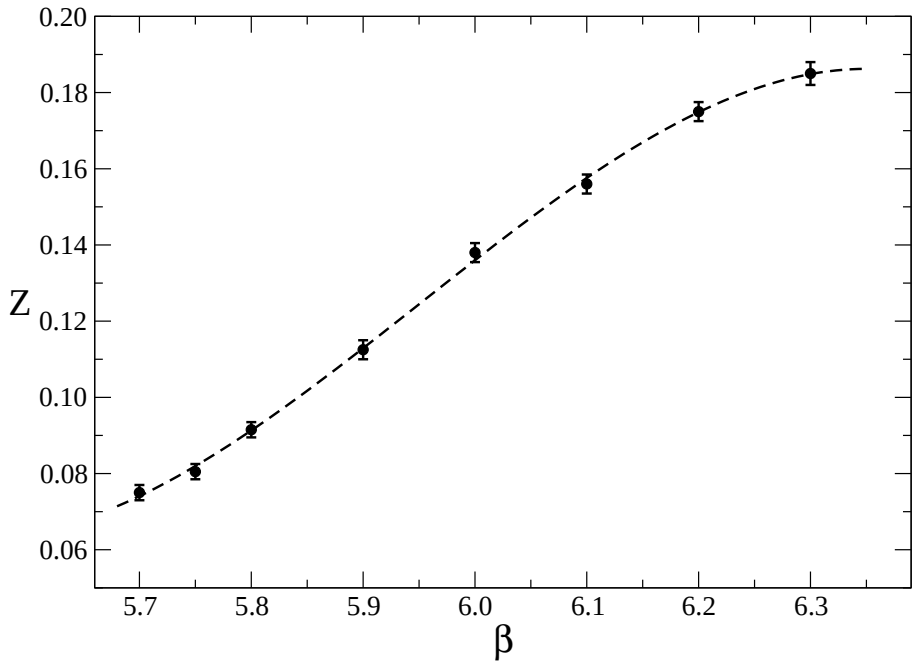
Non-perturbative methods to determine Z exploit the fact that UV fluctuations responsible for renormalization are independent of the topological background:

- $Z = \langle Q_L \rangle_Q / Q$ **in a fixed backgroud Q (heating techniques)**
- $Z = \langle Q_L Q \rangle / \langle Q^2 \rangle$ **over all configurations, where Q is determined by cooling or fermionic methods (Panagopoulos-Vicari 2011)**

We adopt the second strategy

$\frac{\langle Q_L Q \rangle}{\langle Q^2 \rangle}$ **determined on $O(10^5)$ configurations for each β (by cooling)**

$Z(\beta)$ **for intermediate $\beta's$ by cubic interpolation**

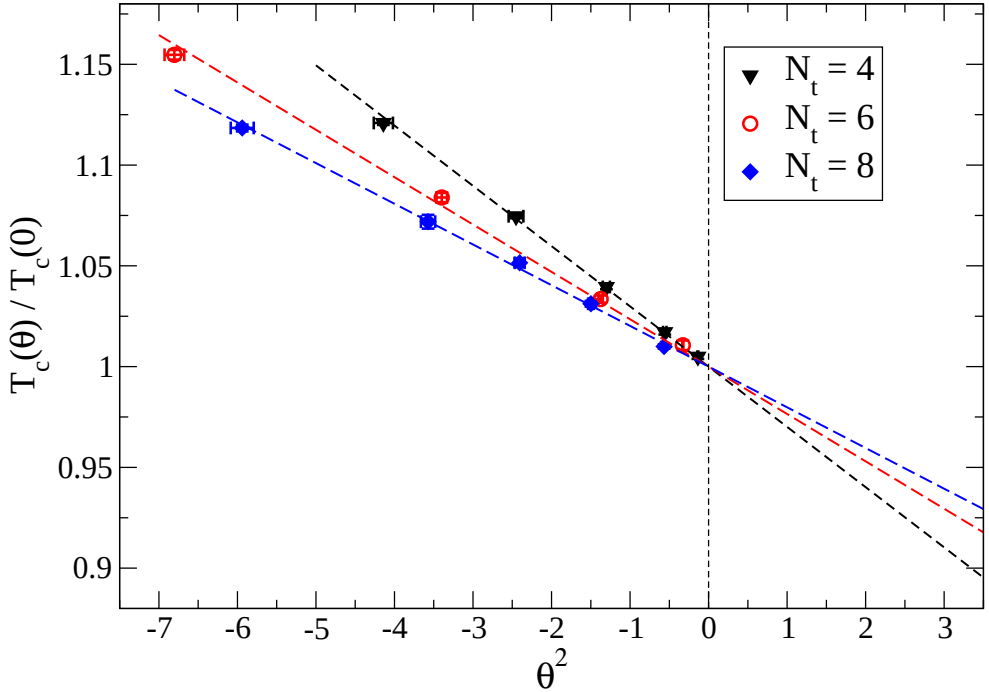


Collection of final results obtained for $T_c(\theta_I)/T_c(0)$

lattice	θ_L	β_c	θ_I	$T_c(\theta_I)/T_c(0)$
$16^3 \times 4$	0	5.6911(4)	0	1
$16^3 \times 4$	5	5.6934(6)	0.370(10)	1.0049(11)
$16^3 \times 4$	10	5.6990(7)	0.747(15)	1.0171(12)
$16^3 \times 4$	15	5.7092(7)	1.141(20)	1.0395(11)
$16^3 \times 4$	20	5.7248(6)	1.566(30)	1.0746(10)
$16^3 \times 4$	25	5.7447(7)	2.035(30)	1.1209(10)
$24^3 \times 6$	0	5.8929(8)	0	1
$24^3 \times 6$	5	5.8985(10)	0.5705(60)	1.0105(24)
$24^3 \times 6$	10	5.9105(5)	1.168(12)	1.0335(18)
$24^3 \times 6$	15	5.9364(8)	1.836(18)	1.0834(23)
$24^3 \times 6$	20	5.9717(8)	2.600(24)	1.1534(24)
$32^3 \times 8$	0	6.0622(6)	0	1
$32^3 \times 8$	5	6.0684(3)	0.753(8)	1.0100(11)
$32^3 \times 8$	8	6.0813(6)	1.224(15)	1.0312(14)
$32^3 \times 8$	10	6.0935(11)	1.551(20)	1.0515(21)
$32^3 \times 8$	12	6.1059(21)	1.890(24)	1.0719(34)
$32^3 \times 8$	15	6.1332(7)	2.437(30)	1.1201(17)

We can now try best fits to interpolate data at $\theta^2 < 0$

Then extrapolate to $\theta^2 > 0$

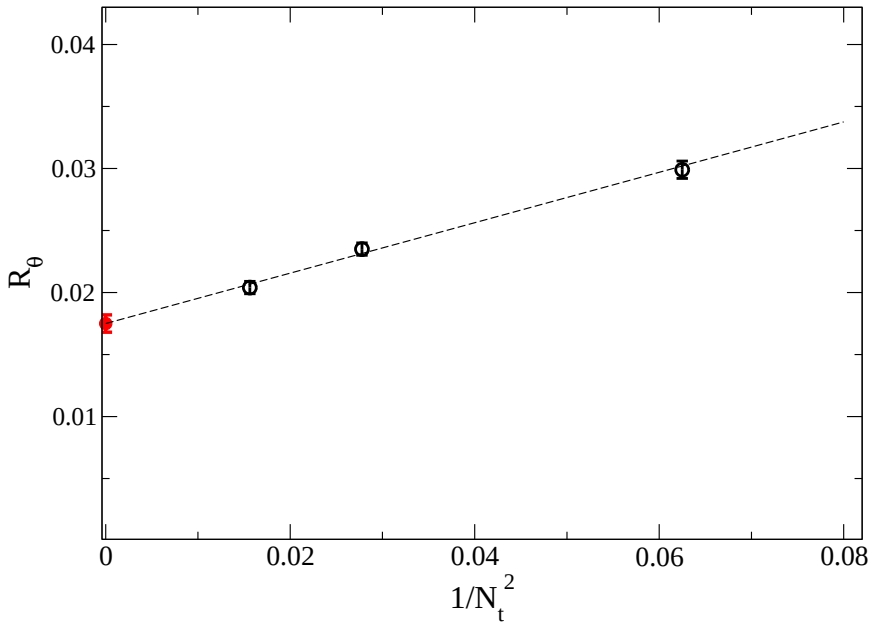


- **The quadratic fit $T_c(\theta)/T_c(0) = 1 - R_\theta\, \theta^2$ works well ($\chi^2/\mathrm{d.o.f.} \lesssim 1$)**
  it is stable in all the explored range.

- **Generic power law $T_c(\theta)/T_c(0) = 1 + A\, |\theta|^\alpha$ works only if $\alpha \simeq 2$**
  Non-analytic behaviors excluded.

- **Non-linear $O(\theta^4)$ terms not visible**
  however finite μ_B teaches that they may be important for $\theta^2 > 0$ when $\theta \sim 1$.

Extrapolation to the continuum limit



$R_\theta = 0.0299(7)$ **for** $N_t = 4$

$R_\theta = 0.0235(5)$ **for** $N_t = 6$

$R_\theta = 0.0204(5)$ **for** $N_t = 8$

We extrapolate to $N_t = \infty$ **assuming** $O(a^2)$ **corrections, works well** **(**$\chi^2/\mathrm{d.o.f.} \simeq 0.97$**)**

Final continuum extrapolated result

$$\frac{T_c(\theta)}{T_c(0)} = 1 - R_\theta\, \theta^2 + O(\theta^4) \qquad R_\theta = 0.0175(7)$$

T_c **decreases in presence of a real** θ

Comments

- **Increase of T_c with θ_I (i.e. decrease with θ) is expected based on naïve arguments**
  - **An imaginary $\theta = i\theta_I$ induces and non trivial background, $\langle Q \rangle \neq 0$.**
  - **Q fluctuations suppressed in the deconfined phase (χ drops).**
  - $\Longrightarrow$ **Stimulating a non-trivial topological background tends to bring you back in the confined phase (T_c increases)**

- **Decrease of T_c compatible with semi-classical computations in dimensionally reduced $SU(2)$ theory** **(M. Unsal, arXiv:1201.6426)**

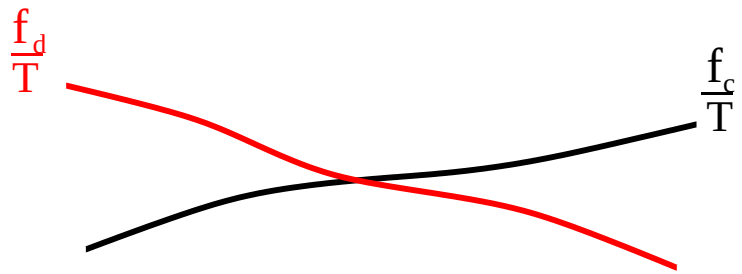
- **We can make a simple model to estimate R_θ for large N_c**

4 – Large N_c estimate

Main idea:

- **Deconfinement transition is first order for $N_c \geq 3$, latent heat $\Delta\epsilon \propto N_c^2$**

- **We have two free energy density sheets (confined and deconfined) crossing at T_c**



- **Around T_c:** $\frac{f_c}{T} = A_c\, t + O(t^2)$ $\quad \frac{f_d}{T} = A_d\, t + O(t^2)$ $\quad t \equiv \frac{T - T_c}{T_c}$

- **Latent heat:** $\Delta\epsilon = -T^2\, [\partial(f_d/T)/\partial T - \partial(f_c/T)/\partial T]_{T_c} = T_c(A_c - A_d)$

- $\theta \neq 0$ **shifts free energy** $f(T,\theta) = f(T,\theta=0) + \chi(T)\,\theta^2/2 + O(\theta^4)$

$\chi(T)$ **differs in the two phases** $\Longrightarrow$ **the two sheets moves separately** $\Longrightarrow$ T_c **moves!**

- **We know that indeed $\chi(T)$ drops at the deconfinement transition!**

  (B. Allés, M. D'E., A. Di Giacomo, 1996)

- **In the large N_c limit the dependence simplifies (step function):**

  (B. Lucini, M. Teper and U. Wenger, 2004; L. Del Debbio, H. Panagopoulos and E. Vicari, 2004)

  - $\chi(T) = \chi(T=0) \equiv \chi$ **in the confined phase**
  - $\chi(T) = 0$ **in the deconfined phase**

- **The equilibrium condition $f_c = f_d$ then reads**

$$A_c t + (\chi/T_c)\,\theta^2/2 \simeq A_d t \qquad \Longrightarrow \qquad t_c(\theta) = \frac{T_c(\theta)}{T_c(0)} - 1 = -\frac{\chi}{2\Delta\epsilon}\theta^2 + O(\theta^4)$$

- **leading N_c estimates** **(B. Lucini, M. Teper, U. Wenger, 2004, 2005; H. Panagopoulos, E. Vicari, 2008)**

$$\frac{\chi}{\sigma^2} \simeq 0.0221(14) \;\; ; \qquad \frac{\Delta\epsilon}{N_c^2 T_c^4} \simeq 0.344(72) \;\; ; \qquad \frac{T_c}{\sqrt{\sigma}} \simeq 0.5970(38)$$

$$R_\theta = \frac{\chi}{2\Delta\epsilon} \simeq \frac{0.253(56)}{N_c^2} + O(1/N_c^4)$$

Comments and Speculations

- **the large N_c estimate gives $R_\theta \simeq 0.0281(62)$ for $N_c = 3$**

  This is larger than our numerical result $R_\theta \simeq 0.0175(7)$, but indeed $\chi(T)$ does not drop to zero at T_c for $N_c = 3$.

- **large N_c estimate in agreement with dependence of the free energy density on $\frac{\theta}{N_c}$**

  (see e.g. Witten, 1998)

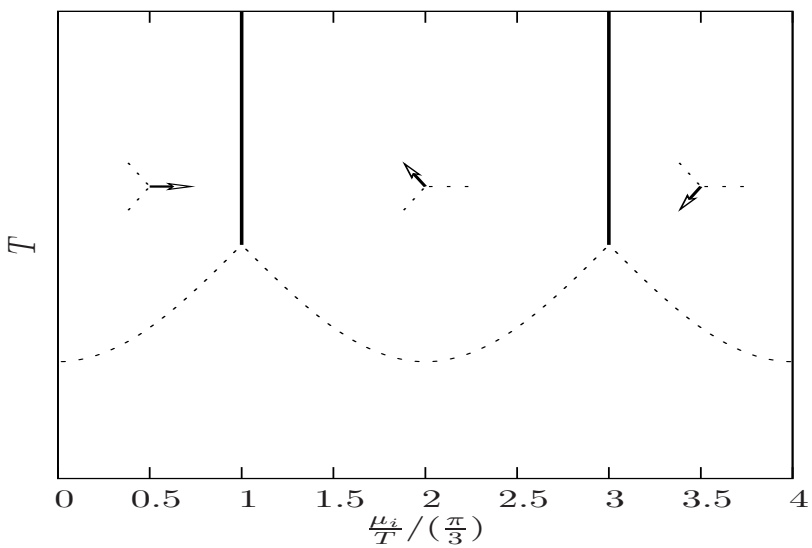
- **According to the same argument, $O(\theta^4)$ corrections to $T_c(\theta)$ should be of $O(1/N_c^4)$.**

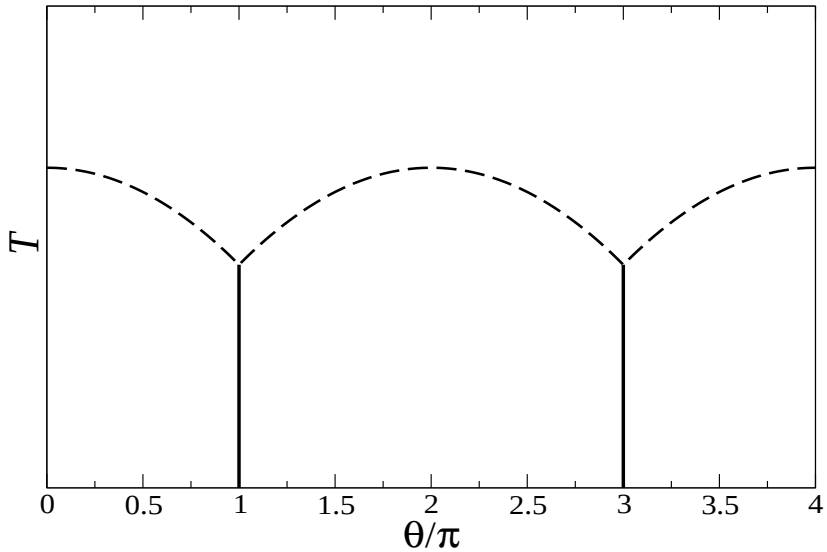
- **$T_c(\theta)$ could then be dominated, for large N$_c$, by the quadratic term down to $\theta = \pi$.**

  To reconcile with periodicity in θ it could be a multibranched function,

$$\frac{T_c(\theta)}{T_c(0)} \simeq 1 - R_\theta \min_k (\theta + 2\pi k)^2$$

What is then, the possible phase structure in the $T - \theta$ plane?





Is it possible that the $T - \theta$ phase diagram (right) looks like the upside-down of the phase diagram for imaginary quark chemical potential (left)?

- **$2\pi/N_c$ periodicity μ_I/T is smoothly realized at low T but phase transitions (Roberge-Weiss lines) present at high T for odd multiples of π/N_c**

- **2π periodicity in θ is realized at low T through phase transitions at odd multiples of π. Smooth periodicity instead at high T?**

5 – Conclusions

- **We have considered $SU(3)$ pure gauge theory and determined $T_c(\theta)$ up to the quadratic order in θ by analytic continuation from simulations at imaginary θ.**

$$T_c(\theta)/T_c(0) = 1 - R_\theta\,\theta^2 + O(\theta^4) \qquad\qquad R_\theta = 0.0175(7)$$

- **We have estimated the curvature R_θ in the large N_c limit.**
  R_θ is of order $1/N_c^2$ and roughly consistent with our numerical determination.

- **We have made speculations about the possible phase structure in the $T-\theta$ plane.**

- **OUTLOOK**
  - **Nature of the transition? Strength seems to increase (decrease) with imaginary (real) θ. Finite size scaling analysis required**
  - **Repeat for larger N_c, to check large N_c prediction, and also for $N_c = 2$.**

Holographic hydrodynamics of systems with broken rotational symmetry

Johanna Erdmenger

Max-Planck-Institut für Physik, München

Based on joint work with M. Ammon, V. Grass, M. Kaminski, P. Kerner, H.T. Ngo, A. O'Bannon, H. Zeller

Motivation

J.E., Haack, Kaminski, Yarom 0809.2488 (JHEP);

Banerjee, Bhattacharya, Bhattacharyya, , Loganayagam, Dutta, Surowka 0809.2596 (JHEP):

Gauge/Gravity Duality at finite charge density requires
5d Chern-Simons term:

Axial contribution to hydrodynamic expansion of current

$$J_\mu = \rho u_\mu + \xi \omega_\mu, \qquad \omega_\mu = \frac{1}{2}\epsilon_{\mu\nu\sigma\rho} u^\nu \partial^\sigma u^\rho$$

Chiral vortical effect in field-theory context

Related to axial anomaly (Son, Surowka 2009)

Motivation:

Gauge/gravity duality: New tools for strongly coupled systems

Famous result: Shear viscosity/Entropy density

Kovtun, Son, Starinets

$$\frac{\eta}{s} = \frac{1}{4\pi} \frac{\hbar}{k_B}$$

From `Planckian time' $\tau_P = \frac{\hbar}{k_B T}$, Universal result

This talk:
Deviations from this result at leading order in λ and N

Holographic proof of universality relies on space-time isotropy

Key ingredient for changes to the universal result: Spacetime anisotropy

Rotational invariance broken

Holographic p-wave superfluids/superconductors

ρ meson condensate breaks rotational symmetry

At finite isospin density (or in external magnetic field)

Outline

- Holographic superconductors
- Transport coefficients in anisotropic systems
- Condensates at finite magnetic field

Holographic Superfluids

- Holographic Superfluids from charged scalar in Einstein-Maxwell gravity (Gubser; Hartnoll, Herzog, Horowitz)

- p-wave superfluid
  Current dual to gauge field condensing (Gubser, Pufu)
  SU(2) Einstein-Yang-Mills model

s-wave superfluid:

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F^{ab}F_{ab} - V(|\psi|) - |\nabla\psi - iqA\psi|^2$$

Operator $\mathcal{O}$ dual to scalar ψ condensing

Herzog, Hartnoll, Horowitz 2008

p-wave superfluid:

$$S = \frac{1}{2\kappa^2}\int d^4x \left[R - \frac{1}{4}(F^a_{\mu\nu})^2 + \frac{6}{L^2}\right]$$

Current J^1_x dual to gauge field component A^{1x} condensing

Gubser, Pufu 2008

P-wave superfluid from probe branes

Ammon, J.E., Kaminski, Kerner 0810.2316, 0903.1864

- A holographic superconductor with field theory in 3+1 dimensions for which
- the dual field theory is explicitly known
- there is a qualitative ten-dimensional string theory picture of condensation

This is achieved in the context of
adding flavour to gauge/gravity duality

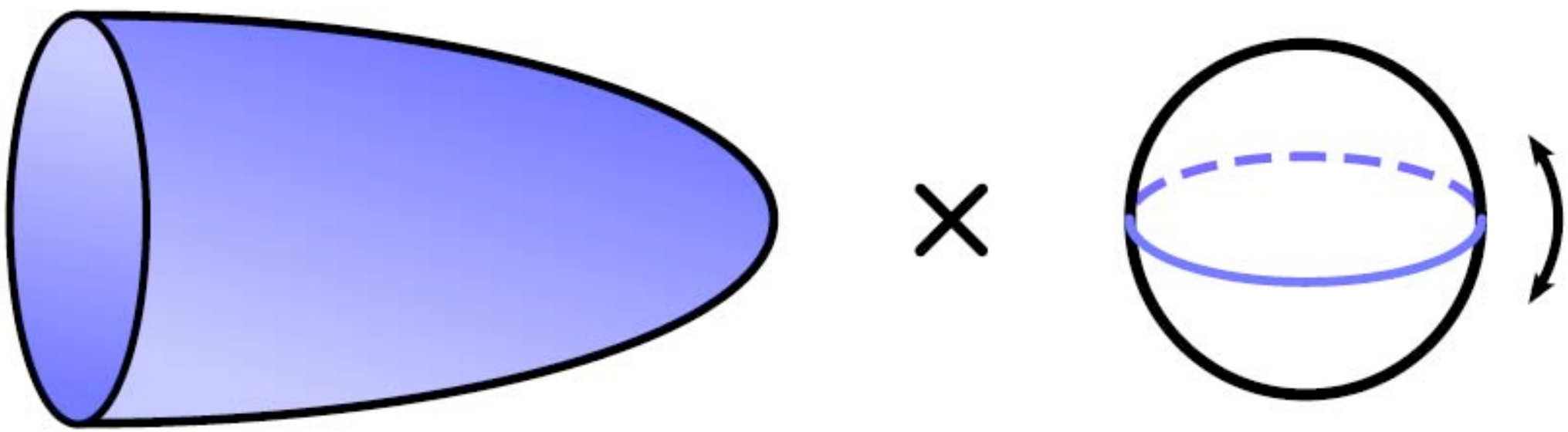
Brane probes added on gravity side $\Rightarrow$
fundamental d.o.f. in the dual field theory (quarks)

Additional D-branes within $AdS_5 \times S^5$ or deformed version thereof

Quarks within Gauge/Gravity Duality

Adding D7-Brane Probe:

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D7	X	X	X	X	X	X	X	X		



Probe brane fluctuations $\Rightarrow$ Masses of mesons ($\bar{\psi}\psi$ bound states)

On gravity side:
Probe brane fluctuations described by Dirac-Born-Infeld action

$$S_{\mathrm{DBI}} = -T_{D7} \int \mathrm{d}^8\xi \, \mathrm{Str} \sqrt{|\det(G + 2\pi\alpha' F)|}$$

On field theory side: Lagrangian explicitly known

$$\mathcal{L} = \mathcal{L}_{\mathcal{N}=4} + \mathcal{L}({\psi_q}^i, {\phi_q}^i)$$

Turn on finite temperature and isospin chemical potential:

Finite temperature: Embed D7 brane in black hole background

Isospin chemical potential: Probe of two coincident D7 branes

Additional symmetry $U(2) = SU(2)_I \times U(1)_B$

$$A_0^3 = \mu - \frac{\tilde{d}_0^3}{2\pi\alpha'}\frac{\rho_H}{\rho^2} + \dots, \qquad A_3^1 = -\frac{\tilde{d}_1^3}{2\pi\alpha'}\frac{\rho_H}{\rho^2} + \dots$$

Condensate $\langle J_3 \rangle, \quad J_3 = \bar{\psi}_d \gamma_3 \psi_u + \ bosons$

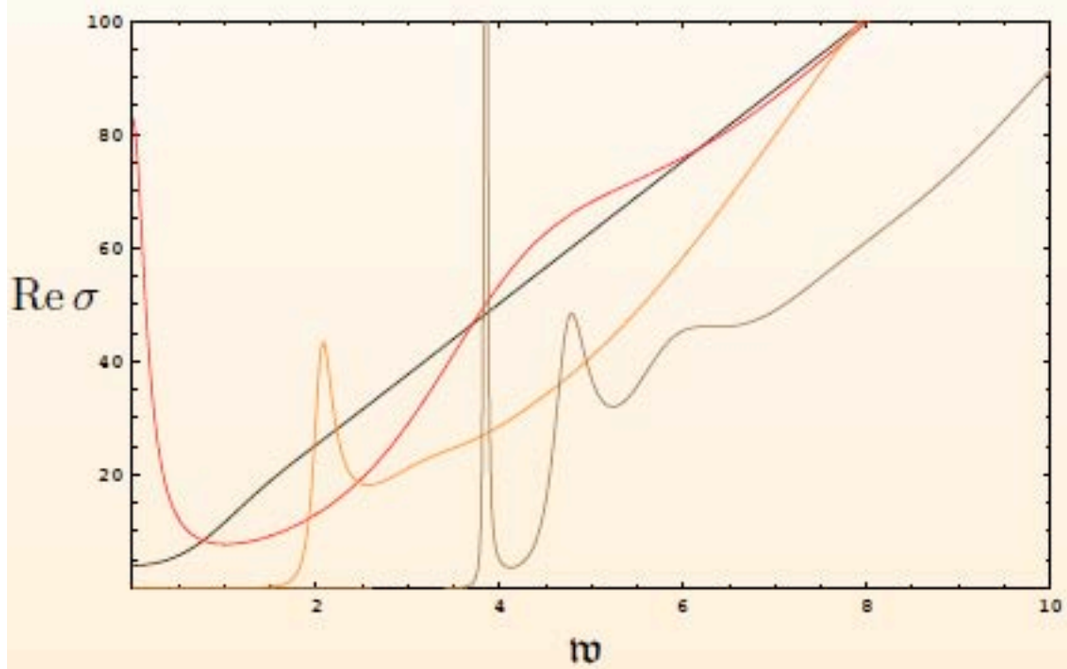
rho meson condensation in Sakai-Sugimoto:
Aharony, Peeters, Sonnenschein, Zamaklar '07

Calculate correlators from fluctuations

Conductivity

Frequency-dependent conductivity $\sigma(\omega) = \frac{i}{\omega} G^R(\omega)$

G^R retarded Green function for fluctuation a_2^3



Ammon, J.E., Kaminski, Kerner '08

$\mathfrak{w} = \omega/(2\pi T)$

T/T_c: Black: ∞, Red: 1, Orange: 0.5, Brown: 0.28.

(Vanishing quark mass)

Interpretation: Frictionless motion of mesons through plasma

Effective 5d model ➡ anisotropic shear viscosity

Bottom-up: Including the backreaction

Ammon, J.E., Graß, Kerner, O'Bannon 0912.3515

Einstein-Yang-Mills-Theory with $SU(2)$ gauge group

$$S = \int d^5x \sqrt{-g} \left[\frac{1}{2\kappa^2} (R - \Lambda) - \frac{1}{4\hat{g}^2} F^a_{\mu\nu} F^{a\mu\nu} \right]$$

$$\alpha = \frac{\kappa_5}{\hat{g}}$$

$\alpha^2 \propto$ number of charged d.o.f./all d.o.f.

In presence of $SU(2)$ chemical potential, same condensation process as before

Hairy black hole solution

- metric ansatz

$$\mathrm{d}s^2 = -N(r)\sigma(r)^2\mathrm{d}t^2 + \frac{1}{N(r)}\mathrm{d}r^2 + \frac{r^2}{f(r)^4}\mathrm{d}x^2 + r^2 f(r)^2\left(\mathrm{d}y^2 + \mathrm{d}z^2\right)$$

with

$$N(r) = -\frac{2m(r)}{r^2} + r^2$$

AdS boundary $r = r_{\mathrm{bdy}} \to \infty$ & black hole horizon $r = r_h$

- gauge field ansatz

$$A = \phi(r)\tau^3\mathrm{d}t + w(r)\tau^1\mathrm{d}x$$

Field Theory	$\Leftrightarrow$	Gravity
chemical potential μ $SU(2) \to U(1)_3$		$A_t^3 = \phi(r) \neq 0$ $SU(2) \to U(1)_3$
$\langle \mathcal{J}_1^x \rangle \neq 0$ $U(1)_3 \to \mathbb{Z}_2,\ SO(3) \to SO(2)$		$A_x^1 = w(r) \neq 0$ $U(1)_3 \to \mathbb{Z}_2,\ SO(3) \to SO(2)$

- $w(r_{\text{bdy}}) = 0 \Rightarrow$ SSB $U(1)_3 \to \mathbb{Z}_2$ & $SO(3) \to SO(2)$

$\Rightarrow$ holographic p-wave superfluid with backreaction

- 5 fields: $\{\phi(r), w(r), \sigma(r), f(r), m(r)\}$

Variation of on-shell action at AdS boundary gives

- energy-momentum tensor

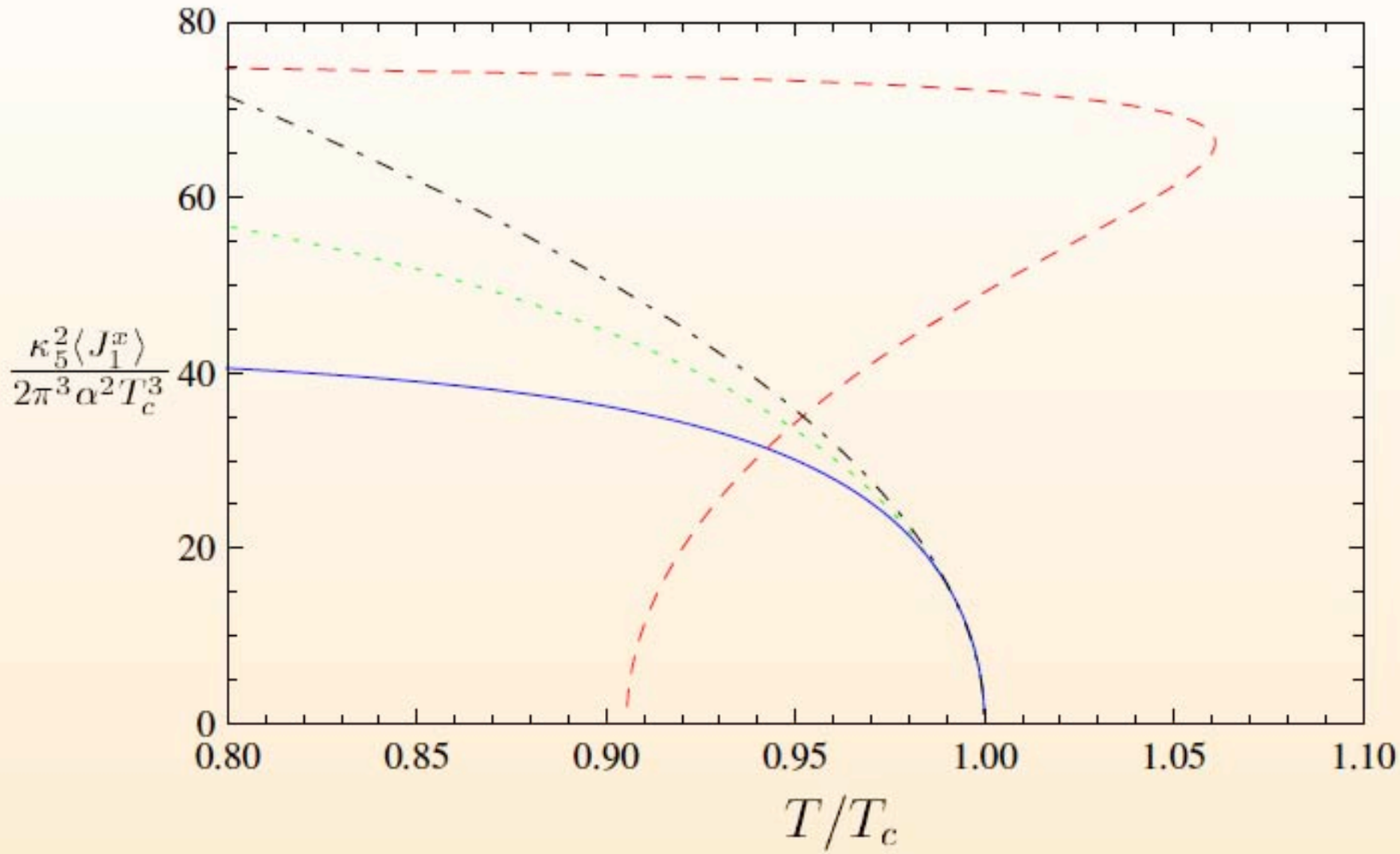
$$\langle \mathcal{T}_{\mu\mu} \rangle \propto T^4 \cdot \mathrm{Func}(m_0^b, f_2^b), \text{ with: } \langle \mathcal{T}_{yy} \rangle = \langle \mathcal{T}_{zz} \rangle \neq \langle \mathcal{T}_{xx} \rangle$$
$$\langle \mathcal{T}_{\mu\nu} \rangle = 0 \text{ for } \mu \neq \nu$$

$$\langle \mathcal{T}_{xx} \rangle = P + \Delta \langle \mathcal{J}_1^x \rangle \langle \mathcal{J}_1^x \rangle$$

- condensate

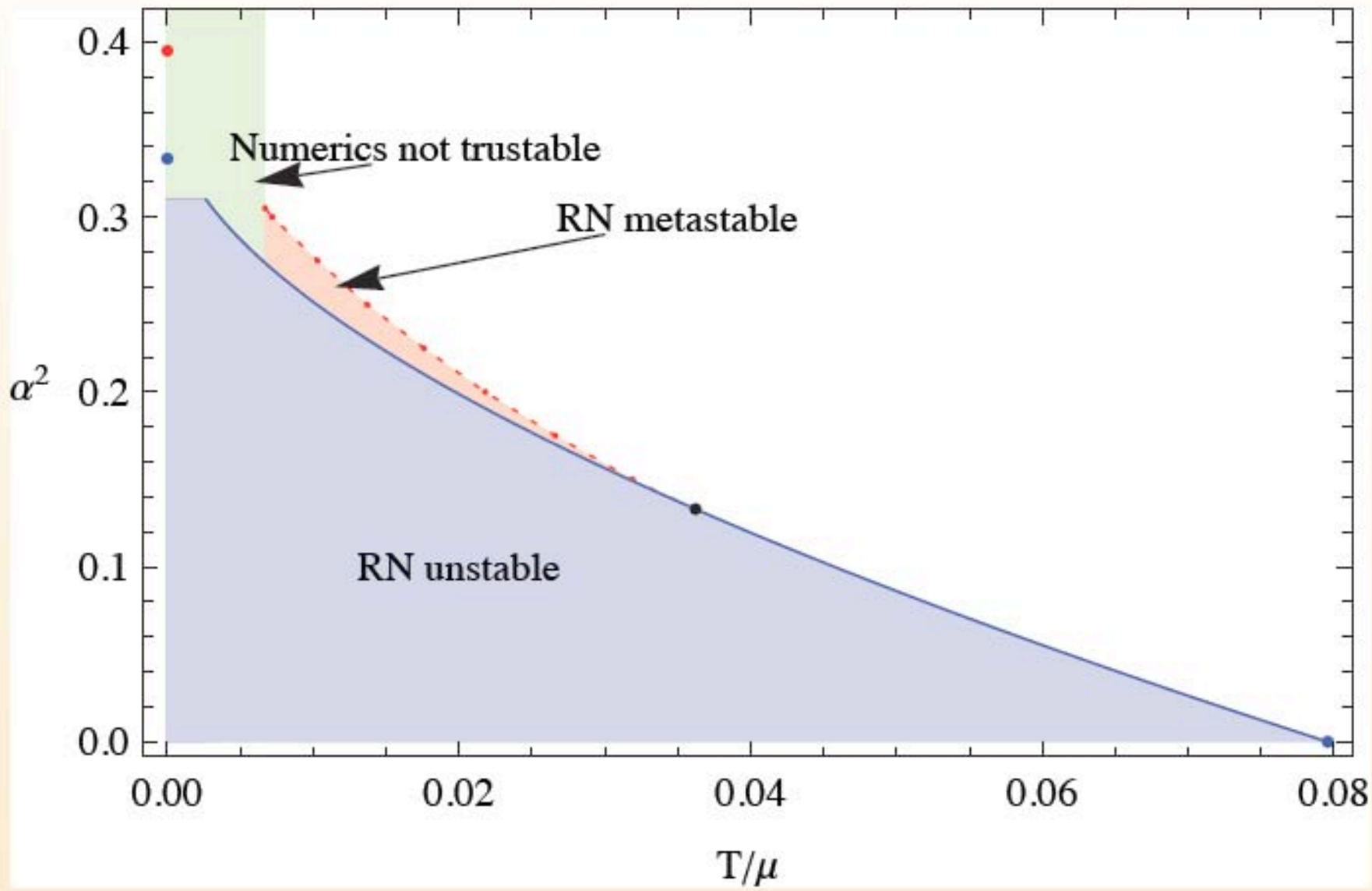
$$\langle \mathcal{J}_1^x \rangle \propto T^3 w_1^b$$

Phase transition



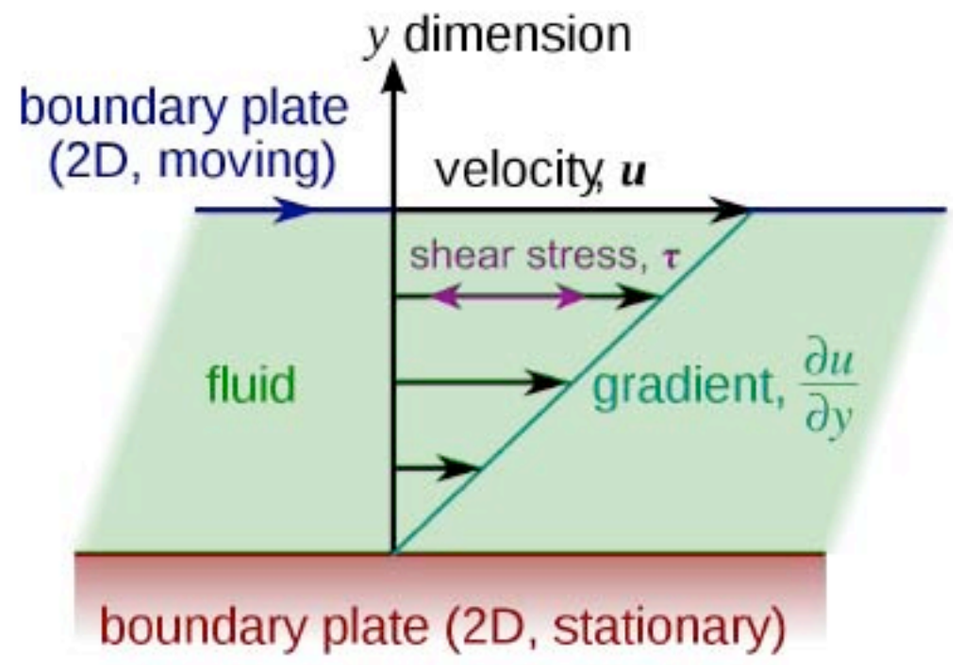
Phase transition becomes first order above α_{crit}

Phase diagram



Anisotropic shear viscosity

- viscosity tensor $\eta_{\alpha\beta\gamma\delta}$
- anisotropic systems:
  21 components
- isotropic systems:
  1 shear viscosity
- transversely isotropic systems:
  2 shear viscosities



Holographic calculation: J.E., Kerner, Zeller 1011.5912; 1110.0007

Classification of Fluctuations

- set $k_\perp = 0$

$\Rightarrow$ classification under $SO(2)$ rotational symmetry around x-axis possible:

	dynamical fields	constraints	# physical modes
helicity 2	$h_{yz}, h_{yy} - h_{zz}$	none	2
helicity 1	$h_{ty}, h_{xy}; a^a_y$	h_{yr}	4
	$h_{tz}, h_{xz}; a^a_z$	h_{zr}	4
helicity 0	$h_{tt}, h_{xx}, h_{yy} + h_{zz}, h_{xt};$ a^a_t, a^a_x	$h_{tr}, h_{xr}, h_{rr}; a^a_r$	4

gauge choice $h_{Mr} = 0$ and $a^a_r = 0 \Rightarrow$ 14 physical modes

Transport coefficients from Green functions

One non-trivial helicity 2 mode ➡

gives well-known result $\eta/s = 1/4\pi$

Helicity 1 modes:

- in $\vec{k} \to 0$ limit additional symmetry:
  $\mathbb{Z}_2$: $x \to -x$, $w \to -w$

$\Rightarrow$ helicity 1 modes decouple in 2 blocks:

even parity: $\{\Psi_t = g^{yy} h_{t\perp}, a^3_\perp, h_{r\perp}\}$

odd parity: $\{\Psi_x = g^{yy} h_{x\perp}, a^1_\perp, a^2_\perp\}$ $\Rightarrow$ 3 independent fields: $\Psi_x, a^1_\perp, a^2_\perp$

$\Rightarrow$ Green's function: 3×3 matrix

Linear response

- choose basis: $a_\perp^\pm = a_\perp^1 \pm \mathrm{i} a_\perp^2$

$\Rightarrow$ transform in fundamental repr. of unbroken $U(1)_3$

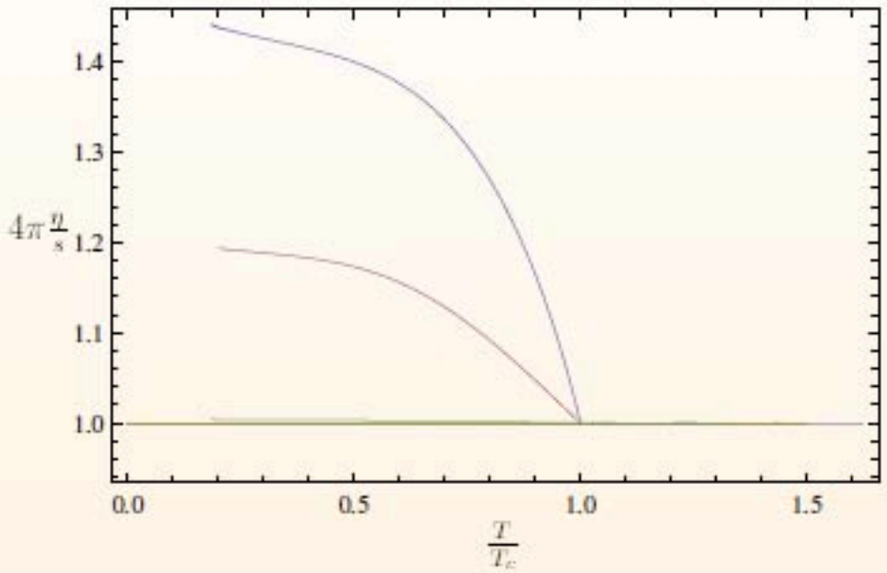
- field theory:

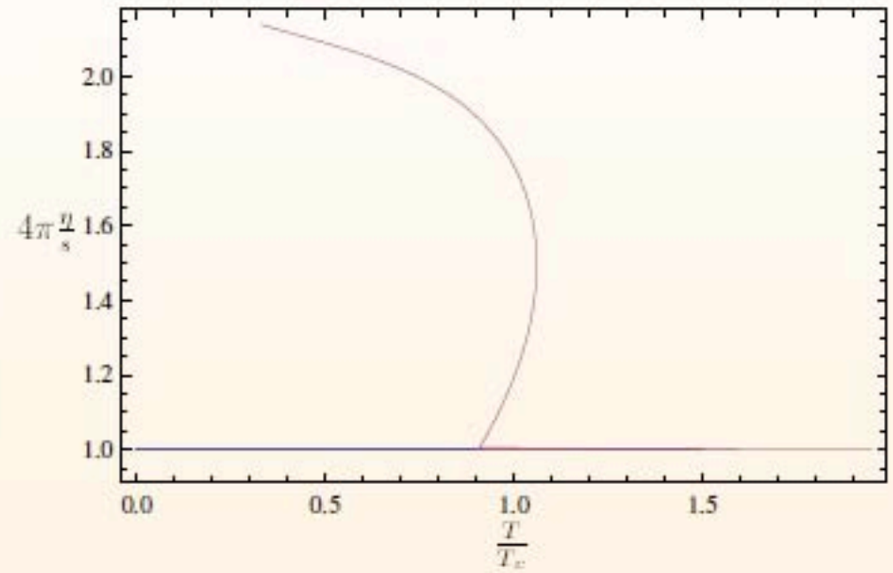
$$\begin{pmatrix} \langle J_+^\perp \rangle \\ \langle J_-^\perp \rangle \\ \langle T^{x\perp} \rangle \end{pmatrix} = \begin{pmatrix} G_{+,+}^{\perp,\perp} & G_{+,-}^{\perp,\perp} & G_+^{\perp\, x\perp} \\ G_{-,+}^{\perp,\perp} & G_{-,-}^{\perp,\perp} & G_-^{\perp\, x\perp} \\ G^{x\perp}{}_+^\perp & G^{x\perp}{}_-^\perp & -\langle T_{xx} \rangle - \mathrm{i}\omega\eta_{x\perp} \end{pmatrix} \begin{pmatrix} a_\perp^+ \\ a_\perp^- \\ h_{x\perp} \end{pmatrix}$$

- with

$$\eta_{x\perp} = -\lim_{\omega\to 0} \frac{1}{\omega} \mathrm{Im}\left(G^{x\perp,x\perp} \right)$$

Anisotropic shear viscosity





$\eta_{yz}/s = 1/4\pi$; η_{xy}/s dependent on T and on α

Critical behaviour: $1 - 4\pi\frac{\eta_{xy}}{s} \propto \left(1 - \frac{T}{T_c}\right)^{\beta}$ with $\beta = 1.00 \pm 3\%$, α-independent

Non-universal behaviour at leading order in λ and N

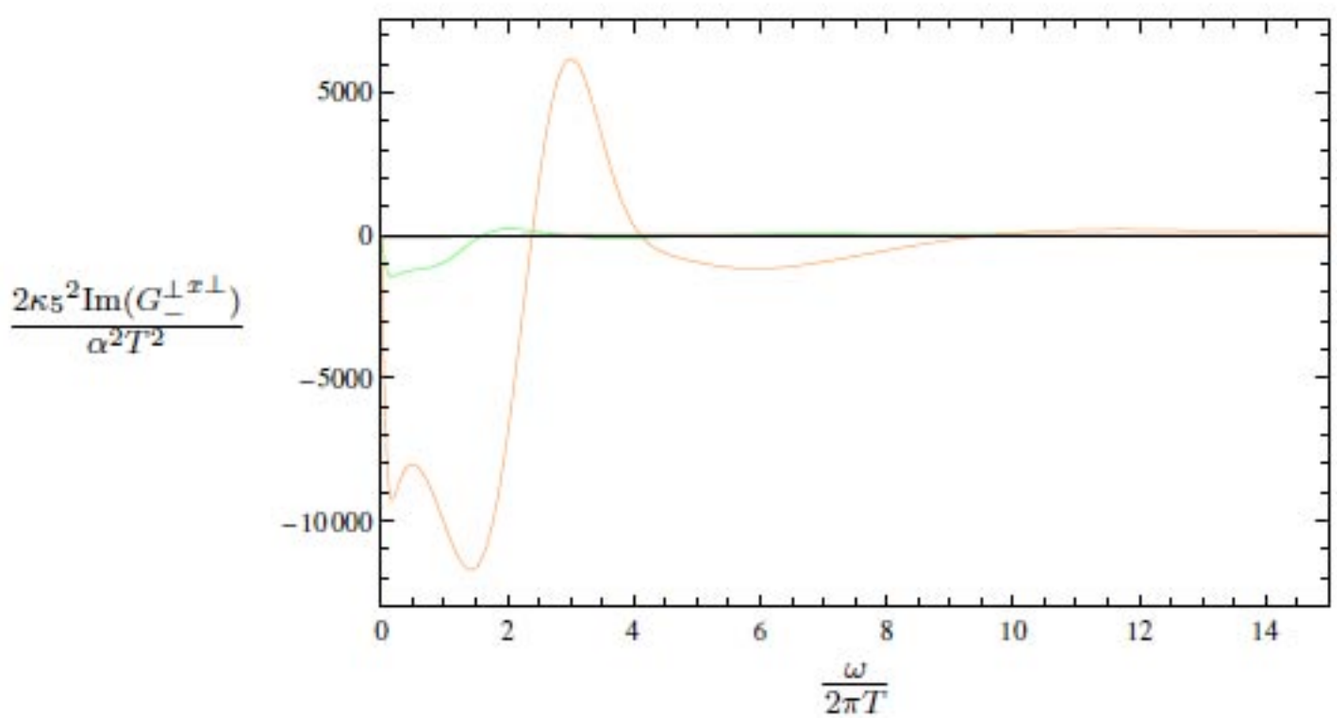
Critical exponent confirmed analytically in Basu, Oh 1109.4592

Flexoelectric Effect

Nematic phase:
A strain introduces spontaneous electrical polarization

In our case:

A strain $h_{x\perp}$ introduces an inhomogeneity in the current $\mathcal{J}_1^x$ which introduces a current $\mathcal{J}_\pm^\perp$



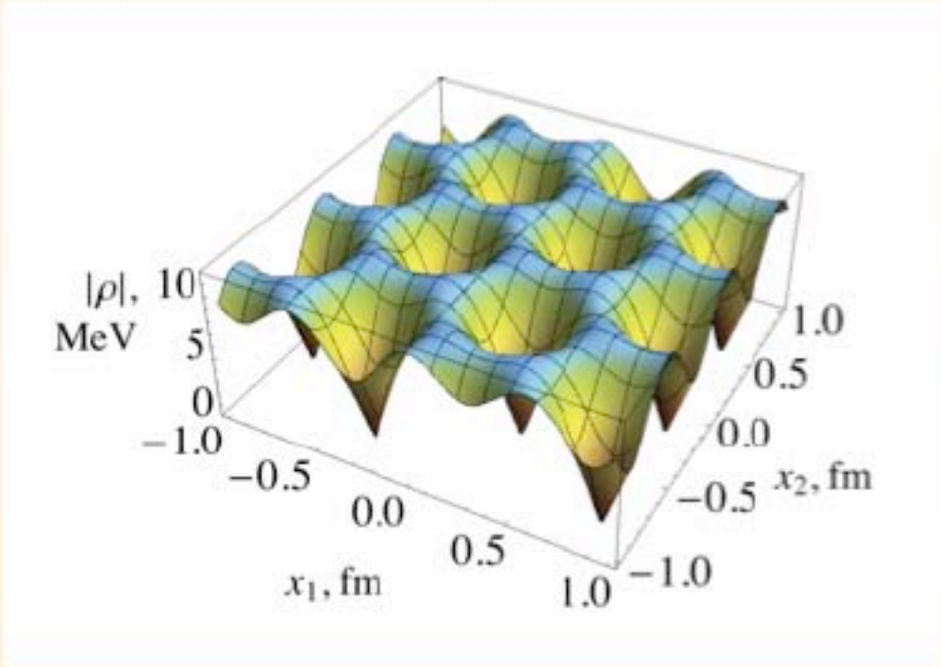
J.E., Kerner, Zeller
1110.0007

External electromagnetic fields

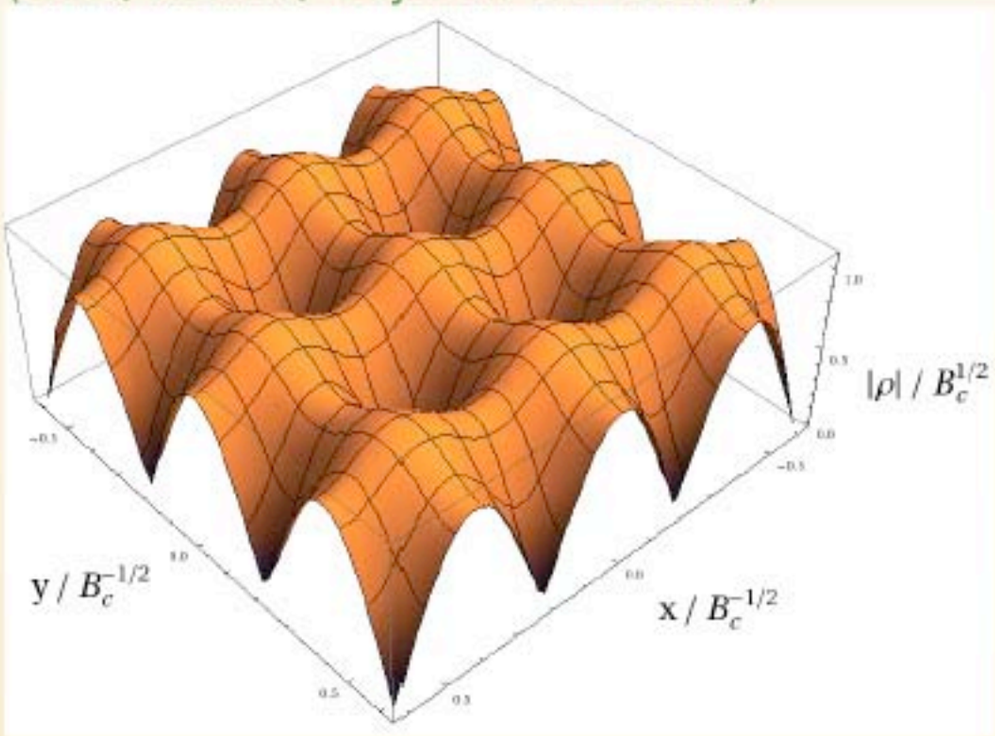
A magnetic field leads to

ρ meson condensation and superconductivity in the QCD vacuum

Effective field theory:
(Chernodub)



Gauge/gravity duality
magnetic field in black hole supergravity background
(J.E., Kerner, Strydom PLB 2011)

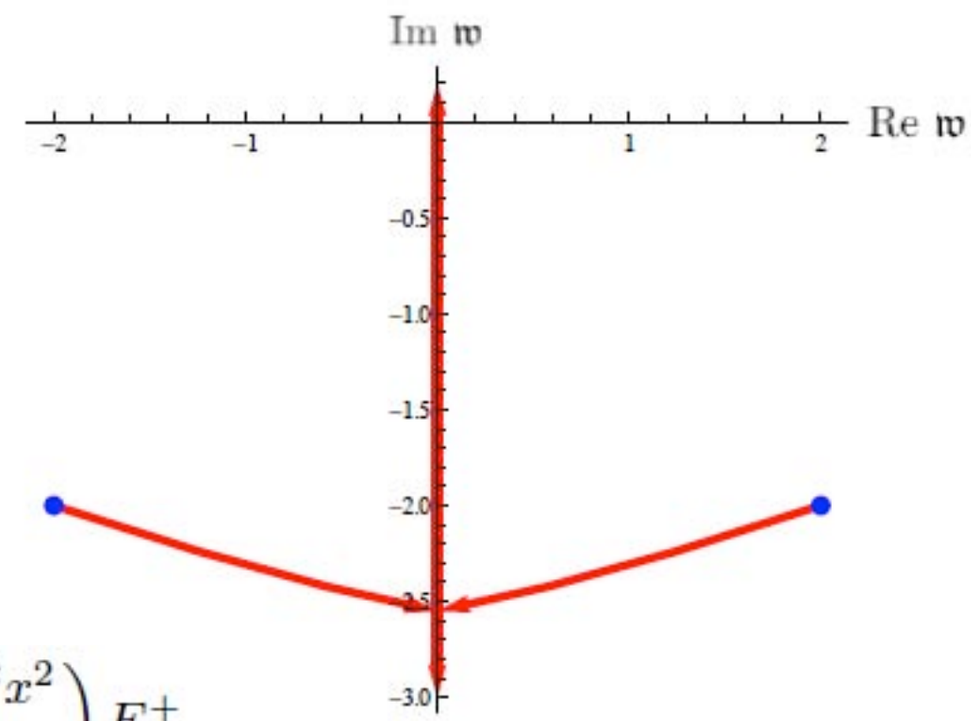


Condensation in magnetic field

$$S = \int \mathrm{d}^5x\sqrt{-g}\left[\frac{1}{2\kappa_5^2}\left(R-2\Lambda\right) - \frac{1}{4\hat{g}^2}F^a_{\mu\nu}F^{a\mu\nu}\right] + S_{\mathrm{bdy}}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon^{abc}A^b_\mu A^c_\nu$$

$$A^3_y = xB$$

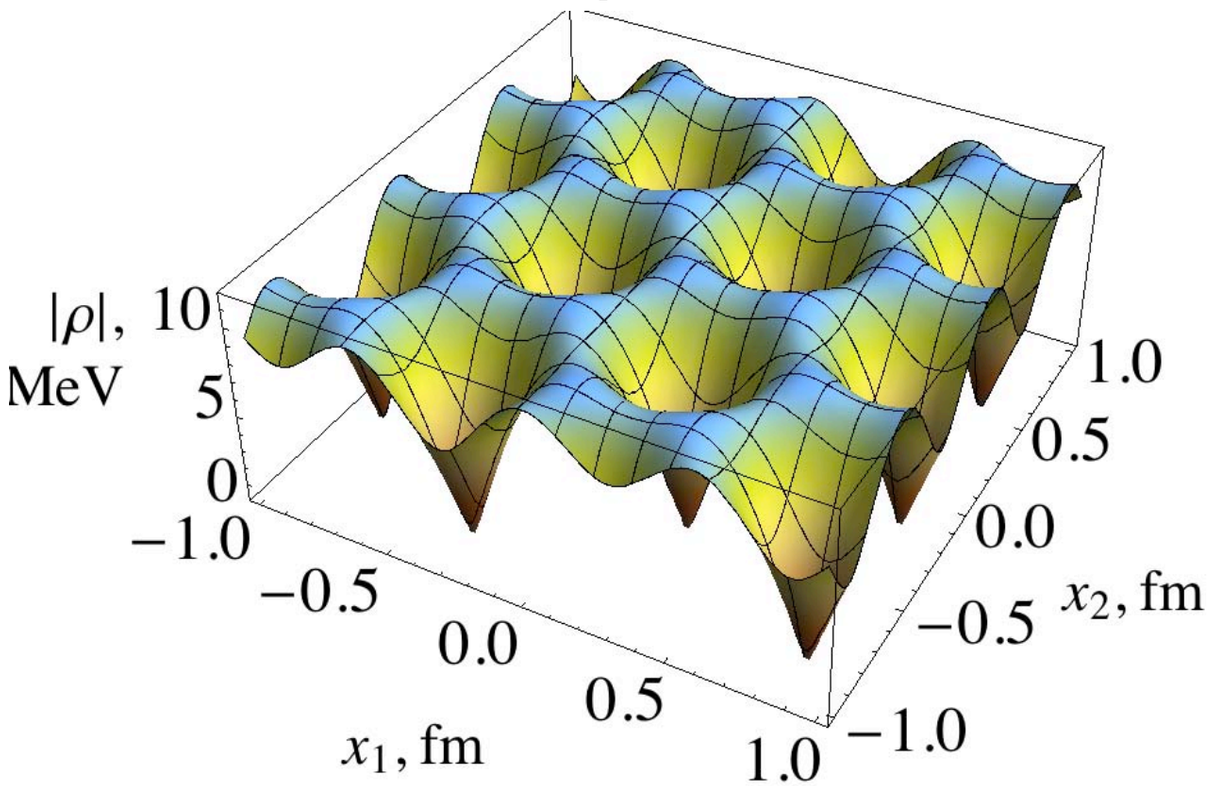


Fluctuations

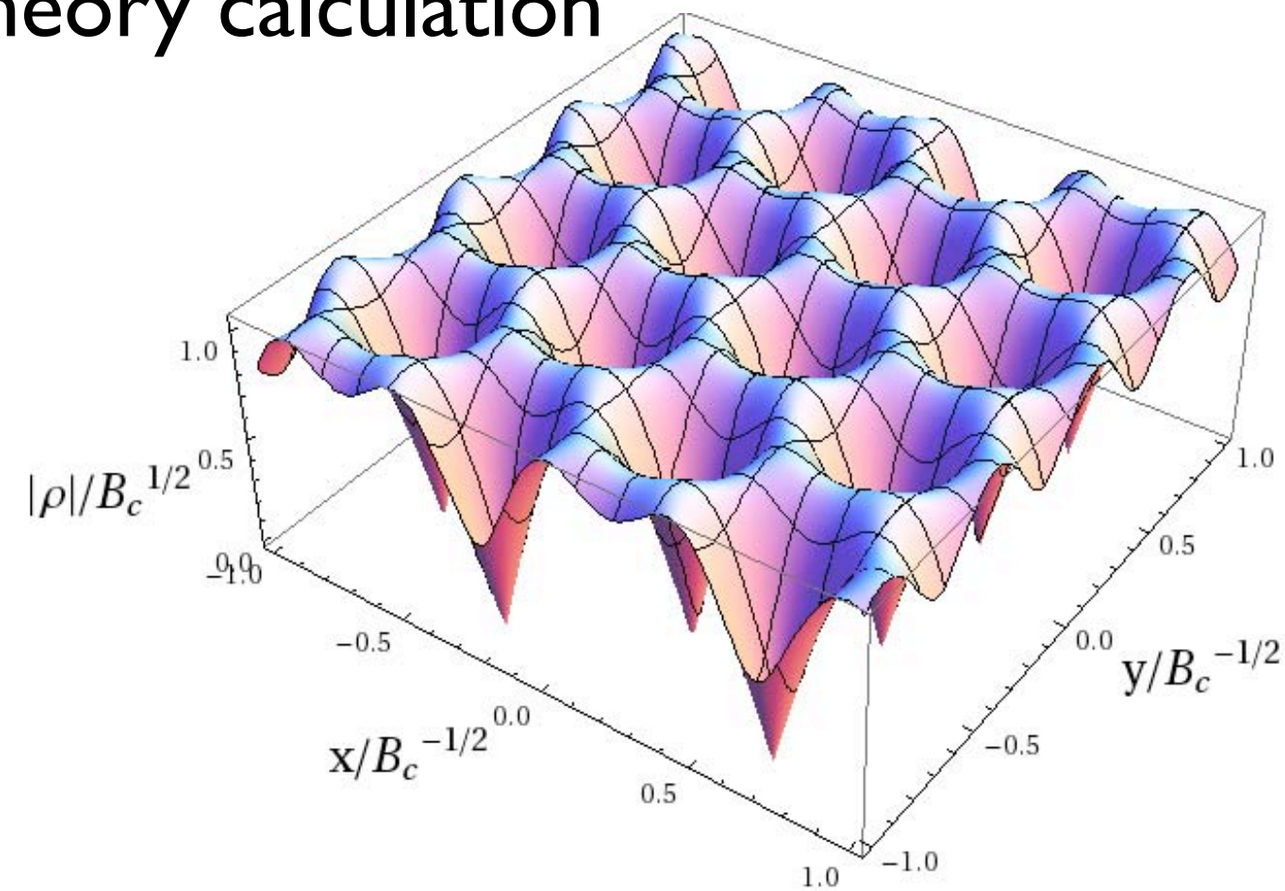
$$0 = \partial_u^2 E_x^+ + \frac{1}{f}\partial_x^2 E_x^+ + \left(\frac{f'}{f} - \frac{1}{u}\right)\partial_u E_x^+ - \frac{2}{xf}\partial_x E_x^+ + \left(\frac{\omega^2}{f^2} - \frac{B^2x^2}{f}\right)E_x^+$$

cf. Chernodub;
Callebaut, Dudas, Verschelde;
Donos, Gauntlett, Pantelidou

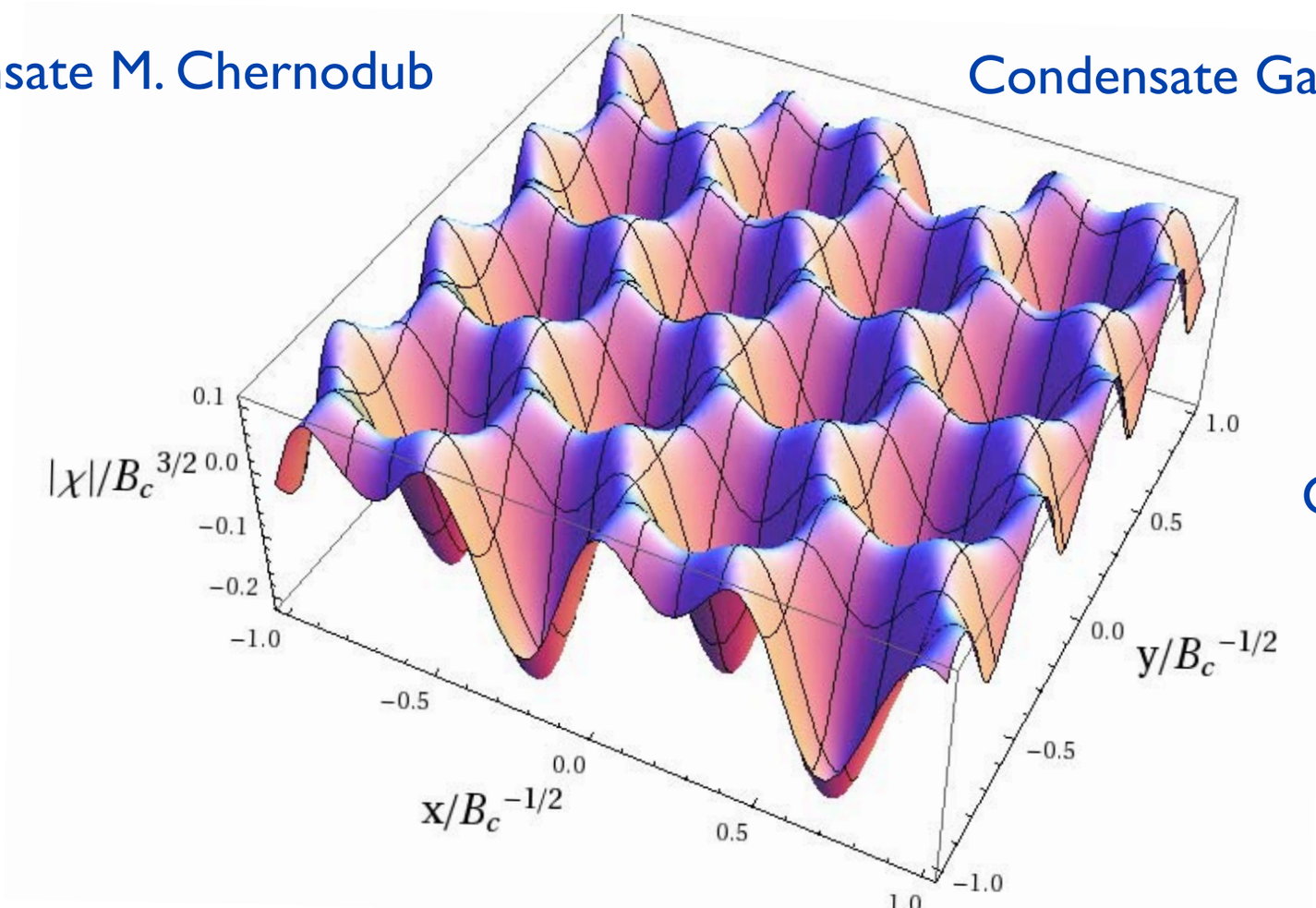
Comparison to field theory calculation



Condensate M. Chernodub



Condensate Gauge/Gravity Duality

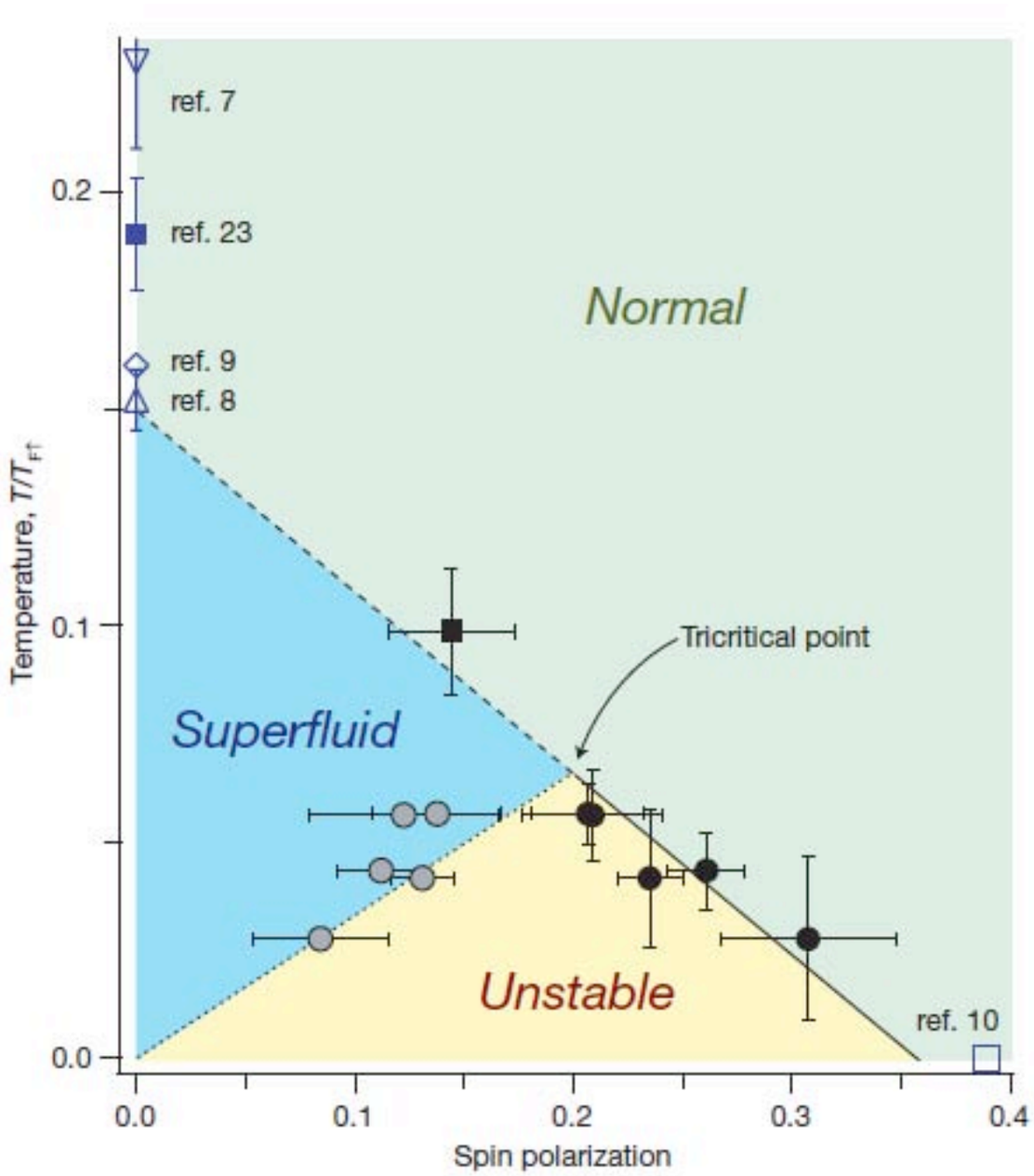


Magnetization
Gauge/Gravity Duality

Conclusions

- D3/D7 with finite isospin: Holographic p-wave superconductor with known dual field theory
- Add backreaction in bottom-up model
- Anisotropic shear viscosity:
  Non-universal contribution at leading order in N and λ
- Flexoelectric effect
- Condensation at finite magnetic field

Superfluidity in imbalanced mixtures



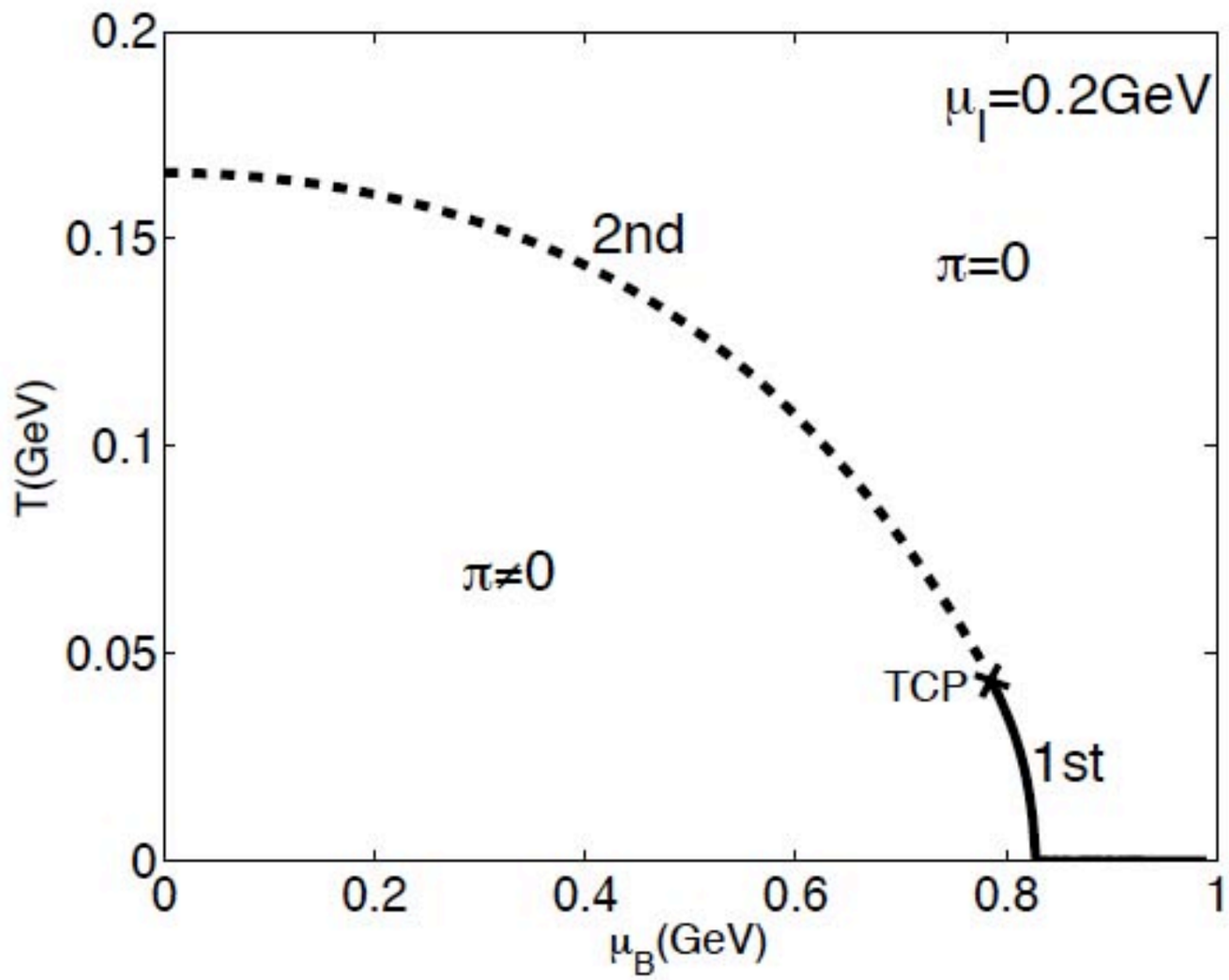
Shin, Schunck, Schirotzek, Ketterle, Nature 2008

Imbalanced mixtures

Contain different number of spin up and spin down particles

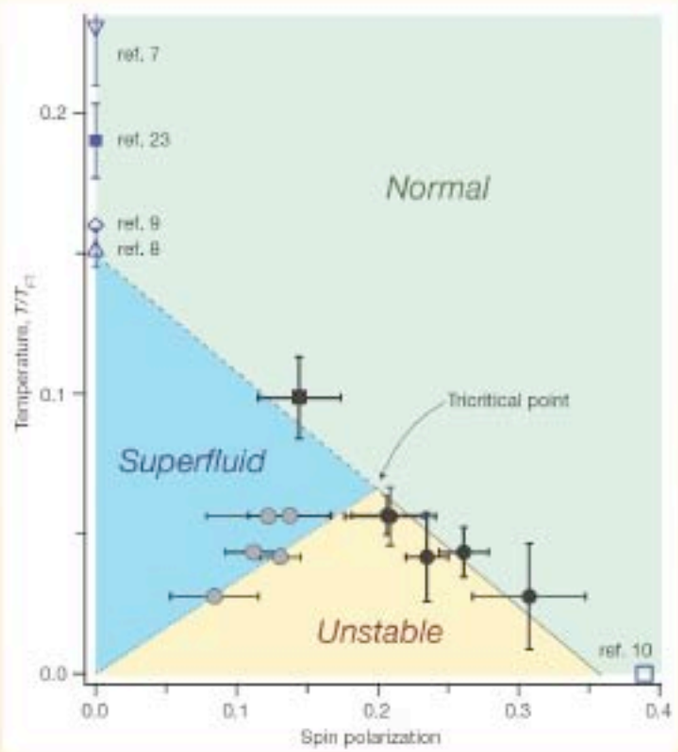
How does an imbalance in numbers (spin polarization) affect the superfluid phase transition?

QCD at finite isospin chemical potential

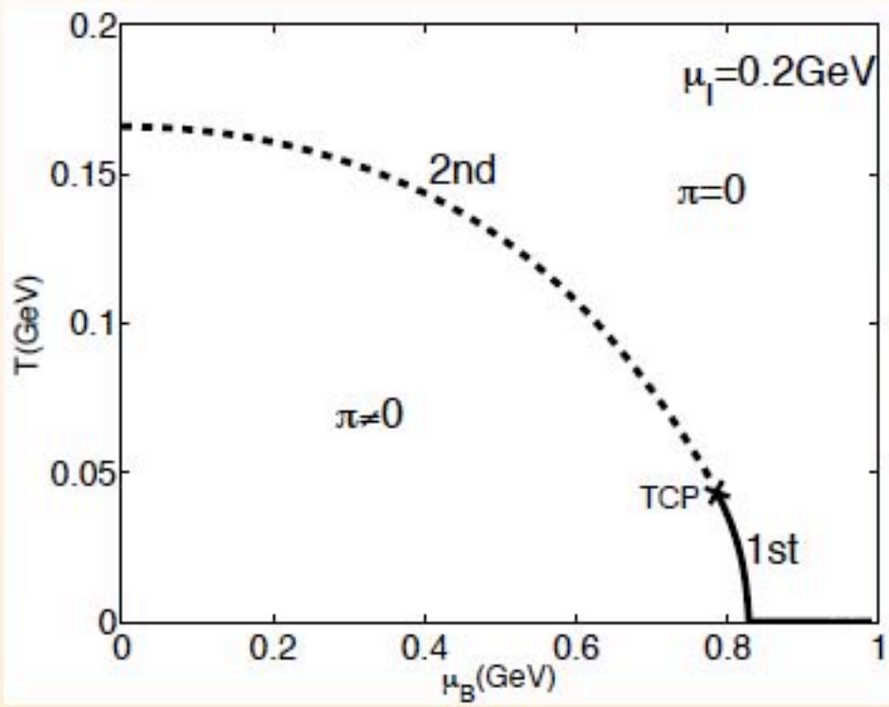


He, Jin, Zhuang, PRD 2005

Inbalanced Mixtures and Quantum Phase Transition



Shin, Schunck, Schirotzek, Ketterle, Nature 2008



He, Jin, Zhuang, PRD 2005

Lithium superfluid

QCD at finite isospin density

There appears to be universal behavior

Can we describe imbalanced mixtures in gauge/gravity duality?

Yes!

Can we obtain a similar phase diagram?

We can, in principle...

Holographic Imbalanced Mixtures

Turn on both isospin and baryon chemical potential

$$U(2) = SU(2)_I \times U(1)_B$$

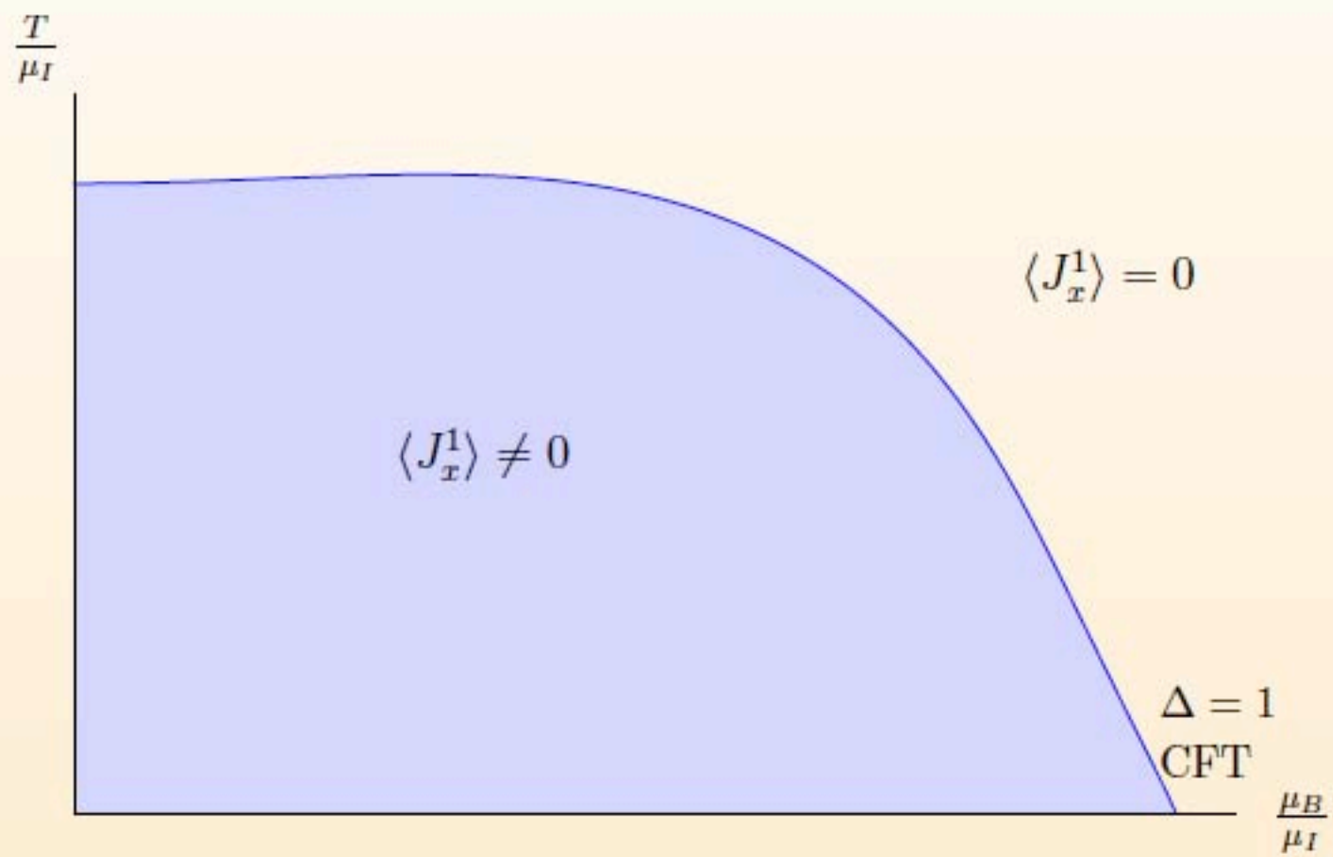
Condensate $\bar{\psi}_d \gamma_3 \psi_u$ (rho meson)

Increasing μ_B turns u into $\bar{u}$ quarks

Inbalanced Mixtures and Quantum Phase Transition

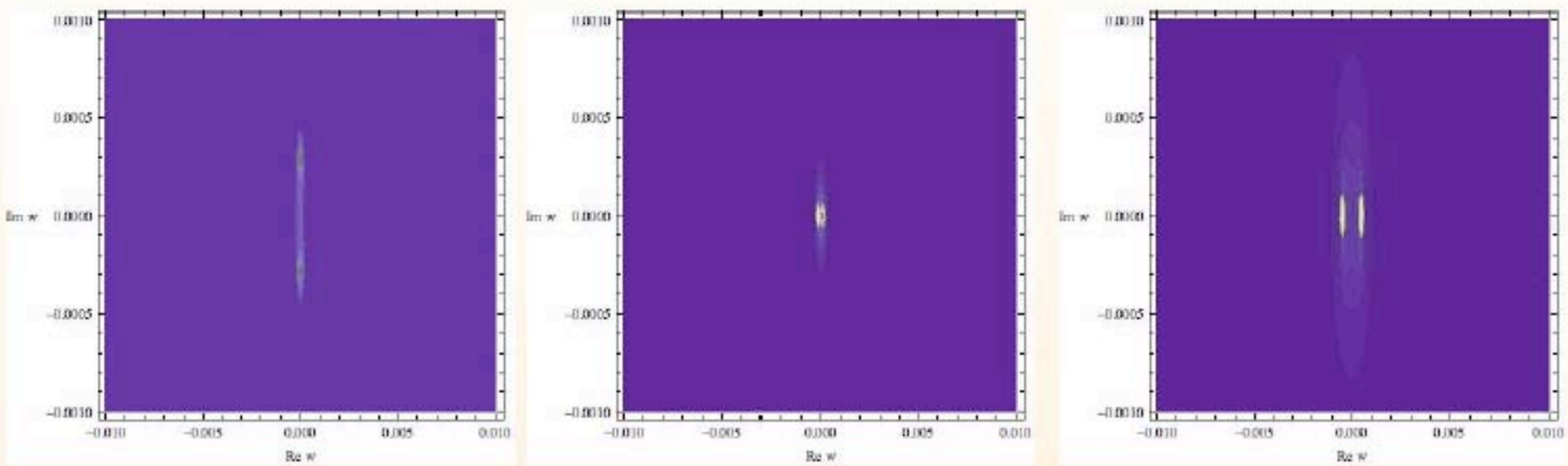
J.E., Graß, Kerner, Ngo 1103.4145

Turn on both isospin and baryon chemical potential in D3/D7 setup



Phase transition second order

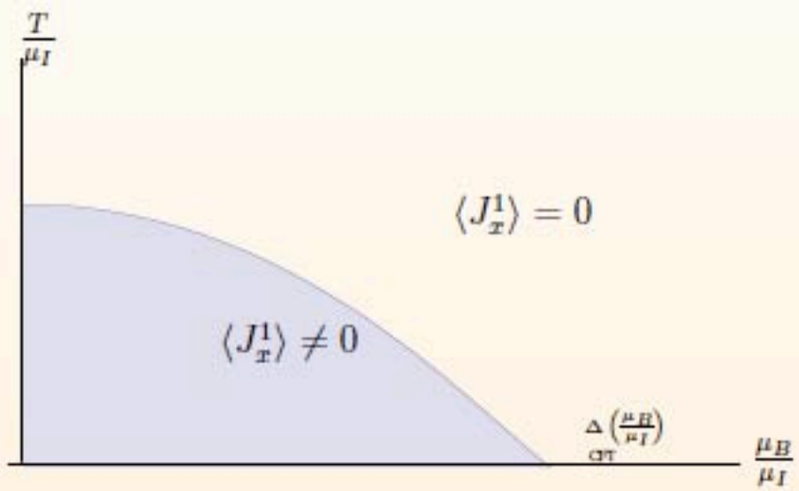
Quantum Phase Transition

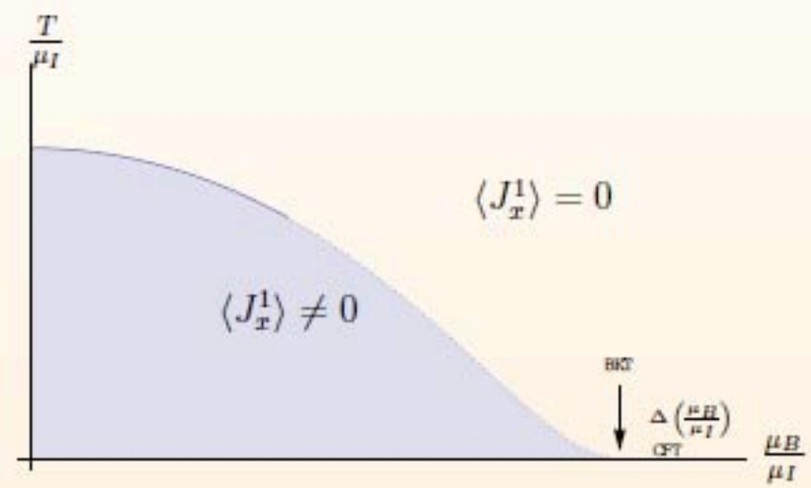


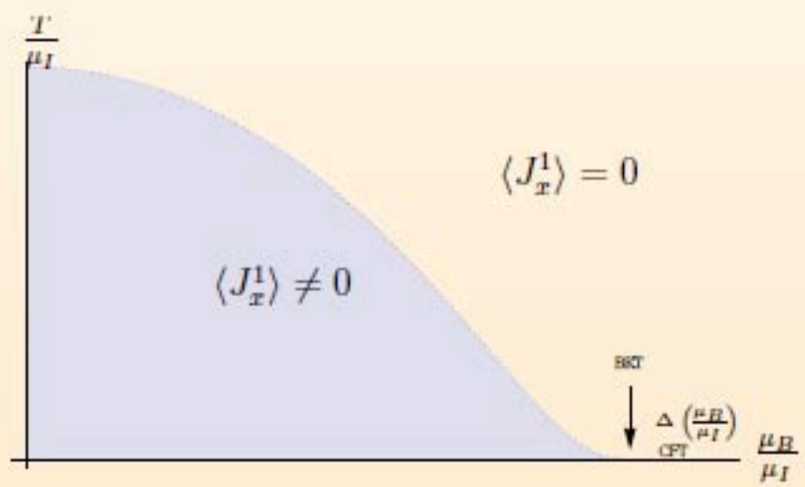
Quantum phase transition

Figure by Patrick Kerner

Example with backreaction: $SU(2)$ Einstein-Yang-Mills Model







J.E., Grass, Kerner, Ngo 1103.4145

BKT transition in gauge/gravity duality

Jensen, Karch, Son, Thompson 2010
Evans, Gebauer, Kim, Magou 2010

Order parameter scales as $\exp(-c/\sqrt{T_c - T})$

Gravity side: violation of the BF bound in the IR

IR $AdS_2 \times S^2$ region

Only possible when the two parameters have the same dimension

D3/D7 vs. backreacted model

D3/D7:

Effective IR mass of A_x^1/r vanishes,
independently of μ_B
BF bound violated along flow, but not in IR
Flavor fields directly interact with each other

Einstein-Yang-Mills:

Effective IR mass depends on μ_B/μ_I
BF bound violated in IR
$AdS_2 \times S^2$ region in IR
Flavor fields interact with gluon fields

Conclusion

- D3/D7 with finite isospin: Holographic p-wave superconductor with known dual field theory
- Add backreaction in bottom-up model
- Anisotropic shear viscosity:
  Non-universal contribution at leading order in N and λ
- Flexoelectric effect
- Add baryon chemical potential: Imbalanced mixtures
- Quantum critical point arising from AdS_2 in IR

Dilepton excess from local parity breaking in baryonic matter

Domènec Espriu

Departament d'Estructura i Constituents de la Matèria &
Institut de Ciències del Cosmos,
Universitat de Barcelona

Brookhaven, 27 June 2012

With A. Andrianov, V. Andrianov and X. Planells

Outline

- Introduction
- Parity breaking at large densities
- Parity breaking from topological fluctuations
- Some consequences of LPB
- How hadronic physics is modified by LPB
- Can this possibility be tested in HIC?
- The CERES/NA60/PHENIX/STAR 'anomaly'
- Enhanced dilepton production explained?
- Summary, conclusions and outlook

Introduction

Parity is one of the well established symmetries of strong interactions. Yet there are reasons to believe that it may be broken in some circumstances

The Vafa-Witten theorem does not apply at $\mu \neq 0$. No fundamental principle forbids spontaneus parity breaking

- *P-and CP-odd condensates = "pion" condensates*
  A Vilenkin, Phys. Rev. D22, 3080 (1980); A.B. Migdal, Zh. Eksp. Teor. Fiz. 61 (1971); T. D. Lee and G. C. Wick, Phys. Rev. D 9, 2291 (1974) (and many others); A.A.Andrianov and D.Espriu, Phys.Lett. B 663 (2008) 450; A.A.Andrianov, V.A.Andrianov and D.Espriu, Phys.Lett. B 678 (2009) 416

Introduction

Local large fluctuations in the topological charge probably exist in a hot environment.

For *peripheral* collisions they lead to the Chiral Magnetic Effect (CME): Large $\vec{B} \Rightarrow$ large $\vec{E} \Rightarrow$ charge separation

For *central* collisions (and light quarks) they correspond to an ephemeral phase with chiral chemical potential $\mu_5 \neq 0$

- *Topological fluctuations*
  D. Kharzeev, R. D. Pisarski and M. H. G. Tytgat, Phys. Rev. Lett. 81, 512 (1998); K. Buckley, T. Fugleberg, and A. Zhitnitsky, Phys. Rev. Lett. 84 (2000) 4814; D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A 803, 227 (2008); A. Andrianov, V. Andrianov, D. Espriu, X. Planells, Phys. Lett. B 710 (2012) 230.

Introduction

Thus the generic situation in heavy ion collisions could be described by a combination of baryon and chiral chemical potentials

$$\mu \Rightarrow V_0, \qquad \mu_5 \Rightarrow A_0$$

Both A_0 and V_0 will be isosinglets but the latter may trigger a pseudoscalar condensate in a non-singlet channel as we will see below.

We will present an effective meson theory description of both phenomena.

Parity breaking at large densities

Take a model with two scalar doublets $H_j = \tilde{\sigma}_j \mathbf{I} + i\hat{\pi}_j$, $j = 1, 2$

$$V_{\text{eff}} = \frac{1}{2}\mathrm{Tr}\{-\sum_{j,k=1}^{2} H_j^\dagger \Delta_{jk} H_k + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 H_1^\dagger H_1 H_2^\dagger H_2$$

$$+\frac{1}{2}\lambda_4 (H_1^\dagger H_2 H_1^\dagger H_2 + H_2^\dagger H_1 H_2^\dagger H_1) + \frac{1}{2}\lambda_5 (H_1^\dagger H_2 + H_2^\dagger H_1) H_1^\dagger H_1$$

$$+\frac{1}{2}\lambda_6 (H_1^\dagger H_2 + H_2^\dagger H_1) H_2^\dagger H_2\} - \bar{q}_R H_1 q_L + \bar{q}_L H_1^\dagger q_R$$

The constants Δ_{ij}, λ_i are constrained by QCD properties at low energies.

At least two scalar doublets are necessary to trigger a pseudoscalar condensate.

Parity breaking at large densities

This model provides a fairly accurate description of many aspects of hadronic and nuclear physics: condensation point, absence of chiral collapse, nuclear compressibility,...

The vacuum structure of this model is quite complex

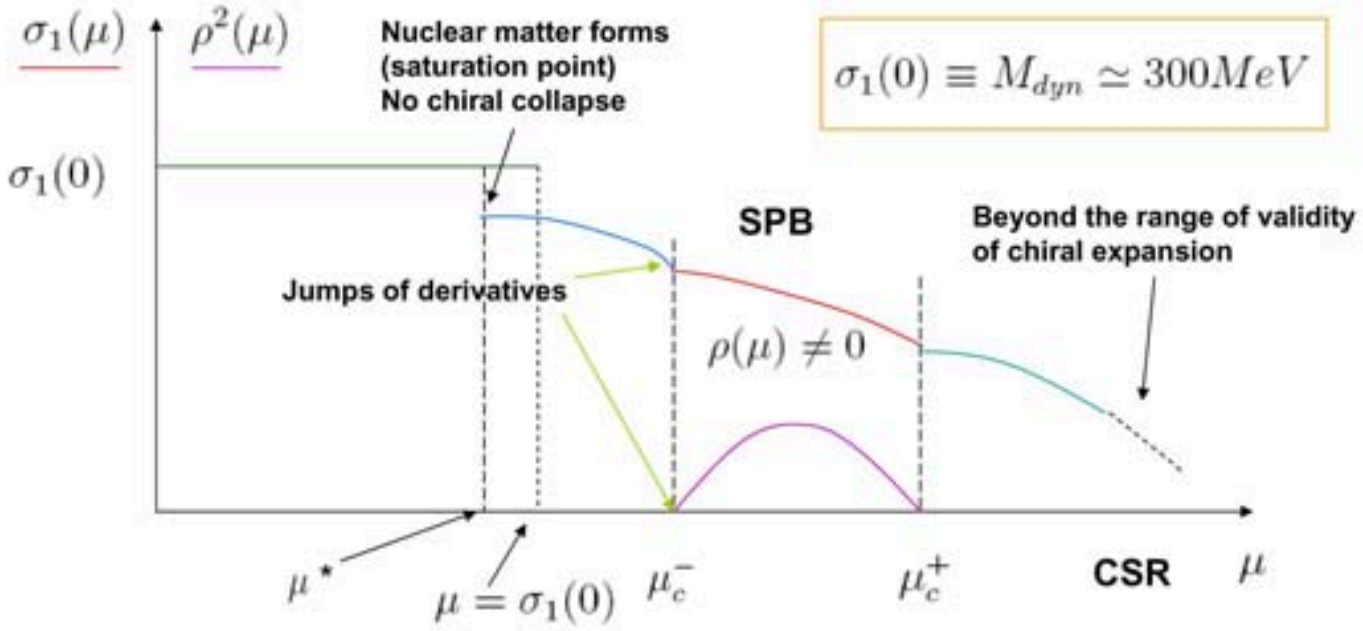
It is a generic feature of the model that, after imposing all QCD constraints, a phase with a pseudoscalar condesate appears

$$\langle \hat{\pi}_2 \rangle = \rho \tau^3 \neq 0$$

The transition of is of second order (may be an artifact of the approximation).

Parity breaking at large densities

Spontaneous P-parity breaking (2nd order phase transition)



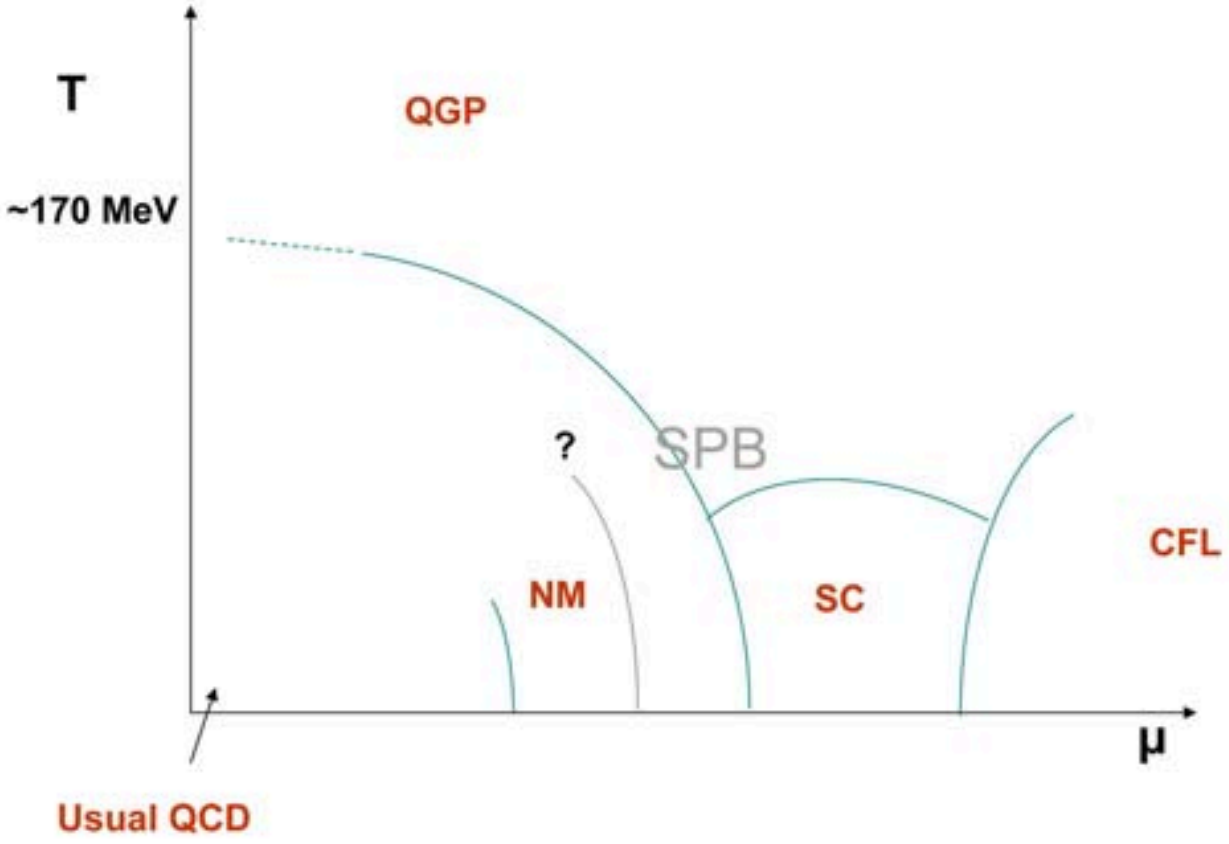
With increasing μ one enters SPB phase and leaves it before (?) encountering any new phase (CSR, CFL ...)

Order of the phase transition?

P and CP-odd effects in hot and dense matter - BNL April 2010 20

Parity breaking at large densities

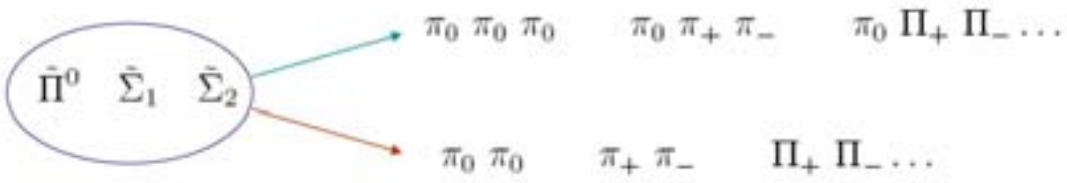
The transition is located at intermediate densities (from 3 to 6 $\times$ ρ_N), where using several isomultiplets seems particularly justified



Some consequences of parity breaking

Some features:

- Parity is no longer a good quantum number in strong interactions: $S \leftrightarrow P$
- Genuine mass eigenstates do not posess a definite parity in the decays



- Isospin symmetry broken: $SU(2) \to U(1)$ if $\langle \hat{\pi}_2 \rangle \neq 0$
- Two new Goldstone bosons may appear
- Likely to influence the equation of state of neutron stars

P-breaking phase

Mixing with massless pions is different for neutral and charged ones because vector isospin symmetry is broken

$$\tilde{\pi}^{\pm} = \pi^{\pm} + \zeta\Pi^{\pm}, \qquad \tilde{\pi}^{0} = \pi^{0} + \frac{F_0^2}{F_0^2 + A_{22}\rho^2}\Big(\zeta\Pi^0 - \frac{\rho}{F_0}\sum_{j=1}^{2} A_{j2}\partial_\mu\Sigma_j\Big).$$

Partially diagonalized kinetic term

$$\begin{aligned}\mathcal{L}^{(2)}_{kin} = {} & \partial_\mu\tilde{\pi}^{\pm}\partial^\mu\tilde{\pi}^{\mp} + (A_{22} - \zeta^2)\partial_\mu\Pi^{\pm}\partial^\mu\Pi^{\mp} + \frac{1}{2}\Big(1 + \frac{A_{22}\rho^2}{F_0^2}\Big)\partial_\mu\tilde{\pi}^0\partial^\mu\tilde{\pi}^0 \\ & + \frac{1}{2}\Big(A_{22} - \frac{F_0^2}{F_0^2 + A_{22}\rho^2}\zeta^2\Big)\partial_\mu\Pi^0\partial^\mu\Pi^0 + \frac{1}{2}\sum_{j,k=1}^{2}\frac{A_{jk}F_0^2 + \rho^2\det A\,\delta_{1j}\delta_{1k}}{F_0^2 + A_{22}\rho^2}\partial_\mu\Sigma_j\partial^\mu\Sigma_k \\ & - \frac{F_0\rho}{F_0^2 + A_{22}\rho^2}\zeta\partial_\mu\Pi^0\sum_{j=1}^{2}A_{j2}\partial^\mu\Sigma_j\end{aligned}$$

Isospin breaking $\quad SU_V(2) \to U(1)$

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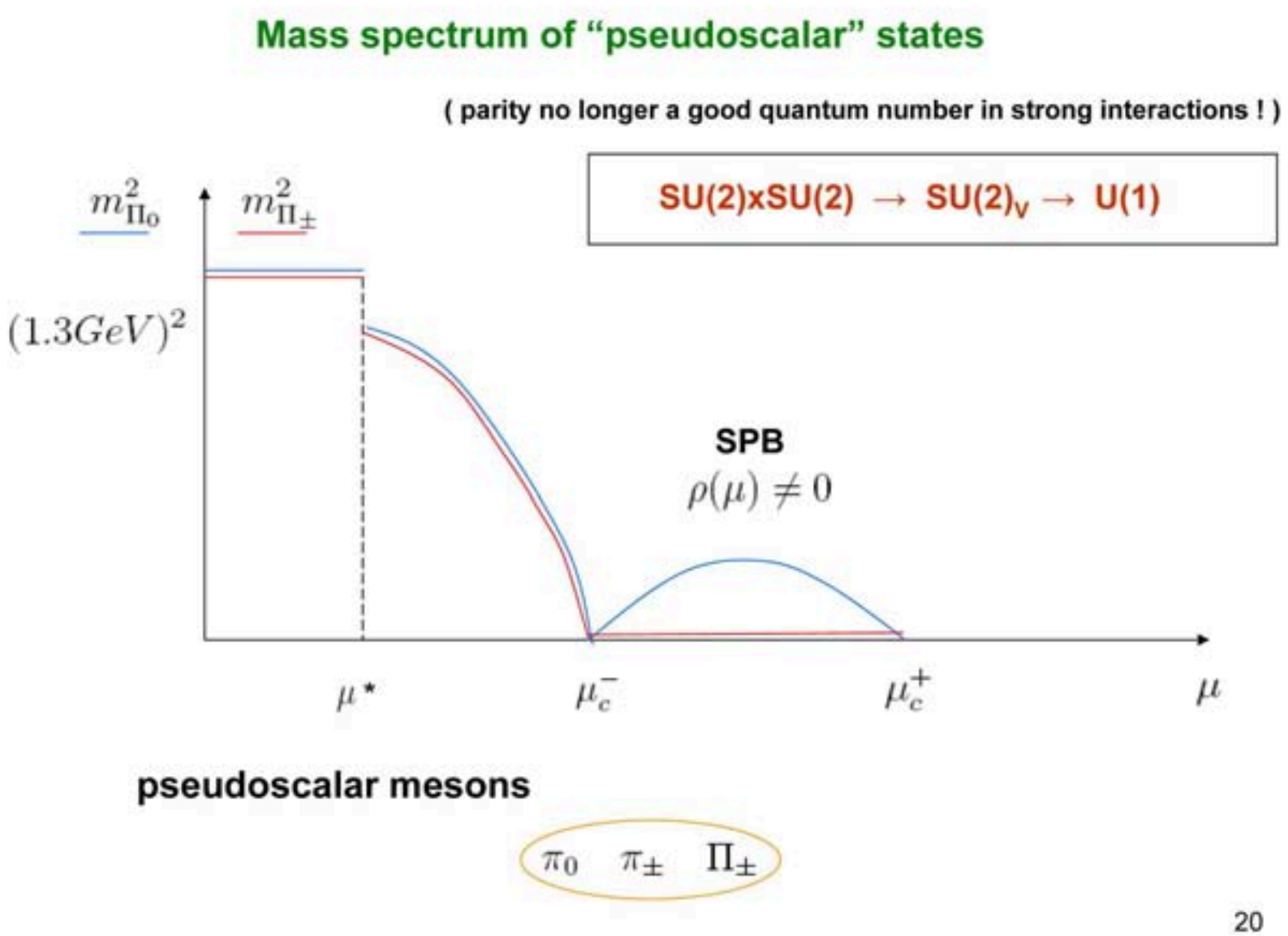
Phenomenological consequences of SPB



Physical scenario

In a HIC the nuclear matter is first compressed and heated during < 0.5 fm, then cools down for 5-10 fm until freeze-out. We shall assume that a situation of quasi-equilibrium is established and that the fireball can be approximately described by a spatially homogeneous but time dependent pseudoscalar background.

The scalar meson effective theory has already been shown (the pseudoscalar condensate will be time dependent: $\langle \hat{\pi}_2 \rangle = \rho(t)\tau^3$)

In addition we get a new contribution via the anomaly

$$\Delta\mathcal{L} \simeq \varepsilon^{\mu\nu\rho\sigma} \mathrm{Tr}\left[\hat{\zeta}_\mu V_\nu V_{\rho\sigma}\right]$$

with $\hat{\zeta}_\mu \simeq (\hat{\zeta}, \vec{0})$ for a homogenous fireball. $\hat{\zeta}_0 \propto \partial_0 \rho(t)$ and it could be either $SU(3)_f$ singlet or diagonal part of $SU(3)_f$ octet or a mixture of the two

This term influences the properties of vector mesons.

Parity breaking from topological fluctuations

During the compression and heating period a topological charge may emerge. This can be treated by including a term

$$\Delta\mathcal{L}_{top} = \mu_\theta \Delta T_5$$

Until freeze-out the topological charge is approximately conserved.

For light quarks the creation of topological charge leads to the generation of an axial charge (anomaly equation). The axial charge is conserved too provided that the quark mass term remains subdominant.

The characteristic oscillation time is governed by inverse quark masses. For u, d quarks $1/\hat{m}_q \sim 1/5$ MeV^{-1} ~ 40 fm $\gg \tau_{\text{fireball}}$ and the left-right quark mixing can be neglected. For strange quarks as $1/m_s \sim 1/200$ MeV^{-1} ~ 1 fm $\ll \tau_{\text{fireball}}$ and the mean value of strange quark axial charge is around zero due to left-right oscillations.

Effective description in terms of chiral charge

For u, d quarks QCD with a topological charge $\langle \Delta T_5 \rangle \neq 0$ can be equally described at the Lagrangian level by topological chemical potential μ_θ or by axial chemical potential μ_5

$$\langle \Delta T_5 \rangle \simeq \frac{1}{2N_f} \langle Q_5^q \rangle \iff \mu_5 \simeq \frac{1}{2N_f} \mu_\theta,$$

$$\Delta \mathcal{L}_{top} = \mu_\theta \Delta T_5 \iff \Delta \mathcal{L}_q = \mu_5 Q_5^q$$

We have to account for the photon contribution to the singlet axial anomaly

$$Q_5^q \to \tilde{Q}_5 = Q_5^q - T_5^{\text{em}}, \quad T_5^{\text{em}} = \frac{N_c}{8\pi^2} \int_{\text{vol.}} d^3x \, \varepsilon_{jkl} \, \text{Tr} \left(\hat{A}^j \partial^k \hat{A}^l \right).$$

The following term appears

$$\Delta \mathcal{L} \simeq \varepsilon^{\mu\nu\rho\sigma} \text{Tr} \left[\hat{\zeta}_\mu V_\nu V_{\rho\sigma} \right]$$

with $\hat{\zeta}_\mu = \hat{\zeta} \delta_{\mu 0}$ for a spatially homogeneous fireball and $\zeta \propto \mu_5$

Effective meson theory with chiral charge

The scalar part of the lagrangian can be estimated by using the spurion technique

$$D_\nu \Longrightarrow D_\nu - i\{\mu_5 \delta_{0\nu}, \cdot\} = D_\nu - 2i\mu_5 \delta_{0\nu}$$

Two new processes are then likely to be most relevant inside the fireball thermodynamics: the decays $\eta, \eta' \to \pi\pi$ that are strictly forbidden in QCD on parity grounds.

Dimension four terms in the chiral lagrangian lead to couplings such as

$$\sim \frac{16\mu_5}{F_\eta f_\pi^2} L\, \partial\eta\, \partial\pi \partial\pi$$

A rough estimate of the partial width gives values comparable to $\Gamma_{\rho\to\pi\pi}$: if ρ's are in thermal equilibrium in the pion bath, so will the η.

New eigenstates of strong interactions with LPB

Effective lagrangian:

$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\left(D_\mu H D^\mu H^\dagger\right) + \frac{b}{2}\mathrm{Tr}\left[M(H+H^\dagger)\right] + \frac{M^2}{2}\mathrm{Tr}\left(HH^\dagger\right)$$

$$-\frac{\lambda_1}{2}\mathrm{Tr}\left[(HH^\dagger)^2\right] - \frac{\lambda_2}{4}\left[\mathrm{Tr}\left(HH^\dagger\right)\right]^2 + \frac{c}{2}(\det H + \det H^\dagger)$$

$$+\frac{d_1}{2}\mathrm{Tr}\left[M(HH^\dagger H + H^\dagger HH^\dagger)\right] + \frac{d_2}{2}\mathrm{Tr}\left[M(H+H^\dagger)\right]\mathrm{Tr}\left(HH^\dagger\right)$$

with

$$H = \xi\Sigma\xi, \quad \xi = \exp\left(i\frac{\Phi}{2f}\right), \quad \Phi = \lambda^a\phi^a, \quad \Sigma = \lambda^b\sigma^b$$

New eigenstates of strong interactions with LPB

$$\Phi = \begin{pmatrix} \eta_q + \pi^0 & \sqrt{2}\pi^+ & 0 \\ \sqrt{2}\pi^- & \eta_q - \pi^0 & 0 \\ 0 & 0 & \sqrt{2}\eta_s \end{pmatrix}, \Sigma = \begin{pmatrix} v_u + \sigma + a_0^0 & \sqrt{2}a_0^+ & 0 \\ \sqrt{2}a_0^- & v_d + \sigma - a_0^0 & 0 \\ 0 & 0 & v_s \end{pmatrix}$$

$$\begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \eta \\ \eta' \end{pmatrix}$$

For $\mu_5 = 0$, we assume $v_u = v_d = v_s = v_0 \equiv f_\pi \approx 93$ MeV.
For non-vanishing μ_5 we assume isospin symmetry and thus, we impose to our solutions to be $v_u = v_d = v_q \neq v_s$.
The coupling constants are fitted to phenomenology (MeV):

$$b = -3510100, M^2 = 1255600, c = 1252.2, d_1 = -1051.7,$$

$$d_2 = 523.21, \lambda_1 = 67.007, \lambda_2 = 9.3126$$

New eigenstates of strong interactions with LPB

$$\mathcal{L} = -\frac{1}{2}\begin{pmatrix}\sigma & \eta_q & \eta_s\end{pmatrix}\begin{pmatrix}-k^2+2\Sigma & i\Theta k_0 & 0\\ -i\Theta k_0 & -k^2+2Q_v & g_n\\ 0 & g_n & -k^2+2S\end{pmatrix}\begin{pmatrix}\sigma\\ \eta_q\\ \eta_s\end{pmatrix}$$

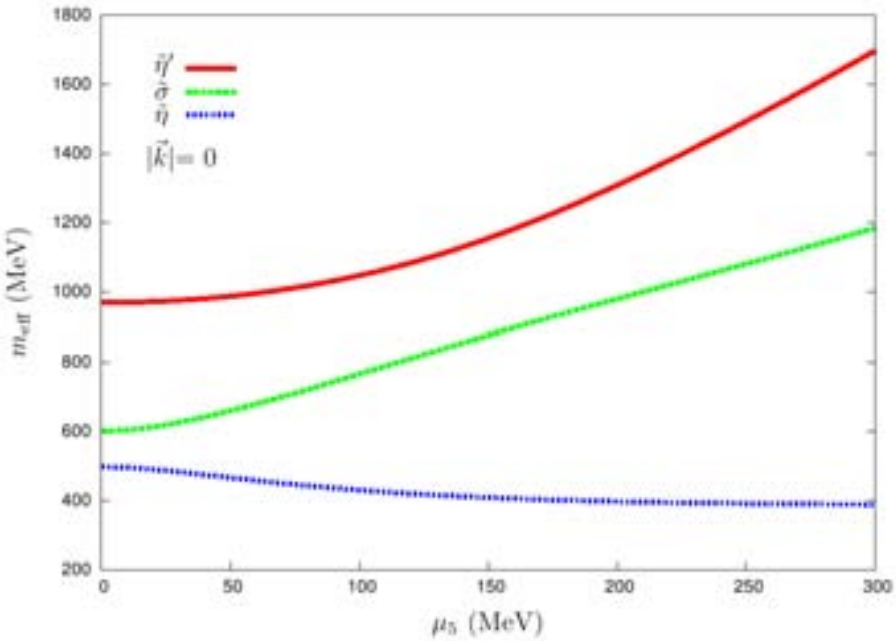
$+$ interaction terms

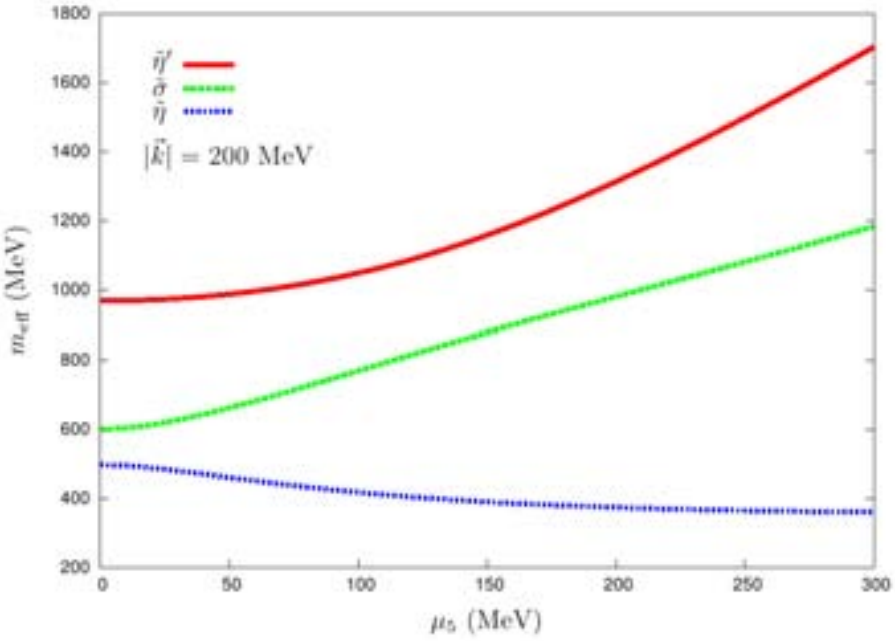
$$\begin{aligned}
\Sigma &= -(M^2 + 6d_1 m v_q + 12 d_2 m v_q + c v_s + 2 d_2 m_s v_s - 6 v_q^2 \lambda_1 \\
&\quad - 6 v_q^2 \lambda_2 - v_s^2 \lambda_2 + 2\mu_5^2) \\
Q_v &= bm/v_q + d_1 m v_q + 2 d_2 m v_q + 2 c v_s + d_2 m v_s^2/v_q \\
S &= b m_s/v_s + c v_q^2/v_s + 2 d_2 m_s v_q^2/v_s + d_1 m_s v_s + d_2 m_s v_s \\
\Theta &= 4\mu_5, \qquad g_n = 2\sqrt{2} c v_q
\end{aligned}$$

After diagonalization new eigenstates are defined: $\tilde{\sigma}$, $\tilde{\eta}$ and $\tilde{\eta}'$.
π and a_0 are also included and lead to new states $\tilde{\pi}$ and $\tilde{a}_0$.

Masses

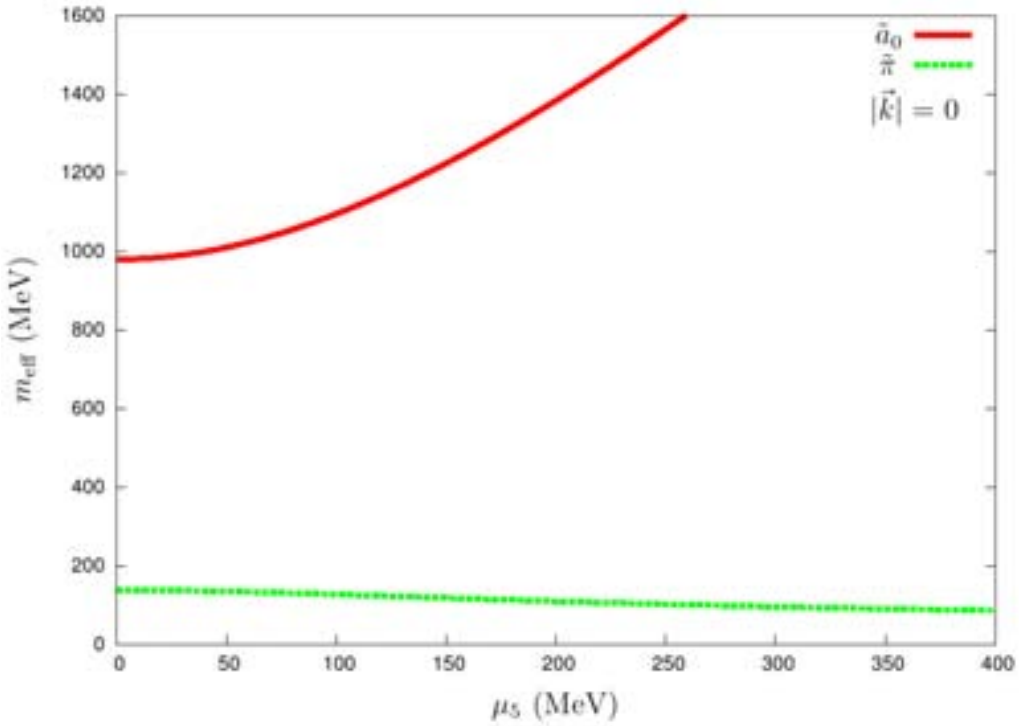
Mass dependence on μ_5 for two values of the 3-momenta



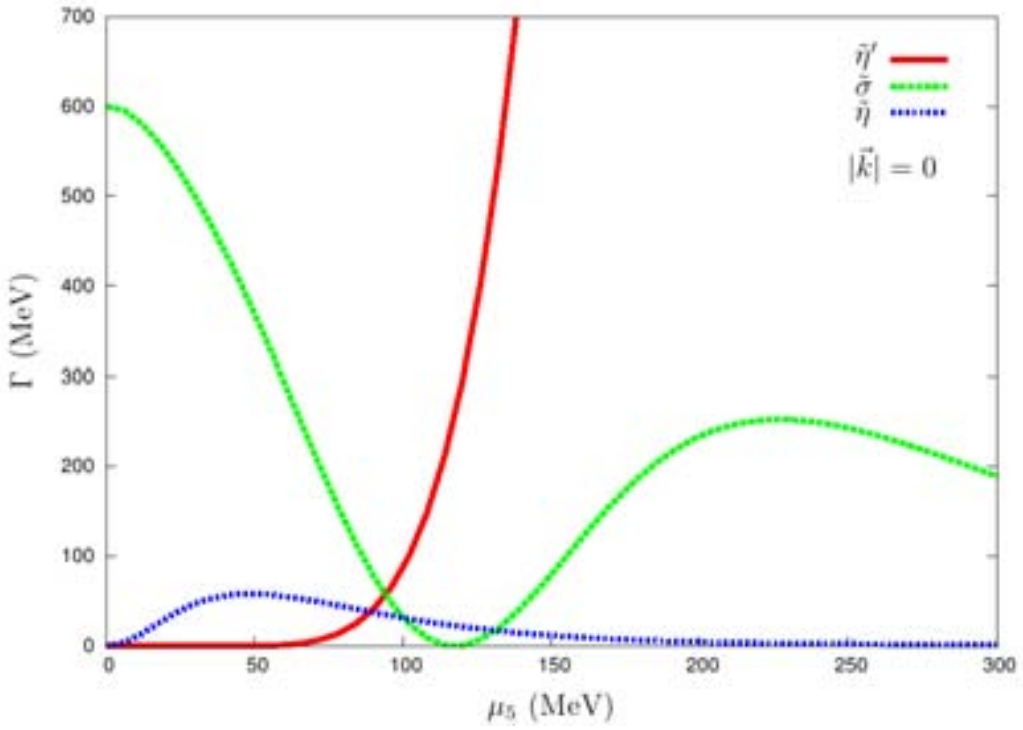


Masses

The dependence of $\tilde{\pi}$ and $\tilde{a}_0$ masses on μ_5 for $\vec{k} = 0$



Widths



Widths depend strongly on μ_5 and (unfortunately) on the details of the effective theory. $\tilde{\eta}'$ shows clear violations of unitarity $\Rightarrow$ more hadronic d.o.f. are needed

VMD and LPB

$$\mathcal{L}_{int} = \bar{q}\gamma_\mu \hat{V}^\mu q; \quad \hat{V}_\mu \equiv -eA_\mu Q + \frac{1}{2} g_\omega \omega_\mu \mathbf{I} + g_\rho \rho_\mu \frac{\tau_3}{2}$$

$$(V_{\mu,a}) \equiv (A_\mu,\, \omega_\mu,\, \rho_\mu \equiv (\rho_0)_\mu)\,,$$

where $Q = \frac{\tau_3}{2} + \frac{1}{6}$; $\quad g_\omega \simeq g_\rho \equiv g \simeq 6$

$$\mathcal{L}_{kin} = -\frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} + \rho_{\mu\nu}\rho^{\mu\nu}\right) + \frac{1}{2} V_{\mu,a} (\hat{m}^2)_{a,b} V^\mu_b,$$

$$(\hat{m}^2)_{a,b} = m_V^2 \begin{pmatrix} \frac{10e^2}{9g^2} & -\frac{e}{3g} & -\frac{e}{g} \\ -\frac{e}{3g} & 1 & 0 \\ -\frac{e}{g} & 0 & 1 \end{pmatrix}, \ \det\left(\hat{m}^2\right) = 0.$$

VMD and SPB

The parity-odd interaction affecting vector mesons is given by the term

$$\mathcal{L}_{mixing}(k) = -\frac{1}{4}\varepsilon^{\mu\nu\rho\sigma}\, tr\, \hat{\zeta}_\mu \hat{V}_\nu(x)\hat{V}_{\rho\sigma}(x) = \frac{1}{2}\zeta\epsilon_{jkl}\, V_{j,a}\, N_{ab}\, \partial_k V_{l,b}$$

For isosinglet pseudoscalar background the mixing matrix is

$$(N_{ab}) \simeq \begin{pmatrix} \frac{10e^2}{9g^2} & -\frac{e}{3g} & -\frac{e}{g} \\ -\frac{e}{3g} & 1 & 0 \\ -\frac{e}{g} & 0 & 1 \end{pmatrix}, \quad \det(N) = 0$$

For iso-triplet condensate (not considered in the following)

$$(N_{ab}) \simeq \begin{pmatrix} \frac{2e^2}{3g^2} & -\frac{e}{g} & -\frac{e}{3g} \\ -\frac{e}{g} & 0 & 1 \\ -\frac{e}{3g} & 1 & 0 \end{pmatrix}, \quad \det(N) = 0$$

VMD and SPB

Physical states:

$$\varepsilon_L^\mu = \frac{\zeta^\mu k^2 - k^\mu(\zeta\cdot k)}{\sqrt{k^2\left((\zeta\cdot k)^2 - \zeta^2 k^2\right)}}; \quad \varepsilon_{\mu,L}\varepsilon_L^\mu = -1,$$

$$K_{\mu\nu}\varepsilon_\pm^\nu = \left(k^2\mathbf{I} - \hat{m}^2 \pm \sqrt{(\zeta\cdot k)^2 - \zeta^2 k^2}\ \hat{N}\right)\varepsilon_\pm^\mu; \quad \varepsilon_-\cdot\varepsilon_+ = -1.$$

After the simultaneous diagonalization of matrices $\hat{m}^2, \hat{N}$

$$N = \mathrm{diag}\left[0,\ 1,\ 1 + \frac{10e^2}{9g^2}\right] \simeq \mathrm{diag}\,[0,\ 1,\ 1]$$

$$\hat{m}^2 = m_V^2\,\mathrm{diag}\left[0,\ 1,\ 1 + \frac{10e^2}{9g^2}\right] \simeq \mathrm{diag}\,[0,\ 1,\ 1]$$

VMD and SPB

After diagonalization

$$m^2_{V,\pm} \equiv m^2_V \pm \zeta|\vec{k}|$$

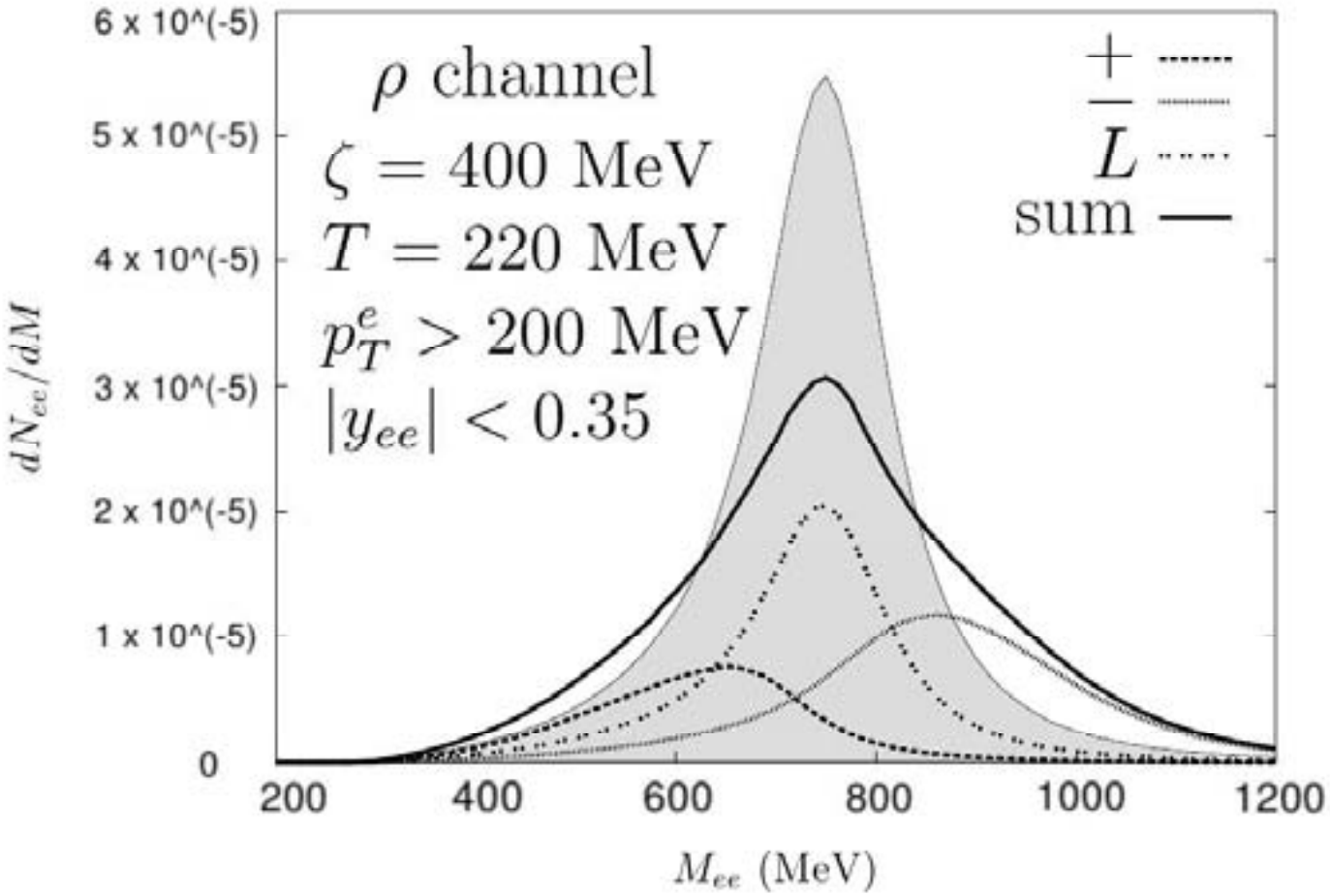
The photon itself is unaffected by a singlet $\hat{\zeta}$

The position of the poles for $\pm$ polarized mesons is moving with wave vector $|\vec{k}|$

Massive vector mesons split into three polarizations with masses $m^2_{V,-} < m^2_{V,L} < m^2_{V,+}$.

This splitting unambiguously signifies LPB. Can it be measured?

Separation of helicities



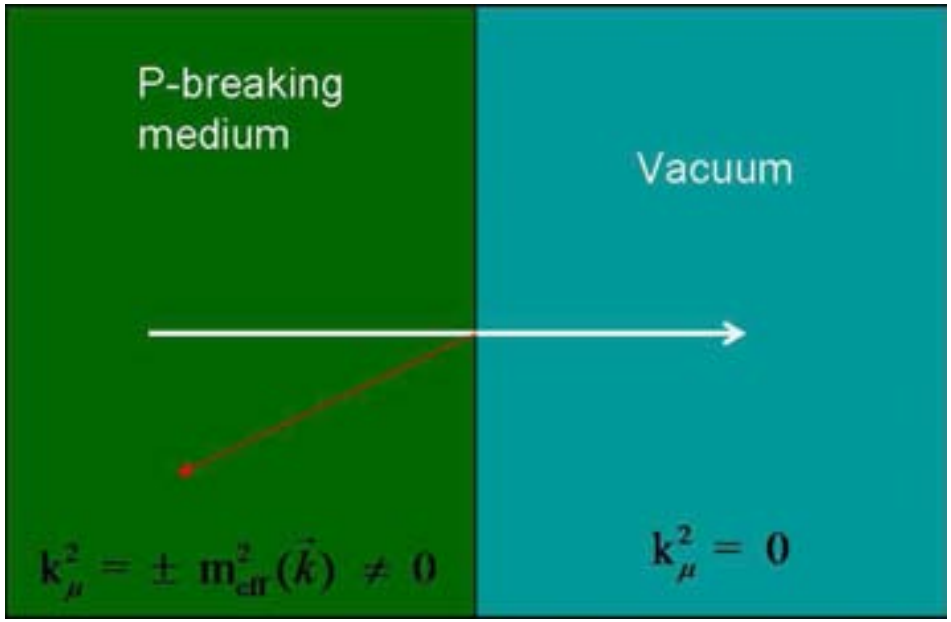
Corresponds to $\mu_5 = 290$ MeV

Bouncing back to the medium

(A.Andrianov, S.Kolevatov, R.Soldati)

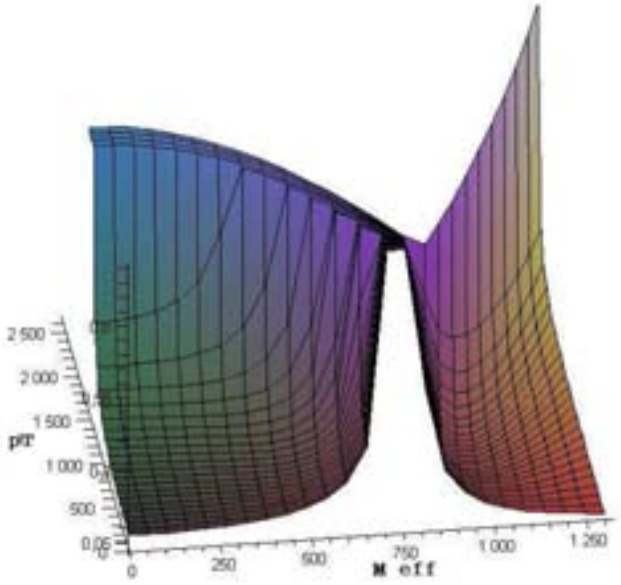
Mean free paths for vector mesons:

- $L_\rho \sim 0.8 fm$
  $\ll L_{fireball} \sim 5 - 10 fm$
- $L_\omega \sim 16 fm \gg L_{fireball}$
  LPB "vacuum"
  $\neq$ empty vacuum
  $=$ coherent state
  of vacuum mesons



Mesons have different dispersion relations on both sides of the wall: continuity of the wave function implies that transmission will be accompannied by reflection back to the medium.

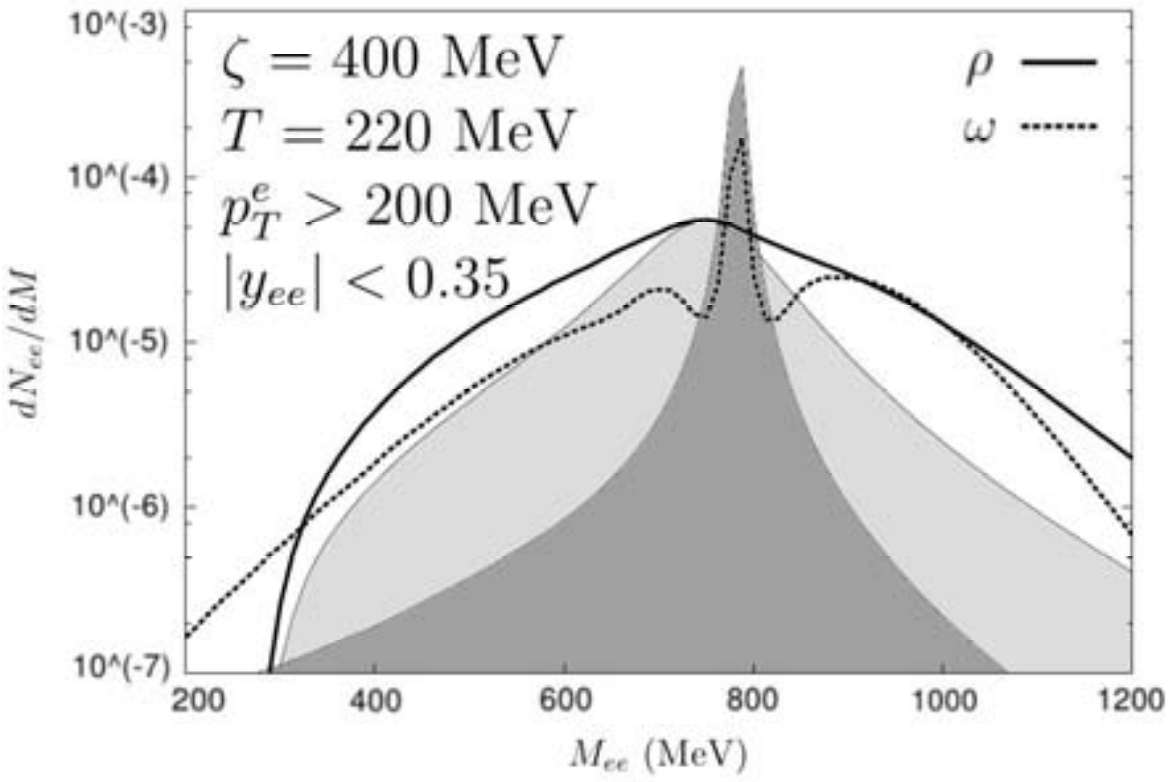
Bouncing back to the medium



The reflection coefficient approaches 1 for large p_T and close to the vacuum on-shell condition. Many ω will decay inside the firewall, in the LPB vacuum.

The ω much like the ρ will show a distorted shape.
The emission of mesons at large p_T should be suppressed.
Can this be experimentally quantitatively verified?

Distorted spectral functions



Mixing of vector mesons with their axial counterparts has not been considered. This mixing is however expected to be relatively small due to the relatively large mass differences and that $m_V \gg \mu_5$.

How to test these ideas?

Electromagnetic probes (electrons, positrons and photons, but also muons) are best suited to extract information about the possible existence of a LPB phase (and also other properties of hot/dense nuclear matter)

Do we understand the electromagnetic properties of hot and dense nuclear matter and/or the QGP?

The CERES/NA60/PHENIX/STAR 'anomaly'

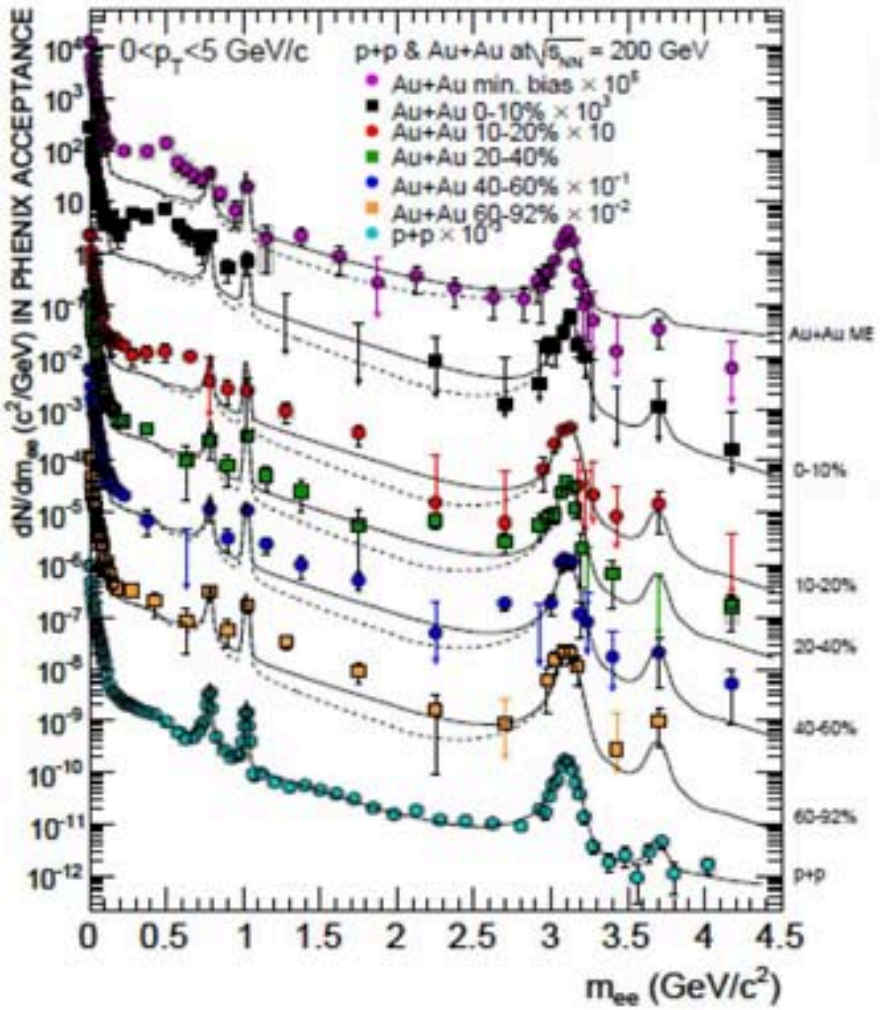


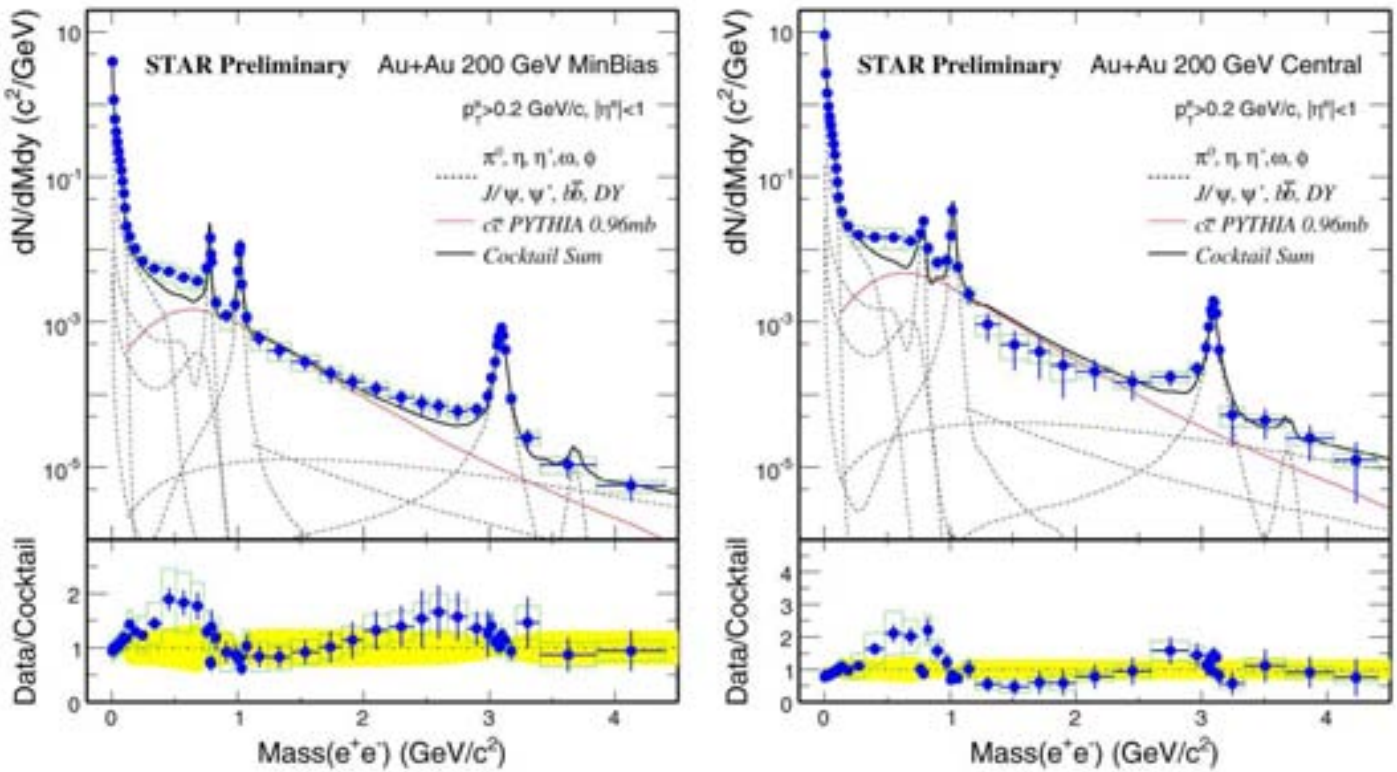
FIG. 27: (Color online) Invariant mass spectrum of e^+e^- pairs inclusive in p_T compared to expectations from the model of hadron decays for $p+p$ and for different Au + Au centrality classes.

From: 0912.0244v1 [nucl-ex]
(PHENIX Collaboration)

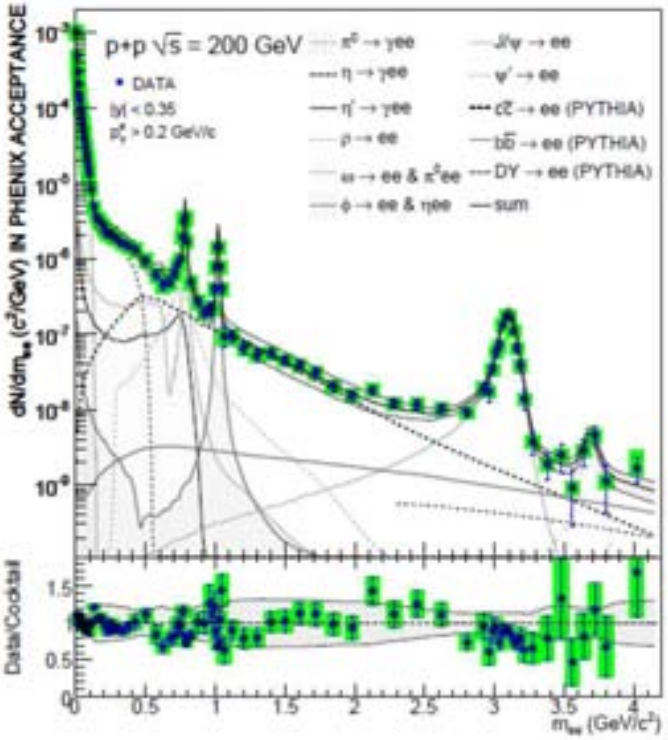
TABLE IX: The enhancement factor, defined as the ratio between the measured yield and the expected yield for $0.15 < m_{ee} < 0.75$ GeV/c^2, for different centrality bins. The meaning of the errors is defined in the text.

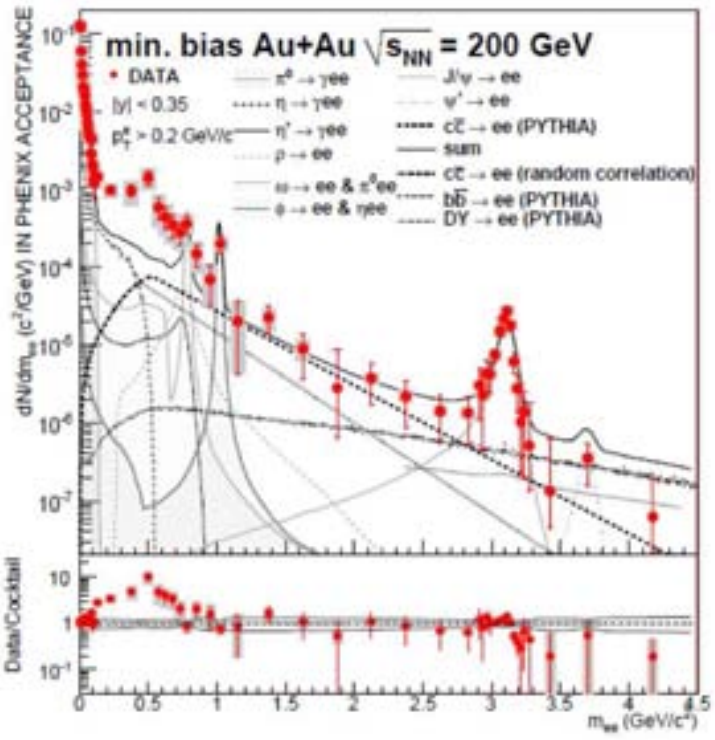
Centrality	Enhancement (±stat ±syst ±model)
00-10 %	7.6 ± 0.5 ± 1.5 ± 1.5
10-20 %	3.2 ± 0.4 ± 0.1 ± 0.6
20-40 %	1.4 ± 1.3 ± 0.02 ± 0.3
40-60 %	0.8 ± 0.3 ± 0.03 ± 0.2
60-92 %	1.5 ± 0.3 ± 0.001 ± 0.3
Min. Bias	4.7 ± 0.4 ± 1.5 ± 0.9

The CERES/NA60/PHENIX/STAR 'anomaly'



The CERES/NA60/PHENIX/STAR 'anomaly'





The CERES/NA60/PHENIX/STAR 'anomaly'

This is an old puzzle...

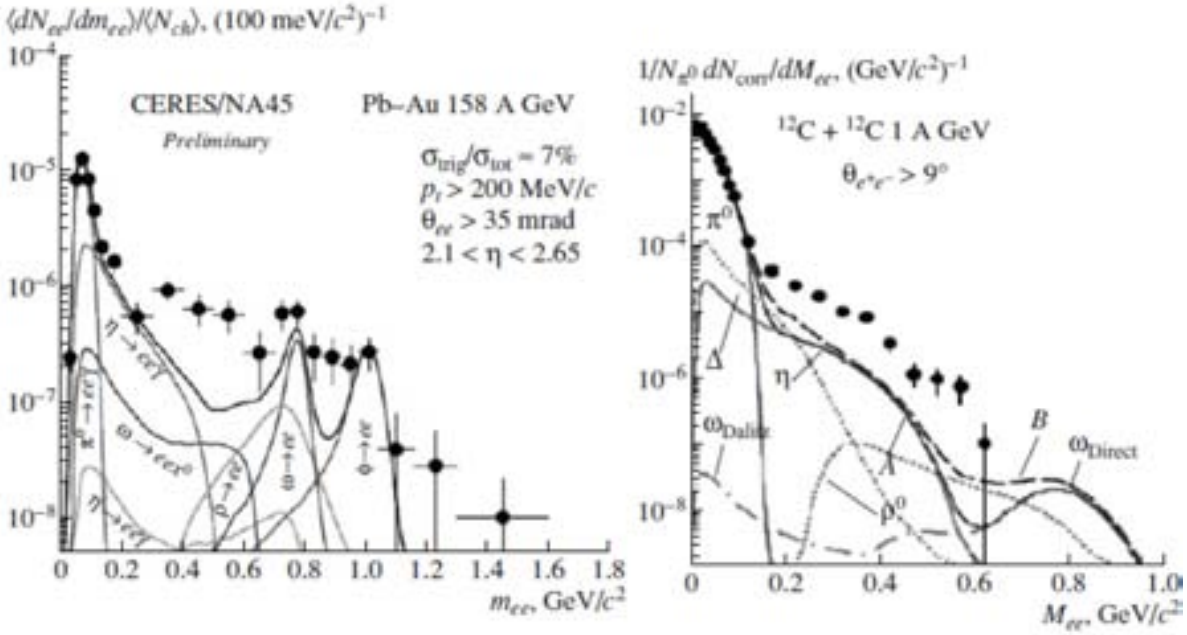


Fig. 2. CERES experiment data collected in 2000 [17].

HADES-experiment data on the dilepton yield

From K. O. Lapidus and V. M. Emel'yanov, Phys. Part. Nucl. 40 (2009) 29

Theoretical interpretations have so far failed...

Enhanced dilepton production

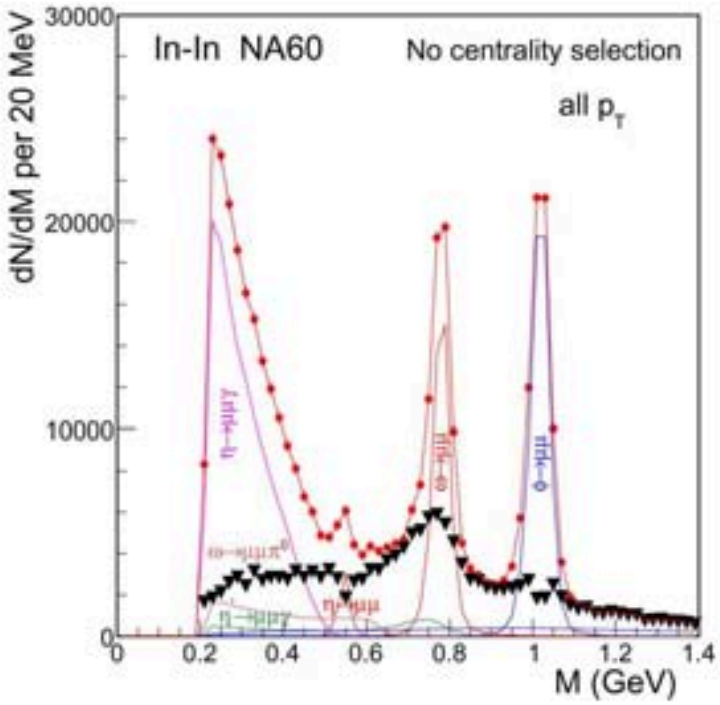


Fig. 1. Background-subtracted mass spectrum before (dots) and after subtraction of the known decay sources (triangles).

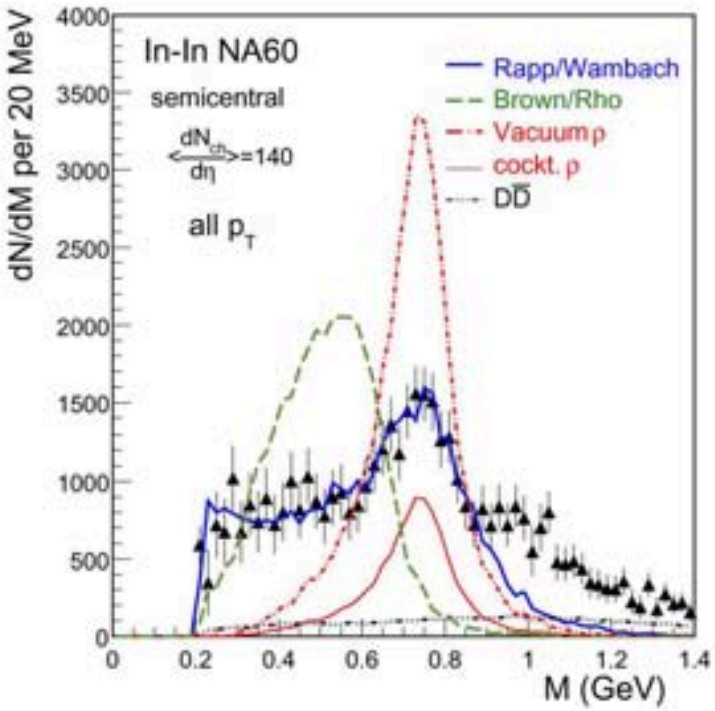


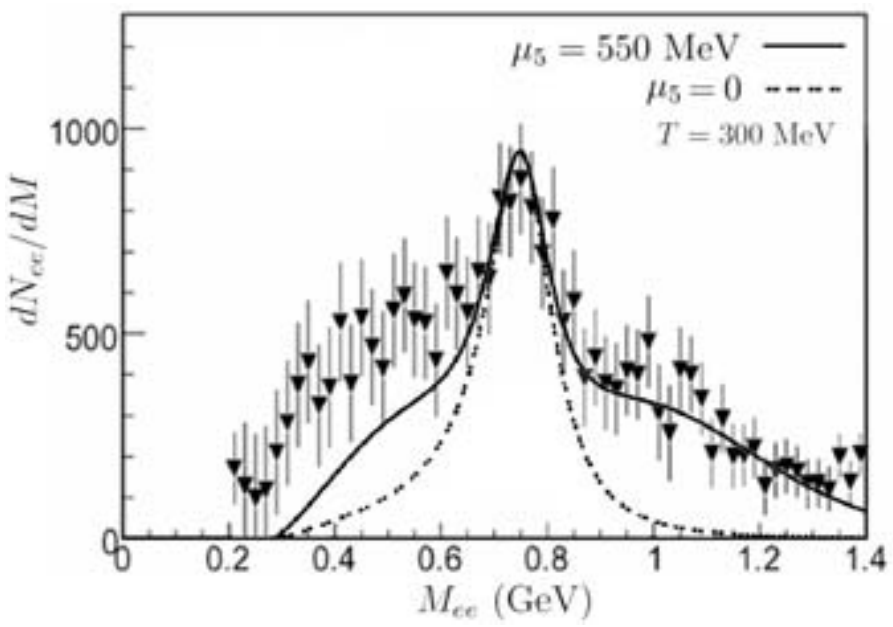
Fig. 2. Excess dimuons compared to theoretical predictions [16], renormalized to the data in the mass interval $M<0.9$ GeV. No acceptance correction applied.

Theoretical calculation 'cocktail' by Rapp and van Hees, Nucl. Phys. A806, 339 (2008)

The contribution from hadron decays is independently normalized based on meson measurements.

Modified $\rho + \omega$ contribution

Comparison with NA60 measurements (no acceptace correction)



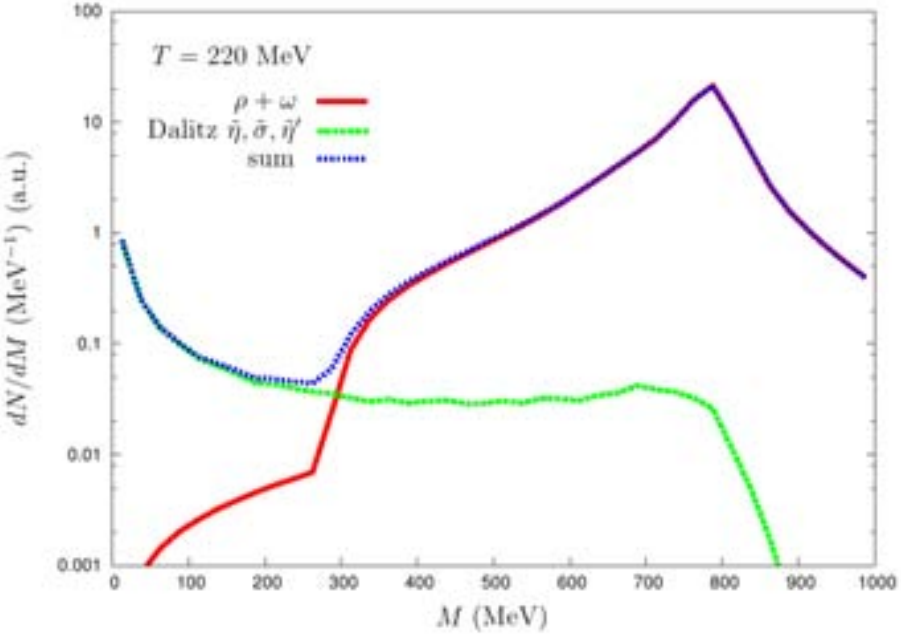
A large value of μ_5 required for best fit. μ_5 is meant to be an average, experiment dependent quantity.

LPB gives a good description for $M^2 > m_V^2$ but 'something is missing' for $M^2 < m_V^2$.

This region could receive additional contributions from the states $\tilde{\eta}, \tilde{\eta}'$ in thermal equilibrium now as well as from ω Dalitz decay.

Modified Dalitz contribution

We only show the contribution of particles in thermal equilibrium.
The ω Dalitz decay is not included
The green line represents the new contribution due to $\mu_5 \neq 0$
On top of this one has to include the contribution from the direct η η' and ω decays

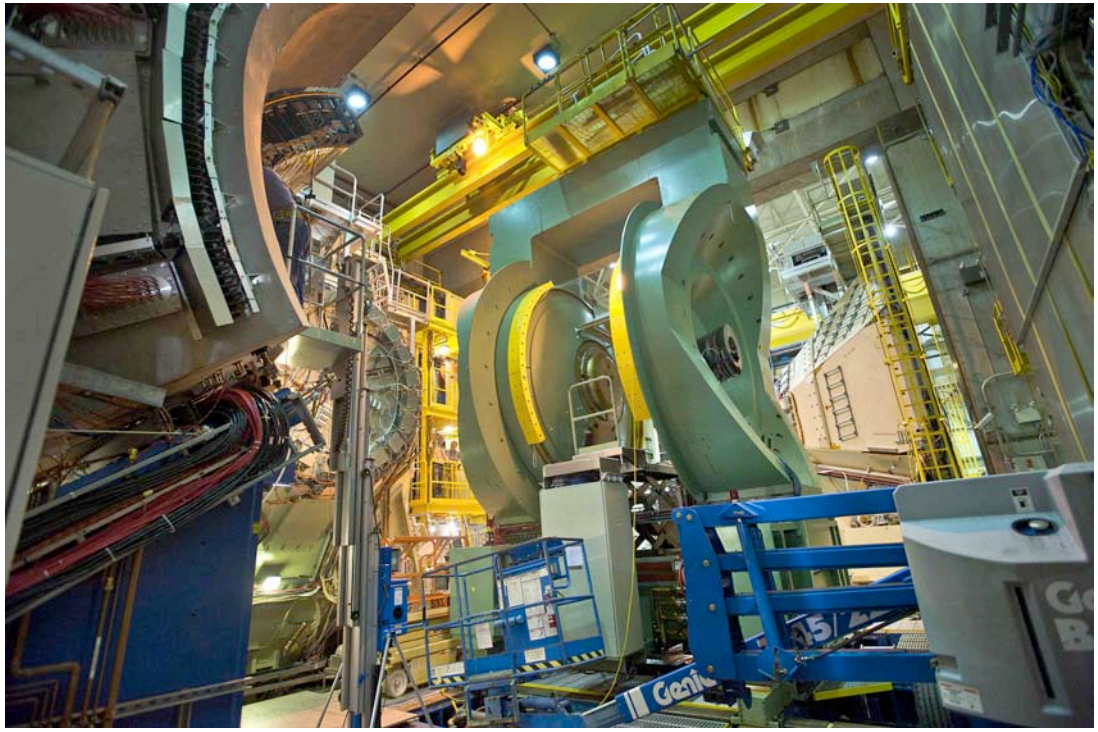


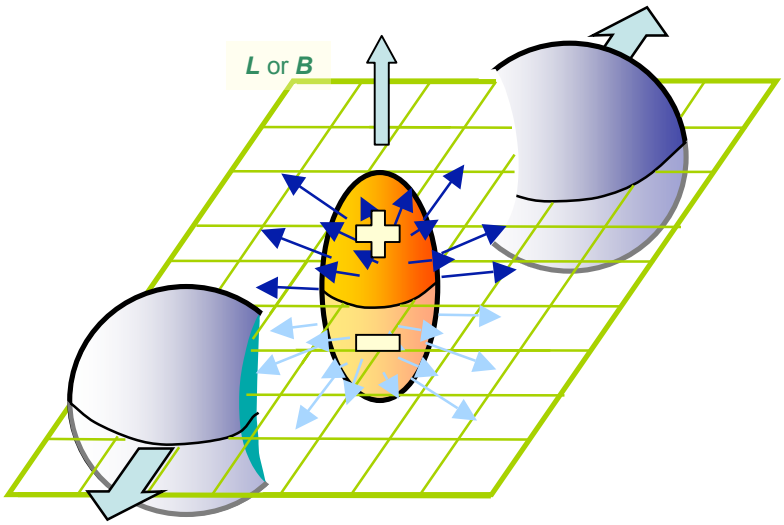
Conclusions and outlook

- Local parity breaking is not forbidden by any physical principle in strong interactions at finite chemical potential
- Topological fluctuations transmit their influence to hadronic physics via a chiral chemical potential
- LPB leads to unexpected modifications of the in-medium properties of scalar and vector mesons
- Measurement event-by-event of the lepton polarization may reveal in an unambiguous way the existence of parity violation
- The 'bouncing back effect' may change the high p_T spectrum and reinforce thermal equilibration
- LPB may help explaining the observed lepton spectrum in the LMR of PHENIX and STAR
- The possibility of breaking a fundamental symmetry is quite open!
- Other implications?

Charge asymmetric correlation measurement as a possible signature of Local Parity Violation in 200GeV Au+Au collisions at RHIC-PHENIX

ShinIchi Esumi for the PHENIX collaboration
Inst. of Physics, Univ. of Tsukuba





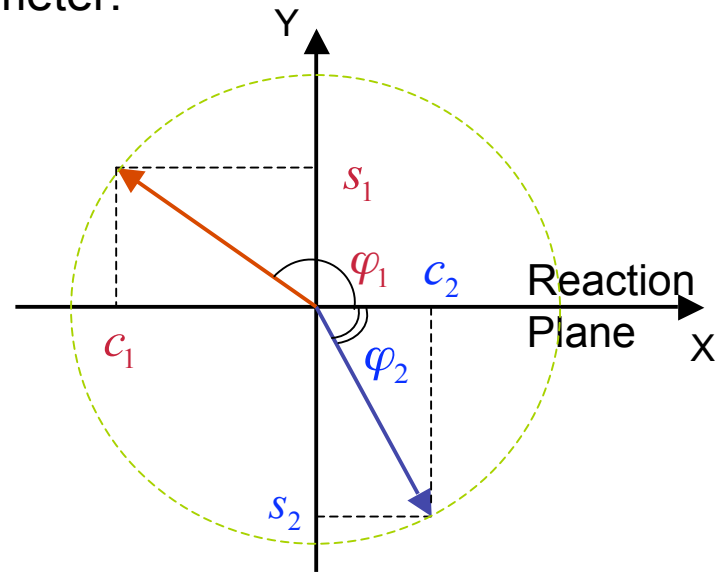
This is two particle correlation measurement with respect to the reaction plane <cos(ϕ_A+ϕ_B-2$\Phi_{R.P.}$)>, which is sensitive to the local parity violation, but this variable would be affected by many other effects. Therefore we are not intending to claim an observation of the violation, but just to present the measured correlation parameter.

directed plane Φ_1 (spectators) : Global P.V.
elliptic plane Φ_2 (participants) : Local P.V.

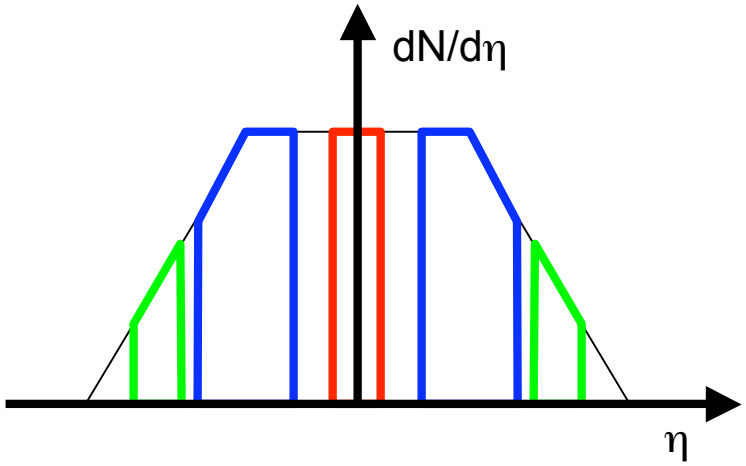
$$\frac{dN_\pm}{d\phi} \propto 1 + 2v_1\cos(\Delta\phi) + 2v_2\cos(2\Delta\phi) + \ldots + 2a_{1,\pm}\sin(\Delta\phi) + \ldots, \quad \Delta\phi = \phi - \Psi_{RP}$$

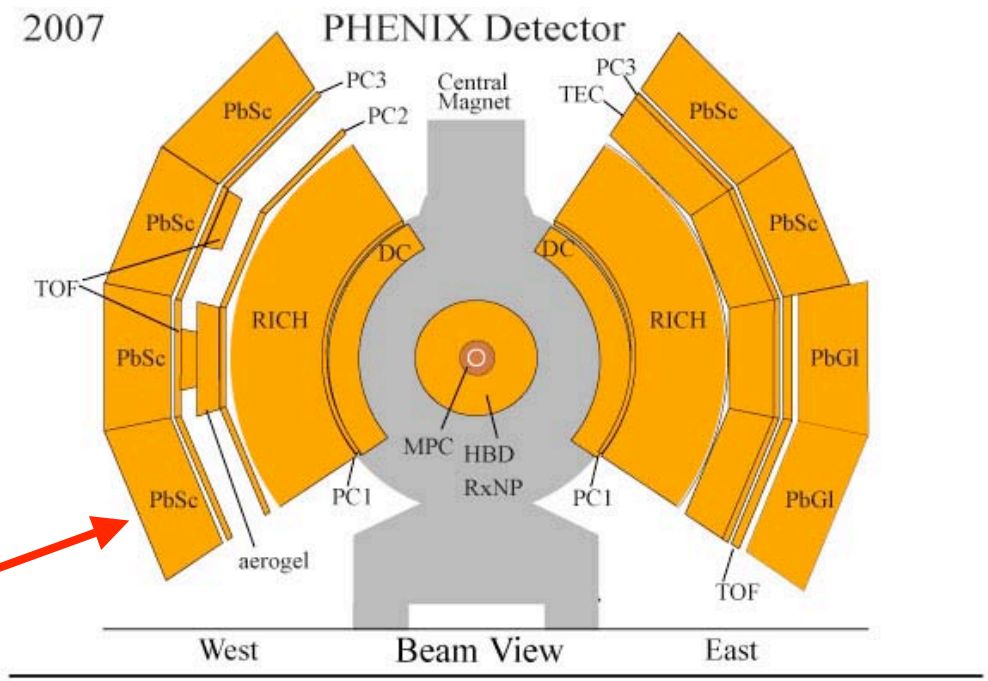
$$\langle\cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP})\rangle = \langle\cos\Delta\phi_\alpha\cos\Delta\phi_\beta\rangle - \langle\sin\Delta\phi_\alpha\sin\Delta\phi_\beta\rangle = [\langle v_{1,\alpha}v_{1,\beta}\rangle + B^{in}] - [\langle a_\alpha a_\beta\rangle + B^{out}].$$

$v_1 = 0$, $B^{in} \approx B^{out}$



acceptance correction with event mixing
reaction plane resolution correction

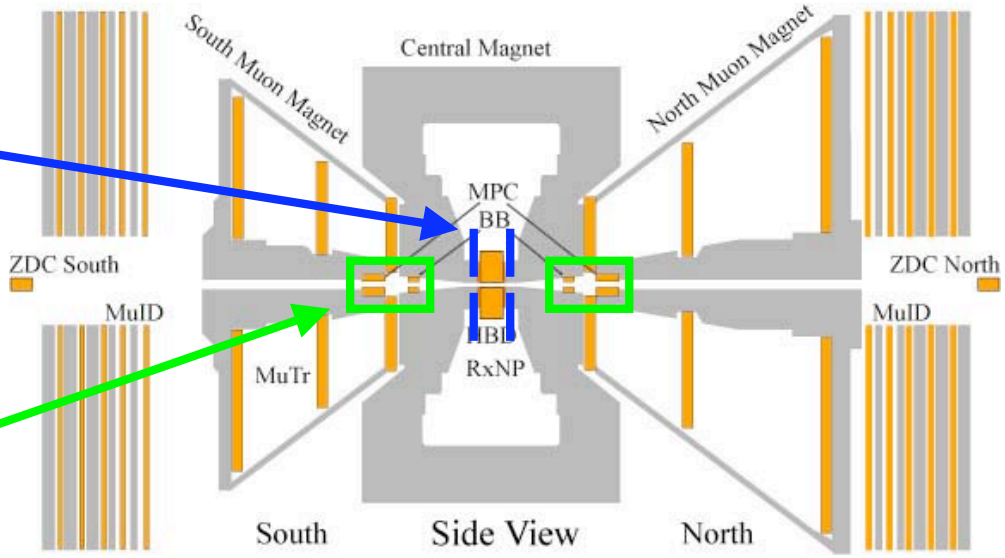




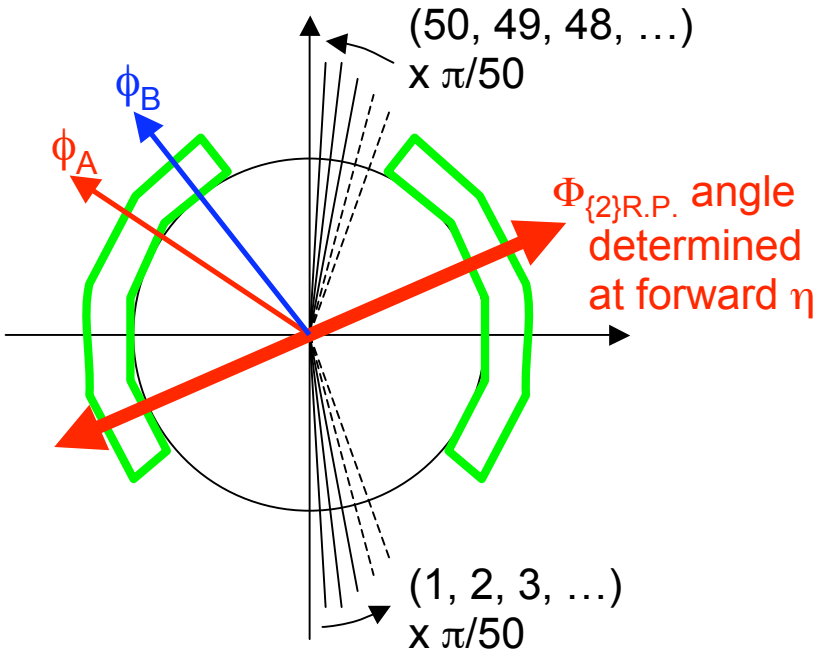
Charged Particle Tracks
|η|<0.35, π/2 x 2 arms
Central Arm Spectrometer (DC,PCs)

Mid Reaction Plane
1.0<|η|<2.8, 0<φ<2π
Reaction Plane Detector (RXN)

Forward Reaction Plane
3.0<|η|<3.8, 0<φ<2π
Beam Beam Counter (BBC)
Muon Piston Calorimeter (MPC)



Event mixing with fine bin in laboratory R.P. angle $\Phi_{\{2\}}^{\{\text{forward } \eta\}}$



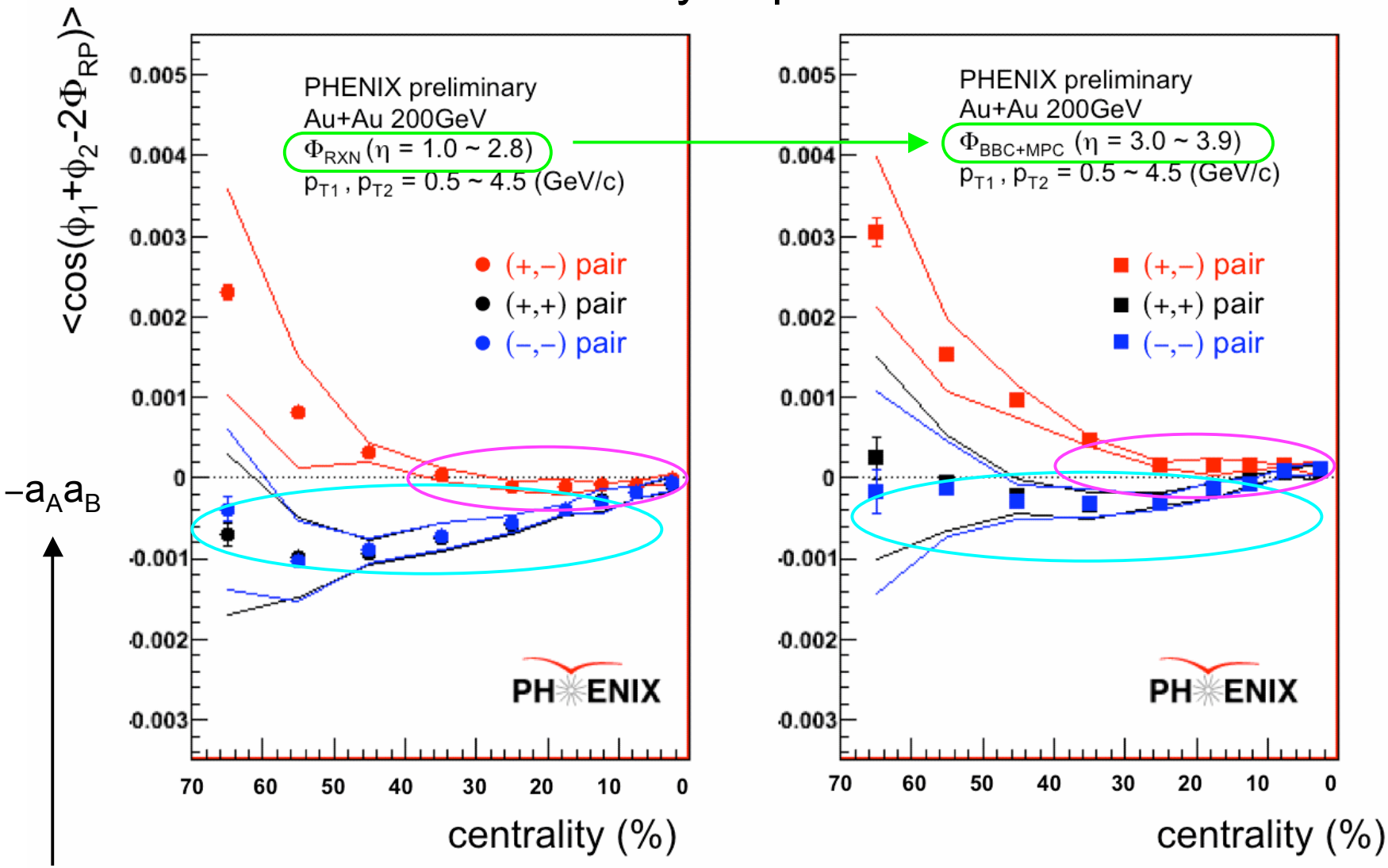
event mixing in
- centrality: 10 bins [0-100%]
- z-vertex: 10 bins [-30~30cm]
- reaction plane: 50 bins [$-\pi/2 \sim \pi/2$]

mixed event within the same event class of (cent, z-vtx, $\Phi_{\{2\}R.P.}$) in order to take into account the acceptance as well as residual flow effects to be removed.

* measure $F_{AB}=\langle\cos(\phi_A+\phi_B-2\Phi_{\{2\}R.P.})\rangle$ and for F_{+-}, F_{++} and F_{--} for both real and mixed pairs
* take a difference between real and mixed, then correct for R.P. resolution: $(F_{real}-F_{mixed}) / Res_{R.P.}$

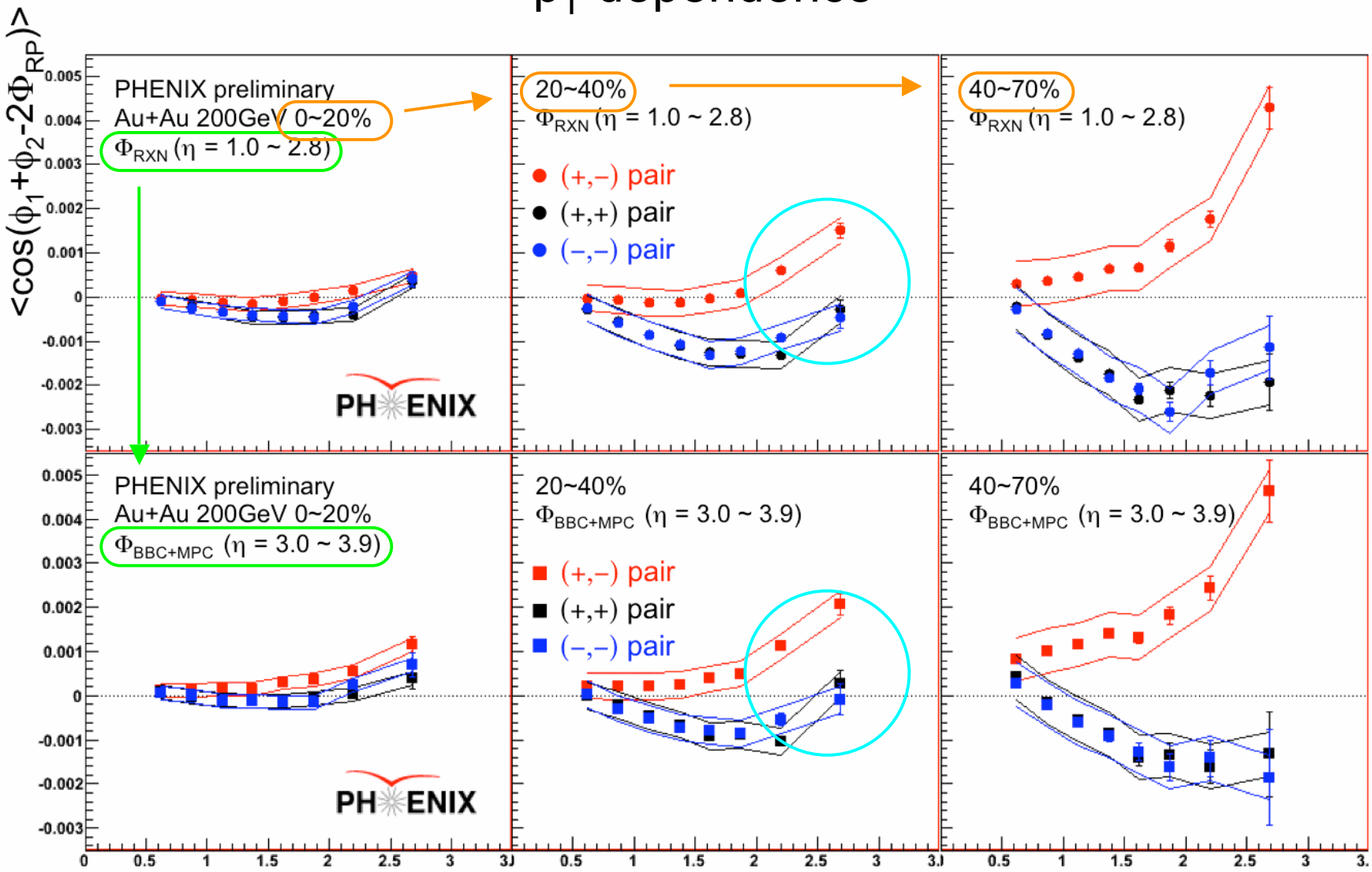
For a given particle ϕ_A in an event with $\Phi_{\{2\}R.P.}$ angle, another particle ϕ_B is taken from a different event with a similar $\Phi_{\{2\}R.P.}$ angle.

Centrality dependence



negative values for like sign pairs
suppressed magnitude for un-like sign pairs in central

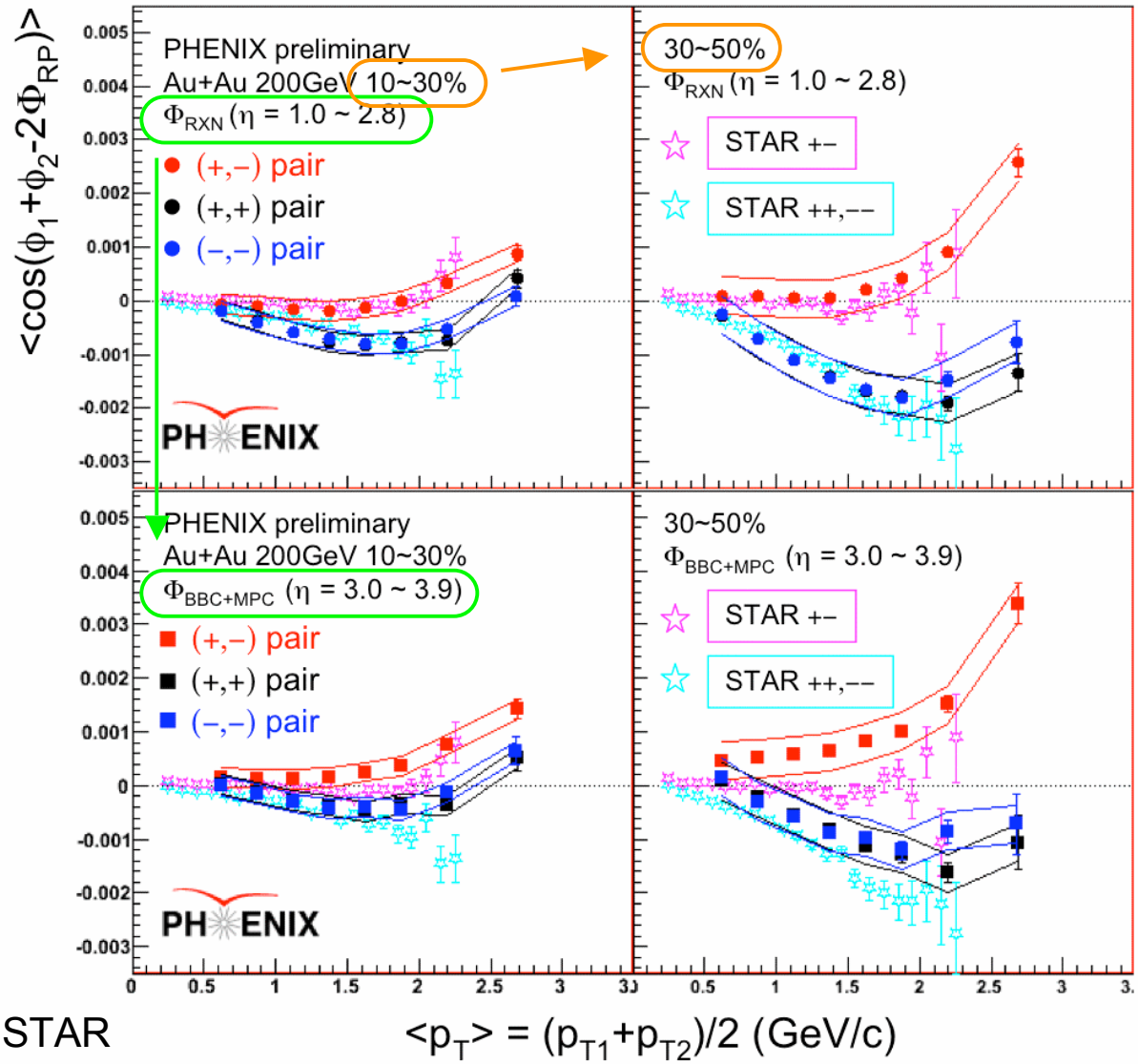
p_T dependence



$<p_T> = (p_{T1}+p_{T2})/2$ (GeV/c) p_{T1} , p_{T2} = 0.5 ~ 4.5 (GeV/c)

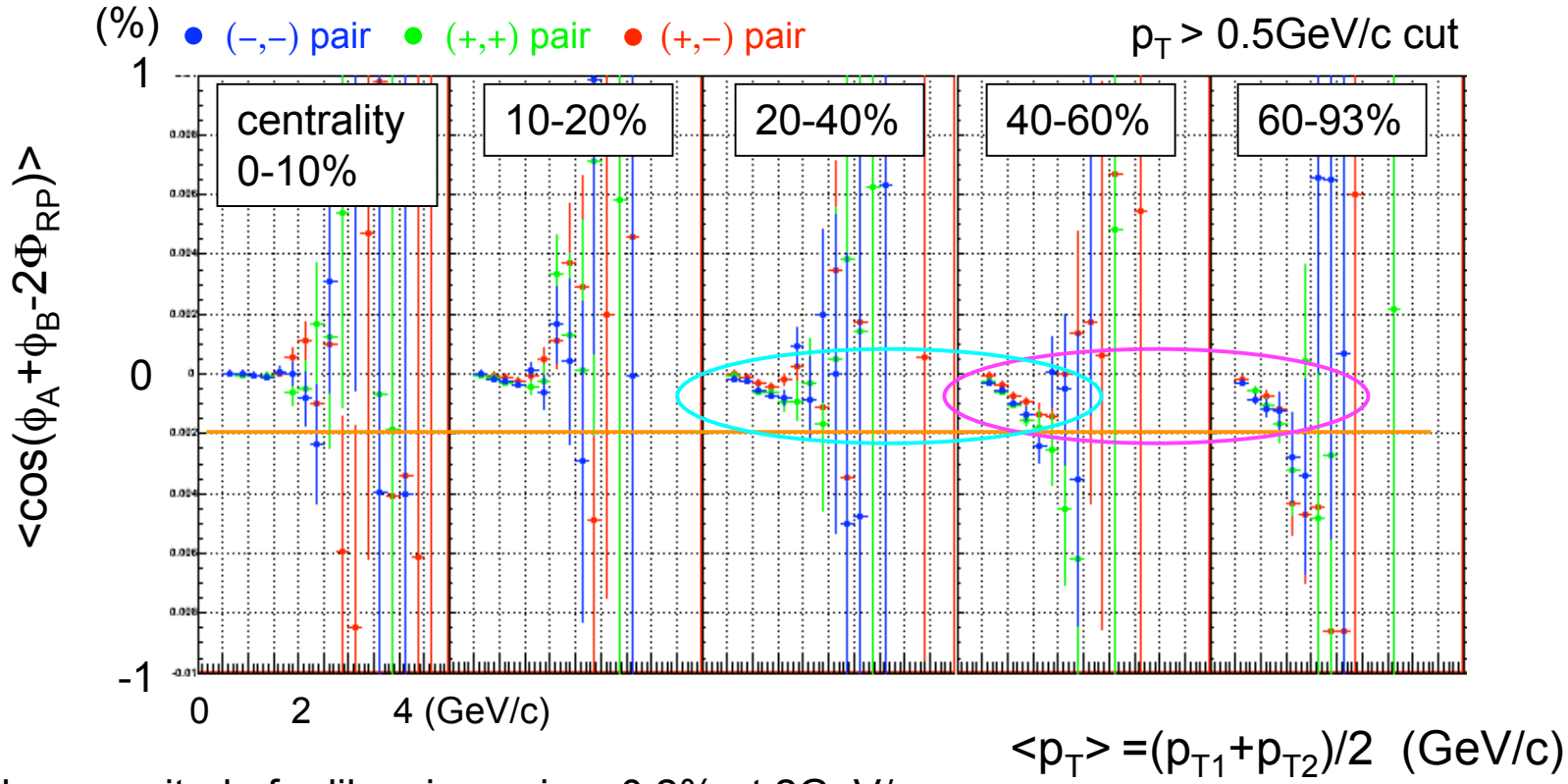
correlations goes up at higher p_T
stronger signal for peripheral

Comparison with STAR results



comparable to STAR measurements

AMPT simulation (with string melting)

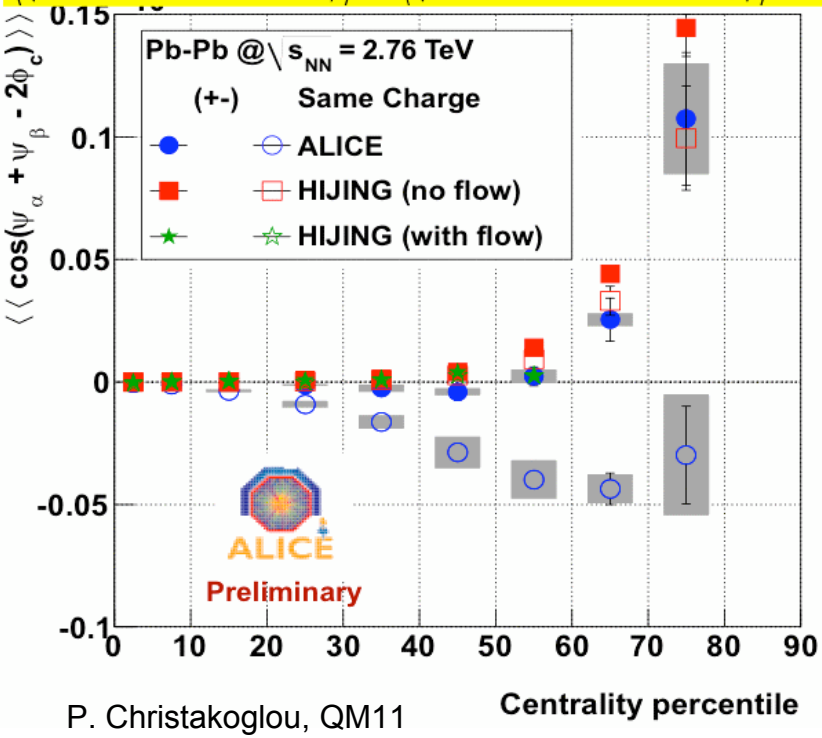


similar magnitude for like-sign pairs ~0.2% at 2GeV/c
negative values also for un-like-sign pairs
some similarity (+− > ++,−−) to experimental data

Comparison with LHC results

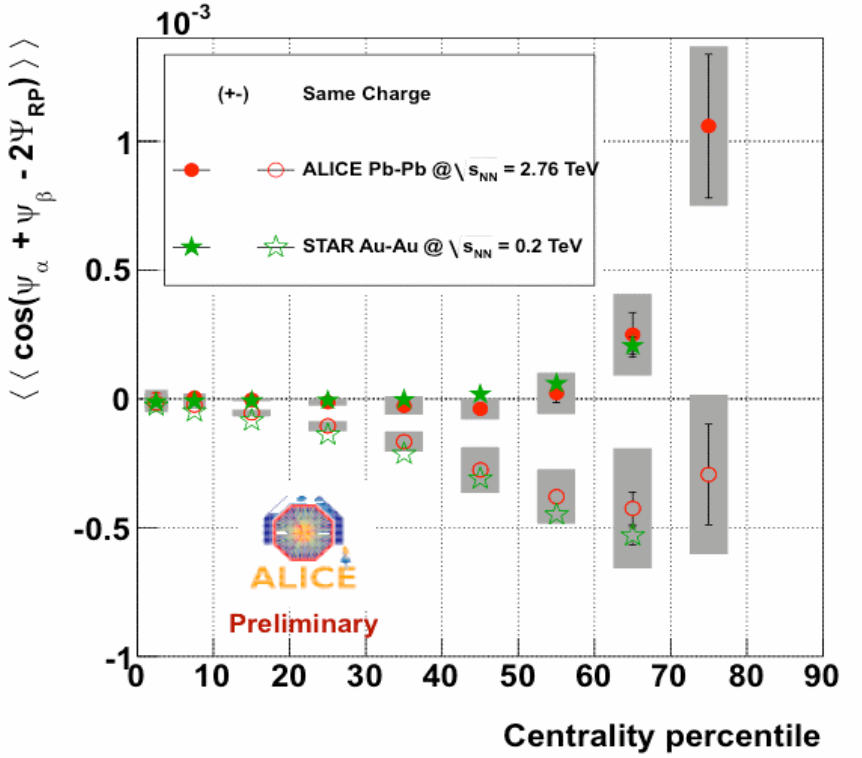
S. A. Voloshin, Phys. Rev. *C* **70**, 057901 (2004).

$$\langle\langle \cos(\psi_a + \psi_b - 2\phi_c) \rangle\rangle = \langle\langle \cos(\psi_a + \psi_b - 2\Psi_{RP}) \rangle\rangle \; v_{2,c}$$



P. Christakoglou, QM11

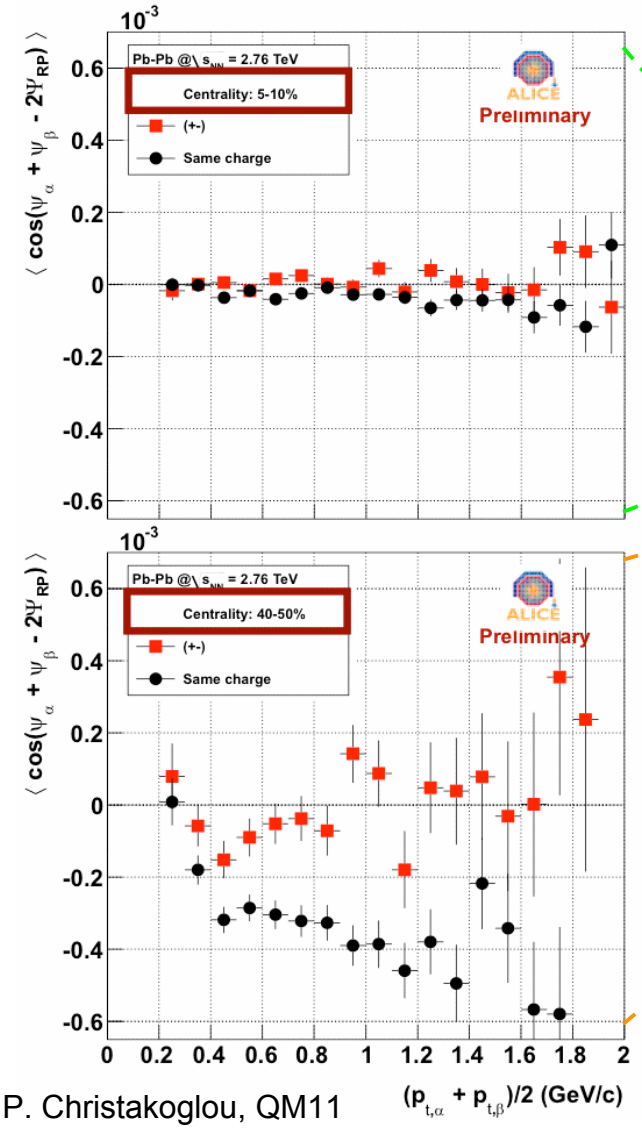
STAR Collaboration: Phys. Rev. Lett. **81**, 251601 (2009)
STAR Collaboration: Phys. Rev. ***C*81**, 054908 (2010)



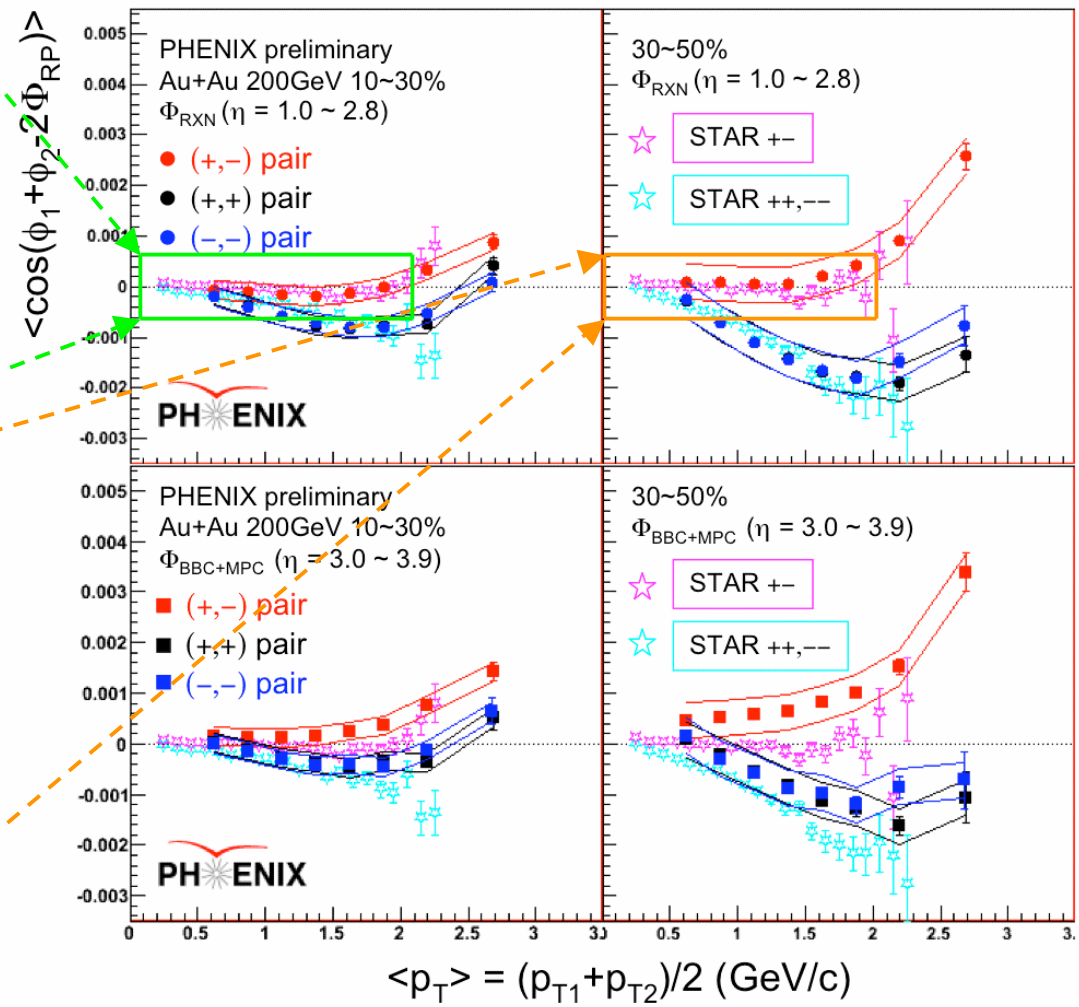
very similar results between RHIC and LHC energies

smaller signal for a given p_T at LHC than at RHIC?

Comparison with LHC results --- p_T dependence ---



P. Christakoglou, QM11

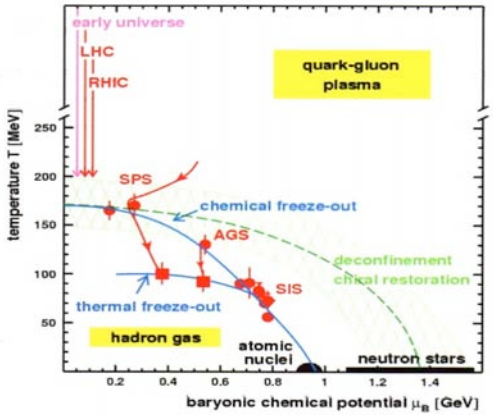


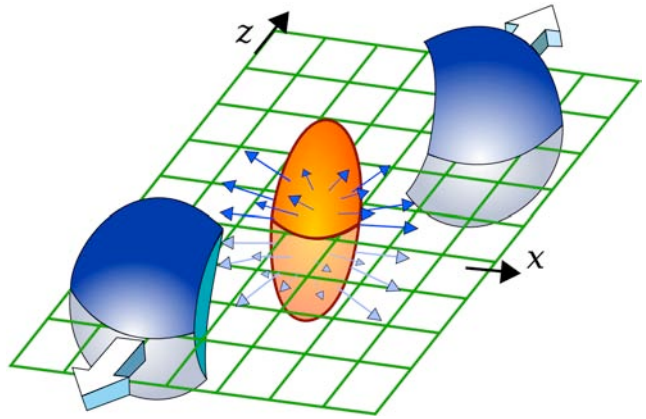
Summary

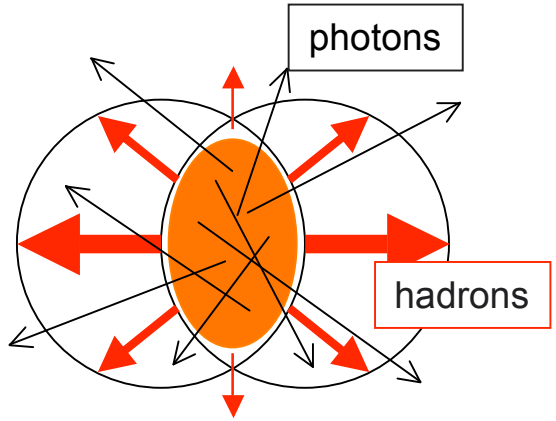
Charge dependences of the 2-particle and R.P. correlation are observed similarly in various experimental studies…

Simulations can not easily explain the measurements…

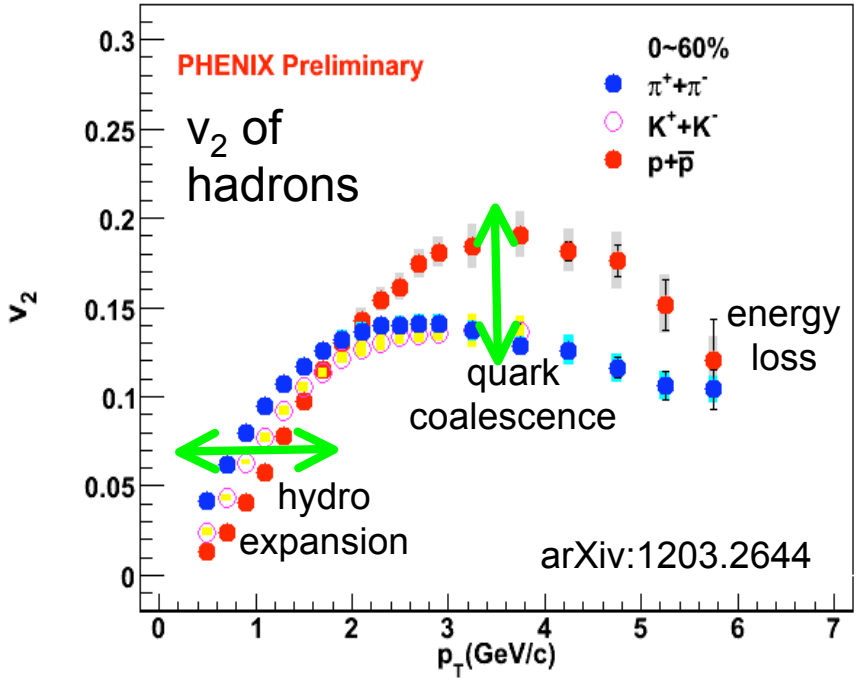
Collective expansion and freeze-out

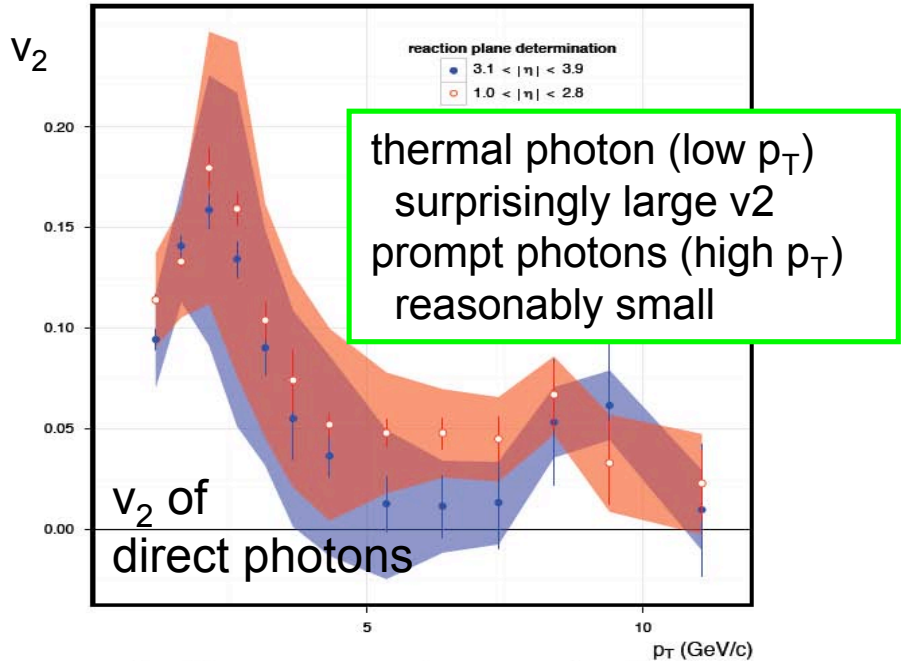




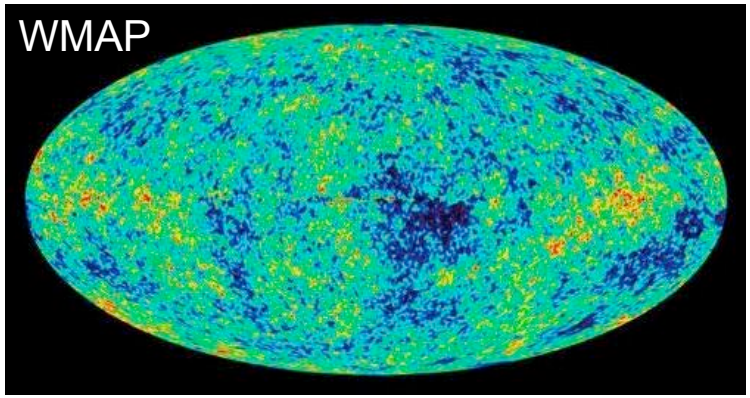


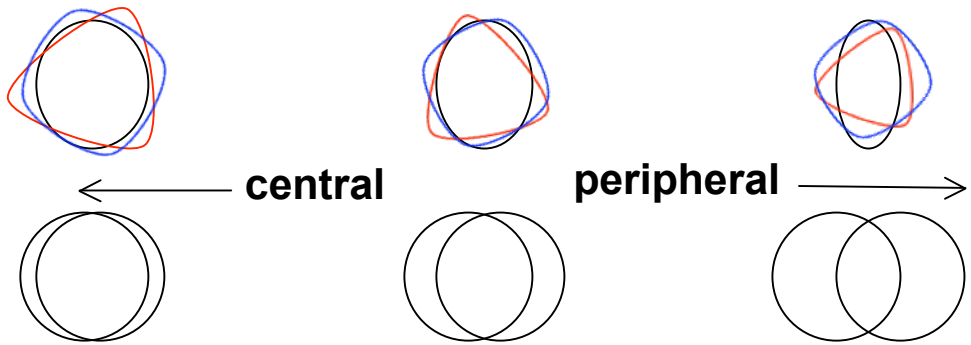
arXiv:1105.4126



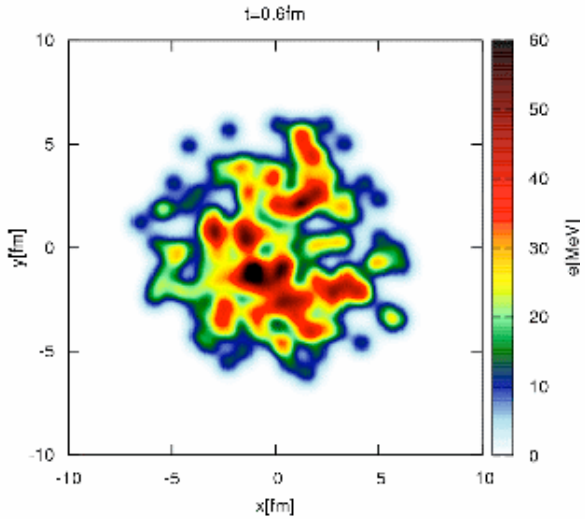


Initial fluctuation followed by collective expansion

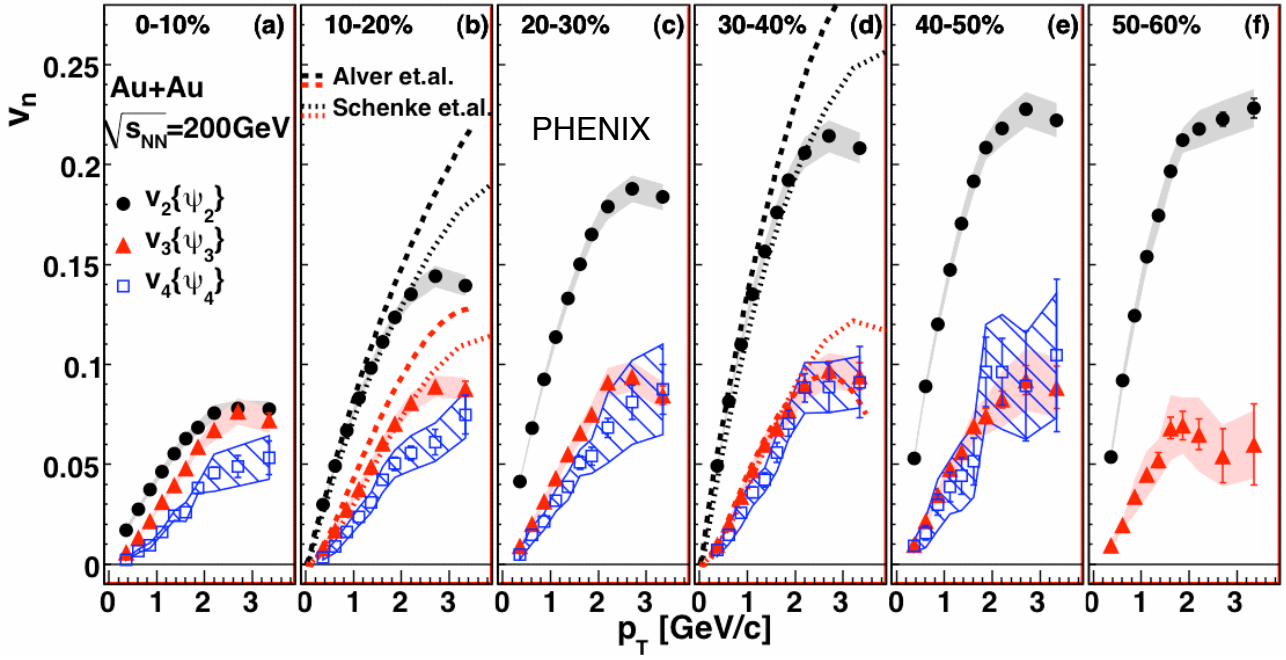




initial density distribution

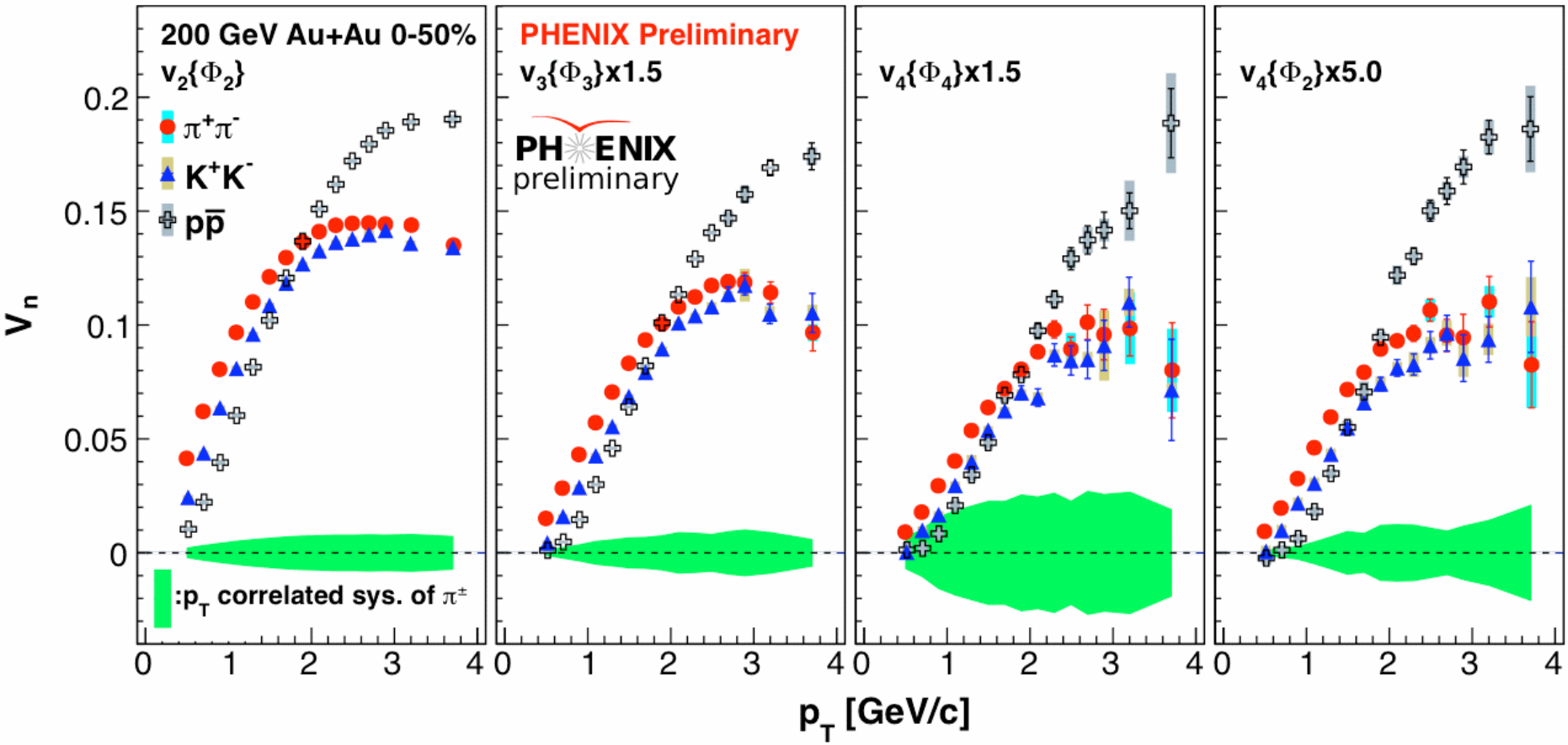


determination of initial conditions and hydro-dynamic parameters of QGP



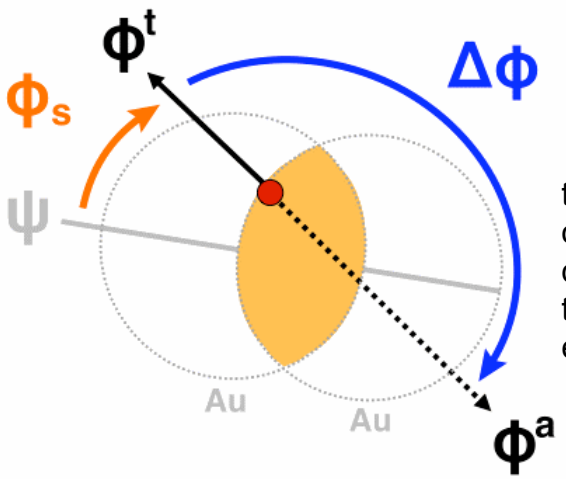
PRL107, 252301 (2011)

Identified particle higher order event anisotropy

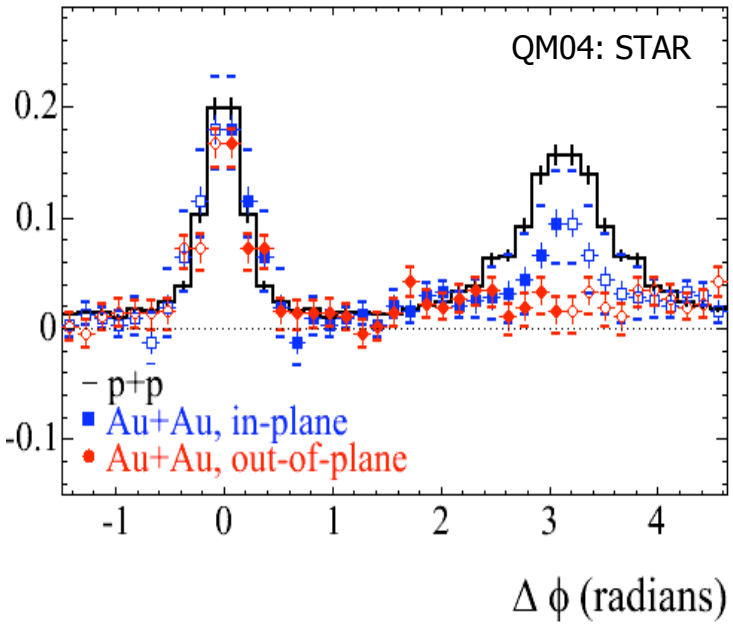


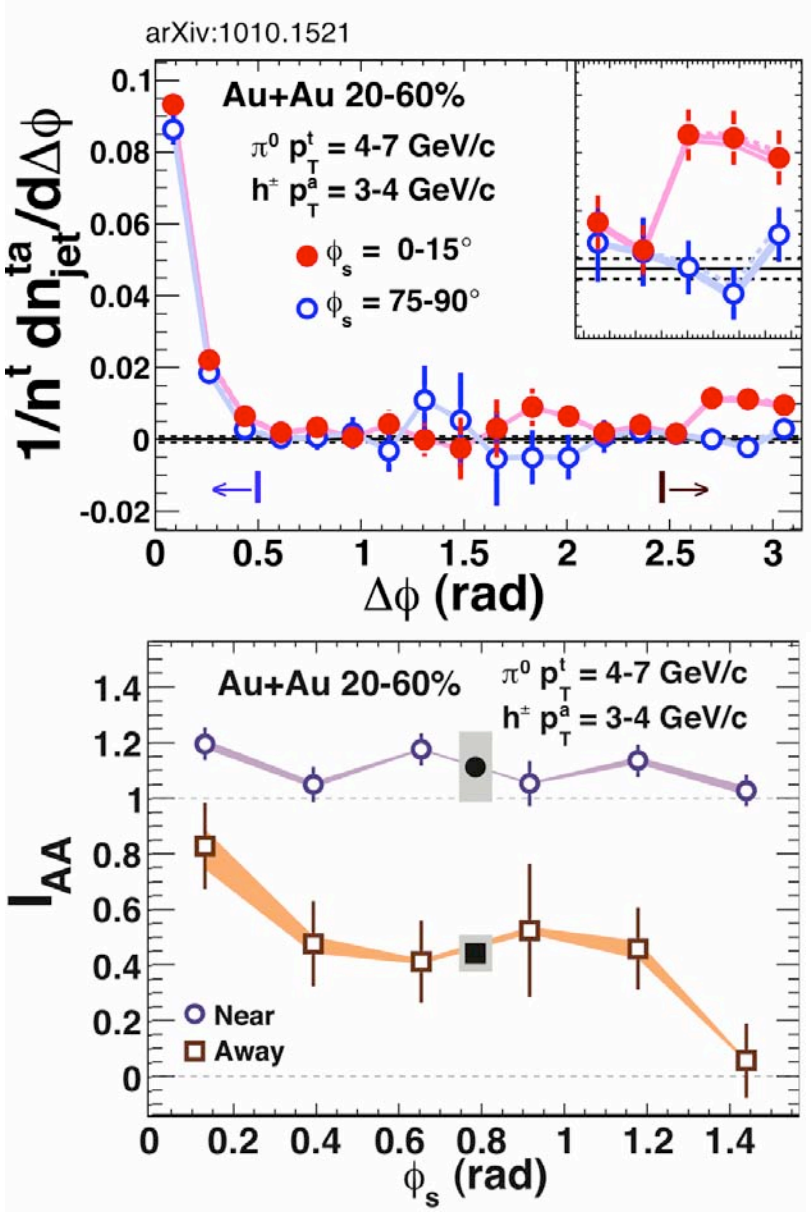
Reaction plane (path length) dependent energy loss

--- one of dominant sources of v_2 at high p_T ---



thickness dependence of penetration is more dominant than tangential surface emission.

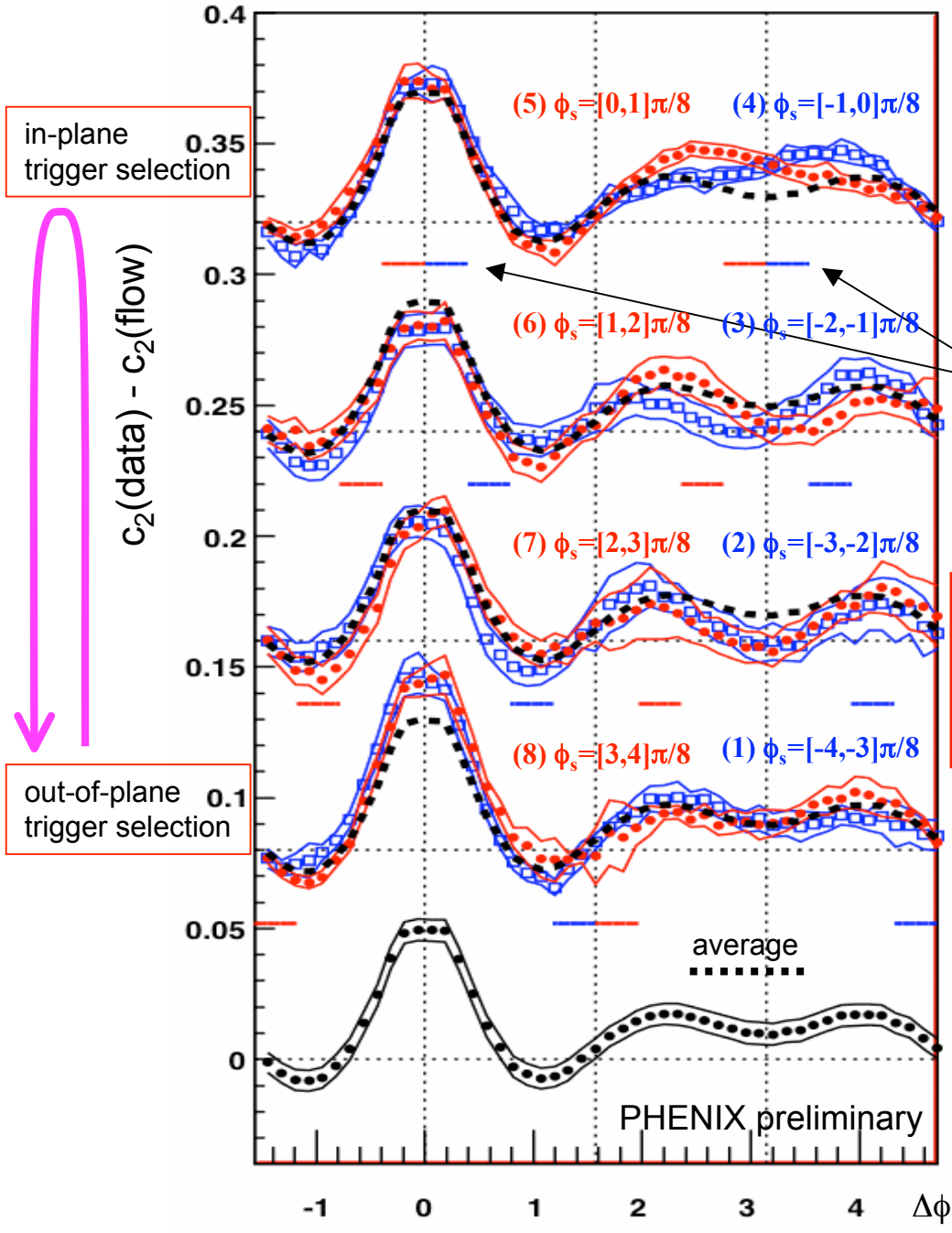




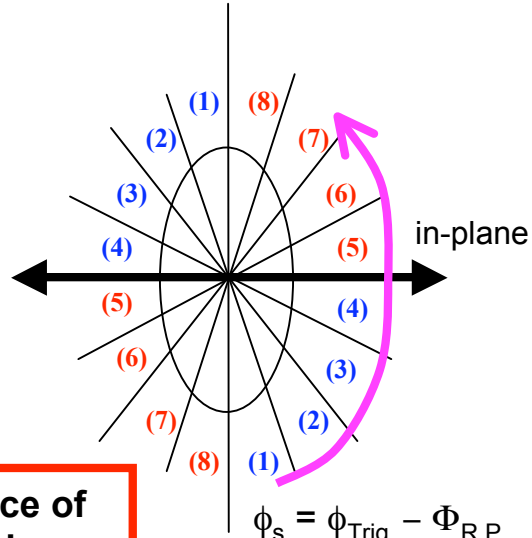
200GeV Au+Au -> h-h (run7)

(p_T^{Trig}=2~4GeV/c, p_T^{Asso}=1~2GeV/c)

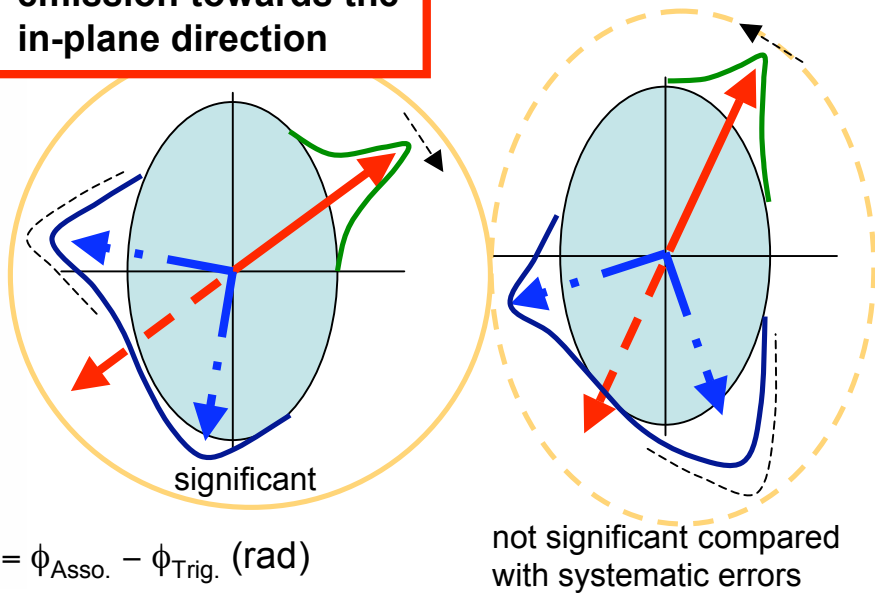
mid-central : 20-50%



in-plane associate regions



strong preference of associate particle emission towards the in-plane direction

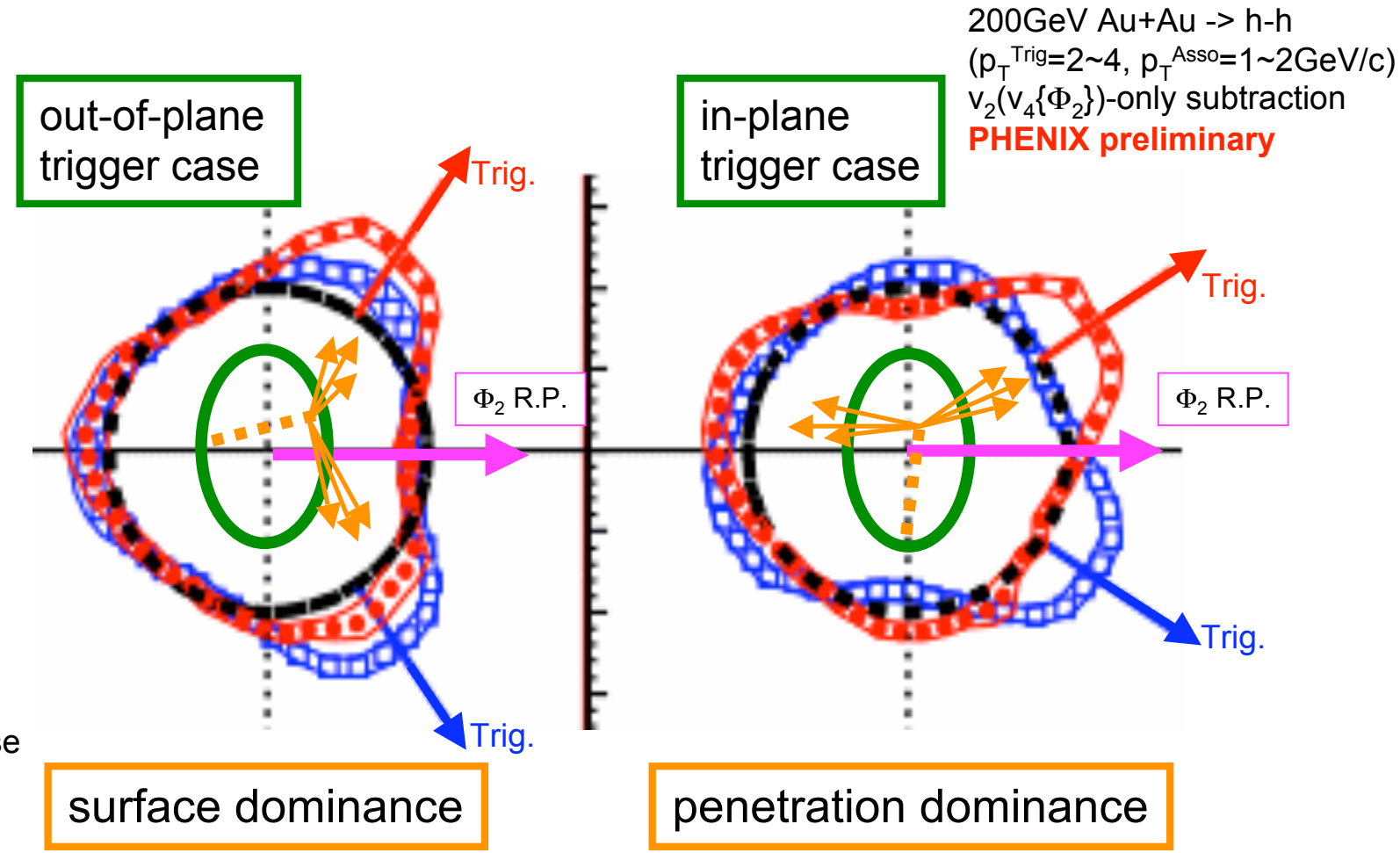


Observed left/right asymmetry remains after "the usual/normal" v_3 subtraction.

Trigger angle selection w.r.t. Φ_2 separately for left(up) / right(down)

Trigger angle selected 2-part. corr. data are plotted in polar coordinate by rotating Φ_2 R.P. angle as X-axis.

Flow subtracted yield is shown radially with base line. ▪▪▪▪▪▪



Two competing processes seen

Photons in the Chiral Magnetic Effect

Kenji Fukushima

Department of Physics, Keio University

Current from the Quantum Anomaly

Anomaly Relation

$$\boldsymbol{j} = N_c \sum_{i=\text{flavor}} \frac{Q_i^2 e^2 \mu_5}{2\pi^2} \boldsymbol{B}$$

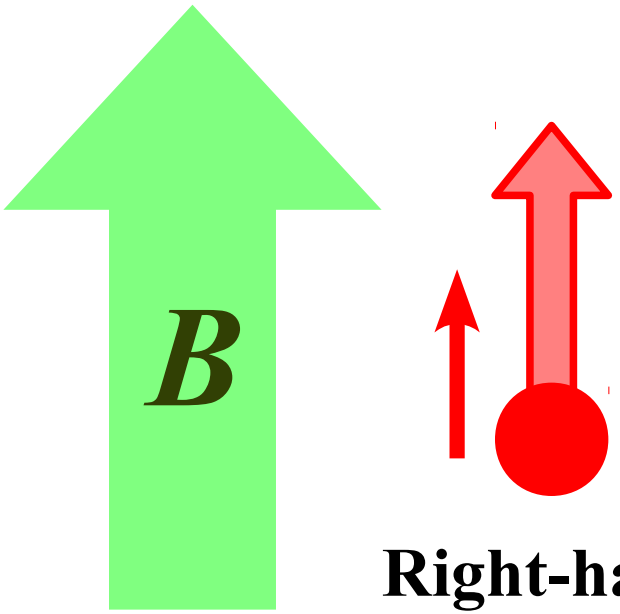
Chiral Magnetic Effect = QCD anomaly × QED anomaly

No correction from higher-order fluctuations
(apart from the RG non-invariance)

Vilenkin (1980) Rediscovered over and over again!

Conventional Explanation

Classical Picture



Left-handed Quarks
= momentum anti-parallel to spin

Right-handed Quarks
= momentum parallel to spin

$$\boldsymbol{J} \neq 0 \quad \text{if} \quad N_5 = N_R - N_L \neq 0$$

Kharzeev-McLerran-Warringa (2007)
Fukushima-Kharzeev-Warringa (2008)

Conventional Interpretation

- Seemingly, *massless* and *deconfined* quarks flow in response to the external magnetic field.

  → Signature for chiral restoration and deconfinement

- Chiral restoration is not a necessary condition.

  CME formula is insensitive to IR quantities
  BUT chiral restoration is necessary for a large N_5

- Deconfinement is not a necessary condition.
  Charged hadrons can generate an electric current
  BUT more topological excitations at high T

Questions

- What is the CME in the hadronic phase ?

- What is the origin and the meaning of μ_5 ?

  What is really necessary for the CME ?

- What is the crucial difference driven by space-time inhomogeneous backgrounds ?

What is the CME in the hadronic phase ?

Calculation in the Hadronic Phase

Chiral Effective Lagrangian

$$L_\chi = \frac{f_\pi^2}{4}\,\mathrm{tr}\left[\partial_\mu U^\dagger \partial^\mu U + 2\underline{B}\left(MU^\dagger + U\underline{M}\right)\right]$$

chiral condensate **quark mass**

NG boson mass term

If $U = e^{i\pi^a t^a / f_\pi}$ is plugged in

$$L_\chi = \frac{1}{2}\left[\partial_\mu \pi^a \partial^\mu \pi^a - m_\pi^2 \pi^a \pi^a + \cdots\right]$$

Not Sufficient

Anomaly Term ← θ dependence

$$L_{U(1)_A} = -\frac{N_f \chi_{\rm top}}{2}\left[\theta - \frac{i}{2}\,{\rm tr}\left(\ln U - \ln U^\dagger\right)\right]^2$$

Di Vecchia-Veneziano (1980)

Correctly capture the coupling between θ and U

$U(1)_A$ symmetry is explicitly broken

η' becomes massive

(Witten-Veneziano mass formula at large N_c)

$U(1)_A$ Rotation

Chiral Rotation

$$q_R \rightarrow V_R q_R \qquad q_L \rightarrow V_L q_L$$

$$U \rightarrow V_R U V_L^\dagger$$

$U(1)_A$ Rotation

$$V_R \simeq \exp(i\alpha)$$

$$V_L \simeq \exp(-i\alpha)$$

$$\theta \rightarrow \theta - 2N_f \alpha$$

$$\boxed{\theta - \frac{i}{2}\mathrm{tr}\left(\ln U - \ln U^\dagger\right) \text{ is invariant}}$$

Not Yet Sufficient

Local Chiral Rotations

Theory can be invariant with external vector fields:

$$D_\mu U = \partial_\mu - i[v_\mu, U] - i\{a_\mu, U\} + \frac{i}{2}\left(\partial_\mu \theta + 2\,\mathrm{tr}(a_\mu)\right)U$$

$$v_\mu + a_\mu \to V_R\left(v_\mu + a_\mu + i\partial_\mu\right)V_R^\dagger$$

$$v_\mu - a_\mu \to V_L\left(v_\mu - a_\mu + i\partial_\mu\right)V_L^\dagger$$

Local $U(1)_A$ Rotation

QCD with v_μ, a_μ

Effective theory with v_μ, a_μ

→ **Same anomaly**

Bardeen (1969)

Wess-Zumino-Witten Action

Simple Form in Two-Flavor (Kaiser 2001)

$\pi^0 \to 2\gamma$

$$L_{\mathrm{WZW}} = -\frac{N_c}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\Big[\mathrm{tr}\Big\{U^\dagger \hat{r}_\mu U \hat{l}_\nu - \hat{r}_\mu \hat{l}_\nu$$

$$+ i\Sigma_\mu\left(U^\dagger \hat{r}_\nu U + \hat{l}_\nu\right)\Big\}\mathrm{tr}\left(v_{\rho\sigma}\right) + \frac{2}{3}\mathrm{tr}\left(\Sigma_\mu \Sigma_\nu \Sigma_\rho\right)\mathrm{tr}\left(v_\sigma\right)\Big]$$

$$r_\mu = v_\mu + a_\mu \qquad l_\mu = v_\mu - a_\mu \qquad \Sigma_{\mu\nu} = U^\dagger \partial_\mu U$$

$$v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - i\left[v_\mu, v_\nu\right]$$

$$\hat{r}_\mu = r_\mu - \frac{1}{2}\mathrm{tr}\left(r_\mu\right) \qquad \hat{l}_\mu = l_\mu - \frac{1}{2}\mathrm{tr}\left(l_\mu\right)$$

Electromagnetic Vector Potential

Choice of the Vector Fields

$$v_\mu = eQ\,A_\mu \qquad a_\mu = 0$$

Q : Electric-charge Matrix

$$L_{\mathrm{WZW}} = -\frac{N_c\,\mathrm{tr}(Q)}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\Big\{\, i\,e^2\,\mathrm{tr}\Big[\big(\Sigma_\mu + \tilde{\Sigma}_\mu\big)\tau_3\Big]A_\nu\partial_\rho A_\sigma$$

$$\pi^0 \to 2\gamma \qquad -\frac{2e}{3}\,\mathrm{tr}\big(\Sigma_\mu\Sigma_\nu\Sigma_\rho\big)A_\sigma\,\Big\}$$

Electric Current I

$$\boxed{j^{\mu}(x) = \frac{\delta}{\delta A_{\mu}(x)} \int d^4 x\, L}$$

$$L_{\chi} \;\rightarrow\; j^{\mu}_{\chi} = -i \frac{e f_{\pi}^2}{4} \mathrm{tr}\left[\left(\Sigma^{\mu} - \tilde{\Sigma}^{\mu}\right)\tau^3\right]$$

$$\simeq \frac{e}{2}\left(\pi^- i \partial^{\mu} \pi^+ - \pi^+ i \partial^{\mu} \pi^-\right) + \cdots$$

Non-anomalous part just gives the pion current

Electric Current II

$$L_{\mathrm{WZW}} \rightarrow j^{\mu}_{\mathrm{WZW}} = -\frac{N_c \,\mathrm{tr}(Q)}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}(\sim\sim\sim\sim)$$

$$\simeq \frac{N_c \,\mathrm{tr}(Q) e^2}{8\pi^2 f_\pi}\epsilon^{\mu\nu\rho\sigma}(\partial_\nu \pi^0) F_{\rho\sigma} + \cdots$$

π^0-domain wall **(Son-Stephanov 2007)**
Skyrmion **(Hashimoto et al 2011)**

Similar to the CME but not exactly it

Still Not Sufficient

Contact Contribution (RG non-invariant)

c.f.
Gorbar-Miransky-Shovkovy
Fukushima-Ruggieri

$$L_{\mathrm{P}} = -\frac{N_c}{8 N_f \pi^2} \epsilon^{\mu\nu\rho\sigma} \left\{ \mathrm{tr}\left[v_\mu \left(\partial_\nu v_\rho - \frac{2i}{3} v_\nu v_\rho \right) \right] \partial_\sigma \theta \right.$$

$$\left. + \mathrm{tr}\left(a_\mu D_\nu^v a_\rho \right) \left(\frac{4}{3} \mathrm{tr}\left(a_\sigma \right) + \partial_\sigma \theta \right) - \frac{2}{3 N_f} \mathrm{tr}\left(a_\mu \right) \mathrm{tr}\left(\partial_\nu a_\rho \right) \partial_\sigma \theta \right\}$$

$$= \frac{N_c e^2 \mathrm{tr}\left(Q^2 \right)}{8 N_f \pi^2} \epsilon^{\mu\nu\rho\sigma} A_\mu \left(\partial_\nu A_\rho \right) \partial_\sigma \theta$$

$$L_{\mathrm{P}} \rightarrow \boxed{j_{\mathrm{P}}^\mu = \frac{N_c e^2 \mathrm{tr}\left(Q^2 \right)}{4 N_f \pi^2} \epsilon^{\mu\nu\rho\sigma} \left(\partial_\nu A_\rho \right) \partial_\sigma \theta}$$

CME

What is the origin

and the meaning of μ_5 ?

Space-time Dependent θ

$$j_{\mathrm{P}}^{\mu} = \frac{N_c e^2 \,\mathrm{tr}\left(Q^2\right)}{4 N_f \pi^2} \epsilon^{\mu\nu\rho\sigma} \left(\partial_\nu A_\rho\right) \partial_\sigma \theta$$

$$+$$

$$\mu_5 = \frac{\partial_0 \theta(t)}{2 N_f}$$

$$\|$$

$$\boldsymbol{j}_{\mathrm{P}} = \frac{N_c e^2 \,\mathrm{tr}\left(Q^2\right)}{2 \pi^2} \mu_5 \boldsymbol{B}$$

Origin of Space-time Dependent θ

$$L_\chi = \frac{f_\pi^2}{4}\,\mathrm{tr}\left[\partial_\mu U^\dagger \partial^\mu U + 2B\left(MU^\dagger + UM\right)\right]$$

$$L_{U(1)_A} = -\frac{N_f \chi_{\mathrm{top}}}{2}\left[\theta - \frac{i}{2}\,\mathrm{tr}\left(\ln U - \ln U^\dagger\right)\right]^2$$

Vacuum state should minimize the energy **(Witten 1980)**

$$U \sim e^{i\eta/f_\eta} \qquad \text{If } BM = 0 \text{ (small enough)}$$

$$\theta + \eta/f_\eta = \theta_{\mathrm{eff}} = 0 \qquad \theta\text{-dep is gone}$$

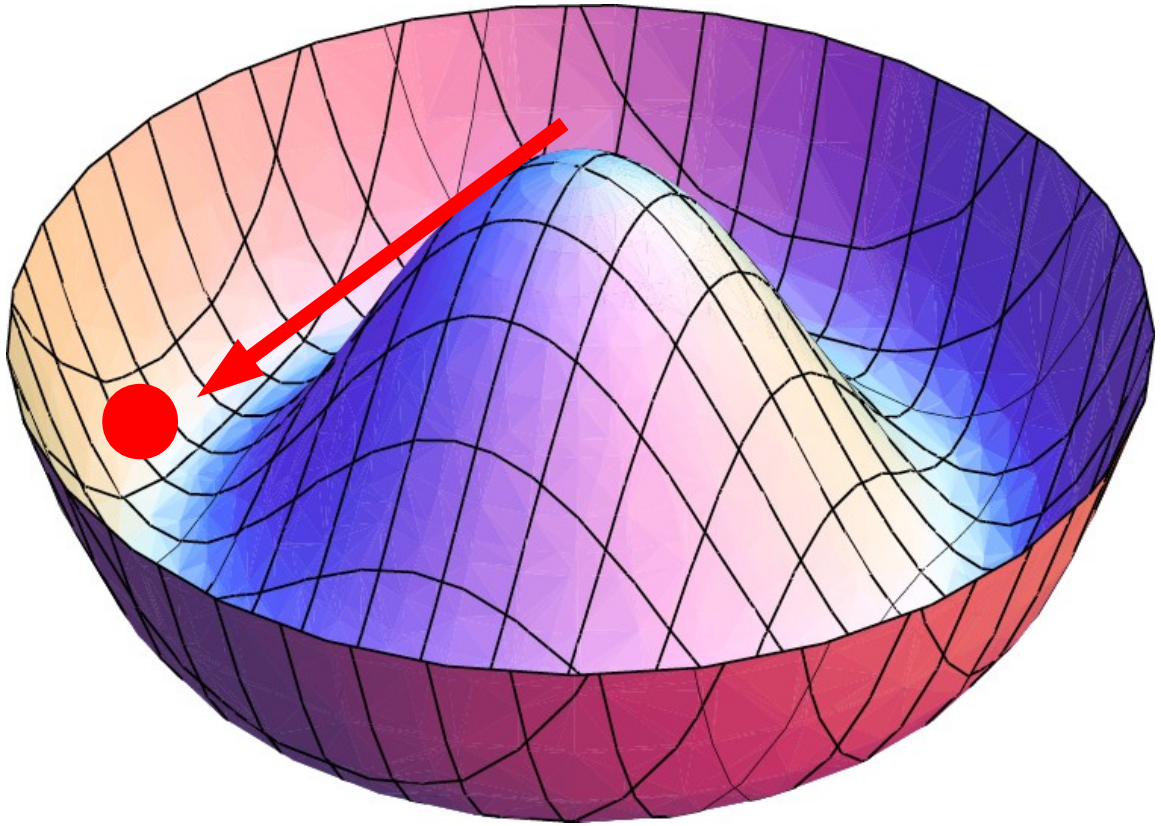
When η condenses, it is absorbed in the redefinition of (Normalization condition of *U*)

c.f. linear-σ model

Chiral Symmetry Breaking

Chiral Condensate and Chiral Circle

$$\sigma \sim \bar{\psi}\psi$$

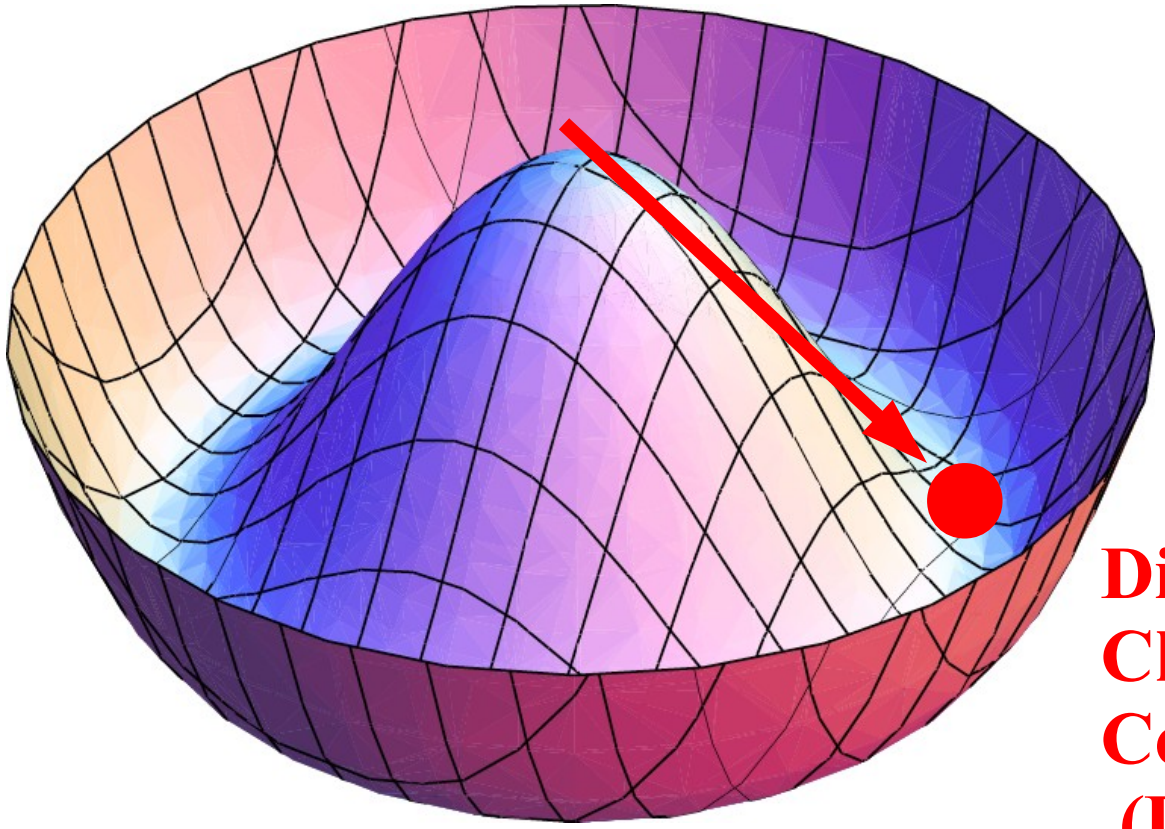


$$\pi \sim \bar{\psi} i \gamma_5 \tau \psi$$

Chiral Symmetry Breaking

Chiral Condensate and Chiral Circle

$$\sigma \sim \bar{\psi}\psi$$



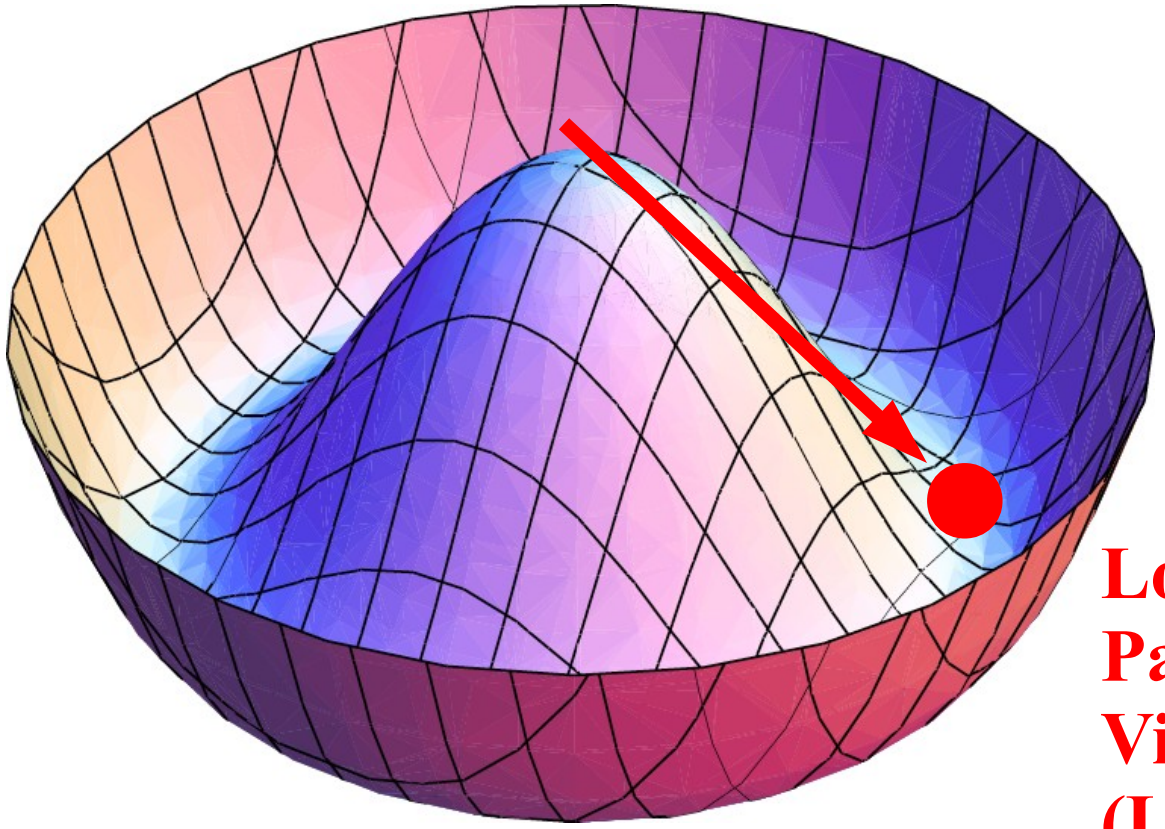
Disoriented Chiral Condensate (DCC)

$$\pi \sim \bar{\psi} i \gamma_5 \tau \psi$$

Chiral Symmetry Breaking

Chiral Condensate and Chiral Circle

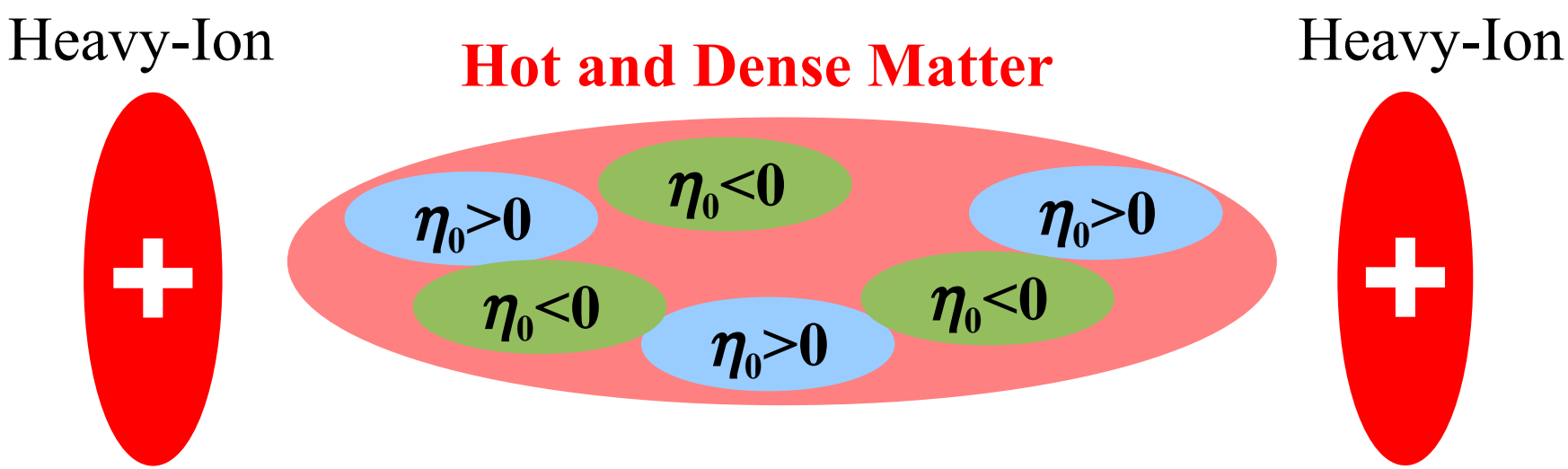
$$\sigma \sim \bar{\psi}\psi$$



Local Parity Violation (LPV)

$$\eta_0 \sim \bar{\psi} i \gamma_5 \psi$$

Local Parity Violation (LPV)



CME is certainly a signature for the LPV.
LPV or η-DCC needs the chiral symmetry restoration.
Thus, CME could be an indirect evidence for it.
Confinement-deconfinement is completely irrelevant.

What is the crucial difference driven by inhomogeneous backgrounds ?

Contact Term Again

$$L_{\mathrm{P}} = \frac{N_c e^2 \mathrm{tr}\left(Q^2\right)}{8 N_f \pi^2} \epsilon^{\mu\nu\rho\sigma} A_\mu \left(\partial_\nu A_\rho\right) \partial_\sigma \theta$$

What is the natural interpretation of this?

Three-point (θ-γ-γ) vertex

Split the vector potential
to the external *B* and the photon

$$A = \bar{A} + \delta A$$

Photon Vertices

$$L_{\mathrm{P}} = \frac{N_c e^2 \,\mathrm{tr}\left(Q^2\right)}{8 N_f \pi^2} \epsilon^{\mu\nu\rho\sigma} [\underbrace{\delta A_\mu \left(\partial_\nu \delta A_\rho\right)}_{\theta\text{-}\gamma\text{-}\gamma} + \underbrace{\delta A_\mu \bar{F}_{\nu\rho}}_{\theta\text{-}B\text{-}\gamma}] \partial_\sigma \theta$$

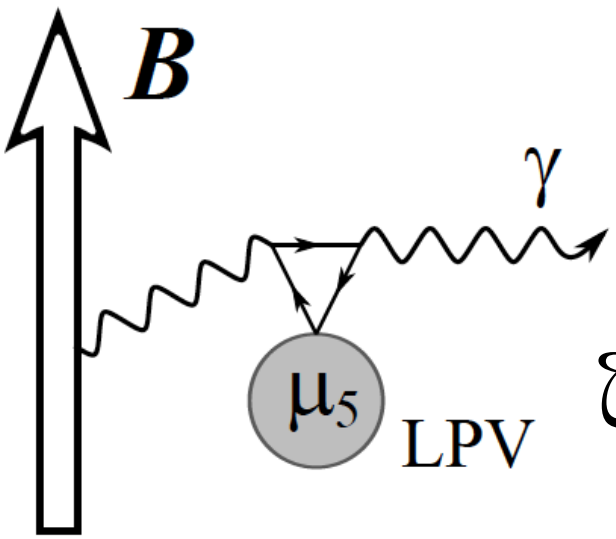
Because |*eB*| is as large as the QCD scale, only the second term is the dominant process.

Chiral Magnetic Effect = Primakoff Effect

$$\gamma + \gamma^* \rightarrow (\text{neutral field})$$

Reverse Primakoff Effect

Neutral field provided by the LPV (η condensate)



$$q_0 \frac{dN_\gamma}{d^3 q} = \frac{\boldsymbol{q}_\perp^2}{2(2\pi)^3 \boldsymbol{q}^2} \cdot \frac{25\,\alpha_e \zeta(\boldsymbol{q})}{9\pi^3}$$

$$\zeta(\boldsymbol{q}) = \left| \underbrace{\int d^4 x\, e^{-iq\cdot x}}_{\text{Momentum Conservation}} \underbrace{eB(x)}_{\text{Calculable}} \underbrace{\mu_5(x)}_{\text{Unknown}} \right|^2$$

c.f. Basar-Kharzeev-Skokov (Trace Anomaly)

Characteristics

- Small, but not hopelessly small...

$$q_0 \left(d N_\gamma / d^3 q \right) \simeq 10^{-7} \sim 10^{-3} \, \mathrm{GeV}^{-2}$$

- Emitted to the directions perpendicular to B.

  Competing with other (pure-B) effects **Hiroshima Group**

- Photon's energy and momentum reflect the typical distribution of the LPV domains.

  Homogeneous backgrounds cannot supply the necessary energy and momentum.

Summary

Summary

- What is the CME ?
  What flows to generate the current with hadrons.

- Microscopic origin of the LPV needs much more investigations (DCC-type sumulation in the hadron phase and/or Glasma simulation with isotropization.)

- Photon related to the strong-*B* background need more investigations. Anomalous and non-anomalous processes both are important.

Spirals of Chiral & Spin DW in Dense QCD in B

Vivian de la Incera
University of Texas at El Paso

Collaborators:
Efrain. J. Ferrer and Angel C. Sanchez

arXiv:1205.4492

Workshop On
P- and CP-odd Effects in Hot and Dense Matter (2012)
RIKEN BNL Research Center Workshop
June 25-27, 2012 at Brookhaven National Laboratory

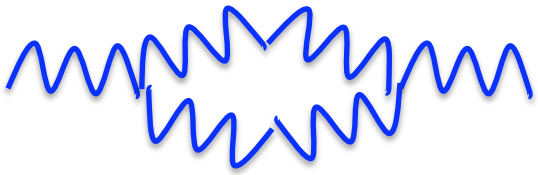
Key Observations

- **Large N_c QCD** (good to describe nonperturbative effects of QCD: meson parameters, parton distributions)

Large Nc Limit: $g_s^2 N_c$ = constant

'tHooft'74, Witten'79

- **Vacuum Gluon and Quark Contributions**



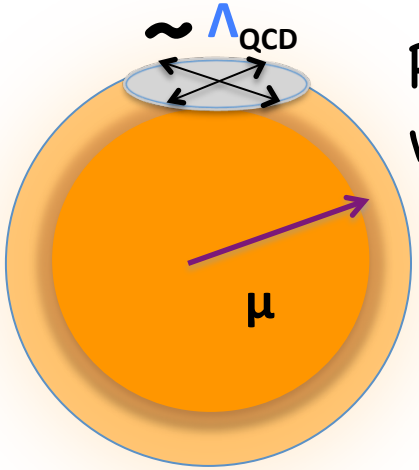
$g_s^2 N_c \sim 1$

$g_s^2 \sim 1/N_c$

- **Finite Density**

Screening effects $\sim \mu^2/N_c$
only important at very high density

Quarkyonic Matter



Relevant at high density & large N_c , where screening effects are negligible

$$m_D << \Lambda_{QCD} << \mu$$

Bulk Properties: perturbative
Excitations at the Fermi surface: confined

Large N_c : Gluon Propagator unaffected by quarks (same as in confined vacuum). Gribov-Zwanziger propagator:

$$D_{44}^{AB}(k) = -\frac{8\pi}{C_F} \times \frac{\sigma}{(\vec{k}^2)^2}\,\delta^{AB}$$

$$D_{ik}(\omega, \mathbf{q}) = \frac{\delta_{ik} - q_i q_k/q^2}{\omega^2 + \mathbf{q}^2}$$

perturbative

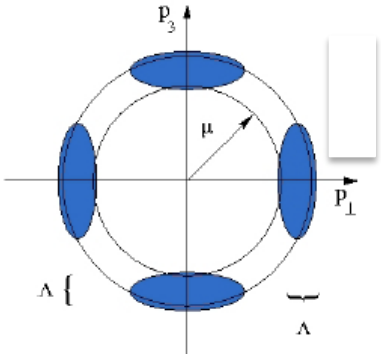
Valid in the Coulomb gauge and for $|p| \leq \Lambda_{QCD}$

McLerran & Pisarski, NPA'07; Kojo, et al NPA'10

Dimensional Reduction at B=0

$$\not{\Sigma}(p) = -\int \frac{d^4k}{(2\pi)^4}\, D^{AB}_{\mu\nu}(p-k)\,(\gamma_\mu t_A)\, S(k)\,(\gamma_\nu t_B)$$

$$\not{\Sigma}(p) = \int \frac{d^4k}{(2\pi)^4}\, \frac{8\pi\sigma}{((\vec{p}-\vec{k})^2)^2}\, \gamma_4\, S(k)\, \gamma_4$$



Can neglect $p_\perp$ in the quark propagator, because in the patch at the Fermi surface $|p_\perp| < \Lambda_{QCD} << \mu$ **and**

$$ip_4 \approx \delta p_z + \frac{\vec{p}_\perp^{\,2}}{2\mu} + \ldots$$ then,

4D QCD in Coulomb gauge reduces to 1+1 D QCD in axial gauge $A_z=0$

$$\not{\Sigma}(p_4, p_z, \vec{0}_\perp) \simeq \frac{N_c g_{2D}^2}{2} \int \frac{dk_4\, dk_z}{(2\pi)^2}\, \gamma_4\, S(k_4, k_z, \vec{0}_\perp)\, \gamma_4\, \frac{1}{(k_z - p_z)^2}$$

Shuster & Son, '99
Kojo et al, '10

Effective 1+1 D Theory at B=0

Chiral rotation $\Phi = \exp\left(-i\,\mu\, z\,\Gamma^5\right)\Phi'$

$$\mathcal{L}_{\rm eff}^{\rm 2D} = \overline{\Phi}\left[i\,\Gamma^\mu\left(\partial_\mu + i g_{\rm 2D}\,A_\mu\right) + \mu\,\Gamma^0\right]\Phi = \overline{\Phi}'\left[i\,\Gamma^\mu\left(\partial_\mu + i g_{\rm 2D}\,A_\mu\right)\right]\Phi'$$

Chiral Spirals

Chiral condensate $\langle\overline{\Phi}'\Phi'\rangle$

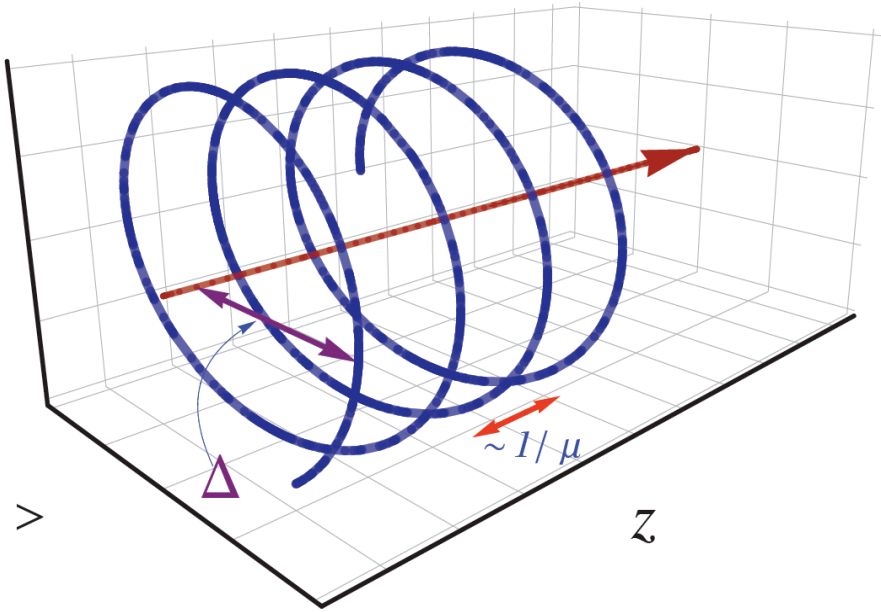
$\langle\overline{\Phi}\Phi\rangle$ $\boxed{\langle\bar{\psi}\psi\rangle}$

$= \langle\overline{\Phi}'\Phi'\rangle\cos(2\mu z)$

$< \overline{\Phi}\,\Phi >$

$\langle\overline{\Phi}\,i\Gamma^5\,\Phi\rangle$ $\boxed{\langle\bar{\psi}\gamma^0\gamma^z\psi\rangle}$

$= \langle\overline{\Phi}'\Phi'\rangle\sin(2\mu z)$ $< \overline{\Phi}\,i\Gamma^5\,\Phi >$



Kojo, et.al NPA'10

Condensates

At B = 0

$$\overline{\Phi}'\Phi' = \overline{\varphi}'_{\uparrow}\varphi'_{\uparrow} + \overline{\varphi}'_{\downarrow}\varphi'_{\downarrow}$$

Inverting the chiral rotation

$$\langle\overline{\Phi}\Phi\rangle = cos(2\mu z)\langle\overline{\Phi}'\Phi'\rangle \quad \langle\overline{\Phi}\Gamma^5\Phi\rangle = -isin(2\mu z)\langle\overline{\Phi}'\Phi'\rangle$$

Contributions of opposite spins cancel out

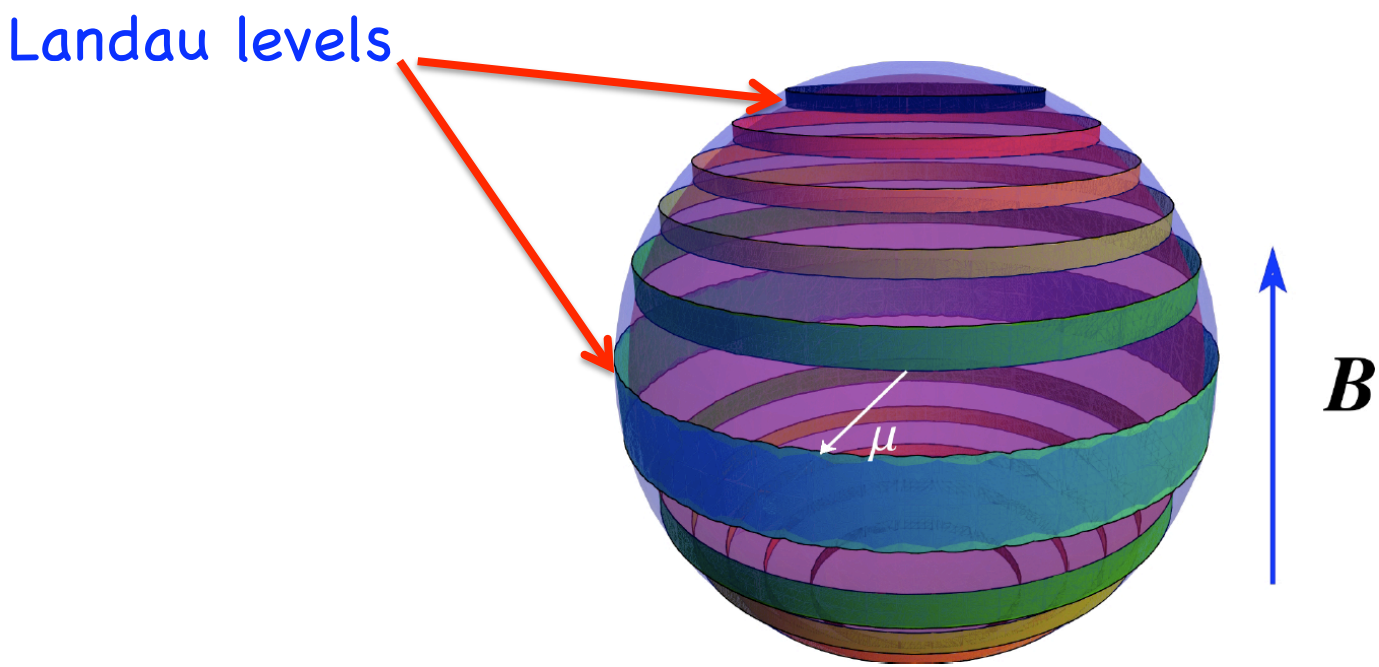
$$\boxed{\overline{\Phi}'\tau^3\Phi' = \overline{\varphi}'_{\uparrow}\varphi'_{\uparrow} - \overline{\varphi}'_{\downarrow}\varphi'_{\downarrow} = 0}$$

A New Needed Ingredient: Nonzero B

Possible realization of QCS:

Heavy ion collision experiments & neutron stars

Both environments typically have very strong magnetic fields, $B \leq \Lambda_{QCD}^2$



SD Eq. at B ≠ 0 in Confining Potential

$$\hat{\delta}_{ll'}\Sigma^l(p)\Pi_l = g^2 t_a t_b \int \frac{d^4q}{(2\pi)^4} e^{-\hat{q}_\perp^2} D^{ab}_{\mu\nu}(q) \Big\{ \Pi_l \gamma_\nu^{||} \tilde{G}^l(\overline{p-q}) \gamma_\mu^{||} \delta_{ll'}$$

$$+ \sum_{\sigma\pm 1} \Delta(\sigma)\gamma_\nu^\perp \Pi_{l+\sigma} \tilde{G}^{l+\sigma}(\overline{p-q}) \gamma_\mu^\perp \Delta(\sigma)\delta_{ll'}$$

$$+ \sum_{\sigma\pm 1} \Big[\Delta(\sigma)\gamma_\nu^\perp \Pi_{l+\sigma} \tilde{G}^{l+\sigma}(\overline{p-q}) \gamma_\mu^\perp \Delta(-\sigma)\delta_{l'l+2\sigma}$$

$$+ \Pi_l \gamma_\nu^{||} \tilde{G}^l(\overline{p-q}) \gamma_\mu^\perp \Delta(\sigma)\delta_{ll'+\sigma} \cdot$$

$$+ \Delta(\sigma)\gamma_\nu^\perp \Pi_{l+\sigma} \tilde{G}^{l+\sigma}(\overline{p-q}) \gamma_\mu^{||} \delta_{l'l+\sigma} \Big] \Big\}$$

Gribov-Zwansiger propagator:

$$D^{ab}_{\mu\nu}(q) = -i\delta_{ab} \frac{8\pi}{C_F} \frac{\sigma}{(\vec{q}^2)^2} g_{0\mu} g_{0\nu}$$

$C_F = (N_c^2 - 1)/2N_c$

$\sigma \sim \Lambda_{QCD}$

SD Eq. at B ≠ 0 in Confining Potential

$$\hat{\delta}_{ll'}\Sigma^l(p)\Pi_l = g^2 t_a t_b \int \frac{d^4q}{(2\pi)^4} e^{-\hat{q}_\perp^2} D^{ab}_{\mu\nu}(q) \Big\{ \Pi_l \gamma_\nu^{||} \tilde{G}^l(\overline{p-q}) \gamma_\mu^{||} \delta_{ll'}$$

$$+ \sum_{\sigma\pm 1} \Delta(\sigma)\gamma_\nu^\perp \Pi_{l+\sigma} \tilde{G}^{l+\sigma}(\overline{p-q})\gamma_\mu^\perp \Delta(\sigma)\delta_{ll'}$$

$$+ \sum_{\sigma\pm 1} \Big[\Delta(\sigma)\gamma_\nu^\perp \Pi_{l+\sigma} \tilde{G}^{l+\sigma}(\overline{p-q})\gamma_\mu^\perp \Delta(-\sigma)\delta_{l'l+2\sigma}$$

$$+ \Pi_l \gamma_\nu^{||} \tilde{G}^l(\overline{p-q})\gamma_\mu^\perp \Delta(\sigma)\delta_{ll'+\sigma}$$

$$+ \Delta(\sigma)\gamma_\nu^\perp \Pi_{l+\sigma} \tilde{G}^{l+\sigma}(\overline{p-q})\gamma_\mu^{||} \delta_{l'l+\sigma} \Big] \Big\}$$

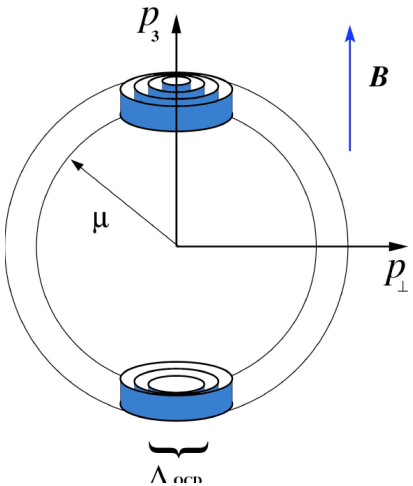
Gribov-Zwansiger propagator:

$$D^{ab}_{\mu\nu}(q) = -i\delta_{ab} \frac{8\pi}{C_F} \frac{\sigma}{(\vec{q}^2)^2} g_{0\mu} g_{0\nu}$$

$C_F = (N_c^2 - 1)/2N_c$

$\sigma \sim \Lambda_{QCD}$

Patch on the z-axis: Dimensional Reduction



Assume $qB \leq (\Lambda_{QCD})^2$

One equation for each LL in the patch. The LLs become flavors l

$$\Sigma^l(p)\Pi(l) = -i\tilde{g}^2\Pi(l)\int\frac{d^2q_{||}}{(2\pi)^2}\gamma_0\tilde{G}^l(\overline{p-q})\gamma_0\int\frac{d^2q_\perp}{(2\pi)^2}\frac{e^{-\hat{q}_\perp^2}}{(q_\perp^2+q_3^2)^2}$$

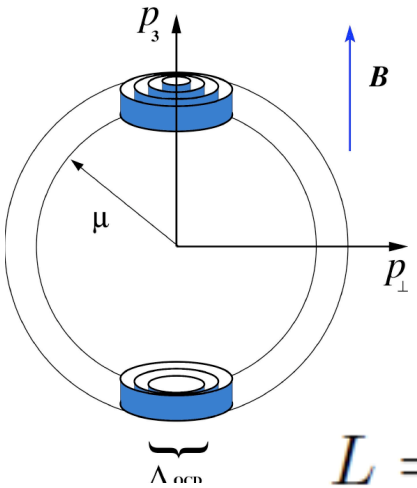
$$= -i\tilde{g}^2\Pi(l)\int\frac{d^2q_{||}}{(2\pi)^2}\gamma_0\tilde{G}^l(\overline{p-q})\gamma_0\frac{1}{4\pi}\int_0^{\Lambda^2}dq_\perp^2\frac{e^{-\hat{q}_\perp^2}}{(q_\perp^2+q_3^2)^2}$$

$$= -i\frac{\tilde{g}^2}{4\pi}\Pi(l)\int\frac{d^2q_{||}}{(2\pi)^2}\gamma_0\tilde{G}^l(\overline{p-q})\gamma_0\frac{1}{q_3^2}$$

Only one spin contributes to the lowest LL

$$\boxed{\Pi(l) = \Delta(+) + \Delta(-)(I - \delta_{l0})}$$

Efffective (1+1)-D Theory at B≠0



Assume $qB \leq (\Lambda_{QCD})^2$

1+1-D theory with lmax flavors

$$L = \int d^2p_{||}\overline{\Phi}_0[\Gamma^\mu(p_\mu + g_{2D}A_\mu) + \mu\Gamma^0]\Phi_0$$
$$+ \sum_{l=1}^{l_{max}} \int d^2p_{||}\overline{\Phi}_l[\Gamma^\mu(p_\mu + g_{2D}A_\mu) + \mu\Gamma^0]\Phi_l$$

$$l_{\max} = [\Lambda^2 / 2eB]$$

zero Landau level

$$\Phi_0 = \begin{pmatrix} \varphi_{0\uparrow} \\ 0 \end{pmatrix}$$

higher Landau levels l>0

$$\Phi_l = \begin{pmatrix} \varphi_{l\uparrow} \\ \varphi_{l\downarrow} \end{pmatrix}$$

Quarkyonic Chiral Spirals at B≠0

Chiral rotation $\Phi_l = e^{(-i\mu z \Gamma_5)}\Phi'_l$

$$L = \overline{\Phi}_0[i\Gamma^\mu(\partial_\mu + ig_{2D}A_\mu) + \mu\Gamma^0]\Phi_0 + \sum_{l=1}^{l_{max}} \overline{\Phi}_l[i\Gamma^\mu(\partial_\mu + ig_{2D}A_\mu) + \mu\Gamma^0]\Phi_l$$

$$= \overline{\Phi}'_0[i\Gamma^\mu(\partial_\mu + ig_{2D}A_\mu)]\Phi'_0 + \sum_{l=1}^{l_{max}} \overline{\Phi'}_l[i\Gamma^\mu(\partial_\mu + ig_{2D}A_\mu)]\Phi'_l$$

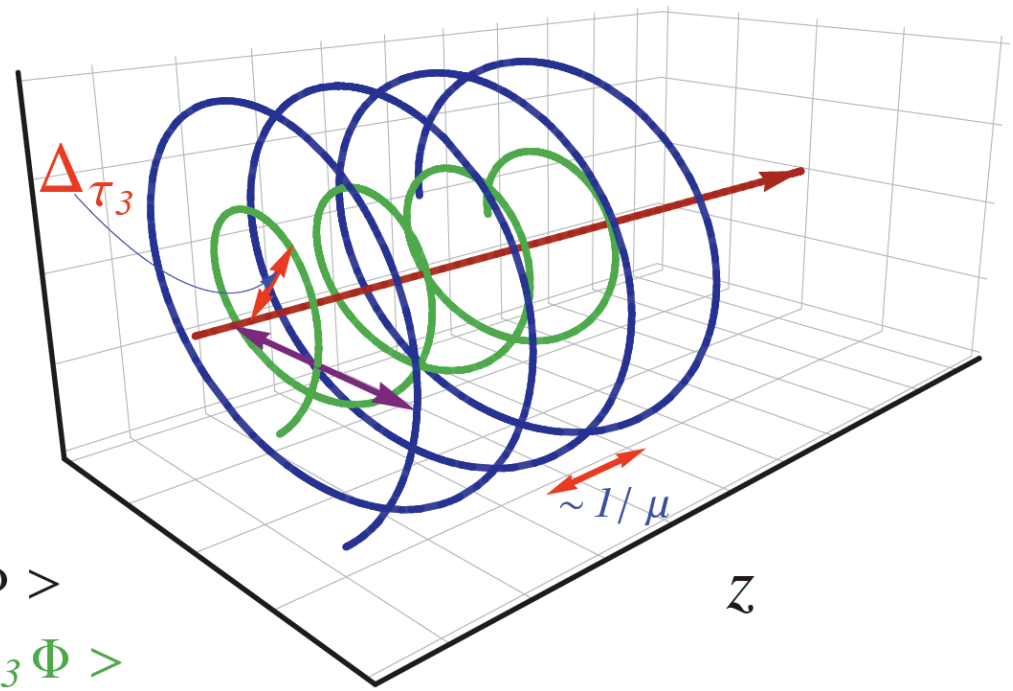
$\boxed{\overline{\Phi}'\Phi'}$ $\boxed{\overline{\Phi}'\tau^3\Phi'}$

$< \overline{\Phi}\ \Phi >$

$< \overline{\Phi}\ \tau_3 \Phi >$

$< \overline{\Phi}\ i\Gamma^5 \Phi >$

$< \overline{\Phi}\ i\Gamma^5 \tau_3 \Phi >$



Kojo, et.al NPA'10

Spirals at B ≠ 0

$$\langle\overline{\Phi}\Phi\rangle = cos(2\mu z)\langle\overline{\Phi}'\Phi'\rangle$$

$$\langle\overline{\Phi}\Gamma^5\Phi\rangle = -isin(2\mu z)\langle\overline{\Phi}'\Phi'\rangle$$

$$\langle\overline{\Phi}\tau^3\Phi\rangle = cos(2\mu z)\langle\overline{\Phi}'\tau^3\Phi'\rangle$$

$$\langle\overline{\Phi}\tau^3\Gamma^5\Phi\rangle = -isin(2\mu z)\langle\overline{\Phi}'\tau^3\Phi'\rangle$$

P and T broken. Rotational sym. explicitly broken by B, translational symmetry spontaneously broken by the spirals

Ferrer, V.I, and Sanchez, arXiv:1205.4492

Chiral Spirals in 3+1-D at B ≠ 0

$$\langle \overline{\Phi}\Phi \rangle \longrightarrow \langle \bar{\psi}\psi \rangle$$

$$\langle \overline{\Phi} i\Gamma^5 \Phi \rangle \longrightarrow \langle \bar{\psi}\gamma^0\gamma^z\psi \rangle$$

Spiral is formed by chiral and electric dipole condensates

$$\langle \overline{\Phi}\tau^3\Phi \rangle \longrightarrow \langle \overline{\psi} i\gamma^1\gamma^2\psi \rangle$$

$$\langle \overline{\Phi}\tau^3\Gamma^5\Phi \rangle \longrightarrow \langle \overline{\psi}\gamma^5\psi \rangle$$

Spiral is formed by pion and magnetic moment condensates

Generation of Parallel & Inhomogeneous Electric and Magnetic fields

Ferrer, V.I, and Sanchez, arXiv:1205.4492

Condensates

At B ≠ 0

$$\overline{\Phi}'\Phi' = \overline{\varphi}'_{0\uparrow}\varphi'_{0\uparrow} + \sum_{l=1}^{l_{max}} [\overline{\varphi}'_{l\uparrow}\varphi'_{l\uparrow} + \overline{\varphi}'_{l\downarrow}\varphi'_{l\downarrow}]$$

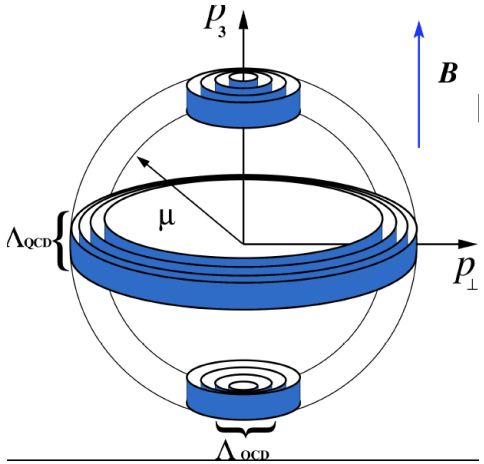
and

$$\overline{\Phi}'\tau^3\Phi' = \boxed{\overline{\varphi}'_{0\uparrow}\varphi'_{0\uparrow}} + \sum_{l=1}^{l_{max}} [\overline{\varphi}'_{l\uparrow}\varphi'_{l\uparrow} - \overline{\varphi}'_{l\downarrow}\varphi'_{l\downarrow}]$$

The LLL makes it nonzero!!

Outlook

- **Equatorial Patch**



Preliminary Findings:

Ferrer, VI, and Sanchez, in prep.

In the equatorial patch the contribution is very similar to the case at B=0 since the LLs are very close to each other. Only one type of quarkyonic spiral in this patch.

- **Has B anything else hidden in the drawer?**

Findings at μ=0:

Galilo & Nedelko '11

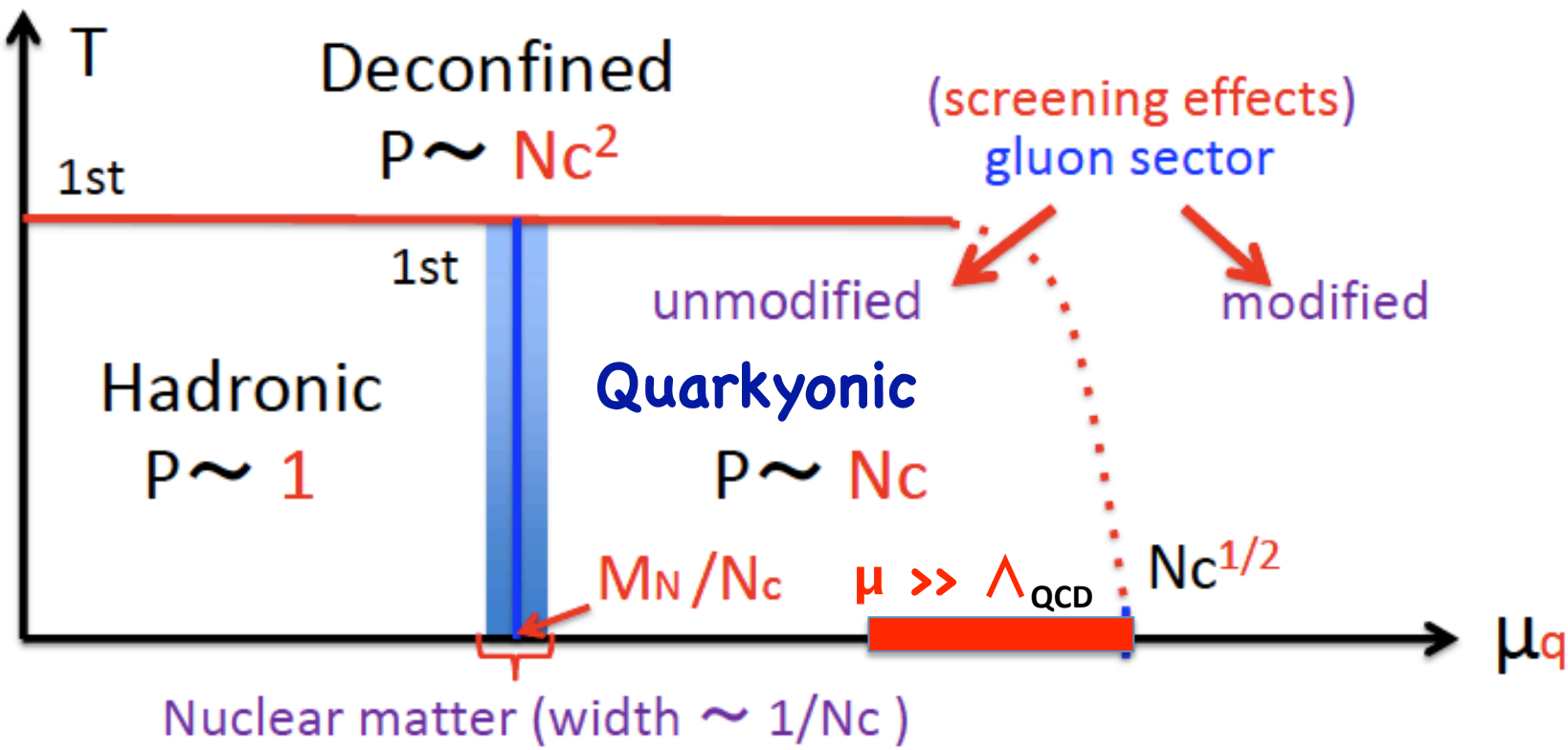
In a strong magnetic field the gluon field configuration that minimize the action does not support confinement of quarks

Miransky & Shovkovy '02

In a strong magnetic the confining scale is smaller (for $N_c < N_c^{thr}$)

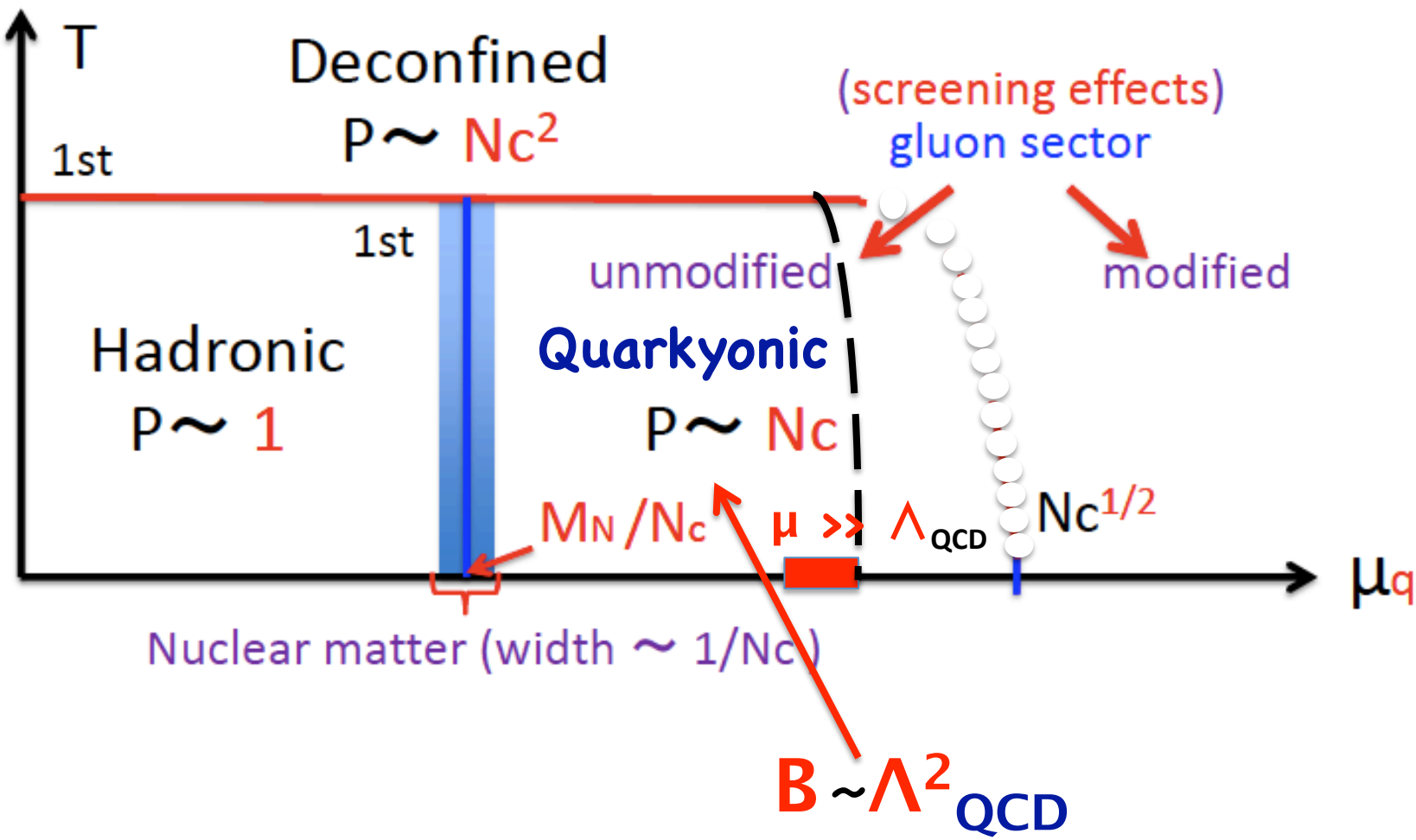
$\lambda_{QCD} < \Lambda_{QCD}$

❖ Can B increase the screening by modifying m_D?



Graph, courtesy of T. Kojo

❖ Can B increase the screening by modifying m_D?



Kalaydzhyan, I.K., PRL 106 (2011) 211601,
Gahramanov, Kalaydzhyan, I.K. (PRD (2012))

Anisotropic Hydrodynamics, Chiral Magnetic Effect and Holography

Ingo Kirsch

DESY Hamburg, Germany

Based on work with

T. Kalaydzhyan, I. Gahramanov

Workshop On
P- and CP-odd Effects in Hot and Dense Matter (2012)
June 25-27, 2012 Brookhaven National Laboratory

Chiral magnetic effect

Chiral magnetic effect:

1. in presence of a magnetic field B, momenta of the quarks align along B

2. topological charge induces chirality

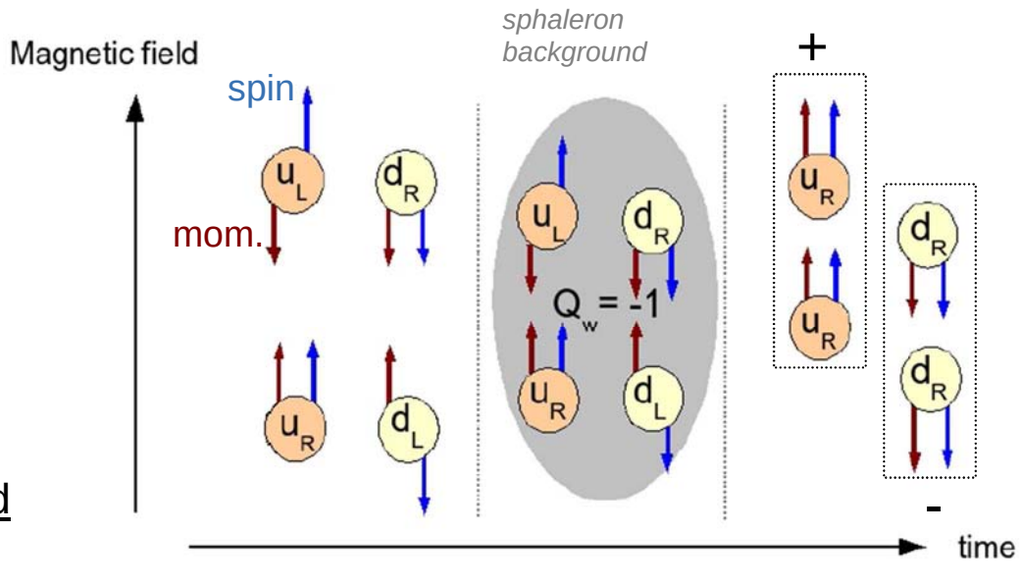
3. positively/negatively charged quarks move up/down (charge separation!)

4. an electric current is induced along the magnetic field B

⇨ $\vec{J} = C\mu_5\vec{B}$

CME in HICs:
Kharzeev, McLerran and Warringa (2008),
Fukushima, Kharzeev and Warringa (2008)

earlier papers:
Vilenkin (1980), Giovannini and Shaposhnikov (1998),
Alekseev, Cheianov and J. Fröhlich (1998)



$$(N_L - N_R)|_\infty - (N_L - N_R)|_{-\infty} = 2N_f Q$$

$$\text{in fig.:} \quad [0-4] - [2-2] = 2 \cdot 2 \cdot (-1)$$

CME is a candidate for explaining an observed charge asymmetry in HIC

Does the CME hold at strong coupling?

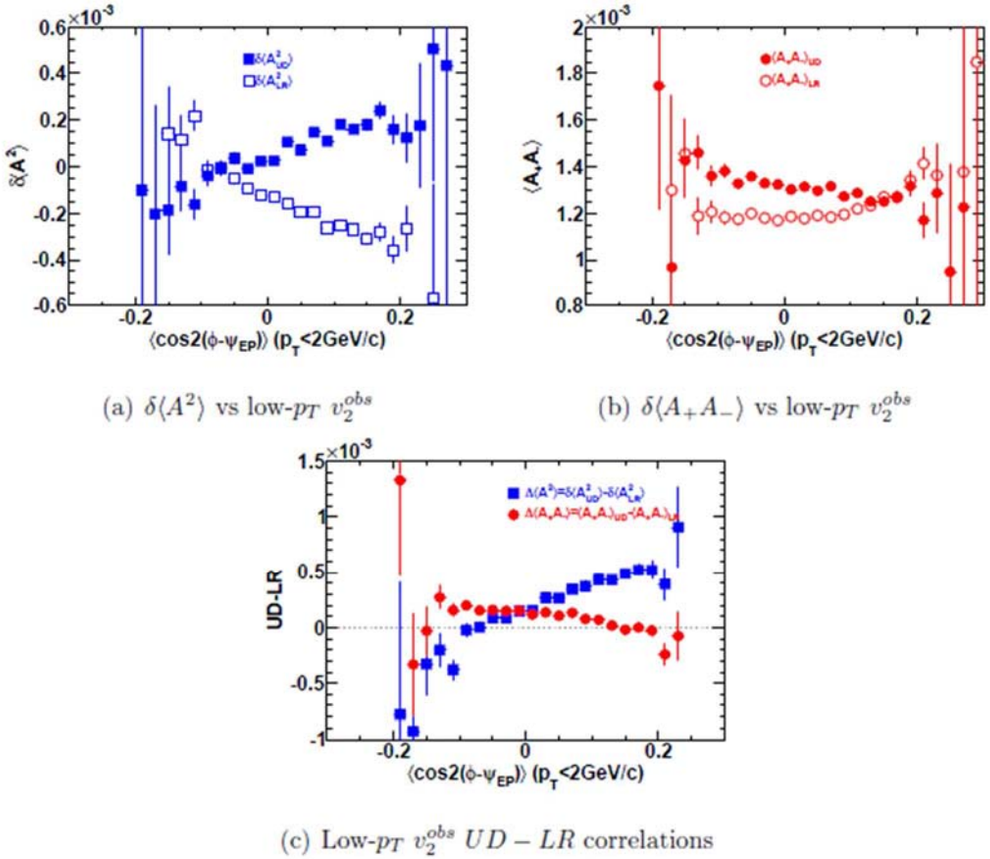
⇨ AdS/CFT

Possible dependence of the charge asymmetry on v_2

Event-by-event anisotropy (v_2^{obs}) dependence (low p_T)

Quan Wang (2012)

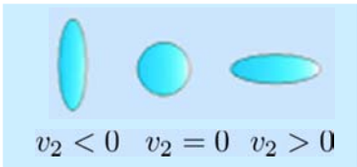
Investigate the charge asymmetry as a function of the anisotropy v_2^{obs} of the measured particles in mid-central 20–40% centrality collisions ($B \approx const.$). Consider (rare) events with different v_2^{obs}.



(a) $\delta\langle A^2\rangle$ vs low-p_T v_2^{obs}

(b) $\delta\langle A_+A_-\rangle$ vs low-p_T v_2^{obs}

(c) Low-p_T v_2^{obs} $UD - LR$ correlations



Observations:

- same-sign particles are emitted more likely in UD direction the larger v_2^{obs}
- same-sign particles are emitted less likely in LR direction the larger v_2^{obs} (the dependence is significantly weaker for opposite-sign particles)
- => strong v_2^{obs} dependence of the difference between UD and LR => charge separation depends approx. linearly on v_2^{obs} (apparently in contradiction with the CME)

Overview

Goals:

i) Construct model for the chiral magnetic effect (CME) at strong coupling (AdS/CFT)
ii) In view of a possible v_2-dependence of the charge separation, I also study the CME in an anisotropic model

Outline:

I. CME in hydrodynamics

II. Fluid/gravity model of the CME

III. CME in anisotropic fluids

(v_2-dependence)

Conclusions

Part I: CME in hydrodynamics

Hydrodynamics vs. fluid/gravity model

Hydrodynamics

Multiple-charge model

$U(1)^n$ plasma with triangle anomalies

Son & Surowka (2009)

reduction ↓

Two-charge model

$U(1)_V \times U(1)_A$ plasma

recover CME (and other effects)

dual ↔

Fluid/gravity model

Holographic *n*-charge model

5D AdS black hole geometry with *n* U(1) charges

↓

Holographic two-charge model

n-charge model reduced to two charges

recover holographic CME, etc.

Hydrodynamical model with *n* anomalous U(1) charges

Son & Surowka (2009)

U(1)^n plasma with triangle anomalies:

$$\partial_\mu T^{\mu\nu} = F^{a\nu\lambda} j^a_\lambda \qquad (a = 1, 2, ..., n)$$
$$\partial_\mu j^{a\mu} = C^{abc} E^b \cdot B^c$$

E- & B-fields, vorticity:

$$E^{a\mu} = F^{a\mu\nu} u_\nu$$
$$B^{a\mu} = \tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu F^a_{\alpha\beta}$$
$$\omega^\mu = \tfrac{1}{2}\epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho$$

stress-energy tensor $T^{\mu\nu}$ and U(1) currents $j^{a\mu}$:

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + \ldots$$
$$j^{a\mu} = \rho^a u^\mu + \xi^a_\omega \omega^\mu + \xi^{ab}_B B^{b\mu} + \ldots$$

"New" transport coefficients (not listed in Landau-Lifshitz)

- vortical conductivities ξ^a_ω *Erdmenger, Haack, Kaminski, Yarom (2008)*
- magnetic conductivities ξ^{ab}_B *Son & Surowka (2009)*

first found in a holographic context (AdS/CFT)

First-order transport coefficients

Son & Surowka (2009), Neiman & Oz (2010)

U(1) currents:

$$j^{a\mu} = \rho^a u^\mu + \xi^a_\omega \omega^\mu + \xi^{ab}_B B^{b\mu} + \dots$$

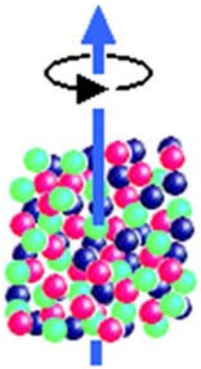
vortical and magn. conductivities:

$$\xi^a_\omega = C^{abc}\mu^b\mu^c - \frac{2}{3}\rho^a C^{bcd}\frac{\mu^b\mu^c\mu^d}{\epsilon + P} + \mathcal{O}(T^2)$$

$$\xi^{ab}_B = C^{abc}\mu^c - \frac{1}{2}\rho^a C^{bcd}\frac{\mu^c\mu^d}{\epsilon + P} + \mathcal{O}(T^2)$$

chemical potentials

Conductivities are non-zero only in fluids with triangle anomalies!



$\xi_\omega \neq 0$

$\xi_\omega = 0$

coffee with sugar
(chiral molecules)

Two charge case (n=2): $U(1)_A$ x $U(1)_V$

Sadofyev and Isachenkov (2010), Kalaydzhyan & I.K. (2011)+(2012)

$U(1)_A$: provides chemical potential μ_5 (chirality)
$U(1)_V$: measures the electric current

Hydrodynamical equations:

$$\partial_\mu T^{\mu\nu} = F^{V\nu\lambda} j_\lambda$$
$$\partial_\mu j_5^\mu = -\frac{C}{4} F^V_{\mu\nu} \tilde{F}^{V\mu\nu}$$
$$\partial_\mu j^\mu = 0$$

axial gauge field switched off! $(A^A_\mu = 0)$

Identifications:

$$A^A_\mu = A^1_\mu\,, \qquad A^V_\mu = A^2_\mu\,,$$
$$j_5^\mu = j^{1\mu}\,, \qquad j^\mu = j^{2\mu}$$
$$\mu_5 = \mu^1\,, \quad \mu = \mu^2\,,$$
$$\rho_5 = \rho^1\,, \quad \rho = \rho^2\,,$$
$$\kappa_\omega = \xi^2_\omega\,, \quad \kappa_B = \xi^{22}_B\,,$$
$$\xi_\omega = \xi^1_\omega\,, \quad \xi_B = \xi^{12}_B$$

Constitutive equations:

CME coefficient

$$j^\mu = \rho u^\mu + \kappa_\omega \omega^\mu + \kappa_B B^\mu$$
$$j_5^\mu = \rho_5 u^\mu + \xi_\omega \omega^\mu + \xi_B B^\mu$$

C-parity allows for:

$$C^{111} = C/3 \quad (AAA)$$
$$C^{122} = C^{221} = C^{212} = C/3 \quad (AVV)$$
$$C^{121} = C^{211} = C^{112} = 0 \quad (VAA)$$
$$C^{222} = 0 \quad (VVV)$$

(Chiral) magnetic and vortical effects

constitutive equations:

$$j^\mu = \rho u^\mu + \kappa_\omega \omega^\mu + \kappa_B B^\mu$$
$$j_5^\mu = \rho_5 u^\mu + \xi_\omega \omega^\mu + \xi_B B^\mu$$

transport coefficients (conductivities):

CVE $\kappa_\omega/\mu = 2C\mu_5 \left(1 - \frac{\mu\rho}{\epsilon+P}[1+\frac{\mu_5^2}{3\mu^2}]\right)$, $\kappa_B = C\mu_5\left(1-\frac{\mu\rho}{\epsilon+P}\right)$ CME

QVE $\xi_\omega/\mu = C\mu\left(1 - 2\frac{\mu_5\rho_5}{\epsilon+P}[1+\frac{\mu_5^2}{3\mu^2}]\right)$, $\xi_B = C\mu\left(1-\frac{\mu_5\rho_5}{\epsilon+P}\right)$ CSE (QME)

C=chiral since CVE, CME prop. to chiral chemical potential μ_5
Q=quark since QVE, QME prop. to quark chemical potential μ
(QME also called chiral separation effect (CSE))

$\mu\vec{\omega}$ creates an effective magnetic field *Kharzeev and Son (2010)*

Part II: Fluid/gravity model of the CME

Kalaydzhyan & I.K., PRL 106 (2011) 211601

Hydrodynamics vs. fluid/gravity model

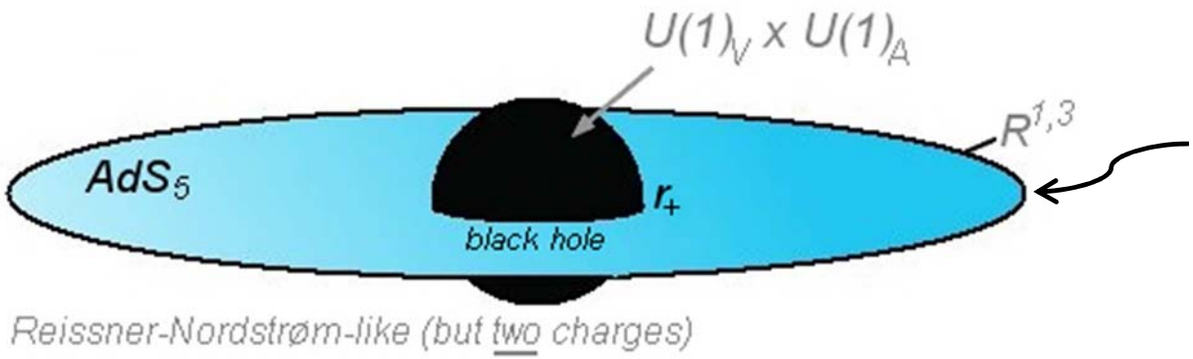
Hydrodynamics		Fluid/gravity model
Multiple-charge model $U(1)^n$ plasma with triangle anomalies Son & Surowka (2009)	dual	Holographic *n*-charge model 5D AdS black hole geometry with *n* U(1) charges
reduction ↓		↓
Two-charge model $U(1)_V \times U(1)_A$ plasma recover CME (and other effects)		Holographic two-charge model *n*-charge model reduced to two charges recover holographic CME, etc.

Gravity: Holographic computation

Kalaydzhyan & I.K.
PRL 106 (2011) 211601

Strategy: quark-gluon plasma is strongly-coupled ⇨ use AdS/CFT to compute the transport coefficients relevant for the anomalous effects (CME, etc.)

- find a 5d charged AdS black hole solution with several U(1) charges



4d fluid (QGP) `lives' on the boundary of the AdS space

- duality:

$$(m, q_a) \longleftrightarrow (T, \mu_a)$$

mass m
U(1) charges q_a

$T \sim r_+$ Hawking temperature
$\mu^a \equiv A_0^a(r_+) - A_0^a(\infty)$

- use fluid-gravity methods to holographically compute the transport coefficients $\kappa_\omega, \kappa_B, \xi_\omega, \xi_B$ (i.e. CME and other effects)

AdS black hole solution with multiple U(1) charges

Five-dimensional $U(1)^n$ Einstein-Maxwell theory
with cosmological term:

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[R + 12 - F^a_{MN} F^{aMN} + \frac{S_{abc}}{6\sqrt{-g}} \varepsilon^{PKLMN} A^a_P F^b_{KL} F^c_{MN} \right]$$

Fields:
- metric g_{MN} ($M, N = 0, ..., 4$)
- n U(1) gauge fields A_M^a ($a = 1, \ldots, n$)
- cosmological constant $\Lambda = -6$

The information of the anomalies is encoded in the Chern-Simons coefficients

$$S_{abc} = 4\pi G_5 \, C_{abc}$$

Son & Surowka (2009)

Boosted AdS black hole solution

5d AdS black hole solution (0th order solution):

$$ds^2 = -f(r)u_\mu u_\nu dx^\mu dx^\nu - 2u_\mu dx^\mu dr + r^2\left(\eta_{\mu\nu} + u_\mu u_\nu\right) dx^\mu dx^\nu$$
$$A^a = \left(A_0^a(r)\, u_\mu + \mathcal{A}_\mu^a\right) dx^\mu$$

background gauge field added (needed to model external B-field)!

with

$$f(r) = r^2 - \frac{m}{r^2} + \sum_a \frac{(q_a)^2}{r^4}$$

$$A_0^a(r) = -\frac{\sqrt{3}q^a}{2r^2}$$

n charges q^a

u_μ = four-velocity of the fluid

First-order transport coefficients

We use the standard procedure of Bhattacharyya et al. (2008) to holographically compute the transport coefficients ξ_ω^a (Torabian & Yee (2009) for n=3) and ξ_B^{ab} :

1. Vary 4-velocity and background fields (up to first order):

$$u_\mu = (-1, x^\nu \partial_\nu u_i), \qquad \mathcal{A}_\mu^a = (0, x^\nu \partial_\nu \mathcal{A}_i^a)$$

The boosted black-brane solution (0th order sol.) is no longer an exact solution, but receives higher-order corrections.

Ansatz for first-order corrections:

$$\begin{aligned} ds^2 &= (-f(r) + \tilde{g}_{tt})\, dt^2 + 2\,(1 + \tilde{g}_{tr})\, dtdr \\ &\quad + r^2 (dx^i)^2 + \tilde{g}_{ij} dx^i dx^j - 2x^\nu \partial_\nu u_i dr dx^i \\ &\quad + 2\left(\left(f(r) - r^2\right) x^\nu \partial_\nu u_i + \tilde{g}_{ti}\right) dt dx^i\,, \\ A^a &= \left(-A_0^a(r) + \tilde{A}_t^a\right) dt + \left(A_0^a(r) x^\nu \partial_\nu u_i + x^\nu \partial_\nu \mathcal{A}_i^a + \tilde{A}_i^a\right) dx^i \end{aligned}$$

Need to determine first order corrections $\tilde{g}_{tt}, \tilde{g}_{tr}, etc.$

First-order transport coefficients (*cont.*)

2. Solve equations of motion (system of Einstein-Maxwell equations) and find the first-order corrections to the metric and gauge fields:

$$\tilde{g}_{tr} = \tilde{g}_{tt} = \tilde{A}^a_t = 0$$

$$\tilde{g}_{ti}(r) = f(r) \int_\infty^r dr' \, \frac{1}{r' \left(f(r')\right)^2} \left(\int_{r_+}^{r'} dr'' \, I(r'') - r_+ f'(r_+) C_i \right)$$

$$\tilde{A}^a_i(r) = \int_\infty^r dr' \frac{1}{r' f(r')} \left[Q^a_i(r') - Q^a_i(r_H) - C_i r_+ A^{a\prime}_0(r_+) + r' \tilde{g}_{ti}(r') A^{a\prime}_0(r') \right]$$

(lengthy calculation, $I(r)$, $Q^a_i(r)$, C_i functions of $A^a_0(r)$, $f(r)$, u_i)

3. Read off energy-momentum tensor and U(1) currents from the near-boundary expansion of the first-order corrected background (e.g. Fefferman-Graham coordinates):

$$T_{\mu\nu} = \frac{g^{(4)}_{\mu\nu}(x)}{4\pi G_5} + c.t$$

$$j^\mu_a = \frac{\eta^{\mu\nu} A^{(2)}_{a\nu}(x)}{8\pi G_5} + \hat{j}^\mu_a$$

$$ds^2 = \frac{1}{z^2} \left(g_{\mu\nu}(z, x) \, dx^\mu dx^\nu + dz^2 \right) ,$$

$$g_{\mu\nu}(z, x) = \eta_{\mu\nu} + g^{(2)}_{\mu\nu}(x) \, z^2 + g^{(4)}_{\mu\nu}(x) \, z^4 + \ldots$$

$$A^a_\mu(z, x) = A^{a(0)}_\mu(x) + A^{a(2)}_\mu(x) \, z^2 + \ldots$$

First-order transport coefficients (*cont.*)

4. Determine the vortical and magnetic conductivities ξ^a_ω and ξ^{ab}_B

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \dots$$

$$j^{a\mu} = \rho^a u^\mu + \xi^a_\omega \omega^\mu + \xi^{ab}_B B^{b\mu} + \dots$$

use identities (from zeroth order solution):

$$\left.\begin{array}{l} P \equiv m/16\pi G_5 \\ \rho_a \equiv \sqrt{3} q_a / 16\pi G_5 \end{array}\right\} \Rightarrow \frac{\sqrt{3} q^a}{4m} = \frac{\rho^a}{\epsilon + P} \qquad (\epsilon = 3P)$$

transport coefficients:

$$\xi^a_\omega = \frac{4}{16\pi G_5}\left(S^{abc}\mu^b\mu^c - \frac{2}{3}\frac{\rho^a}{\varepsilon + P} S^{bcd}\mu^b\mu^c\mu^d \right)$$

$$\xi^{ab}_B = \frac{4}{16\pi G_5}\left(S^{abc}\mu^c - \frac{1}{2}\frac{\rho^a}{\varepsilon + P} S^{bcd}\mu^c\mu^d \right)$$

$$\mu^a \equiv A^a_0(r_H) - A^a_0(\infty)$$

$S_{abc} = 4\pi G_5 C_{abc}$ ⇨ **We recover the hydrodynamic result!**

Holographic magnetic and vortical effects

Using the same identifications as in hydrodynamics, but now for the holographically computed transport coefficients, we get

CVE $\kappa_\omega = 2C\mu\mu_5\left(1-\mu\frac{\sqrt{3}q}{4m}[1+\frac{\mu_5^2}{3\mu^2}]\right)$, $\kappa_B = C\mu_5\left(1-\mu\frac{\sqrt{3}q}{4m}\right)$ CME

QVE $\xi_\omega = C\mu^2\left(1-2\mu_5\frac{\sqrt{3}q_5}{4m}[1+\frac{\mu_5^2}{3\mu^2}]\right)$, $\xi_B = C\mu\left(1-\mu_5\frac{\sqrt{3}q_5}{4m}\right)$ QME

<u>Result:</u> CME, CVE, etc. are realized in an n-charged AdS black hole model (plus background gauge field), when appropriately reduced to a two-charge model (n=2).

Other AdS/QCD models:
Lifschytz and Lippert (2009), Yee (2009), Rebhan, Schmitt and Stricker (2010), Gorsky, Kopnin and Zayakin (2010), Rubakov (2010), Gynther, Landsteiner, Pena-Benitez and Rebhan (2010), Hoyos, Nishioka, O'Bannon (2011)

Part III: CME in anisotropic fluids

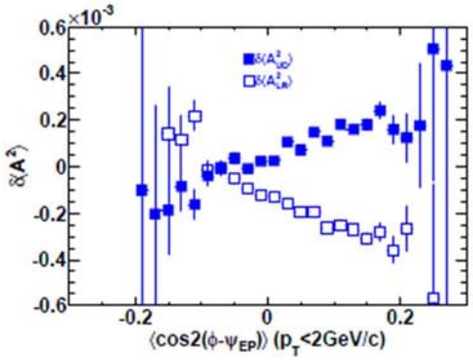
Gahramanov, Kalaydzhyan & I.K., arXiv: 1203.4259,
accepted for publication in PRD (2012)

Possible dependence of the charge asymmetry on v_2

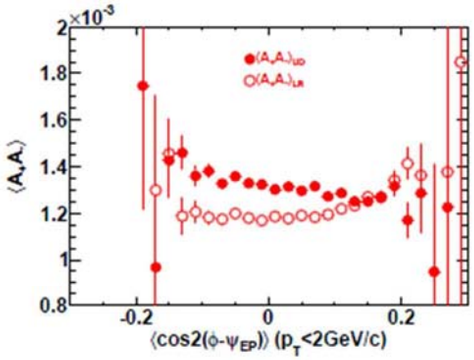
Event-by-event anisotropy (v_2^{obs}) dependence (low p_T)

Quan Wang (2012)

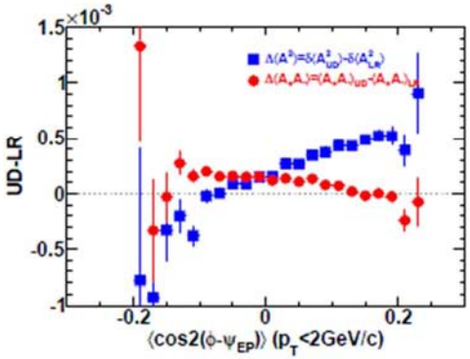
Investigate the charge asymmetry as a function of the anisotropy v_2^{obs} of the measured particles in mid-central 20–40% centrality collisions ($B \approx const.$). Consider (rare) events with different v_2^{obs}.



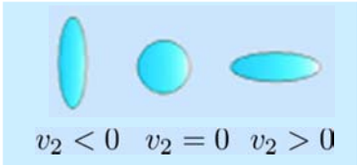
(a) $\delta\langle A^2\rangle$ vs low-p_T v_2^{obs}



(b) $\delta\langle A_+A_-\rangle$ vs low-p_T v_2^{obs}



(c) Low-p_T v_2^{obs} $UD - LR$ correlations



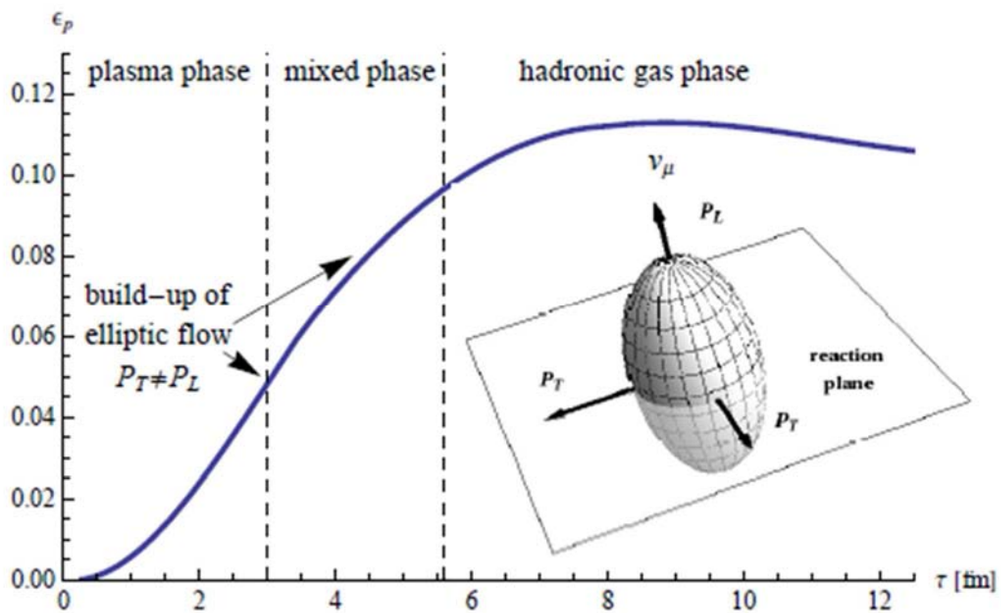
Observations:

- same-sign particles are emitted more likely in UD direction the larger v_2^{obs}
- same-sign particles are emitted less likely in LR direction the larger v_2^{obs} (the dependence is significantly weaker for opposite-sign particles)
- => strong v_2^{obs} dependence of the difference between UD and LR => charge separation depends approx. linearly on v_2^{obs} (apparently in contradiction with the CME)

Build-up of the elliptic flow and momentum anisotropy

Central question: In anisotropic fluids, does the chiral conductivity depend on v_2?

Sketch of the time-evolution of the momentum anisotropy ε_P:



$$\varepsilon_P = \frac{P_T - P_L}{P_T + P_L}$$

at freeze-out:
$v_2 \approx \varepsilon_P/2$

sketch based on
Huovinen, Petreczky (2010)

Our model describes a state after thermalization with unequal pressures $P_T \neq P_L$.
We do not model the full evolution of ε_P.

Hydrodynamics of an anisotropic fluid

Florkowski (various papers)

<u>Anisotropic fluid with *n* anomalous U(1) charges</u>

stress-energy tensor $T^{\mu\nu}$ and U(1) currents $j^{a\mu}$:

$$T^{\mu\nu} = (\epsilon + P_T)u^\mu u^\nu + P_T g^{\mu\nu} - (P_T - P_L)v^\mu v^\nu + \tau^{\mu\nu}$$
$$j^{a\mu} = \rho^a u^\mu + \nu^{a\mu}$$

orthogonality and normalization:

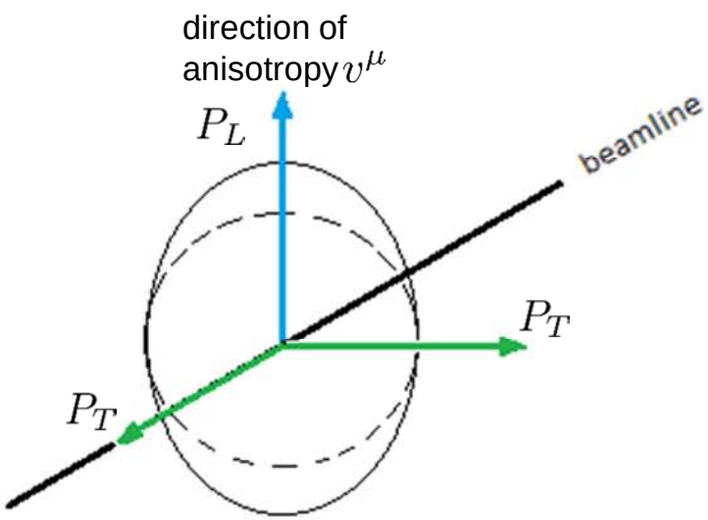
$$u_\mu u^\mu = -1\,, \quad v_\mu v^\mu = 1\,, \quad u_\mu v^\mu = 0$$

local rest frame:

$$u^\mu = (1,0,0,0) \qquad v^\mu = (0,0,0,1)$$
$$T^{\mu\nu} = \mathrm{diag}(\varepsilon, P_T, P_T, P_L)$$

thermodynamic identity:

$$\varepsilon + P_T = Ts + \mu\rho$$



Chiral magnetic and vortical effects

Anisotropic fluid with one axial and one vector U(1)

Repeating the hydrodynamic computation of Son & Surowka (for *n=2*), we find the following result for the chiral magnetic effect:

$$\Delta j^\mu = \kappa_B B^\mu \,, \qquad \kappa_B = C\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + P_T}\right)$$

simplifying assumptions:
$\partial_\mu v^\mu = 0\,, \quad v^\mu \partial_\mu \Delta = 0$
$(\Delta = P_T - P_L)$

or, for small ε_p,

$$\kappa_B \approx \underbrace{C\mu_5}_{\text{as before}} \underbrace{\left(1 - \frac{\mu\rho}{\epsilon + \bar{P}}\left[1 - \frac{\varepsilon_p}{6}\right]\right)}_{\text{anisotropy-dependent}} \qquad (\bar{P} = \tfrac{2P_T + P_L}{3})$$

multiple charge case (*n* arb.):

$$\xi_B^{ab} = C^{abc}\mu^c - \frac{1}{2}\frac{\rho^a}{\varepsilon + P_T} C^{bcd}\mu^c\mu^d$$

Boosted anisotropic AdS black hole solution (w/ *n* U(1)'s)

5d AdS black hole solution (ansatz):

$$ds^2 = \left(r^2 w_T(r) P_{\mu\nu} - f(r) u_\mu u_\nu\right) dx^\mu dx^\nu - 2u_\mu dx^\mu dr$$
$$- r^2 (w_T(r) - w_L(r)) v_\mu v_\nu dx^\mu dx^\nu \,,$$
$$A^a = (A_0^a(r) u_\mu + \mathcal{A}_\mu^a) dx^\mu$$

asymptotic solution (close to the boundary):

$$f(r) = r^2 - \frac{m}{r^2} + \sum_a \frac{(q_a)^2}{r^4} + \mathcal{O}(r^{-6})$$

anisotropy parameter $\zeta \sim \varepsilon_P$

$$w_T(r) = 1 + \frac{m\zeta}{4r^4} + \mathcal{O}(r^{-8})$$

$$w_L(r) = 1 - \frac{m\zeta}{2r^4} + \mathcal{O}(r^{-8})$$

$$A_0^a(r) = -\frac{\sqrt{3} q^a}{2r^2} + \mathcal{O}(r^{-10})$$

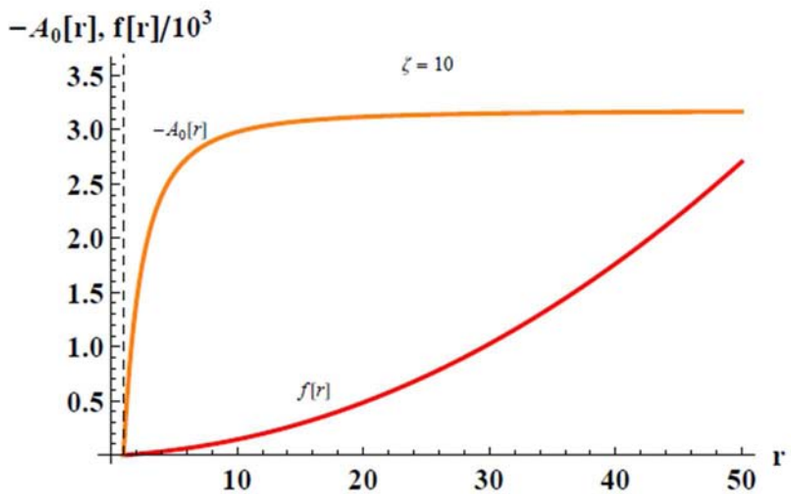
$$\varepsilon = 2P_T + P_L \quad \rho^a = \frac{\sqrt{3} q^a}{16\pi G_5}$$

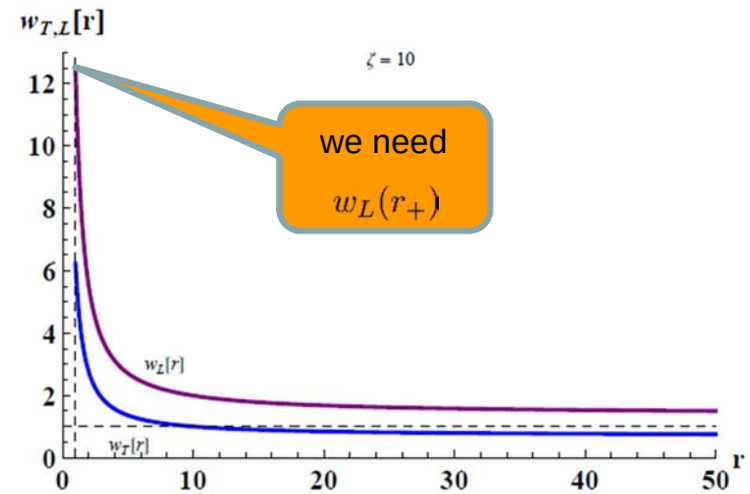
$$P_T = \frac{m(1+\zeta)}{16\pi G_5} \,, \quad P_L = \frac{m(1-2\zeta)}{16\pi G_5}$$

no analytic solution ⇨ numerical solution

Numerical solution for the AdS black hole background

Shooting techniques provide numerical plots for the functions
f(r), $A_0(r)$, $w_T(r)$, $w_L(r)$ for $\zeta=10$:





outer horizon: $r_+ = 1$

First-order transport coefficients

The holographic computation of the transport coefficients is very similar to that in the isotropic case.

Magnetic conductivities (result):

$$\xi_B^{ab} = \frac{4}{16\pi G_5}\left(S^{abc}\mu^c - \frac{1}{2}C(r_+)S^{bcd}\mu^c\mu^d\right)$$

$$\mu^a \equiv A_0^a(r_H) - A_0^a(\infty)\,, \qquad S_{abc} = 4\pi G_5 C_{abc}$$

$$C(r_+) = \frac{r_+\partial_{r_+}A_0^a(r_+)}{r_+(f'(r_+) - 4\sum_a A_0^a(r_+)\partial_{r_+}A_0^a(r_+))}\frac{1}{w_L(r_+)^{1/2}}$$

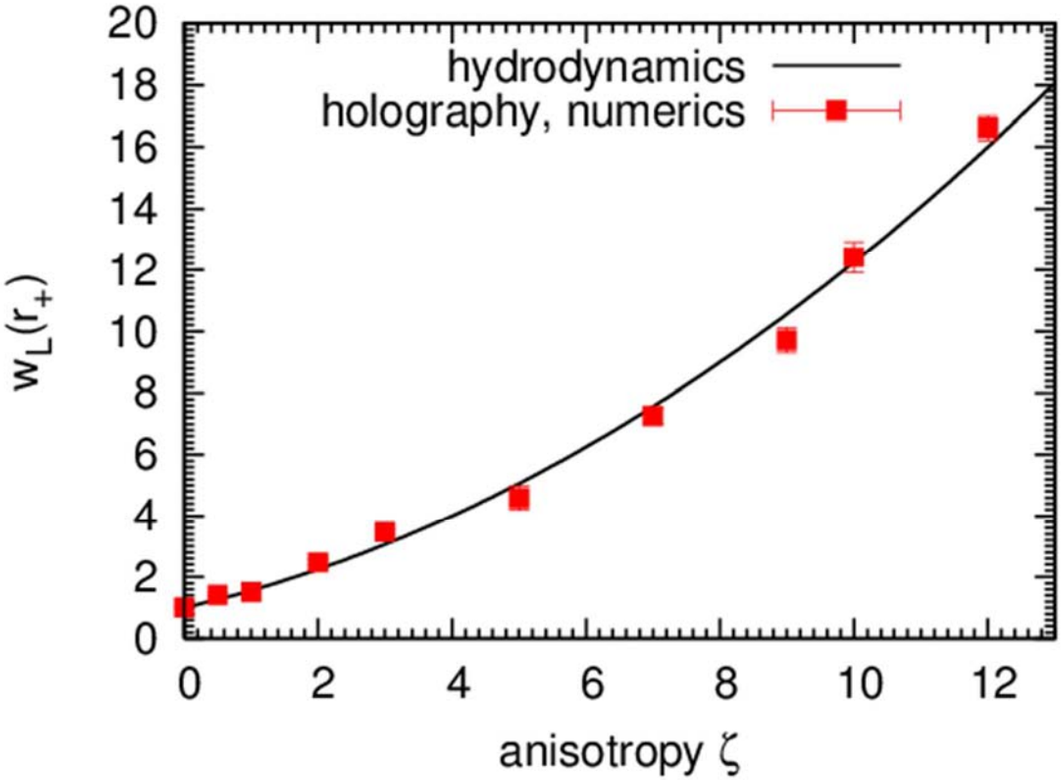
$$\stackrel{!}{=} \frac{\sqrt{3}}{4m}\frac{1}{1+\frac{1}{4}\zeta}q_a = \frac{\rho^a}{\varepsilon + P_T}$$

⇨ find agreement with hydrodynamics if the orange factors agree

(needs to be shown numerically)

Numerical agreement with hydrodynamics

Numerical plot $w_L(r_+)$ as a function of the anisotropy:



agreement with hydrodynamics since

$$w_L(r_+) = (1 + \frac{1}{4}\zeta)^2$$

Conclusions

I presented two descriptions of the CME (and related effects) in

a) isotropic plasmas ($P=P_T=P_L$):

i) hydrodynamic model: $U(1)_A$ x $U(1)_V$ fluid with triangle anomaly
ii) holographic fluid-gravity model: 5d AdS-Reissner-Nordstrom-like solution with two U(1) charges
Agreement was found between both models.

b) anisotropic plasmas ($P_T \neq P_L$):

- experimental data suggests possible v_2 – dependence of the charge separation
- Does the chiral magn. conductivity κ_B depend on v_2?
- we constructed anisotropic versions of the above $U(1)_A$ x $U(1)_V$ models and found

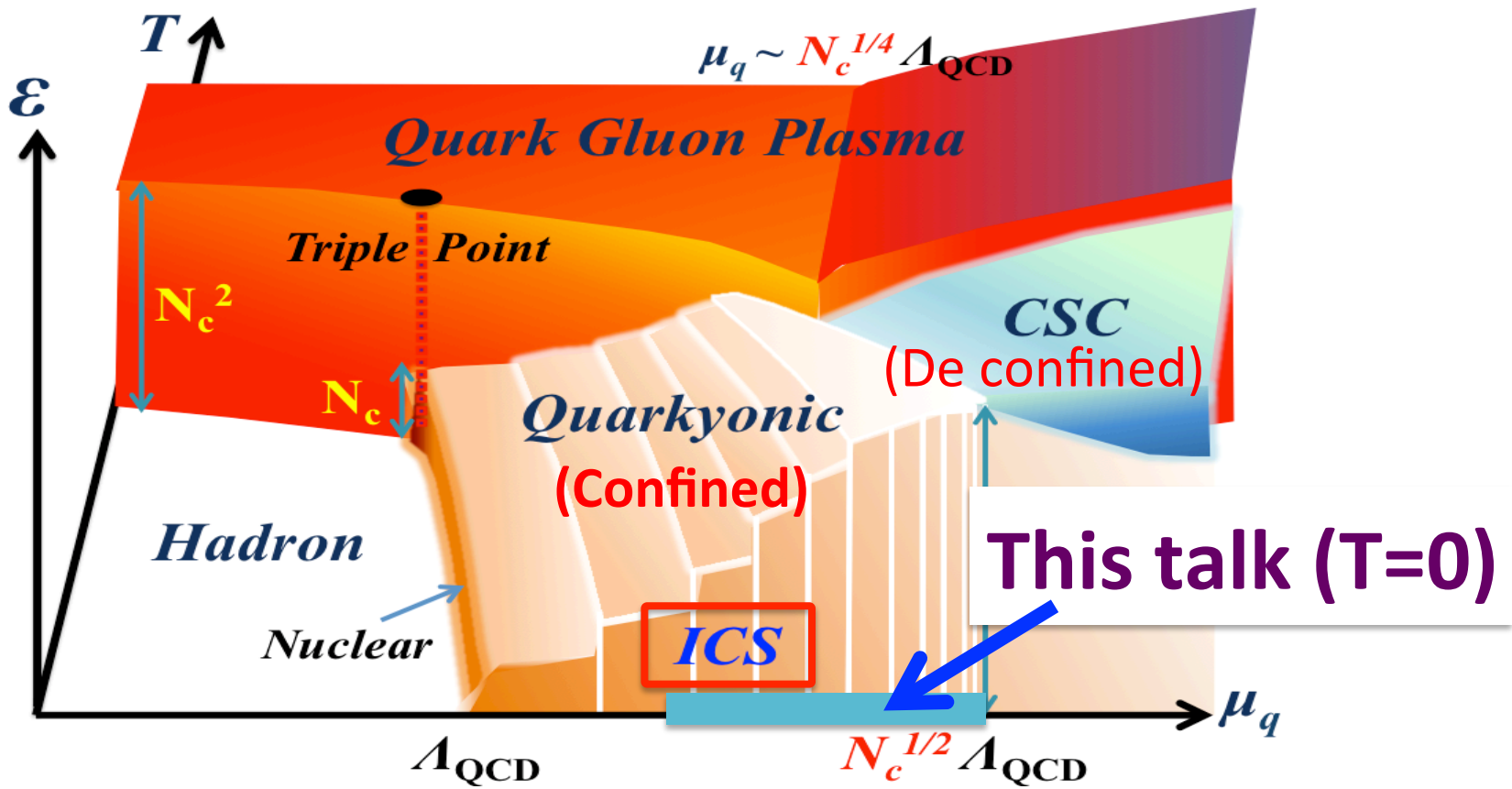
$$\kappa_B \approx \underbrace{C\mu_5}_{\text{CME}}\left(1 - \underbrace{\frac{\mu\rho}{\epsilon+\bar{P}}\left[1-\frac{\varepsilon_p}{6}\right]}_{\text{depends on } \varepsilon_P}\right) \qquad (\bar{P} = \tfrac{2P_T+P_L}{3})$$

Is the observed charge asymmetry a combined effect of the CME (1st term in κ_B) and the dynamics of the system (2nd term)?

Interweaving Chiral Spirals

Toru Kojo (Bielefeld U.)

with: **K. Fukushima, Y. Hidaka, L. McLerran, R.D. Pisarski**



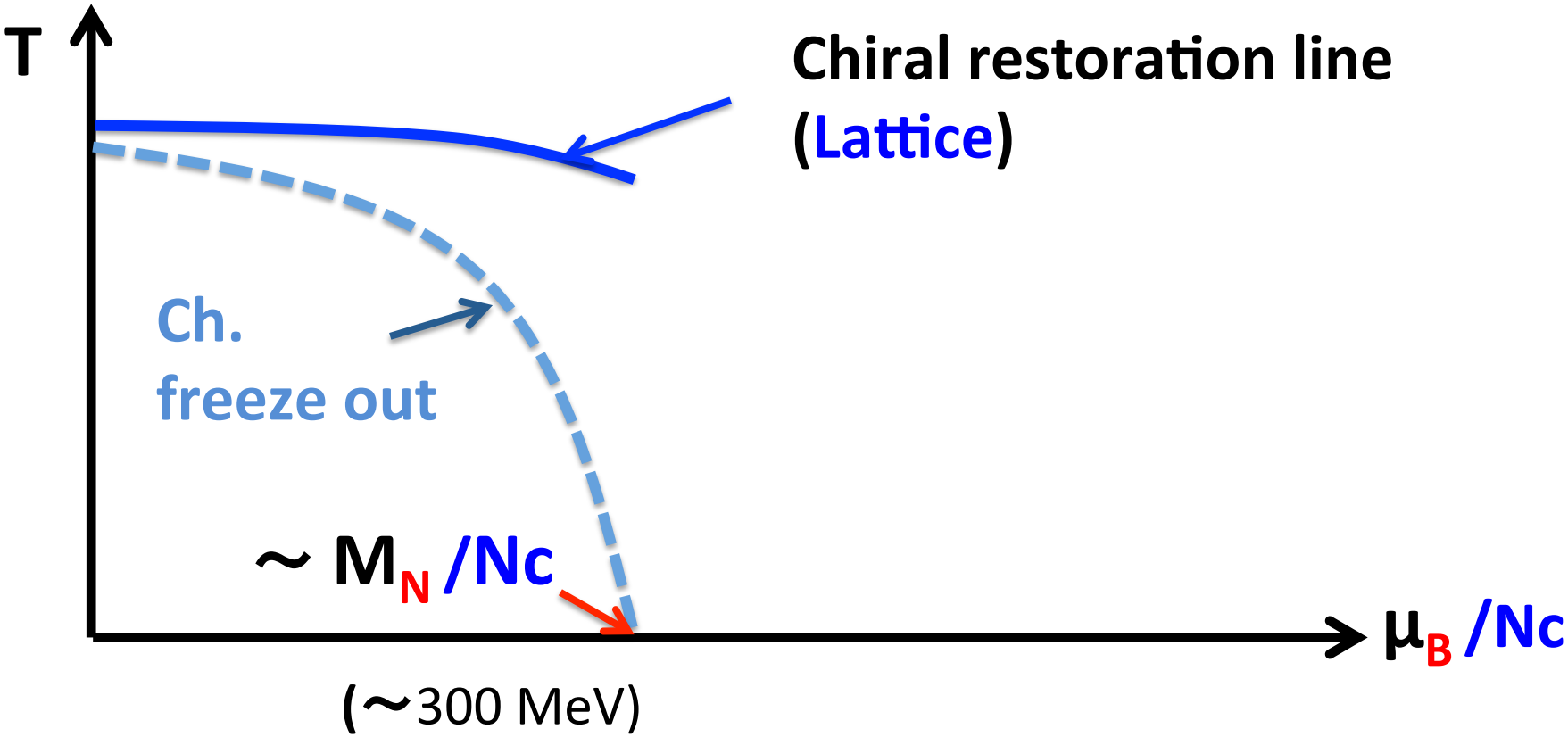
Contents

1, Possible Phenomenological impact

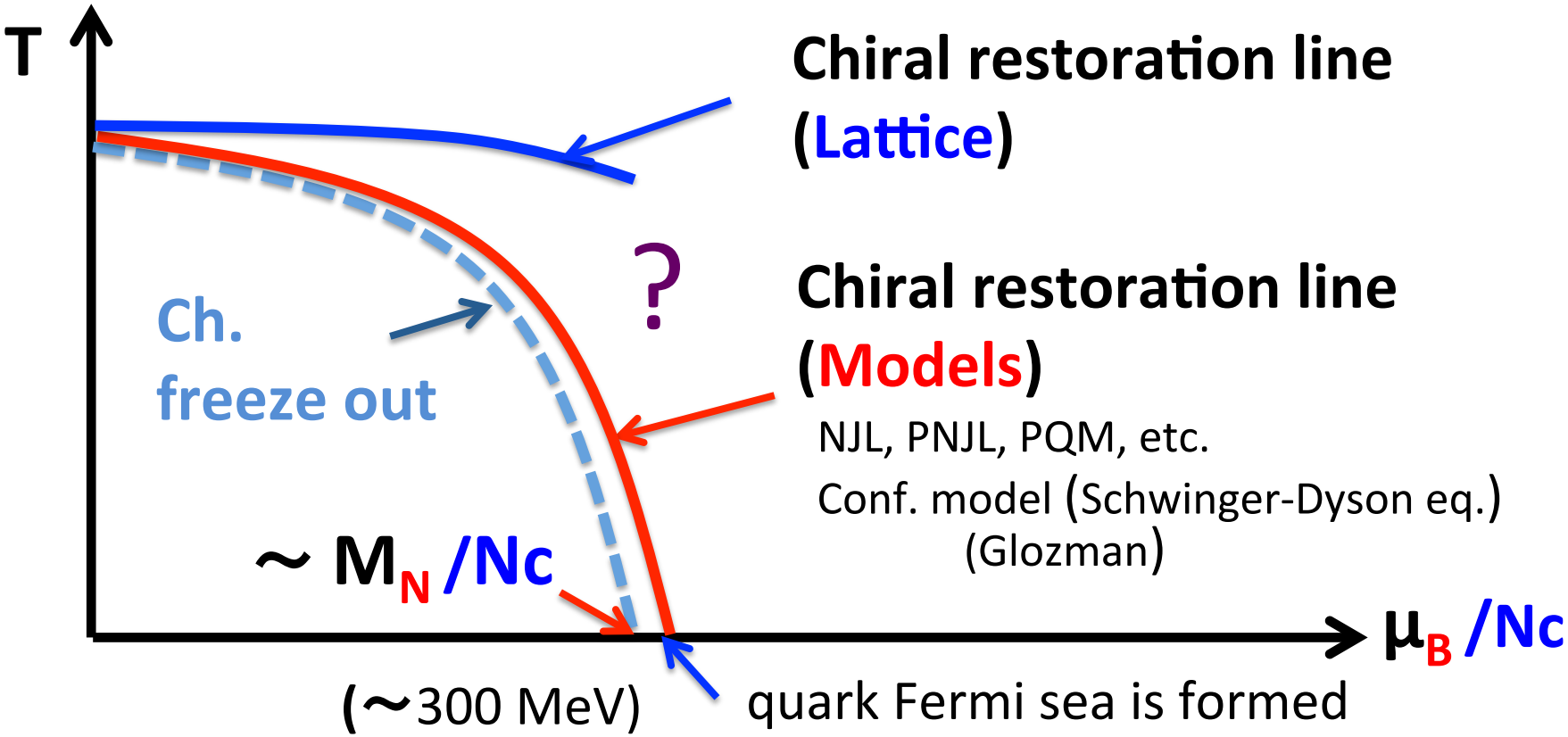
2, Quick review of Single Chiral Spiral (CS)

3, How to interweave many CSs

Chiral restoration



Chiral restoration



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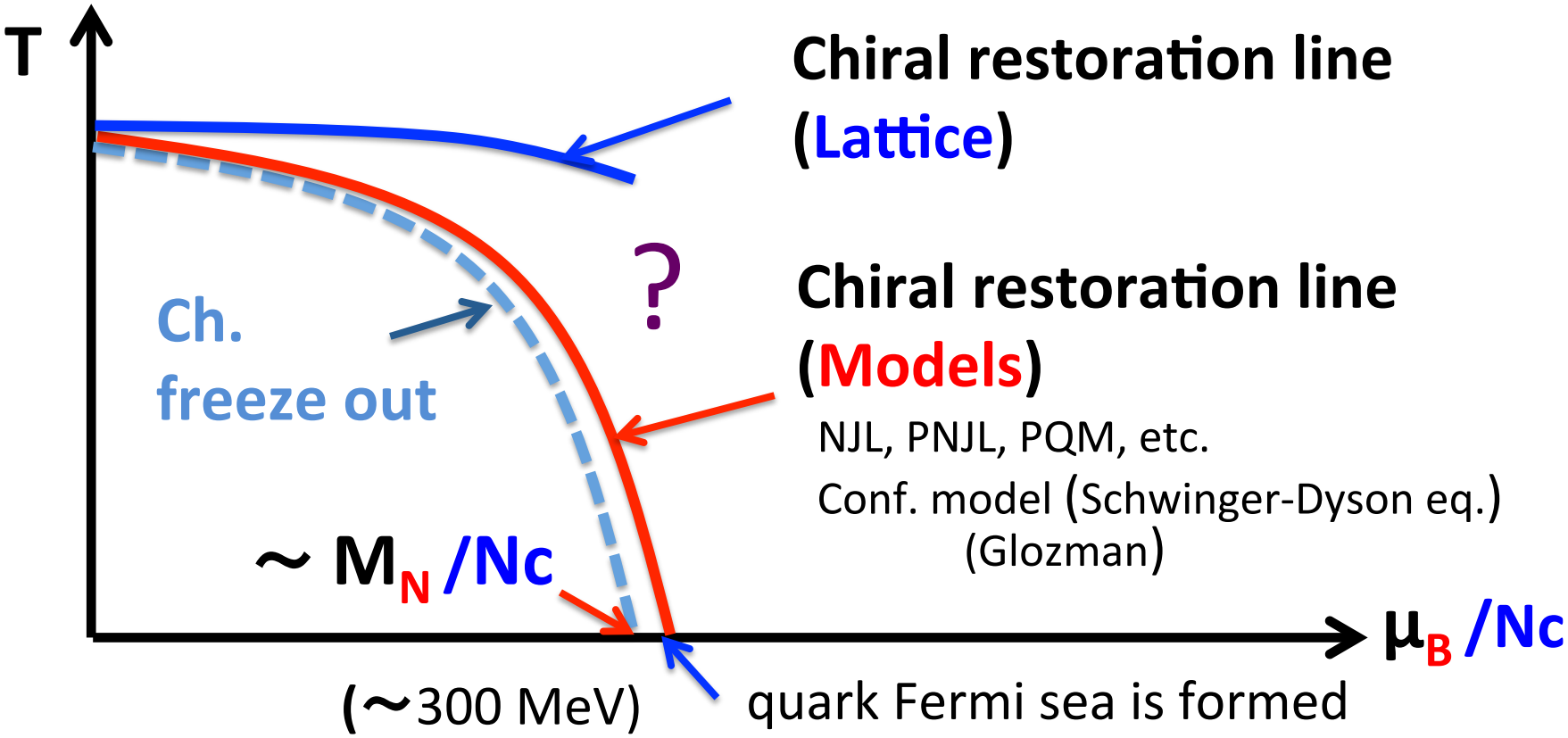
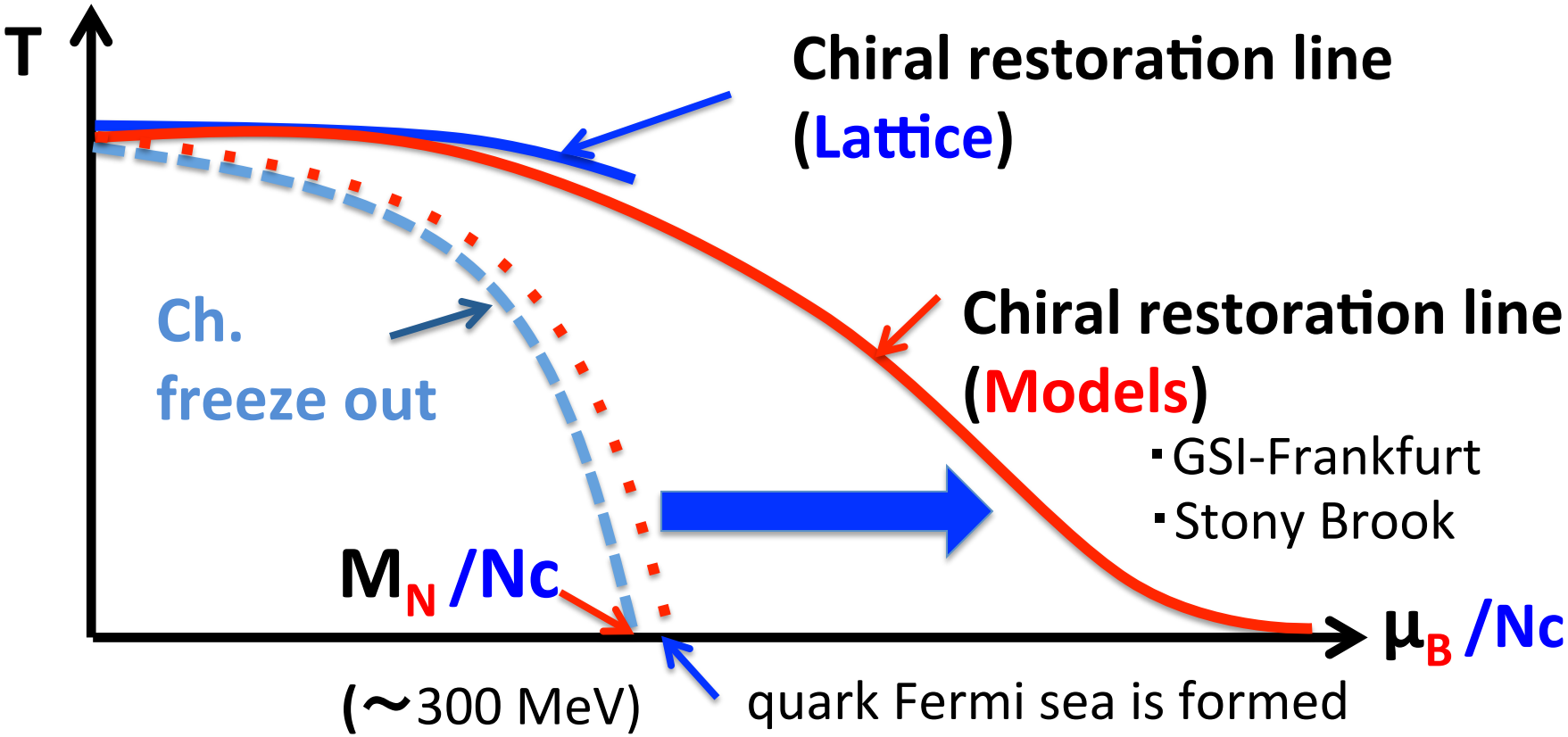


1, In model studies, chiral restoration happens **quickly after the formation of the quark Fermi sea.**

2, Assumption: Chiral condensate is **const. everywhere.**

If we allow non-uniform condensates...



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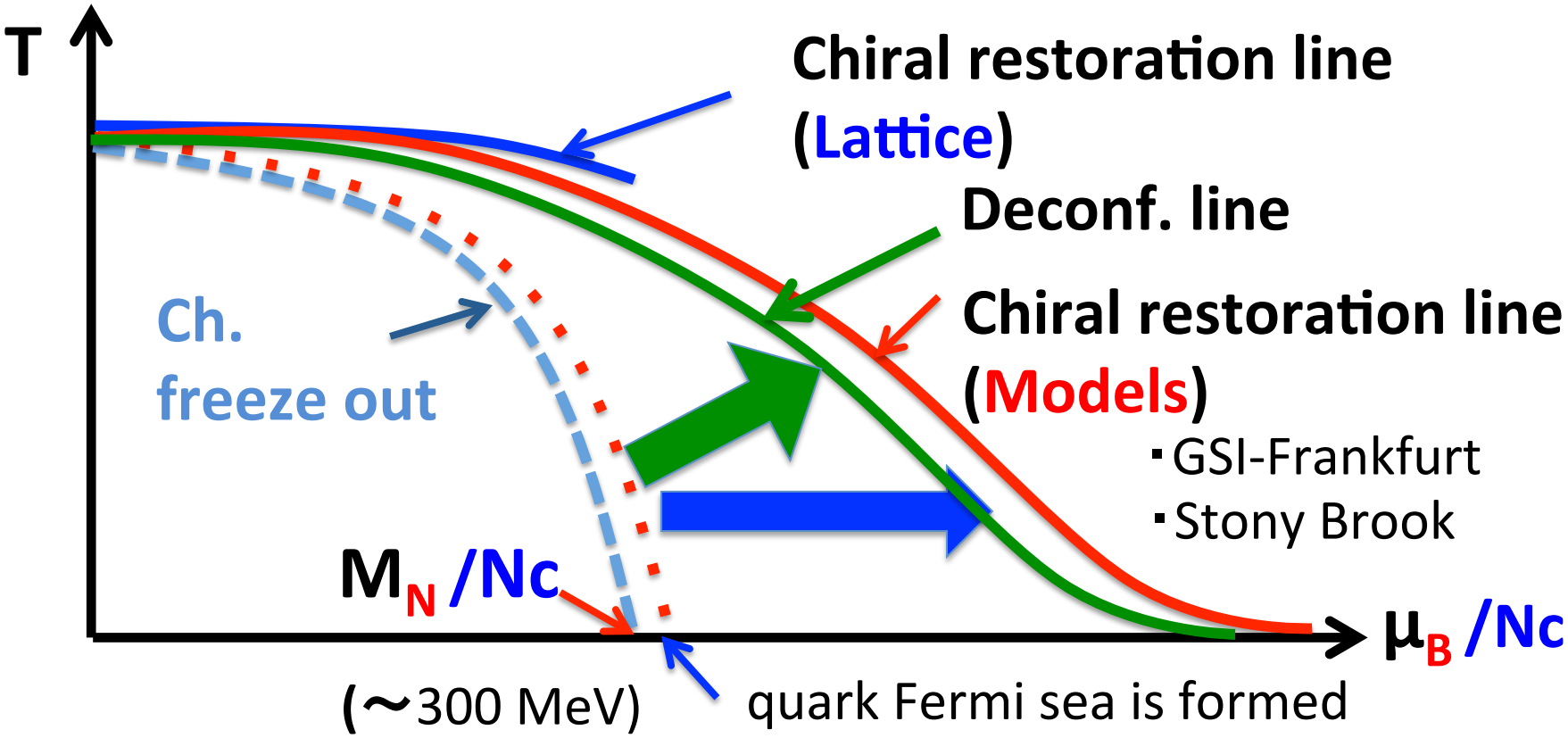
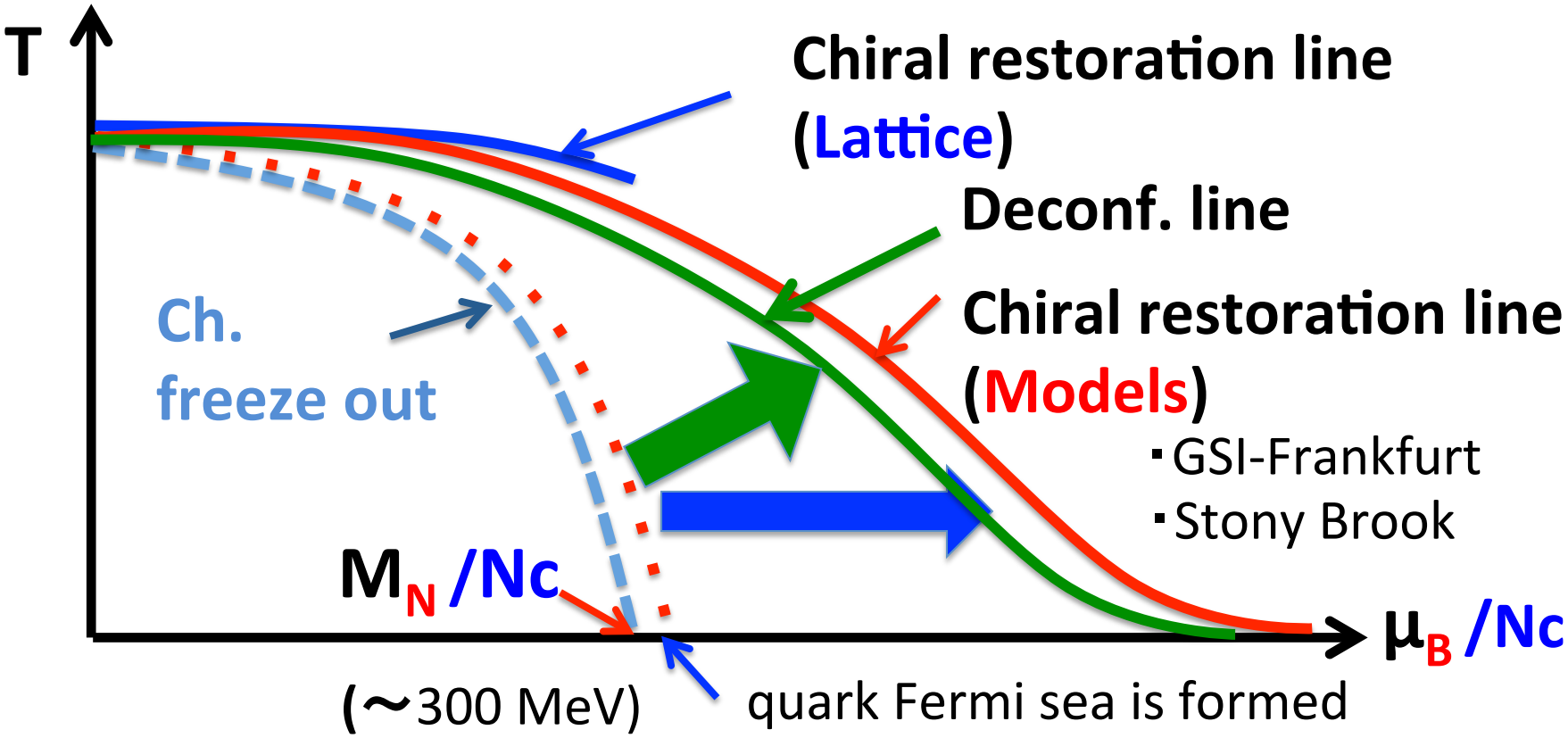


Deconfinment line would be also shifted because:

If we allow non-uniform condensates...



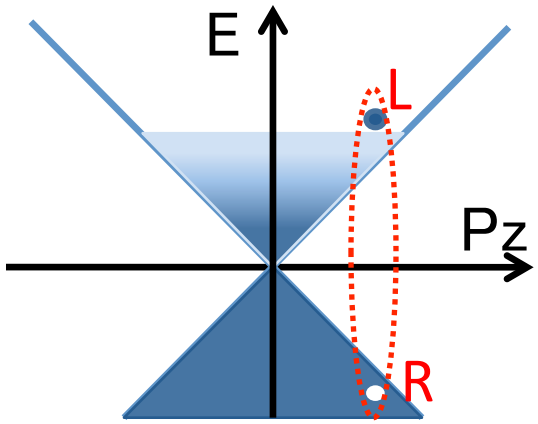
Deconfinment line would be also shifted because:

Non-uniform chiral condensate opens the **mass gap** of **quarks** near the Fermi surface.

→ Deconf. line approaches to the **pure glue results**

Candidates of Chiral Pairing (T=0)

Dirac Type



$P_{Tot}=0$ (uniform)

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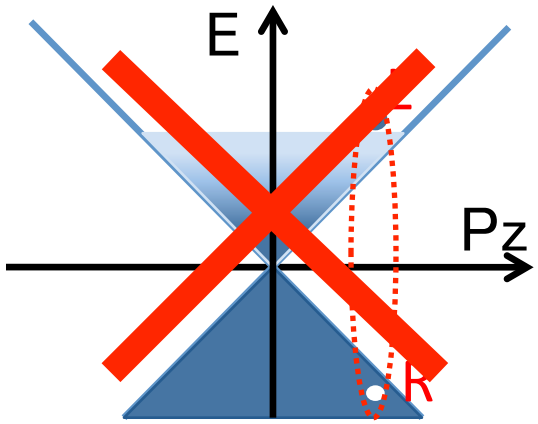

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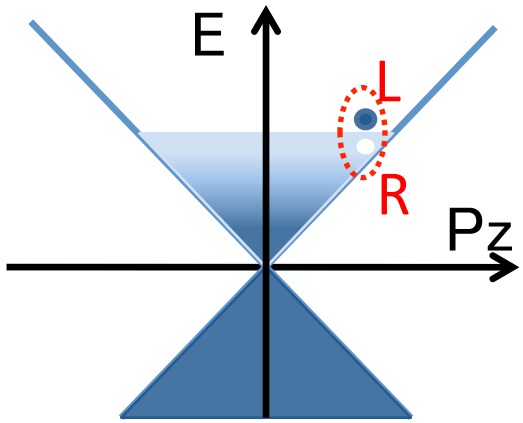
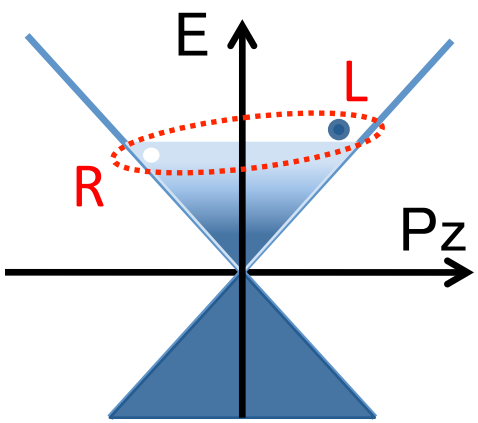
Exciton Type



$P_{Tot}=0$ (uniform)

Density wave



$P_{Tot}=2\mu$ (non-uniform)

Candidates of Chiral Pairing (T=0)

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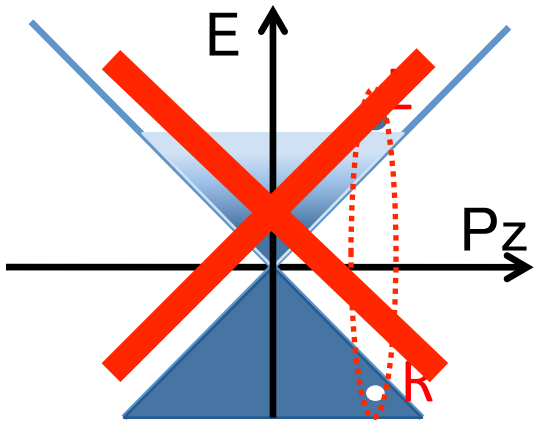
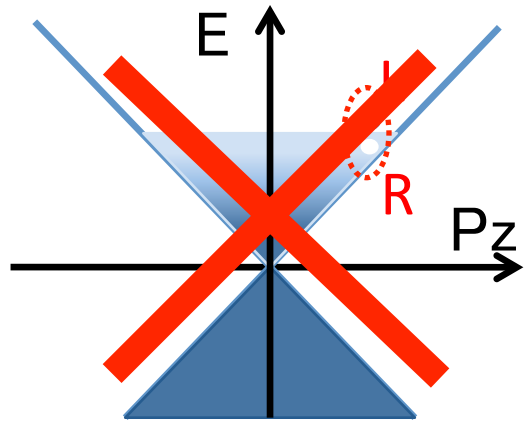


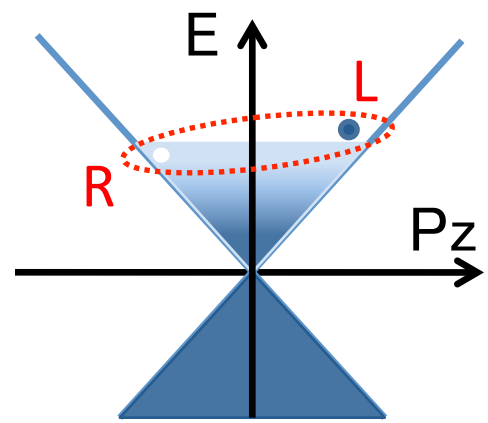
$P_{Tot}=0$ (uniform)

Exciton Type



$P_{Tot}=0$ (uniform)

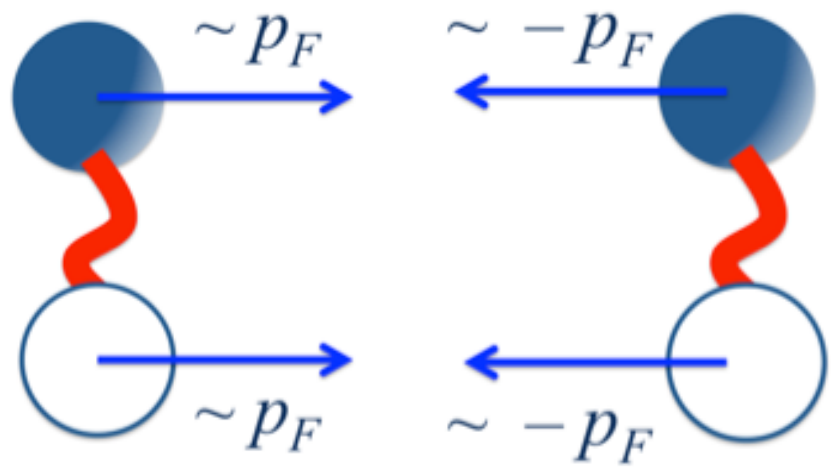
Density wave



$P_{Tot}=2\mu$ (non-uniform)

We consider only

Co-moving paring:



Single Chiral Spiral in z-direction

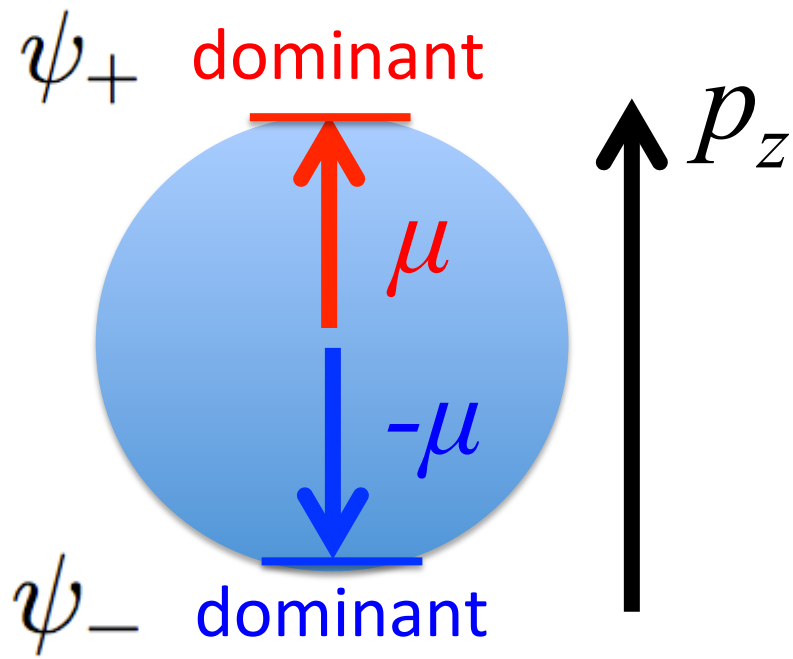
- ***Projection***: $$\psi_\pm = \frac{1 \pm \underline{\gamma_0\gamma_z}}{2}\psi$$ like (1+1) D chirality

- ***Kin. terms***: $$\mathcal{L}_{kin} \simeq \psi_\pm^\dagger\Big(\mathrm{i}(\partial_0 \mp \partial_z) + \mu\Big)\psi_\pm + \psi_\pm^\dagger \frac{\partial_\perp^2}{2\mu}\psi_\mp$$

(near the Fermi surface) **longitudinal** **transverse**

- ***Dispersion***:

$$\epsilon_\pm(p_z) \simeq (\mu \mp p_z) + \frac{p_\perp^2}{2\mu}$$



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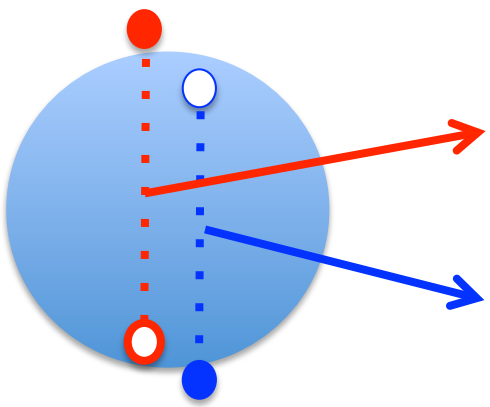


$p \quad h$

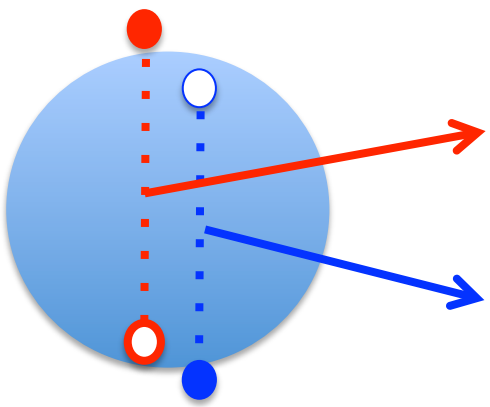
Phase (due to finite mom.)

$$\langle \bar{\psi}_+ \psi_- \rangle = \Delta \, \mathrm{e}^{2\mathrm{i}p_F z}$$

$$\langle \bar{\psi}_- \psi_+ \rangle = \Delta \, \mathrm{e}^{-2\mathrm{i}p_F z}$$

($\Delta \sim \Lambda_{QCD}^3$)

Single Chiral Spiral in z-direction



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($\Delta \sim \Lambda_{QCD}^3$)

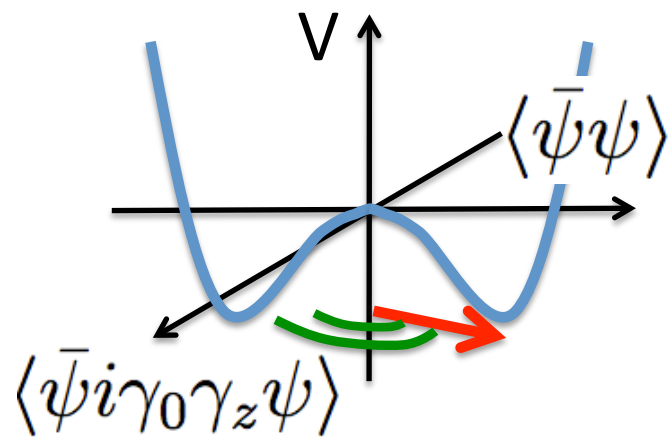
Linear combination :

space-dep.

Sum : $$\langle \bar{\psi}\psi \rangle = \Delta \cos(2p_F z)$$

P-odd

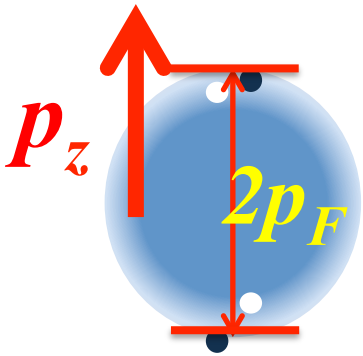
diff : $$\langle \bar{\psi} i\gamma_0 \gamma_z \psi \rangle = \Delta \sin(2p_F z)$$



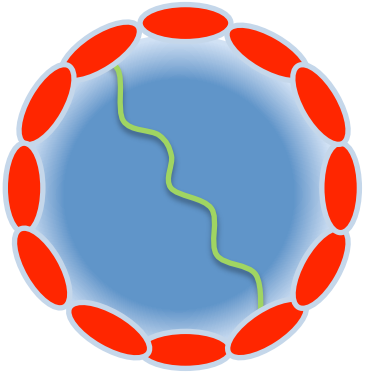
2 CDW → Single Chiral Spiral

Interweaving Chiral Spirals

So far we have considered only
the Chiral Spiral in **one direction**.



Is it possible to have CSs in multiple directions?



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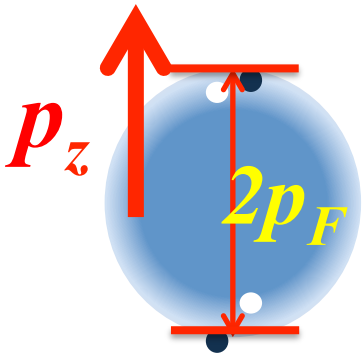
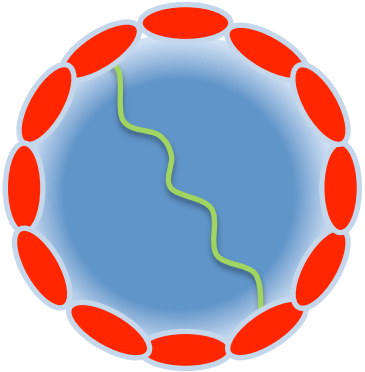


Is it possible to have CSs in multiple directions?

YES!

(**if** theory is **asymptotic free**)

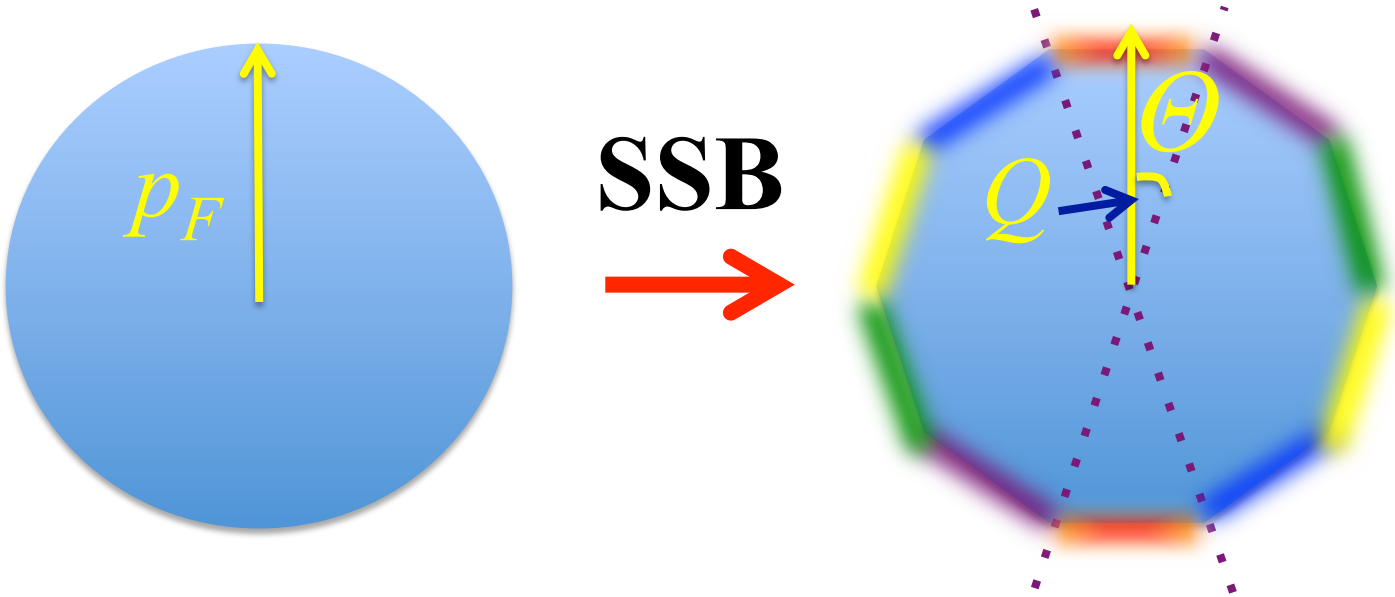


Pairs around the **entire Fermi surface** can condense.

The system acquires very large energetic gain.

(2+1) D Example

Rotational Sym. : $U(1) \longrightarrow Z_{2N_p}$ (N_p : Num. of patches)

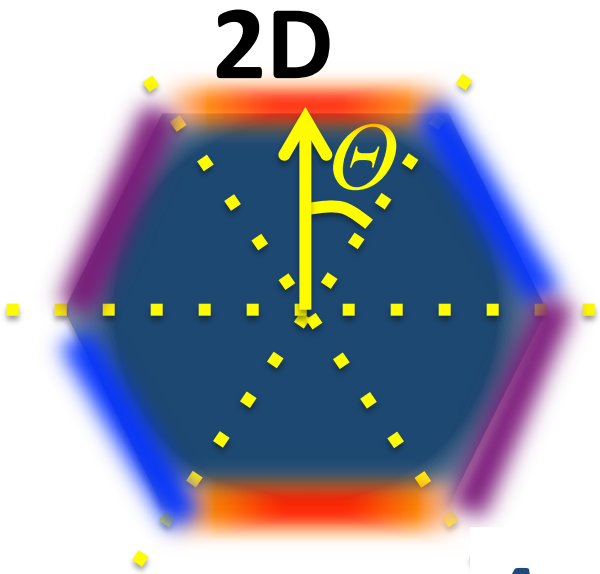


Variational parameter : angle $\Theta \sim 1/N_p$

We use **canonical** ensemble : $Q \rightarrow Q\,(\Theta, p_F)$

- **Energy density :** $E\,(\Theta\,;\,p_F)$

Our goal



- To express the energy density as a function of theta, and to determine the best shape.

Approximations to be used

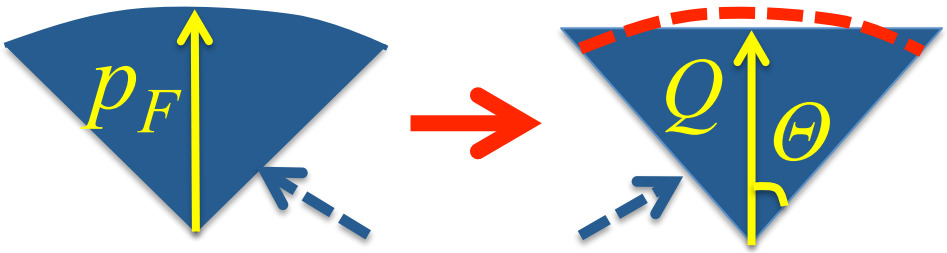
- Simple Interaction with asym. free nature
- Large Nc (MF treatments & ignore screening)
- Large density (Λ_{QCD} / p_F expansion, T=0)

- Although not very realistic for dense QCD, but:

→ Simple analytic insights

Energetic gain v.s. cost

- **Cost : Deformation**
  (dominant for **large** Θ)



equal vol. (particle num.)

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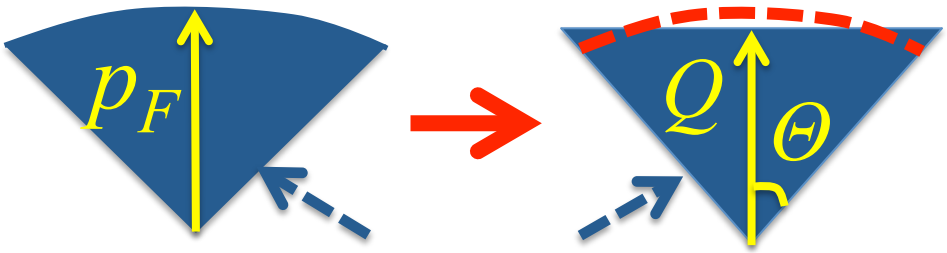
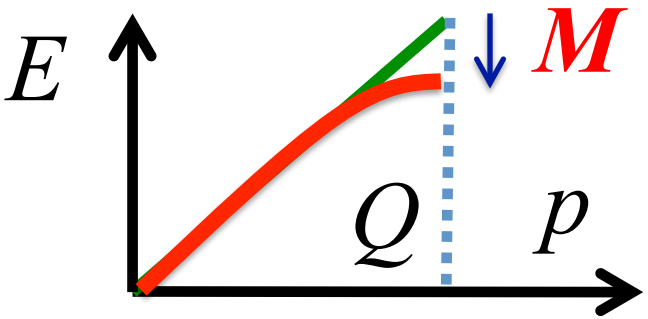


equal vol. (particle num.)

- **Gain : Mass gap origin**

Condensation effects



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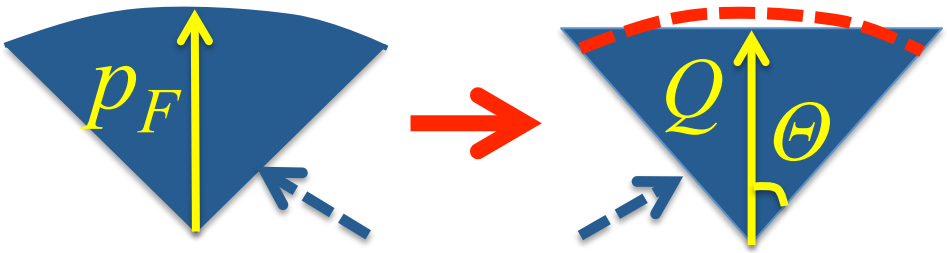

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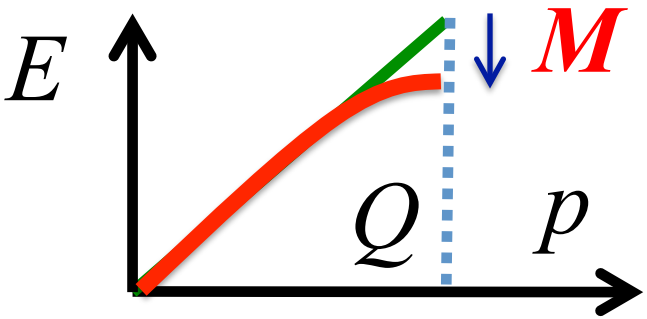
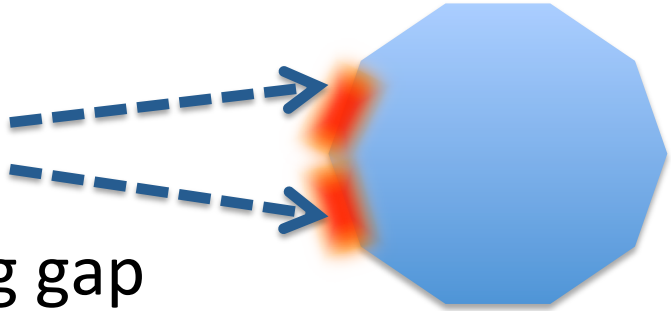


- **Cost : Interactions among CSs** (**Model dep. !!**)
  (dominant for **small** Θ)

Condensate – Condensate int.



→ destroy one another, reducing gap

Previous studies

Numerical studies for many Chiral Density Waves

NJL & Instanton induced interaction in (3+1) D

Rapp, Shuryak, Zahed : PRD63(2001)034008

NJL model in (2+1) D

Carignano, Buballa (Conference report, 2011)

→ **Many CDWs do not reduce the energy of the system so much, compared to single CDW .**

→ **Interactions among CDWs are repulsive.**

But unlike NJL model, the model with asym. free nature can considerably reduce such repulsive effects.

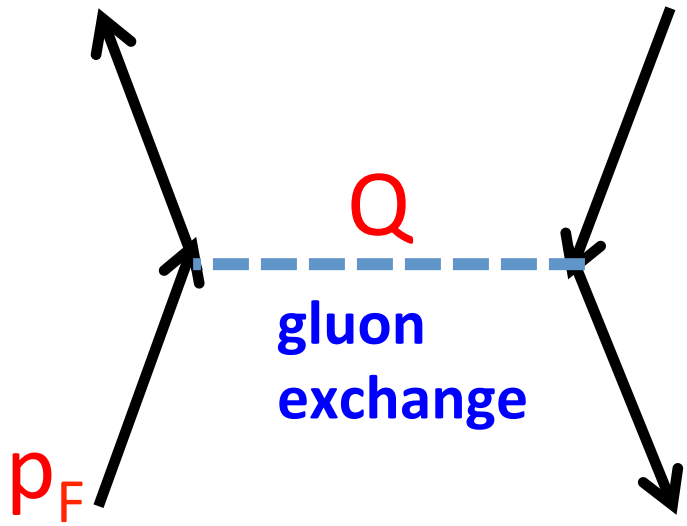
(see below)

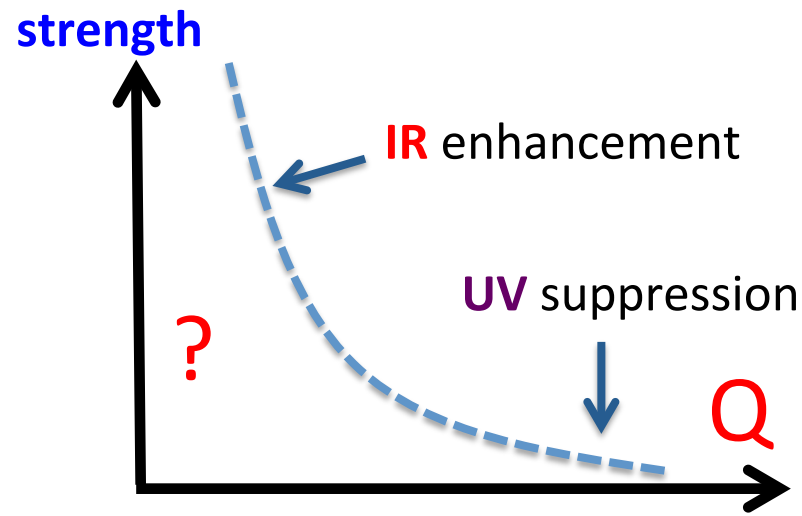
A schematic model

Strength of interactions is determined by

Momentum transfer, **NOT** by **quark momenta.**

→ **Even at high density, int. is strong** for **some processes**.





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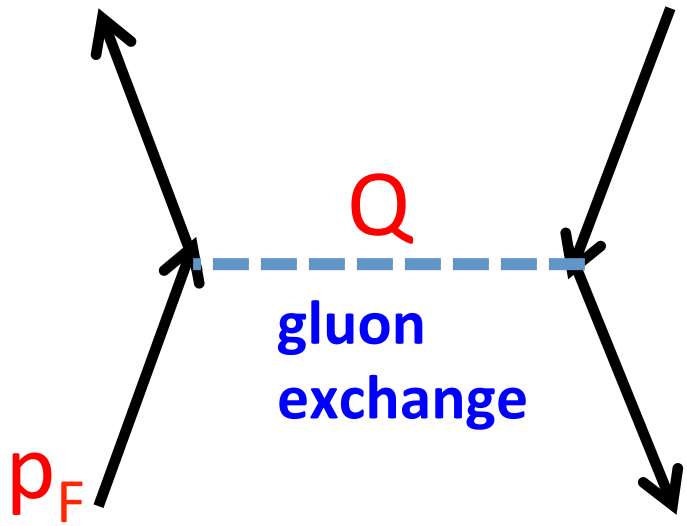
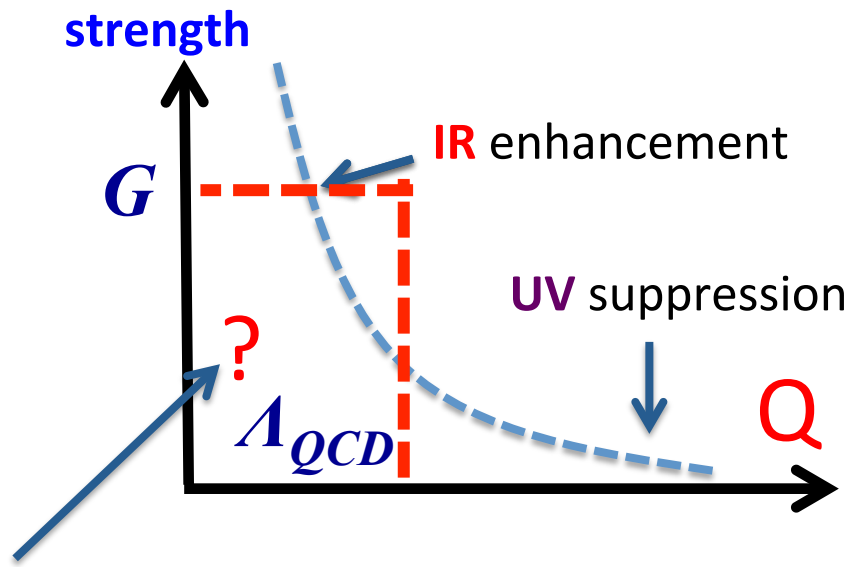




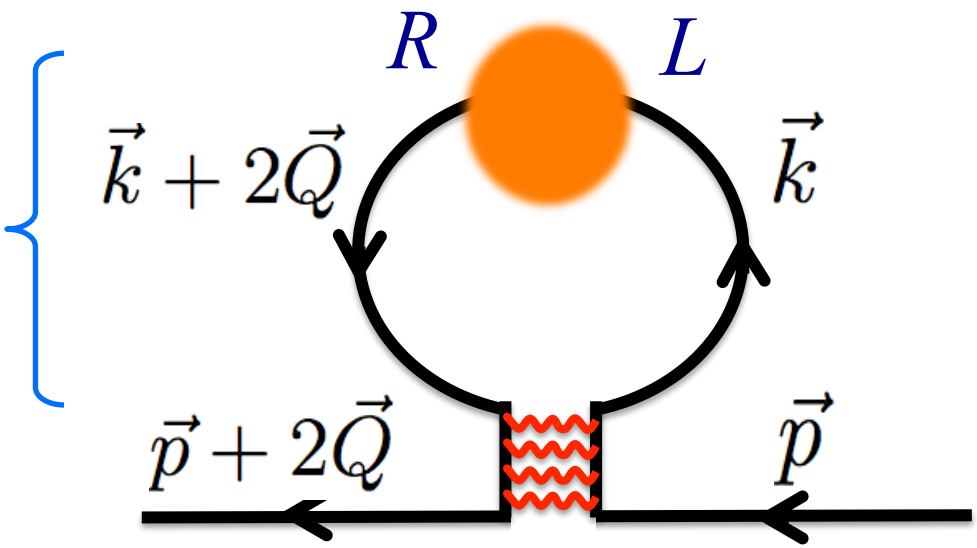
- Detailed form in the **deep IR region: We don't care**

Consequences of the model at large Nc

Contributions to the mass gap at leading Nc:

Inhomogeneous condensate

$$\langle \bar{\psi}_R(\vec{k} + \underline{2\vec{Q}})\psi_L(\vec{k})\rangle$$



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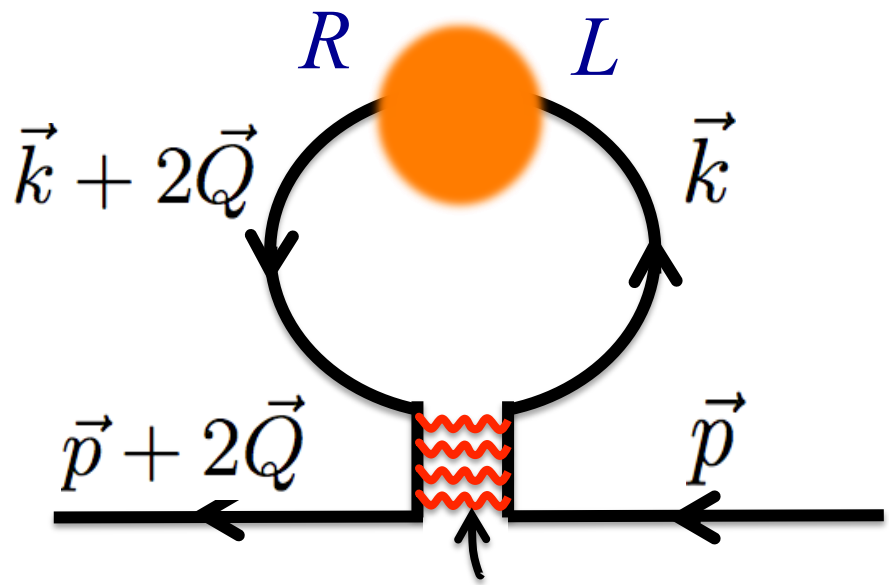

Strong for small mom. transfer

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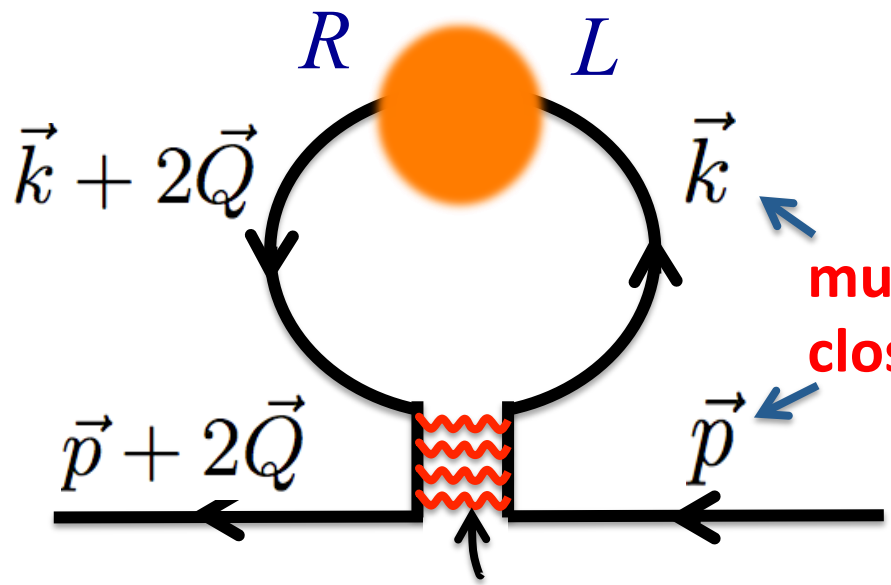

must be close

Strong for small mom. transfer

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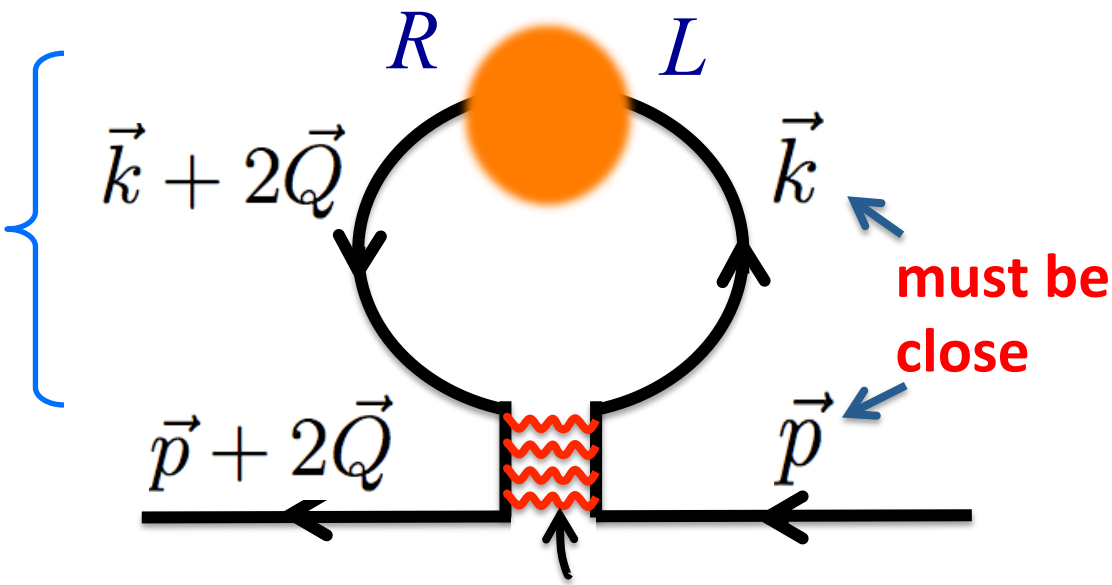

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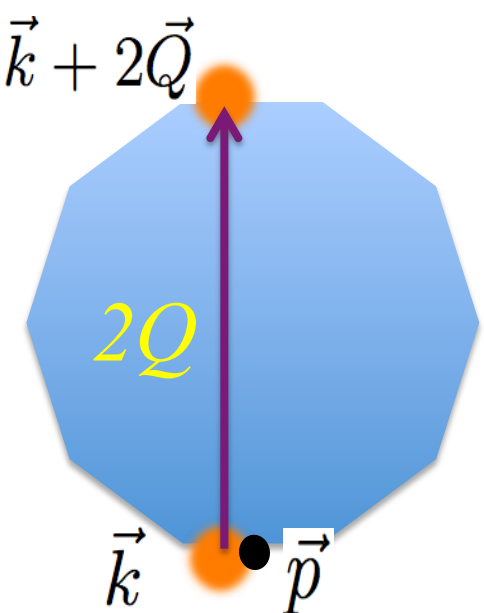
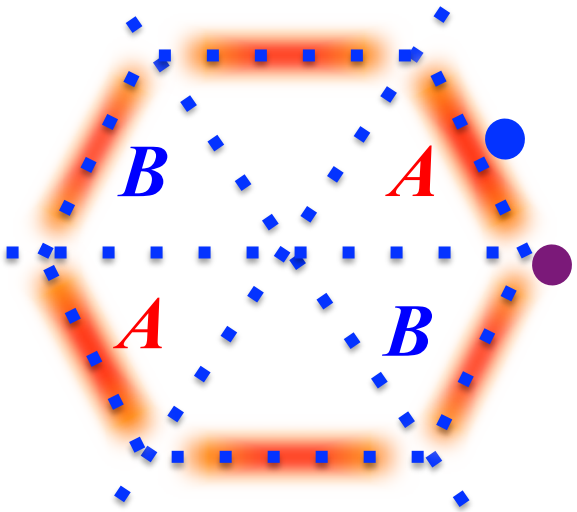


Quark-condensate scattering occurs **only if** **k** & **p** are close.

Condensate & gap distributions

Condensate contribute to the quark mass gap
only if their momentum domains are close one another.



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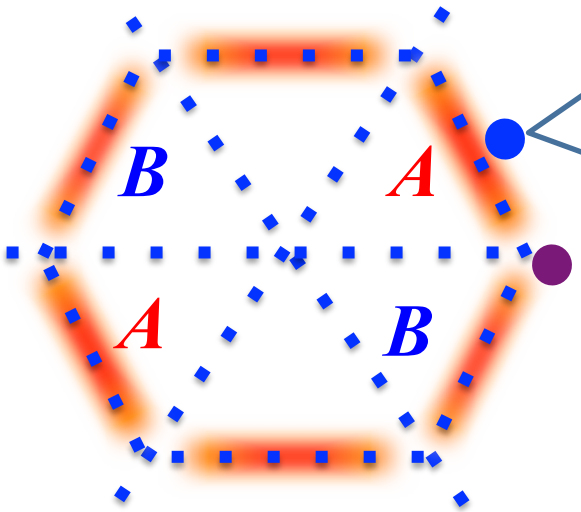
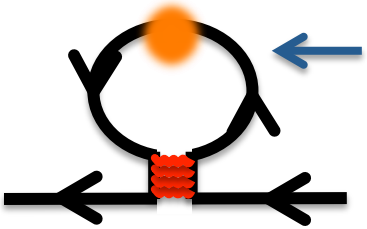


Quarks Away from the patch boundary:

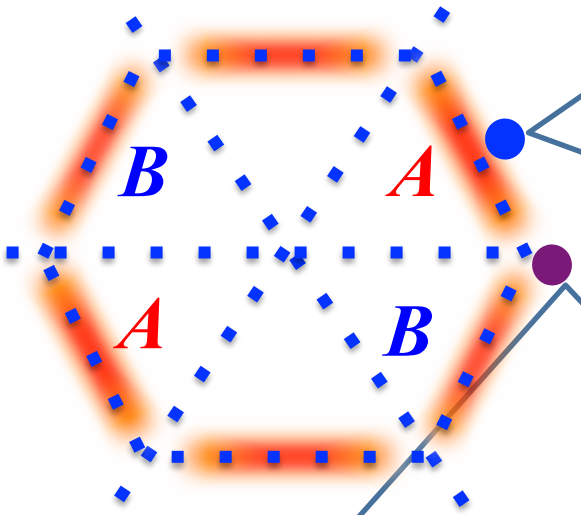


feel only **one** CS (A) condensate in the gap eq.

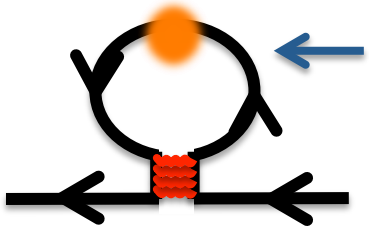
The gap eq. can be solved within one patch treatment.

Condensate & gap distributions

Condensate contribute to the quark mass gap
only if their momentum domains are close one another.



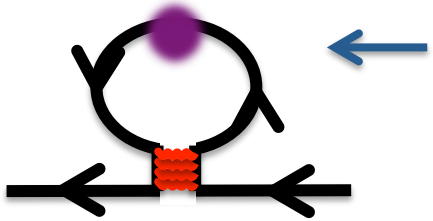
Quarks Away from the patch boundary:



feel only **one** CS (A) condensate in the gap eq.

The gap eq. can be solved within one patch treatment.

Quarks Near the patch boundaries:



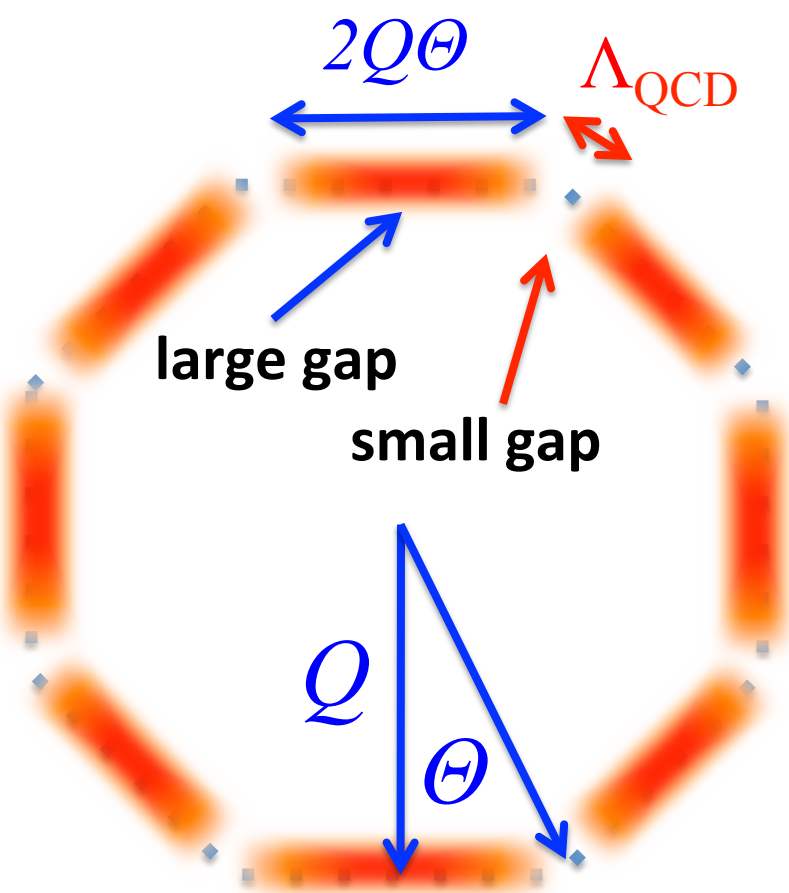
feel **Two** CSs (A & B)

The gap eq. involve **Two** CSs background.

Results: reduction of the gap & condensate

Too many CSs is not favored

Gap or Condensate distribution



If Θ is **too small**

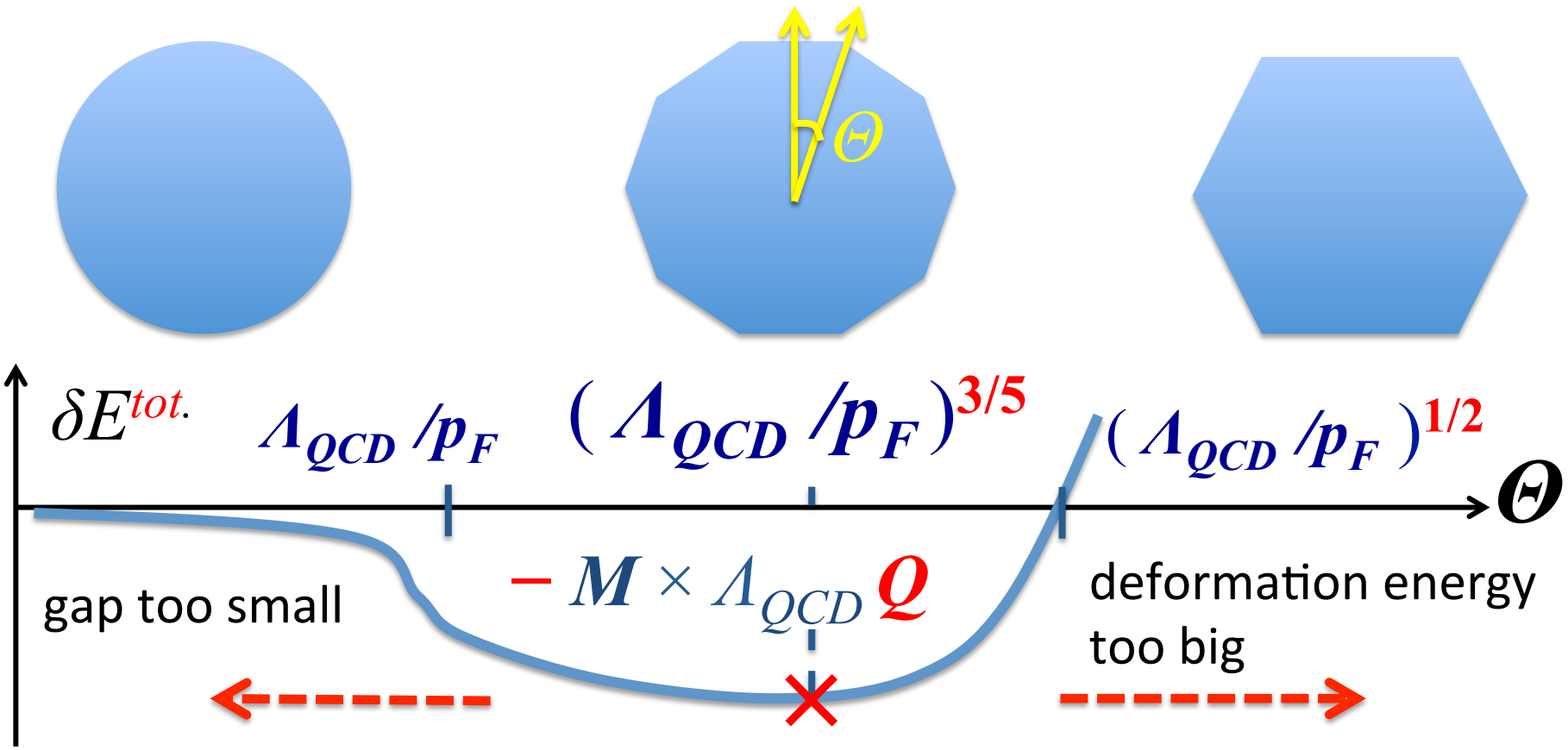
↓

The domain of **large** condensate or gap decreases.

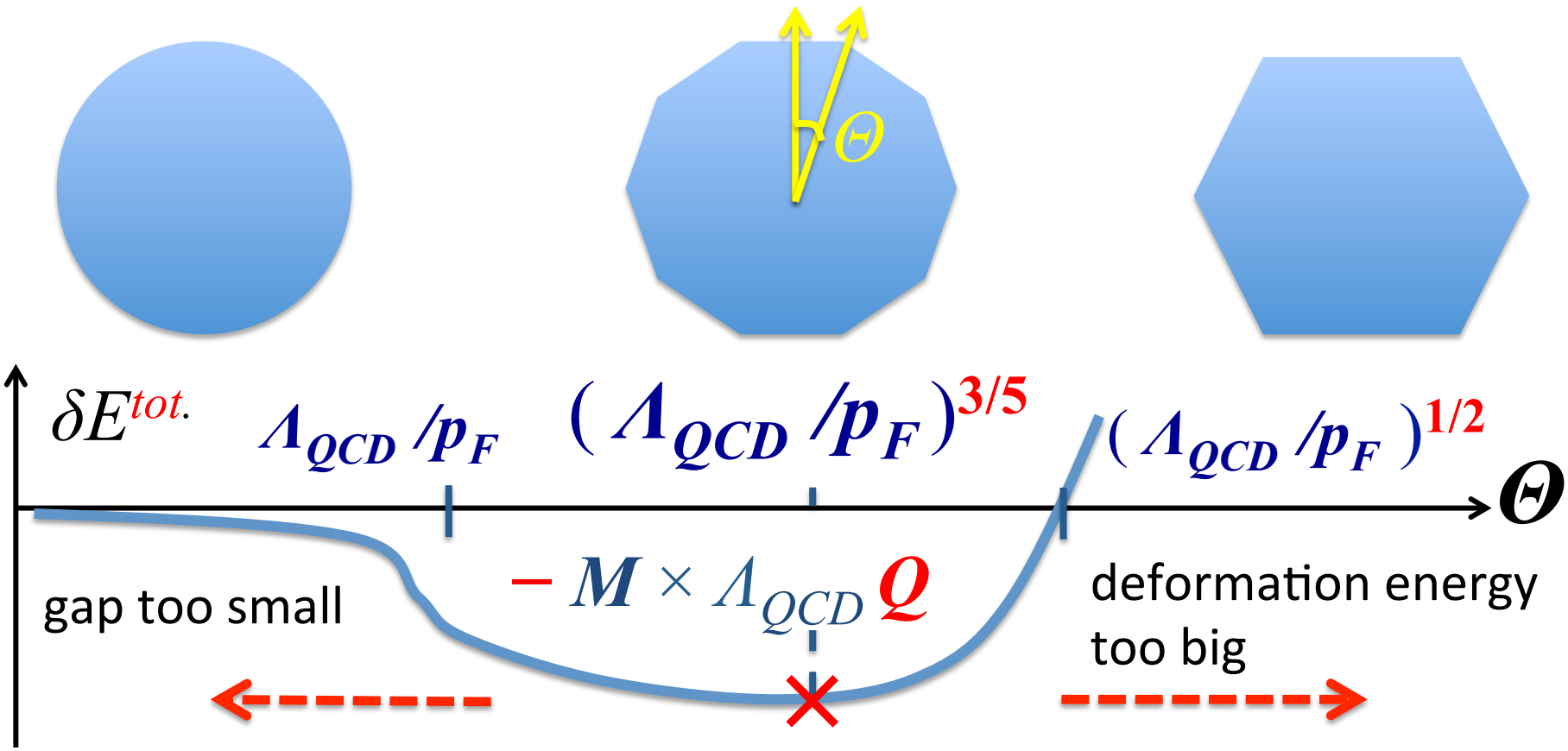
↓

We cannot reduce the energy of the system so much.

Energy Landscape (for fixed p_F)



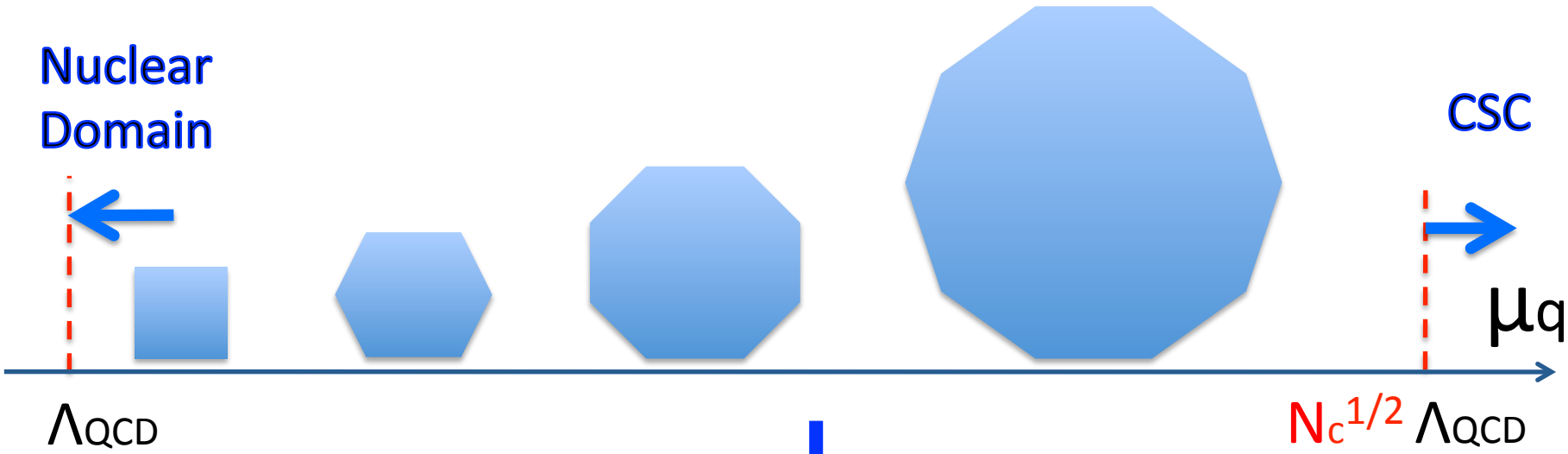
Energy Landscape (for fixed p_F)

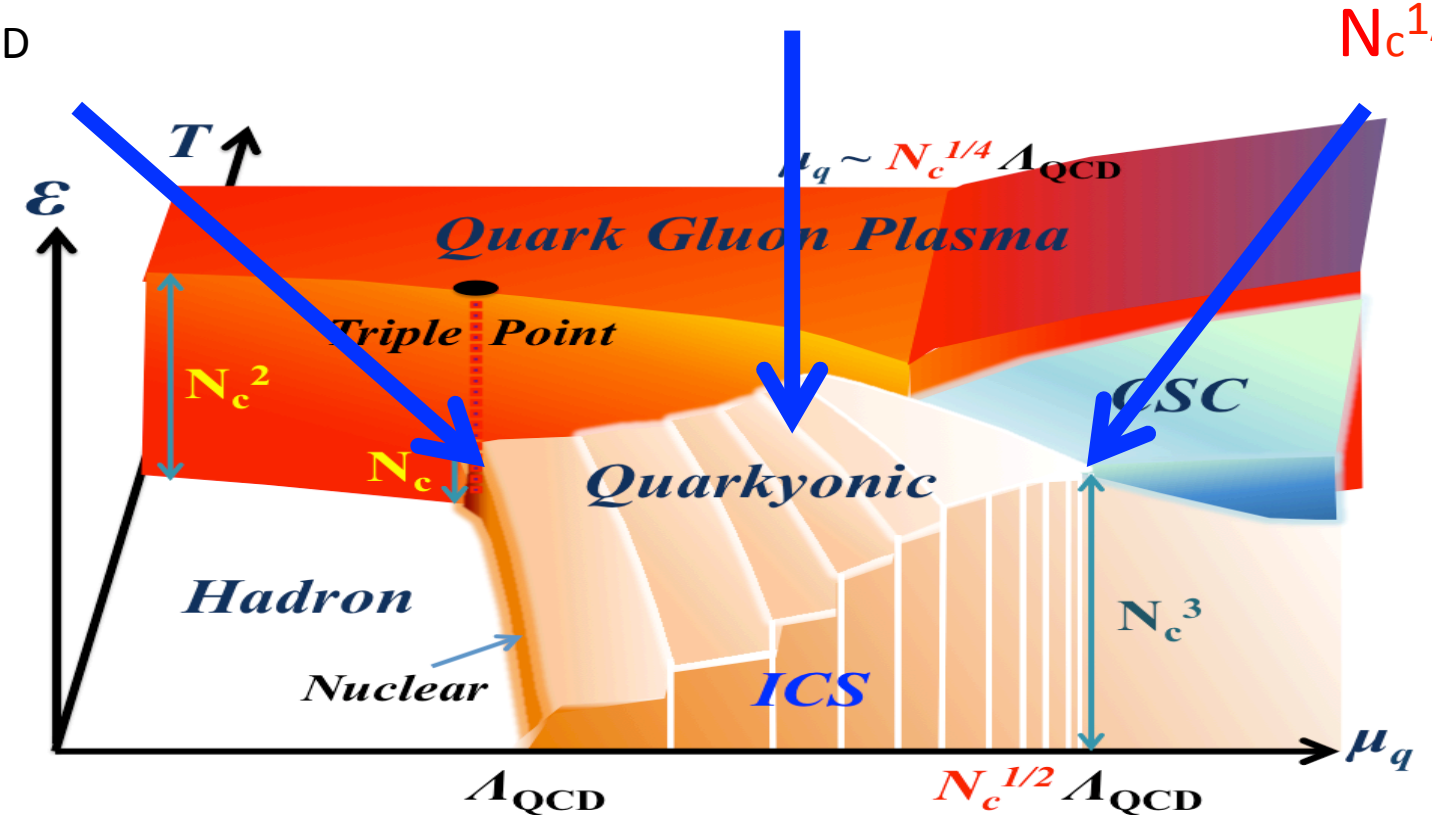


$$N_p \sim 1/\Theta \sim (p_F/\Lambda_{QCD})^{3/5}$$

- **Patch num. depends upon density.**

Sequential Phase transition





Summary

- 1, The ICS creates **Quark Mass gap near the Fermi surface**.

  (→ **shifts** Chiral & Deconf. transitions line to higher density)

- 2, The ICS offers a **source** of **local P violation**.

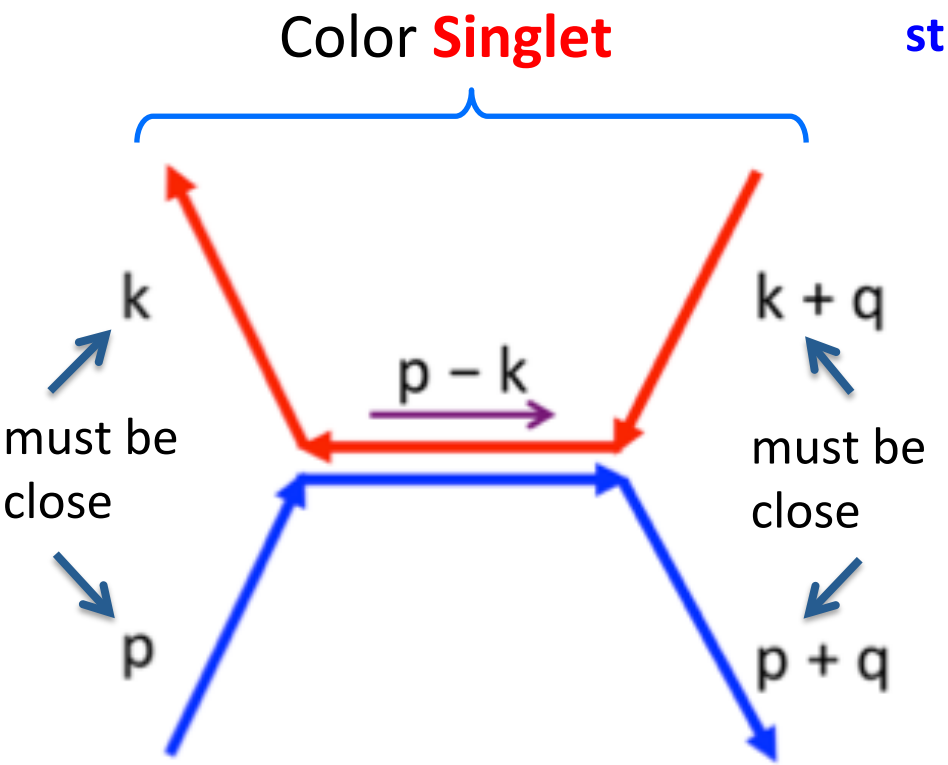
- 3, The ICS is likely to be possible only for asym. free theory with IR enhancement & UV suppression of the interaction.

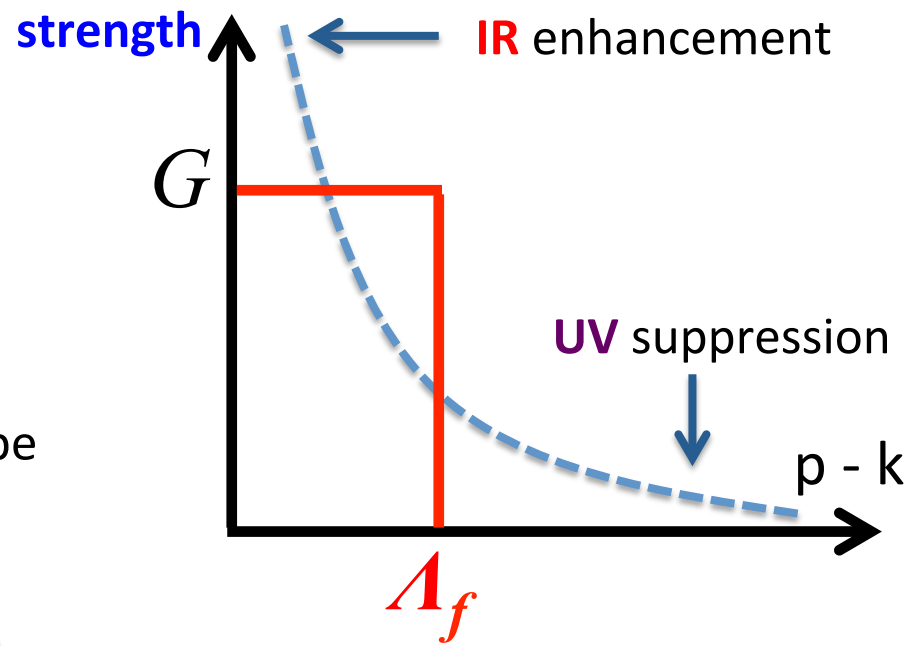
  (W.O. this property, it is difficult to suppress the patch-patch int.)

- 4, The extension of this study from (2+1) D to (3+1) D is **conceptually** straightforward.

  (Maximizing the size of patch as much as possible,
  and minimizing a num. of intersection points (lines))

A crude model with asymptotic freedom



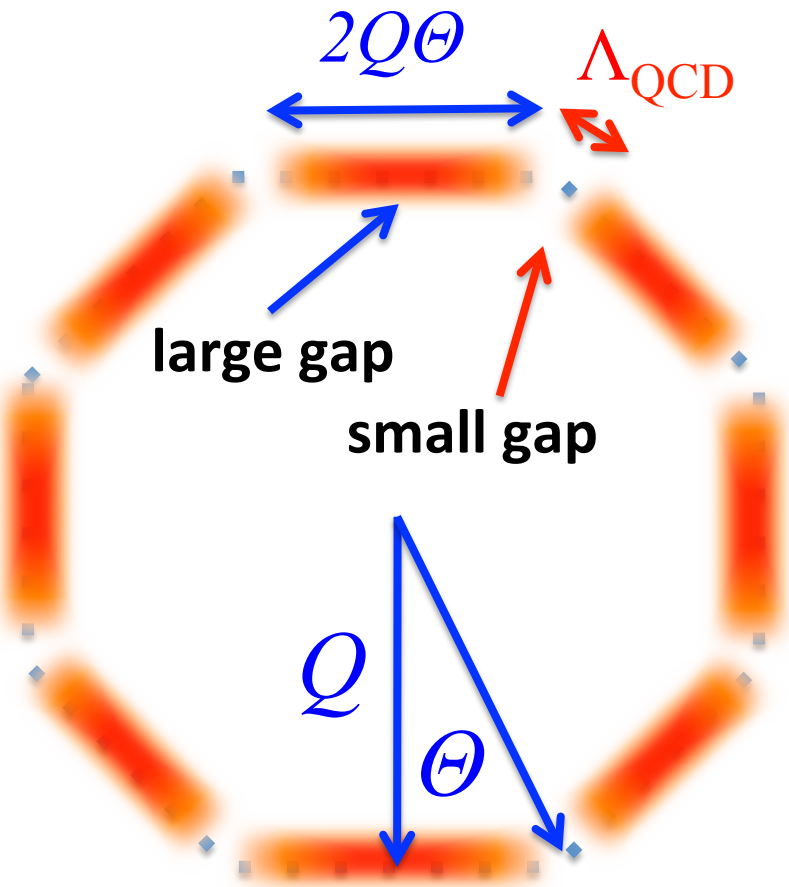


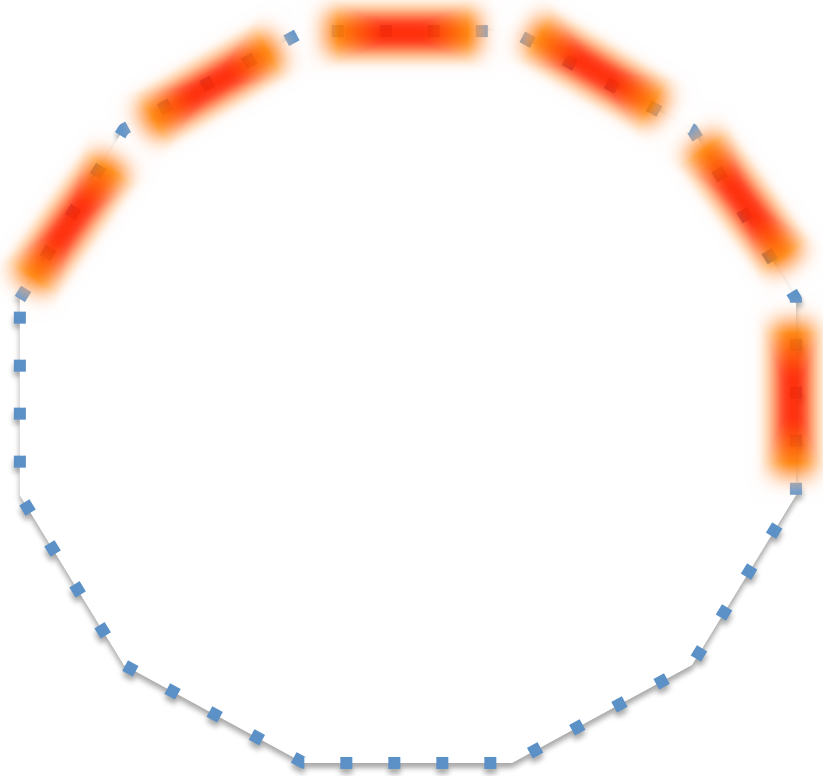
$$\theta_{p,k} \equiv \theta\left(\Lambda_f^2 - (\vec{p} - \vec{k})^2\right)$$

- ex) Scalar - Scalar channel

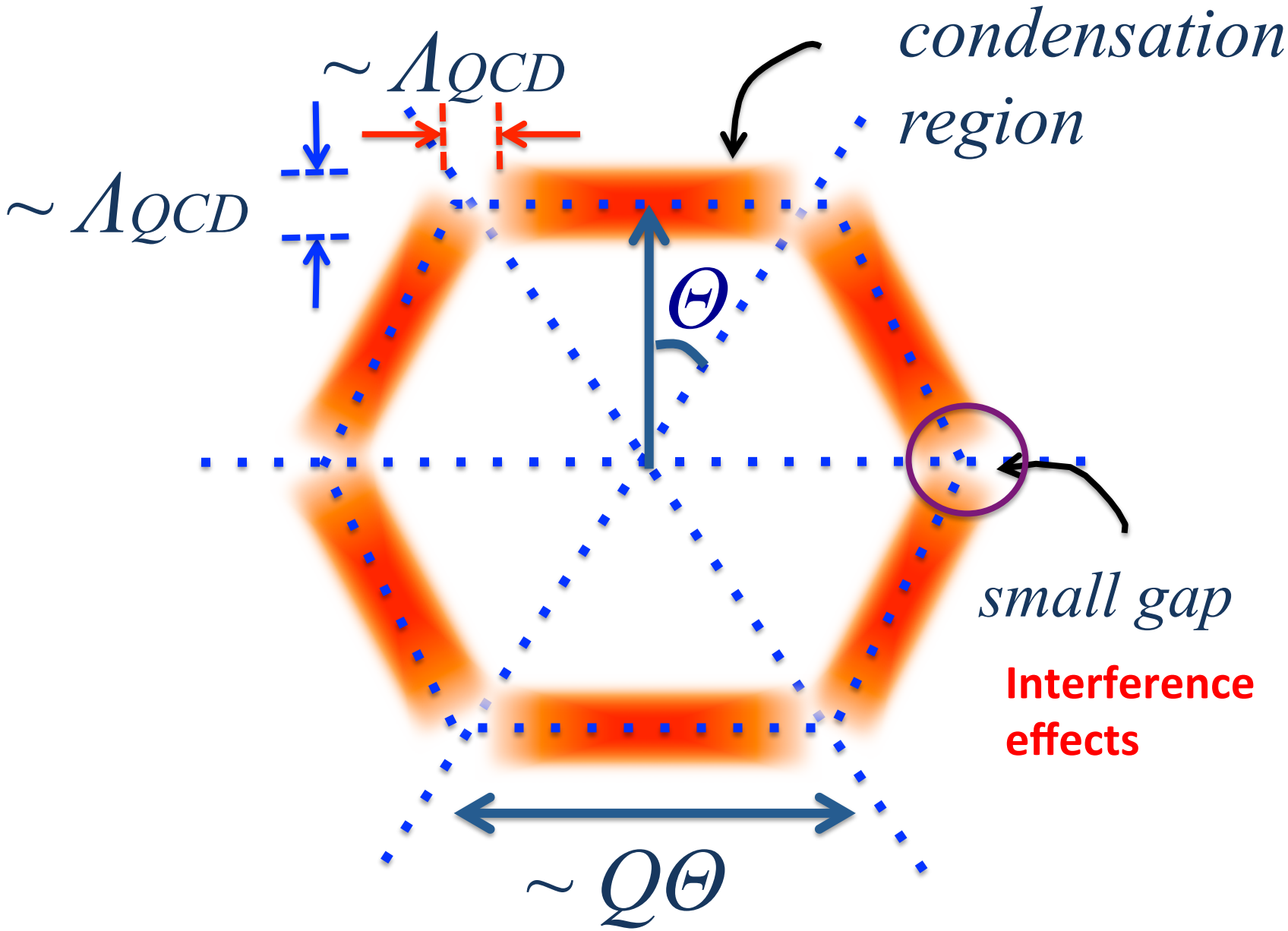
$$\frac{G}{N_c} \int dx^0 \int_{q,p,k} \left(\bar{\psi}(\vec{p}+\vec{q})\psi(\vec{p})\right)\left(\bar{\psi}(\vec{k})\psi(\vec{k}+\vec{q})\right)\theta_{p,k}$$

Gap distributions at optimized Θ

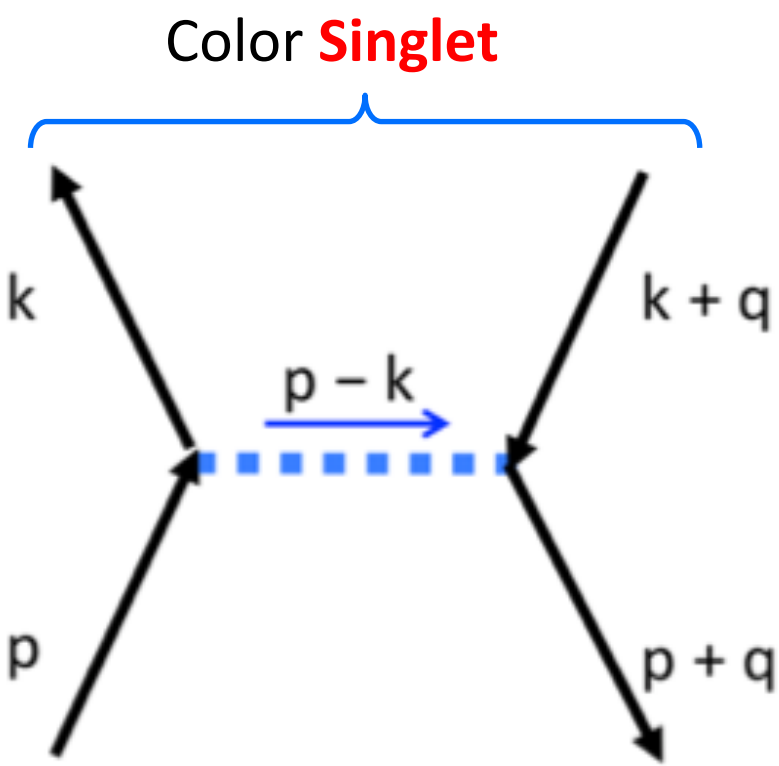


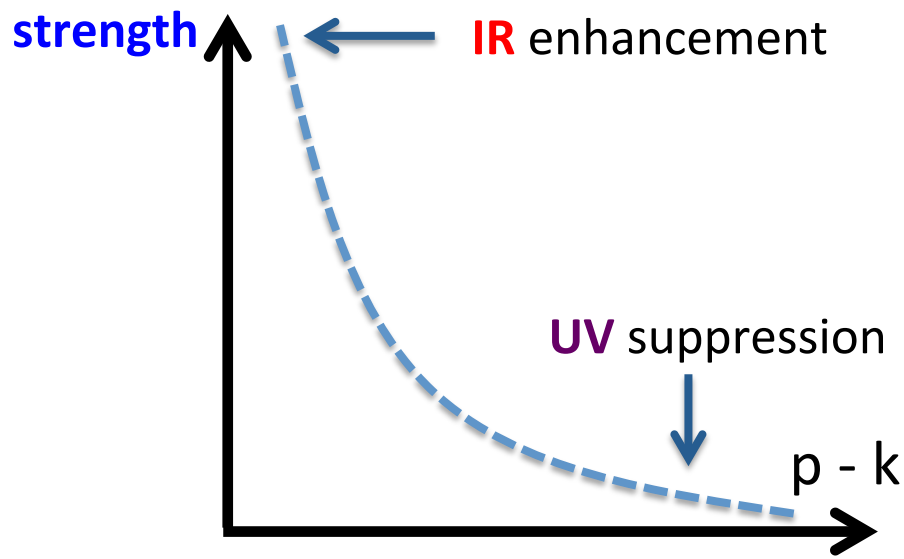


Gap distribution will be



A crude model with asymptotic freedom

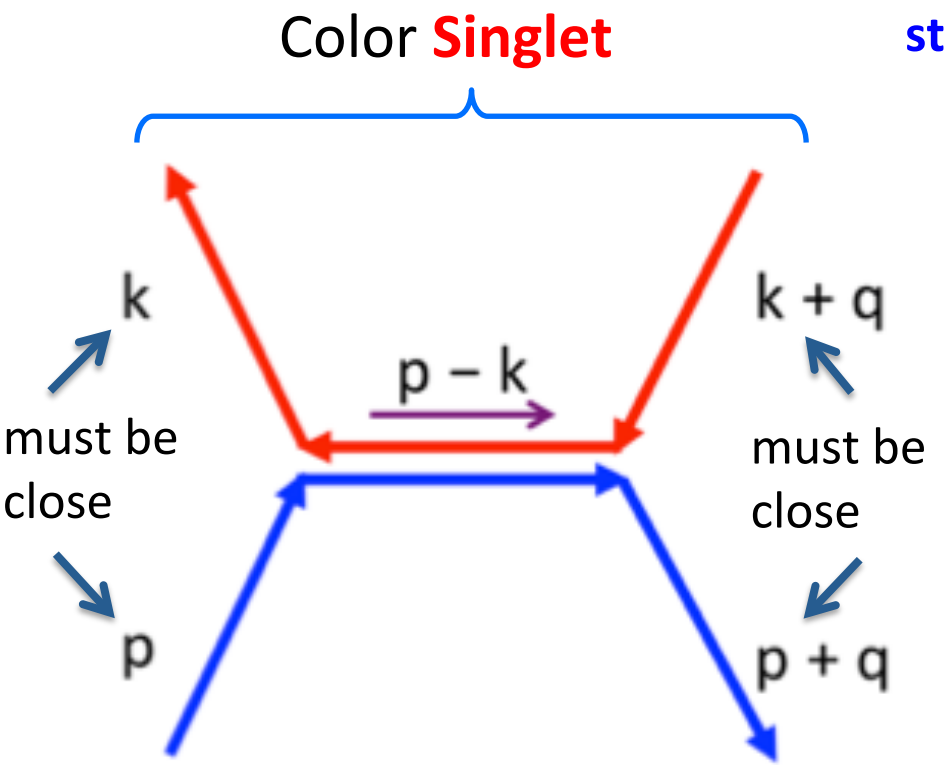


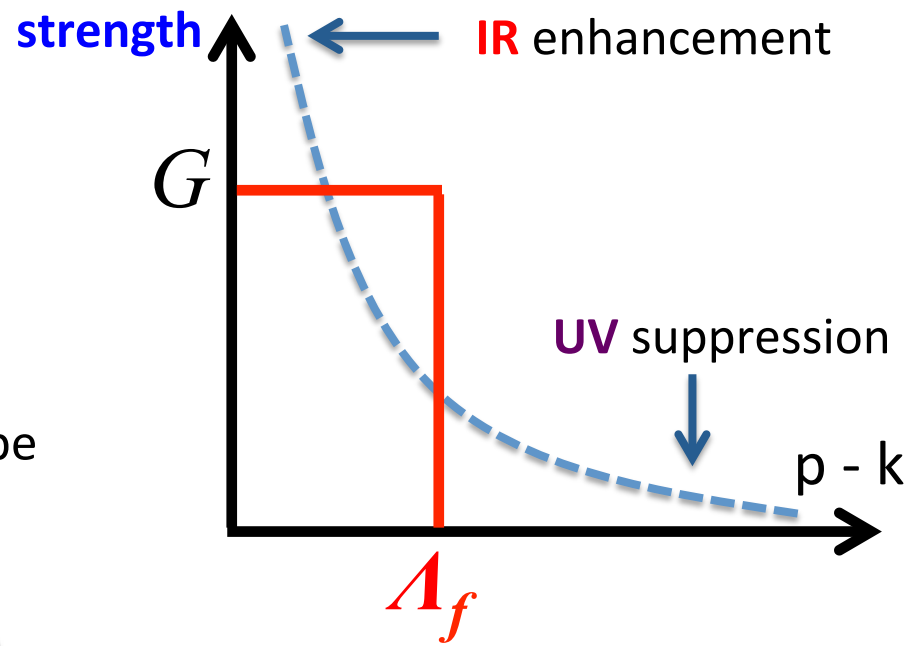


- ex) **Scalar** - **Scalar** channel

$$\frac{G}{N_c}\int dx^0 \int_{q,p,k} \left(\bar{\psi}(\vec{p}+\vec{q})\psi(\vec{p})\right)\left(\bar{\psi}(\vec{k})\psi(\vec{k}+\vec{q})\right)\theta_{p,k}$$

A crude model with asymptotic freedom





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Comparison with other form factor models

	Typical model	Ours
function of :	**quark mom.**	**mom. transfer**
Strength at large μ :	weaken	unchange (at large Nc)

- As far as we estimate **overall** size of free energy,
  two pictures would not differ so much, because:

  Hard quarks → ***Typical*** *int. : Hard*
  (dominant in free energy)

- **However,** if we compare **energy difference b.t.w. phases**,
  typical part **largely cancel out**,
  so we **must** distinguish these two pictures.

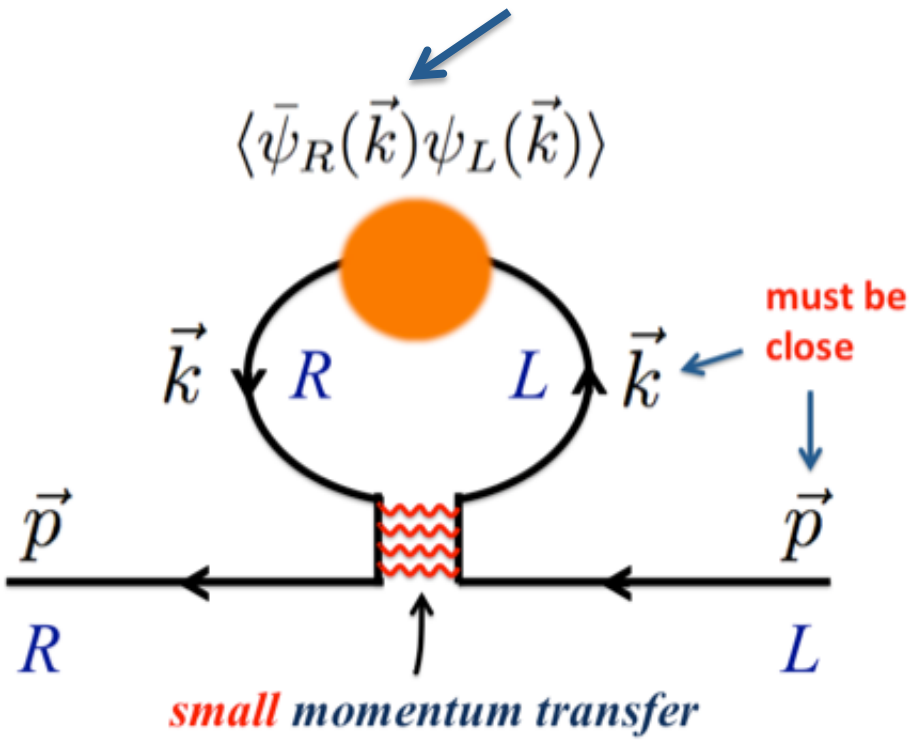
A key consequence of our form factor. 1

Quark Mass Self-energy (vacuum case)

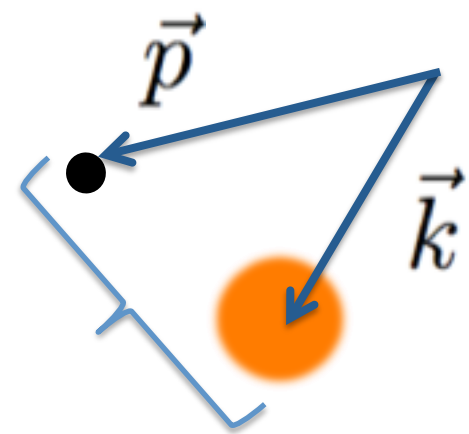
At Large Nc, largely comes from **Quark - Condensate int.**

(large amplitude ~ Nc)

(**Composite** objects
with **internal** momenta)



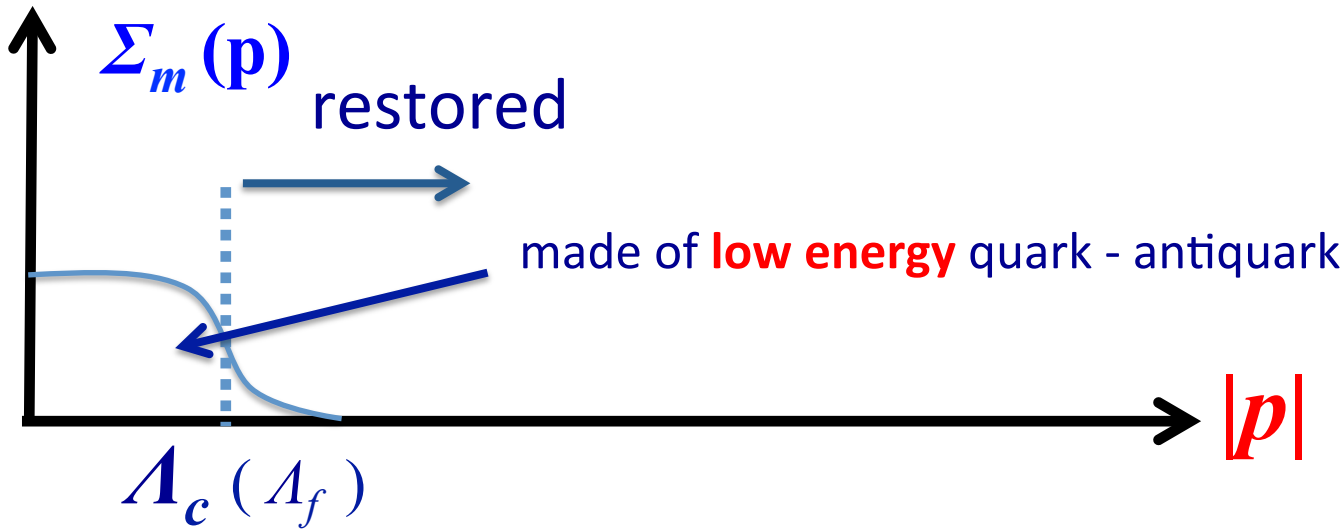
Mom. space



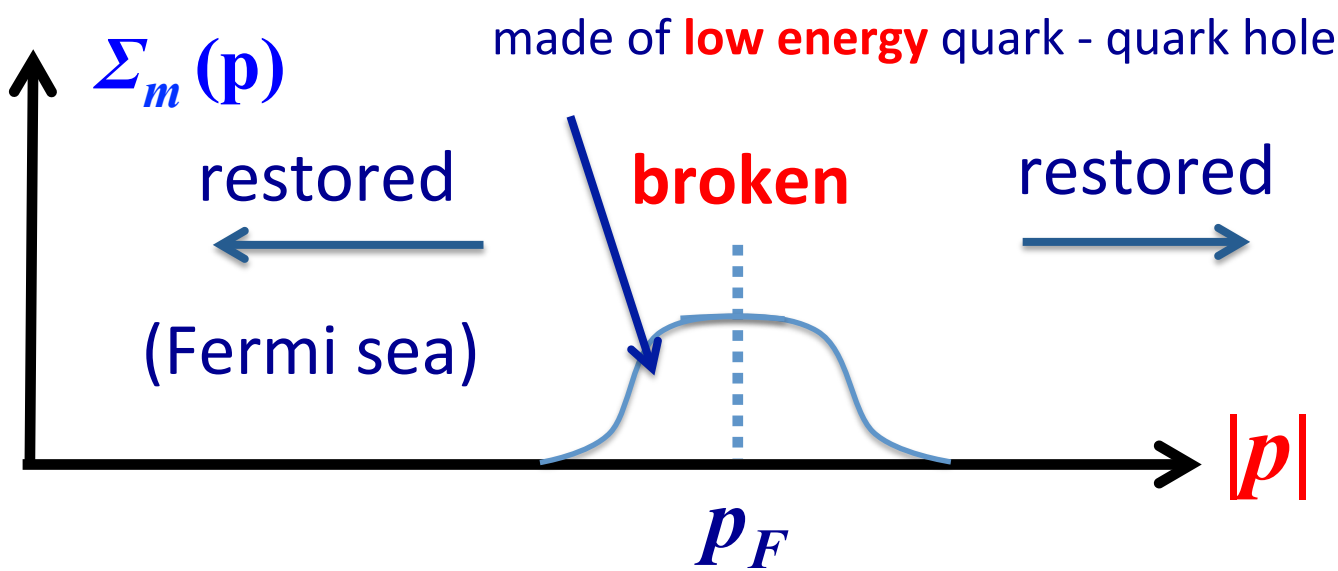
Decouple if p & k
are **very different**

Relevant domain of Non-pert. effects

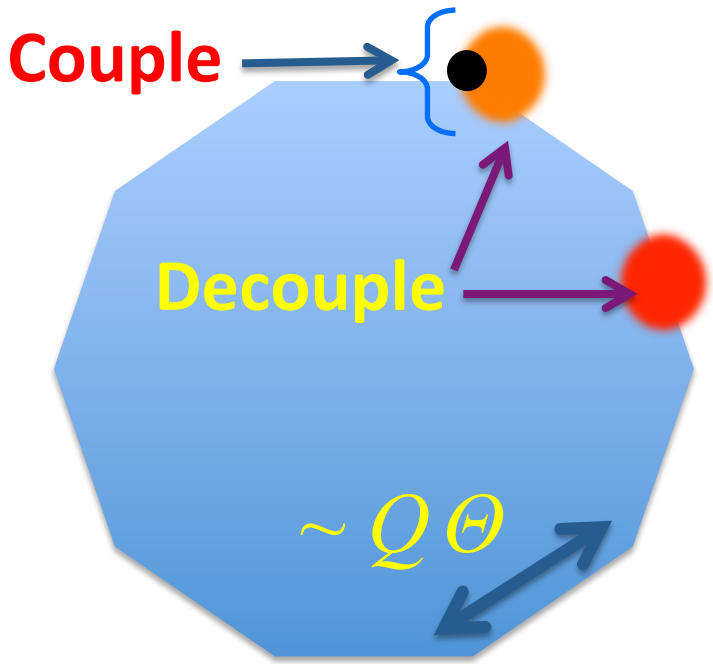
Vac.



Finite Density



3 Messages in this section



- 1, Condensates exist **only near the Fermi surface.**
- 2, Quark-Condensate int. **& Condensate-Condensate int.** are **local in mom. space.**

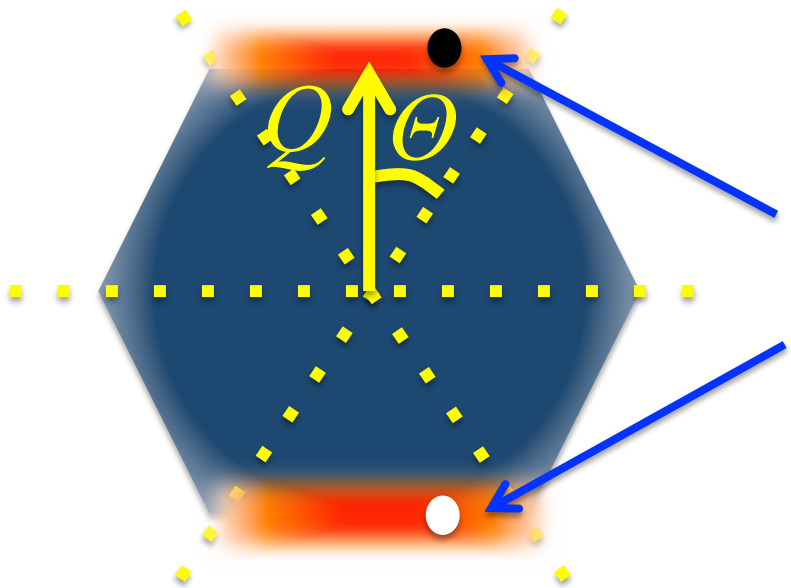
  (Range $\sim \Lambda_f$)

- 3, **Interferences** among **differently oriented** CSs happens only **at the patch-patch boundaries**.

If $Q\Theta >> \Lambda_f$

Boundary int. is **rare process**, and can be treated as Pert.

One Patch : Bases for Pert. Theory

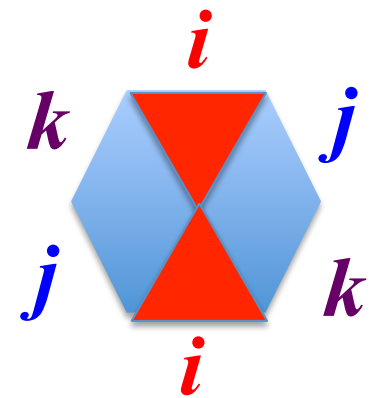


Particle-hole combinations

for one patch chiral spirals

Picking out one patch Lagrangian

ψ_i : momentum belonging to i -th patch



- **Kin. terms:** trivial to decompose

$$\mathcal{L}^{kin} \rightarrow \sum_i \bar{\psi}_i i\partial\!\!\!/ \psi_i \equiv \sum_i \mathcal{L}_i^{kin}$$

- **Int. terms:** Different patches can couple

$$\frac{G}{N_c} \sum_{i,j,k,l} \left((\bar{\psi}_i \psi_j)(\bar{\psi}_k \psi_l) + (\bar{\psi}_i i\gamma_5 \psi_j)(\bar{\psi}_k i\gamma_5 \psi_l) \right)$$

All fermions belong to the i -th patch

Patch - Patch int.

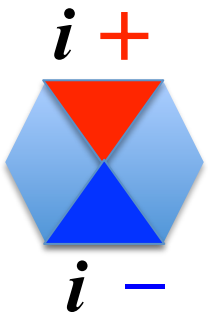
$$\mathcal{L} = \sum_i \mathcal{L}_i^{1patch} + \Delta\mathcal{L}$$

Dominant terms in One Patch, 1

"(1+1) D" "**chirality**" in ***i -th*** patch

$$\Gamma_{i5} \equiv \gamma_0 \gamma_{i\|} \qquad \psi_{i\pm} \equiv \frac{1 \pm \Gamma_{i5}}{2} \psi_i$$

eigenvalue: **Moving direction**



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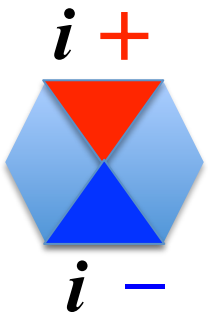

- **Fact :** "Chiral" **Non** - sym. terms → suppressed by $1/Q$

Dominant terms in One Patch, 1

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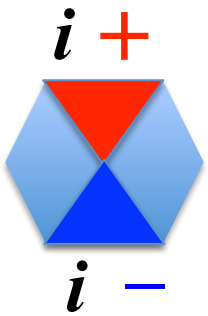

- **Fact :** "Chiral" **Non** - sym. terms → suppressed by $1/Q$

ex) free theory

- **Longitudinal Kin. (Sym.)**

$$\psi^\dagger_{i+} i(\partial_0 - \partial_{i\parallel}) \psi_{i+}$$

- **Transverse Kin. (Non-Sym.)**

$$\bar{\psi}_{i+} i \partial\!\!\!/_\perp \psi_{i-}$$

Dominant terms in One Patch, 1

"(1+1) D" "**chirality**" in ***i -th*** patch

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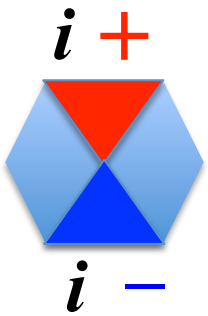
eigenvalue: **Moving direction**



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excitation energy

$$\epsilon^{\text{free}}(\delta\vec{p}) = |\delta p_\parallel| + \frac{\delta p_\parallel^2 + p_\perp^2}{2Q} + \cdots$$

momentum **measured from Fermi surface**

Dominant terms in One Patch, 2

$$(\bar{\psi}\psi)^2$$

"Chiral" sym. part

$$\frac{1}{2}\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\Gamma_5\psi)^2\right)$$

IR dominant

(must be **resummed** → **MF**)

Non - sym. part

$$\frac{1}{2}\left((\bar{\psi}\psi)^2 - (\bar{\psi}i\Gamma_5\psi)^2\right)$$

$1/Q$ suppressed

(can be treated in **Pert**.)

Dominant terms in One Patch, 2

$$(\bar{\psi}\psi)^2$$

"Chiral" sym. part

$$\frac{1}{2}\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\Gamma_5\psi)^2\right)$$

IR dominant

(must be **resummed** → **MF**)

Non - sym. part

$$\frac{1}{2}\left((\bar{\psi}\psi)^2 - (\bar{\psi}i\Gamma_5\psi)^2\right)$$

$1/Q$ suppressed

(can be treated in **Pert**.)

- **IR** dominant : **Unperturbed Lagrangian**

  Longitudinal Kin. + "Chiral" sym. 4-Fermi int.

  → Gap eq. can be reduced to (1+1) D
  (P_T - factorization)

- **IR** suppressed : **Perturbation**

  Transverse Kin. + Non - sym. 4-Fermi int.

Quick Summary of 1-Patch results

At leading order of Λ_{QCD}/μ

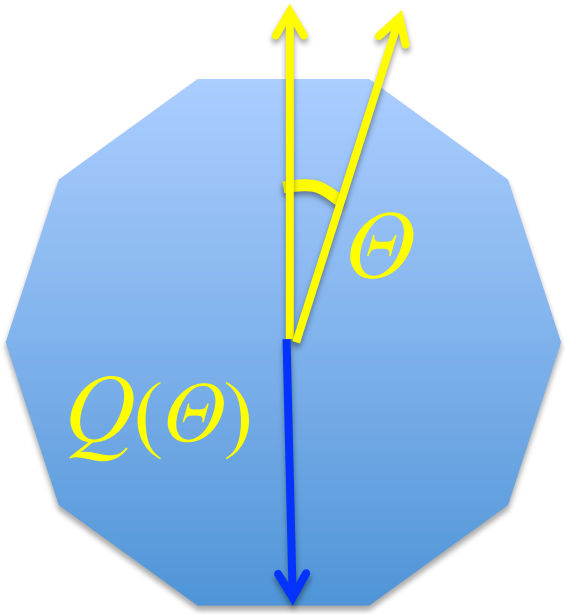
- **Integral** eqs. such as Schwinger-Dyson, Bethe-Salpeter, can be reduced from (**2**+1) D to (**1**+1) D. cf) *kT factorization*
- **Chiral Spirals** emerge, generating **large** quark mass gap. (even larger than vac. mass gap)
- **Quark num**. is **spatially uniform**. (in contrast to chiral density)

Pert. corrections

- **Quark num**. **oscillation**.
- **CSs** : **Plane wave** → **Solitonic**

} approach to **Baryonic Crystals**

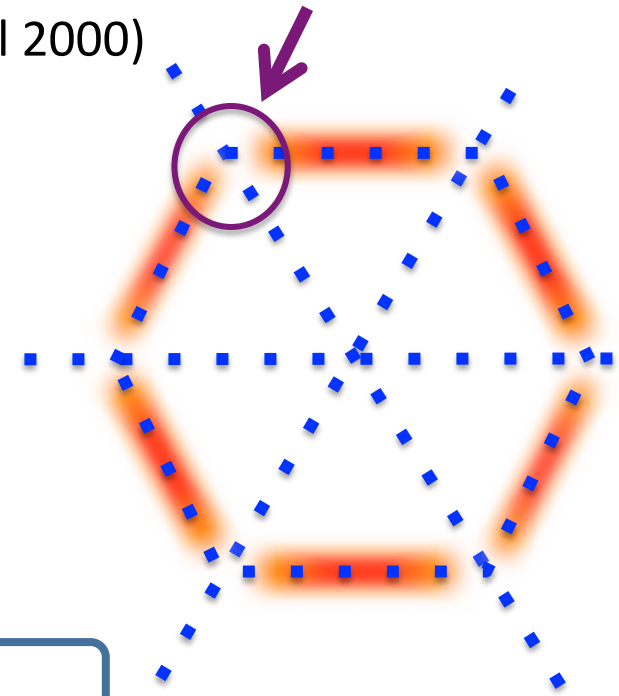
Multi-patches & Optimizing Θ



Multi-Patches: Boundary Effects

- Interferences among **differently oriented** CSs
  destroy one another, **reducing the mass gap**.
  (Checked by Pert. Numerical study by Rapp.et al 2000)

- Such effects arise
  only around patch boundaries.



reduction of gap : $\sim \Lambda_f$

Phase space : $\sim N_p \times \Lambda_f^{\,2}$

$$\boxed{\textit{Energetic Cost}: \sim N_p \times \Lambda_f^{\,3}}$$

$(N_p \sim 1/\Theta)$

(Remark: Such deconstruction effects are bigger if CS'swave vectors take closer value.)

Chiral Spirals (CSs)

- One can find (**1**+1) D **solution** for the gap equation.
  (**except** boundaries of patches)
- The size of mass gap is $\sim \Lambda_f$, if we choose $G \sim 1/\Lambda_f$.
- The form of chiral condensates: **Spirals**

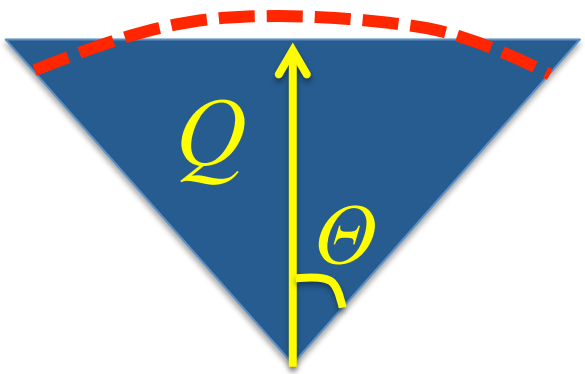
$$\langle \bar{\psi}_{i+}\psi_{i-}\rangle = \Delta\, \underline{e^{-2iQx_{i\|}}} \quad \& \quad \langle \bar{\psi}_{i-}\psi_{i+}\rangle = \Delta\, \underline{e^{2iQx_{i\|}}}$$

$$\Updownarrow$$

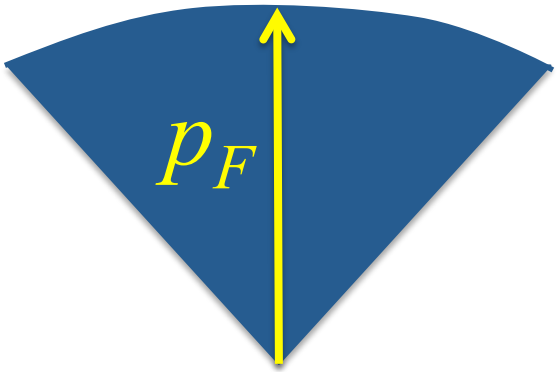
$$\langle \bar{\psi}_i\psi_i\rangle = \Delta \cos 2Qx_{i\|} \quad \& \quad \langle \bar{\psi}_i \underline{\gamma_0\gamma_{i\|}} \psi_i\rangle = \Delta \sin 2Qx_{i\|}$$

- The subdominant terms can be computed
  systematically as $1/Q$ or Θ expansion .

Energetic cost of deformed Fermi sea



v.s.



- **Constraint**: **Canonical** ensemble → **Fermi vol. fixed**

$$\rightarrow \quad Q(\Theta) = p_F\left(1 - \frac{\Theta^2}{6} - \frac{\Theta^4}{40} + \cdots\right)$$

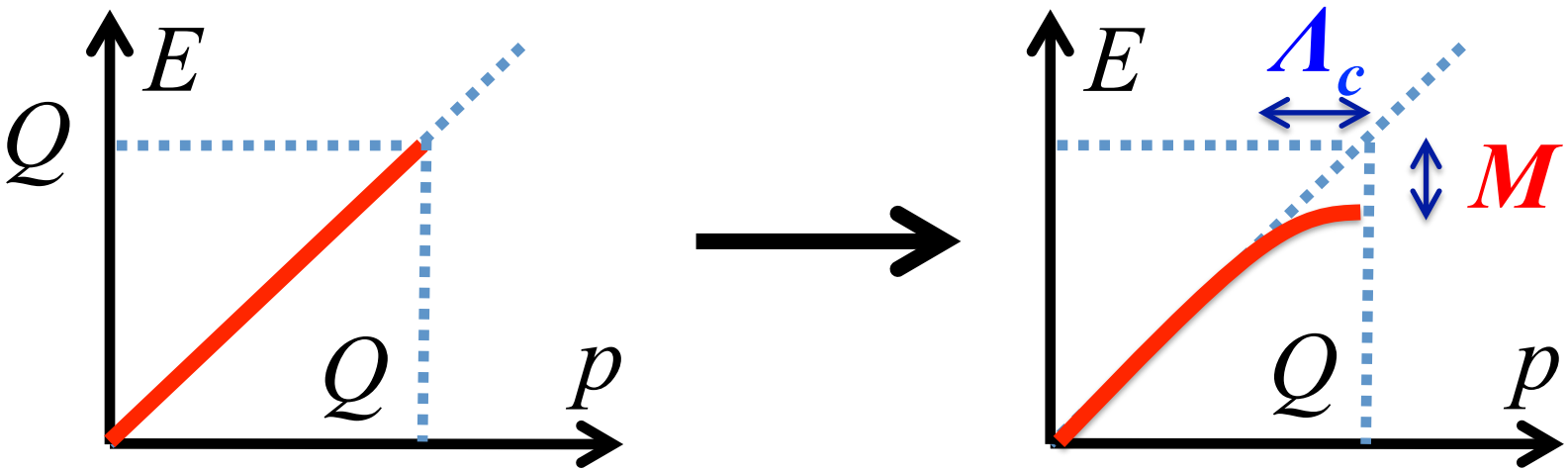
- **Energetic difference : (deformation energy)**

$$\delta E^{deform.} \sim N_p \times p_F{}^3 \times \Theta^5 \, (1 + O(\Theta^2))$$

(This expression holds even if condensations occur.)

Condensation effects 1.

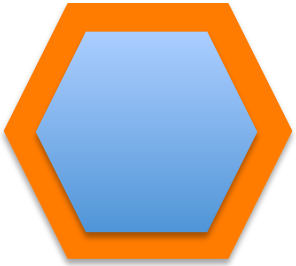
- **Gain** : **Less** single particle contributions
  (due to mass gap generated by condensates)



Fermions occupy energy levels only up to $Q - M$.

$$\delta E^{1particle} \sim -M \times \underbrace{\Lambda_c \times Q}_{\text{(phase space)}}$$

(after adding condensation energy)



Condensation effects 2.

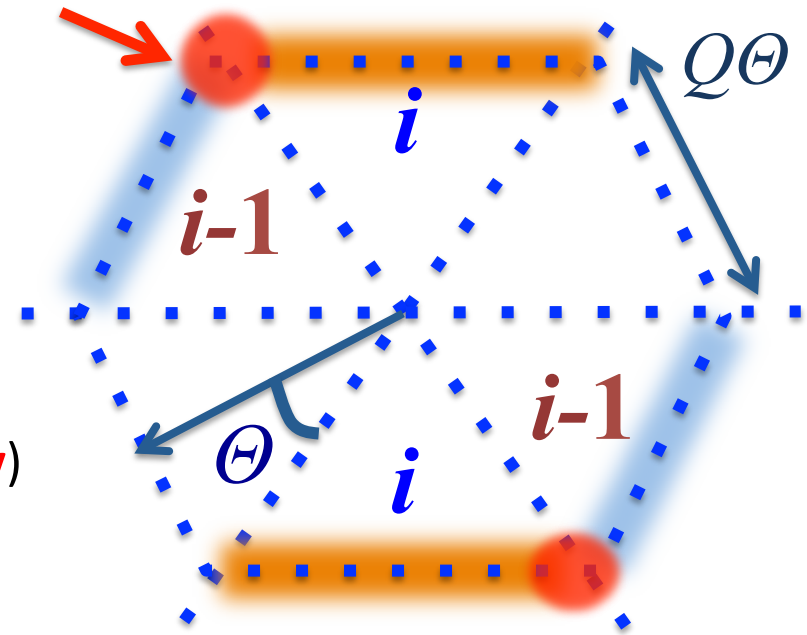
- **Cost** : **Induced interactions** b.t.w. CSs

Patch-Patch boundary

1, Int. between CSs happen **only** within phase space, $\sim \Lambda_f{}^2$

2, The strength becomes smaller with smaller size of M_B (mass gap near the **boundary**)

3, The sign is **positive**.



$$\delta E^{int.} \sim + N_p \times f_{int.}(M_B, \Theta)$$

(Num. of boundary points)

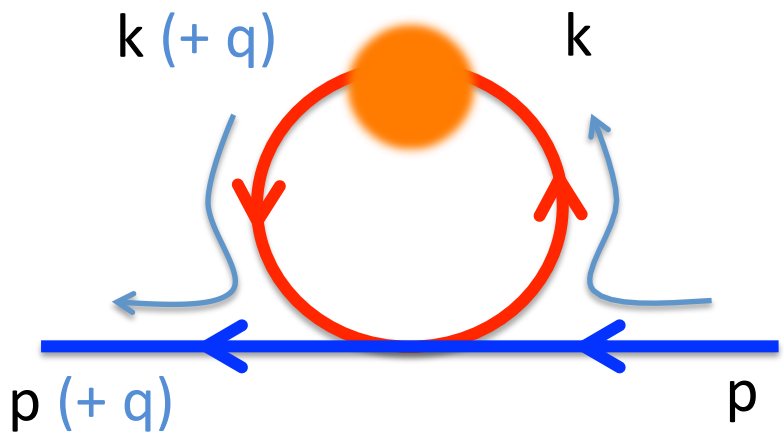
($f \to 0$ as $M_B \to 0$)

Consequences of form factor. 1

For **quarks – condensates int.** to happen,

their momentum domains **must be close** each other.

- **Schwinger-Dyson eq. for mass gap: (q=0 for vacuum)**



$$\Sigma_m(\vec{p}) = \int \frac{d\vec{k}}{(2\pi)^2} \underbrace{\frac{\Sigma_m(\vec{k})}{2\epsilon(\vec{k})}}_{\text{loop (condensate)}} \theta_{p,k}$$

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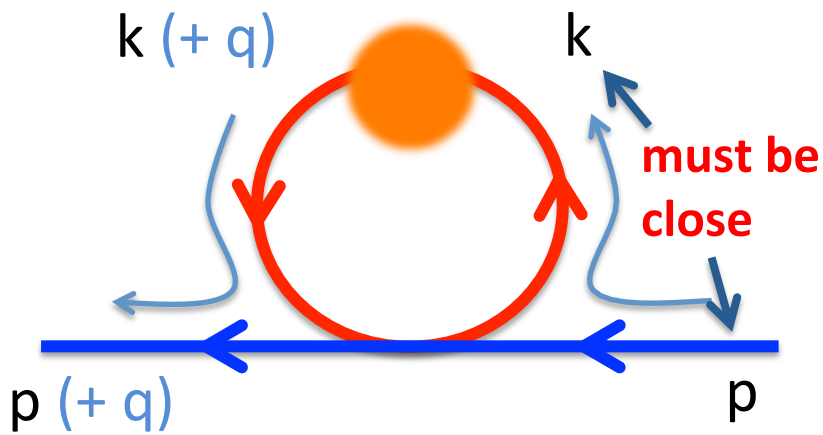

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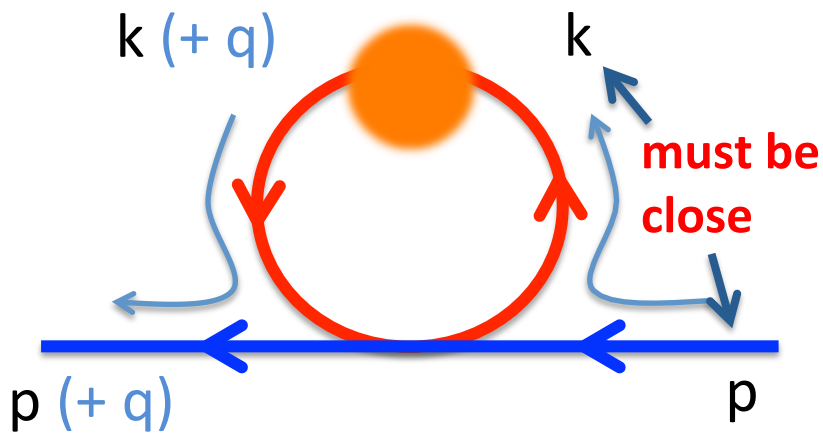

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- Condensate created by fermions around momenta k
  can couple **only** to fermions with momenta $p \sim k$.
- UV cutoff for k is measured from p, **NOT from** 0.

Consequences of form factor. 2

Dominant contributions to condensates : Low **energy** modes

(for vacuum)

$$\Sigma_m(\vec{p}) = \int \frac{d\vec{k}}{(2\pi)^2} \frac{\Sigma_m(\vec{k})}{2\epsilon(\vec{k})} \theta_{p,k}$$

When $\boldsymbol{p} \longrightarrow \infty$:

- $\boldsymbol{k}$ must also go to ∞, so $\boldsymbol{\varepsilon(k)} \longrightarrow \infty$.
- Phase space is **finite** : Nothing compensates denominator.

Consequences of form factor. 2

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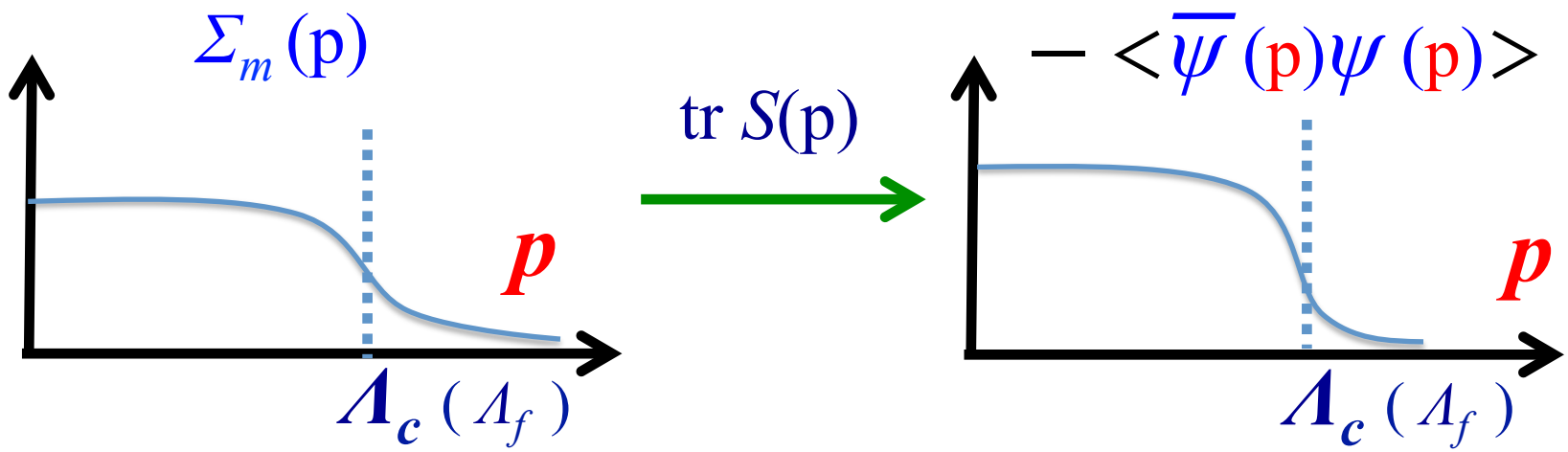
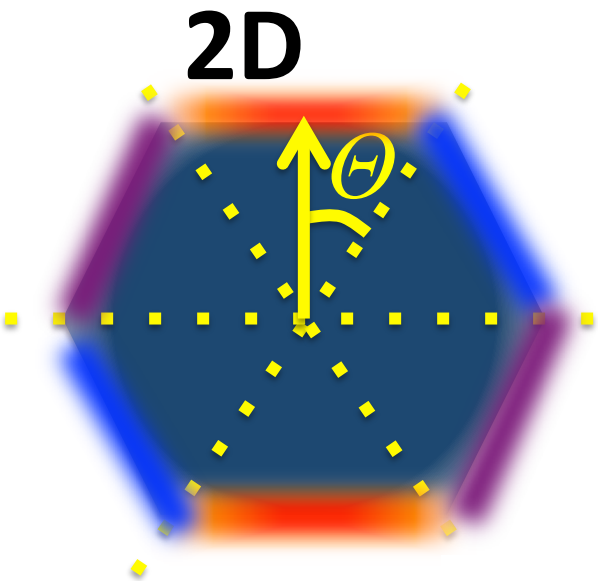


Remark)

- finite density: Low **energy** modes appear **near the Fermi surface**.

Our goal



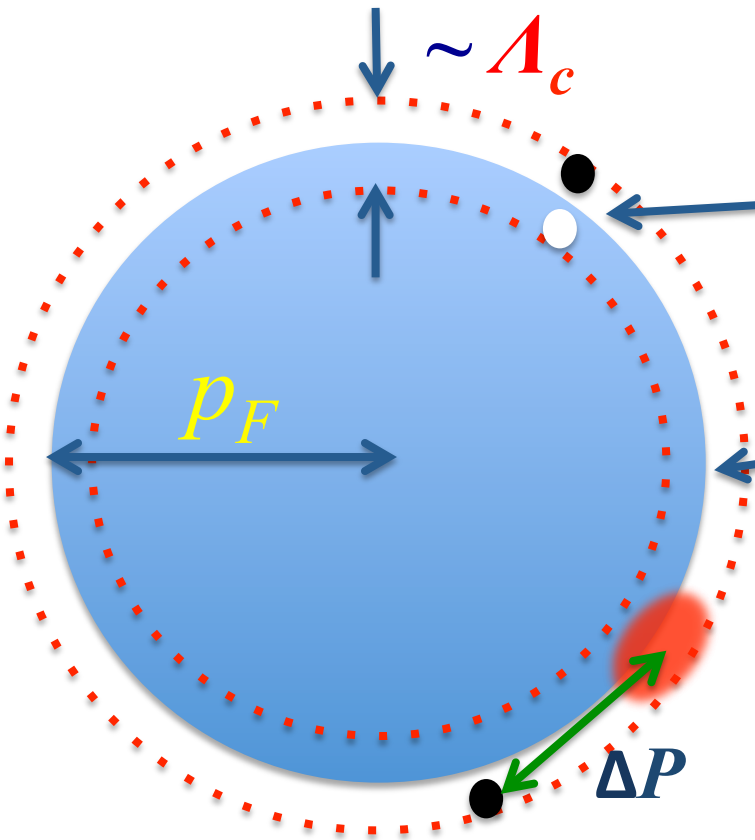
- To express the energy density as a function of theta, and to determine the best shape.

Approximations to be used

- **4-Fermi int. with a strong form factor**
- **Large Nc** (**MF** treatments)
- **Large density** (high density expansion, T=0)
- **(2+1) D** (simple **shape** of the Fermi surface)

→ **Simple analytic insights**

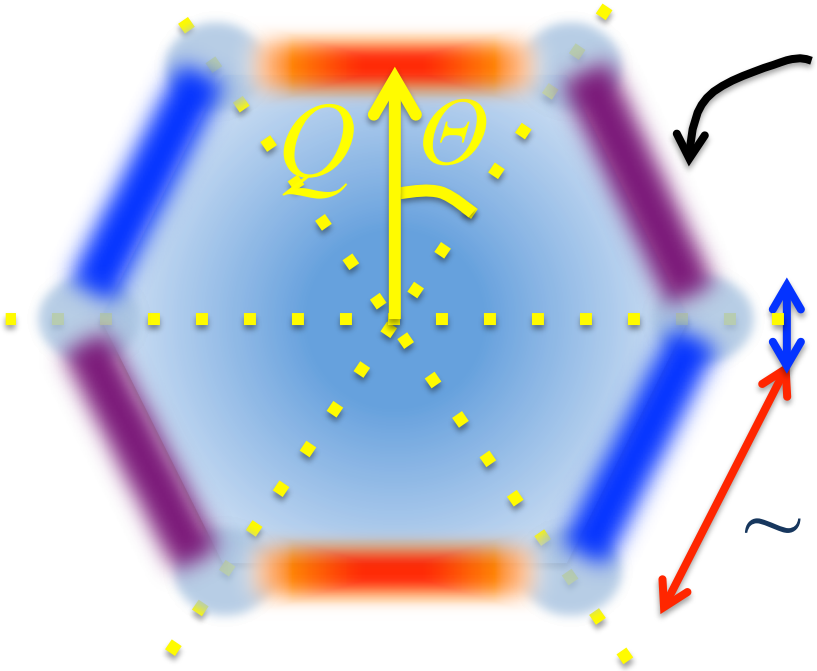
At very high density: $P_F >> \Lambda_f$



- **Low energy modes:**
  particle - holes
  (near the Fermi surface)
- **Domain of condensations:**
  limited to Fermi surface region
- **Decoupling:** For $\Delta P >> \Lambda_f$
  Quarks **do not** couple to condensates in **very** different momentum domain.

Quark-condensate int. is **local** in **momentum space**.

Do we need to treat many CSs simultaneously ?



Domain of condensation

phase space

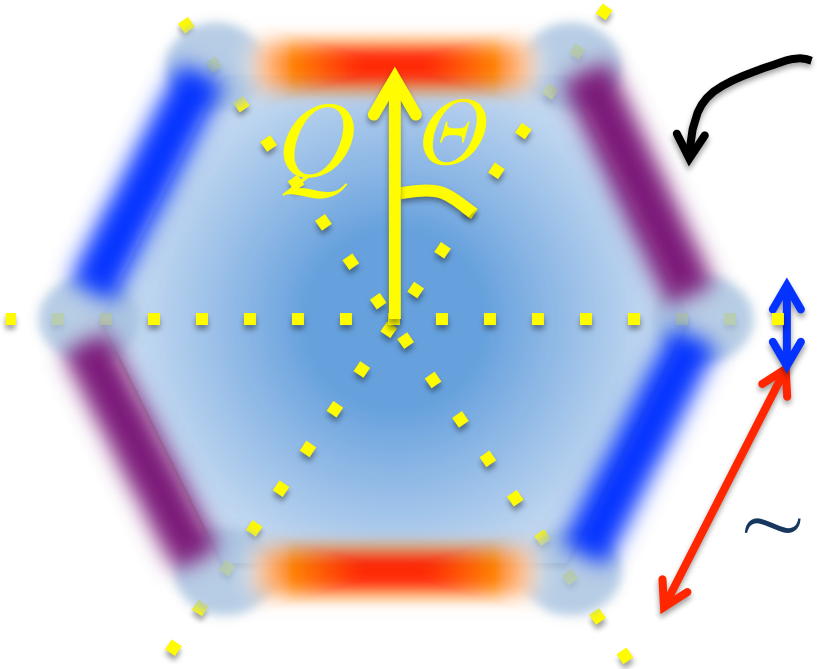
$$\sim \Lambda_f \rightarrow \sim \Lambda_c \Lambda_f$$

(for **1-boundary**)

$$\sim Q\Theta \rightarrow \sim \Lambda_c Q\Theta$$

(for **1- patch**)

Do we need to treat many CSs simultaneously ?



Domain of condensation

phase space

$$\sim \Lambda_f \rightarrow \sim \Lambda_c \Lambda_f$$

(for **1-boundary**)

$$\sim Q\Theta \rightarrow \sim \Lambda_c Q\Theta$$

(for **1- patch**)

We consider $\boxed{\Lambda_f / p_F << \Theta << 1}$ where

Phase space: **1- patch** >> **1- boundary**

Boundary effects
(Patch-Patch interactions) → **Small Perturbations** to the **1-patch problem**

Chiral Spirals (CSs)

- One can find (**1**+1) D **solution** for the gap equation.
  (**except** boundaries of patches)
- The size of mass gap is $\sim \Lambda_f$, if we choose $G \sim 1/\Lambda_f$.
- The form of chiral condensates: **Spirals**

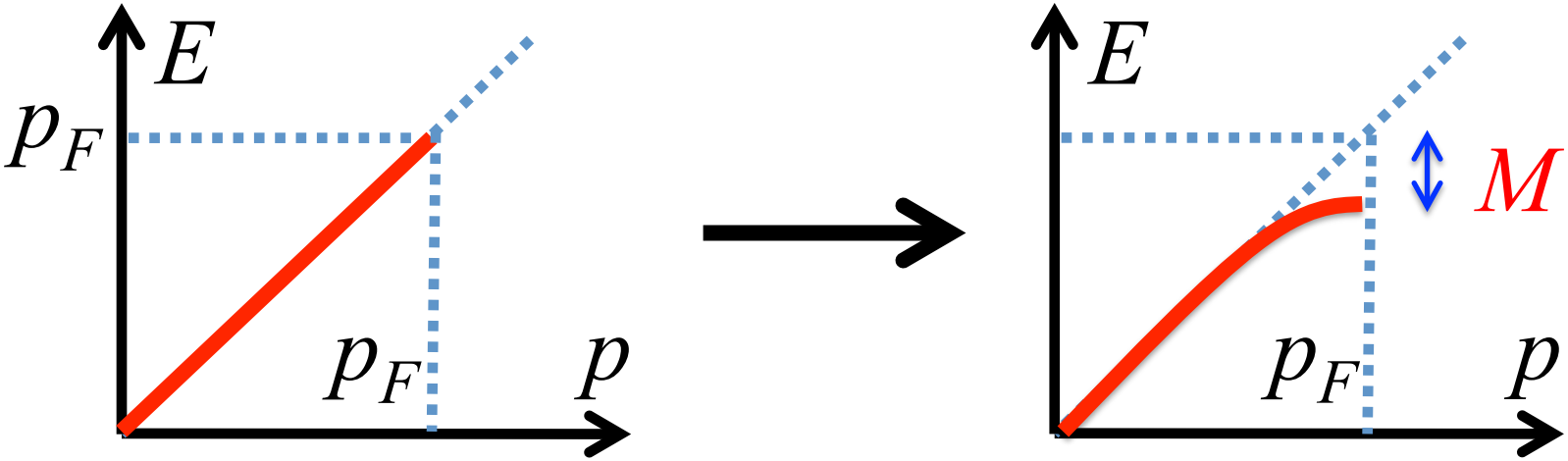
$$\langle \bar{\psi}_{i+}\psi_{i-}\rangle = \Delta\, \underline{e^{-2iQx_{i\|}}} \quad \& \quad \langle \bar{\psi}_{i-}\psi_{i+}\rangle = \Delta\, \underline{e^{2iQx_{i\|}}}$$

$$\Updownarrow$$

$$\langle \bar{\psi}_i\psi_i\rangle = \Delta \cos 2Qx_{i\|} \quad \& \quad \langle \bar{\psi}_i \underline{\gamma_0\gamma_{i\|}} \psi_i\rangle = \Delta \sin 2Qx_{i\|}$$

What is the best shape ?

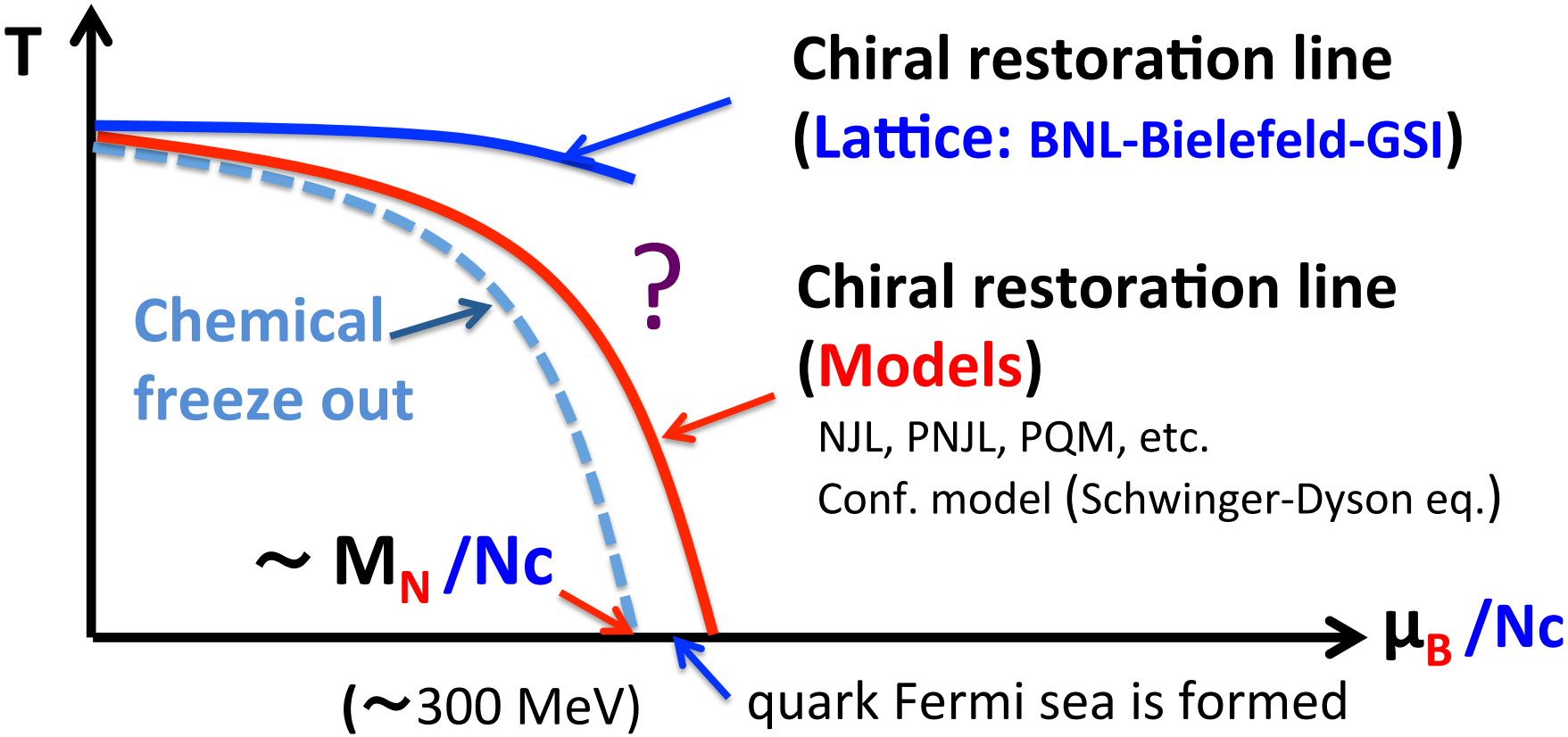
- **Gain** : Less single particle contributions
  (due to mass gap generated by condensates)



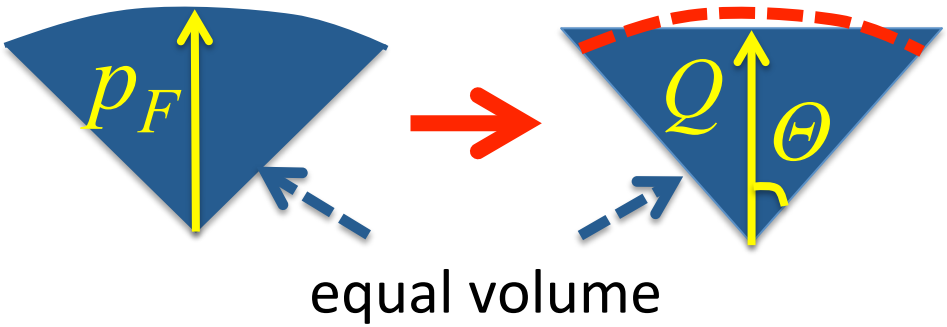
Fermions occupy levels only up to $\boldsymbol{p_F - M}$.

$$\delta E^{1\text{-paticle}} \sim -\boldsymbol{M} \times \boldsymbol{\Lambda} \times \boldsymbol{Q}$$

Chiral restoration

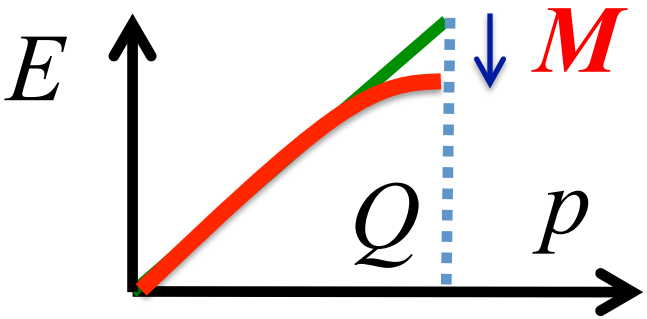


- **Cost : Deformation**
  (dominant for **large** Θ)



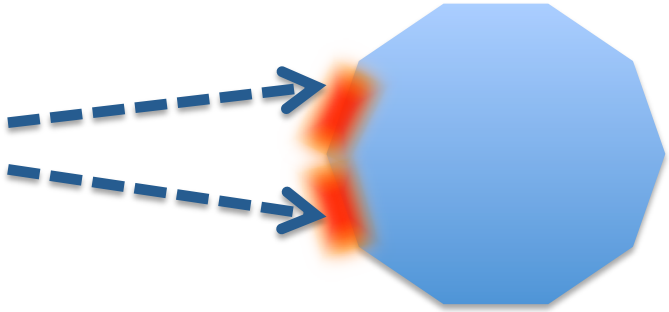
- **Gain : Mass gap origin**

Condensation effects



- **Cost : Interactions among CSs**
  (dominant for **small** Θ)

Condensate – Condensate int.



Anomalies and the helicity of the thermal state

R. Loganayagam

Harvard University, U.S.A..

P and CP Odd Effects in Hot and Dense Matter
RIKEN BNL Research centre workshop
Brookhaven National Laboratory
26th June, 2012.

The Thermal state

- Take a Poincare-invariant QFT on an **even dimensional** flat Minkowski spacetime $\mathbb{R}^{2n-1,1}$.
- Consider a thermal state in this theory at some $\{T, \mu\}$.
- Will assume that this thermal state does not break rotational/translational invariance in its rest frame.
- So the momentum/angular-momentum of the thermal state is zero. If $\vec{\mathcal{P}}$ denotes the momentum and $\mathfrak{L}_{2k-1,2k}$ denote the Cartan angular momenta then

$$\langle \vec{\mathcal{P}} \rangle = 0$$

$$\langle \mathfrak{L}_{2k-1,2k} \rangle = 0$$

Thermal Helicity

- Of course we can easily have $\langle \mathfrak{L}^2 \rangle \neq 0$ in the thermal state.
- **Such thermal expectation values are closely related to non-dissipative transport coefficients in the hydrodynamics.**
- In this talk we will focus on one interesting observeable - what we will call *the thermal helicity*

$$\left\langle \left(\prod_{k=1}^{n-1} \mathfrak{L}_{2k-1,2k} \right) \mathcal{P}_{2n-1} \right\rangle = \langle \mathfrak{L}_{12} \mathfrak{L}_{34} \ldots \mathfrak{L}_{2n-3,2n-2} \mathcal{P}_{2n-1} \rangle$$

- In $3+1$d think of $\left\langle \vec{\mathfrak{L}}.\vec{\mathcal{P}} \right\rangle$: the helicity of the thermal state.

Why Thermal Helicity ?

- An interesting measure of how much the thermal state breaks parity. (Obvious)
- Seems closely linked to anomalies in the underlying quantum theory (Not Obvious at all - content of this talk)
- Aim of this talk - To convince you that **the thermal helicity is actually a polynomial in** $\{T, \mu\}$.
- In most examples a stronger statement seems to be true : **the coefficients in this polynomial are the coefficients in the corresponding anomaly polynomial**.
- Stronger statement has no proof yet - higher spin counterexamples indicate that proof (if there) is probably subtle. [See Yarom's talk]

Thermal Helicity via Sphere Partition function

- A very useful way to compute thermal helicity : the thermal partition function on $S^{2n-1} \times \mathbb{R}$.
- Define a thermal partition function on the sphere with chemical potentials turned on for the angular momenta on the sphere.
- Trying to do this in flat space runs into various IR problems (c.f. Vilenkin's work on rotating ensembles)
- Let L_a with $a = 1, \ldots, n$ be the mutually commuting angular momenta on S^{2n-1}. Define

$$\mathcal{Z}[\Omega] \equiv \mathrm{Tr}_{S^{2n-1}\times\mathbb{R}} \, \mathrm{Exp}\left[-\frac{\left(H - \mu Q - \sum_{a=1}^{n} \Omega_a L_a\right)}{T}\right]$$

Sphere Partition function II

- Flat space thermal helicity per unit spatial volume partition function is

$$\frac{1}{\mathrm{Vol}_{2n-1}}\langle \mathfrak{L}_{12}\mathfrak{L}_{34}\ldots\mathfrak{L}_{2n-3,2n-2}\mathcal{P}_{2n-1}\rangle = \lim_{R\to\infty}\frac{1}{R\forall_{2n-1}}\left[\left(\prod_{a=1}^{n} T\frac{\partial}{\partial\Omega_a}\right)\ln\mathcal{Z}[\Omega]\right]_{\Omega=0} \tag{1}$$

  where $\forall_{2n-1}$ is the volume of a sphere S^{2n-1} with radius R.
- The derivatives wrt Ω bring down n sphere angular momenta and the factor of R converts one sphere angular momentum into a flat space momentum.

$$SO(2n) \mapsto ISO(2n-1)$$

Sphere Partition function III

- Will write this formula in terms of a function

$$\mathfrak{F}^{\omega}_{anom}[T,\mu] = \lim_{R\to\infty}\left[\left(\prod_{a=1}^{n}\frac{1}{2\pi R^2}\frac{\partial}{\partial\Omega_a}\right)[-T\ln\mathcal{Z}[\Omega]]\right]_{\Omega=0}$$

so that

$$\frac{1}{\mathrm{Vol}_{2n-1}}\langle\mathfrak{L}_{12}\mathfrak{L}_{34}\ldots\mathfrak{L}_{2n-3,2n-2}\mathcal{P}_{2n-1}\rangle = -(n-1)!(2T)^{n-1}\mathfrak{F}^{\omega}_{anom}[T,\mu]$$

- Strong claim : a remarkable relation between $\mathfrak{F}^{\omega}_{anom}[T,\mu]$ and the anomaly polynomial $\mathcal{P}_{anom}[F,\mathfrak{R}]$

$$\mathfrak{F}^{\omega}_{anom}[T,\mu] = \mathcal{P}_{anom}\left[F\mapsto\mu, p_1(\mathfrak{R})\mapsto -T^2, p_{k>1}(\mathfrak{R})\mapsto 0\right]$$

Cardy CFT

- Take a simple example of a 2d CFT heated to a finite $\{T, \mu\}$.
- In 2d, the thermal 'helicity' is just the momentum of the thermal state $\langle \mathcal{P} \rangle$.
- Take a 2d CFT with a $U(1)_L \times U(1)_R$ global symmetry placed on a circle of radius R. Will eventually be interested in the flat space limit $R \to \infty$ with $T, \mu_{L,R}$ fixed.
- Thermal helicity can be computed by looking at the partition function on the cylinder

$$\mathcal{Z}[\Omega] \equiv \mathrm{Tr}_{S^1}\ \mathrm{Exp}\left[-\frac{(H - \mu_L Q_L - \mu_R Q_R - \Omega L)}{T}\right]$$
$$\equiv \mathrm{Tr}\ y^{J_0} q^{L_0 - \frac{c_L}{24}}\ \tilde{y}^{\tilde{J}_0} \tilde{q}^{\tilde{L}_0 - \frac{c_R}{24}}$$

Cardy CFT II

- We can get this if we assume a Cardy growth of entropy

$$S \approx 4\pi \left[\sqrt{\frac{c_L}{24}\left(\langle L_0\rangle - \frac{c_L}{24} - \frac{1}{4k_L}\langle J_0\rangle^2\right)} + \sqrt{\frac{c_R}{24}\left(\langle \tilde{L}_0\rangle - \frac{c_R}{24} - \frac{1}{4k_R}\langle \tilde{J}_0\rangle^2\right)} \right]$$

  where $c_{L,R}$ are the central charges and $k_{L,R}$ are the Kac-Moody levels
- As is well-known, a wide variety of 2d CFTs (and AdS_3 duals) satisfy this assumption.

Cardy CFT III

- Cardy approximation can be phrased as

$$\ln \mathcal{Z}[\Omega] \approx \frac{1}{T}\left[p\frac{\forall_{S^1}}{1-R^2\Omega^2} - \mathfrak{F}^{\omega}_{anom}\frac{2\pi R^2\Omega}{1-R^2\Omega^2}\right]$$

- p is the Cardy-pressure given by

$$p \approx 2\pi\left[\frac{c_R+c_L}{24}T^2 + k_R\left(\frac{\mu_R}{2\pi}\right)^2 + k_L\left(\frac{\mu_L}{2\pi}\right)^2\right]$$

$$\mathfrak{F}^{\omega}_{anom} = -2\pi\left[\frac{c_R-c_L}{24}T^2 + k_R\left(\frac{\mu_R}{2\pi}\right)^2 - k_L\left(\frac{\mu_L}{2\pi}\right)^2\right]$$

Thermal Helicity of Cardy CFT

- Compare this against the anomaly polynomial of this 2d CFT

$$\mathcal{P}_{anom}[F,\mathfrak{R}] = -2\pi\left[k_R\left(\frac{F_R}{2\pi}\right)^2 - k_L\left(\frac{F_L}{2\pi}\right)^2 - p_1(\mathfrak{R})\frac{c_R - c_L}{24}\right]$$

to get a remarkable replacement rule

$$\mathfrak{F}^{\omega}_{anom} = \mathcal{P}_{anom}[F_R \mapsto \mu_R, F_L \mapsto \mu_L, p_1(\mathfrak{R}) \mapsto -T^2]$$

- Without much ado we can now directly compute the momentum density in the flat space thermal state.

$$\frac{1}{\text{Vol}}\langle\mathcal{P}\rangle = \lim_{R\to\infty}\left[\frac{1}{R\forall_{S^1}}\langle L\rangle\right]_{\Omega=0} = -\mathfrak{F}^{\omega}_{anom}$$

Cardy CFT -Lessons

1. Thermal helicity can be read off from a particular term in the sphere partition function.
2. The coefficient of this term is a polynomial in T and μ
3. This polynomial is obtained from the anomaly polynomial by a simple substitution rule !
4. Question : How general are these statements ? Do they hold in higher dimensions ?
5. Answer turns out to be yes - but to see this we need to recast Cardy approximation in a hydrodynamic language.

Hydrodynamics of Cardy CFT

- Cardy thermodynamics can be thought of as coming from a spinning conformal fluid with

$$\mathcal{G}^\alpha = -\hat{p}u^\alpha + \hat{\mathfrak{F}}^\omega_{anom} V^\alpha$$
$$T^{\alpha\beta} = \hat{p}\left[g^{\alpha\beta} + 2u^\alpha u^\beta\right] - \hat{\mathfrak{F}}^\omega_{anom}(u^\alpha V^\beta + u^\beta V^\alpha)$$
$$J^\alpha = -\frac{\partial \mathcal{G}^\alpha}{\partial \hat{\mu}} \quad \text{and} \quad J^\alpha_S = -\frac{\partial \mathcal{G}^\alpha}{\partial \hat{T}}$$

- Hatted quatities denote the local rest frame temperature/pressure etc. which are blue-shifted with repect to lab frame quatities.
- u^μ is the 2-velocity and $V_\nu \equiv \epsilon_{\mu\nu} u^\mu$.

Higher dimensional generalisation

- We note that there is a perfect fluid term along with an anomalous correction. And the partition function is just dominated by a fluid configuration.
- This can be generalised to spinning conformal fluids in higher dimensions $d = 2n$

$$\mathcal{G}^\alpha = -\hat{p}u^\alpha + \hat{\mathfrak{F}}^\omega_{anom} V^\alpha$$

$$T^{\alpha\beta} = \hat{p}\left[g^{\alpha\beta} + 2nu^\alpha u^\beta\right] - n\hat{\mathfrak{F}}^\omega_{anom}(u^\alpha V^\beta + u^\beta V^\alpha)$$

$$J^\alpha = -\frac{\partial \mathcal{G}^\alpha}{\partial \hat{\mu}} \quad \text{and} \quad J^\alpha_S = -\frac{\partial \mathcal{G}^\alpha}{\partial \hat{T}}$$

with

$$V^\mu \equiv \epsilon^{\mu\nu\lambda_1\lambda_2\ldots\lambda_{2n-2}} u_\nu (\nabla u)^{n-1}_{\lambda_1\lambda_2\ldots\lambda_{2n-2}}.$$

Higher dimensional generalisation II

- We can now calculate the partition function dominated by the fluid configuration

$$\ln \mathcal{Z}[\Omega] \approx \frac{1}{T}\left[p\frac{\forall_{S^{2n-1}}}{\prod_{b=1}^{n}(1-R^2\Omega_b^2)}+\cdots \right.$$
$$\left. -\mathfrak{F}^{\omega}_{anom}\prod_{b=1}^{n}\frac{2\pi R^2\Omega_b}{1-R^2\Omega_b^2}+\cdots\right]$$

which can be thought of as a higher dimensional generalisation of the results in Cardy CFT before.

Higher dimensional generalisation III

- From the partition function we get the thermal helicity exactly in terms of the transport coefficient $\mathfrak{F}^{\omega}_{anom}$

$$\frac{1}{\mathrm{Vol}_{2n-1}}\langle \mathfrak{L}_{12}\mathfrak{L}_{34}\ldots\mathfrak{L}_{2n-3,2n-2}\mathcal{P}_{2n-1}\rangle = -(n-1)!(2T)^{n-1}\mathfrak{F}^{\omega}_{anom}[T,\mu]$$

- Calculating/constraining the thermal helicity of a theory is same as calculating/constraining this transport coefficient.
- What do we know about $\mathfrak{F}^{\omega}_{anom}$ from hydrodynamics ?

Anomaly induced transport

- $\mathfrak{F}^{\omega}_{anom}$ is the Chiral vortical coefficient.
- Was argued to be a polynomial in $\{T, \mu\}$ using adiabaticity in

  Loganayagam.R. [arXiv:1106.0277]

- Generalises various previous entropy arguments.

  Dam T. Son, Piotr Surowka [arXiv:0906.5044]

  Yasha Neiman, Yaron Oz [arXiv:1011.5107]

  Dmitri E. Kharzeev, Ho-Ung Yee [arXiv:1105.6360]

- Subsequently confirmed by partition function arguments

  N. Banerjee, J. Bhattacharya, S. Bhattacharyya, S. Jain, S. Minwalla and Tarun Sharma [arXiv:1203.3544]

  Nabamita Banerjee, Suvankar Dutta, Sachin Jain, R. Loganayagam and Tarun Sharma [to be published]

- So we arrive at the interesting statement that the thermal helicity is a polynomial in T and μ !

Relation to Anomaly Polynomial

- We can actually go further and argue that in an arbitrary free fermion theory adibaticity argments show that the substitution rule is true.

  R. Loganayagam, Piotr Surowka [arXiv:1201.2812]

- Based on a lot of earlier work in 1+1d and 3+1d :

  A. Vilenkin, [Phys.Lett. B80 (1978)150 - 152].

  K. Landsteiner, E. Megias, L. Melgar and F. Pena-Benitez [arXiv:1111.2823].

- The substitution rule true for arbitrary free theories of fermions/form fields (proof via sphere partition functions).

  R. Loganayagam [to be published]

- The rule holds in all known holographic computations till date.
- However, the rule fails for higher spin particles like chiral gravitini.

Summary

- Began by looking at the thermal helicity
- Related it to the sphere partition function and then to anomaly-induced term in hydrodynamics
- Argued that it is a polynomial - in various examples it is actually obtained by simple substitutions in the anomaly polynomial.
- Worked out in detail the example of a Cardy CFT
- Similar story for Chiral Magnetic coefficient as magnetisation helicity of thermal state $\left\langle \vec{\mathfrak{M}}.\vec{\mathcal{P}} \right\rangle$.

Questions

- Does this relation to anomaly polynomial mean that thermal helicity is not renormalised ?
- How does it behave away from equilibrium ? A useful P/CP Odd observeable ?
- Is there a general proof in holography that thermal helicity of BHs do not get corrected ?
- A general dictionary between thermal angular momentum observeables and non-dissipative hydrodynamic coefficients ? Constructing more CP-odd observeables from angular momentum ?

AMPT results on charge azimuthal correlations at RHIC and LHC

Guo-Liang Ma

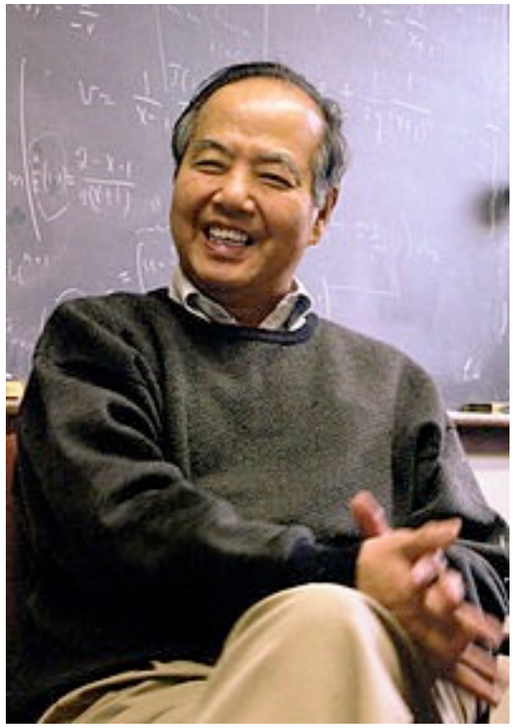
Shanghai Institute of Applied Physics,
Chinese Academy of Sciences

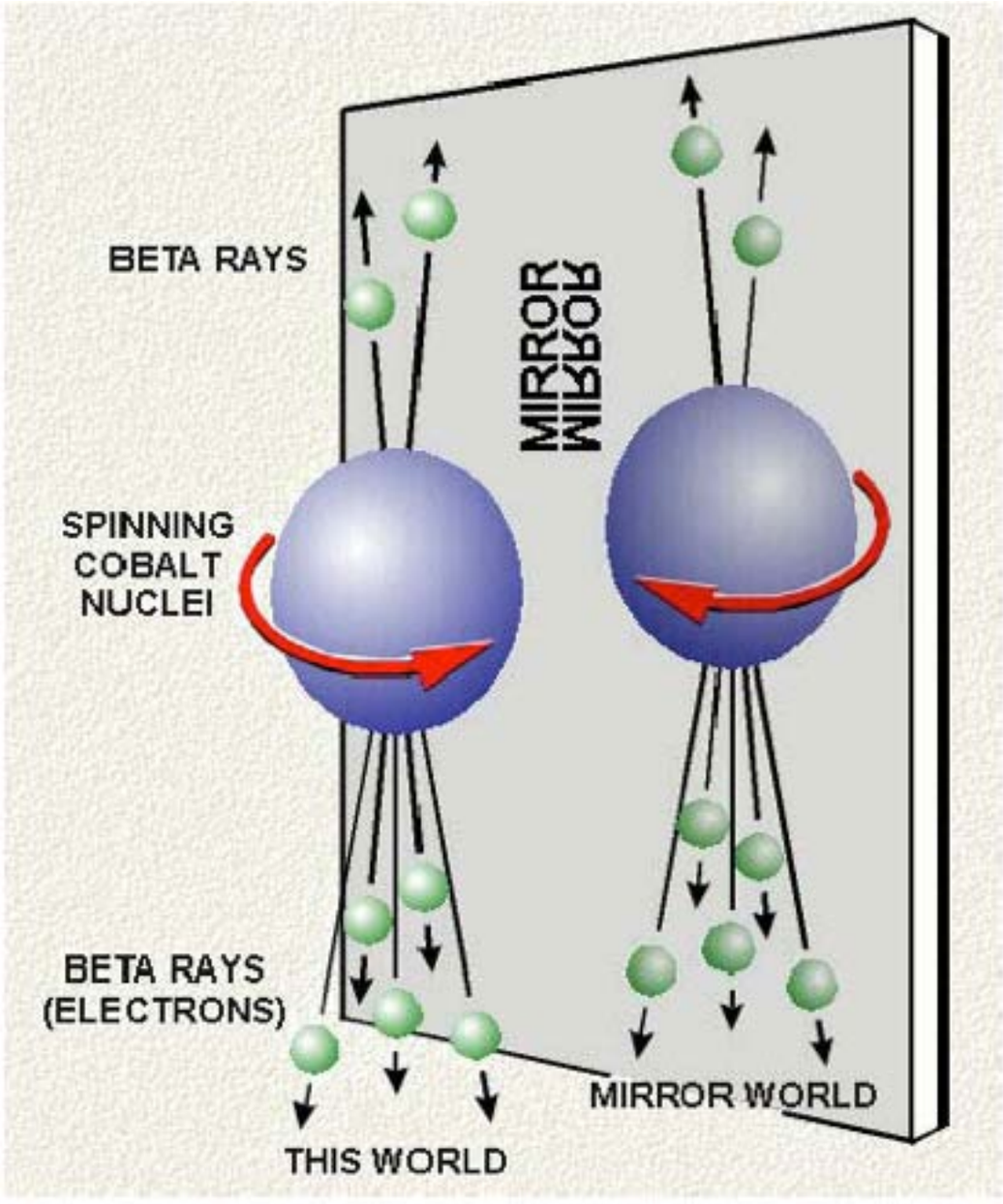
This work is in collaboration with Dr. Bin Zhang (ASU).

Outline

- Introduction
- Charge correlations at top RHIC energy
- Charge correlations at low RHIC energies
- Charge correlations at LHC energy
- Summary

Parity Violation in Weak Interactions

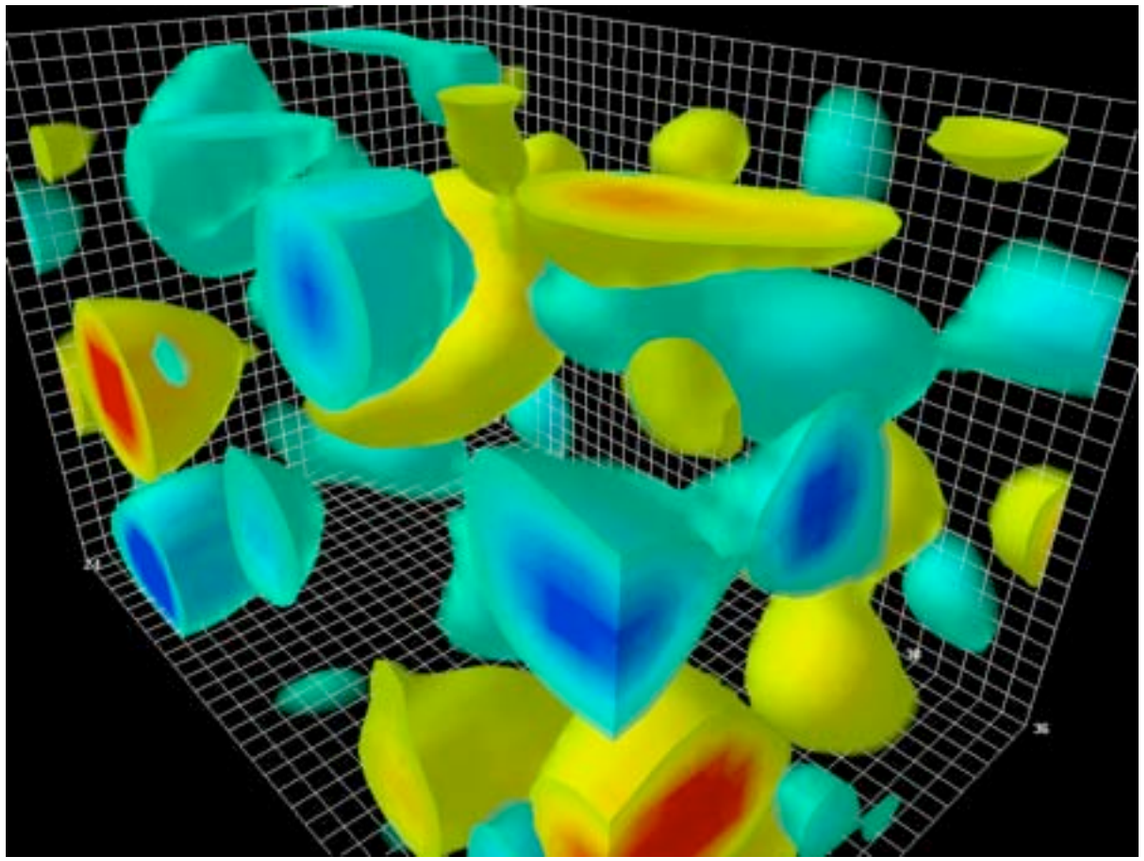


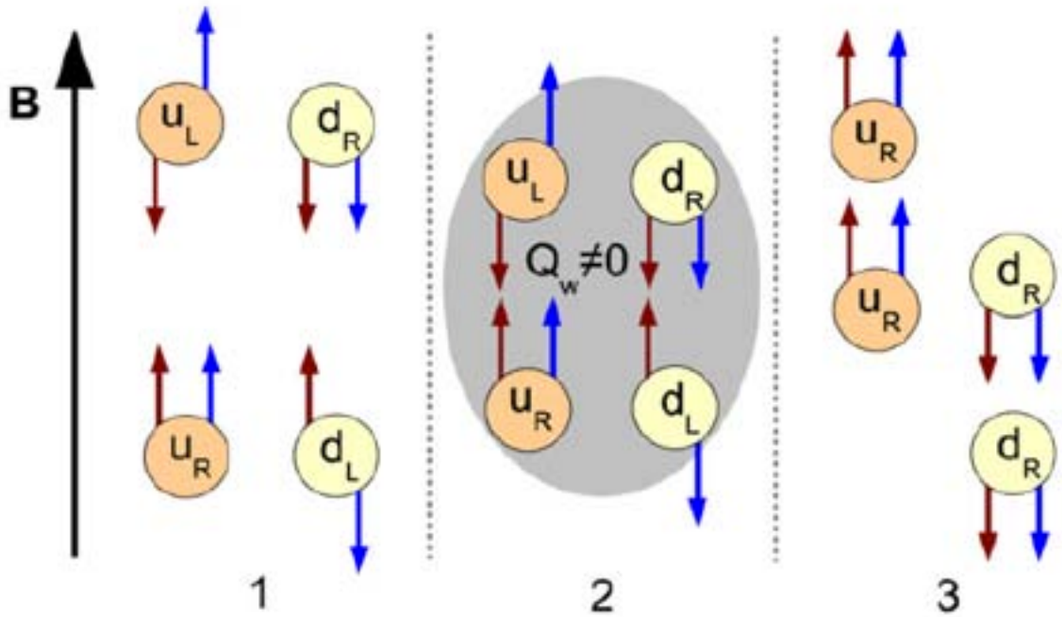


C.S. Wu, 1912-1997

- Lee and Yang won Nobel Prize in 1957, because of prediction about parity violation in weak interactions and confirmation by Wu's Cobalt experiment.
- How about parity conservation in strong interactions?

PV in SI: Chiral Magnetic Effect





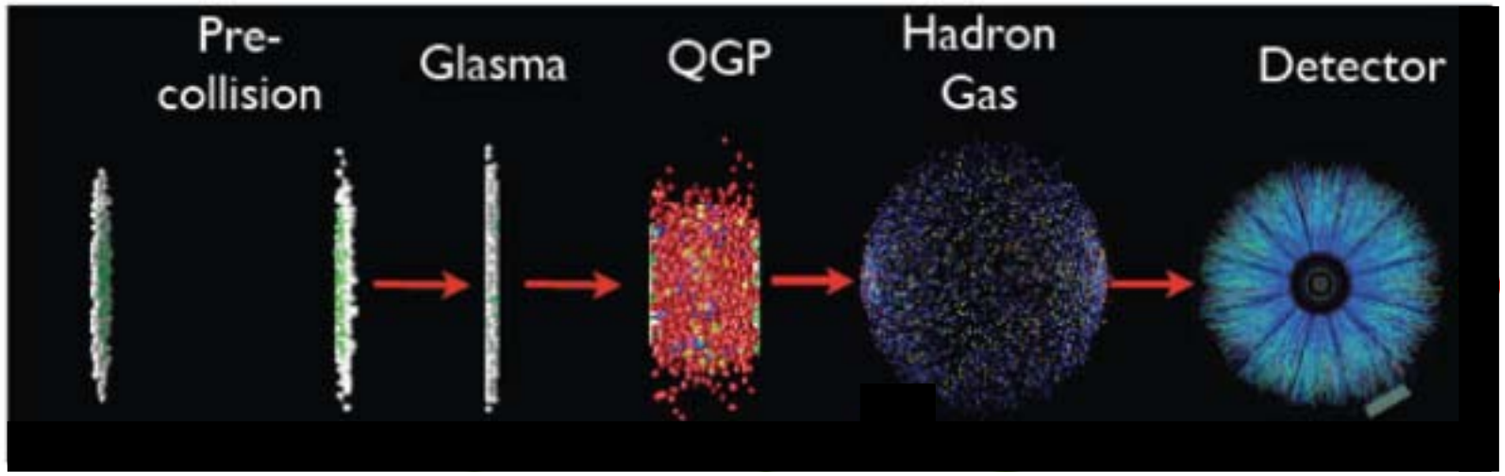
DE Kharzeev

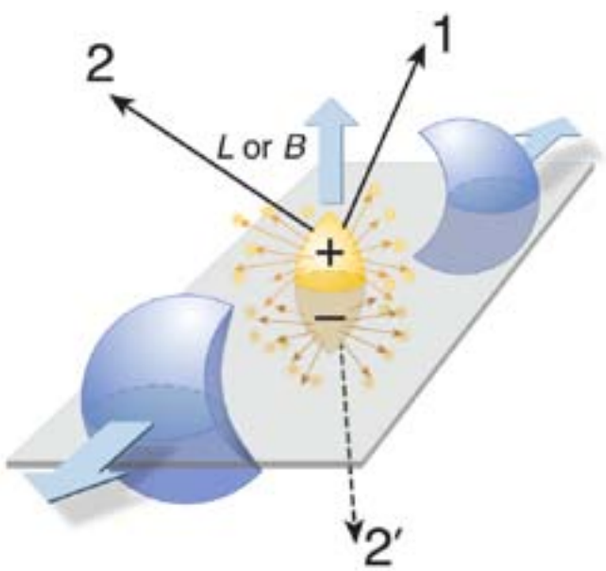
Red arrow - momentum; blue arrow - spin;
In the absence of topological charge no asymmetry between left and right (fig.1) ;the fluctuation of topological charge (fig.2) in the presence of magnetic field induces electric current (fig.3)

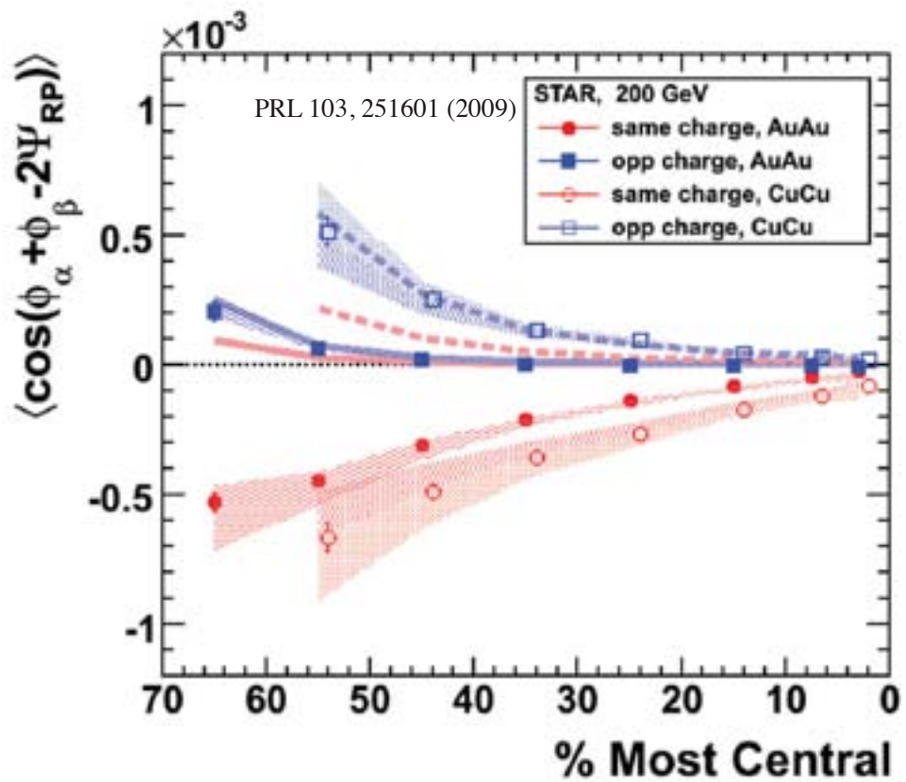
- Initial fluctuations of topological charge in QCD vacuum → P and CP odd metastable domains → Charge separation in the direction of magnetic field → CME indicates that parity can be locally violated in strong interactions.

How to observe CME experimentally?

Exp. observable: charge azimuthal correlation







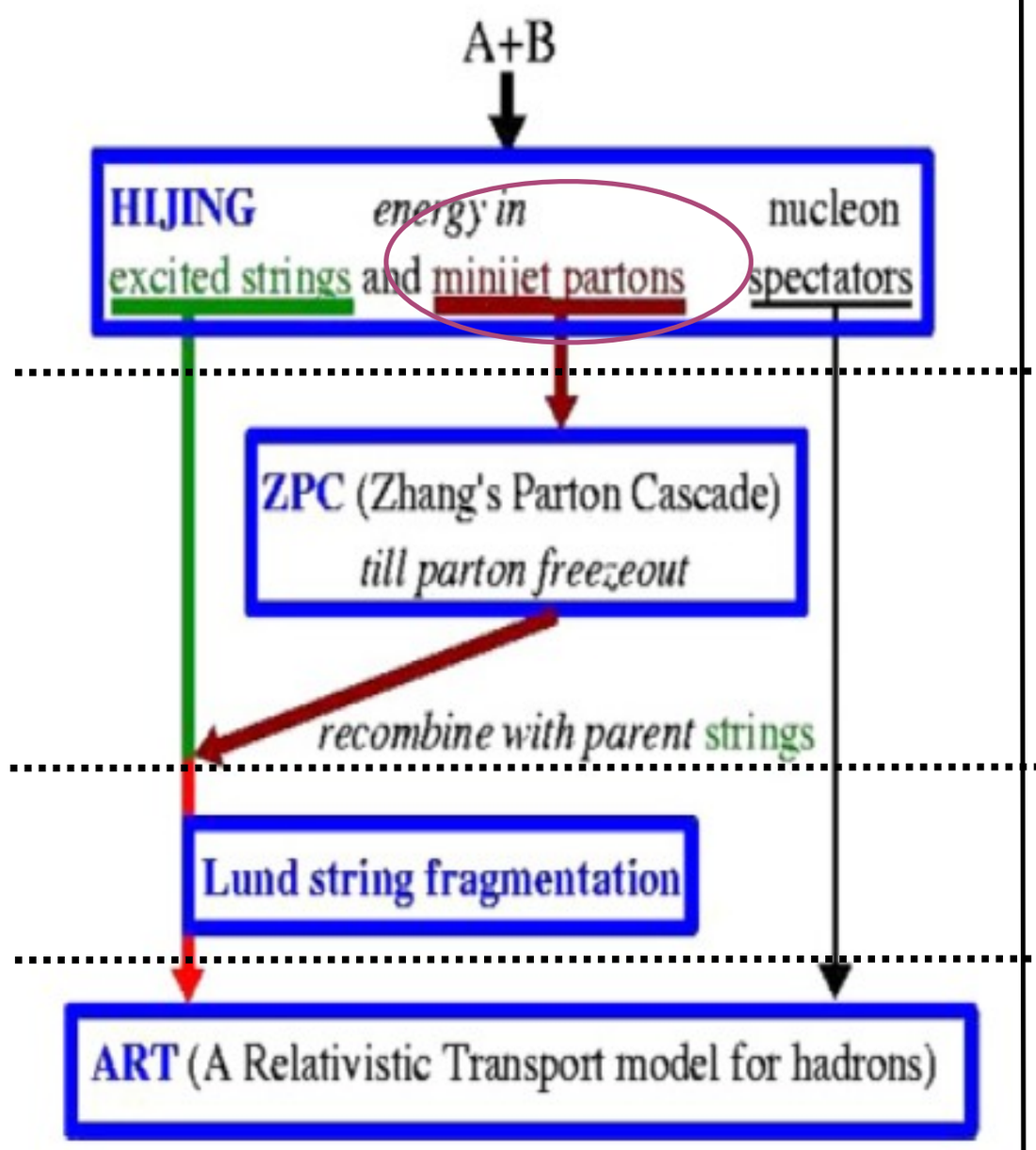
•RHIC data may indicate charges could be distributed asymmetrically w.r.t reaction plane, i.e. charge separation.

Can initial charge separation survive from final strong interactions?

AMPT model introduction

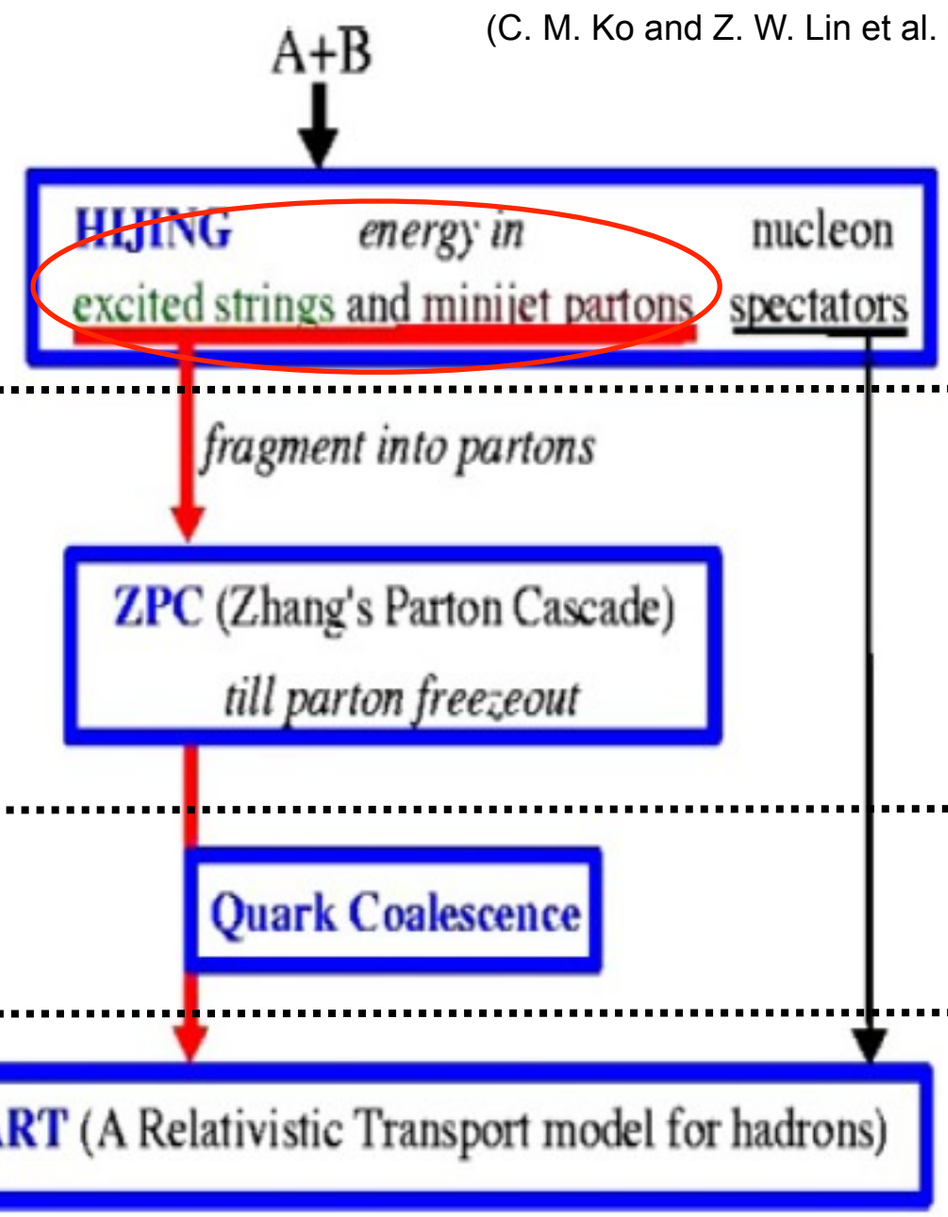
a multi-phase transport model

(1) Default AMPT Model



(2) Melting AMPT Model

(C. M. Ko and Z. W. Lin et al. PRC 72, 064901 (2005))



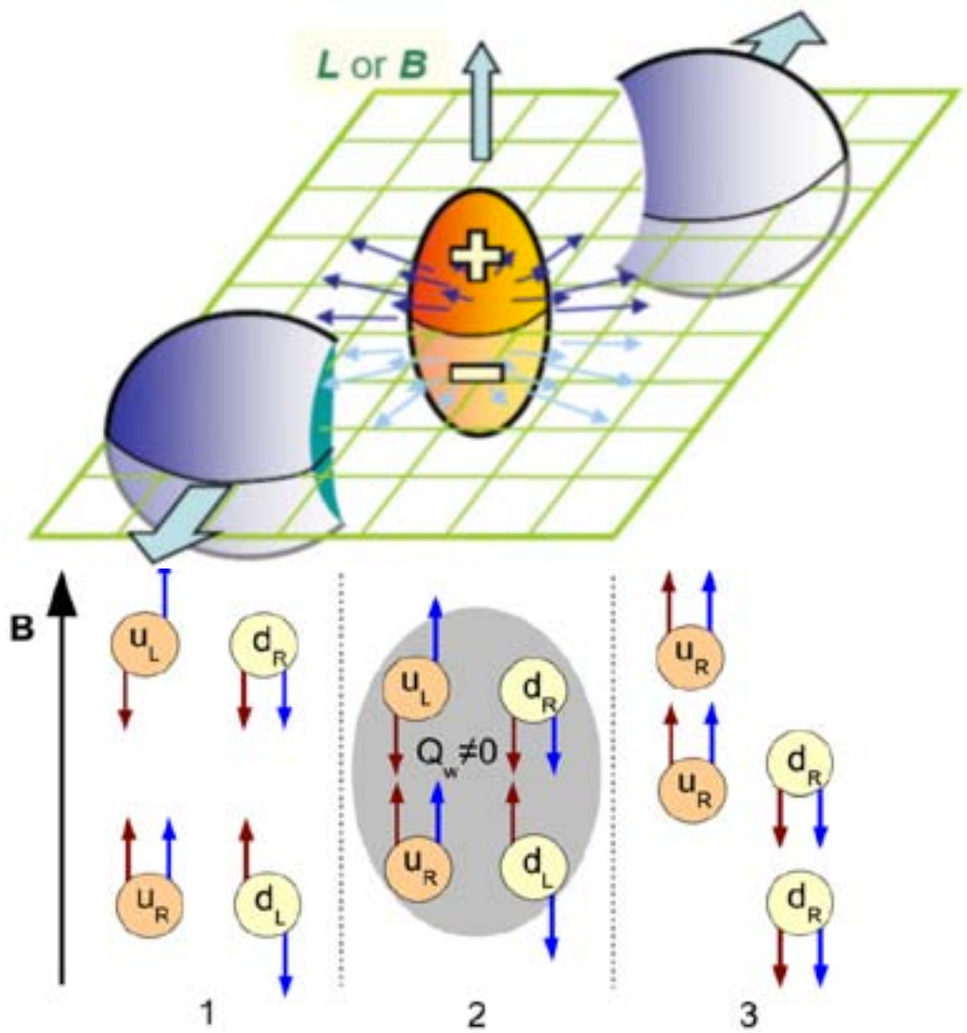
(1) initial condition

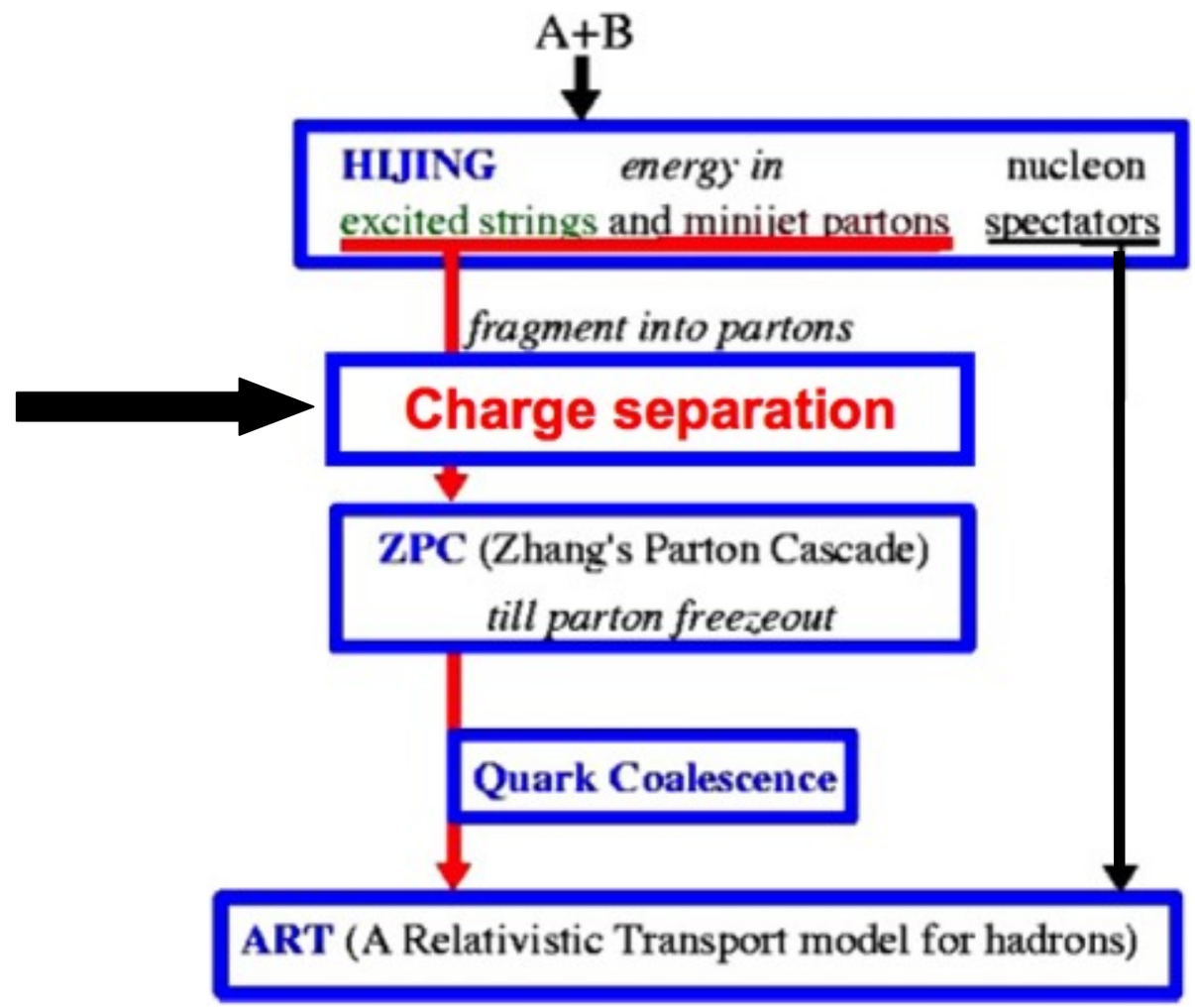
(2) parton cascade

(3) hadroni-zation

(4) hadronic rescattering

How to study charge separation by AMPT model





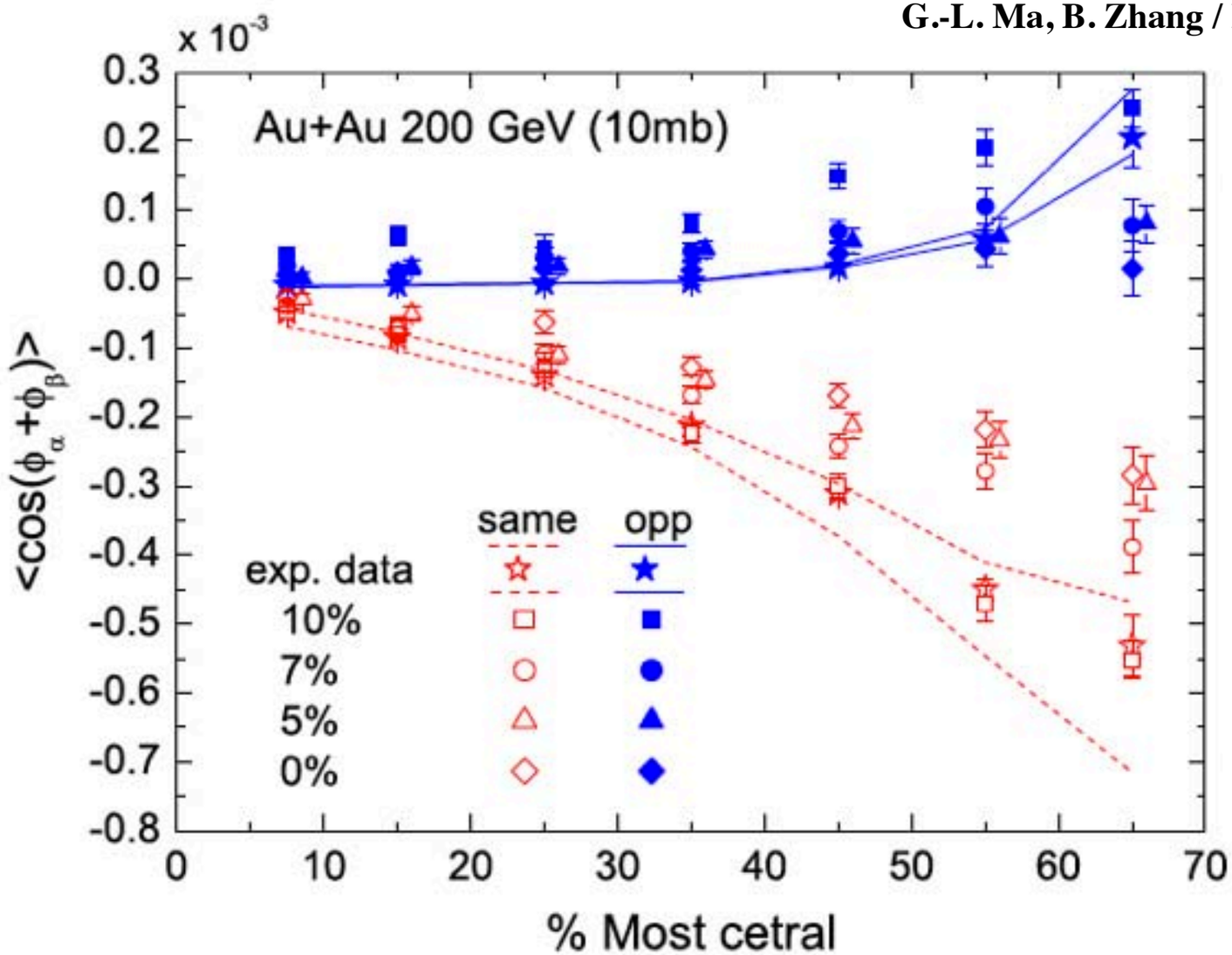
- **We include initial charge separation mechanism into AMPT model.**

We switch the p_y values of a fraction of the downward moving u quarks with those of the upward moving u-bar quarks, and likewise for d-bar and d quarks.

- **We will focus on final interaction effects on the charge separation, including parton cascade, hadronization, resonance decays.**
- **Resonance decays are implemented to ensure charge conservation.**

AMPT results about <cos(φα+φβ)>

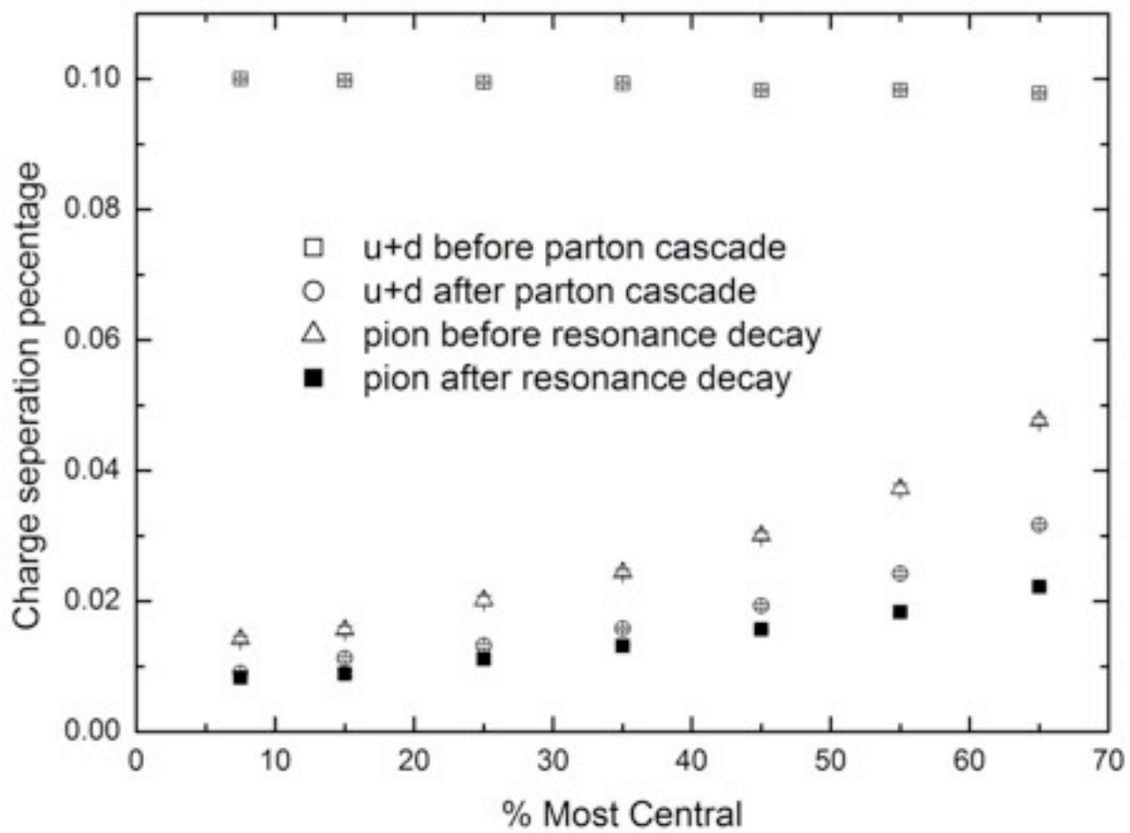
G.-L. Ma, B. Zhang / Physics Letters B 700 (2011) 39–43

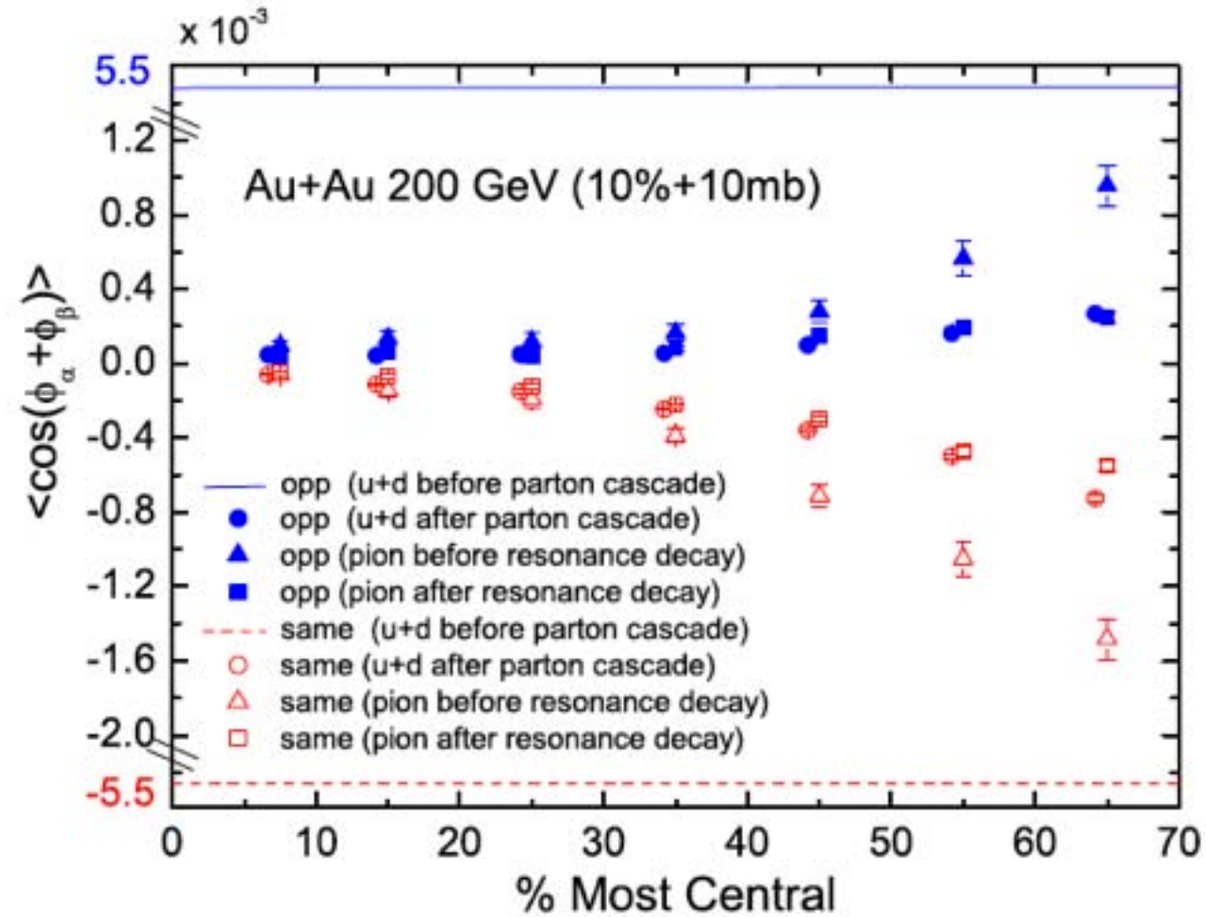


- For same-charge, 10% initial charge separation can describe data.
- For opposite-charge, initial charge separation is not necessary for all centralities except 60-70%.
- For centrality of 60%-70%, 10% initial charge separation can describe both same-charge and opposite charge.

It is challenging to observe an initial charge separation of <5% in the presence of strong final state interactions.

Final state effects on <cos(φα+φβ)>

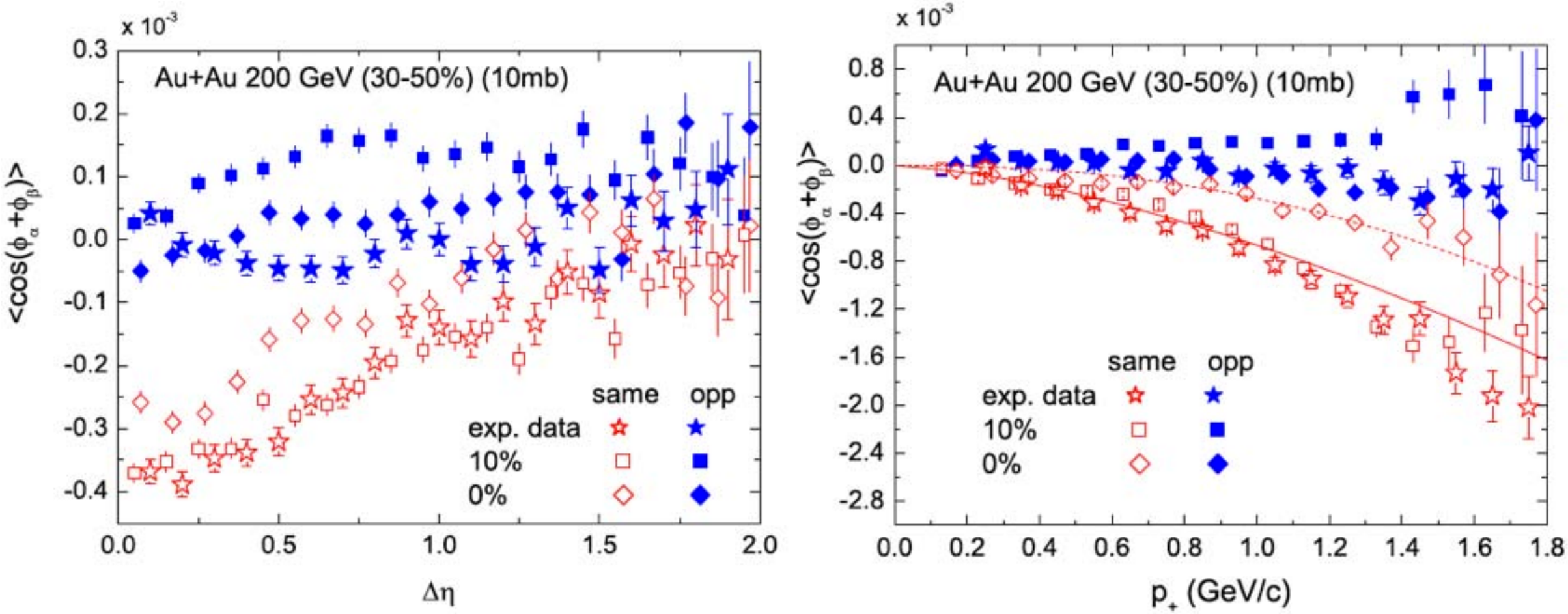




- Parton cascade reduces charge separation significantly.
- Coalescence recovers some charge separation in part because it reduces the number of particles after combining quarks into hadrons.
- Resonance decays reduce charge separation.

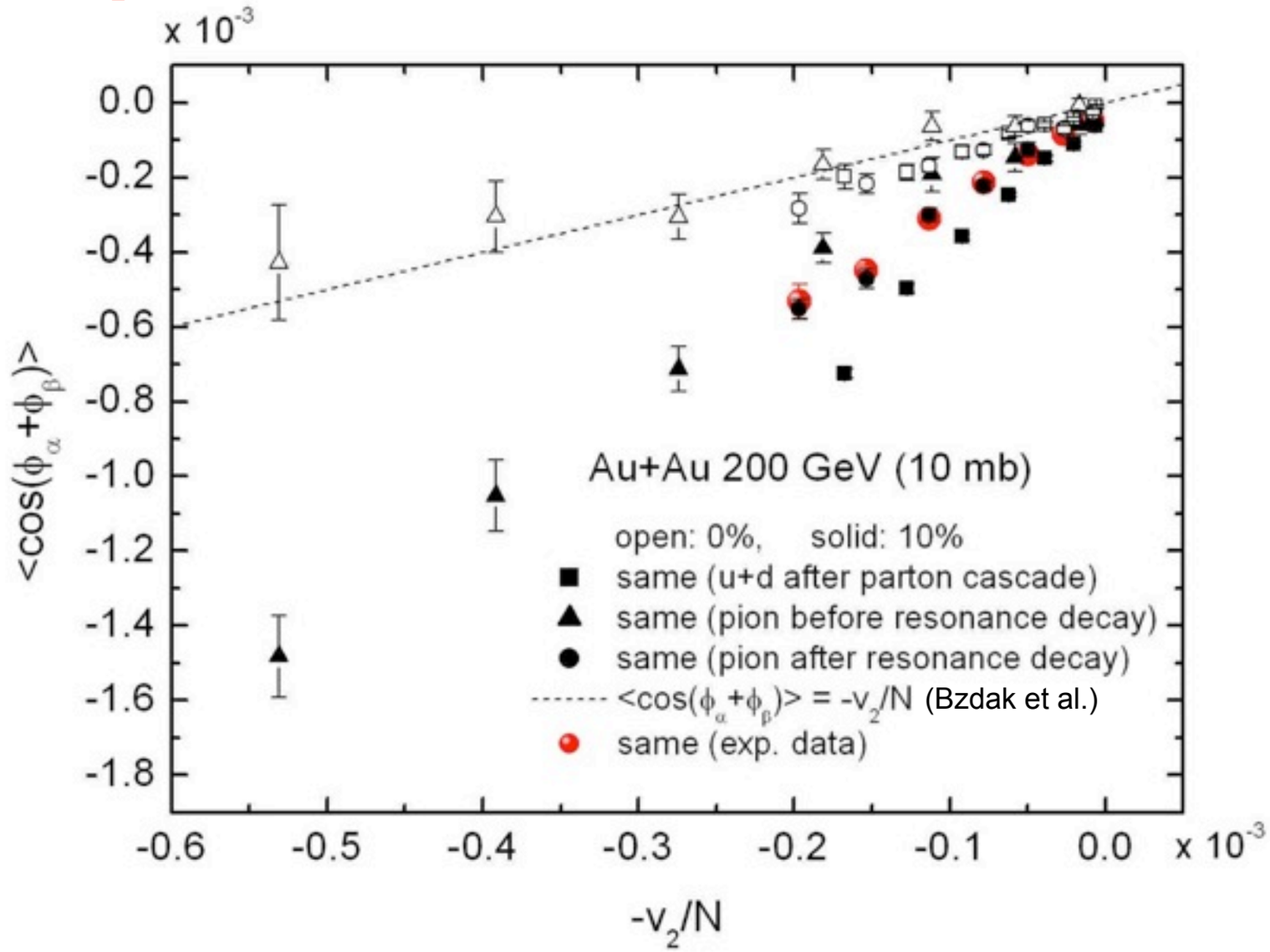
From a percentage of charge separation of 10% in the beginning, only 1-2% percentage remains at the end.

$\Delta\eta$ and p_T dependences of $<\cos(\varphi_\alpha+\varphi_\beta)>$



- AMPT results with initial charge separation can well describe same-charge data.
- AMPT results without initial charge separation are consistent with the expectation from transverse momentum conservation [dashed: $<\cos(\varphi_\alpha+\varphi_\beta)> \propto p_+^n$ (n=2~3)].

Charge separation vs trans. mom. conservation



- AMPT results without initial charge separation are consistent with the expectation of transverse momentum conservation [dashed: $\langle\cos(\varphi_\alpha+\varphi_\beta)\rangle=-v_2/N$].

- Transverse momentum conservation can partly account for data, therefore initial charge separation or other mechanisms are needed.

AMPT results about <cos(φα-φβ)>

- AMPT gives the same trends as data.
- Initial charge separation is not enough to make up for the large difference between AMPT and data.
- Other mechanisms?

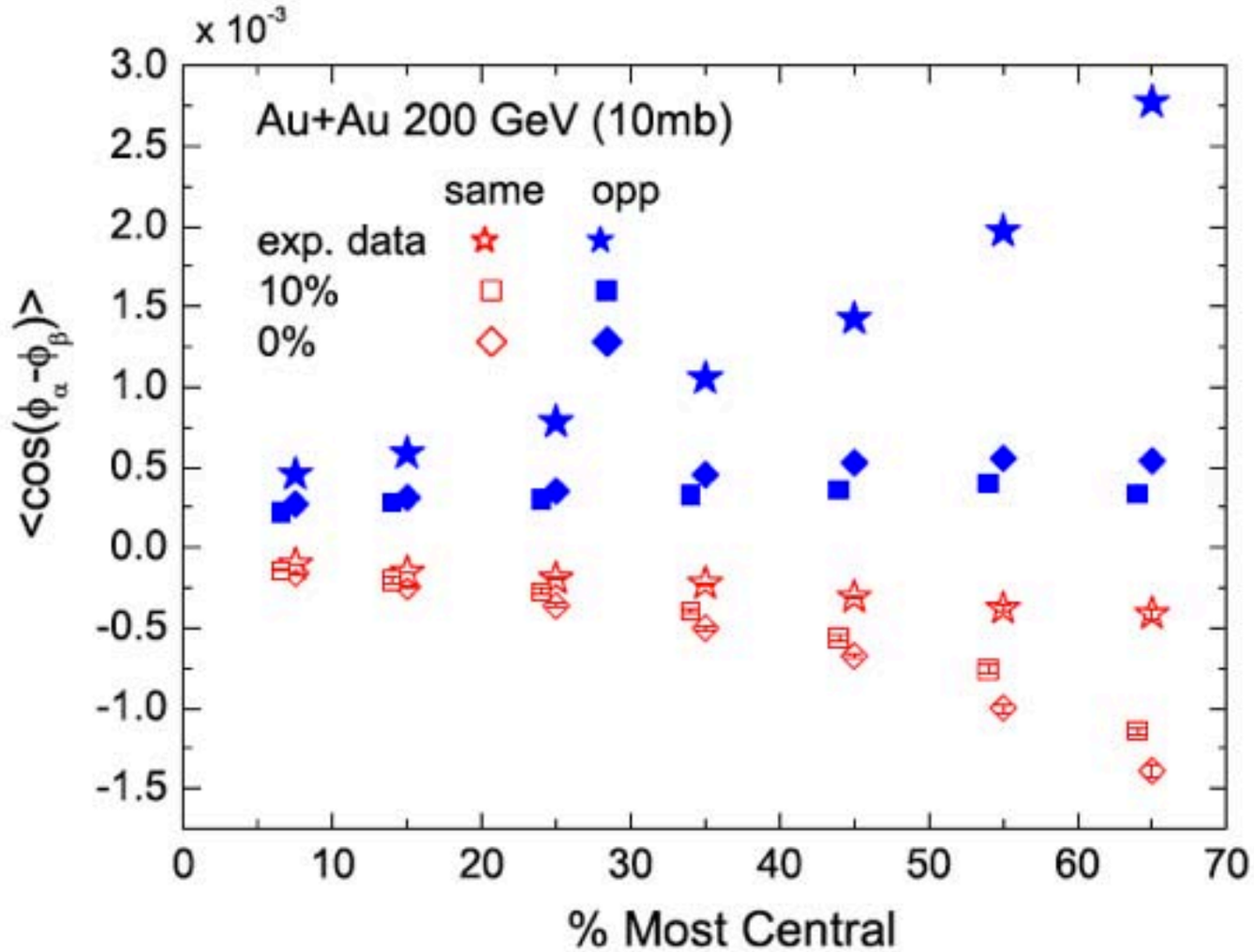
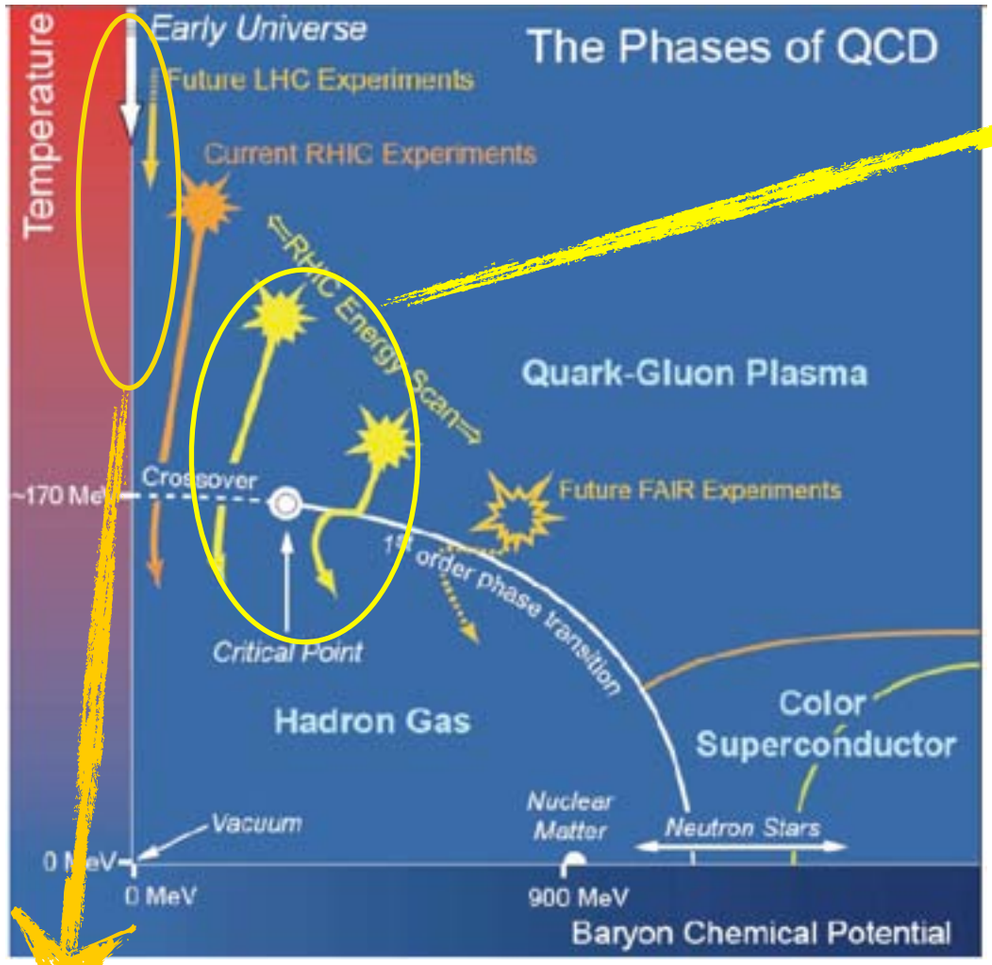


TABLE I. Estimated contributions to azimuthal correlations from various effects and comparison with data. The DATA are from the STAR measurement for AuAu 200-GeV collisions at ~50%–60% centrality.

Bzdak et. al., PRC **83**, 014905 (2011)

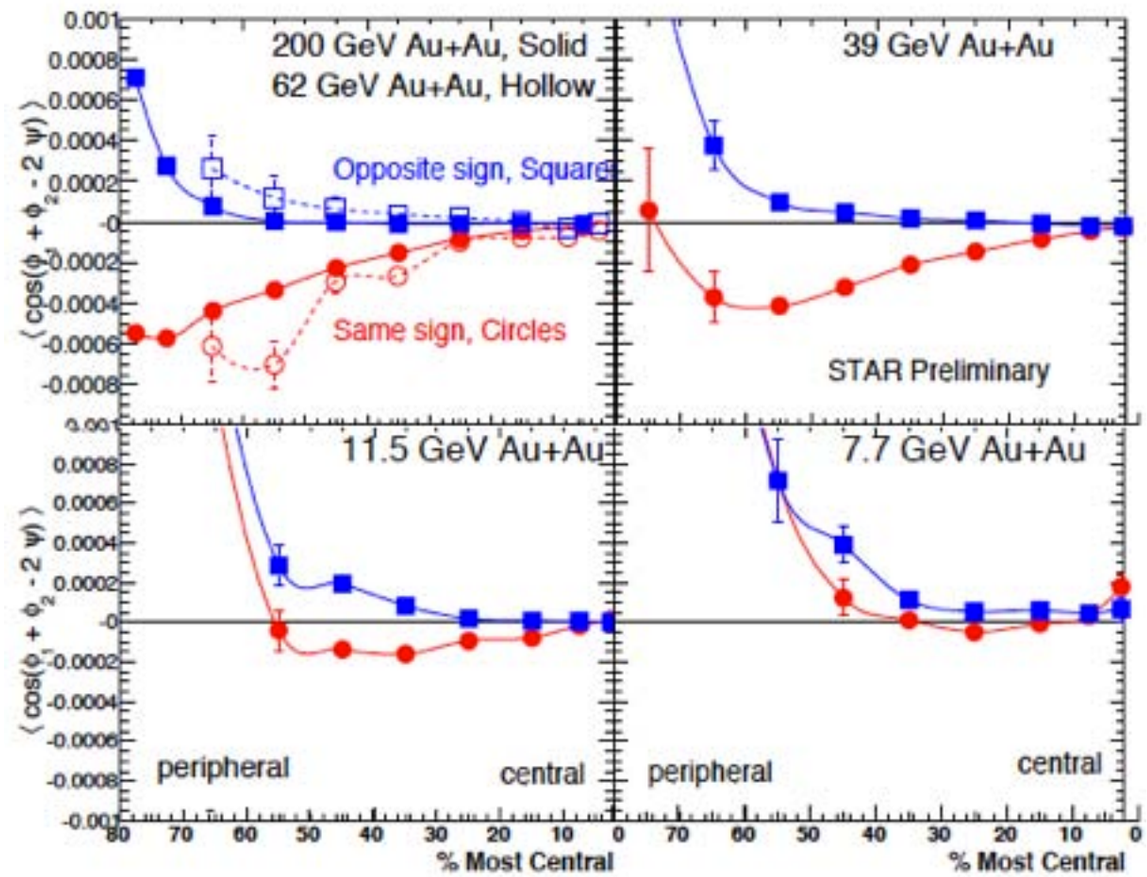
$\hat{O} \times 10^3$	$\langle\cos(\phi_1+\phi_2)\rangle_{++}$	$\langle\cos(\phi_1+\phi_2)\rangle_{+-}$	$\langle\cos(\phi_1-\phi_2)\rangle_{++}$	$\langle\cos(\phi_1-\phi_2)\rangle_{+-}$
CME	$-(0.1-1)$	$+(0.01-0.1)$	$+(0.1-1)$	$-(0.01-0.1)$
LCC	~ 0	$+(0.1-1)$	~ 0	$+(1-10)$
TMC	~ -0.1	~ -0.1	~ -1	~ -1
DATA	-0.45	$+0.06$	-0.38	$+1.97$

NEW DATA：two energy directions

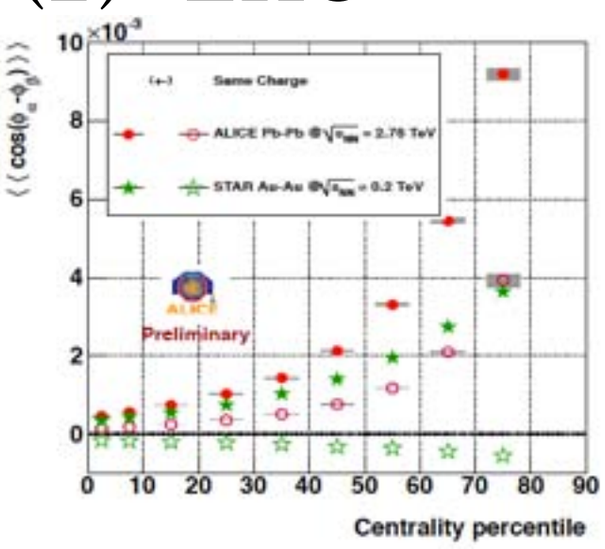


（1） RHIC

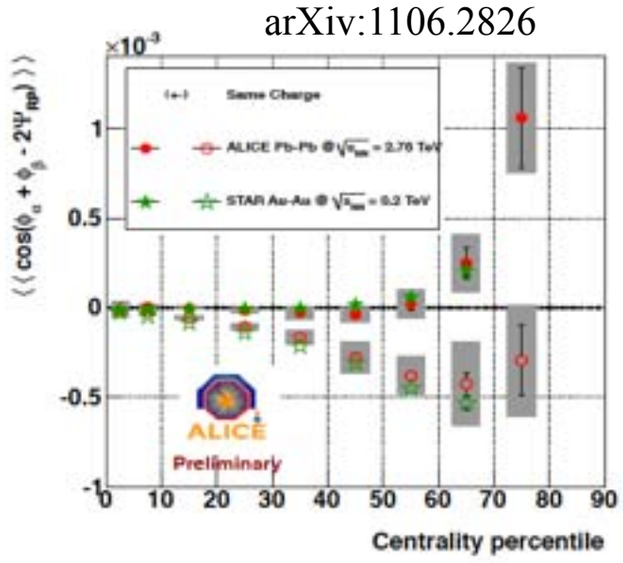
arXiv:1106.5902



（2） LHC

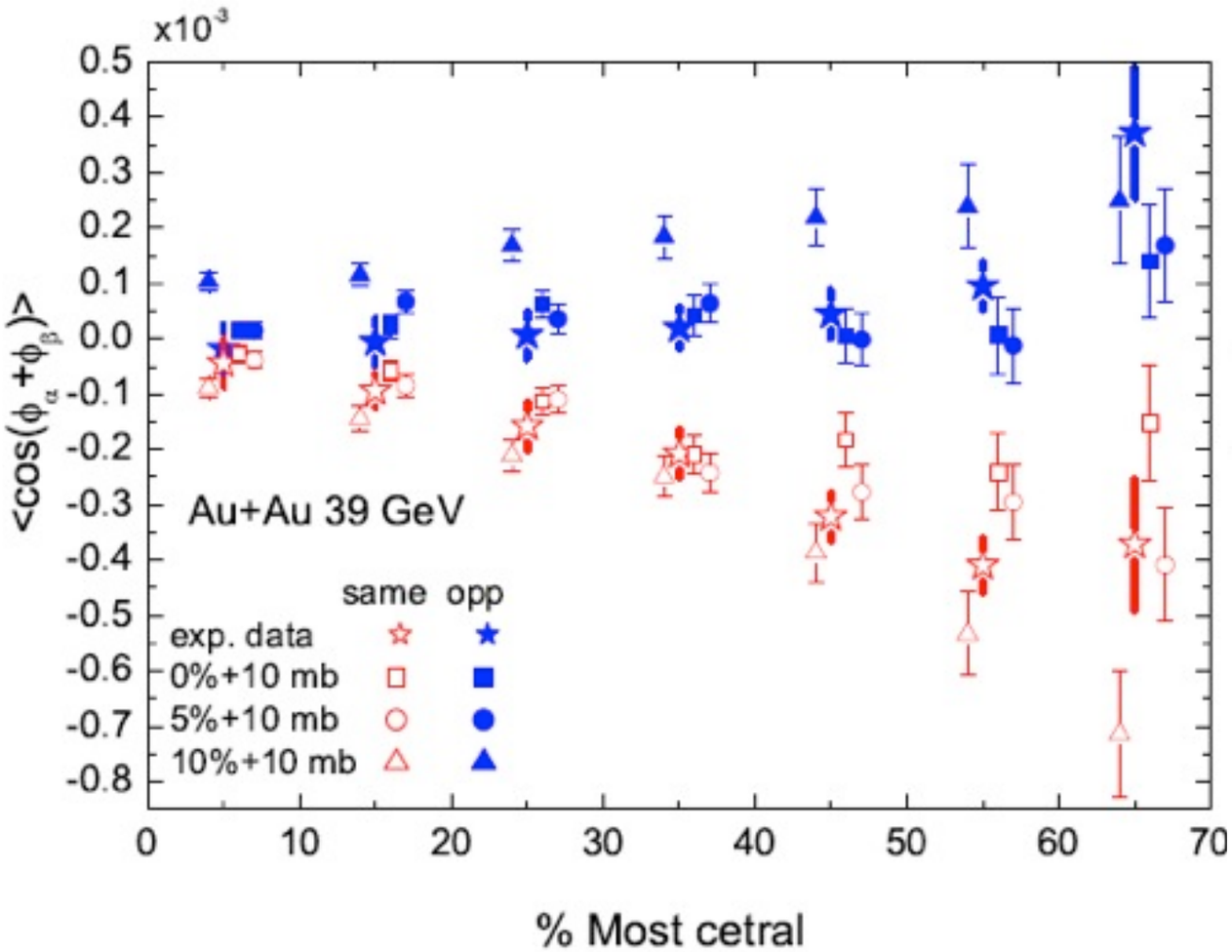


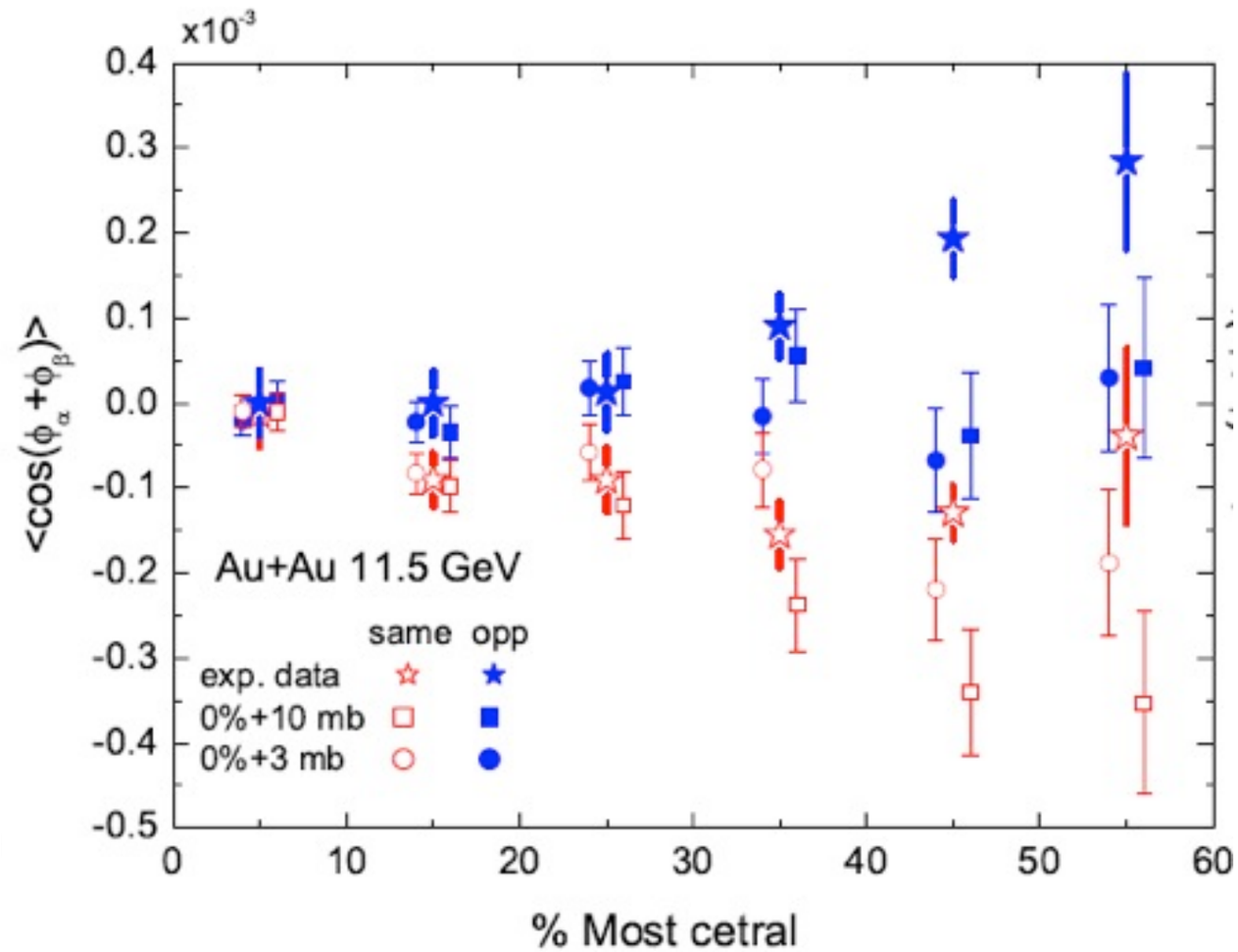
arXiv:1106.2826



- With decreasing of RHIC energy, same- and opposite-charge correlations become closer and positive.
- $\langle\cos(\varphi_\alpha+\varphi_\beta)\rangle$ at LHC is consistent with that at RHIC, but $\langle\cos(\varphi_\alpha-\varphi_\beta)\rangle$ at LHC is different from that at RHIC。
- The new exp. data require model explanations.

<cos(φα+φβ)> at low RHIC energies

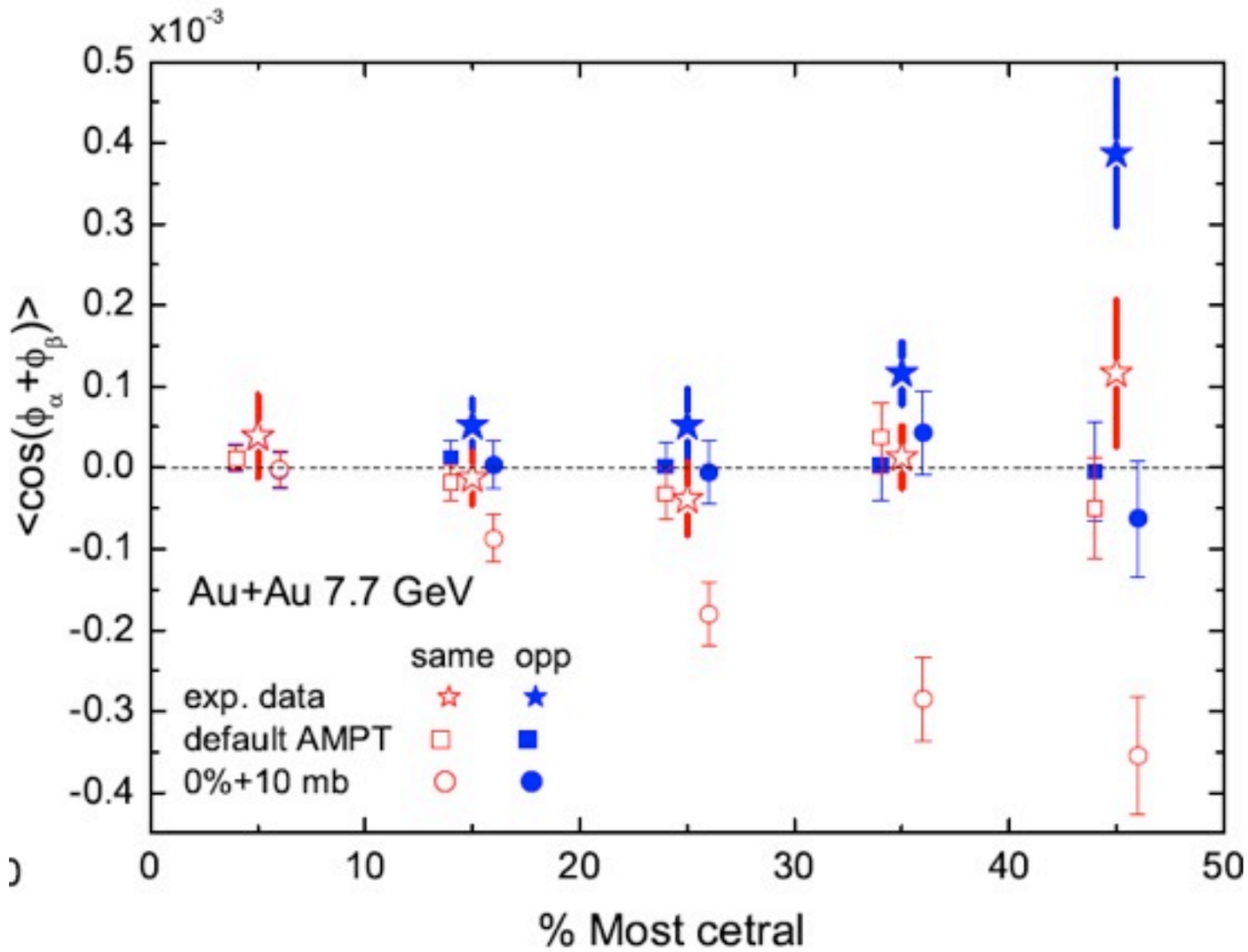




- The percentage of initial charge separation decreases from ~10% for 200 GeV , ~5% for 39 GeV (circles), to ~0% for 11.5 GeV(circles).

Initial charge separation effect decreases with the decreasing of energy.

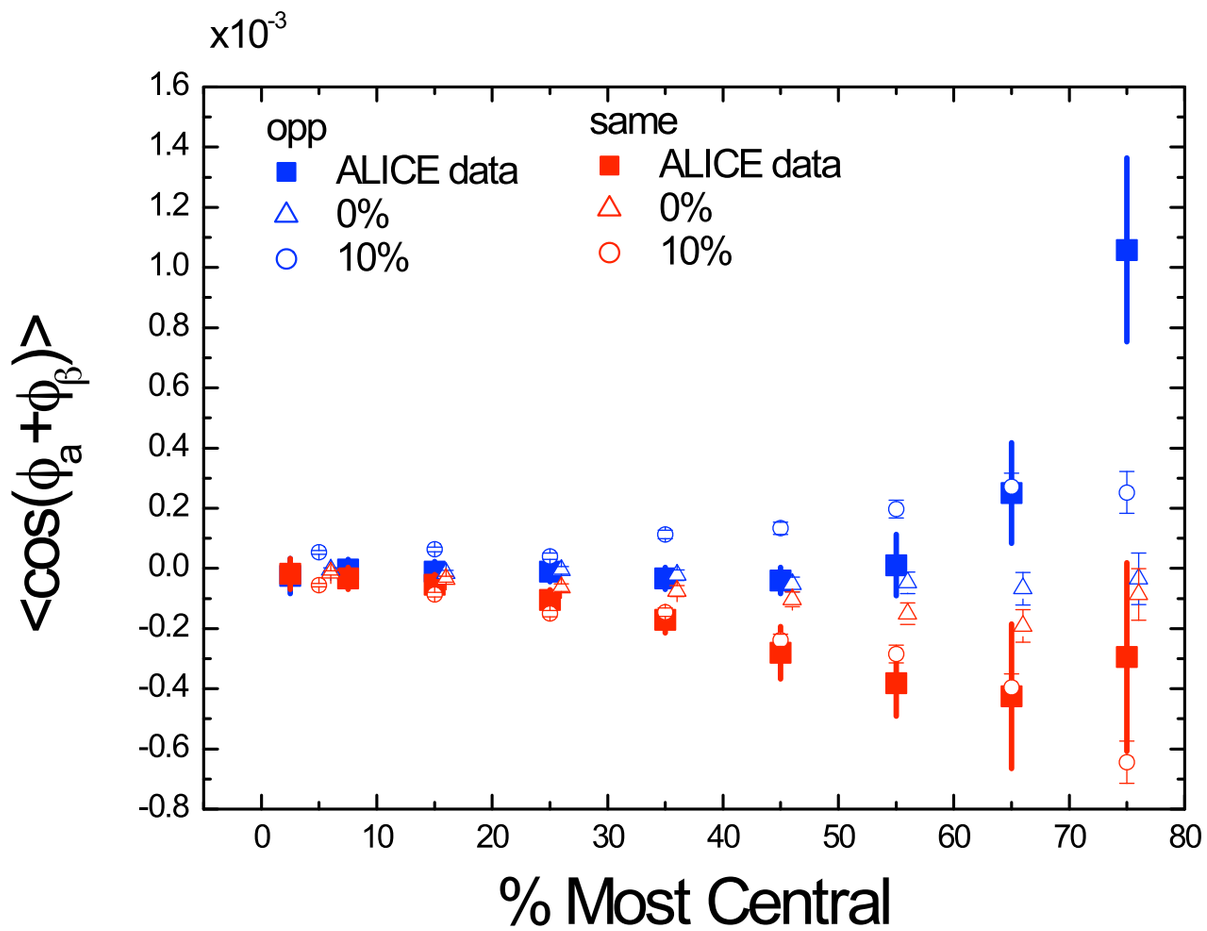
<cos(φα+φβ)> for Au+Au 7.7 GeV

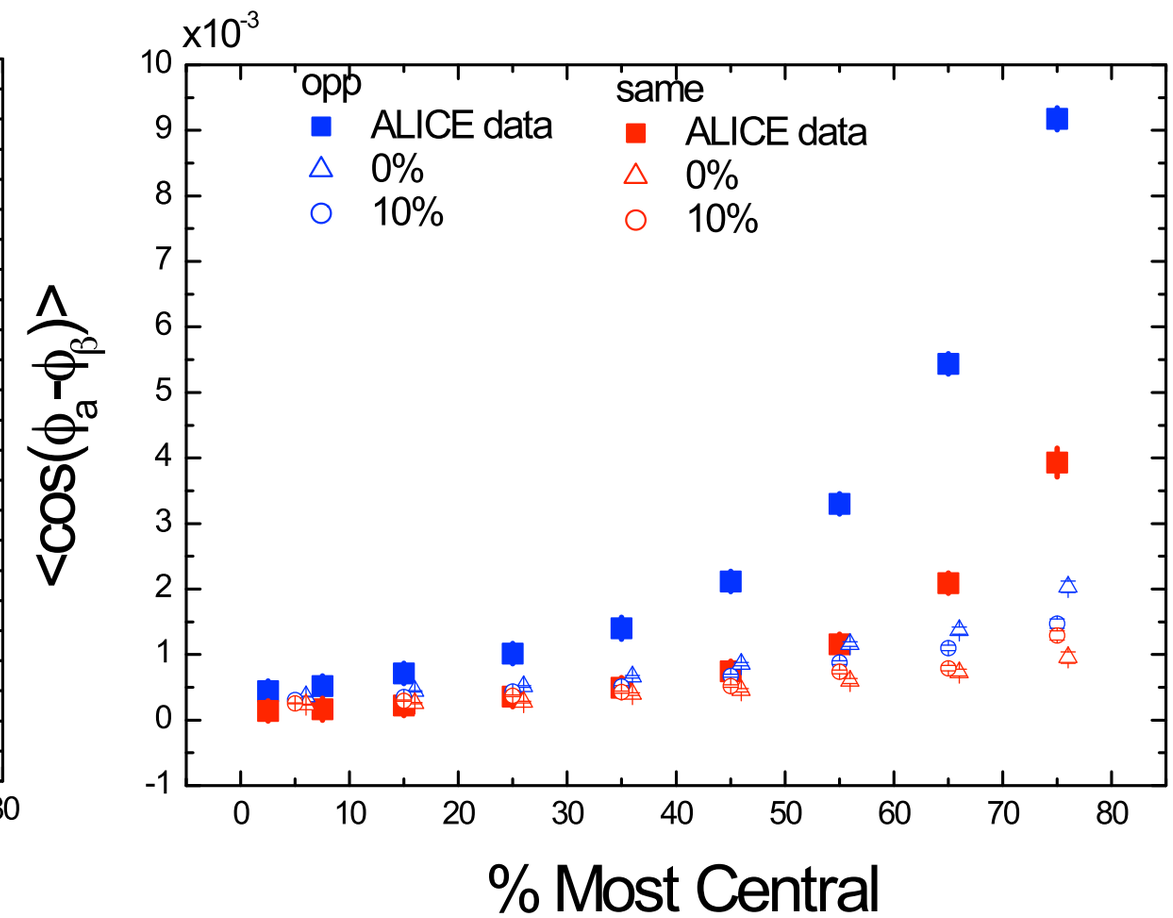


- The partonic interaction cross section decreases from ~10mb for 200 GeV, ~3mb for 11.5 GeV, to no partonic but hadronic interactions only for 7.7 GeV.

The partonic degree of freedom decreases with the decreasing of energy.

<cos(φα±φβ)> at LHC energy





- For same-charge <cos(φα+φβ)>, 10% initial charge separation can describe LHC data.
- For opposite-charge <cos(φα+φβ)>, initial charge separation is not necessary for all centralities except 60-70% and 70-80%.
- For <cos(φα-φβ)>, we only give the experimental trends, fails for the magnitudes.

Initial charge separation at LHC seems similar as that at top RHIC energy.

Summary

- final interactions play an important role, which can reduce the charge separation from 10% in the initial state to 1-2% in the final state.
- The initial charge separation mechanism or other mechanisms are needed in order to describe data for top RHIC and LHC energies.
- Charge azimuthal correlation is a helpful observable to learn phase transition for RHIC energy scan program.
- However, much more studies are required to understand all of the RHIC and LHC data.

Magnetic catalysis and Quantum Hall Effect in Monolayer and Bilayer Graphene

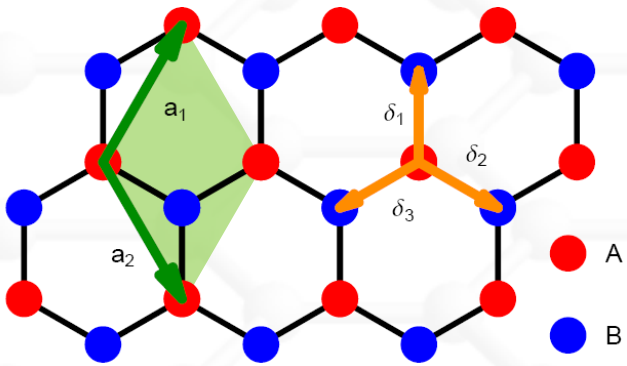
Volodya Miransky

Western University

Lattice in coordinate & reciprocal space

- Translation vectors

$$\mathbf{a}_1 = a\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), \quad \mathbf{a}_2 = a\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

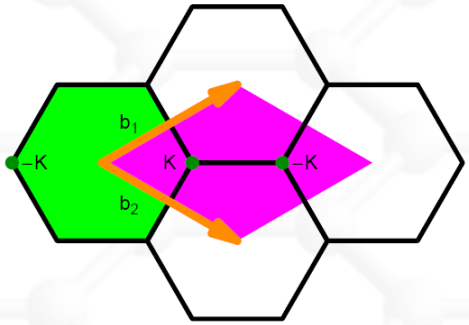


where a is the lattice constant

- Two carbon atoms per primitive cell

- Reciprocal lattice vectors

$$\mathbf{b}_1 = 2\pi/a(1, 1/\sqrt{3}), \; \mathbf{b}_2 = 2\pi/a(1, -1/\sqrt{3})$$



Tight binding model

- Strong covalent sigma-bonds between nearest neighbors (carbon atoms)
- Hamiltonian

$$H = -t \sum_{\mathbf{n},\boldsymbol{\delta}_i,s} \left[a^\dagger_{\mathbf{n},s} \exp\left(\frac{ie}{\hbar c} \boldsymbol{\delta}_i \mathbf{A} \right) b_{\mathbf{n}+\boldsymbol{\delta},s} + \text{h.c.} \right]$$

where $a_{\mathbf{n},s}$ and $b_{\mathbf{n}+\delta,s}$ are the annihilation operators of electrons with spin $s=\uparrow,\downarrow$

- The nearest neighbor vectors are

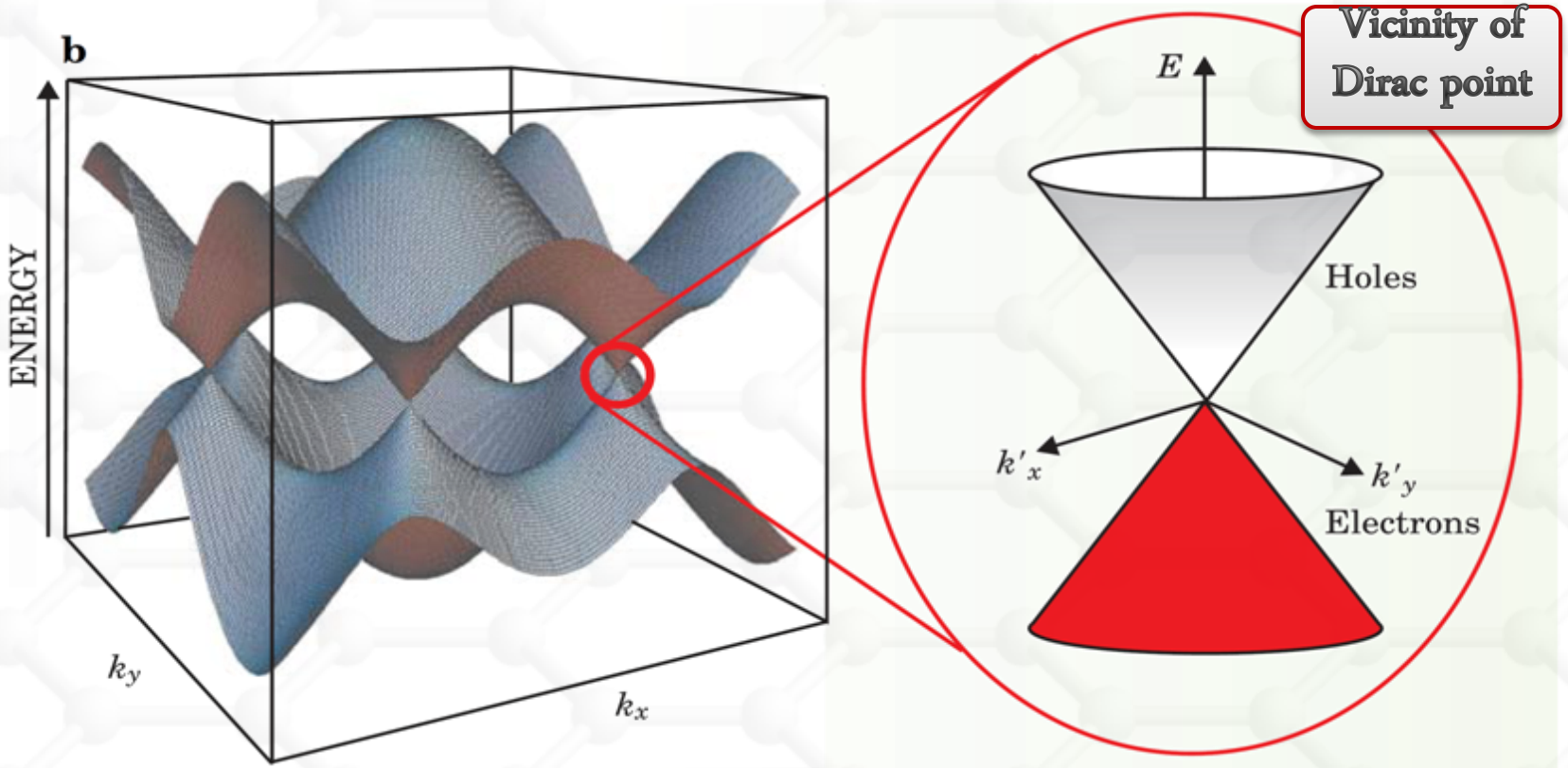
$$\boldsymbol{\delta}_1 = (\mathbf{a}_1 - \mathbf{a}_2)/3\,, \quad \boldsymbol{\delta}_2 = \mathbf{a}_1/3 + 2\mathbf{a}_2/3\,,$$
$$\boldsymbol{\delta}_3 = -\boldsymbol{\delta}_1 - \boldsymbol{\delta}_2 = -2\mathbf{a}_1/3 - \mathbf{a}_2/3$$

Low energy Dirac fermions

$$\mathcal{L} = \sum_{s=\pm 1} \bar{\Psi}_s(t,\mathbf{r})\left[i\gamma^0(\hbar\partial_t - i\mu_s) + i\hbar v_F \gamma^1 D_x + i\hbar v_F \gamma^2 D_y\right]\Psi_s(t,\mathbf{r})$$

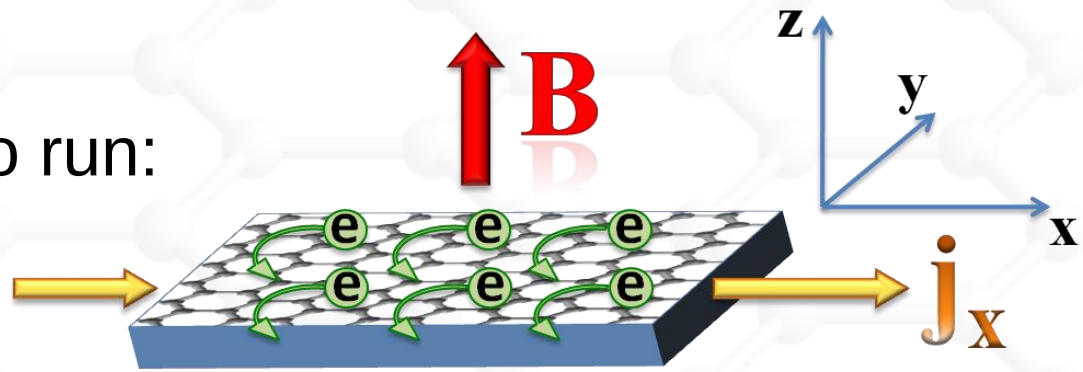
P. R. Wallace, Phys. Rev. **71,** 622 (1947)
G.W. Semenoff, Phys. Rev. Lett. **53**, 2449 (1984)

$$\Psi_s^T = (\psi_{KAs}, \psi_{KBs}, \psi_{K'Bs}, \psi_{K'As})$$

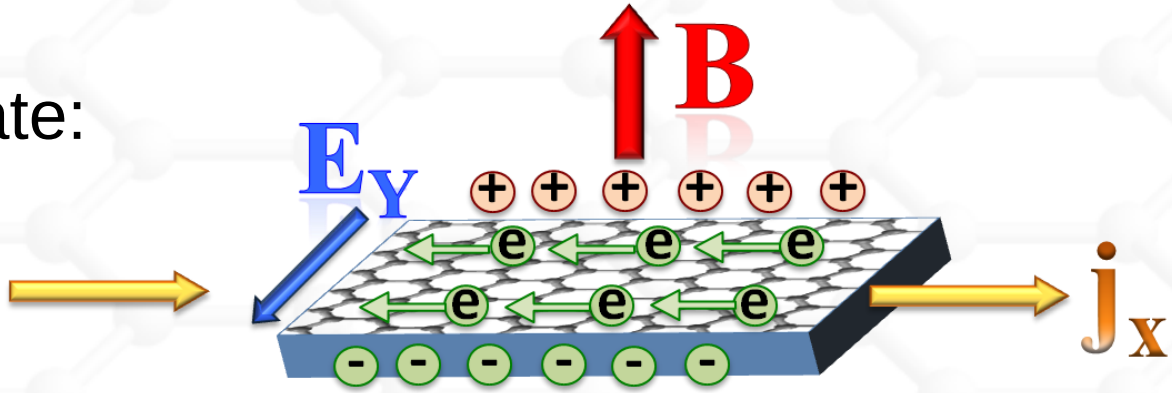


Quantum Hall effect

- General setup
  - Current starts to run:



  - Steady state:



Classical HE: $j_x = \sigma_{xx} E_x + \sigma_{xy} E_y, \quad j_y = \sigma_{yy} + \sigma_{yx} E_x = 0,$

$\sigma_{yx} = -\sigma_{xy}$

Quantum HE: $\sigma_{xx} = \sigma_{yy} = 0, \quad j_x = \sigma_{xy} E_y, \quad j_y = \sigma_{yx} E_x = 0 \;\Rightarrow\; E_x = 0,$

$\rho_{xx} = \rho_{yy} = 0 \quad \text{for} \quad \sigma_{xy} \neq 0$

Quantum Hall effect in graphene

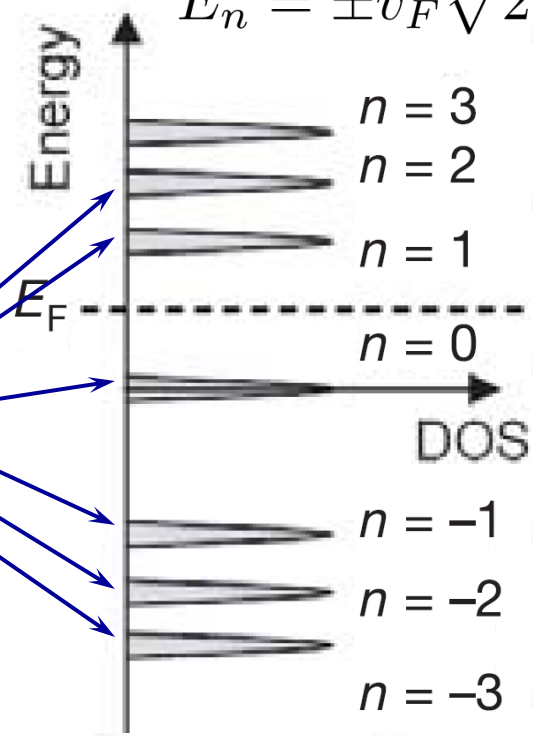
[1] Zheng & Ando, PRB **65**, 245420 (2002)
[2] Gusynin & Sharapov, PRL **95**, 146801 (2005)
[3] Peres, Guinea, & Castro Neto, PRB **73**, 125411 (2006)
[4] Novoselov et al., Nature **438**, 197 (2005)
[5] Zhang et al., Nature **438**, 201 (2005)

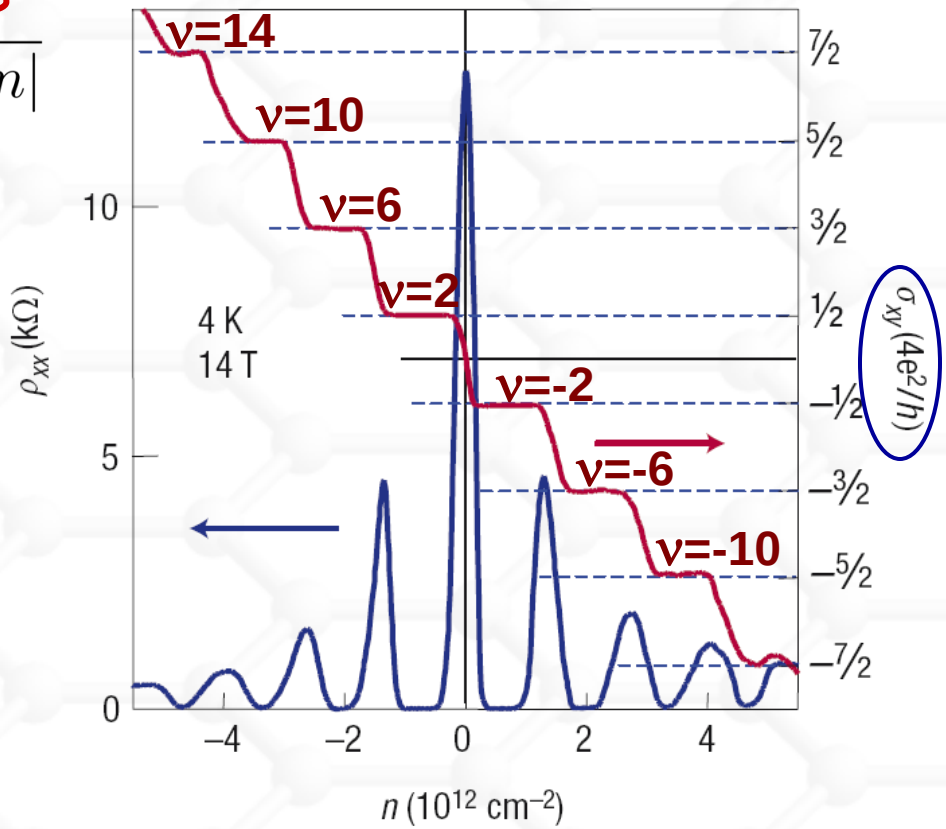
Experiment

$$\sigma_{xy} = \frac{\nu e^2}{h} = \frac{4e^2}{h}\left(n + \frac{1}{2}\right)$$

4–fold degenerate levels

$$E_n = \pm v_F \sqrt{2e\hbar B|n|}$$



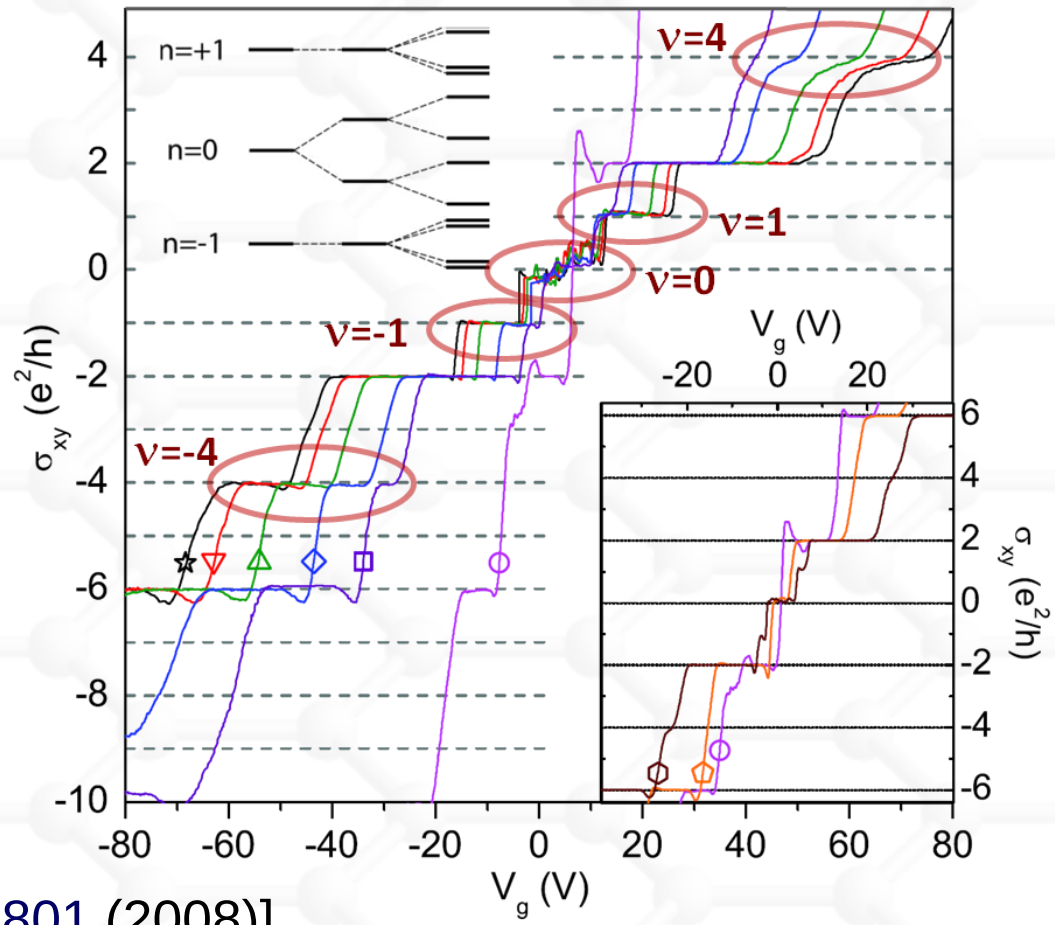


Anomalous QHE

- New plateaus are observed at

$$\nu = 0$$
$$\nu = \pm 1$$
$$\nu = \pm 3$$
$$\nu = \pm 4$$

Zhang et al., PRL **96,** 136806 (2006)



[Novoselov et al., Science **315**, 1379 (2007)]

[Abanin et al., PRL **98**, 196806 (2007)]

[Checkelsky et al., PRL 100, 206801 (2008)]

[Xu Du et al., Nature **462**, 192 (2009)]

Magnetic Catalysis

- Electron wave functions in magnetic field are localized [Gusynin, V.M., Shovkovy, PRL **73** (1994) 3499]
- Effective dimensional reduction

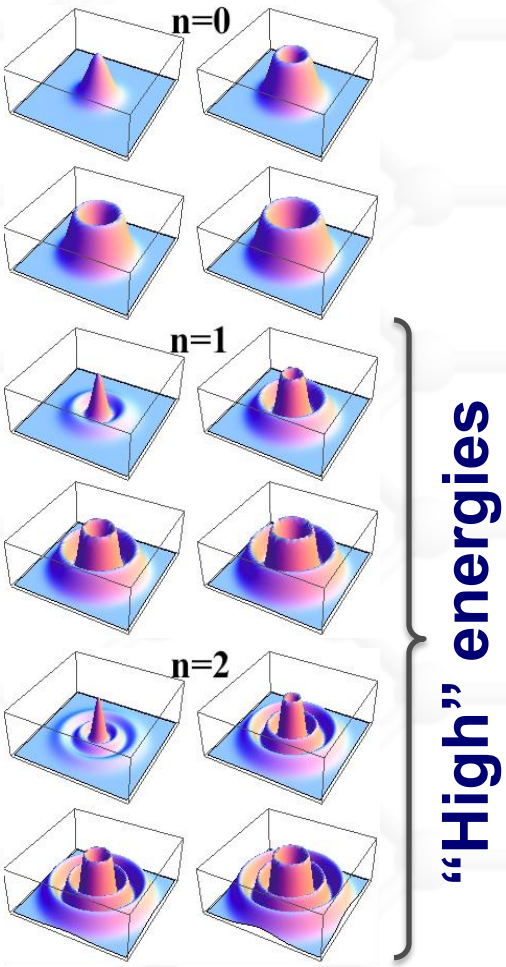
  D space directions ➡ ***D-2*** space directions
- Quantized energy levels:

$$E_n = \pm\sqrt{m^2c^4 + 2n\,\hbar c^2|eB|}$$

$$\simeq \pm\sqrt{2n\,\hbar c^2|eB|} \quad \text{for} \quad m \simeq 0$$

n = 0: zero energy

n ≥ 1 states have "high" energies



Spontaneous Chiral Symmetry Breaking

- At $m_0=0$, a dynamical Dirac mass m_{dyn} is generated

  2D: $m_{\rm dyn} \propto \alpha\sqrt{|eB|}$

  3D: $m_{\rm dyn} \propto \sqrt{|eB|}\; e^{\pi/[\alpha \ln(\alpha N_f)]}$

- This happens even at the *weakest* interaction ("catalysis")

- The phenomenon is *universal* (specific details of interaction are mostly irrelevant)

- Dimensional *reduction* is the key ingredient

Magnetic Catalysis in Graphene

- Charge carriers are Dirac fermions with $m=0$
- Theoretically, $m_{dyn} \neq 0$ must be generated in a sufficiently strong magnetic field

  [Gorbar, Gusynin, V.M., Shovkovy, PRB **66** (2002) 045108; Gusynin, V.M., Sharapov, Shovkovy, PRB **74** (2006) 195429]
- Possible complications:
  - several types of Dirac masses may exist in **2D**
  - competition with quantum Hall ferromagnetism (?)

    [Nomura and MacDonald, PRL **96** (2006) 256602]
  - nonzero electron/hole density
  - impurities, lattice defects, ripples, etc.

General Approach

Model Hamiltonian

[Gorbar, Gusynin, V. M., Shovkovy, Phys. Rev. B **78** (2008) 085437; Phys. Scr. **T146** (2012) 014018]

$$H = H_0 + H_C + \int d^2\mathbf{r}\,[\underbrace{\mu_B B \Psi^\dagger \sigma^3 \Psi}_{\textit{Zeeman term}} - \mu_0 \Psi^\dagger \Psi]$$

where

$$H_0 = v_F \int d^2\mathbf{r}\,\overline{\Psi}\left(\gamma^1 \pi_x + \gamma^2 \pi_y\right)\Psi,$$

is the Dirac Hamiltonian, and

$$H_C = \frac{1}{2}\int d^2\mathbf{r} d^2\mathbf{r}' \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r}) U_C(\mathbf{r}-\mathbf{r}')\Psi^\dagger(\mathbf{r}')\Psi(\mathbf{r}')$$

is the Coulomb interaction term.

Note that $\Psi_s^T = (\psi_{KAs}, \psi_{KBs}, \psi_{K'Bs}, \psi_{K'As}), \quad s = \pm$

Spin index

$v_F \approx 10^6$ m/s

Symmetry

- The Hamiltonian $H = H_0 + H_C$ possesses "flavor" *U(4)* symmetry (no Zeeman term)
- 16 generators read (*spin* ⊗ *valley*)

$$\frac{\sigma^\alpha}{2} \otimes I_4, \quad \frac{\sigma^\alpha}{2i} \otimes \gamma_3, \quad \frac{\sigma^\alpha}{2} \otimes \gamma_5 \text{ and } \frac{\sigma^\alpha}{2} \otimes \gamma_3\gamma_5$$

$$\text{where } \gamma^3\gamma^5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \text{ – diagonal valley matrix}$$

- A ferromagnetic like order parameter breaks spin degeneracy.
  Thus, *U(4)* breaks down to $U^{(+)}(2)_v \times U^{(-)}(2)_v$
- QHF order parameter and/or Dirac mass breaks *U(4)* down to $U^{(K)}(2)_s \times U^{(K')}(2)_s$

Full propagator

- One can use the following general ansatz:

$$iG_s = \left[(i\hbar\partial_t + \underline{\mu_s} + \underline{\tilde{\mu}_s}\gamma^3\gamma^5)\gamma^0 - v_F(\boldsymbol{\pi}\cdot\boldsymbol{\gamma}) - \underline{\tilde{\Delta}_s} + \underline{\Delta_s}\gamma^3\gamma^5\right]^{-1}$$

Electron chemical potential

"Pseudospin" chemical potential

Dirac mass

T-odd mass

- Physical meaning of the order parameters

$$\mu_3: \quad \langle\Psi^\dagger\sigma^3\Psi\rangle = \sum_{\kappa=K,K'}\sum_{a=A,B}\langle\psi^\dagger_{\kappa a+}\psi_{\kappa a+} - \psi^\dagger_{\kappa a-}\psi_{\kappa a-}\rangle$$

$$\tilde{\mu}_s: \quad \langle\Psi^\dagger\gamma^3\gamma^5 P_s\Psi\rangle = \langle\psi^\dagger_{KAs}\psi_{KAs} - \psi^\dagger_{K'As}\psi_{K'As} + \psi^\dagger_{KBs}\psi_{KBs} - \psi^\dagger_{K'Bs}\psi_{K'Bs}\rangle$$

$$\Delta_s: \quad \langle\bar{\Psi}\gamma^3\gamma^5 P_s\Psi\rangle = \langle\psi^\dagger_{KAs}\psi_{KAs} - \psi^\dagger_{KBs}\psi_{KBs} - (\psi^\dagger_{K'As}\psi_{K'As} - \psi^\dagger_{K'Bs}\psi_{K'Bs})\rangle$$

$$\tilde{\Delta}_s: \quad \langle\bar{\Psi}P_s\Psi\rangle = \langle\psi^\dagger_{KAs}\psi_{KAs} + \psi^\dagger_{K'As}\psi_{K'As} - \psi^\dagger_{KBs}\psi_{KBs} - \psi^\dagger_{K'Bs}\psi_{K'Bs}\rangle$$

$$P_\pm = \frac{1\pm\sigma^3}{2}$$

Energy scales in graphene

- Large Landau energy scale

$$\epsilon_B \equiv \sqrt{2\hbar|eB_\perp|v_F^2/c} \simeq 424\sqrt{|B_\perp[\mathrm{T}]|}\ \mathrm{K}$$

- Small Zeeman energy

$$Z \simeq \mu_B B = 0.67 B[\mathrm{T}]\ \mathrm{K}$$

- Intermediate dynamical scales

$$Z \ll A \leq M \ll \epsilon_B\,; \quad A \to \mu_s, \tilde{\mu}_s\,; \quad M \to \Delta_s, \tilde{\Delta}_s$$

$$A \leqslant M \sim 10^{-2}\epsilon_B$$

- In a model calculation [Phys. Rev. B **78** (2008) 085437]

$$M = 4.84 \times 10^{-2}\epsilon_B \text{ and } A = 3.90 \times 10^{-2}\epsilon_B$$

Dispersion relations

The dispersion relations for LLs with $n \geq 1$ are

$$\omega_{ns}^{(\sigma)} = -\mu_s + \sigma\tilde{\mu}_s \pm \sqrt{n\epsilon_B^2 + (\tilde{\Delta}_s + \sigma\Delta_s)^2}$$

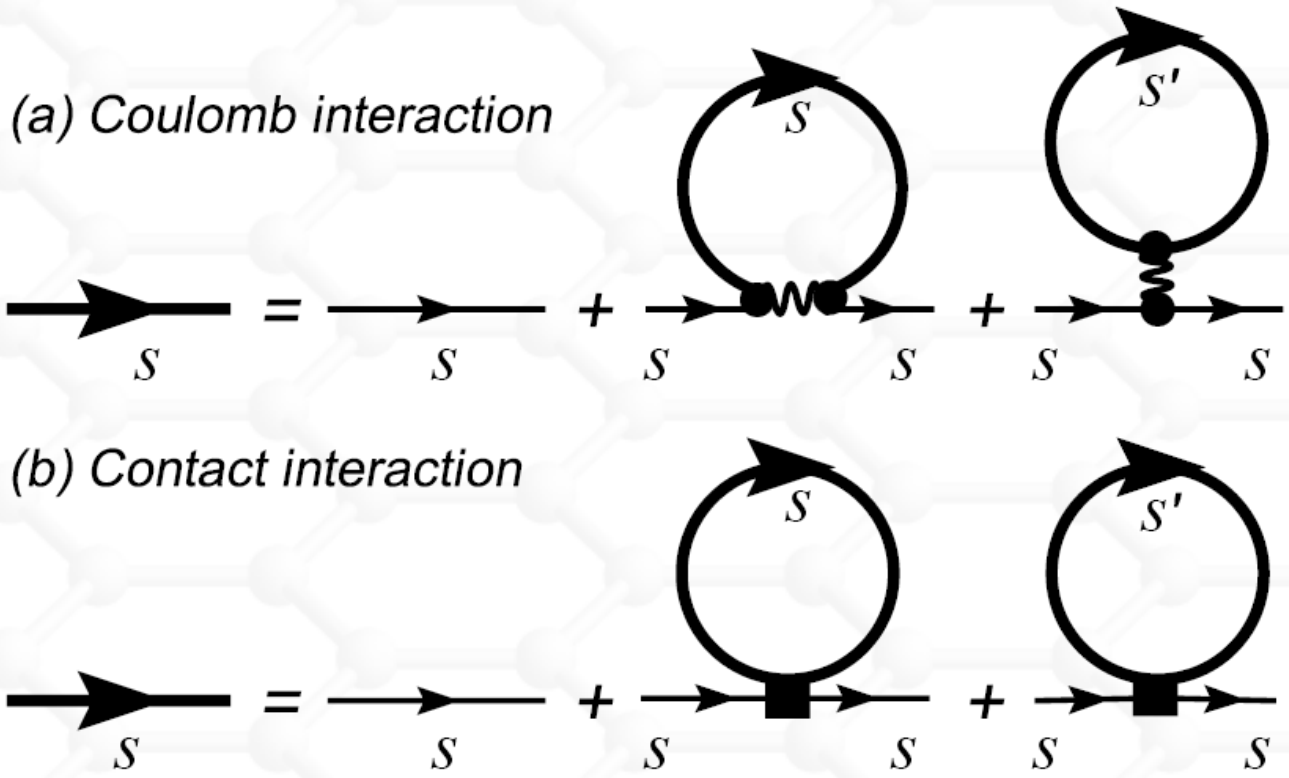
where $\sigma = \pm 1$ is connected with the eigenvalues of the diagonal pseudospin matrix $\gamma^3\gamma^5$.

For the LLL, the dispersion relations read

$$\omega_s^{(\sigma)} = -\mu_s + \sigma\left(\tilde{\mu}_s \,\mathrm{sign}(eB_\perp) + \tilde{\Delta}_s\right) + \Delta_s \,\mathrm{sign}(eB_\perp).$$

Schwinger-Dyson (gap) equation

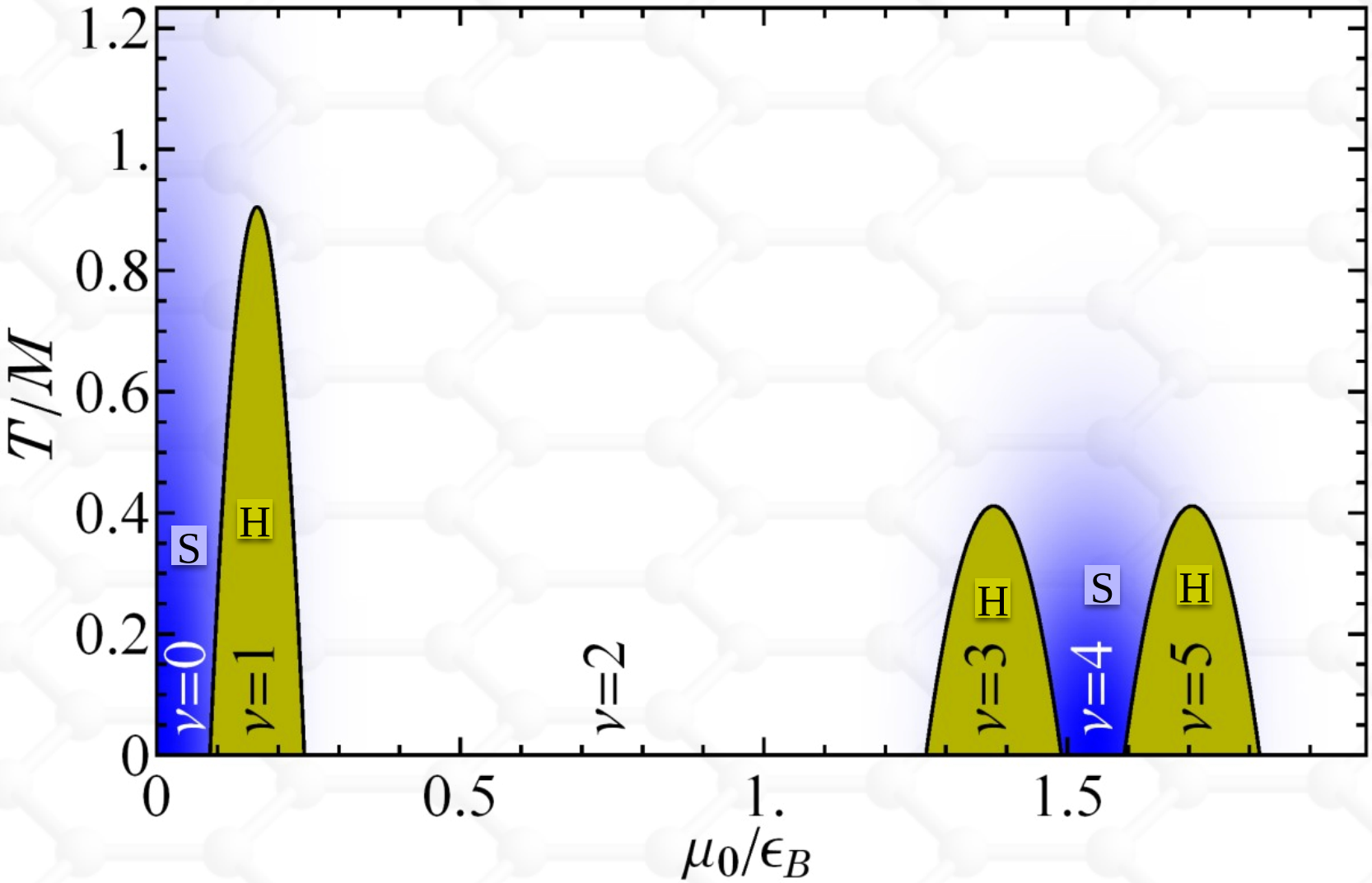
- Hartree-Fock (mean field) approximation:



Three types of solutions

- ***S*** (*singlet* with respect to $U^{(+)}(2)_v \times U^{(-)}(2)_v$); $\nu=0$
  - Order parameters: μ_s and Δ_s
  - Symmetry: $U^{(+)}(2)_v \times U^{(-)}(2)_v$
- ***T*** (*triplet* with respect to $U^{(\pm)}(2)_v$); $\nu=0$
  - Order parameters: $\tilde{\mu}_s$ and/or $\tilde{\Delta}_s$
  - Symmetry: $U^{(K)}(2)_s \times U^{(K')}(2)_s$
- ***H*** (*hybrid*, i.e., singlet + triplet); $\nu=\pm 1$
  - Order parameters: mixture of ***S*** and ***T*** types
  - Symmetry: $U^{(+)}(2)_v \times U^{(-)}(1)_K \times U^{(-)}(1)_{K'}$ or $U^{(+)}(1)_K \times U^{(+)}(1)_{K'} \times U^{(-)}(2)_v$

Phase diagram



Bilayer graphene

The effective low energy Hamiltonian [McCann & Falko, PRL, 96, 086805 (2006)]

Free Hamiltonian:

$$H_0 = -\frac{1}{2m}\int d^2x \Psi^+_{Vs}(x) \begin{pmatrix} 0 & (\pi^\dagger)^2 \\ \pi^2 & 0 \end{pmatrix} \Psi_{Vs}(x)$$

$$m \sim 10^{-2} m_e \sim 10^8 \mathrm{K/c^2}, \pi = \widehat{p}_{x_1} + i\widehat{p}_{x_2}, \hat{\mathbf{p}} = -i\hbar\nabla + e\mathbf{A}/c.$$

Bernal (A_2-B_1) stacking: $\Psi^T_{Ks} = (\psi_{A1}, \psi_{B2})_{Ks}$,whereas $\Psi^T_{K's} = (\psi_{B2}, \psi_{A1})_{K's}$.

$E = \pm\frac{p^2}{2m}$ without magnetic field B and $E_n = \pm\hbar\omega_c\sqrt{n(n-1)}$, $\omega_c = |eB|/mc$ with magnetic field B.

Interaction Hamiltonian:

$$H_{int} = \mu_B B \int d^2x \Psi^+(x)\sigma^3\Psi(x) + \frac{e^2}{2\kappa}\int d^3x d^3x' \frac{n(\mathbf{x})n(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} + \tilde{\Delta}_0 \int d^2x \Psi^+(x)\xi\tau_3\Psi(x),$$

where $n(\mathbf{x}) = \delta(z-\frac{d}{2})\rho_1(x) + \delta(z+\frac{d}{2})\rho_2(x)$ is the three dimensional charge density ($d \simeq 0.35$ nm is the distance between the layers). The Pauli matrix τ^3 in the voltage imbalance term acts on layer components, and $\xi = \pm 1$ for the valleys *K* and *K'*, respectively.

Quantum Hall effect in bilayer graphene

FIRST EXPERIMENTS:

Novoselov et al., Nature Phys. **2**, 177 (2006)

Henriksen et al., PRL **100**, 087403 (2008)

- Quantum Hall states with the filling factor $\nu = \pm 4n,\ n = 1, 2...$ predicted in the framework of the one electron problem were revealed.

RECENT EXPERIMENTS:

Suspended graphene:

Feldman, Martin, Yacoby, Nature Phys. **5**, 889 (2009)

Graphene on SiO_2/Si substrates:

Zhao, Cadden-Zimansky, Jiang, Kim, PRL **104**, 066801 (2010)

- Complete lifting the eightfold degeneracy in the LLL: $\nu = 0, \pm 1, \pm 2, \pm 3.$
- The $\nu = 0$ state is insulating.
- Suspended graphene: $\Delta E \sim 3.5 - 10.5 B_\perp[\mathrm{T}]\mathrm{K},\ B_\perp \lesssim 10\mathrm{T}$;

QHE in bilayer graphene: theory

Barlas, Cote, Nomura, MacDonald, PRL **101**, 097601 (2008)

Gorbar, Gusynin, V. M., JETP Lett. **91**, 314 (2010); PRB **81**, 155451 (2010);

Gorbar, Gusynin, Junji Jia, V. M., PRB **84**, 235449 (2011)

Nandkishore and Levitov, arXiv:0907.5395, arXiv:1002.1966

Tőke, Fal'ko, PRB **91**, 115455 (2011)

Symmetries

H_{int} can be rewritten as

$$H_{int} = \mu_B B \int d^2x \Psi^+(x)\sigma^3\Psi(x) + \tilde{\Delta}_0 \int d^2x \Psi^+(x)\xi\tau_3\Psi(x)$$
$$+\frac{1}{2}\int d^2x d^2x' \left[V(x-x')\left(\rho_1(x)\rho_1(x') + \rho_2(x)\rho_2(x')\right) + 2V_{12}(x-x')\rho_1(x)\rho_2(x')\right] .$$

Intralayer potential *V(x)* $\xrightarrow{\text{FT}} \tilde{V}(k) = 2\pi e^2/\kappa k$

interlayer potential $V_{12}(x)$ $\xrightarrow{\text{FT}} \tilde{V}_{12}(k) = (2\pi e^2/\kappa)(e^{-kd}/k).$

The two-dimensional charge densities $\rho_1(x)$ and $\rho_2(x)$ are:

$$\rho_1(x) = \Psi^+(x)P_1\Psi(x)\,, \quad \rho_2(x) = \Psi^+(x)P_2\Psi(x)\,,$$

where $P_1 = \frac{1+\xi\tau^3}{2}$ and $P_2 = \frac{1-\xi\tau^3}{2}$ are projectors on states in the layers 1 and 2, respectively.

If both the Zeeman and $\tilde{\Delta}_0$ terms are ignored, the bilayer symmetry is

$$G_2 = U^{(K)}(2)_S \times U^{(K')}(2)_S \times Z_{2V}^{(+)} \times Z_{2V}^{(-)} \quad ,$$

where $Z_{2V}^{(s)}$ describes the valley transformation $\xi \to -\xi$ for a fixed spin $s = \pm$.

G_2 is much lower than $G_1 = U(4)$ in monolayer graphene.

Order parameters

Although the G_1 and G_2 symmetries are quite different, it is noticeable that their spontaneous breakdowns can be described by the same QHF and MC order parameters. G_1 and G_2 define the same four conserved commuting currents whose charge densities (and four corresponding chemical potentials) span the QHF order parameters:

$$\mu_3: \quad \langle \Psi^\dagger \sigma^3 \Psi \rangle = \sum_{\kappa=K,K'} \sum_{a=A_1,B_2} \left\langle \psi^\dagger_{\kappa a+}\psi_{\kappa a+} - \psi^\dagger_{\kappa a-}\psi_{\kappa a-} \right\rangle ,$$

$$\tilde{\mu}_s: \quad \langle \Psi^\dagger_s \xi \Psi_s \rangle = \left\langle \psi^\dagger_{KA_1s}\psi_{KA_1s} - \psi^\dagger_{K'A_1s}\psi_{K'A_1s} + \psi^\dagger_{KB_2s}\psi_{KB_2s} - \psi^\dagger_{K'B_2s}\psi_{K'B_2s} \right\rangle .$$

While the first order parameter describes $SU^{(K)}(2)_s \times SU^{(K')}(2)_s$ spin symmetry breakdown, the second one breaks the discrete subgroup $Z_{2V}^{(s)}$. Their MC cousins are

$$\Delta_s: \quad \langle \Psi^\dagger_s \tau_3 \Psi_s \rangle = \left\langle \psi^\dagger_{KA_1s}\psi_{KA_1s} - \psi^\dagger_{KB_2s}\psi_{KB_2s} - \left(\psi^\dagger_{K'A_1s}\psi_{K'A_1s} - \psi^\dagger_{K'B_2s}\psi_{K'B_2s}\right) \right\rangle ,$$

$$\tilde{\Delta}_s: \quad \langle \Psi^\dagger_s \xi \tau_3 \Psi_s \rangle = \left\langle \psi^\dagger_{KA_1s}\psi_{KA_1s} + \psi^\dagger_{K'A_1s}\psi_{K'A_1s} - \psi^\dagger_{KB_2s}\psi_{KB_2s} - \psi^\dagger_{K'B_2s}\psi_{K'B_2s} \right\rangle .$$

LLL quasiparticle propagator

Bare LLL propagator

$$\tilde{S}_{\xi s}(\mathbf{r};\omega) = \frac{1}{2\pi l^2}\exp\left(-\frac{\mathbf{r}^2}{4l^2}\right)\left[L_0\left(\frac{\mathbf{r}^2}{2l^2}\right) + L_1\left(\frac{\mathbf{r}^2}{2l^2}\right)\right] S_{\xi s}(\omega) P_-,$$

where

$$S_{\xi s}(\omega) = \frac{1}{\omega + \bar{\mu}_s + \xi\tilde{\Delta}_0 + i\delta\,\mathrm{sgn}\,\omega}, \quad \bar{\mu} = \mu_0 - sZ, \quad P_\pm = \frac{1 \pm \tau_3}{2}, \quad l = \sqrt{\frac{\hbar c}{|eB|}}.$$

Full LLL propagator

$$\tilde{G}_{\xi s}(\mathbf{r};\omega) = \frac{1}{2\pi l^2}\exp\left(-\frac{\mathbf{r}^2}{4l^2}\right)\left[G_{\xi s0}(\omega) L_0\left(\frac{\mathbf{r}^2}{2l^2}\right) + G_{\xi s1}(\omega) L_1\left(\frac{\mathbf{r}^2}{2l^2}\right)\right] P_-,$$

where

$$G_{\xi sn}(\omega) = \frac{1}{\omega - E_{\xi ns} + i\delta\mathrm{sgn}\omega}, \text{ and } E_{\xi ns} = -(\mu_s(n) + \Delta_s(n)) + \xi(\tilde{\mu}_s(n) - \tilde{\Delta}_s(n)), \quad n = 0, 1,$$

are the energies of the LLL states depending on the order parameters $\mu_s(n), \tilde{\mu}_s(n), \Delta_s(n), \tilde{\Delta}_s(n)$.

Exchange (FOCK) interactions:

$$\tilde{V}_{eff}(\omega, k) = \frac{2\pi e^2}{\kappa} \frac{1}{k + \frac{4\pi e^2}{\kappa}\Pi(\omega, \mathbf{k}^2)}$$, κ -- dielectric constant

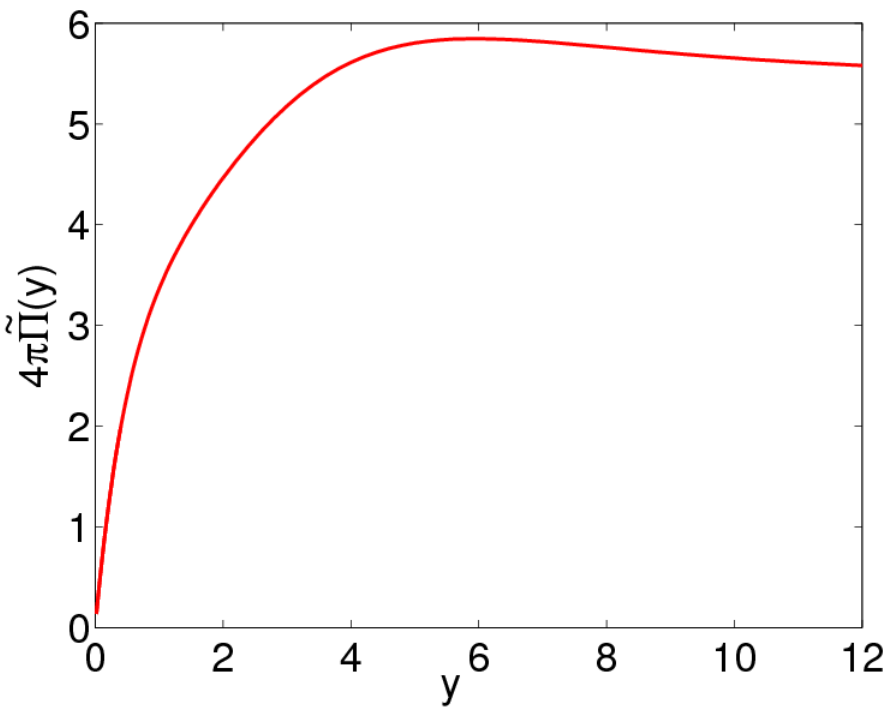
The static polarization function $\Pi(0, \mathbf{k}^2) \equiv \frac{m}{\hbar^2}\tilde{\Pi}(y), \quad y \equiv \frac{\mathbf{k}^2 l^2}{2}$.

Because

$$m \sim 10^{-2} m_e \sim 10^8 \mathrm{K}/c^2 \gg \hbar^2/e^2 l,$$

the polarization function dominates in $\tilde{V}_{eff}$:

$$\tilde{V}_{eff}(k) = C(y)\hbar^2/ml^2k^2 .$$



HARTREE interactions:

$$\tilde{V}_H = -\frac{2\pi e^2 d}{\kappa}$$

$\nu = 0$ QH state

Two competing solutions of the gap equation at the neutrality point $\nu = 0$

(I) spin polarized solution

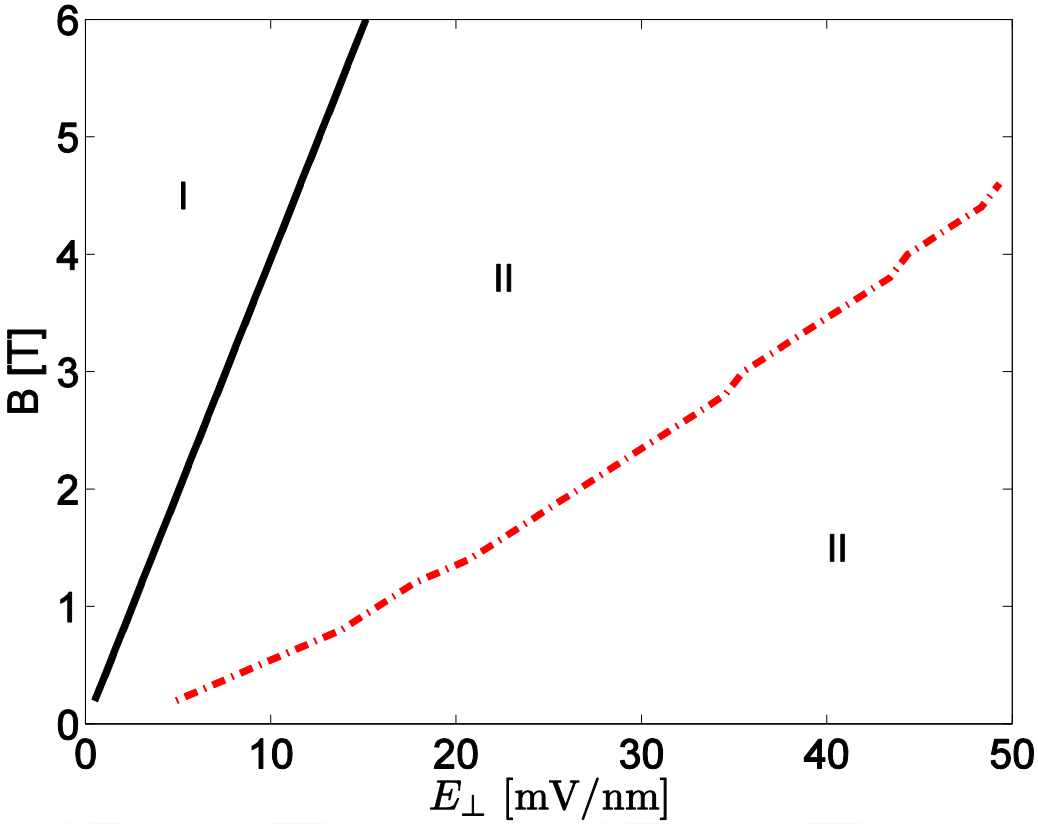
$$\Delta E^{(I)} = (E^{(I)}_{\xi 1+} - E^{(I)}_{\xi 1-})/2 \simeq 14.4 B[\mathrm{T}]\mathrm{K}$$

(II) layer polarized solution

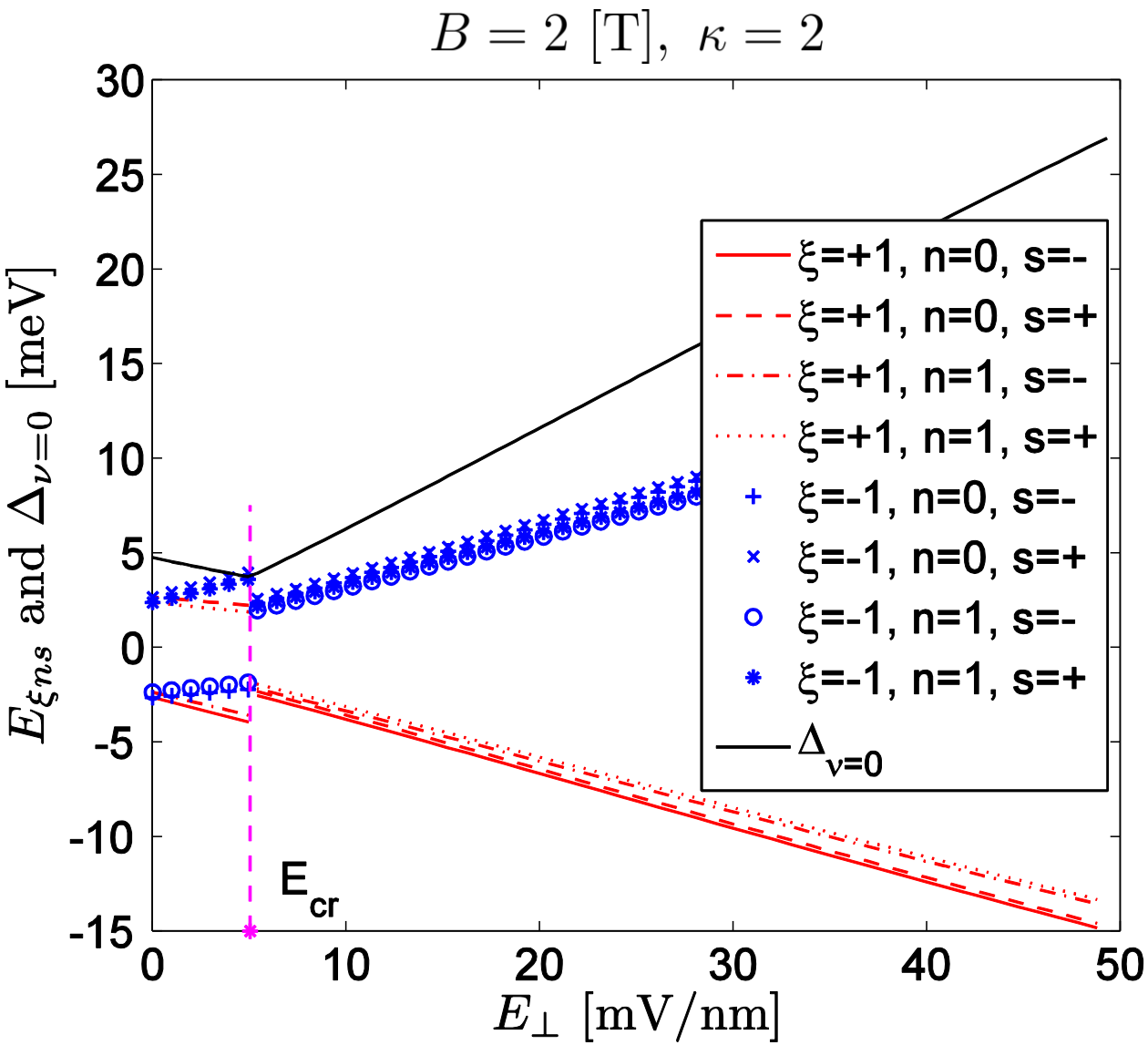
$$\Delta E^{(II)} = (E^{(II)}_{-1-} - E^{(II)}_{+1+})/2 \simeq 9.3 B[\mathrm{T}]\mathrm{K} + \tilde{\Delta}_0$$

$$\tilde{\Delta}_0 = e E_\perp d/2$$

$$E_\perp^{cr}\left[\frac{\mathrm{mV}}{\mathrm{nm}}\right] = C B\,[\mathrm{T}], \quad C \lesssim 5$$



Energy spectrum and gap of the $\nu = 0$ QH state



The values of the external field $E_\perp$ where the conductance is not quantized correspond to the minima of the gaps.

Latest Experiments

Weitz, Allen, Feldman, Martin, Yacoby, Science **330**, 812 (2010)

Martin, Feldman, Weitz, Allen, Yacoby, PRL **105**, 256806 (2010)

Kim, Lee, Tutuc, PRL **107**, 016803 (2011)

Freitag, Trbovic, Weiss, Schonenberger, PRL **108**, 076602 (2012)

J. Velasco *et al*., Nature Nanotechnology **7**, 156 (2012).

The confirmation of the phase transition between the ferromagnetic (spin polarized) and layer asymmetric (layer polarized) QH states

$$E_{\perp}^{cr}\left[\frac{\mathrm{mV}}{\mathrm{nm}}\right] \sim 10B\,[\mathrm{T}]$$

Higher, $\nu = \pm 1, \ \pm 2$ and ± 3 LLL plateaus

Main experimental results:

a) For $\nu = \pm 1$, there are two phases separated by approximately the same critical line as that in the $\nu = 0$ QH state.

b) There is only one phase for $\nu = \pm 2$ and $\nu = \pm 3$ QH states.

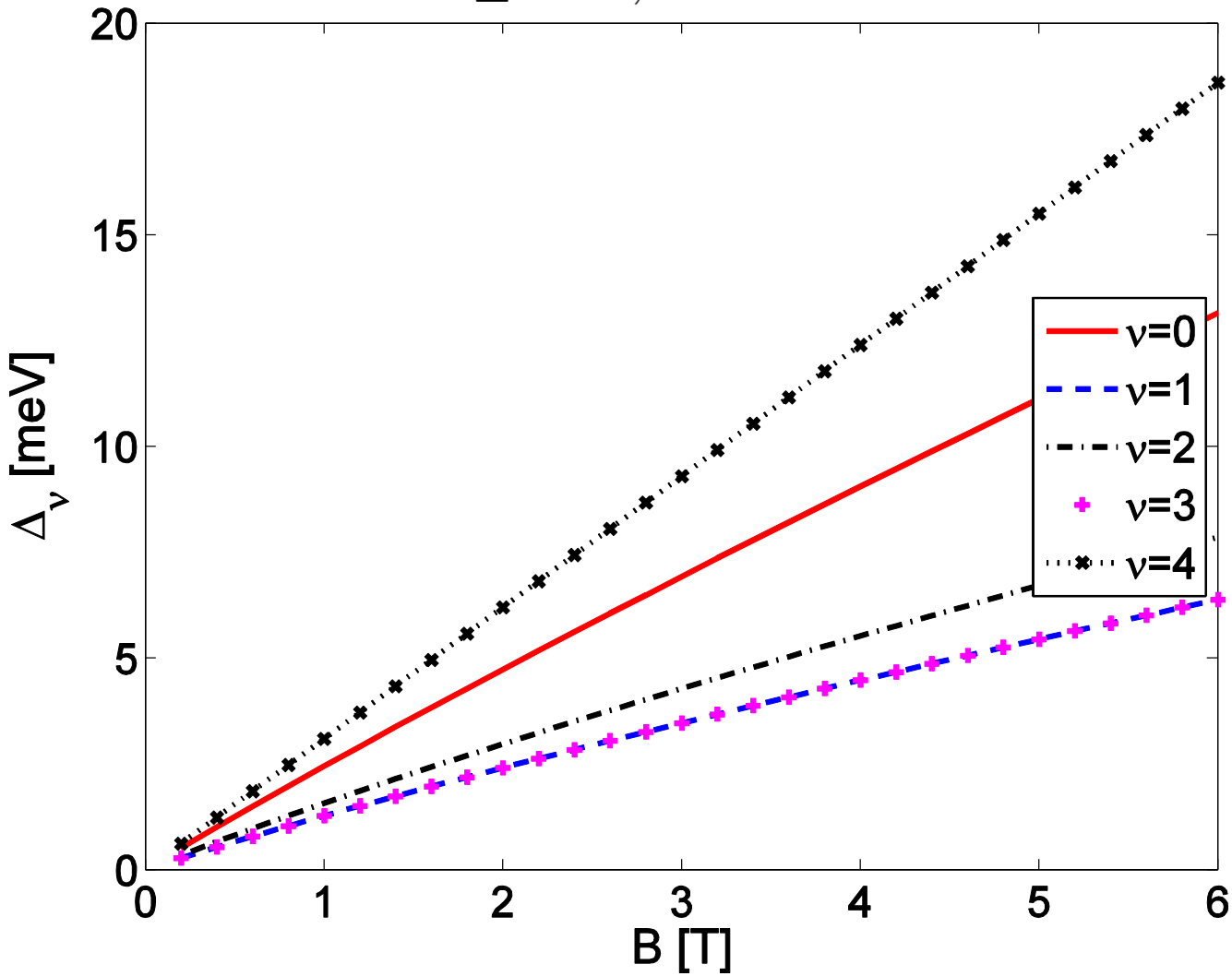
c) The $\nu = 0$ gap is approximately 30-40% larger than the $\nu = \pm 2$ one and significantly (by factor 10) exceeds the $\nu = \pm 1$ gap.

a) For $\nu = 0, \ \pm 1, \ \pm 2$ and $\nu = \pm 3$, the conductance is quantized except at particular values of the electric field $E_\perp$.

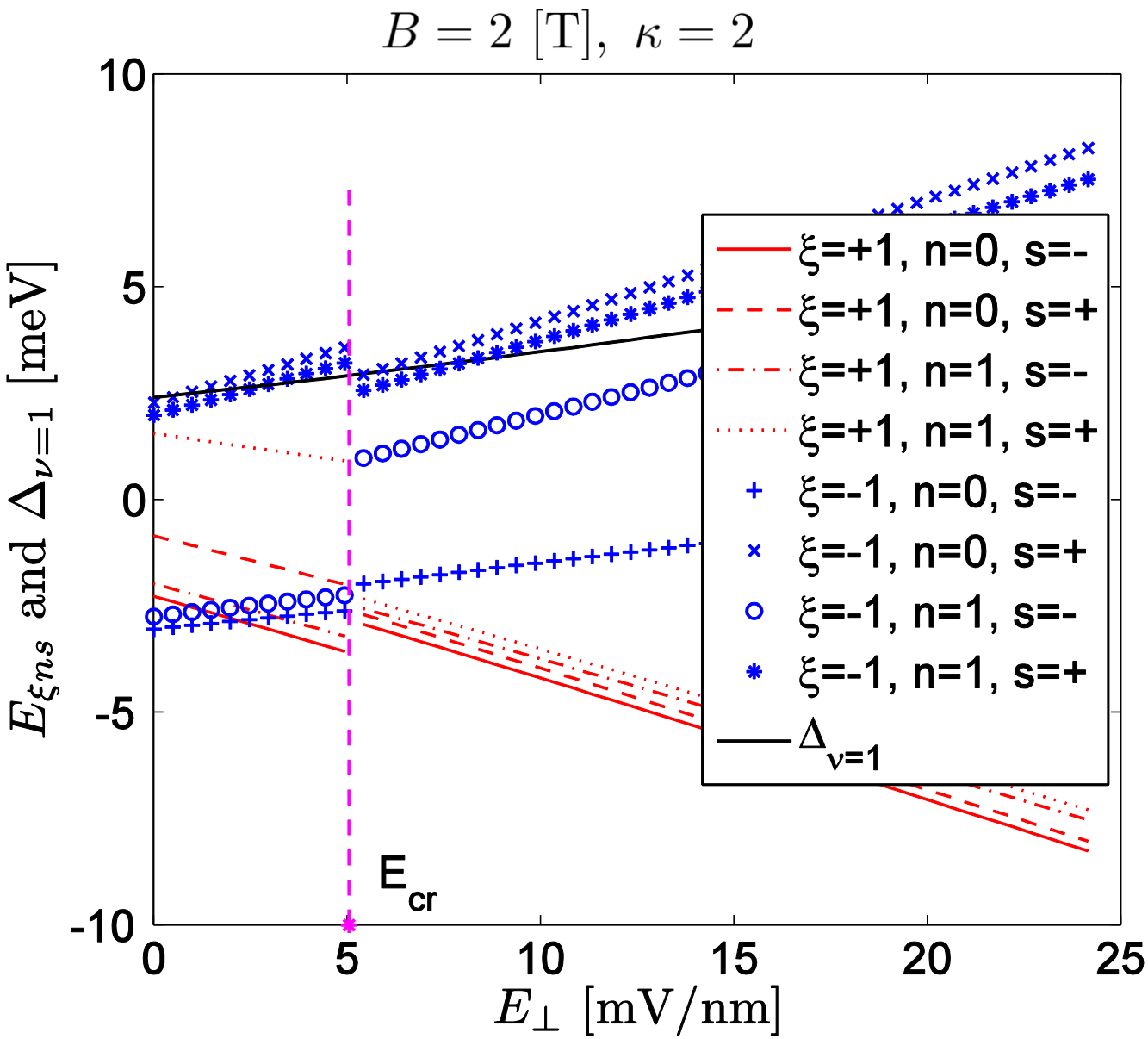
Theory (Gorbar, Gusynin, Junji Jia, V.M., PRB **84**, 235449 (2011))

The energy gaps as functions of magnetic field at zero electric field for different filling factors:

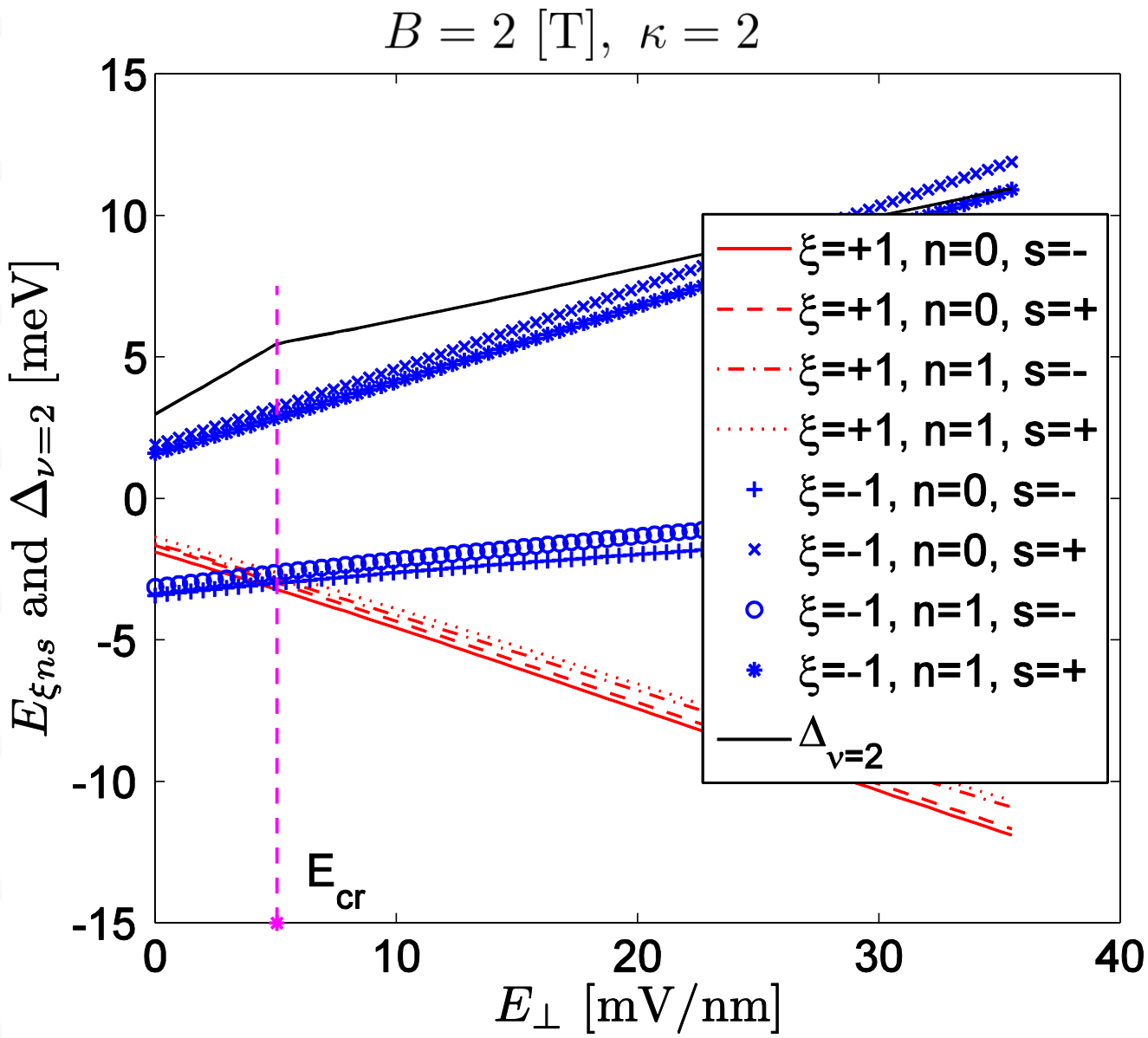
$$E_\perp = 0, \ \kappa = 2$$



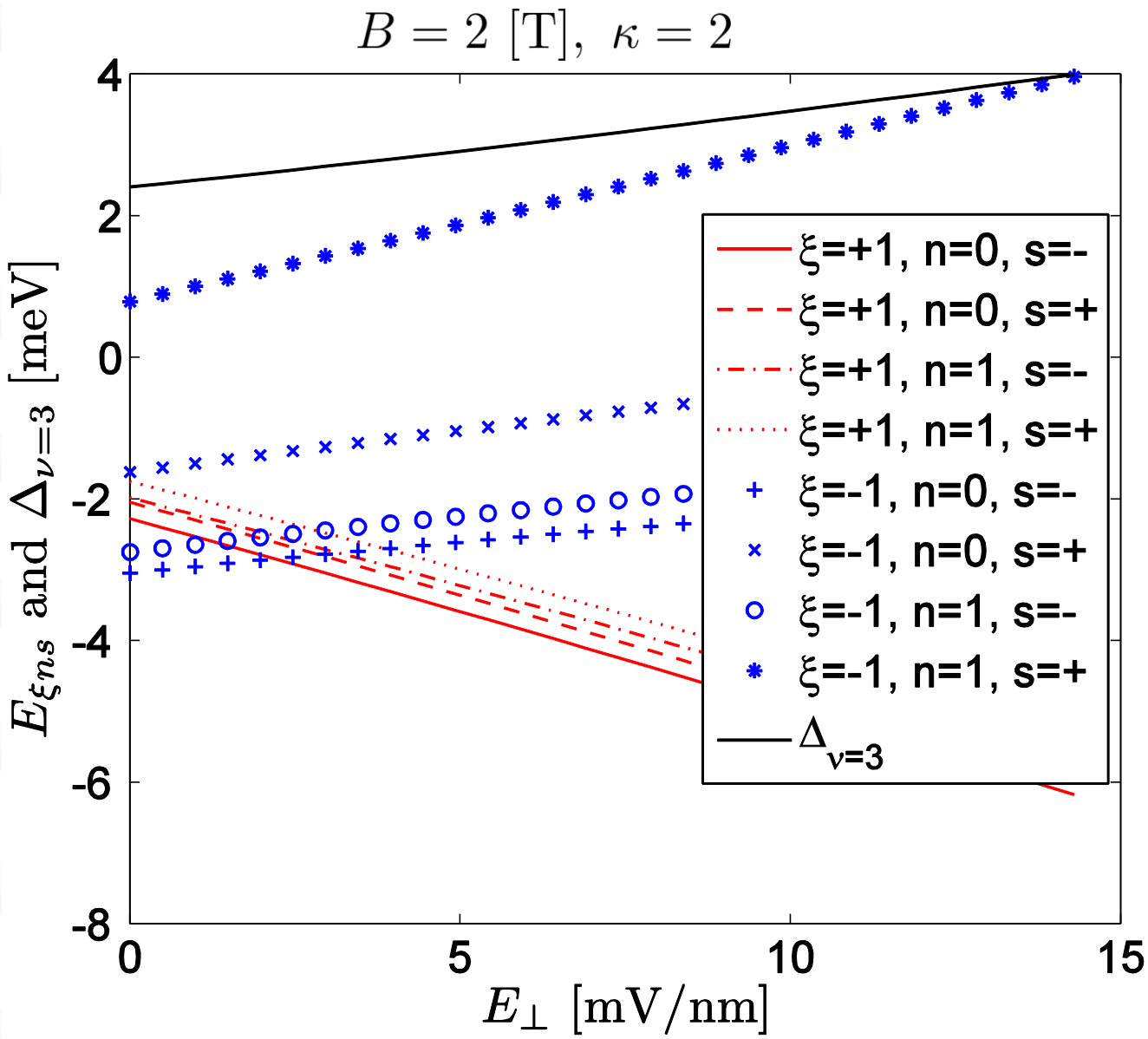
Energy spectrum and gap of the $\nu = 1$ QH state



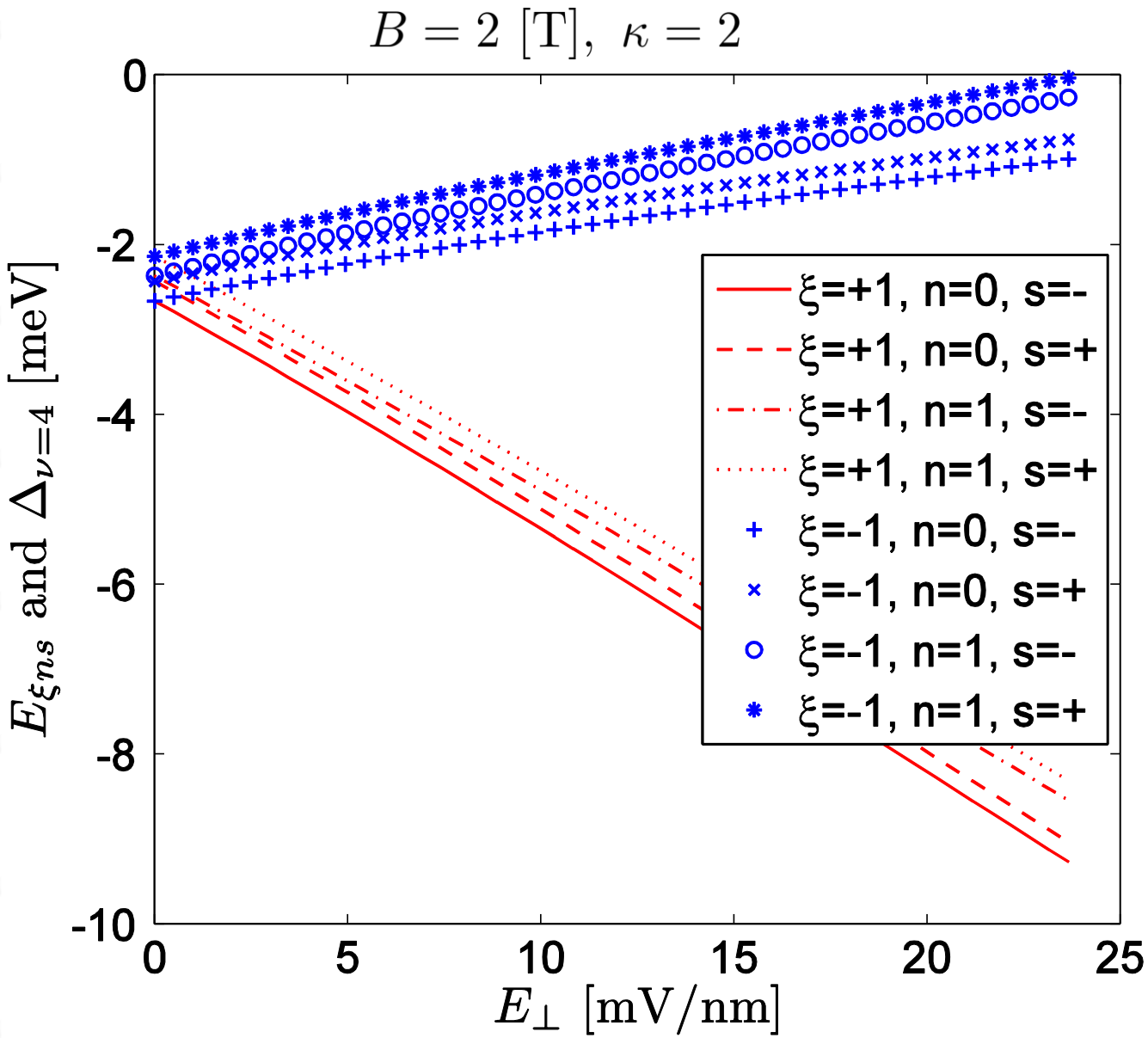
Energy spectrum and gap of the $\nu = 2$ QH state



Energy spectrum and gap of the $\nu = 3$ QH state



Energy spectrum and gap of the $\nu = 4$ QH state



Outlook

(i) The present ansatz with the sixteen order parameters is the minimal one for describing the breakdown of the $U^{(K)}(2)_S \times U^{(K')}(2)_S \times Z_{2V}^{(+)} \times Z_{2V}^{(-)}$ symmetry in bilayer graphene.
It could be extended in order to look for other solutions of the gap equation. A natural extension would be to include order parameters that mix the *n=0* and *n=1* LLL states.

(ii) It would be important to analyze the gap equation with a non-static polarization function.

(iii) It would be interesting to describe explicitly the dynamics around the threshold value B_{thr}, when the crossover between the regimes with the nonrelativistic-like scaling $\Delta E \sim |eB|$ and the relativistic-like one $\Delta E \sim \sqrt{|eB|}$ should take place.

arXiv:1201.4872v1 [cond-mat.mes-hall]

Broken-symmetry $\nu = 0$ quantum Hall states in bilayer graphene: Landau level mixing and dynamical screening

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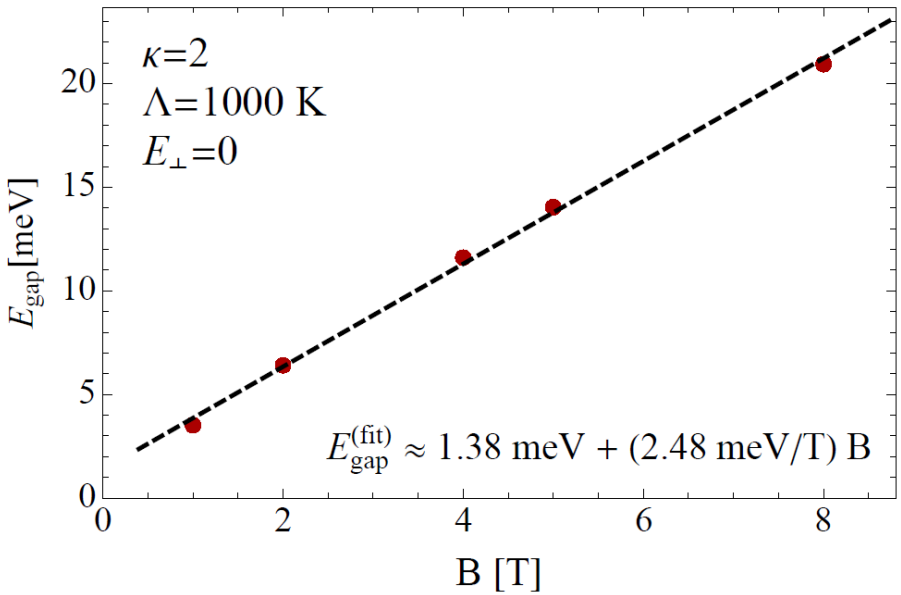
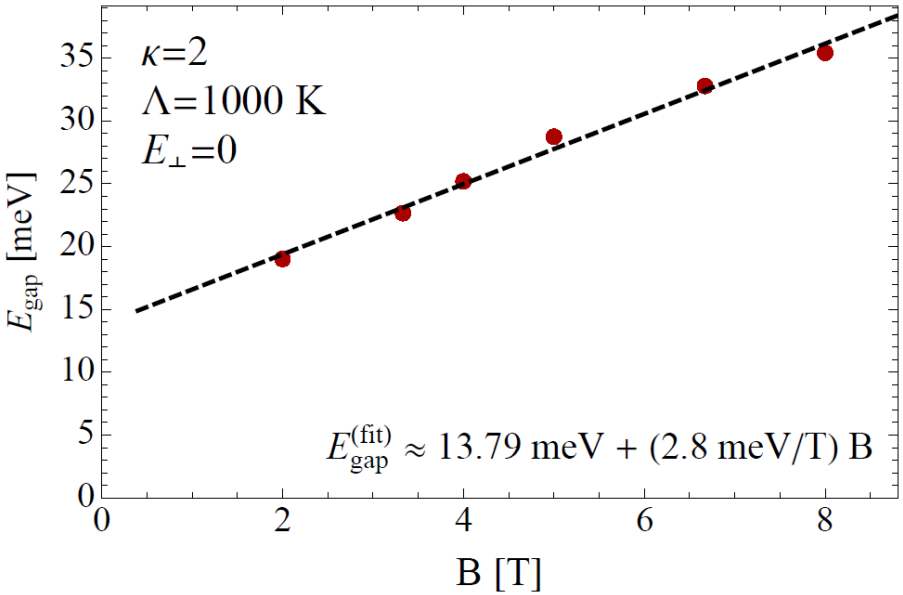
[2] *Bogolyubov Institute for Theoretical Physics, 03680, Kiev, Ukraine*
[3] *Department of Applied Mathematics, University of Western Ontario, London, Ontario N6A 5B7, Canada*
[4] *Department of Applied Sciences and Mathematics, Arizona State University, Mesa, Arizona 85212, USA*

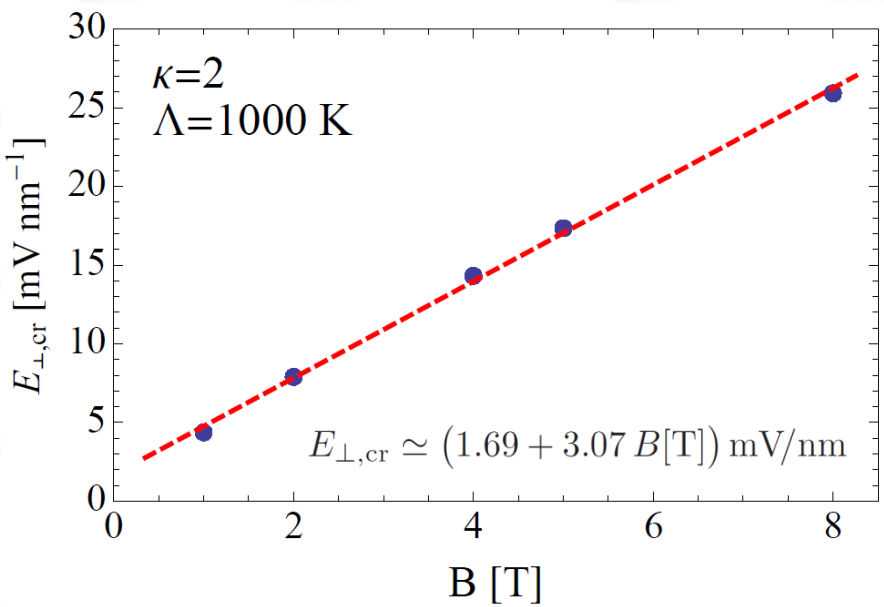
(Dated: January 25, 2012)

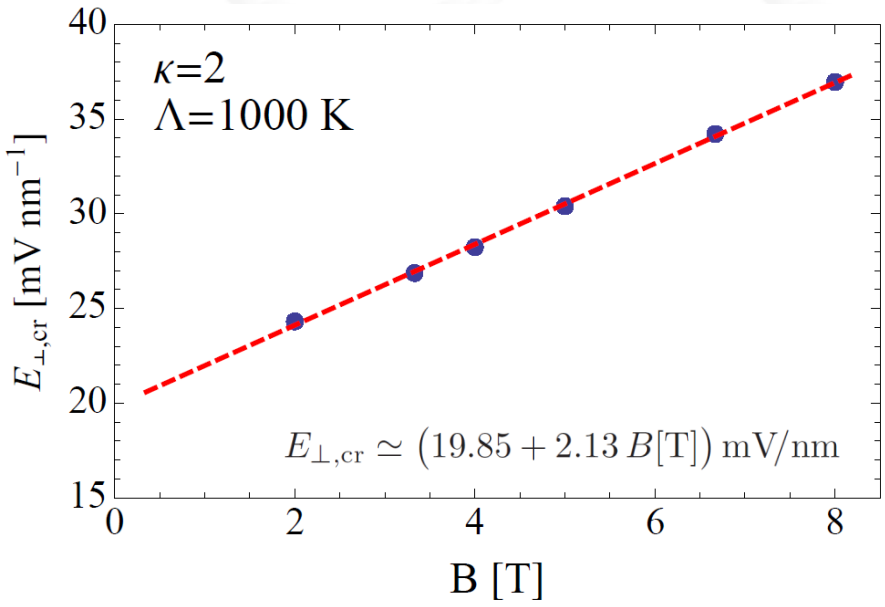
For bilayer graphene in a magnetic field at the neutral point, we derive and solve a full set of gap equations including all Landau levels and taking into account the dynamically screened Coulomb interaction. There are two types of the solutions for the filling factor $\nu = 0$: (i) a spin-polarized type solution, which is the ground state at small values of perpendicular electric field $E_\perp$, and (ii) a layer-polarized solution, which is the ground state at large values of $E_\perp$. The critical value of $E_\perp$ that determines the transition point is a linear function of the magnetic field, i.e., $E_{\perp,\mathrm{cr}} = E_\perp^{\mathrm{off}} + aB$, where $E_\perp^{\mathrm{off}}$ is the offset electric field and a is the slope. The offset electric field and energy gaps substantially increase with the inclusion of dynamical screening compared to the case of static screening. The obtained values for the offset and the energy gaps are comparable with experimental ones. The interaction with dynamical screening can be strong enough for reordering the levels in the quasiparticle spectrum (the $n = 2$ Landau level sinks below the $n = 0$ and $n = 1$ ones).



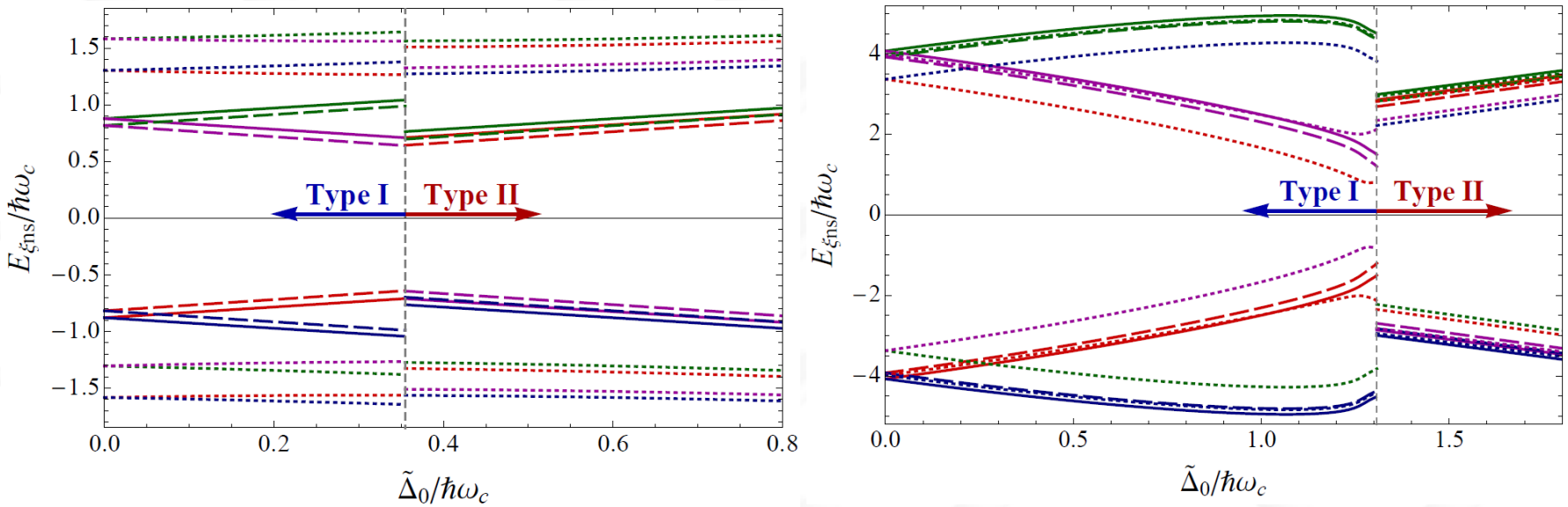


The energy gaps as functions of magnetic field for the static screening (left panel) and for the dynamical screening (right panel)

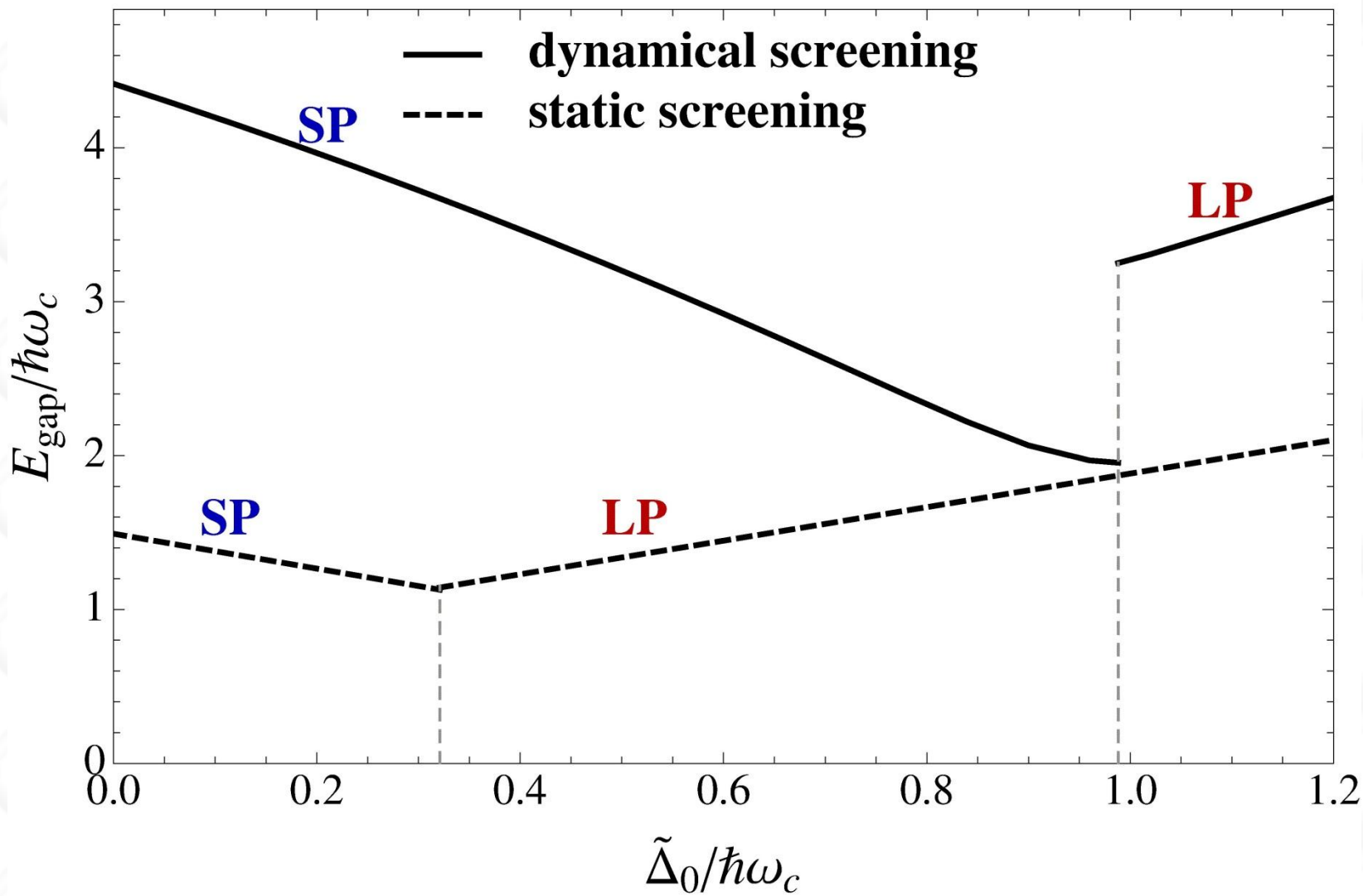




The dependence of the critical value of $E_{\perp}$ on magnetic field for the static screening (left panel) and for the dynamical screening (right panel)



Quasiparticle energies in the lowest three Landau levels, n = 0 (solid lines), n = 1 (dashed lines) and n = 2 (dotted lines), for Type I and Type II solutions obtained in the approximation with static screening (left panel) and dynamical screening (right panel) at B = 1T. Colors of the lines correspond to specific values of quantum numbers (ξ,s): red to (–,↓), green to (–, ↑), blue to (+,↓), and purple to (+,↑).


dynamical screening
static screening
SP
LP
SP
LP
$E_{gap}/\hbar\omega_c$
$\tilde{\Delta}_0/\hbar\omega_c$
0
1
2
3
4
0.0
0.2
0.4
0.6
0.8
1.0
1.2

Conclusion

- It seems that a dynamical Dirac mass (masses) is (are) necessarily produced in graphene in a strong magnetic field

- The set of order parameters which describes the QHE in graphene is quite large

- Feedback of QHE in graphene for particle physics: dynamics in dense Quantum Chromodynamics in a strong magnetic field [Gorbar, V.M., Shovkovy, PRD **83**, 085003 (2011)]

In 3+1 dimensions, the analog of the Haldane mass term, $\Delta\bar{\Psi}\gamma^3\gamma^5\Psi,$ describes an axial-vector current density, rather than a mass.

Strong Interaction Effects

of Strong Magnetic Fields

Berndt Mueller

CPODD Workshop 2012

RIKEN BNL, 25-27 June 2012

Overview

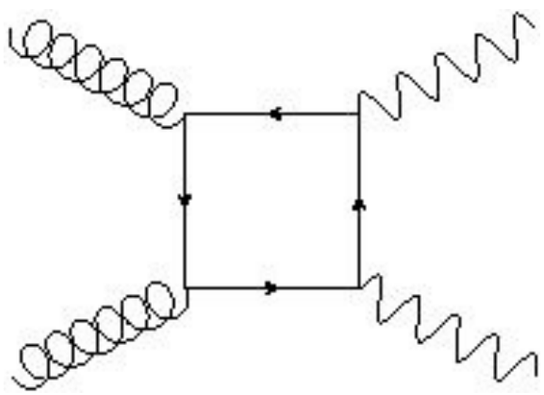
- Pseudoscalar QED-QCD couplings
- CME phenomenology
- Results

M. Asakawa, A. Majumder & BM, PRC 81 (2010) 064912

BM & A. Schäfer, PRC 82 (2010) 057902

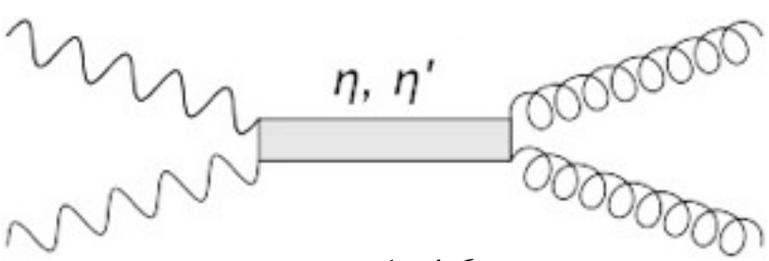
QED-QCD pseudoscalar sector

Gluon $\mathbf{E}^a \cdot \mathbf{B}^a$ interacts with $\mathbf{E}\cdot\mathbf{B}$ of the electromagnetic field via a quark loop:



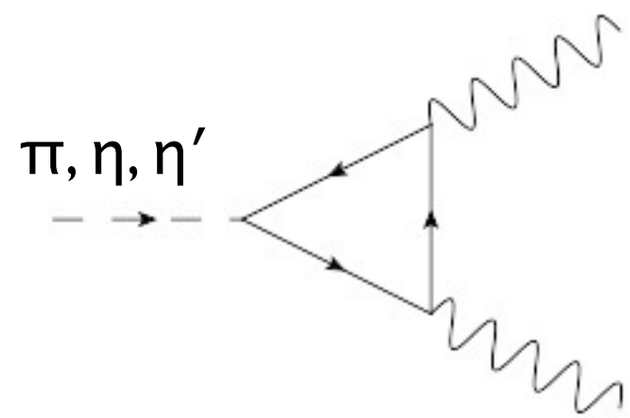
$$\mathcal{L}_{\mathrm{QECD}} = \kappa\alpha\alpha_s(\mathbf{E}^a \cdot \mathbf{B}^a)\,(\mathbf{E}\cdot\mathbf{B})$$

Effective interaction from η, η′ exchange:



$$\kappa \approx \frac{1.46}{\pi^2 f_\eta^2 m_{\eta'}^2}$$

Pseudoscalar mesons interact with electro-magnetic $\mathbf{E}\cdot\mathbf{B}$ via a triangular quark loop:



$$\mathcal{L}'_{\mathrm{QECD}} = \sum_{i=\pi^0,\eta,\eta'} \frac{\alpha}{\pi f_i}\phi_i\,\mathbf{E}\cdot\mathbf{B}$$

f_i is the nonperturbative "meson decay constant"

Anomalous current

Maxwell's equations dictate that the effective interaction $\mathcal{L}_{\rm QECD} = \mathcal{P}\,\mathbf{E}\cdot\mathbf{B}$

gives rise to an anomalous current $\mathbf{j}_{\rm an} = (\partial_t \mathcal{P})\,\mathbf{B}$ = $\sigma_\chi\,\mathbf{B}$ (D'Hoker & Goldstone - 1985).

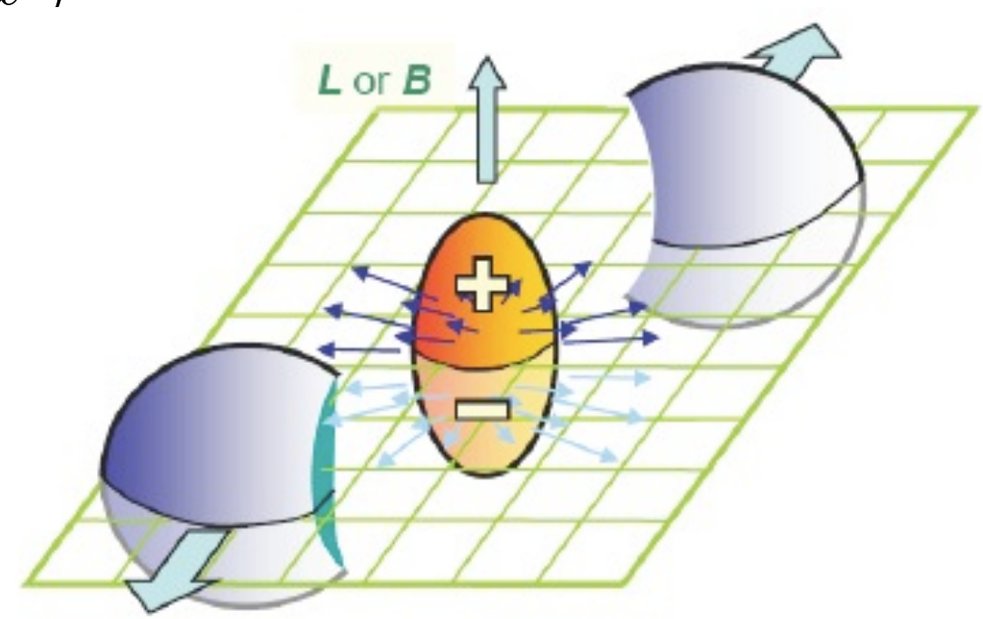
If $(\partial_t\,\mathscr{P})$ has a <u>nonzero expectation value</u>, this implies a new kind of conductivity σ_χ (the chiral conductivity), which "violates" parity, because $\mathbf{B}$ is <u>parity-even</u> and $\mathbf{j}$ is <u>parity-odd</u>.

But even if $(\partial_t\,\mathscr{P})$ has a vanishing expectation value, <u>nonvanishing fluctuations</u> can exist:

$$\langle j_i(x)\,j_k(x')\rangle = \langle \partial_t\mathcal{P}(x)\,\partial_t\mathcal{P}(x')\rangle\, B_i(x)B_k(x')$$

Strong *B*-fields exist in non-central relativistic heavy ion collisions; maybe the fluctuations of the anomalous current can be observed !

(Kharzeev, McLerran, Warringa '07)



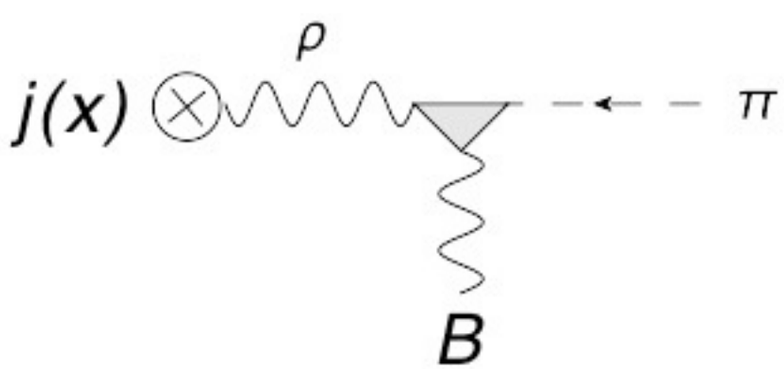
Hadronic anomalous current

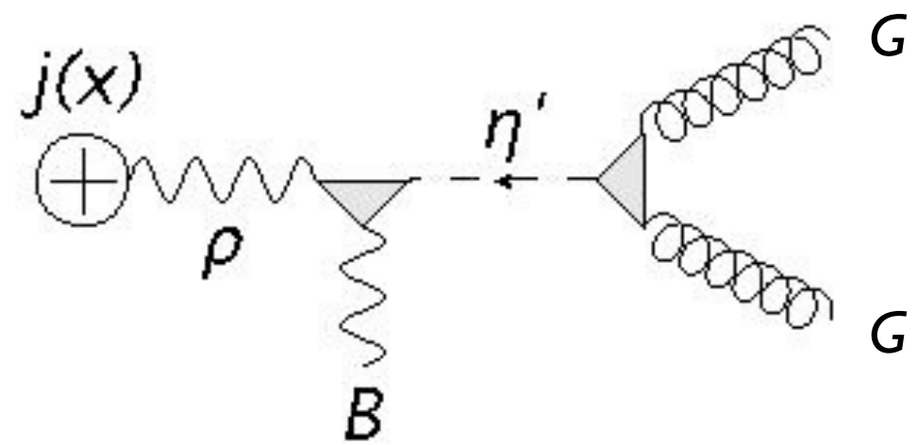
Vector meson dominance (VMD) relates the electromagnetic hadronic current to the neutral rho-meson field:

$$j^\mu = -\frac{e\, m_\rho^2}{g_{\rho\pi\pi}}\, \rho^\mu$$

Thus, to generate an "anomalous" current, the B-field needs to convert a pseudoscalar meson into a rho-meson (Asakawa, Majumder, BM '10):

(γππππ coupling)





The relevant interactions are well known from radiative decays:

$$\Gamma_{\rho^0\to\pi^0\gamma} = 3\alpha\, g^2_{\rho\pi\gamma}\, \frac{p^3_{\rm cm}}{m^2_\rho} \approx 90 \pm 12 \text{ keV} \qquad \Gamma_{\eta'\to\rho\gamma} = \alpha\, g^2_{\rho\eta'\gamma}\, \frac{p^3_{\rm cm}}{m^2_{\eta'}} \approx 60 \pm 5 \text{ keV}$$

Suppressed, if *B* is constant, but not if *B* is strongly time dependent.

CME effective action

True CME = Anomalous current induced by winding number change in B-field:

$$\vec{j} = \sigma_\chi \vec{B} \quad \text{with} \quad \sigma_\chi = \frac{3e^2 n_5}{2\pi^2 T^2} = -\sum_f \frac{3e_f^2 g^2}{16\pi^4 T^2} \int dt \left(\vec{E}^a \cdot \vec{B}^a \right)$$

Compare with Ohm's law: $\vec{j} = \sigma \vec{E}$

$$L_{eff}^{(Ohm)} = -\int \left\langle \vec{j} \right\rangle \cdot d\vec{A} = \tfrac{1}{2}\sigma \int_{-\infty}^{t} dt' \vec{E}(t')^2 \quad \Leftrightarrow \quad \tilde{L}_{eff} = \frac{i\sigma}{2\omega} \left| \vec{E}(\omega) \right|^2$$

Nonlocal (in time) effective action for CME:

$$L_{eff}^{(\mathrm{CME})} = -\sum_f \frac{e_f^2 g^2}{8\pi^4 T^2} \int_{-\infty}^{t} dt' \left(\vec{E} \cdot \vec{B} \right) \int_{-\infty}^{t'} dt'' \left(\vec{E}^a \cdot \vec{B}^a \right)$$

CME phenomenology

Requirements

- Strong B-field:
  - Available in heavy ion collisions
  - What are the relevant time and length scales?

- Pseudoscalar QCD source:
  - Chern-Simons number fluctuation
  - 2-gluon scattering in the pseudoscalar channel
  - Pseudoscalar meson condensate domain
  - Pseudoscalar meson excitation
  - What are the relevant length and time scales?

Mechanisms

- ***CGC mechanism***: Two gluons from the initial nuclei fuse in the pseudoscalar channel and generate an anomalous current in the strong magnetic field;
- ***Glasma mechanism***: Gluons in the "glasma" generate an anomalous current in the strong magnetic field via a winding number fluctuation;
- ***QGP mechanism***: Gluons in the equilibrated quark-gluon plasma generate an anomalous current in the strong magnetic field via a winding number fluctuation ("sphaleron");
- ***Hadronic mechanism***: A π^0 in the hadronic gas phase generates an anomalous current by converting into a ρ-meson in the strong B-field. Effect can be amplified by local π^0 or η (η′) condensate;
- ***Corona mechanism***: A π^0 in the hadronic corona generates an anomalous current by converting into a ρ-meson in the strong B-field.

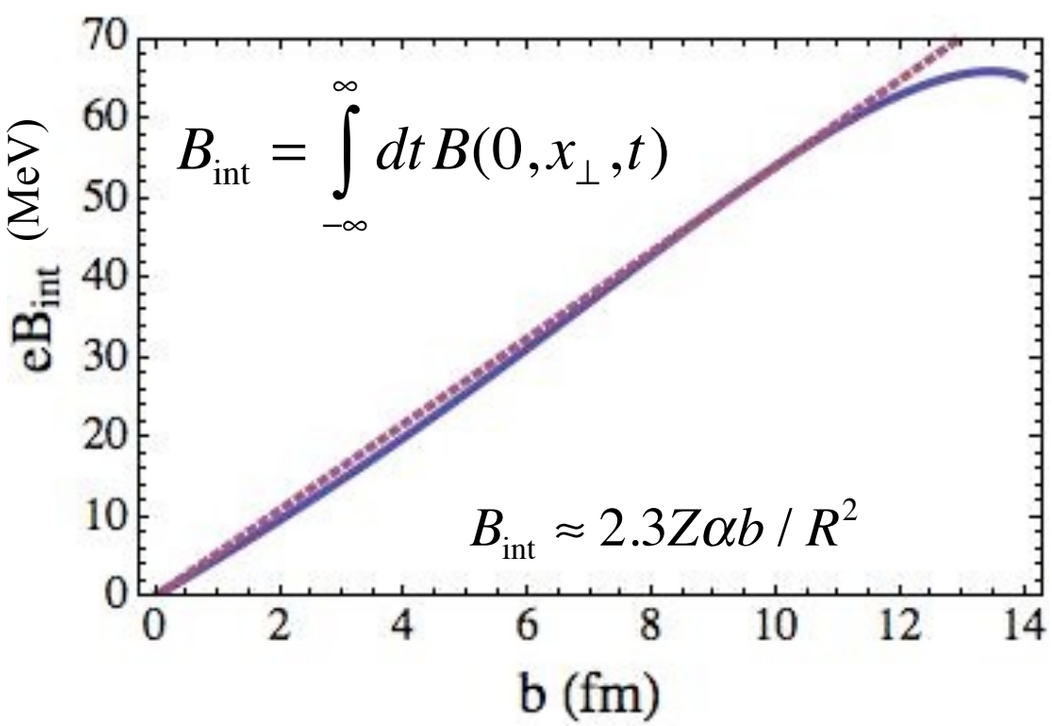
Time scales

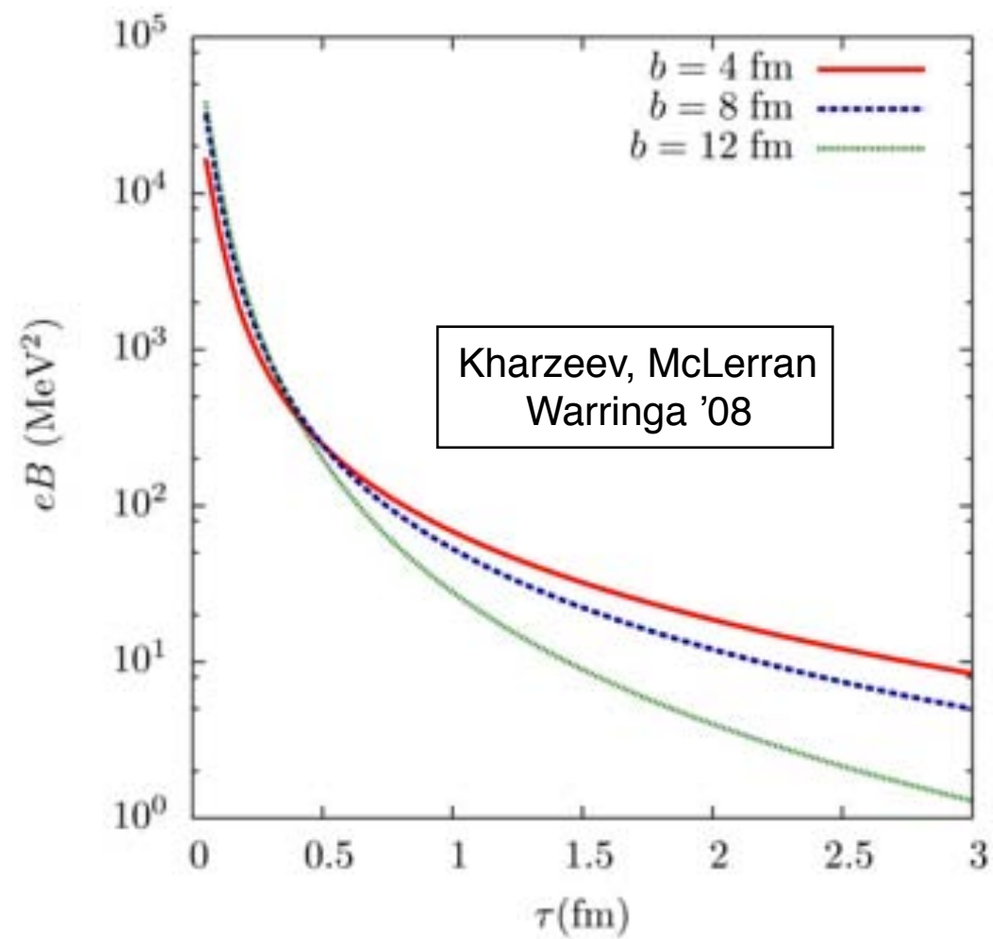
Life-time of the strong magnetic field: $\tau_B \approx 2R/\gamma \approx 0.1$ - 0.2 fm/c.

Time scale for winding number transitions: $\tau_{sph} \approx 1/T \approx 0.5$ - 1 fm/c.

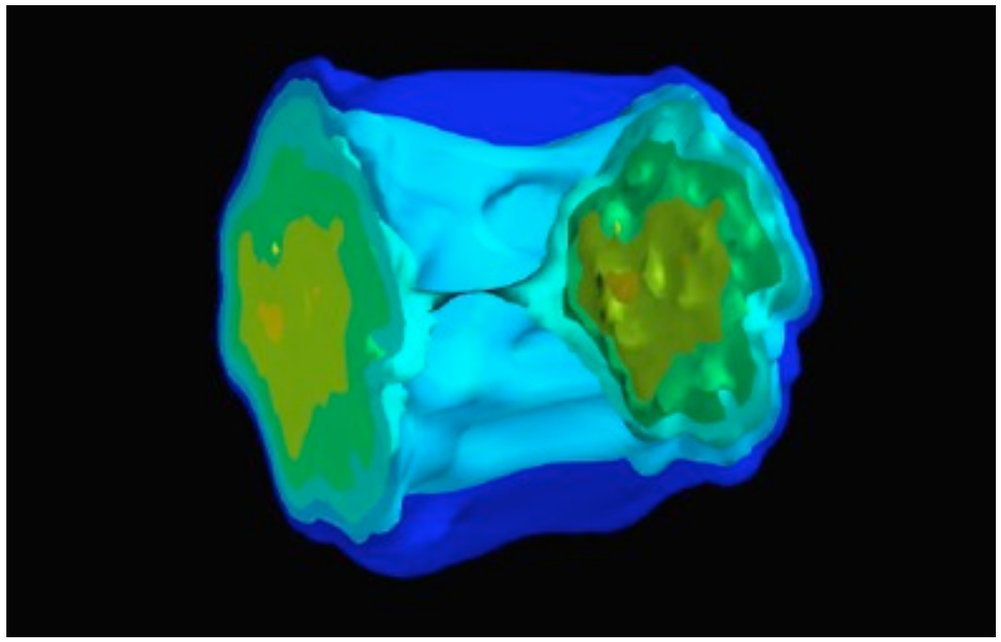
Time scale of hadronic pseudoscalar interactions: $\tau_{had} \approx 1/m_{\eta'} \approx 0.2$ fm/c.

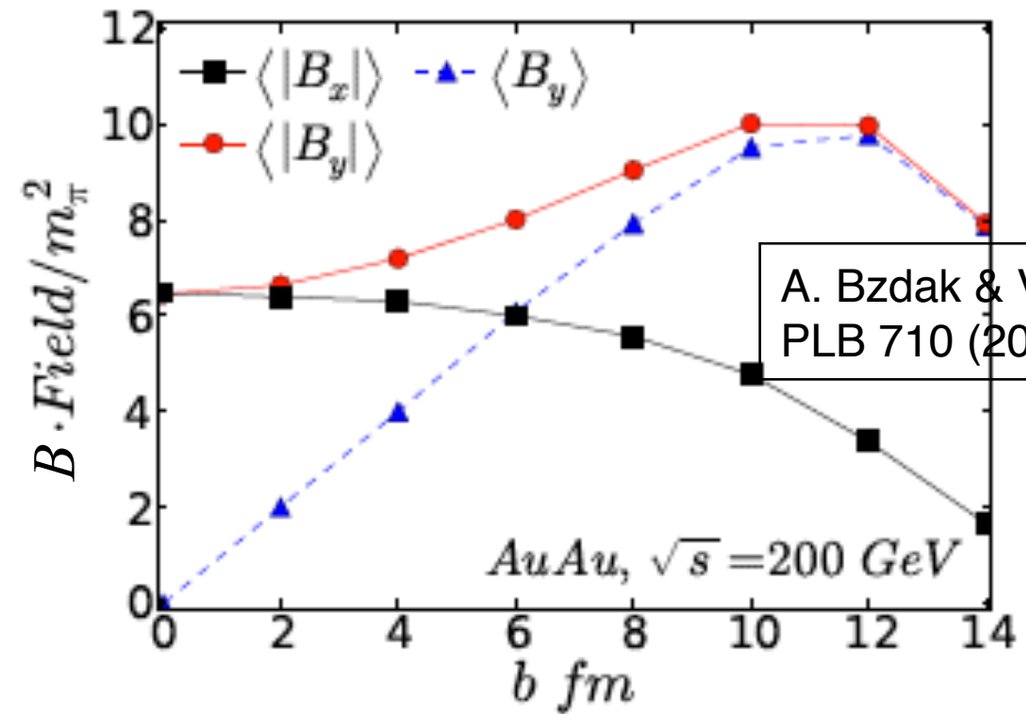
Weak, long lived B-field component due to baryon stopping and "frozen" B-fields



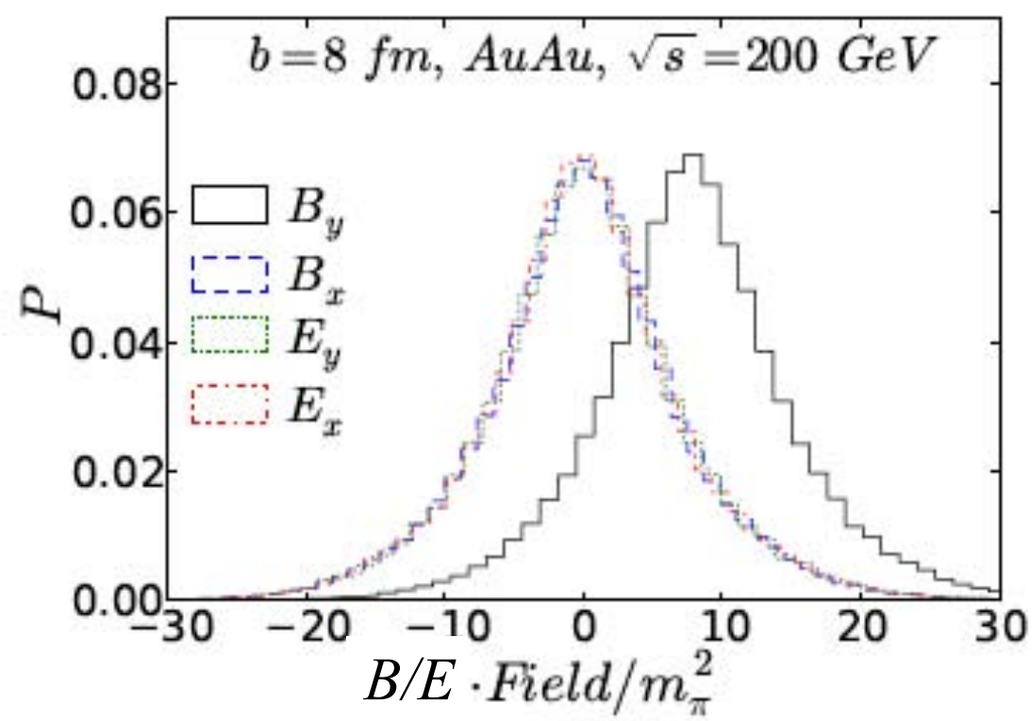


Length scales (B-field)





A. Bzdak & V. Skokov, PLB 710 (2012) 171



Lumpy nuclear charge distributions in individual events $\Longrightarrow$ nonzero but randomly oriented B-fields even in central collisions.

Magnetic field domains are larger than coherence length of pseudoscalar QCD observables → "homogeneous" B-field

Vector boson condensation?

$$E^2 = k_z^2 + (2n+1-2S_z)eB + m^2$$ in homogeneous B-field

$E^2 < 0$ for $S_z = 1$, $n = 0$ and $eB > m^2 + k_z^2$. → ρ, W condensation ?

Ambjorn & Olesen, PLB 257 (1991) 201

But strong B-field is highly localized: Δz, Δt ≈ R/γ: $\int B\,dz \approx \int B\,dt \approx \text{in dep of } \gamma.$

B-field can be idealized as $b_0\delta(z)$ "magnetic sheet"

Exact Green function can be obtained: no vector boson condensation!
Reason: Landau level only partially interacts with magnetic field.

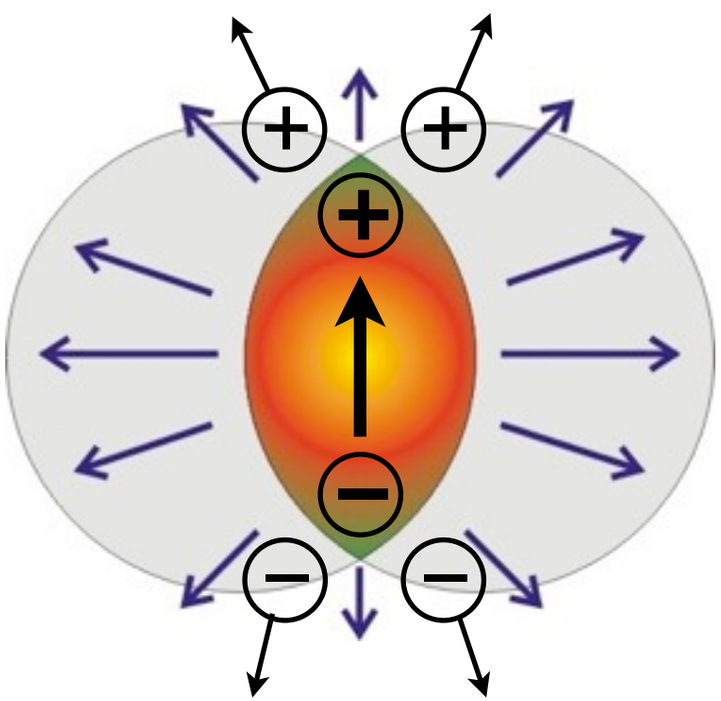
S. Schramm, BM, A. Schramm, PLB 277 (1992) 512; PLA 164 (1992) 28.

From J to ΔQ

The anomalous current is not directly observable. What is observed is the final charged particle distribution and its asymmetry with respect to the reaction plane:

$$\Delta Q = \int d^3p \int_{\Sigma_f} \frac{d\sigma_\mu p^\mu}{E} \sum_i e_i f_i(\mathbf{x}, \mathbf{p}) \, \mathrm{sgn}(p_z) \theta(\sigma_\mu p^\mu)$$

Transformation of spatial charge asymmetry into a momentum space asymmetry requires either **collective flow** or **opacity** during freeze-out (or both):



Charge asymmetry is created early; it must be transported to the freeze-out surface. Locally separated charged be depleted by diffusion processes.

Altogether, it is a complex transport problem, but one describable with current technology (viscous hydro + diffusion & hadron cascade).

From J to ΔQ (II)

Two ways to proceed:

1. For isochronous freeze-out, a position-momentum correlation requires collective flow.

$$\Delta Q = \int d^3x \int d^3p \sum_i e_i f_i(\mathbf{x}, \mathbf{p}; \tau_f) \mathrm{sgn}(p_z)$$

where $$f_i(\mathbf{x}, \mathbf{p}; \tau_f) = \exp\left[-u_\mu(\mathbf{x}) p^\mu / T_f + e_i \mu_Q(\mathbf{x}) / T_f\right]$$

For weak flow, expand f(x,p) in first order in **v**. For charged pions only one finds the simple result:

$$\Delta Q \approx \tfrac{3}{2} \int d^3x \, \rho(\mathbf{x}, \tau_f) \, v_z(\mathbf{x})$$

then use the continuity equation $\partial_t \rho = - \nabla \cdot \mathbf{j}_{an}$ to obtain:

$$\begin{aligned} \left\langle (\Delta Q)^2 \right\rangle &\approx \frac{9}{4} \int_0^{\tau_f} dt \, dt' \int d^3x \int d^3x' \\ &\times \nabla v_z(\mathbf{x}, \tau_f) \cdot \langle \mathbf{j}_{\mathrm{an}}(\mathbf{x}, t) \mathbf{j}_{\mathrm{an}}(\mathbf{x}', t') \rangle \cdot \nabla' v_z(\mathbf{x}', \tau_f). \end{aligned}$$

From J to ΔQ (III)

2. In the geometric approximation, one simply assumes that all charges in the upper hemisphere are emitted upwards, and vice versa:

$$\Delta Q = \int_{z>0} d^3x\, \rho(\mathbf{x}) = \int d^4x\, \delta(z) j_z(x)$$

Predictions using the two approaches differ by a factor ~30 (#1 is smaller)!

The weak flow approximation differs from the geometric approximation by a factor

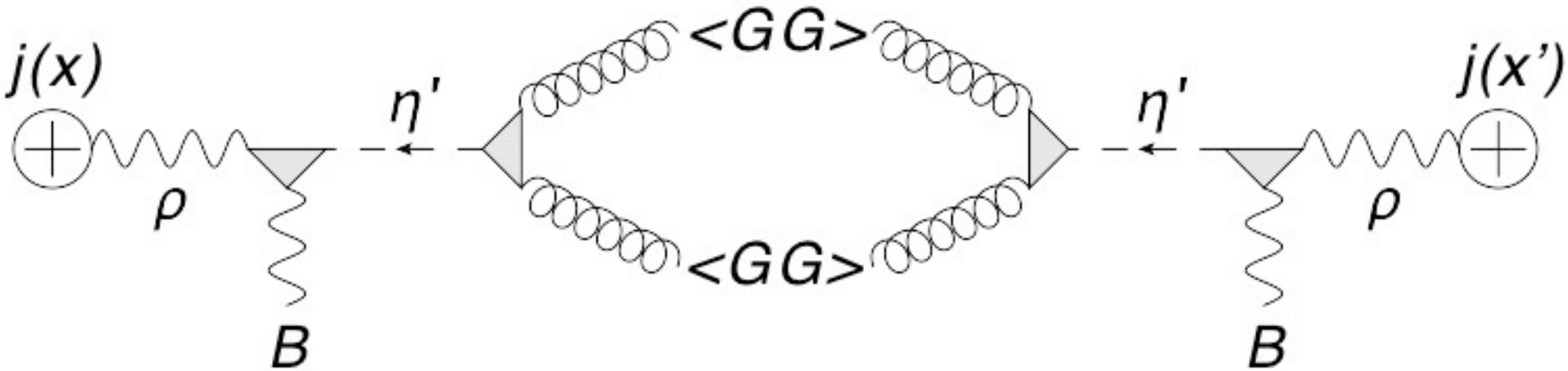
$$\Theta = \frac{\langle (\Delta Q)^2 \rangle_{\rm flow}}{\langle (\Delta Q)^2 \rangle_{\rm geo}} \approx v_f^2\, \xi_j / R$$

where ξ_j is the anomalous current correlation length.

For $\xi_j \approx 1$ fm, $v_f \approx 0.5$ and $R \approx 7$ fm, one has $\Theta \approx 0.035$ $\Rightarrow$ large uncertainty.

Results

CGC mechanism



$$\langle j^3(x) j^3(x') \rangle \approx e^2\, C \langle (\mathbf{E}^a \cdot \mathbf{B}^a)(x)(\mathbf{E}^b \cdot \mathbf{B}^b)(x') \rangle$$

with $C = \dfrac{g^2_{\rho\eta'\gamma}}{g^2_{\rho\pi\pi}} \dfrac{3(Z\alpha)^2 \alpha_s^2 \cos^2\theta}{(2\pi f_\eta)^2 m^2_{\eta'} m^2_\rho} \dfrac{b^2\gamma^2}{R^6}$

Because CGC color fields are nearly transverse, the fields in $\mathbf{E}^a \cdot \mathbf{B}^a$ must come from different nuclei. The gluon matrix element can be expressed in terms of the nuclear gluon distribution:

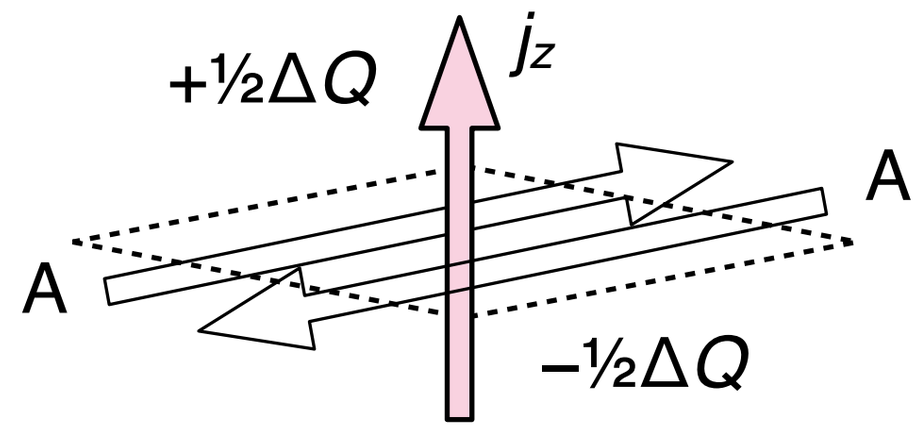
$$\left\langle (\mathbf{E}^a \cdot \mathbf{B}^a)(\mathbf{E}^b \cdot \mathbf{B}^b) \right\rangle \propto [x_0 G(x_0)]^2\, T_{AA}(\bar{\mathbf{x}}_\perp; \mathbf{b})$$

This mechanism can be enhanced if the η′ mass is lowered by medium interactions.

CGC mechanism (II)

The nuclear gluon density can be related to the CGC saturation scale Q_s by:

$$A\left[\xi_0 G(\xi_0)\right] = \frac{(N_c^2-1)\,R^2 Q_s^2}{8\pi^2\alpha_s}$$



To calculate the up-down charge asymmetry fluctuations, we calculate the current through the reaction plane and assume that all charges above are emitted upwards and all charges below are emitted downwards (an overestimate!). After a lengthy calculation involving various additional “reasonable” approximations one finds:

$$\left\langle(\Delta N_{\rm ch})^2\right\rangle = \begin{cases} C\,\frac{9(N_c^2-1)v_f^2}{(8\pi)^3\alpha_s^2}\,Q_s^2\,f(b) \approx 5\times10^{-5}\,v_f^2\,\frac{b^2}{R^2}\,f(b) & \text{(weak flow)} \\ C\,\frac{3(N_c^2-1)}{32\pi^4\alpha_s^2}\,Q_s^3 R\,f(b) \approx 1.7\times10^{-3}\,\frac{b^2}{R^2}\,f(b) & \text{(geometric)} \end{cases}$$

with $f(b) \approx 1-(b^2/R^2)(1-b/4R)^2$

CME effective action

$$L_{eff}^{(\mathrm{CME})} = -\sum_f \frac{e_f^2 g^2}{8\pi^4 T^2} \int_0^{\tau_B} dt' \left(\vec{E}\cdot\vec{B}\right) \int_0^{\tau_{sph}} dt'' \left(\vec{E}^a \cdot \vec{B}^a\right)$$

Minimize action:

$$L = \tfrac{1}{2} f(B) E^2 + L_{eff}^{(\mathrm{CME})} \quad \Rightarrow \quad eE_{\min}$$

Implies momentum shift: $\Delta p = eE_{\min}\tau_B$

Thermal spectrum with transverse flow: $E\dfrac{d^3N}{dp^3} \sim \exp\left(-\gamma[E - \vec{\mathrm{v}}\cdot(\vec{p} - \Delta\vec{p})]/T_f\right)$

Asymmetry: $\dfrac{dN^{(+)}}{dN^{(-)}} \approx e^{2\mathrm{v}\tau_B eE_{\min}/T_f} \quad \Rightarrow \quad \Delta^{\pm} = \dfrac{dN^{(+)} - dN^{(-)}}{dN^{(+)} + dN^{(-)}} \approx \dfrac{\tau_B \mathrm{v}\, eE_{\min}}{T_f\sqrt{N_{\mathrm{domains}}}}$

CME estimate

Using $\tau_B |e\bar{B}| \approx \frac{2.3 Z\alpha b}{R^2}$ gives $|eE_{\min}| \approx \frac{18.4\, Z\alpha^2 b}{\pi f(B) T^2 R^2} \frac{|Q_5|}{V}$

Assume: $|Q_5|/V = \rho^{-3} \approx (0.5\ \text{fm})^{-3}$, $T = 350$ MeV, $Z = 79$, $f(B) = 1$:

$$|eE_{\min}| \approx (20\ \text{MeV})^2 (b/R)$$

Further: $\tau_B = 2R/\gamma_{cm} \approx 0.15$ fm/c, $v = 0.5$ c, $T_f \approx 150$ MeV

Number of domains: $N = \pi R^2/\rho^2 \approx 600$

Final result: $\Delta^{\pm} \approx 3.5 \times 10^{-6} \frac{b}{R}$ (compare with $\Delta^{\pm}_{\text{exp}} \leq 6 \times 10^{-4}$)

“To Do” list

- Other experimental observables need to be studied, e.g.
  - E-by-E charge dipole.
- Beam energy dependence?
  - maximal ***B*** is proportional to energy, but time-integrated ***B*** is constant;
- System size dependence?
  - Central U+U can have $v_2 > 0$, but ***B*** = 0.
- Comprehensive overview of all experimental investigations needed
- If CME is much smaller than STAR effect, how small an effect could be seen with the most sensitive observable?

- Theoretical studies:
  - Realistic calculations of all mechanisms and backgrounds, or at least improved estimates, are needed.
  - Technically feasible, but challenging.

Numerical study of the monolayer graphene phase diagram

P.V. Buividovich, O.V. Pavlovsky, M.V. Ulybyshev, E.V. Luschevskaya, M.A. Zubkov, V.V. Braguta, M.I. Polikarpov

ArXiv:1204.0921; ArXiv:1206.0619

- Charge carriers in graphene and effective field theory
- Calculations on hypercubic lattice
- Calculations on hexagonal lattice

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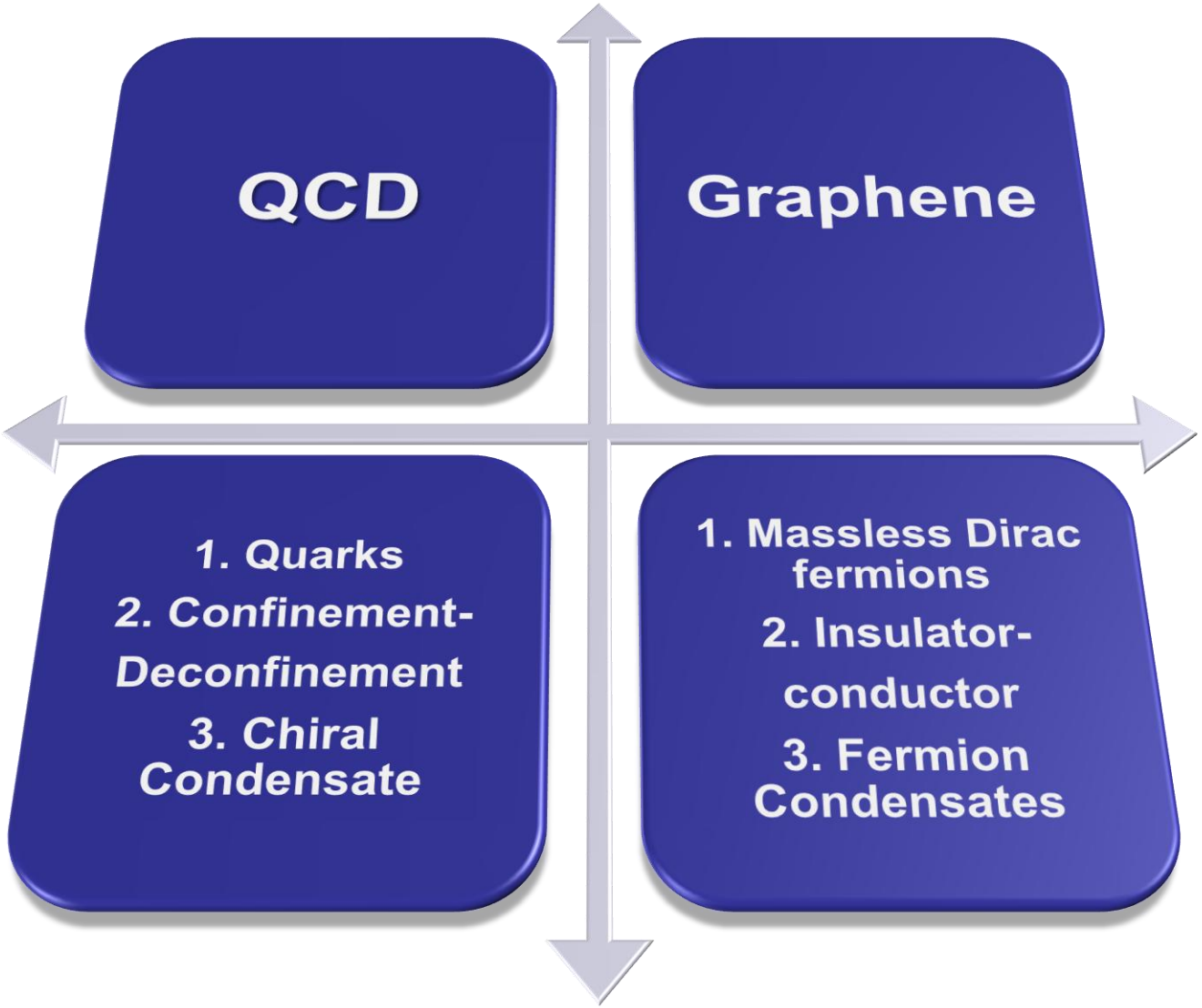
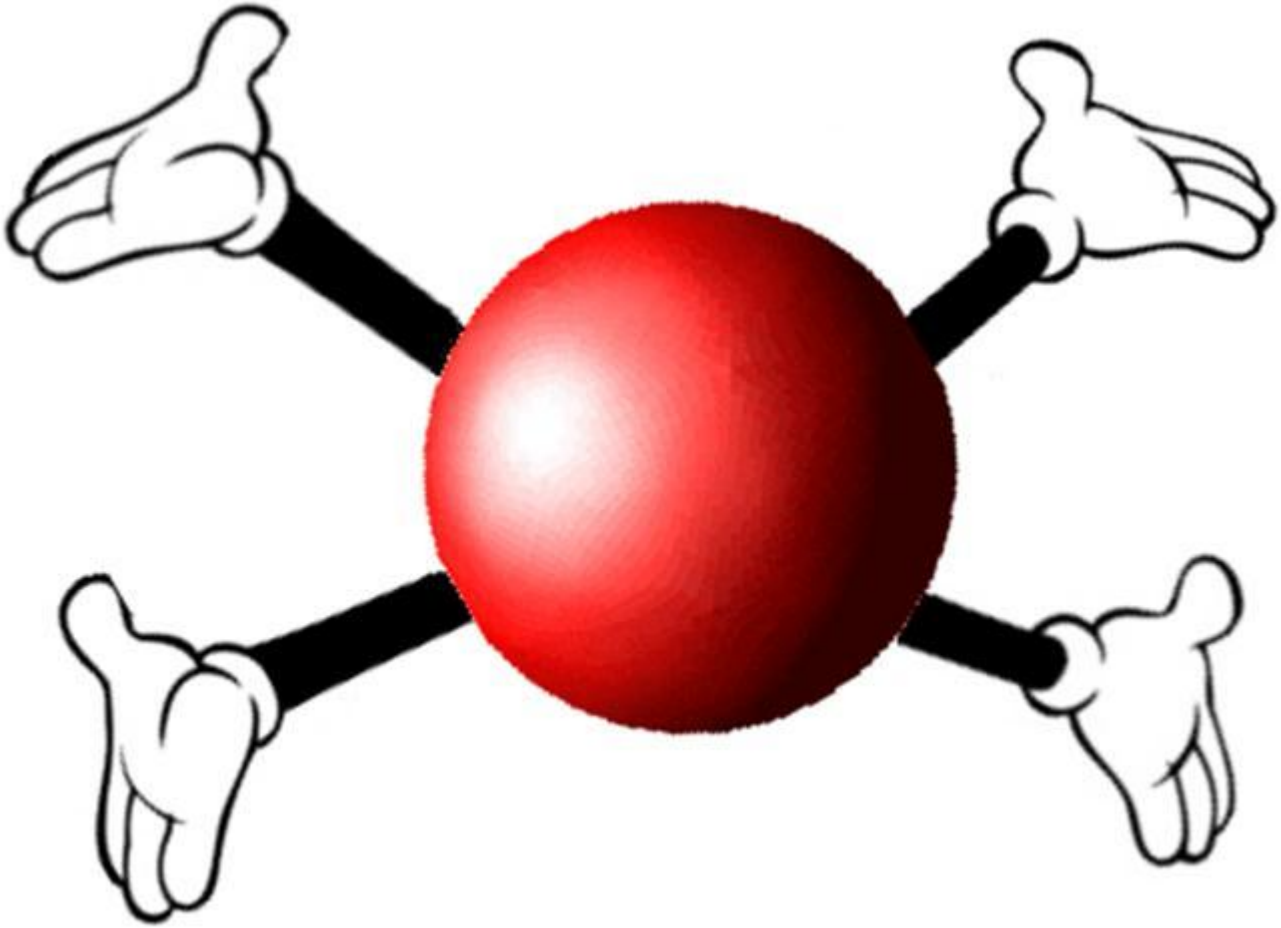
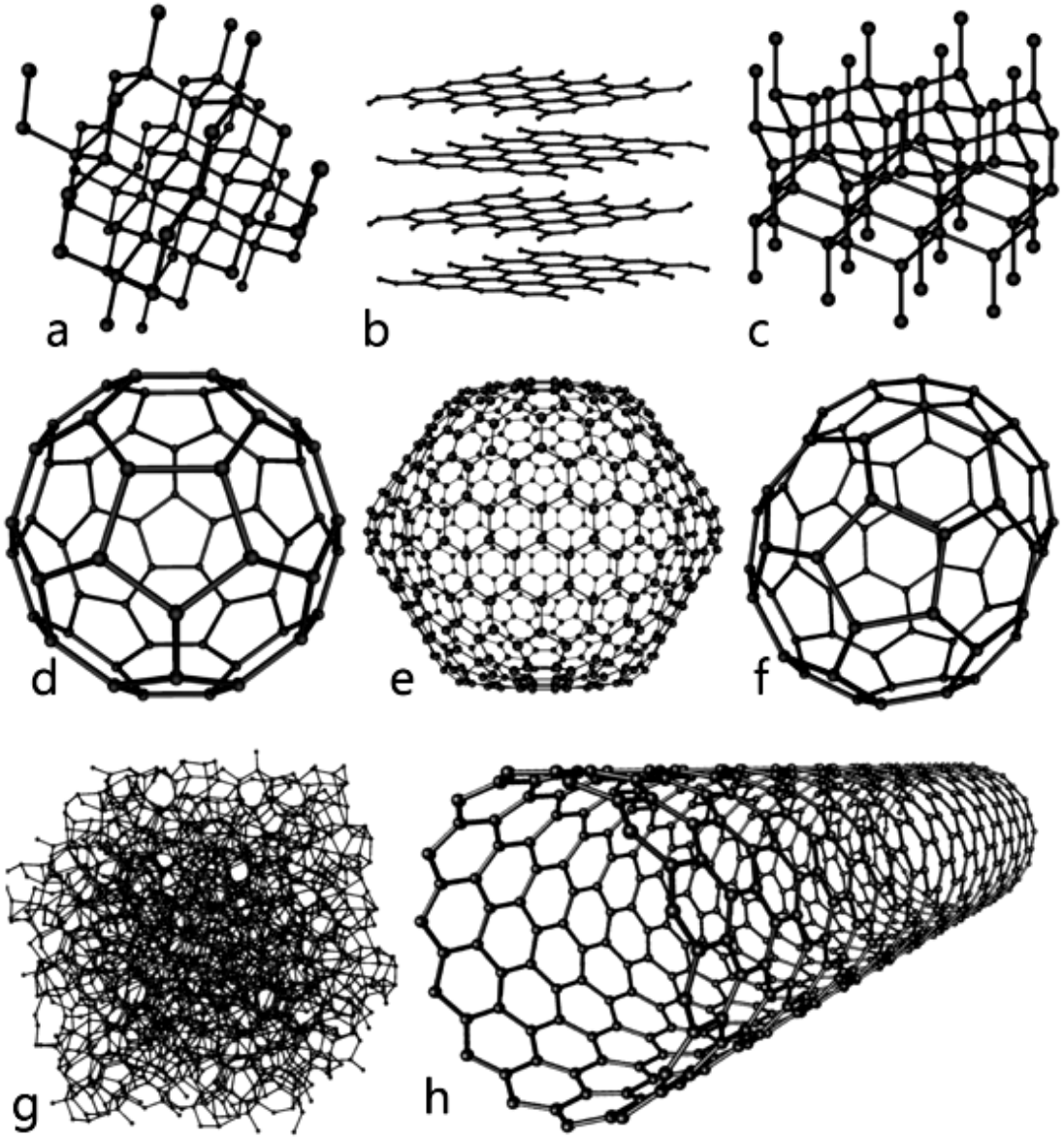
QCD and Graphene



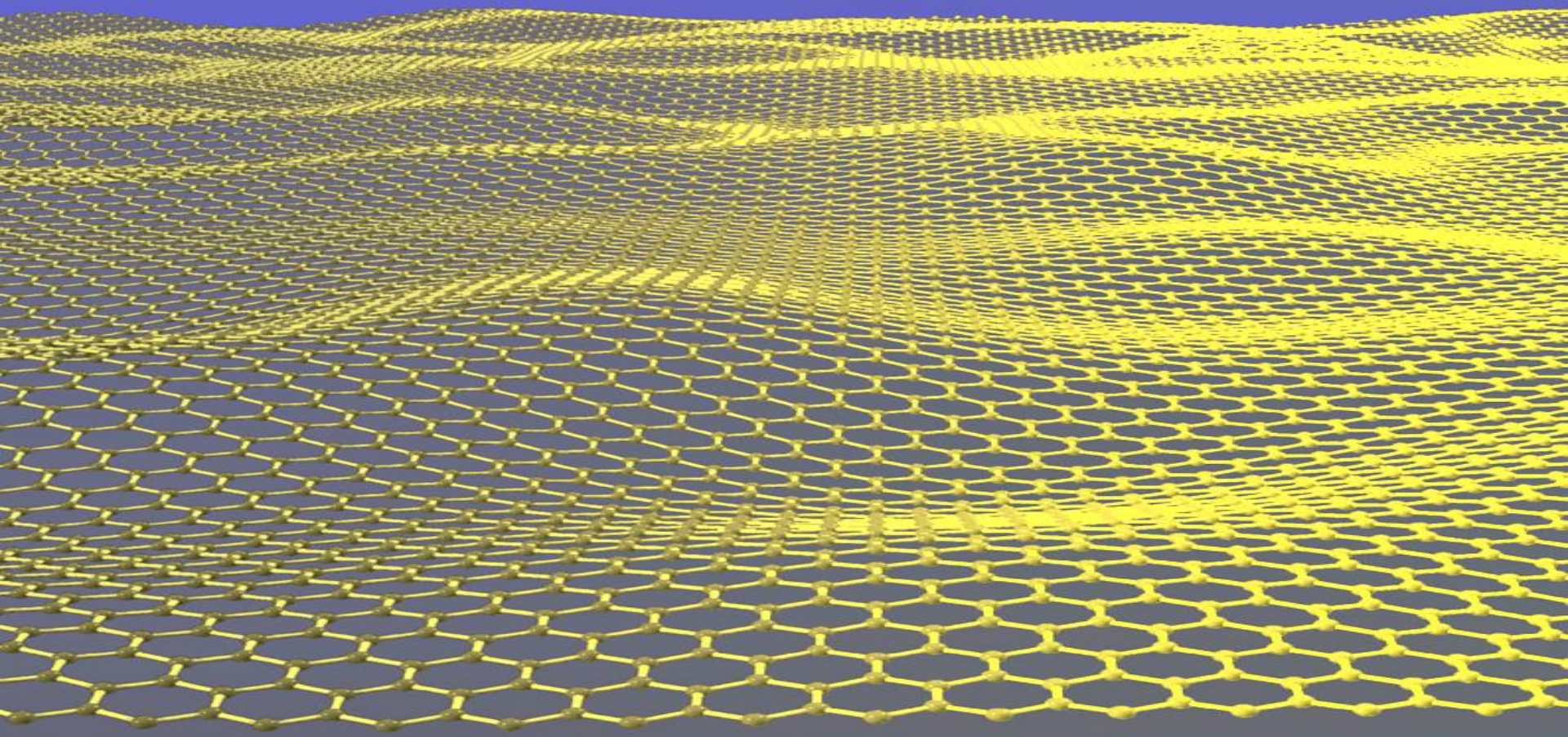
Carbon atom



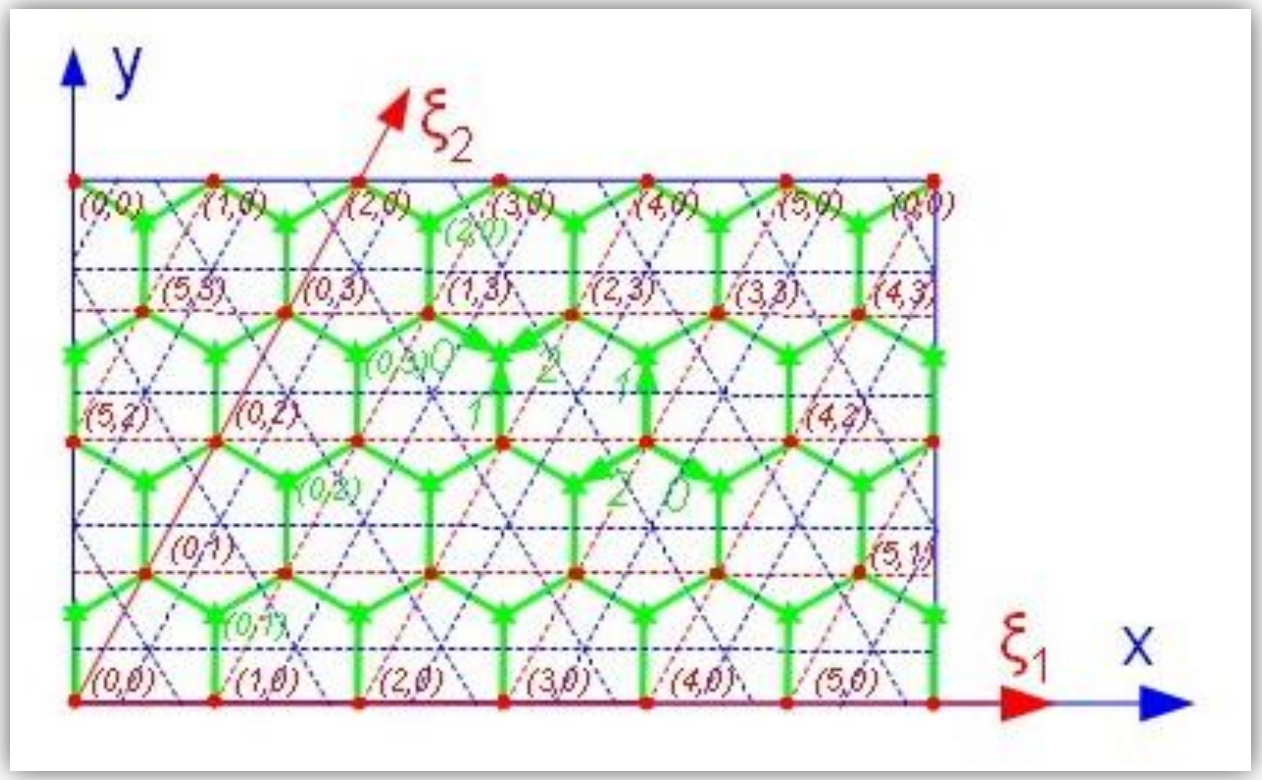


Some allotropes of carbon: a) diamond; b) graphite; c)lonsdaleite; d–f) fullerenes (C_{60}, C_{540}, C_{70}); g) amorphous carbon; h) carbon nanotube.

Graphene



Hexagonal lattice = triangular lattice + triangular lattice



lattice spacing

$$a \approx 0.142\, nm$$

hopping parameter

$$\kappa \approx 2.7\, eV$$

Tight binding Hamiltonian

$$\hat{H}_{tb} = -\kappa \sum_{\sigma=\uparrow,\downarrow} \sum_{<XY>} \left(\hat{a}^{\dagger}_{\sigma,X}\, \hat{a}_{\sigma,Y} + \hat{a}^{\dagger}_{\sigma,Y}\, \hat{a}_{\sigma,X} \right)$$

Relativistic particle

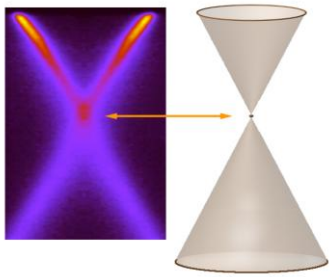
$$E = \sqrt{m^2c^4 + p^2c^2}$$

Massless particle

$$E = cp$$

Charge carrier in Graphene

$$E = v_F p; \quad v_F = \frac{c}{300}$$



Effective field theory for graphene

Four component Dirac fermions + Coulomb field

$$D[A_0] = \gamma_0(\partial_0 + iA_0) + v_F \gamma_i \partial_i, \quad i = 1,2$$

After transformation $t' = v_F t, \quad A_0' = A_0 / v_F.$

we can neglect A_i; $\alpha \to \dfrac{\alpha}{v_F} \approx 300\alpha = \alpha_g$

$$S_E \equiv \frac{1}{2g^2} \int d^3x\, dt\, (\partial_i A_0)^2 - \sum_{a=1}^{N_f} \int d^2x\, dt\, \bar{\psi}_a D[A_0] \psi_a,$$

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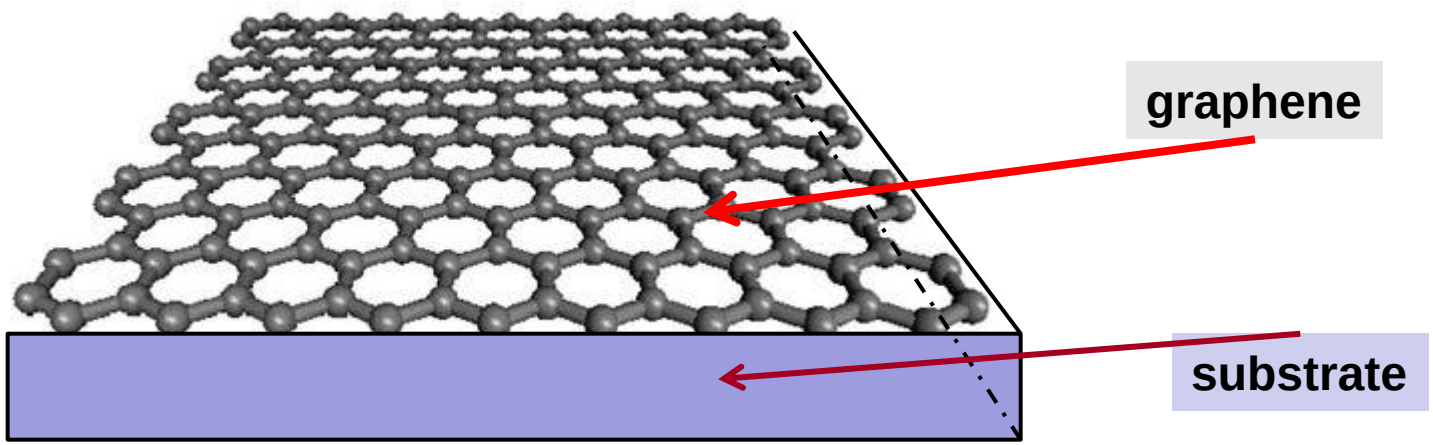
We can vary the effective coupling in graphene!

Graphene in the dielectric media $\alpha_g \to \dfrac{\alpha_g}{\varepsilon}$

Graphene on substrate $\alpha_g \to \dfrac{2}{1+\varepsilon}\alpha_g$

There exists the additional renormalization *T.O. Wehling et al.* arXiv: 1101.4007

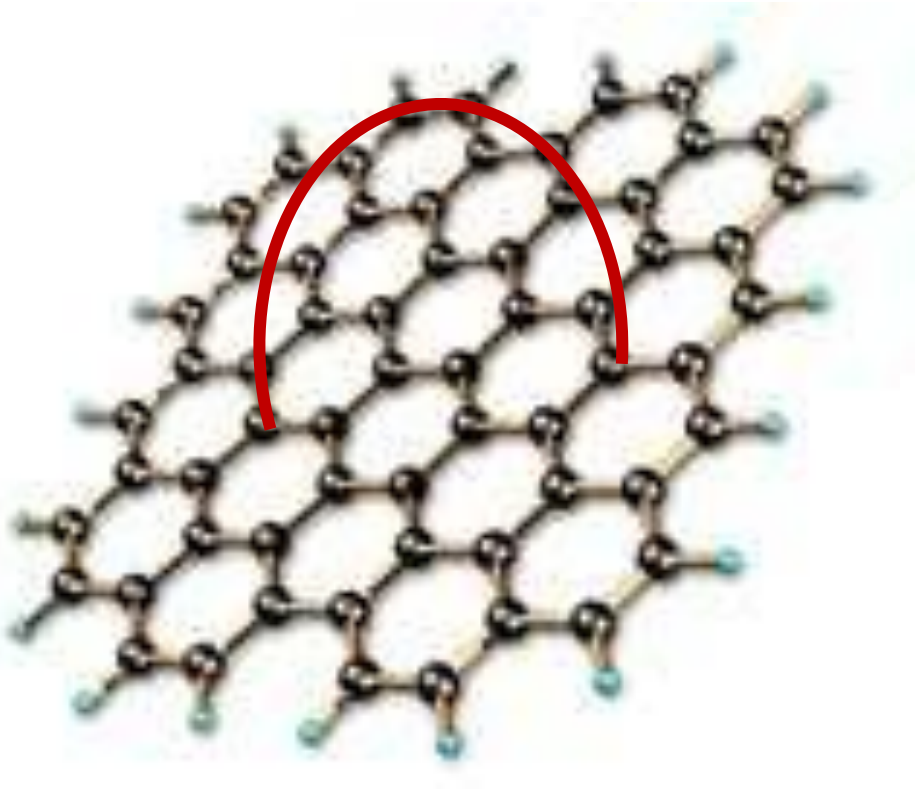


$$\text{if} \quad \frac{2}{1+\varepsilon}\alpha_g > \alpha_g^{crit}(\approx 1.11) \quad \text{graphene is insulator (?)}$$

$$S_E \equiv \frac{1}{2g^2}\int d^3x\,dt\,(\partial_i A_0)^2 - \sum_{a=1}^{N_f}\int d^2x\,dt\,\bar{\psi}_a D[A_0]\psi_a,$$

On substrate

$$\alpha_g \to \frac{2}{1+\varepsilon}\alpha_g$$



(2+1)D fermions

(3+1)D Coulomb

Effective theory of charge carriers in graphene

1. "Massless" four component Dirac fermions

$$D[A_0] = \gamma_0(\partial_0 + iA_0) + v_F \gamma_i \partial_i, \quad i = 1,2$$

2. Fermi velocity is

$$v_F = c/300$$

3. The effective charge is

$$\alpha_g \approx 300\alpha \approx 2.16 > 1$$

4. We can vary the effective charge if we vary the dielectric permittivity of the substrate

$$\alpha_g \to \frac{2}{1+\varepsilon}\alpha_g$$

Vacuum	**ε=1**
SiO_2	**ε ~ 3.9**
SiC	**ε ~ 10.0**

There exists the additional renormalization
T.*O. Wehling et al.* arXiv: 1101.4007

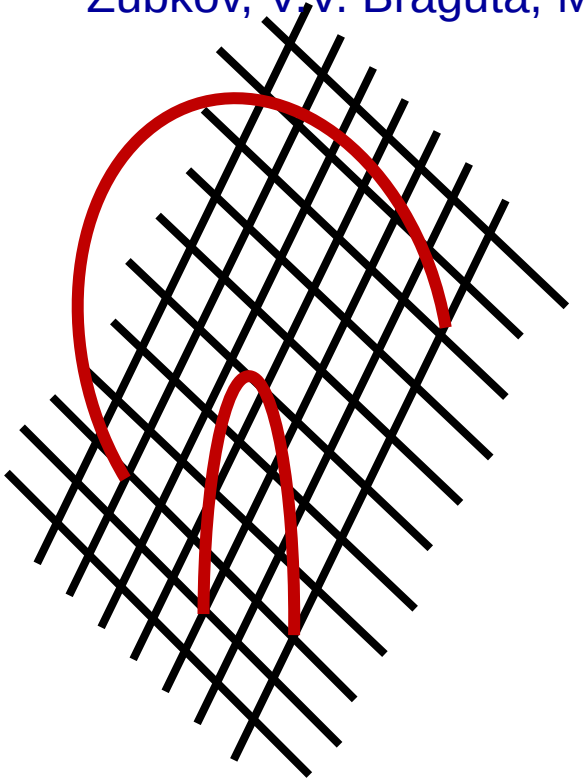
Simulation of the effective graphene theory

Approach 1, hypercubic lattice

J. E. Drut, T. A. Lahde, and E. Tolo (2009-2011)

W. Armour, S. Hands, and C. Strouthos (2008-2011)

P.V. Buividovich, O.V. Pavlovsky, M.V. Ulybyshev, E.V. Luschevskaya, M.A. Zubkov, V.V. Braguta, M.I. P. (2012)



(2+1)D fermions

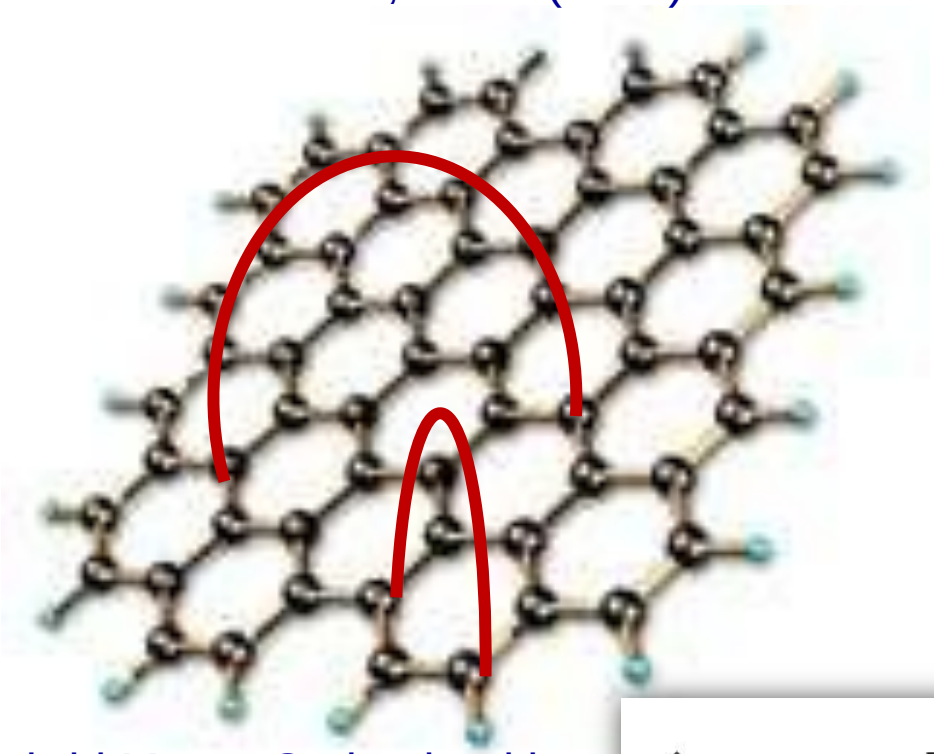
(3+1)D Coulomb

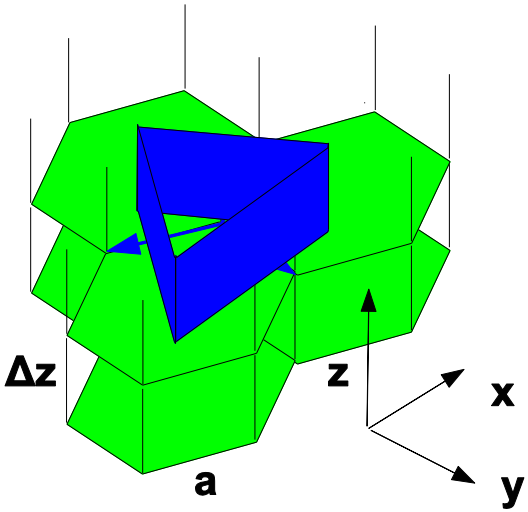
$$S_E \equiv \frac{1}{2g^2}\int d^3x\,dt\,(\partial_i A_0)^2 - \sum_{a=1}^{N_f}\int d^2x\,dt\,\bar{\psi}_a D[A_0]\psi_a,$$

Simulation of the effective graphene theory

Approach 2, 2D hexagonal lattice and rectangular lattice in Z and time dimensions

R. Brower, C. Rebbi, and D. Schaich (2011-2012)
P.V. Buividovich, M.I.P. (2012)





Hybrid Monte-Carlo algorithm for fermions and heat bath for gauge field

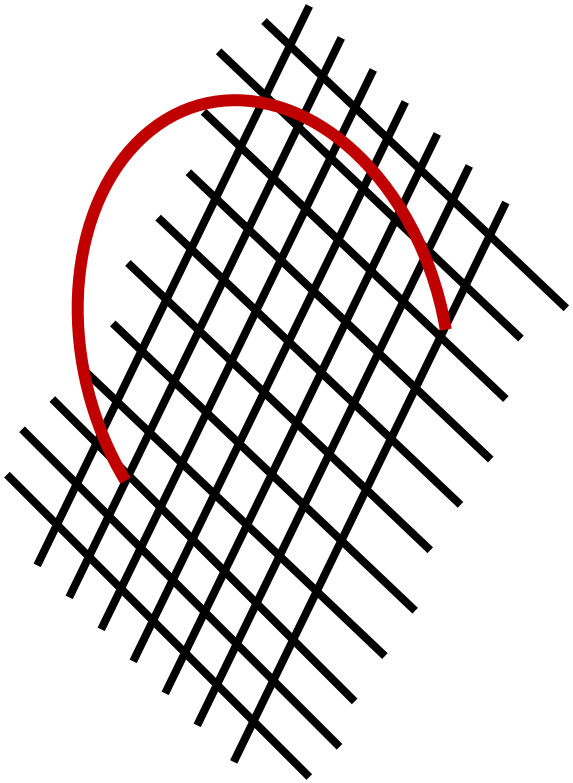
$$\hat{H}_{tb} = -\kappa \sum_{\sigma=\uparrow,\downarrow} \sum_{<XY>} \left(\hat{a}^{\dagger}_{\sigma,X}\, \hat{a}_{\sigma,Y} + \hat{a}^{\dagger}_{\sigma,Y}\, \hat{a}_{\sigma,X} \right)$$

Numerical Results

ArXiv:1204.0921;
ArXiv:1206.0619

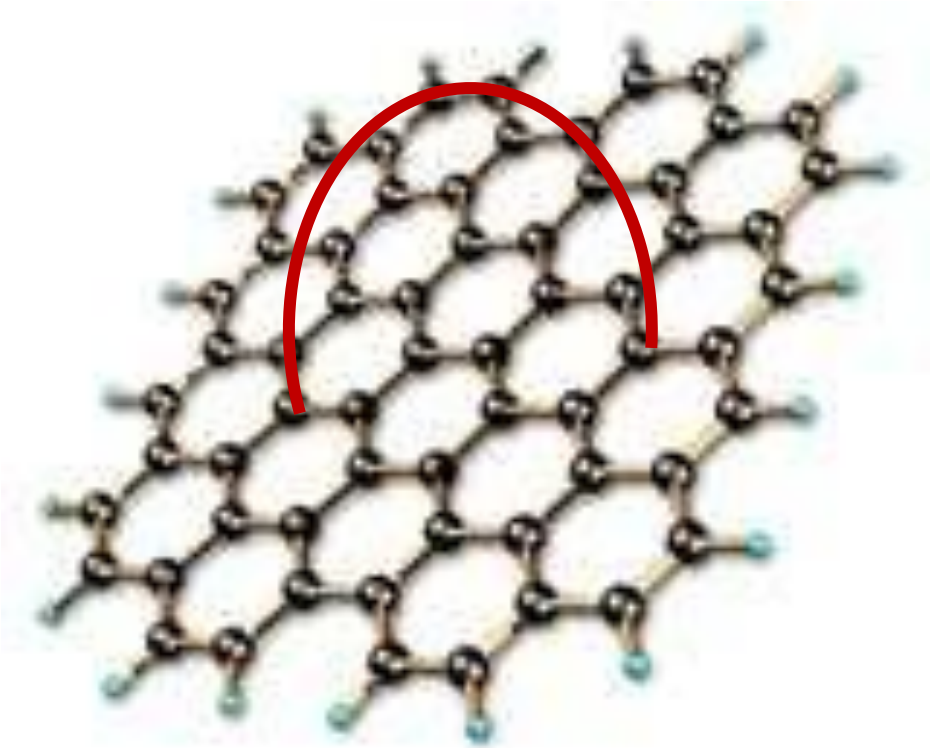
ArXiv:1204.0921

Approach 1

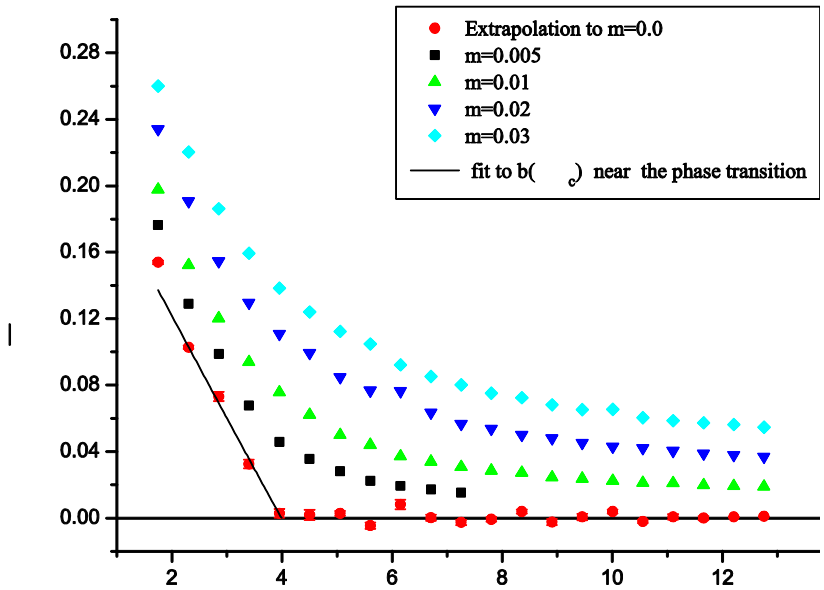


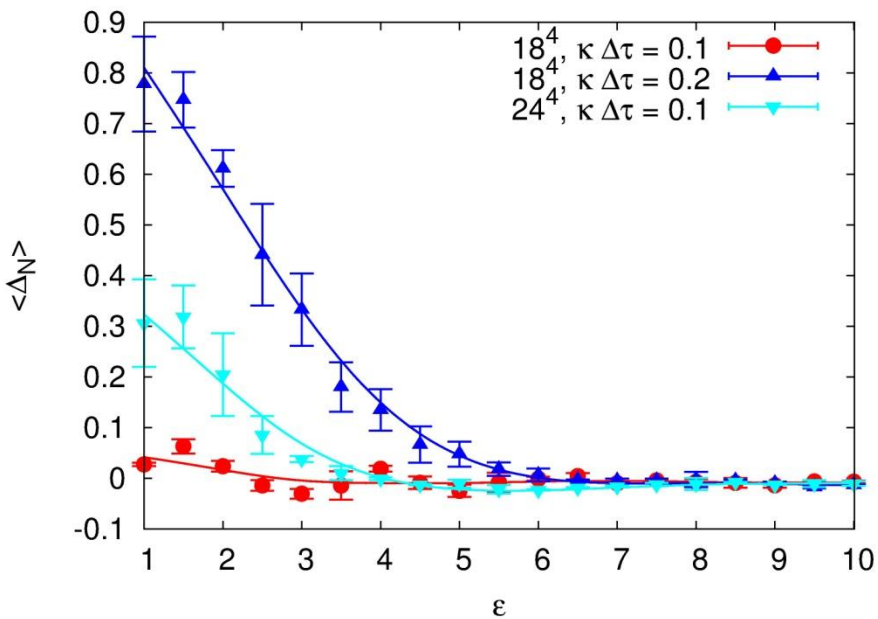
ArXiv:1206.0619

Approach 2



Fermion condensate as the function of substrate dielectric permittivity

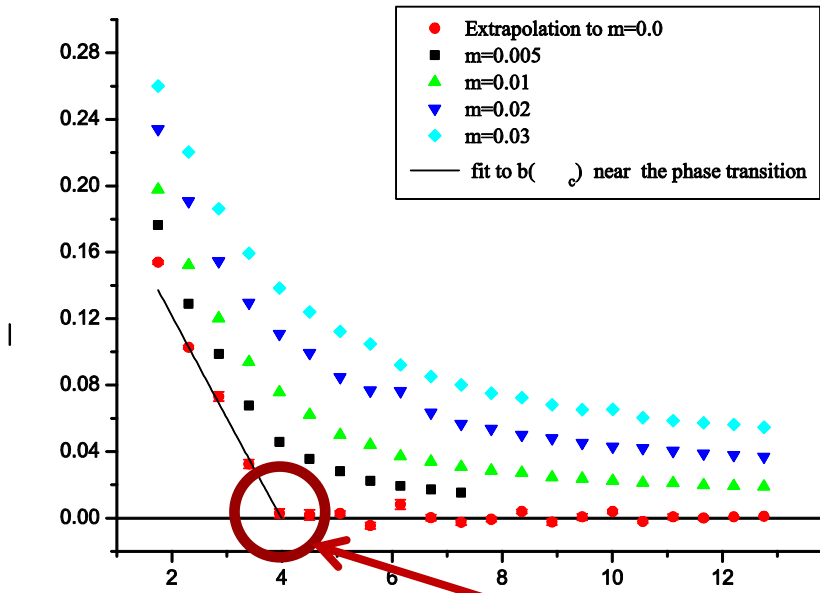


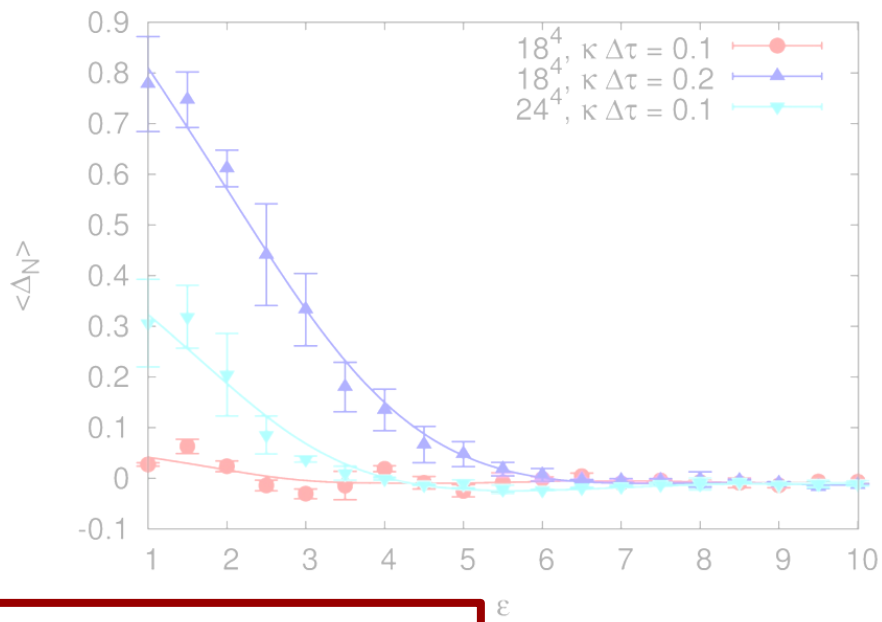


Approach 1
Hypercubic lattice

Approach 2
Hexagonal lattice

Fermion condensate as the function of substrate dielectric permittivity



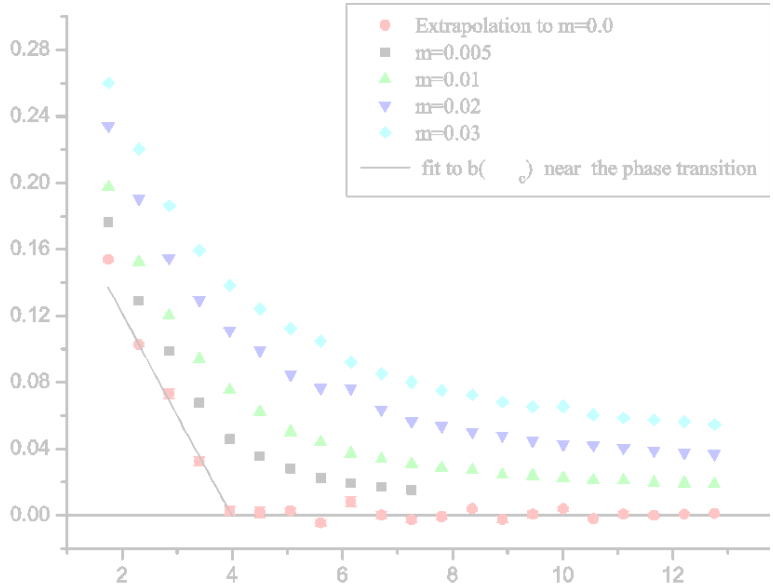


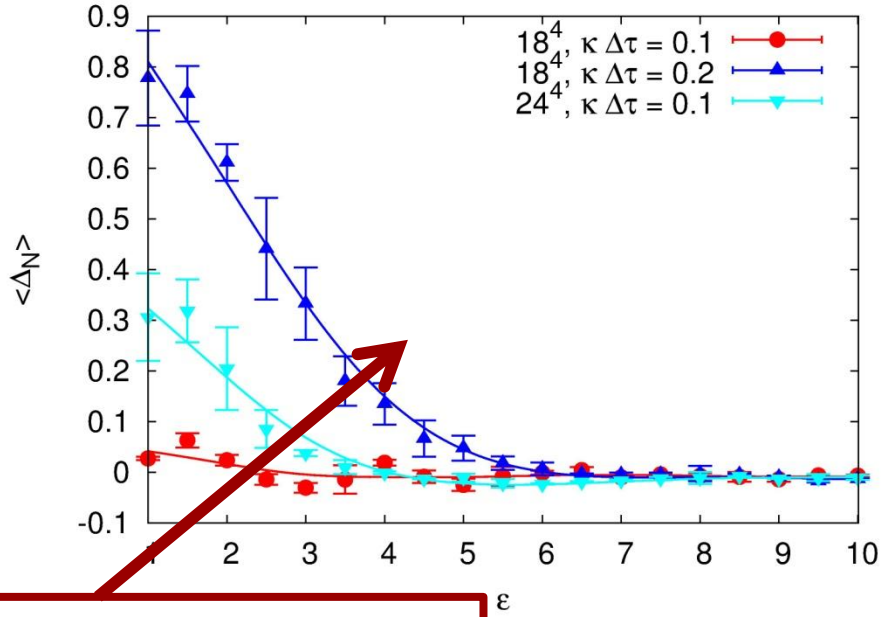
Second order phase transition?

Approach 1
Hypercubic lattice

Approach 2
Hexagonal lattice

Fermion condensate as the function of substrate dielectric permittivity



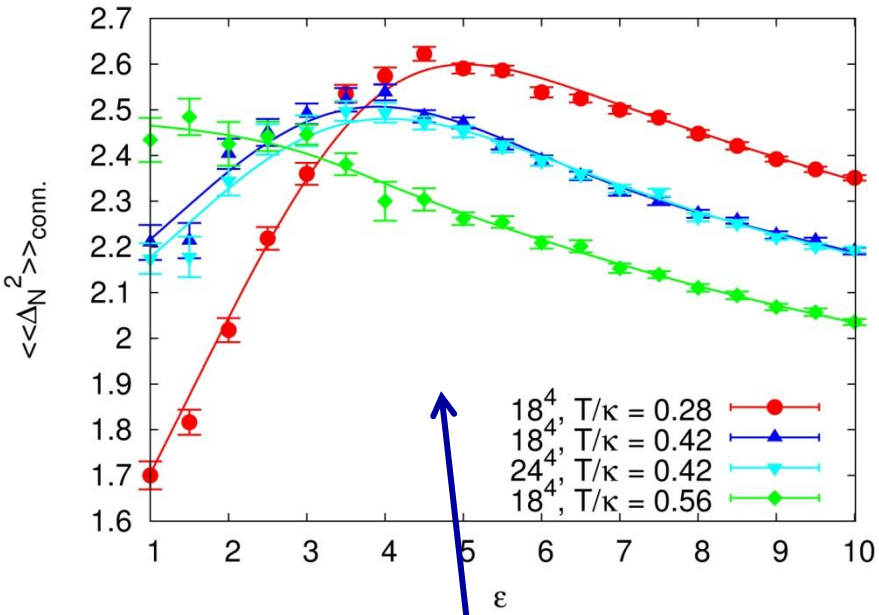


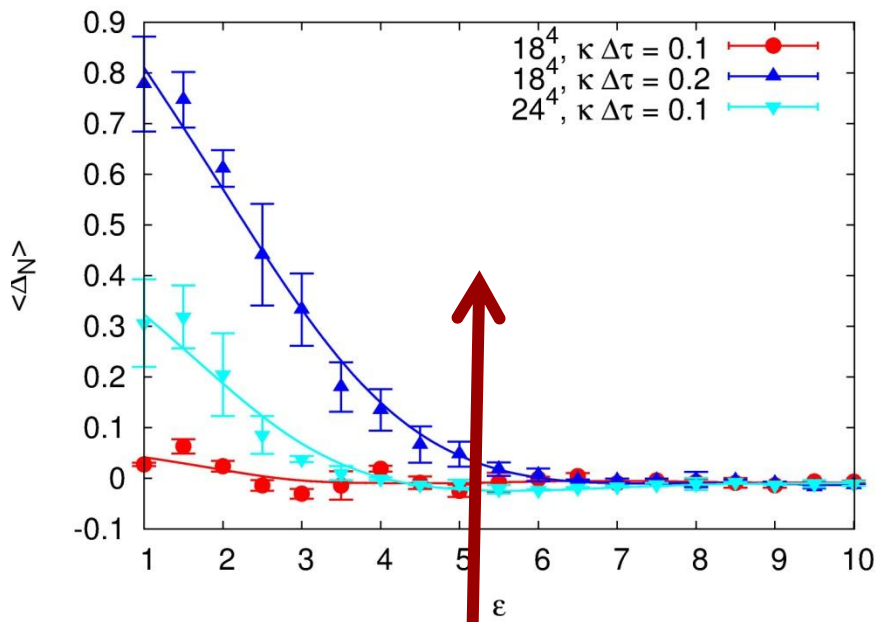
Order of the phase transition?

Approach 1
Hypercubic lattice

Approach 2
Hexagonal lattice

Fermion condensate as the function of substrate dielectric permittivity Hexagonal lattice (Approach 2)

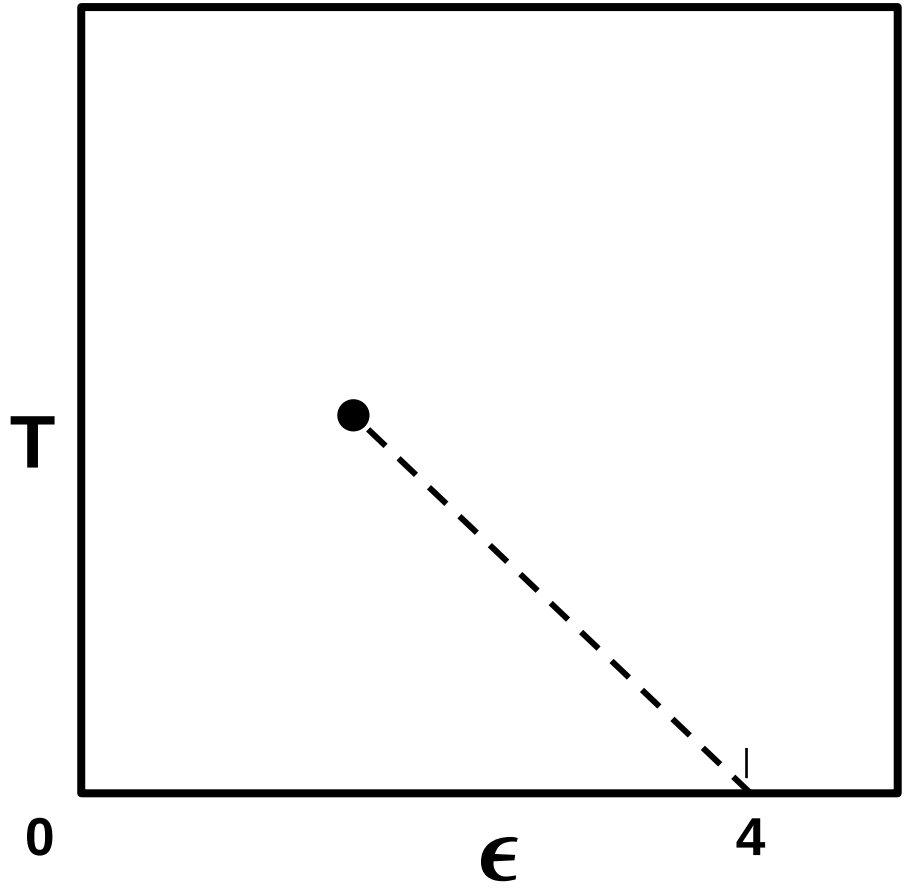




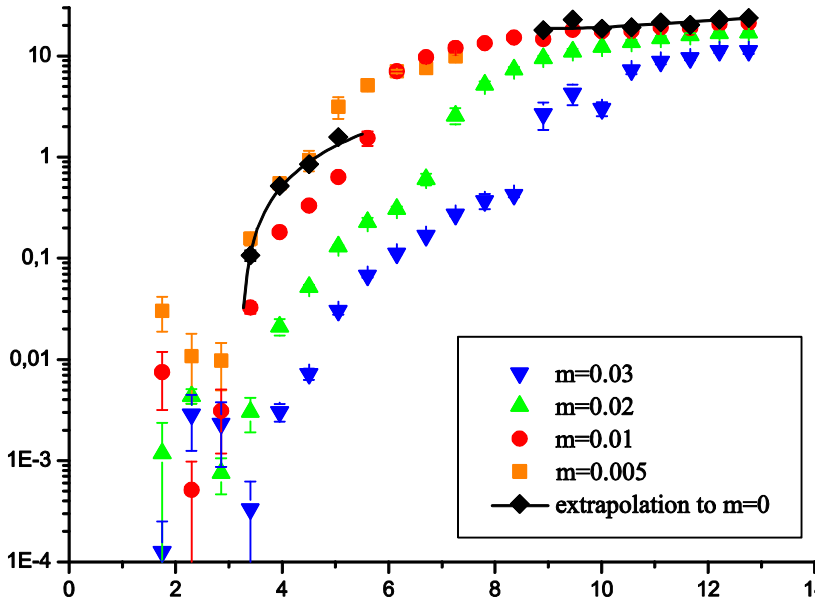
Connected part of the susceptibility of the fermion condensate (no volume dependence!) Crossover?

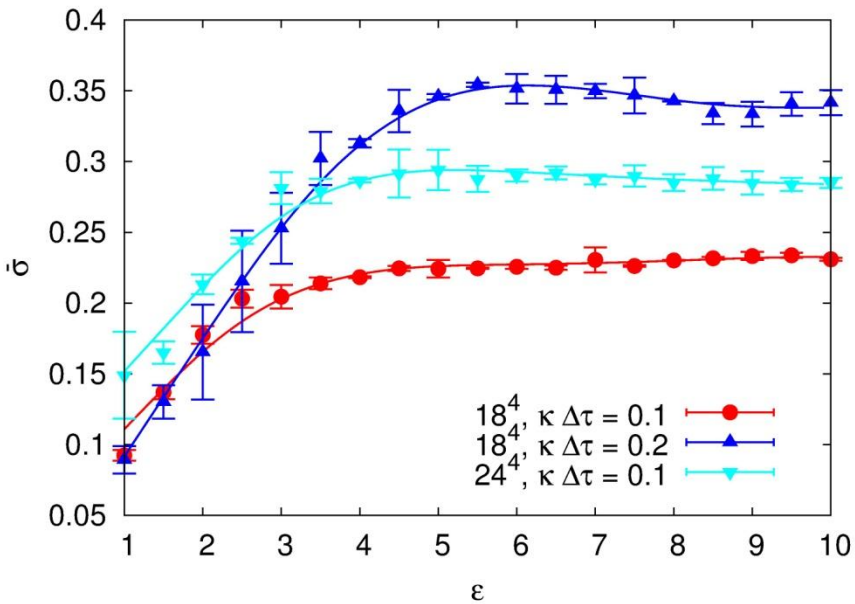
Order of the phase transition? Crossover?

Phase diagram Temperature - dielectric permittivity Hexagonal lattice (Approach 2)



Conductivity as a function of substrate dielectric permittivity



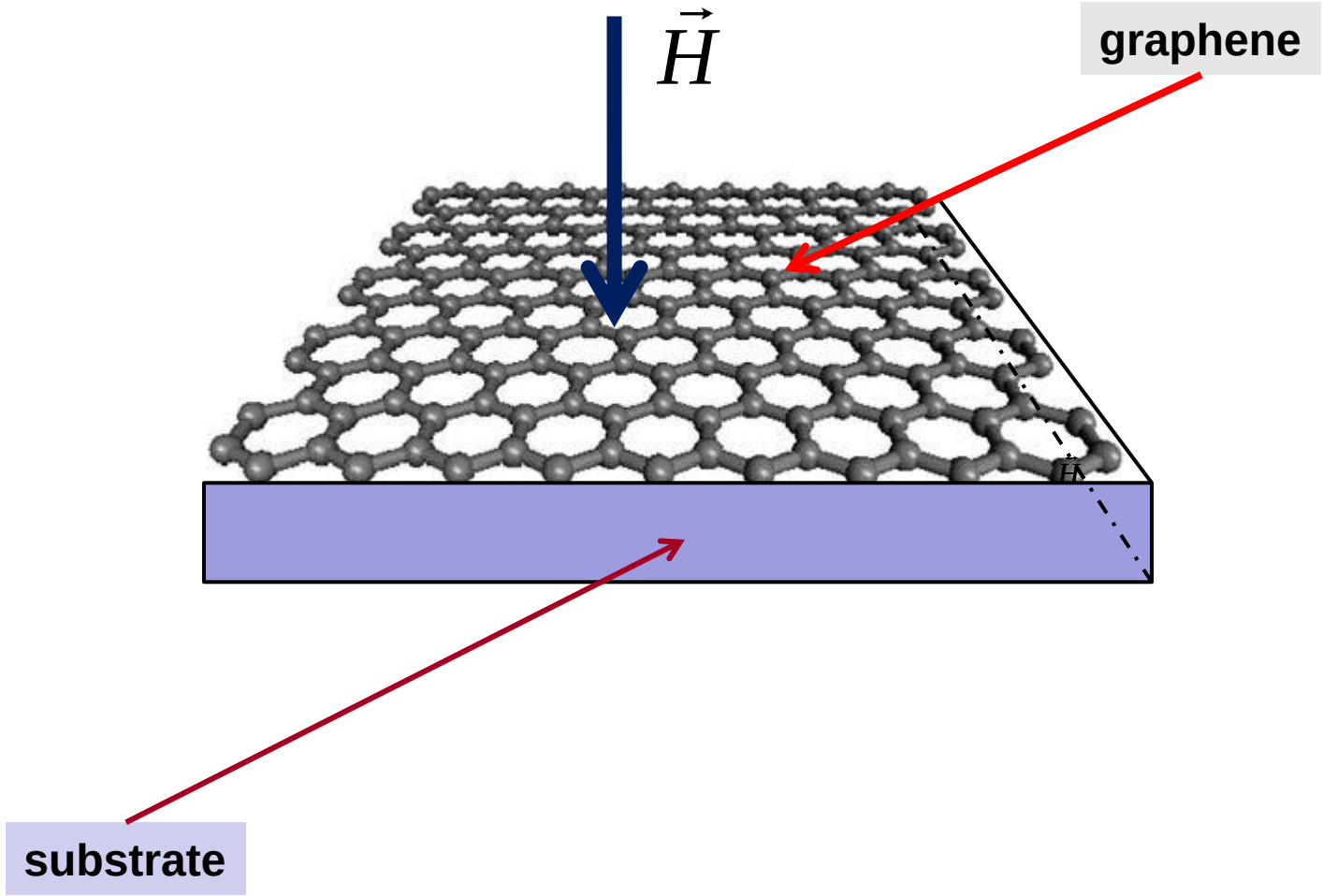


Approach 1
Hypercubic lattice

Approach 2
Hexagonal lattice

Magnetic Field and Graphene

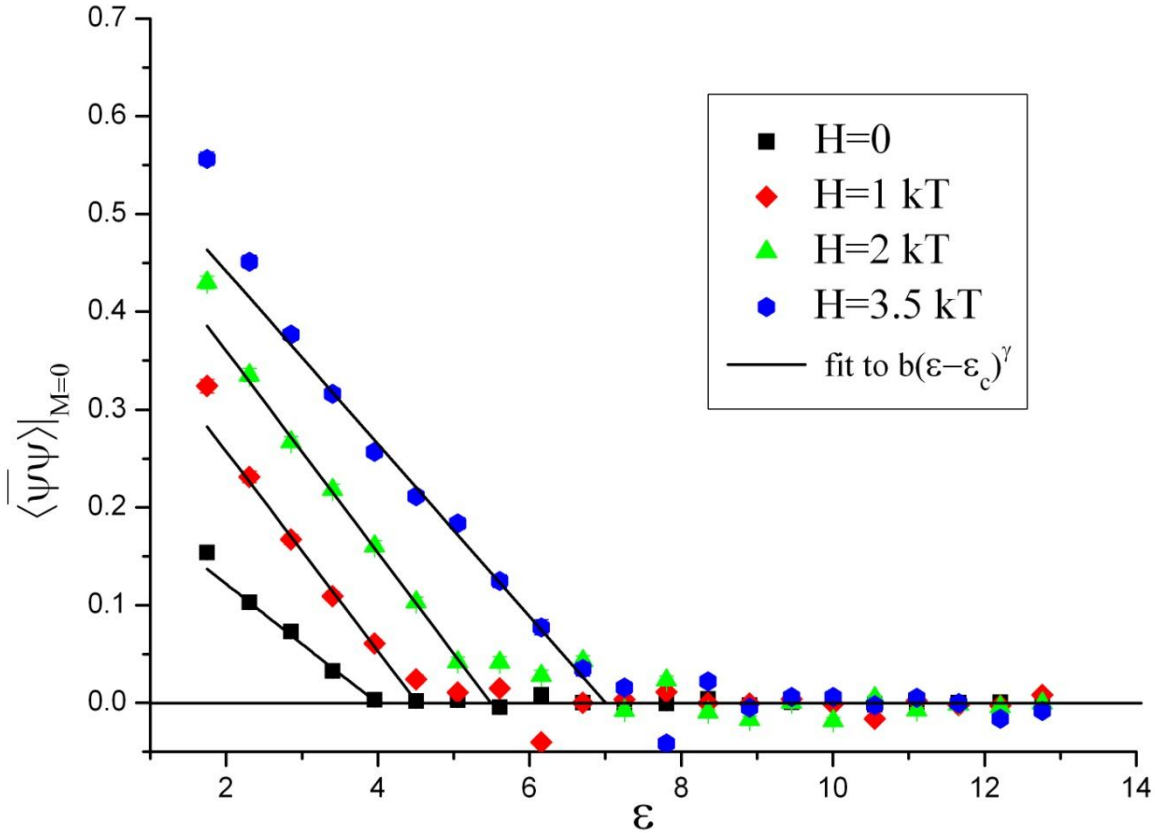
Perpendicular magnetic field



Graphene changes its properties when an external magnetic field is applied, we can numerically simulate all that

Substrate dielectric permittivity - Magnetic field phase diagram

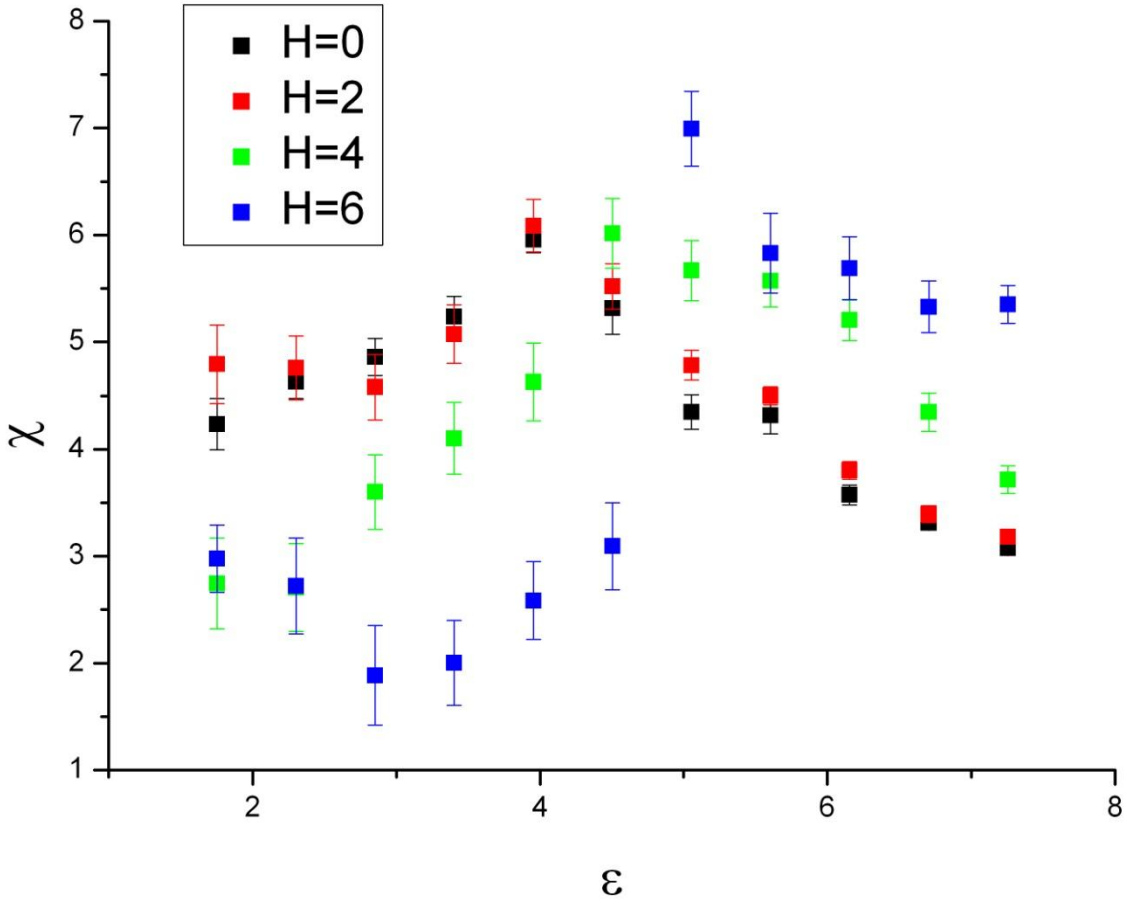
Approach 1 hypercubic lattice *(preliminary)*



Quark condensate vs permittivity for various values of magnetic field

Substrate dielectric permittivity - Magnetic field phase diagram

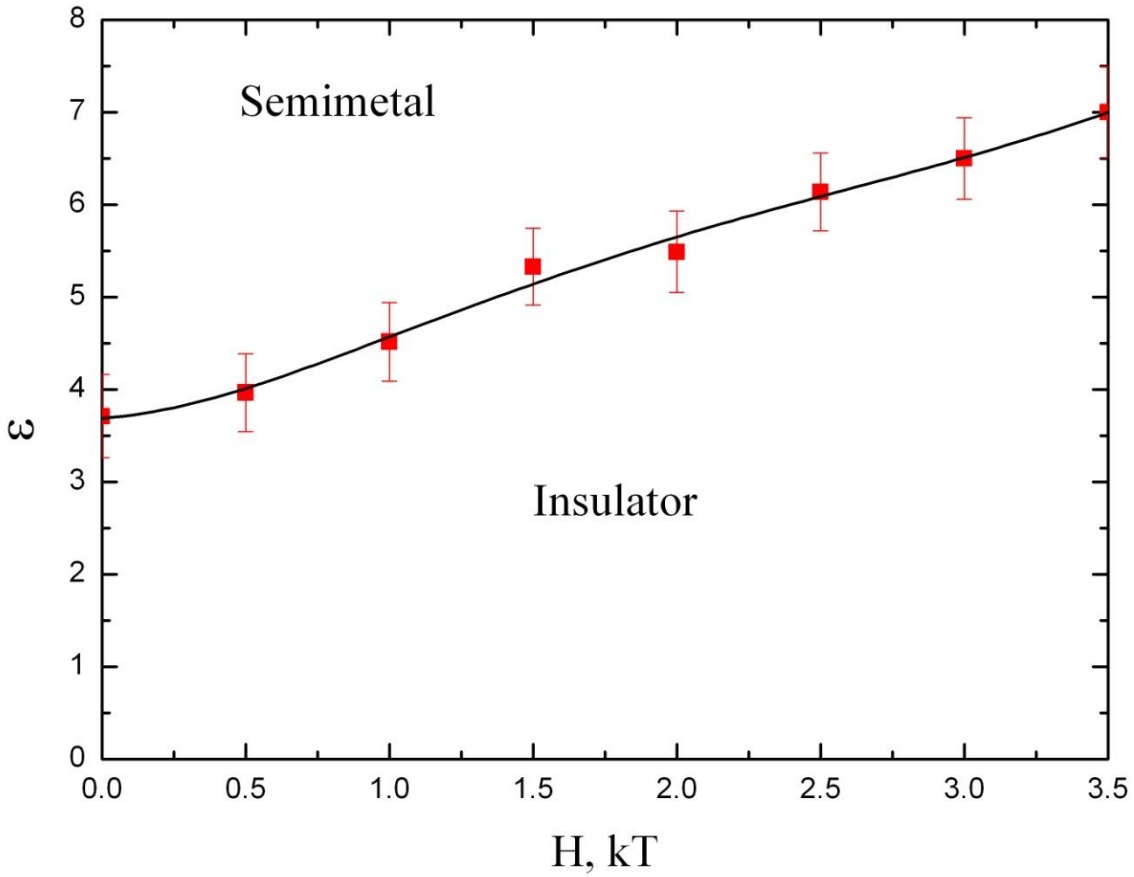
Approach 1 hypercubic lattice *(preliminary)*



Quark condensate suscepsibility vs permittivity for various values of magnetic field

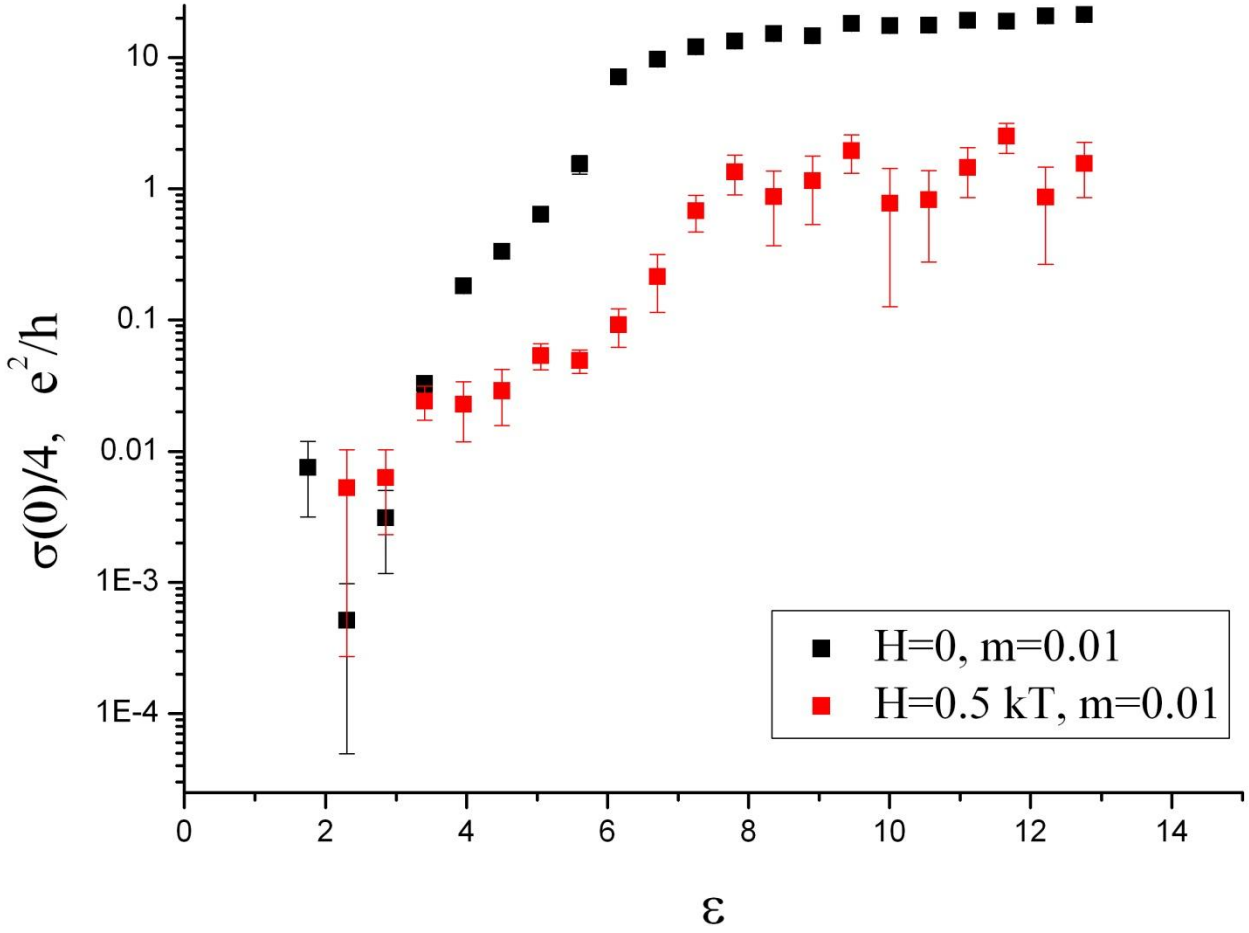
Substrate dielectric permittivity - Magnetic field phase diagram

Approach 1 hypercubic lattice *(preliminary)*



Conductivity at finite magnetic field

Approach 1 hypercubic lattice *(very preliminary)*



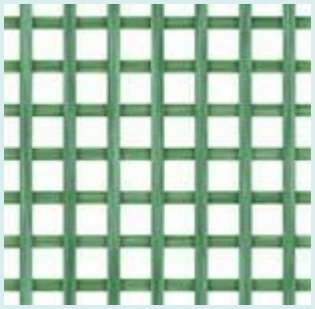
Main Results
(hypercubic and hexagonal lattices)

APPROACH 1

HYPERCUBIC LATTICE

TRANSITION AT $\varepsilon = 4 \pm 1$

SECOND ORDER (?)



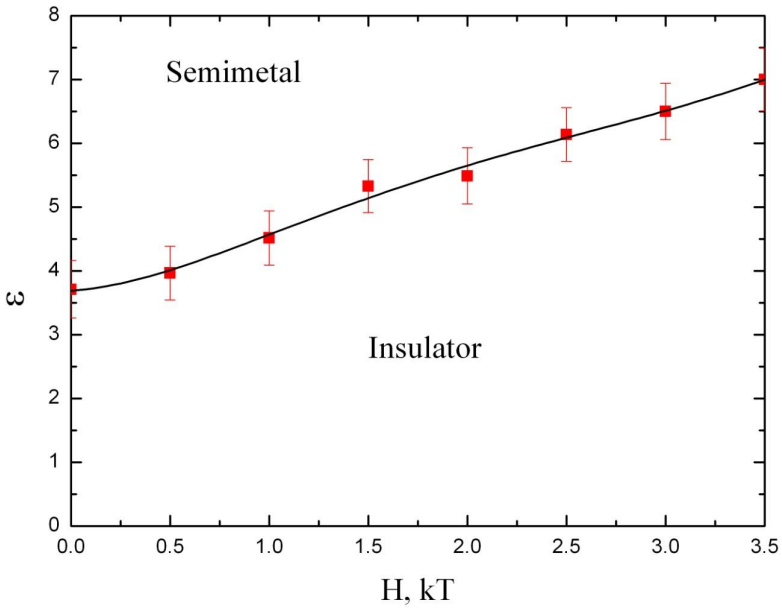
APPROACH 2

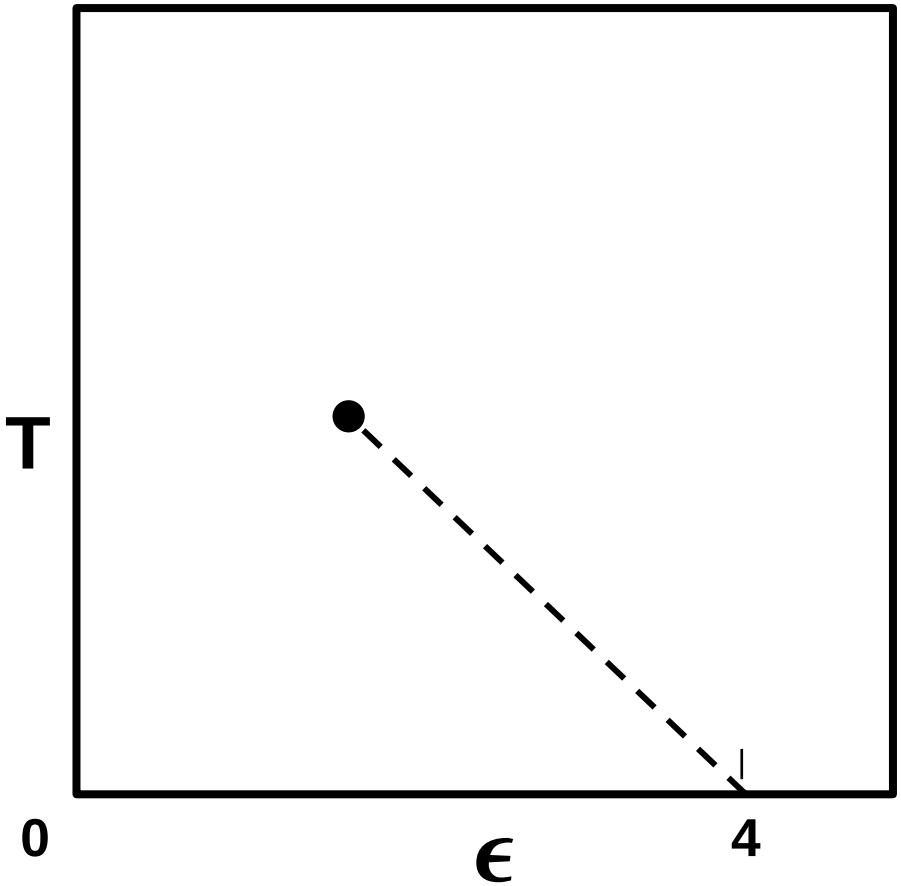
HEXAGONAL LATTICE

TRANSITION AT $\varepsilon = 4 \pm 1$

CROSSOVER (?)

Main Results
(hypercubic and hexagonal lattices)





Our plans

(hypercubic and hexagonal lattices)

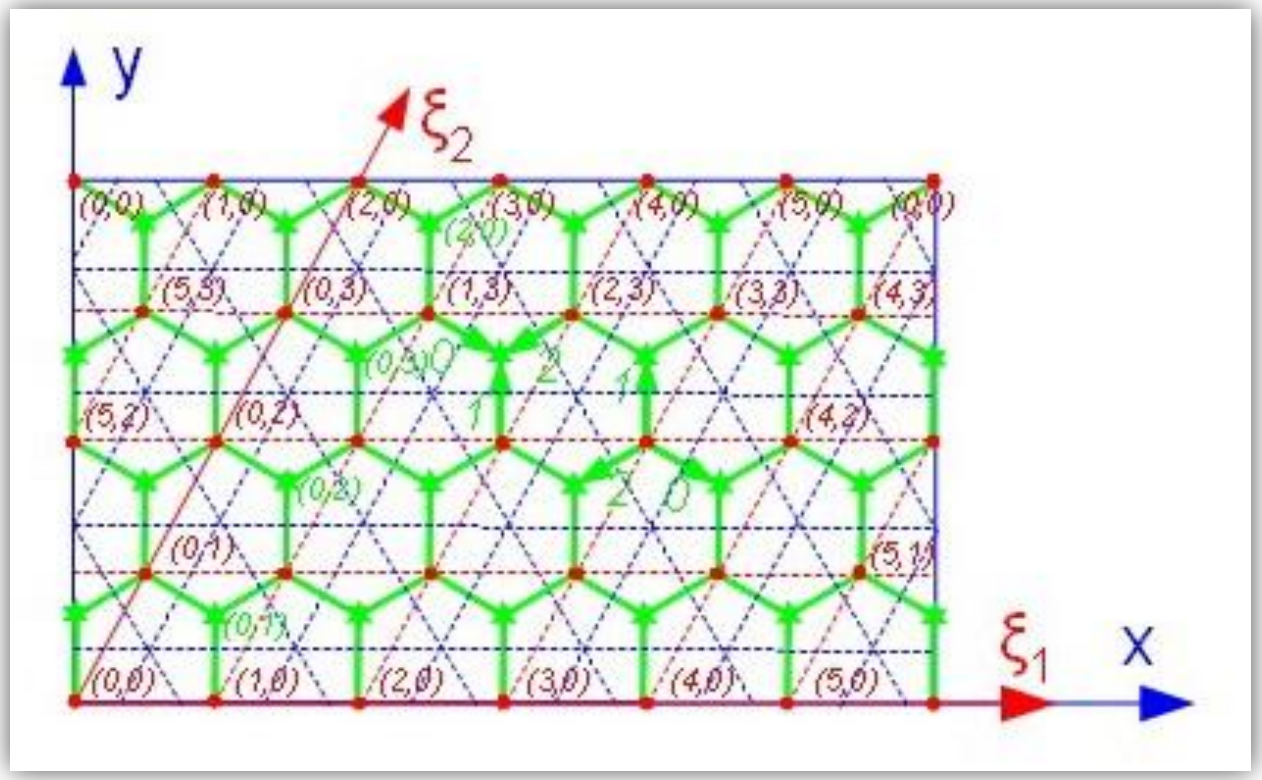
Magnetic field
Finite temperature
Impurities
2-3-4 layers
Conductivity
Viscosity – Entropy
Optical properties
Critical indices

$$E \propto v_F^2$$

Special thanks to
ITEP multiprocessor computers
GRAPHYN and STAKAN

Appendix A
Hamiltonian for charge carriers

Hexagonal lattice = triangular lattice + triangular lattice



lattice spacing

$$a \approx 0.142\, nm$$

hopping parameter

$$\kappa \approx 2.7\, eV$$

Tight binding Hamiltonian

$$\hat{H}_{tb} = -\kappa \sum_{\sigma=\uparrow,\downarrow} \sum_{<XY>} \left(\hat{a}^{\dagger}_{\sigma,X}\, \hat{a}_{\sigma,Y} + \hat{a}^{\dagger}_{\sigma,Y}\, \hat{a}_{\sigma,X} \right)$$

$$\{\hat{a}^{+}_{\sigma,X}, \hat{a}_{\sigma',Y}\} = \delta_{\sigma\sigma'}\delta_{X,Y}$$

Redefinition of creation and annihilation operators

$$\hat{\psi}_{\uparrow,X} = \hat{a}_{\uparrow,X}, \quad \hat{\psi}_{\downarrow,X} = \hat{a}^{\dagger}_{\downarrow,X},$$

Vacuum

$$\hat{\psi}_{\uparrow,X}\,|0\rangle = 0,\ \hat{\psi}_{\downarrow,X}\,|0\rangle = 0$$

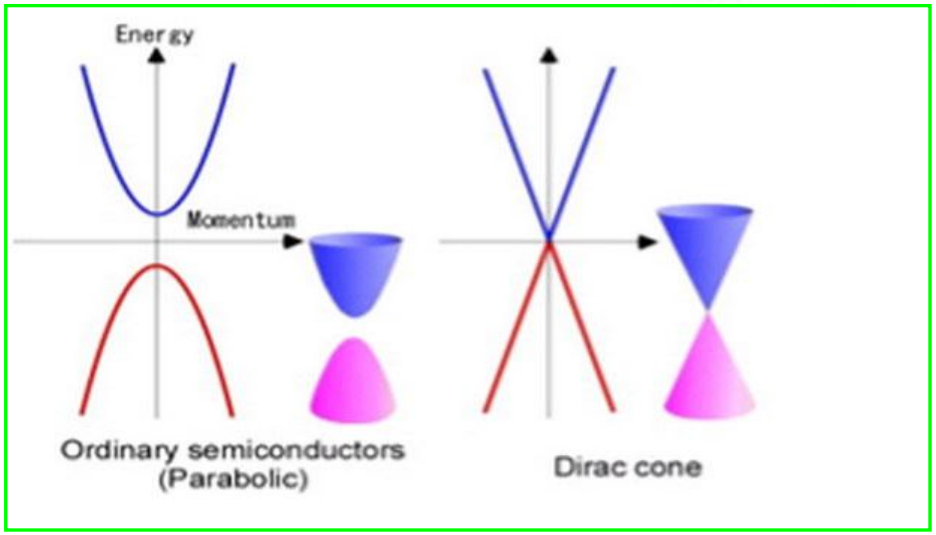
Charge operator

$$\hat{q}_X = \hat{\psi}^{\dagger}_{\uparrow,X}\,\hat{\psi}_{\uparrow,X} - \hat{\psi}^{\dagger}_{\downarrow,X}\,\hat{\psi}_{\downarrow,X}.$$

Free Hamiltonian with regularization (staggered potential *m*)

$$\hat{H}_{tb} = -\kappa \sum_{\sigma=\uparrow,\downarrow} \sum_{<XY>} \left(\hat{\psi}^{\dagger}_{\sigma,X} \hat{\psi}_{\sigma,Y} + \; + \hat{\psi}^{\dagger}_{\sigma,Y} \hat{\psi}_{\sigma,X} \right) +$$

$$\sum_{\sigma=\uparrow,\downarrow} \sum_{X_1} m \, \hat{\psi}^{\dagger}_{\sigma,X_1} \hat{\psi}_{\sigma,X_1} - \sum_{\sigma=\uparrow,\downarrow} \sum_{X_2} m \, \hat{\psi}^{\dagger}_{\sigma,X_2} \hat{\psi}_{\sigma,X_2}$$

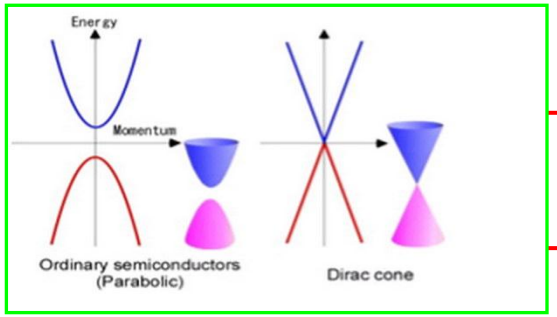


Eigenvalues of the regularized TB Hamiltonian

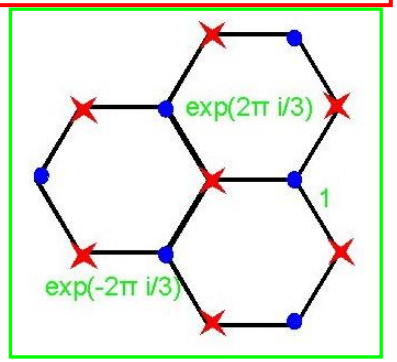
$$E_\zeta(q) \equiv \zeta E(q) = \zeta\sqrt{m^2 + \kappa^2 |\Phi(q)|^2},$$

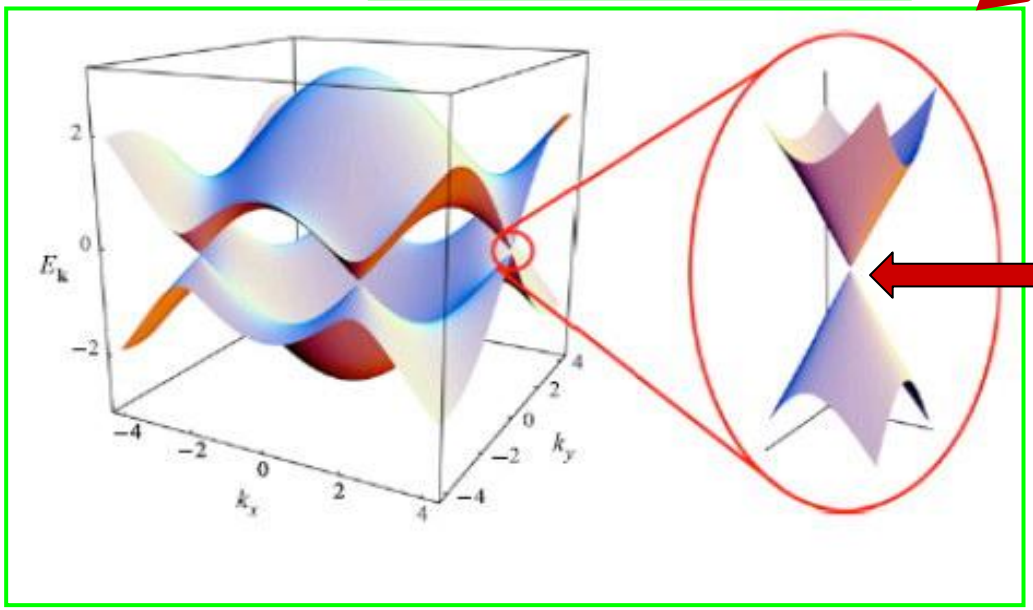
$$\Phi(q) = \sum_{a=1}^{3} e^{i\vec{q}\cdot\vec{e}_a}$$

$$\varsigma = \pm 1$$



$$m \to 0$$





$$E(q) = v_F |\vec{q}|$$

$$v_F = \frac{3}{2}\kappa a$$

Fermi velocity (velocity at Fermi point)

$$E(q) = v_F \left|\vec{q}\right|$$

$$v_F = \frac{3}{2}\kappa a \approx c/300$$

$$a \approx 0.142\, nm \qquad \kappa \approx 2.7\, eV$$

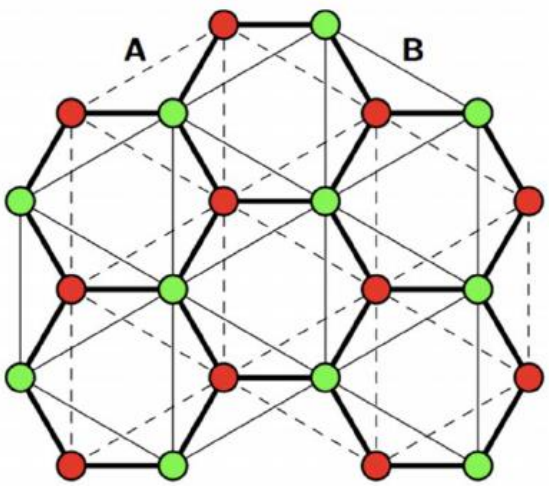
Hamiltonian with Coulomb interaction

$$\hat{H} = \hat{H}_{tb} + \hat{H}_{I} + \hat{H}_{em}$$

$$\hat{H}_I = \sum_{X,Y} e^2/r\,(X,Y)\;\hat{q}_X\,\hat{q}_Y\,,$$

$$\hat{H}_{em} = \frac{1}{8\pi}\int d^3\vec{r}\left(\hat{E}^2\,(\vec{r}) + \left(\mathrm{rot}\,\hat{A}\,(\vec{r})\right)^2\right)$$

$$\hat{H}_{tb} = -\kappa \sum_{\sigma=\uparrow,\downarrow}\sum_{<XY>}\left(\hat{\psi}^{\dagger}_{\sigma,X}\,\hat{\psi}_{\sigma,Y} + +\hat{\psi}^{\dagger}_{\sigma,Y}\,\hat{\psi}_{\sigma,X}\right) + \sum_{\sigma=\uparrow,\downarrow}\sum_{X_1} m\,\hat{\psi}^{\dagger}_{\sigma,X_1}\hat{\psi}_{\sigma,X_1} - \sum_{\sigma=\uparrow,\downarrow}\sum_{X_2} m\,\hat{\psi}^{\dagger}_{\sigma,X_2}\hat{\psi}_{\sigma,X_2}$$



Appendix B
Temperature of the charge carriers
Approach 2

Temperature of the charge carriers

$$T = \frac{1}{\Delta\tau\, L_t}$$

$L^4 = 18^4;\ \kappa\Delta\tau = 0.1;\ T = 0.56\ \kappa = 1.51\ \text{eV} = 1.8\cdot 10^4\,\text{K}$

$L^4 = 18^4;\ \kappa\Delta\tau = 0.2;\ T = 0.28\ \kappa = 0.76\ \text{eV} = 8.8\cdot 10^3\,\text{K}$

$L^4 = 24^4;\ \kappa\Delta\tau = 0.1;\ T = 0.42\ \kappa = 1.13\ \text{eV} = 1.3\cdot 10^4\,\text{K}$

Appendix C
Green functions in momentum space
Approach 1; arXiv:1205.0888

Green functions

M. V. Ulybyshev, M. A. Zubkov; **arXiv:1205.0888**

$$\mathbf{G} = \langle \psi_x^\dagger \psi_y \rangle = \frac{1}{Z} \int D\psi^\dagger D\psi DA \psi_x^\dagger \psi_y \exp\Big(-\frac{1}{2} \int d^4x [\partial_I A_4]^2$$
$$- \int d^3x \bar{\psi}([\partial_4 - igA_4]\Gamma^4 + \partial_a \Gamma^a \psi \Big),$$

$$\tilde{\mathbf{G}} = -i\mathbf{G}\Gamma_4. \qquad \tilde{\mathbf{G}} = g_1\Gamma_1 + g_2\Gamma_2 + g_3\Gamma_4 + g_0$$

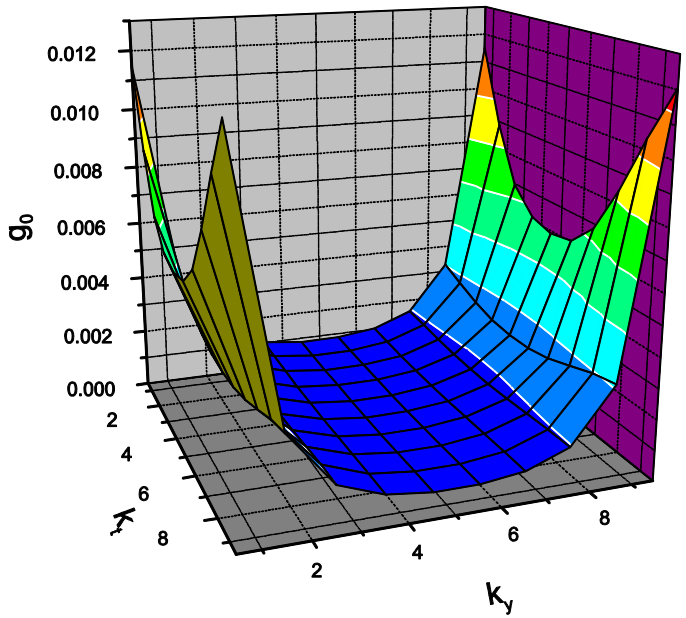
$$g_a(k) = \frac{i}{16N_1^2N_2^2N_t^2} \sum_{y,z} e^{ik(z-y)} \sum_{\eta,\eta'} (-1)^{\eta_1+\ldots+\eta_{a-1}}$$
$$\delta(\eta_i' - [\eta_i + \delta_{ia}] \bmod 2) \langle G(2y+\eta, 2z+\eta') \rangle$$

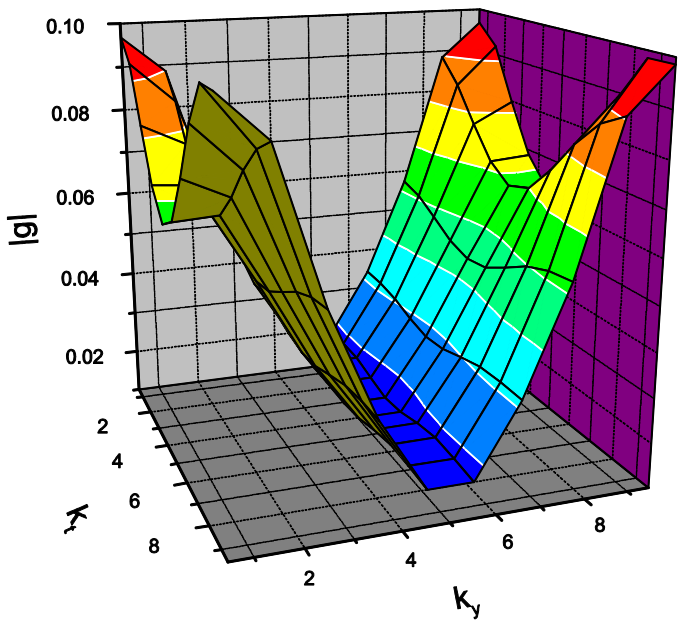
$$g_0(k) = \frac{i}{16N_1^2N_2^2N_t^2} \sum_{y,z} e^{ik(z-y)} \sum_{\eta,\eta'} \delta(\eta_i' - [\eta_i + \delta_{ia}] \bmod 2) \langle G(2y+\eta, 2z+\eta') \rangle$$

Deep in the Semi-metal phase

arXiv:1205.0888

M. V. Ulybyshev, M. A. Zubkov

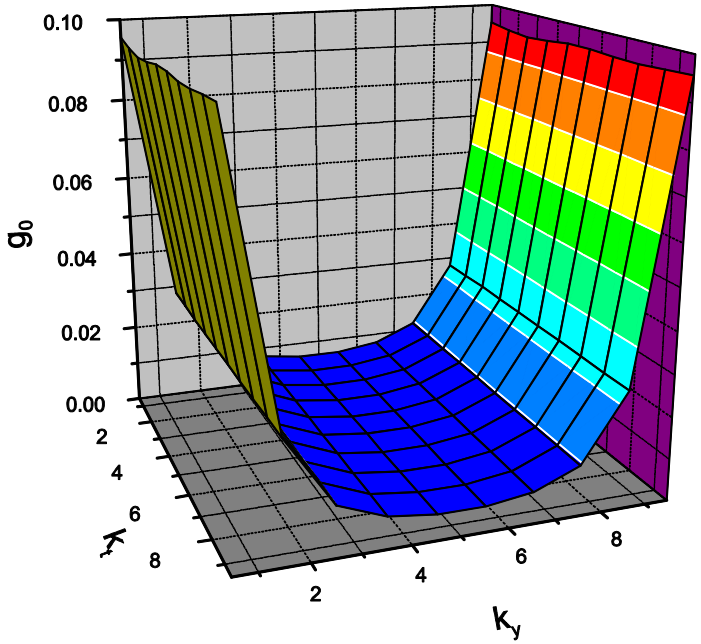


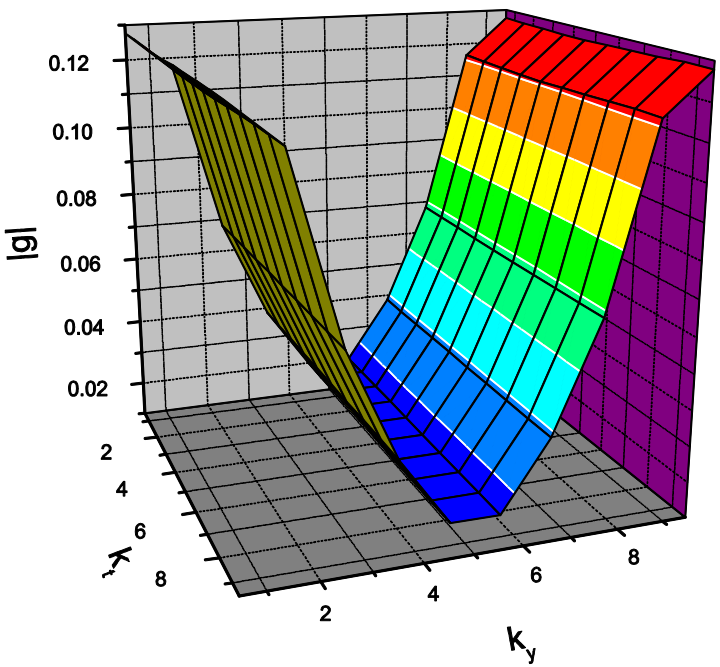


$$|g| = \sqrt{g_a g_a}, \quad a = 1, 2, 3$$

Deep in the insulator phase

no dependence on energy →different time slices do not correlate →energy of the fermion excitation is infinite





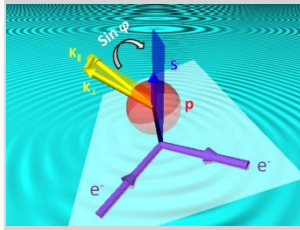
$$|g| = \sqrt{g_a g_a}, \quad a = 1, 2, 3$$

Local $\mathcal{P}$-Odd Effects Through QCD Instantons

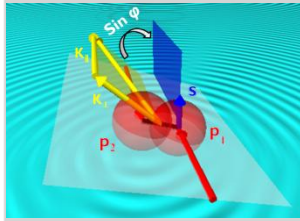
Authors: Yachao Qian
Ismail Zahed
Speaker: Yachao Qian
Stony Brook University

Guideline

Semi-Inclusive Deep Inelastic Scaterring



$p_{\uparrow}p$ Collision



Sivers Effects Induced by Instanton

P-Odd Pion Azimuthal Charge Correlations

Conclusion & Acknowledgements

Guideline

- Instanton induced local $\mathcal{P}$-odd contributions to the Single Spin Asymmetries

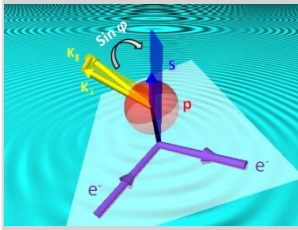
  ---Semi-Inclusive Deep Inelastic Scattering

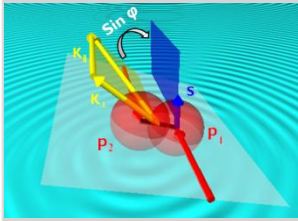
  --- $p_{\uparrow}p$ Collision

- P-Odd Pion Azimuthal Charge Correlations

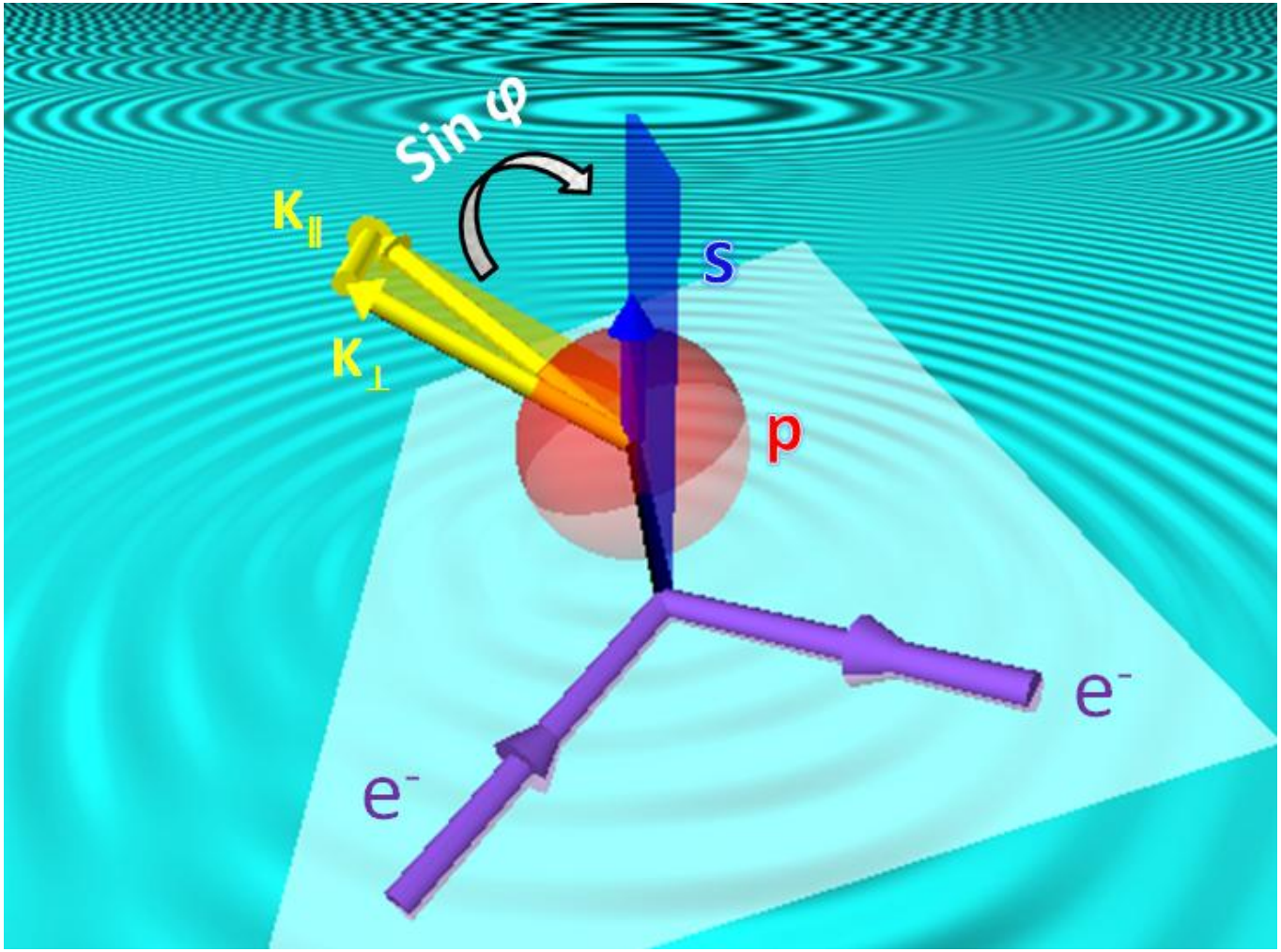
  --- pp Collision

  --- Peripheral Heavy Ion Collisions



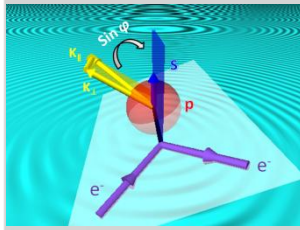


Semi-Inclusive Deep Inelastic Scattering

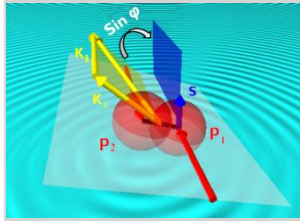


Guideline

Semi-Inclusive Deep Inelastic Scaterring



$p_{\uparrow}p$ Collision

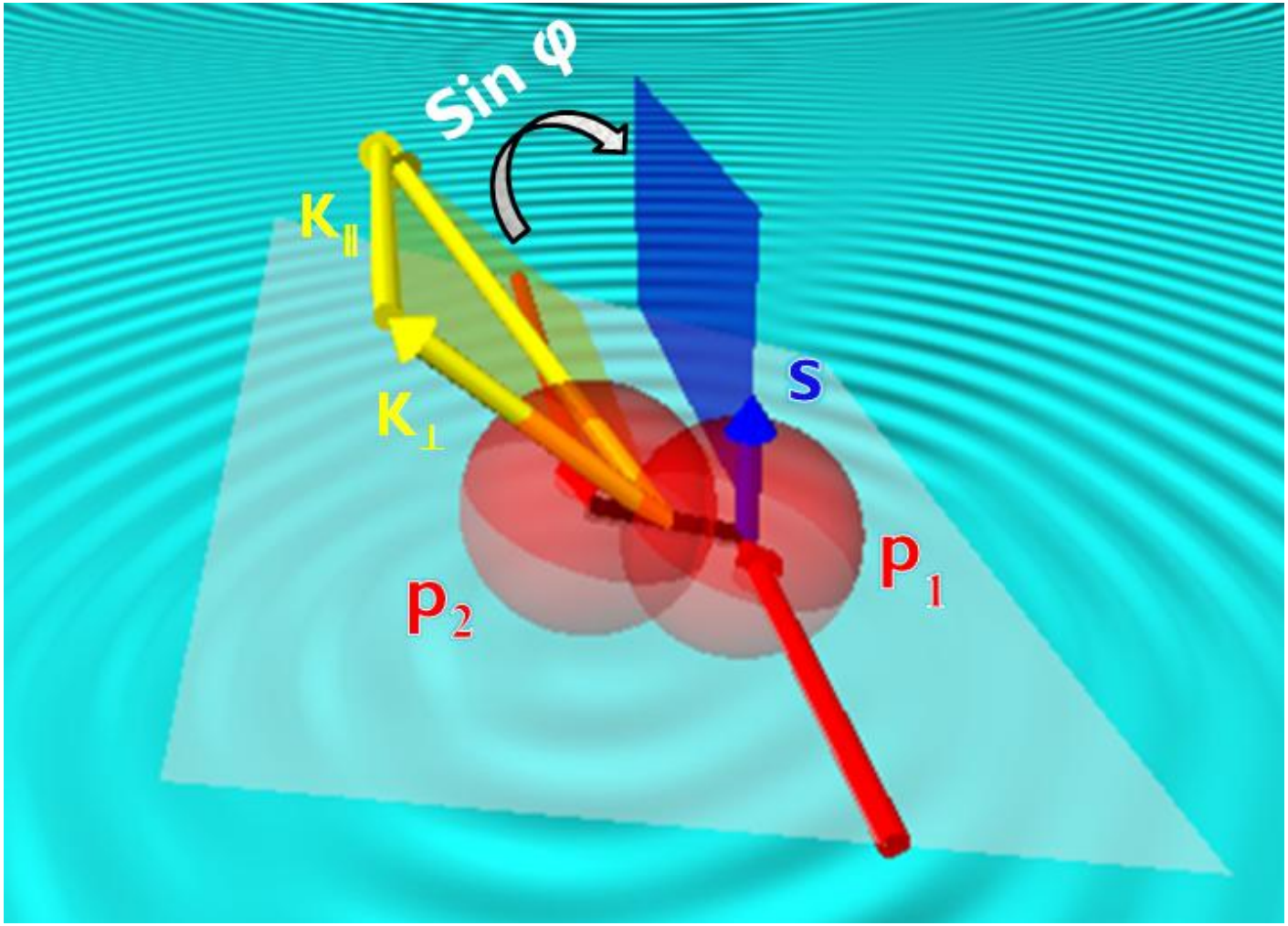


Sivers Effects Induced by Instanton

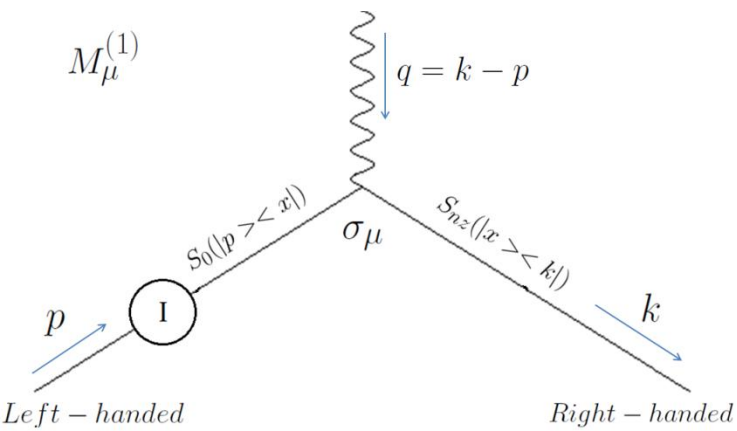
P-Odd Pion Azimuthal Charge Correlations

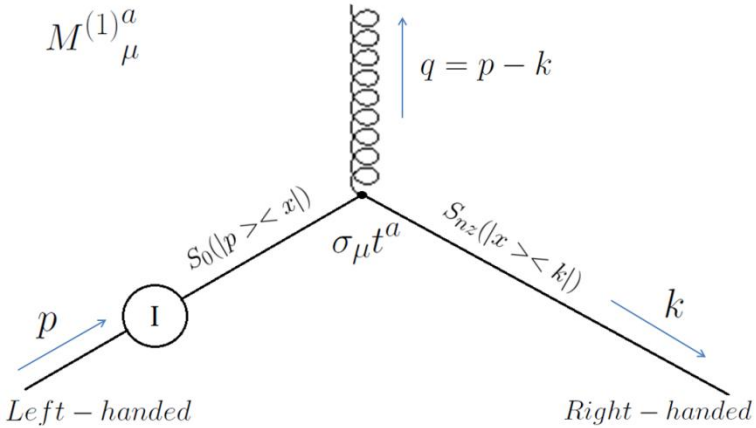
Conclusion & Acknowledgements

$p_{\uparrow}p$ Collision



Photon Exchange & Gluon Exchange





$$\sum_{color} M_\mu^{(1)} = +i[\gamma_\mu \not{k} + \not{p}\gamma_\mu]\Phi(\rho|q|)$$

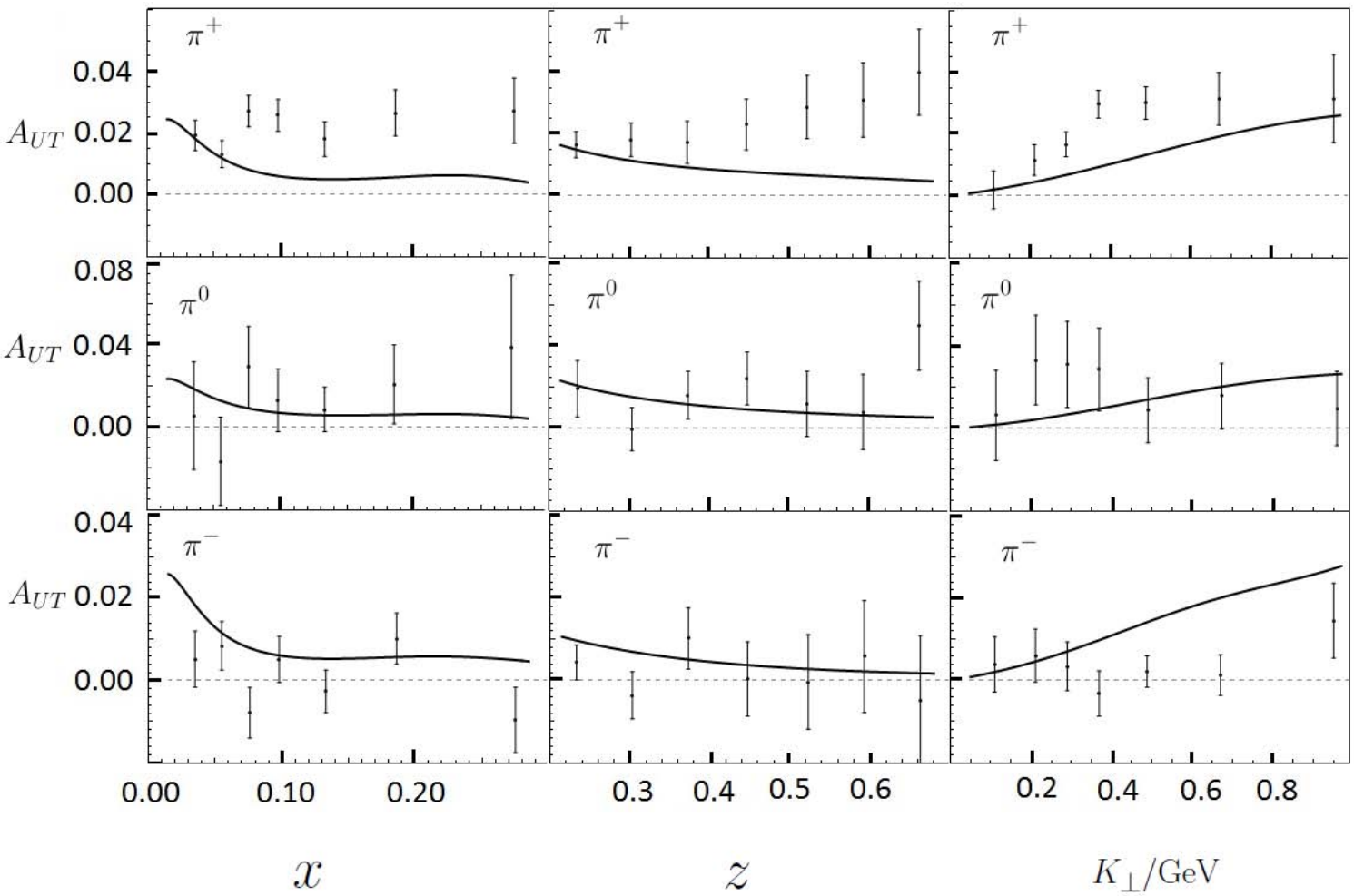
$$\sum_{color} M_{\mu,a}^{(1)} t_b \delta^{ab} = -i(\gamma_\mu \not{k} + \not{p}\gamma_\mu)\Psi(\rho|q|)$$

$$\Phi(a) \equiv \frac{4\pi^2\rho^4}{\lambda a^2}[1 - aK_1(a)]$$

$$\Psi(a) \equiv \frac{2\pi^2\rho^4}{\lambda a^2}[\frac{4}{a^2} - \frac{4}{3}aK_1(a) - 2K_2(a) + \frac{1}{3}]$$

- ρ is the instanton size
- λ the renormalized virtuality of the instanton quark zero mode.

Semi-Inclusive Deep Inelastic Scattering



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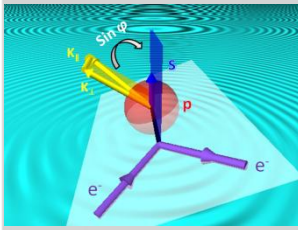
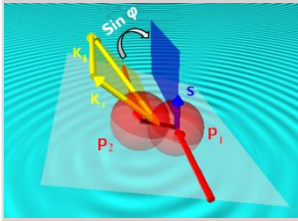
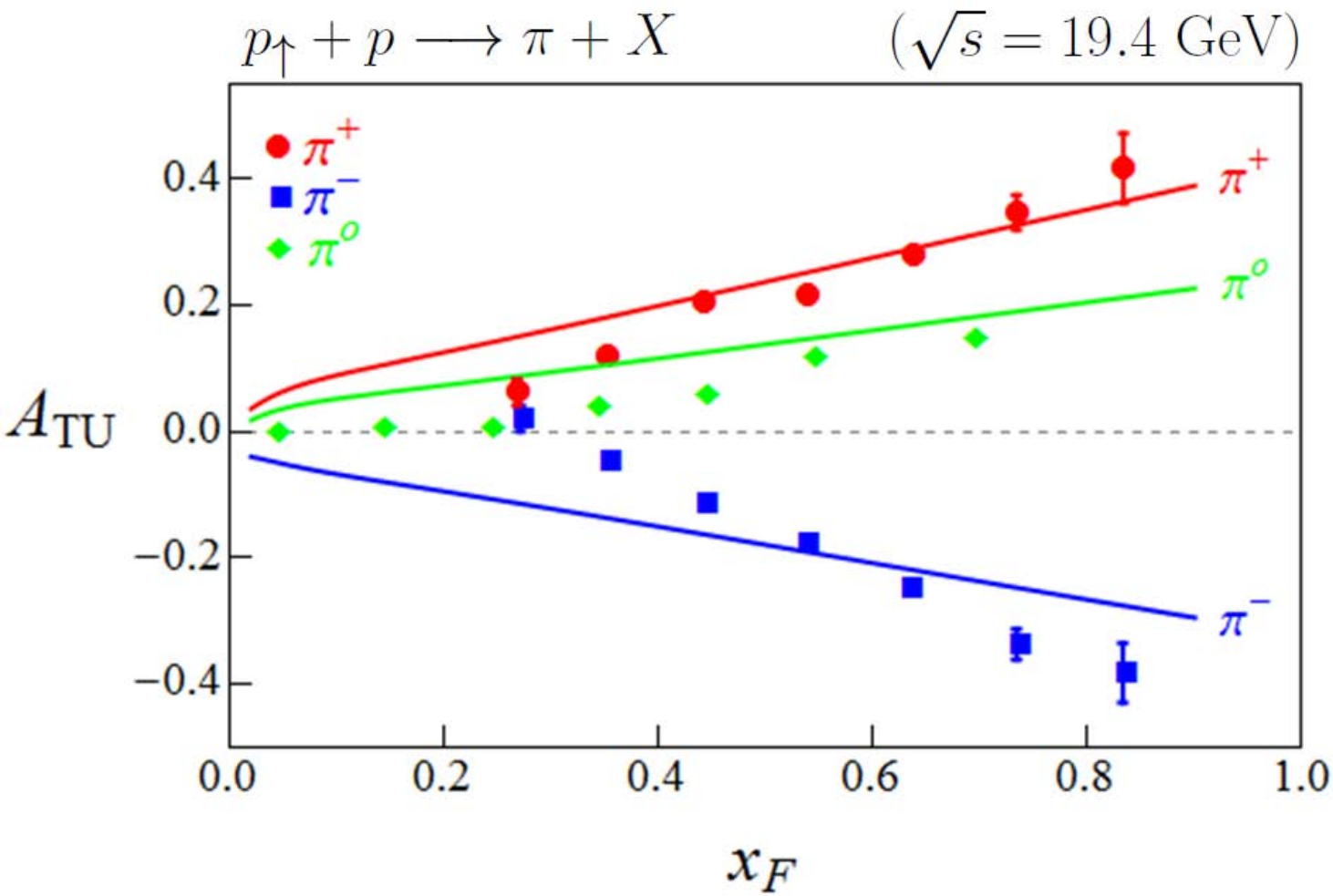
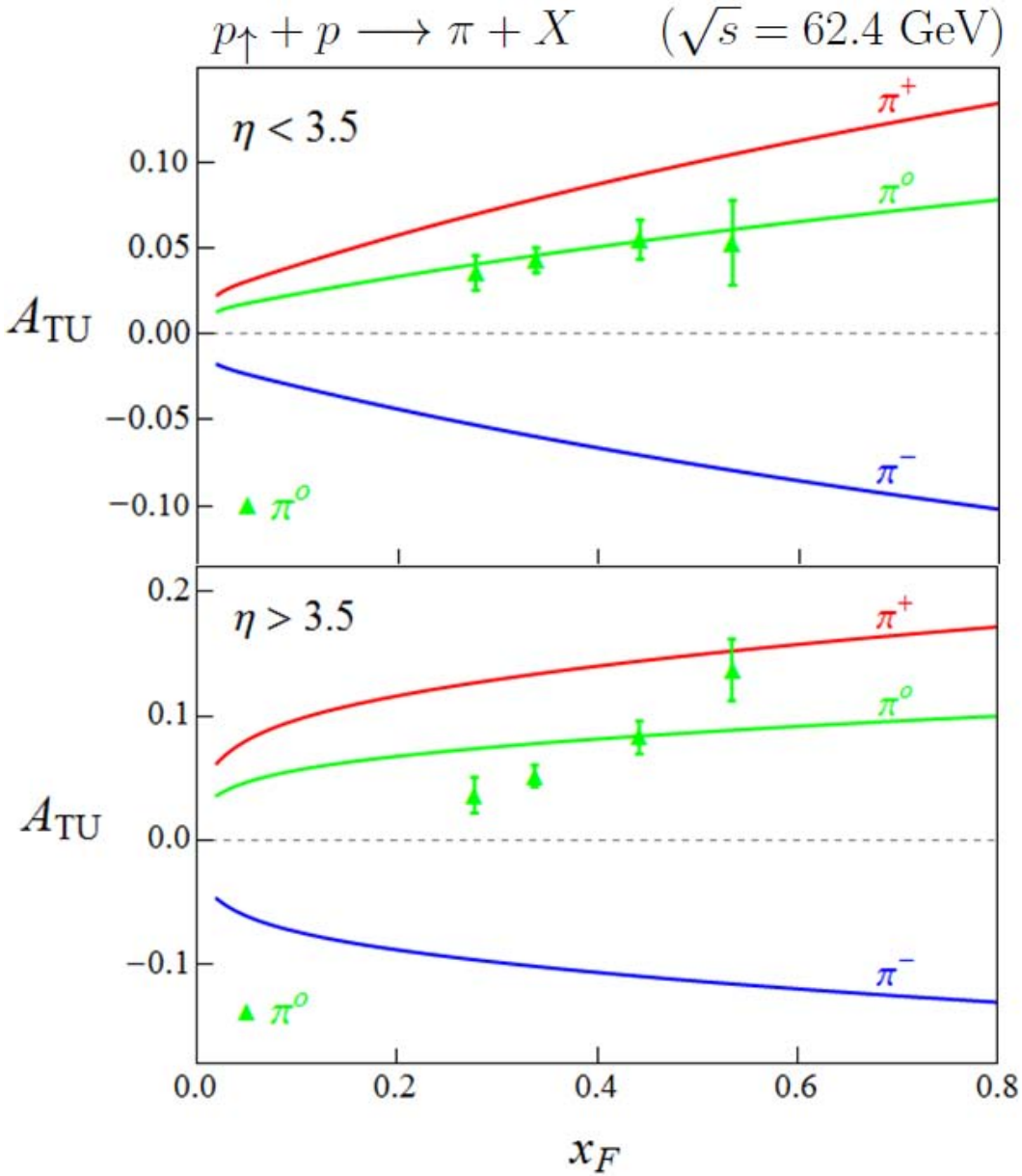




$p_\uparrow p$ Collision



D. Adams et al. (FNAL E704 Collaboration), Physics Letters B **264,** 462 (1991).



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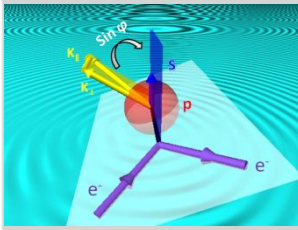

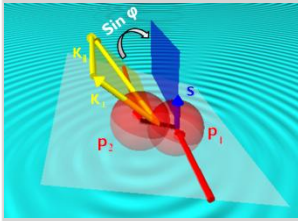

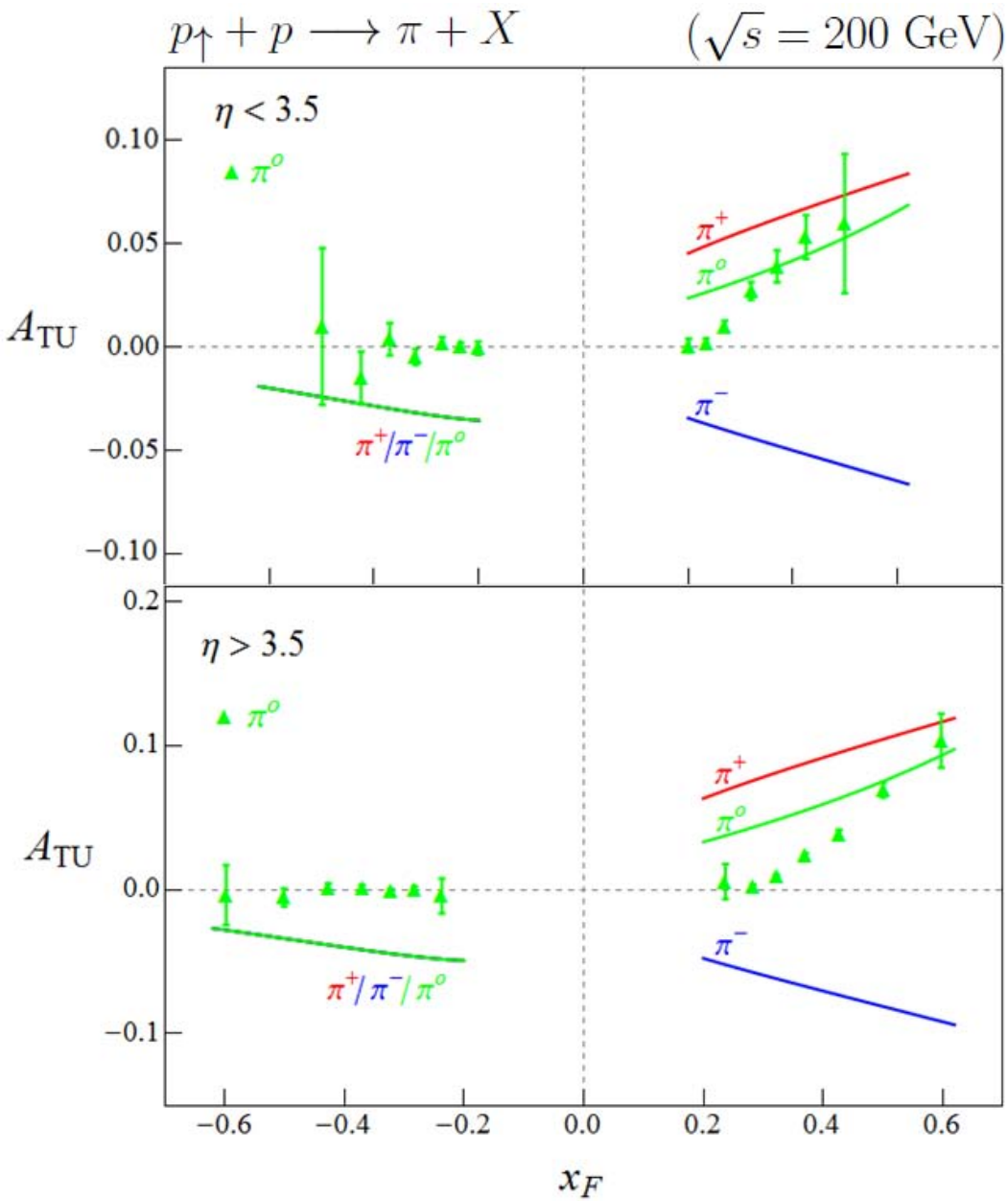

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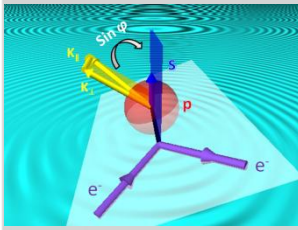
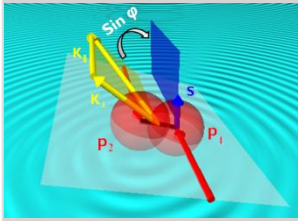
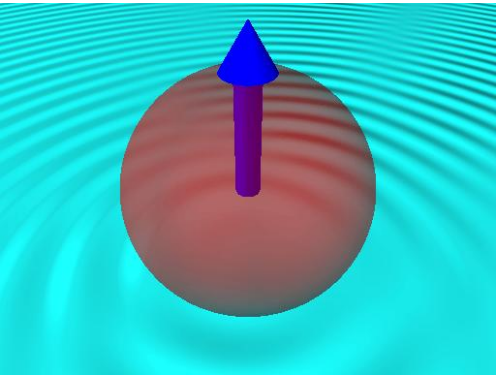
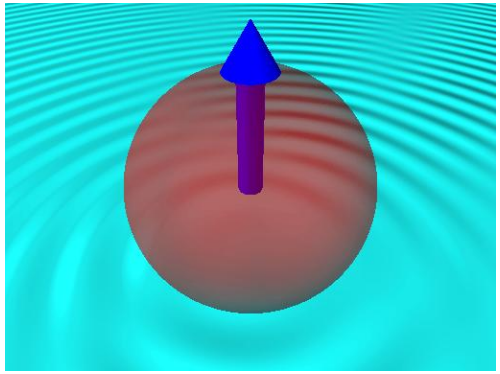


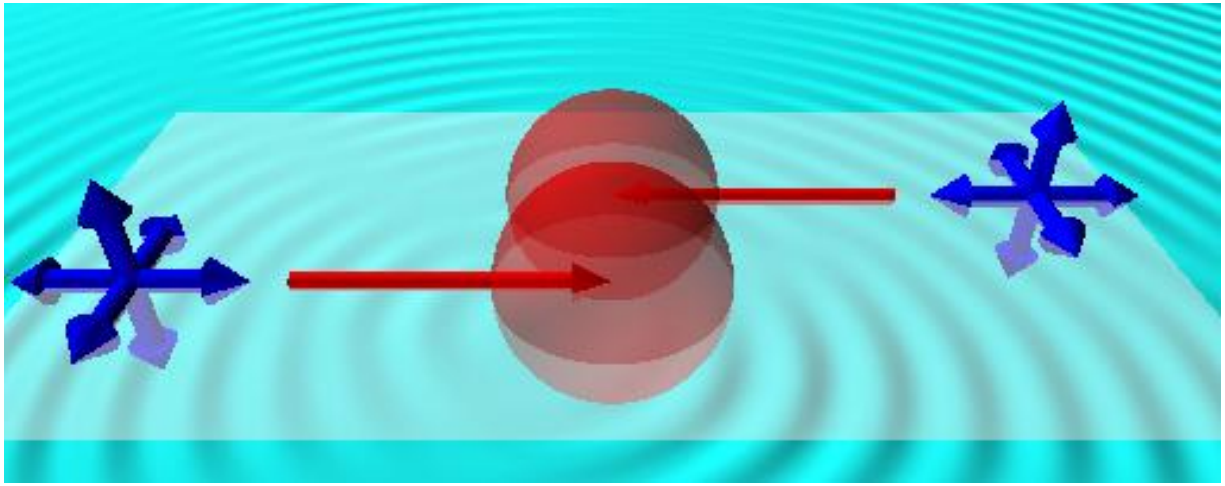


P-Odd Pion Azimuthal Charge Correlations

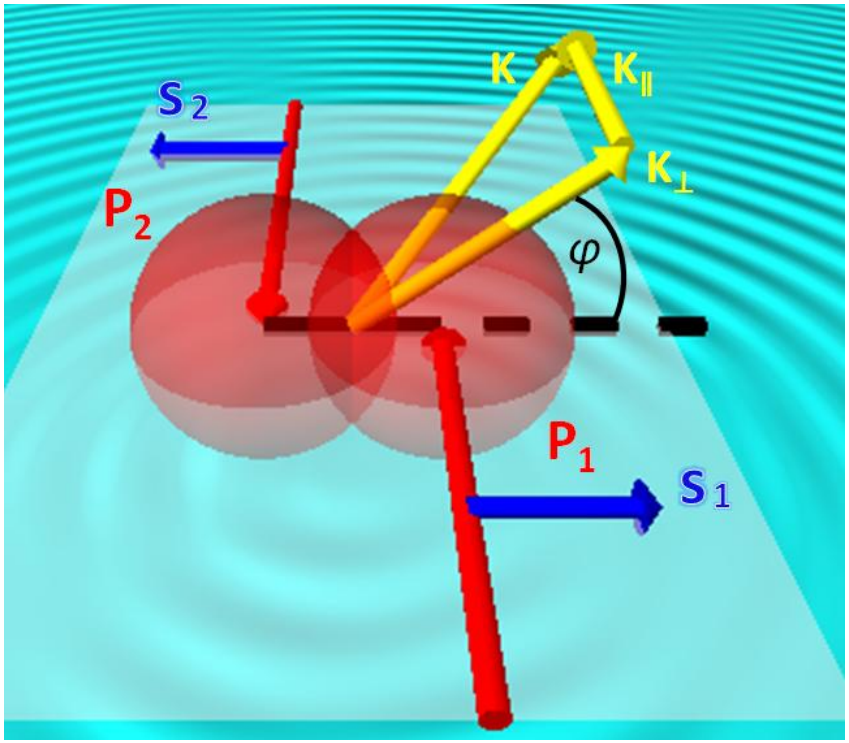




$$6 \times 6 = 36$$



One Collision Configuration



$$A_{\pi^+} = +\ 2 \times 0.0414\ \sin\phi$$
$$A_{\pi^-} = -\ 2 \times 0.0356\ \sin\phi$$

$$(< K_\perp > = 3\ \text{GeV})$$

P-Odd Pion Azimuthal Charge Correlations in *pp* collision at LHC

$$\frac{dN_\alpha}{d\phi} \sim 1 + 2v_\alpha \sin\phi$$

Probability	$v_{\pi+}$	$v_{\pi-}$
1/36	0.0414	-0.0356
1/36	-0.0414	0.0356
2/9	0.0207	- 0.0178
2/9	-0.0207	0.0178
1/2	0	0

$$< v_{\pi+} v_{\pi+} > \; = \; +\frac{2}{36} \times 0.0414 \times 0.0414 + \frac{4}{9} \times 0.0207 \times 0.0207 = +0.286 \times 10^{-3}$$

$$< v_{\pi-} v_{\pi-} > \; = \; +\frac{2}{36} \times 0.0356 \times 0.0356 + \frac{4}{9} \times 0.0178 \times 0.0178 = +0.211 \times 10^{-3}$$

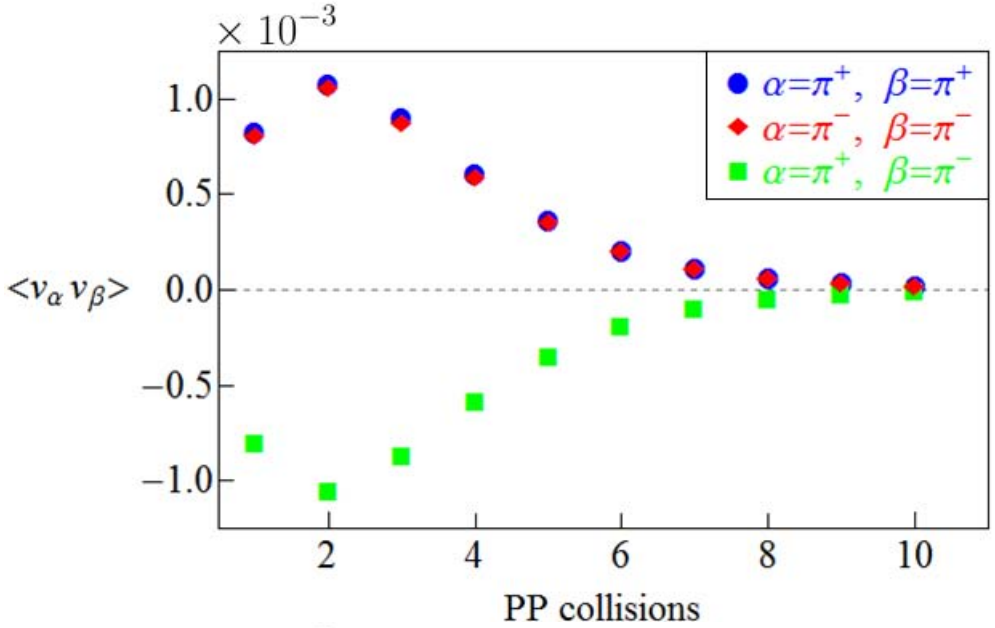
$$< v_{\pi+} v_{\pi-} > \; = \; -\frac{2}{36} \times 0.0414 \times 0.0356 - \frac{4}{9} \times 0.0207 \times 0.0178 = -0.246 \times 10^{-3}$$

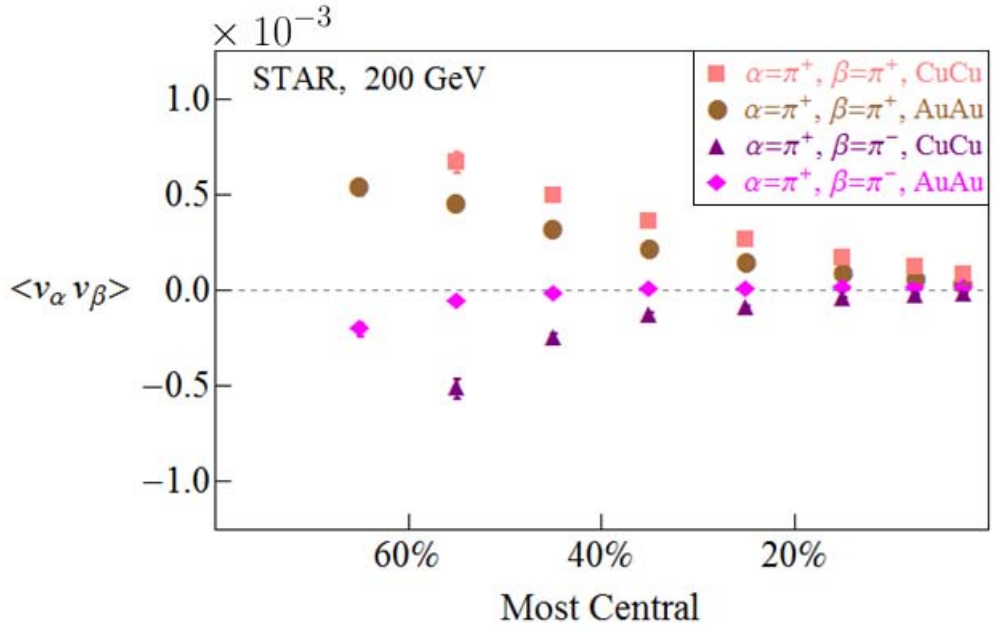
P-Odd Pion Azimuthal Charge Correlations

- The charge-dependent azimuthal correlations in unpolarized pp collisions as detailed before may survive in peripheral heavy-ion collisions both at RHIC and LHC. To estimate this effect, consider --- **n** independent hard pp collisions in a heavy ion event.

- $< K_\perp > = 1$ GeV

- Asymptotic $\sqrt{s}$





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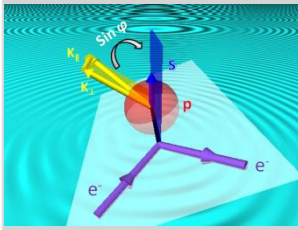
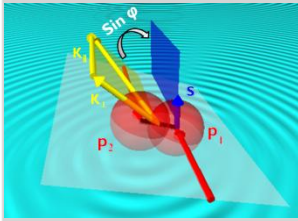
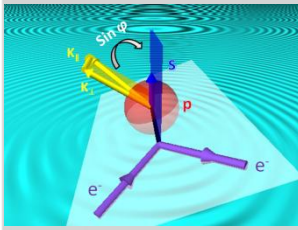
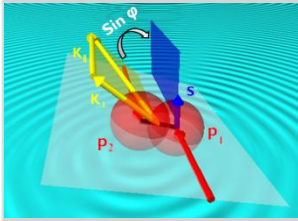




Conclusion

- QCD instantons may contribute significantly to the transverse Single Spin Asymmetry in Semi-Inclusive Deep Inelastic Scattering.
- The instanton mechanism provides a natural explanation for the Sivers effect in $p_{\uparrow}p$ collision.
- Instanton induced local P-odd contributions to the Sivers effects in $p_{\uparrow}p$ scattering, may cause substantial charge-dependent azimuthal correlations for pion production in pp scattering at the LHC.
- Instanton induced local P-odd contributions to the Sivers effects may still be substantial in peripheral heavy ion collisions both at RHIC and LHC.





Thank You!

Some Field Theoretical Issues of the Chiral Magnetic Effect

Hai-cang Ren
The Rockefeller University & CCNU
with De-fu Hou, Hui Liu

JHEP 05(2011)046

The contents

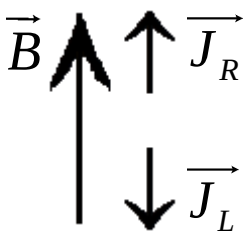
- An introduction of CME
- Axial anomaly in QCD
- General properties of CME
- One loop calculation
- Summary
- Current project

I. An introduction to CME

(Fukushima, Kharzeev and Warringa)

1. A charged massless quark in a magnetic field $\vec{B} = \uparrow$

Helicity $\Leftrightarrow -\gamma_5$	R		L	
charge	+	—	+	—
Magnetic moment	↑	↑	↑	↑
Momentum	↑	↓	↓	↑
Current **J**	↑	↑	↓	↓



In a quark matter of net axial charge

Q_5

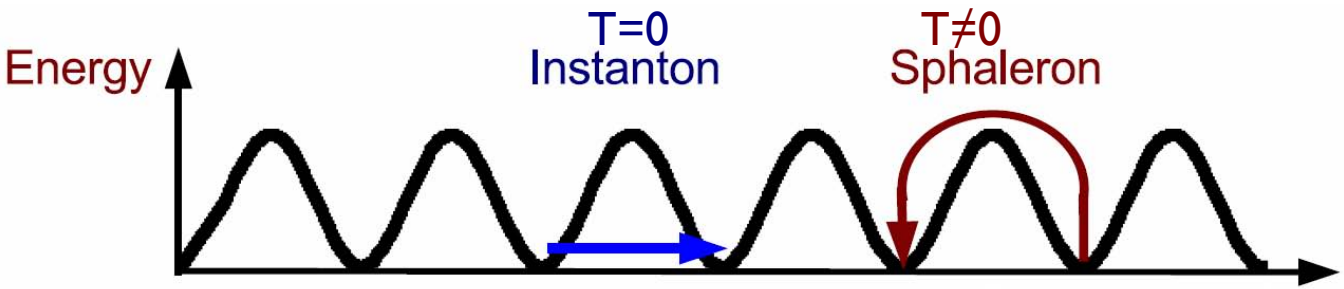
$$\mathbf{J} = \mathbf{J}_R + \mathbf{J}_L = \eta \frac{e^2}{2\pi^2} \mu_5 \mathbf{B}$$

$\eta = N_C \sum_f q_f^2 =$ Color-flavor factor

2. RHIC Implementation

i) Excess axial change $\mu_5 \neq 0$

Transition between different topologies of QCD

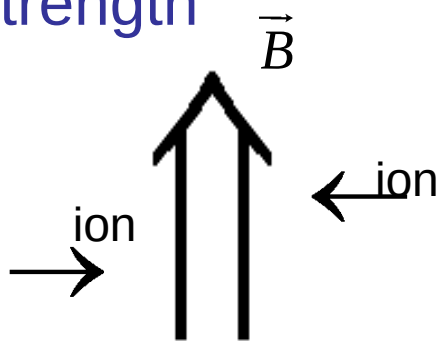


Axial anomaly $$\Delta N_5 = -\frac{N_f g^2}{32\pi^2}\int d^4x \varepsilon_{\mu\nu\rho\lambda} F^l_{\mu\nu} F^l_{\rho\lambda} = n_W$$

$n_W =$ the wind number $F^l_{\mu\nu} =$ QCD field strength

ii) Magnetic field

Generated by an off-central collision



iii) May provide a new signal of QCD phase transition.

iv) Theoretical approach:

---- *Field theory* (*Fukushima et. al*., *Kharzeev et. al*.)

---- *Holographic theory* (*Yee, Rebhan et. al*.)

v) There are experimental evidences, remains to be solidified.

vi) Complication in RHIC:

* *Inhomogeneous & time dependent magnetic field*

* *Inhomogeneous temperature and chemical potentials*

local equilibrium

* *Beyond thermal equilibrium*

3. The robustness of

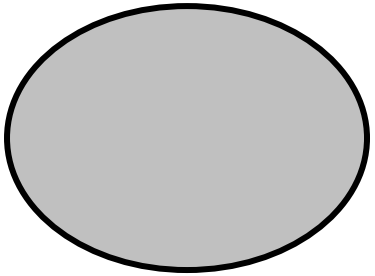
$$\mathbf{J}_{\mathrm{CME}} = \eta \frac{e^2}{2\pi^2} \mu_5 \mathbf{B}$$

under the Infrared limit of

$$\mu_5 \left(\mathbf{k}, k_0 \right)$$

$$\Downarrow$$

$$\mathbf{J}\left(\mathbf{q} + \frac{1}{2}\mathbf{k}\ , \omega + \frac{1}{2}k_0 \right) \Leftarrow$$



$$\Leftarrow \mathbf{B}\left(\mathbf{q} - \frac{1}{2}\mathbf{k}, \omega - \frac{k_0}{2} \right)$$

i.e. $(\mathbf{k}, k_0) \to 0$ and $(\mathbf{q}, \omega) \to 0$

A relativistic quantum field theory at nonzero temperature and/or chemical potentials

- UV divergence is no worse than vacuum

- IR is more problematic because:

---- The appearance of the ratio $\frac{p_0}{|\mathbf{p}|}$

limits $p_0 \to 0$ and $\mathbf{p} \to 0$ may depend on orders

---- The appearance of the ratios $\frac{|\mathbf{p}|}{T}$ etc.

$T \to 0$ and $\mathbf{p} \to 0$ may depend on orders

---- Linde's problem with gluons.

II. Axial anomaly in QCD

- Naïve Ward identities:

$$\partial_\mu J_\mu = 0 \qquad \text{and} \qquad \partial_\mu J_{5\mu} = 0$$

$$J_\mu = i\overline{\psi}\gamma_\mu \hat{q}\psi = \text{EM current} \qquad J_{5\mu} = i\overline{\psi}\gamma_\mu\gamma_5\psi = \text{axial current}$$

$$\hat{q} = \text{charge matrix in flavor space}$$

- UV divergence demands regularization (e.g. Pauli Villars)

------ Not all Ward identities can be preserved

------ The ones related to gauge symmetries have to be maintained

PV regulators:

$$\overline{\psi}_{\text{PV}} \text{ with mass } M_{\text{PV}} \text{ and negative metric in Hilbert space}$$

$$\partial_\mu J_\mu^{\text{PV}} = 0 \qquad \text{but} \qquad \partial_\mu J_{5\mu}^{\text{PV}} = 2iM_{\text{PV}}\overline{\psi}_{\text{PV}}\gamma_5\psi_{\text{PV}}$$

$$\text{Fermion loop of } \psi \rightarrow$$

$$\text{Fermion loop of } \psi - \sum_{\text{PV}} C_{\text{PV}} \times \text{fermion loop of } \psi_{\text{PV}}$$

$$\sum_{\text{PV}} C_{\text{PV}} = 1$$

- Ward identities post regularization:

$$M_{\mathrm{PV}} \to \infty$$

$$\partial_\mu J_\mu = 0$$

$$\partial_\mu J_{5\mu} = i\frac{N_f g^2}{32\pi^2}\varepsilon_{\mu\nu\rho\lambda}F^l_{\mu\nu}F^l_{\rho\lambda} + i\eta\frac{e^2}{16\pi^2}\varepsilon_{\mu\nu\rho\lambda}F_{\mu\nu}F_{\rho\lambda}$$

anomalous Ward identity

$F^l_{\mu\nu}$ = YM field strength $F_{\mu\nu}$ = EM field strenth

$$\eta = N_c\sum_f q_f^2 = \text{color - flavor factor}$$

- Applications of the axial anomaly:

※ The explanation of the rate of $\pi^0 \to 2\gamma$

※ The solution of $U_A(1)$ problem

※ Link the change of the axial charge and the change of topology.

※ Chiral magnetic effect, chiral vortical effect, etc.

III General properties of CME

i) Naïve axial charge & conserved axial charge

$$\rho_5 = \psi^\dagger \gamma_5 \psi \qquad N_5 = \int d^3\mathbf{r}\rho_5$$

$$\frac{dN_5}{dt} = \int d^3\mathbf{r}\frac{\partial \rho_5}{\partial t} = -\int d^3\mathbf{r}\nabla\cdot\mathbf{J}_5 + \eta\frac{e^2}{4\pi^2}\int d^3\mathbf{r}\mathbf{E}\cdot\mathbf{B} = \eta\frac{e^2}{4\pi^2}\int d^3\mathbf{r}\mathbf{E}\cdot\mathbf{B}$$

not conserved because of the anomaly

$$\tilde{N}_5 \equiv N_5 + \eta\frac{e^2}{4\pi^2}\int d^3\mathbf{r}\mathbf{A}\cdot\mathbf{B}$$

$$\frac{d\tilde{N}_5}{dt} = 0 \quad \Rightarrow \quad \tilde{N}_5 \text{ is conserved!}$$

$\tilde{N}_5$ **should be used in thermodynamics equilibrium** (*Rubakov*)

ii) Grand partition function:

$$Z = \mathrm{Tr}\exp\left(-\frac{H-\mu N-\mu_5\tilde{N}_5}{T}\right)$$

$$N = \text{quark number}, \quad \mu, \mu_5 = \text{chemical potentials}$$

iii) Linear response

$$J_i(Q) = K_{ij}(Q)A_j(Q) \qquad Q = (\vec{q}, \omega)$$

$$K_{ij}(Q) = -\Pi_{ij}(Q) - i\eta\frac{e^2}{4\pi^2}\mu_5\varepsilon_{ijk}q_k + O(A^2)$$

$\Pi_{ij}(Q) =$ *The usual photon self-energy tensor, subject to higher order corrections*

iv) The Taylor expansion in μ_5

$$K_{ij}(Q) = K_{ij}^{(0)}(Q) + \mu_5 K_{ij}^{(1)}(Q) + O(\mu_5^2)$$

Chiral magnetic current $\vec{J}_{CME}$

$$K_{ij}^{(1)}(Q) = -\frac{\partial}{\partial \mu_5}\Pi_{ij}(Q)\Big|_{\mu_5=0} - i\eta\frac{e^2}{2\pi^2}\varepsilon_{ijk}q_k$$

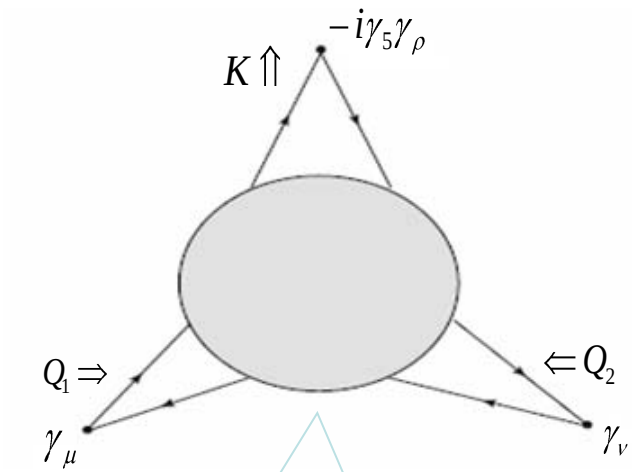
Normal term *Anomaly term*

$$Q_1 = \left(\mathbf{q} + \frac{1}{2}\mathbf{k}, \omega + \frac{1}{2}k_0\right)$$

$$Q_2 = \left(-\mathbf{q} + \frac{1}{2}\mathbf{k}, -\omega + \frac{1}{2}k_0\right)$$

$$K = (\mathbf{k}, ik_0)$$

$$= -\lim_{K\to 0}$$



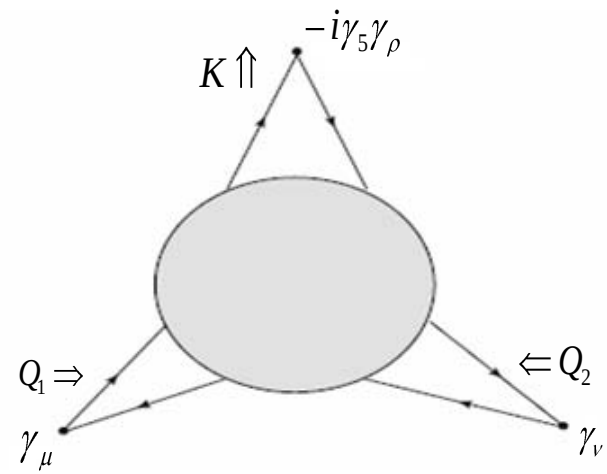
$$-i\eta\frac{e^2}{2\pi^2}\varepsilon_{ijk}q_k$$

$$\Delta_{\mu\nu}(Q_1, Q_2)$$

The limit $K=(0,k_0)\to 0$

$$Q_1=\left(\mathbf{q},\omega+\frac{1}{2}k_0\right)$$

$$Q_2=\left(-\mathbf{q},-\omega+\frac{1}{2}k_0\right)$$



$$\lim_{k_0\to 0}\lim_{\mathbf{k}\to 0}\Delta_{ij}(Q_1,Q_2)=-i\lim_{k_0\to 0}\frac{1}{k_0}K_\lambda\Delta_{ij\lambda}(Q_1,Q_2)=-i\eta\frac{e^2}{2\pi^2}\varepsilon_{ijk}q_k$$

$$\Uparrow$$

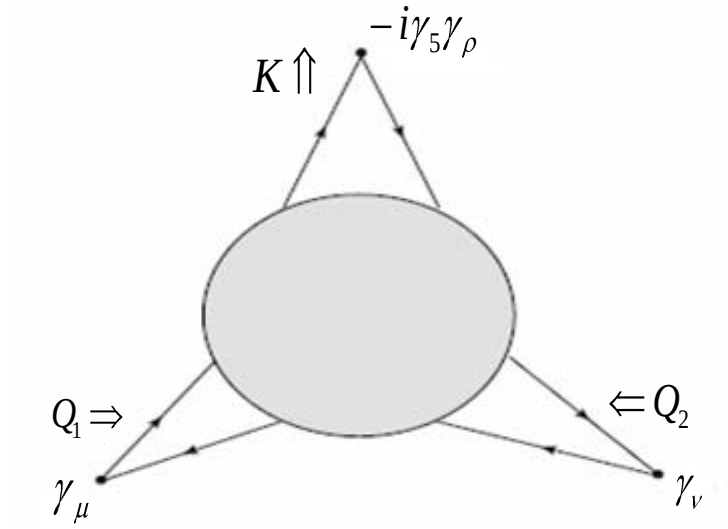
anomalous Ward identity

$\mathbf{J}_{\mathbf{CME}}=0$ to all orders and all T and μ

The limit $K = (\mathbf{k},0) \to 0$

$$Q_1 = (\mathbf{q}_1, \omega) = \left(\mathbf{q} + \frac{1}{2}\mathbf{k}, \omega\right)$$

$$Q_2 = (\mathbf{q}_2, -\omega) = \left(-\mathbf{q} + \frac{1}{2}\mathbf{k}, -\omega\right)$$



General tensor structure with Bose symmetry:

$$\Delta_{ij}(Q_1,Q_2) = i\eta\frac{e^2}{2\pi^2}[C_0(q_1^2,q_2^2,\mathbf{q}_1\cdot\mathbf{q}_2;\omega)\epsilon_{ijk}q_{1k} - C_0(q_2^2,q_1^2,\mathbf{q}_1\cdot\mathbf{q}_2;-\omega)\epsilon_{ijk}q_{2k}$$
$$+ C_1(q_1^2,q_2^2,\mathbf{q}_1\cdot\mathbf{q}_2;\omega)\epsilon_{jkl}q_{1k}q_{2l}q_{1i} - C_1(q_2^2,q_1^2,\mathbf{q}_1\cdot\mathbf{q}_2;-\omega)\epsilon_{ikl}q_{1k}q_{2l}q_{2j}]$$

$$\Delta_{4k}(Q_1,Q_2) = \eta\frac{e^2}{2\pi^2}C_2(q_1^2,q_2^2,\mathbf{q}_1\cdot\mathbf{q}_2;\omega)\epsilon_{ijk}q_{1i}q_{2j} = \Delta_{k4}(Q_2,Q_1)$$

The electromagnetic gauge invariance:

$$Q_{1\mu}\Delta_{\mu\nu}(Q_1,Q_2) = Q_{2\nu}\Delta_{\mu\nu}(Q_1,Q_2) = 0$$

$$C_0(q_1^2,q_2^2,\mathbf{q}_1\cdot\mathbf{q}_2;\omega) = -q_2^2 C_1(q_2^2,q_1^2,\mathbf{q}_1\cdot\mathbf{q}_2;-\omega) + \omega C_2(q_2^2,q_1^2,\mathbf{q}_1\cdot\mathbf{q}_2;-\omega)$$

$$C_0(q_2^2,q_1^2,\mathbf{q}_1\cdot\mathbf{q}_2;-\omega) = -q_1^2 C_1(q_1^2,q_2^2,\mathbf{q}_1\cdot\mathbf{q}_2;\omega) - \omega C_2(q_1^2,q_2^2,\mathbf{q}_1\cdot\mathbf{q}_2;\omega).$$

$$Q_1 \to -Q_2 \equiv (\mathbf{q},\omega) \qquad K_{ij}^{(1)}(Q) = -i\eta\frac{e^2}{2\pi^2}\mu_5 \; F(Q) - 1 \; \varepsilon_{ijk}q_k$$

$$\begin{aligned} F(Q) &= -C_0(q^2,q^2,-q^2;\omega) - C_0(q^2,q^2,-q^2;-\omega) \\ &= q^2\left[C_1(q^2,q^2,-q^2;\omega) + C_1(q^2,q^2,-q^2;-\omega)\right] \\ &+ \omega\left[C_2(q^2,q^2,-q^2;\omega) - C_2(q^2,q^2,-q^2;-\omega)\right] \end{aligned}$$

If the infrared limit exists: $\lim_{Q\to 0} F(Q) = 0$

$$K_{ij}^{(1)}(Q) = -i\eta\frac{e^2}{2\pi^2}\mu_5\varepsilon_{ijk}q_k \qquad \mathbf{J}_{CME} = \eta\frac{e^2}{2\pi^2}\mu_5\mathbf{B}$$ ***to all orders***

III. One loop calculation

$$K_{ij}^{(1)}(Q) = \eta T \sum_{p_0} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[\Xi_{ij}(P,Q\,|\,0) - \sum_{\mathrm{PV}} C_{\mathrm{PV}} \Xi_{ij}(P,Q\,|\,M_{\mathrm{PV}}) \right]$$

$$\Xi_{ij}(P,Q\,|\,m) \equiv \mathrm{tr} S(P+Q\,|\,m)\gamma_i S(P\,|\,m)\gamma_j$$

$$S(P\,|\,m) = \frac{i}{-i\gamma_\rho P_\rho + \mu\gamma_4 + \mu_5\gamma_4\gamma_5 - m} \qquad P = (\mathbf{p}, p_0), \quad Q = (\mathbf{q}, \omega)$$

$$\sum_{\mathrm{PV}} C_{\mathrm{PV}} = 1$$

Continuation of imaginary Matsubara q_0 *to* $\omega \pm i0^+$ *with* ω *real for retarded (advanced) response function* ***after*** *the summation over Matsubara* p_0

Subtlety of IR limit:

$$K_{ij}^{(1)}(Q) = -i\eta \frac{e^2}{2\pi^2} \mu_5 \; F(Q) - 1 \; \varepsilon_{ijk} q_k$$

$$F(Q) = q^2[C_1(q^2, q^2, -q^2; \omega) + C_1(q^2, q^2, -q^2; -\omega)] + \omega[C_2(q^2, q^2, -q^2; \omega) - C_2(q^2, q^2, -q^2; -\omega)]$$

$T \neq 0$ *and / or* $\mu \neq 0$

$$\lim_{\mathbf{q}\to 0} \lim_{\omega \to 0} F(Q) = 0 \qquad \mathbf{J}_{CME} = \eta \frac{e^2}{2\pi^2} \mu_5 \mathbf{B}$$

$$\lim_{\omega \to 0} \lim_{\mathbf{q}\to 0} F(Q) = \frac{2}{3} \qquad \mathbf{J}_{CME} = \eta \frac{e^2}{6\pi^2} \mu_5 \mathbf{B}$$

Kharzeev & Warringar

IR singularity:

$$C_2(0,0,0;\omega) = \frac{1}{3\omega}$$

$T = 0$ *and* $\mu = 0$

$$\lim_{\mathbf{q}\to 0} \lim_{\omega \to 0} F(Q) = \lim_{\omega \to 0} \lim_{\mathbf{q}\to 0} F(Q) = 1 \qquad \mathbf{J}_{CME} = 0$$

IR singularity:

$$C_1(q^2, q^2; -q^2; \omega) = \frac{1}{2(q^2 - \omega^2)} \quad \text{and} \quad C_2(q^2, q^2; -q^2; \omega) = -\frac{\omega}{2(q^2 - \omega^2)}$$

III. Summary

$$\mathbf{J}_{CME}\left(\mathbf{q}+\frac{1}{2}\mathbf{k},\omega+\frac{1}{2}k_0\right) \quad \text{vs.} \quad \mu_5\left(\mathbf{k},k_0\right)\&\,\mathbf{B}\left(\mathbf{q}-\frac{1}{2}\mathbf{k},\omega-\frac{1}{2}k_0\right)$$

	IR limit		$\mathbf{J}_{CME}$	Higher order
$T=0$ $\mu=0$	$\lim\limits_{k_0\to 0}\lim\limits_{\vec{k}\to 0}$		0	none
	$\lim\limits_{\vec{k}\to 0}\lim\limits_{k_0\to 0}$			
$T\neq 0$ and/or $\mu\neq 0$	$\lim\limits_{\vec{k}\to 0}\lim\limits_{k_0\to 0}$	$\lim\limits_{\vec{q}\to 0}\lim\limits_{\omega\to 0}$	$\eta\dfrac{e^2}{2\pi^2}\mu_5\mathbf{B}$	none if IR safe
		$\lim\limits_{\omega\to 0}\lim\limits_{\vec{q}\to 0}$	$\dfrac{1}{3}\times\eta\dfrac{e^2}{2\pi^2}\mu_5\mathbf{B}$	yes
	$\lim\limits_{k_0\to 0}\lim\limits_{\vec{k}\to 0}$		0	none

Current project

Anomalous transport coefficients

The Lagrangian density

$$= -\bar{\psi}\gamma_\mu \partial_\mu - iA_\mu \gamma_5 \psi - \mu_5 \bar{\psi}\gamma_5 \psi + \ldots$$

Anomalous Ward identity :

$$\partial_\mu J_{5\mu} = \frac{1}{3} \times \frac{i}{48\pi^2} \varepsilon_{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda}$$

Anomaly & thermodynamic laws dictates *Son & Surowka*

$$\vec{J}_5 = \xi_B \vec{B} + \xi \vec{\omega}$$

$\vec{B}$ = axial magnetic field $\qquad \vec{\omega}$ = fluid vorticity

One loop without regulators *Landsteiner et. al.*

$$\xi_B = \frac{1}{2\pi^2} \mu_5 \qquad \xi = \frac{1}{2\pi^2} \mu_5^2 + \frac{T^2}{2}$$

One loop with PV regulators :

Using the conserved axial charge dictated by anomaly :

$$\bar{\psi}\gamma_5\psi \rightarrow \bar{\psi}\gamma_5\psi + \frac{1}{3}\times\frac{1}{4\pi^2}\mathbf{A}\cdot\mathbf{B}(\text{anomaly te rm})$$

$$\langle \xi_B, \xi \rangle = \text{PV regulated one loop} + \text{anomaly te rm}$$

	Regulated one loop	Anomaly term	Total
ξ_B	$\frac{1}{3\pi^2}\mu_5$	$\frac{1}{6\pi^2}\mu_5$	$\frac{1}{2\pi^2}\mu_5$
ξ	$\frac{1}{2\pi^2}\mu_5^2 + \frac{1}{6}T^2$	0	$\frac{1}{2\pi^2}\mu_5^2 + \frac{1}{6}T^2$

Anomaly and thermodynamics

Anomalous Ward identity and energy - momentum conservation

$$\partial_\mu J_5^\mu = \kappa E_\mu B^\mu \qquad \text{and} \qquad \partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_{5\lambda}$$

$$\kappa = \text{anomaly coefficient}$$

$$u^\nu \partial_\mu T^{\mu\nu} + \mu_5 \partial_\mu J_5^\mu = F^{\nu\lambda} J_{5\lambda} + \kappa E_\mu B^\mu$$

But we found for PV regulators

$$\partial_\mu \langle J_{5\mathrm{PV}} \rangle^\mu = -2M_{\mathrm{PV}} \overline{\psi}_{\mathrm{PV}} \gamma_5 \psi_{\mathrm{PV}} \quad \text{but}$$

$$\partial_\mu \langle T_{\mathrm{PV}} \rangle^{\mu\nu} = F^{\nu\lambda} \langle J_{5\mathrm{PV}} \rangle_\lambda + 2A^\nu \overline{\psi}_{\mathrm{PV}} \gamma_5 \psi_{\mathrm{PV}}$$

$$u^\nu \partial_\mu \langle T_{\mathrm{PV}} \rangle^{\mu\nu} + \mu_5 \partial_\mu \langle J_{5\mathrm{PV}} \rangle^\mu = F^{\nu\lambda} \langle J_{5\mathrm{PV}} \rangle_\lambda$$

Does the anomaly still show up in the regulated

$$u^\nu \partial_\mu T^{\mu\nu} + \mu_5 \partial_\mu J_5^\mu \quad ?$$

Thank you!

Kent Riley
Yale University
6/25/2012

CHARGE DEPENDENT AZIMUTHAL CORRELATIONS OF K-PI PAIRS

Motivation

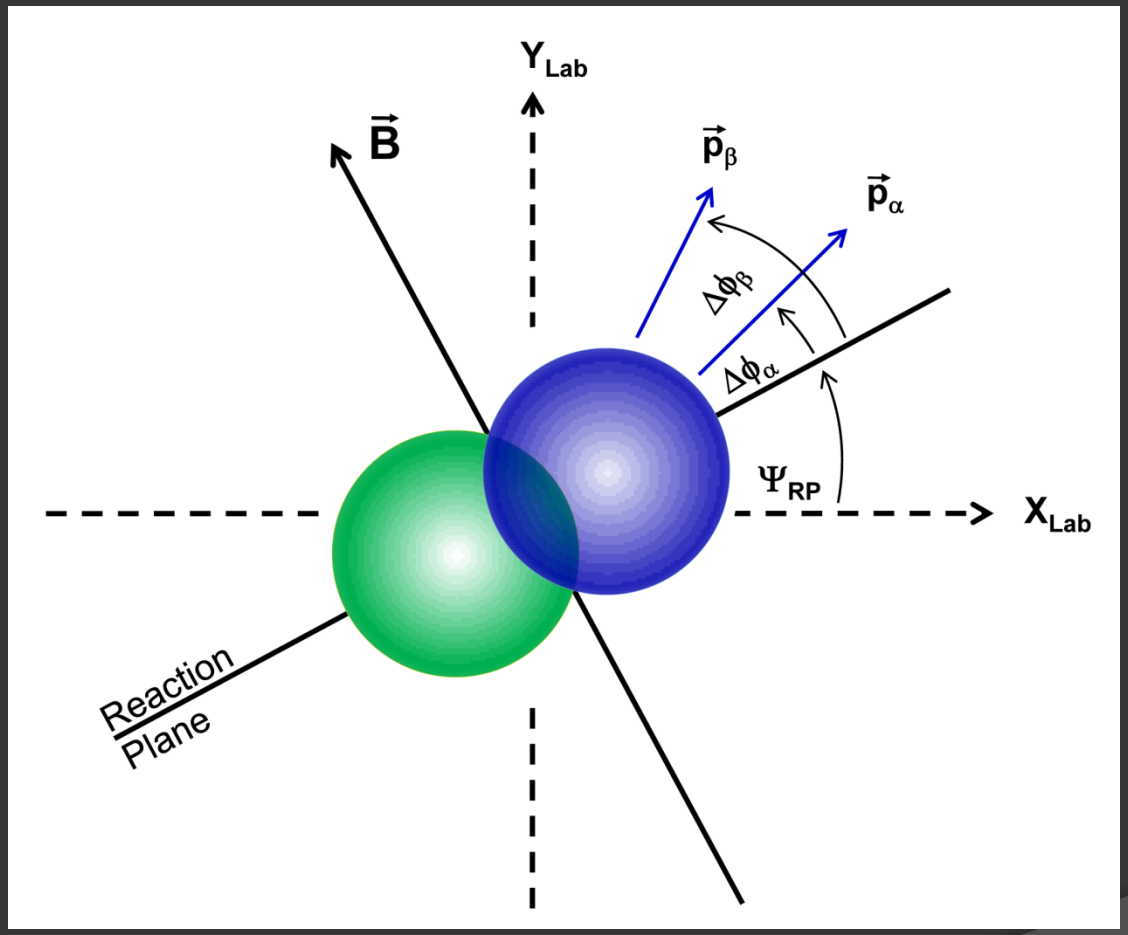
- Azimuthal correlations of charged particles
  - A probe for Local Parity Violation (LPV)
  - Charge separation effects due to the Chiral Magnetic Effect (CME) should be reflected in P-even correlations
    - $\langle\cos(\varphi_a+\varphi_b-2\psi_{RP})\rangle = (\langle\cos\Delta\varphi_a\cos\Delta\varphi_b\rangle - \langle\sin\Delta\varphi_a\sin\Delta\varphi_b\rangle)$
- Paper on subject published on 2004 data
  - B. I. Abelev, et al., Phys. Rev. C 81, 054908 (2010)
- Kaon-Pion analysis is a next step

Motivation

- CME
  - Separation of charged quarks due to strong magnetic field right after collision
    - Strong interaction effects or CME
- Kaon-Pion correlations
  - Majority of charged particles are pions – many pion pairs are produced later in the hadronization process after flow is established
    - Leads to Pi-Pi pairs being produced at small angles – clouds signal
  - Naïve assumption of CME is similar in K-Pi and all-charge combinations

Measuring the Charge Separation

- 2-particle correlator:



- $\langle\cos(\varphi_a+\varphi_b-2\psi_{RP})\rangle = (\langle\cos\Delta\varphi_a\cos\Delta\varphi_b\rangle - \langle\sin\Delta\varphi_a\sin\Delta\varphi_b\rangle)$

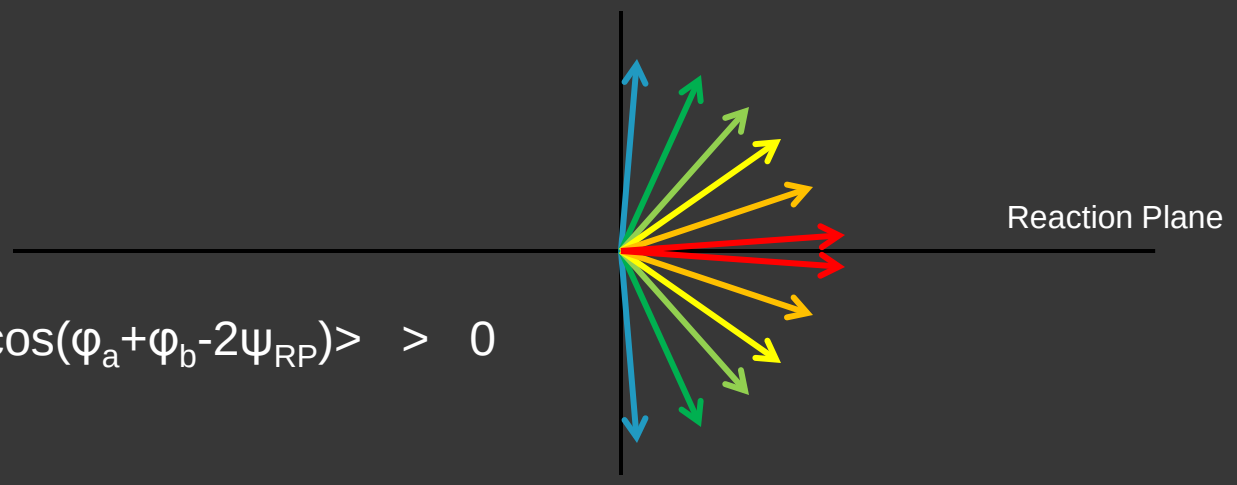
Image from: STAR collaboration
arXiv:0909.1717v2 [nucl-ex]

Measuring the Charge Separation

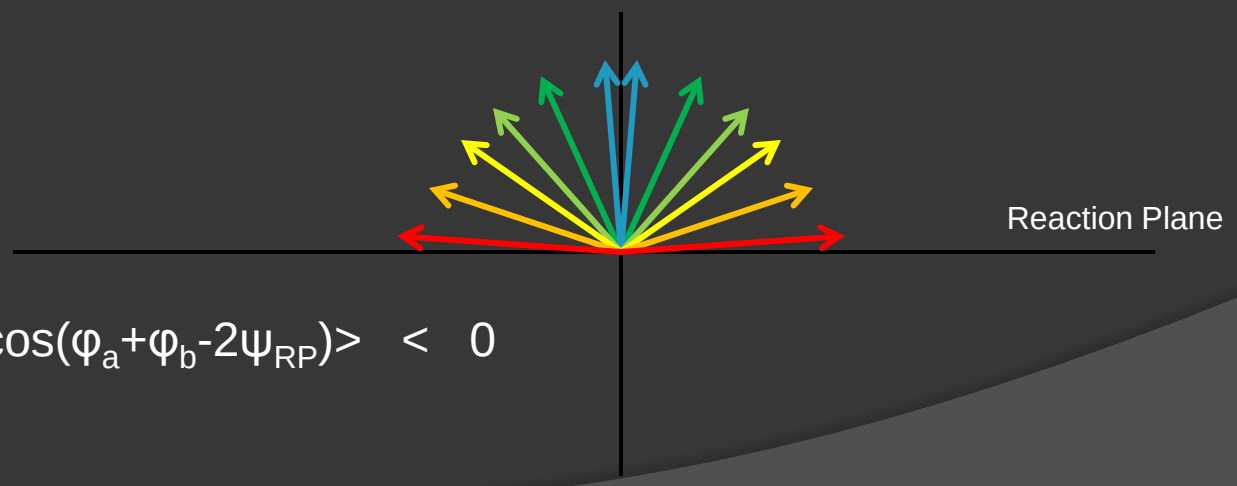
$$\langle\cos(\varphi_a+\varphi_b-2\psi_{RP})\rangle = (\langle\cos\Delta\varphi_a\cos\Delta\varphi_b\rangle - \langle\sin\Delta\varphi_a\sin\Delta\varphi_b\rangle)$$

- $\langle\cos\Delta\varphi_a\cos\Delta\varphi_b\rangle$
  - In-plane term:
    - Positive for particles emitted in the same direction in plane
    - Negative for particles emitted in the opposite direction in plane
- $\langle\sin\Delta\varphi_a\sin\Delta\varphi_b\rangle$
  - Out-of-plane term:
    - Positive for particles emitted in the same direction out of plane
    - Negative for particles emitted in the opposite direction out of plane
- The correlator looks at whether particles are emitted in the same or opposite directions
  - But its difficult to distinguish between whether its in or out of plane

Measuring the Charge Separation



$\langle\cos(\varphi_a+\varphi_b-2\psi_{RP})\rangle \quad > \quad 0$



$\langle\cos(\varphi_a+\varphi_b-2\psi_{RP})\rangle \quad < \quad 0$

Measuring the Charge Separation

- Can't actually measure the reaction plane
  - Must estimate => event plane (EP)

$$\langle\cos(\varphi_a+\varphi_b-2\psi_{RP})\rangle = \langle\cos(\varphi_a+\varphi_b-2\psi_{EP})\rangle/\langle\cos(2(\psi_{EP}-\psi_{RP}))\rangle$$

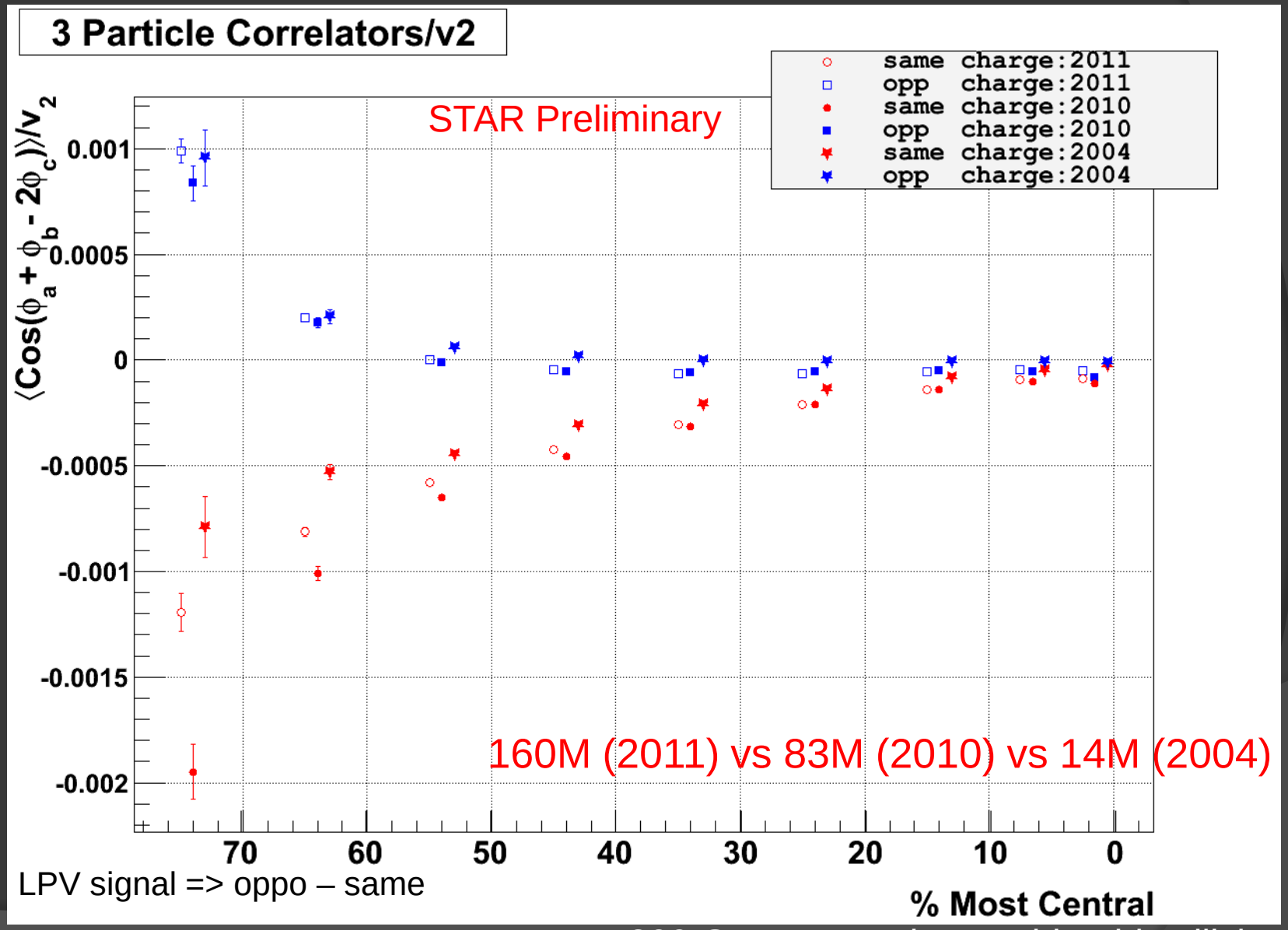
  - where $\langle\cos(2(\psi_{EP}-\psi_{RP}))\rangle$ is the full event plane resolution

- Another method…
  - The 3-particle correlator

$$\langle\cos(\varphi_a+\varphi_b-2\varphi_c)\rangle/v_2 = \langle\cos(\varphi_a+\varphi_b-2\psi_{RP})\rangle$$

  - Will use instead of 2-particle correlator here

Comparing 3-particle correlations - a,b,c are charged particles



LPV signal => oppo – same

200 GeV per nucleon gold-gold collisions

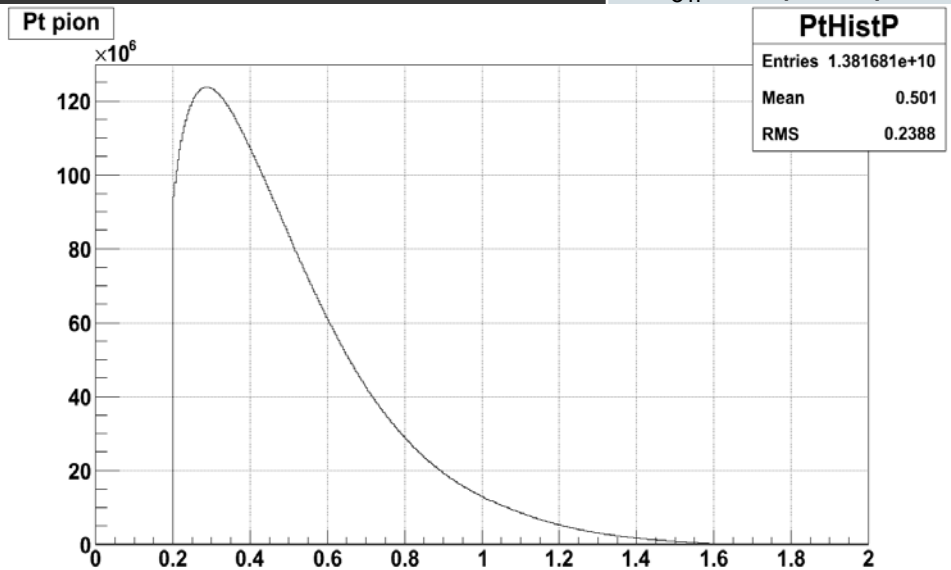
K-Pi correlation

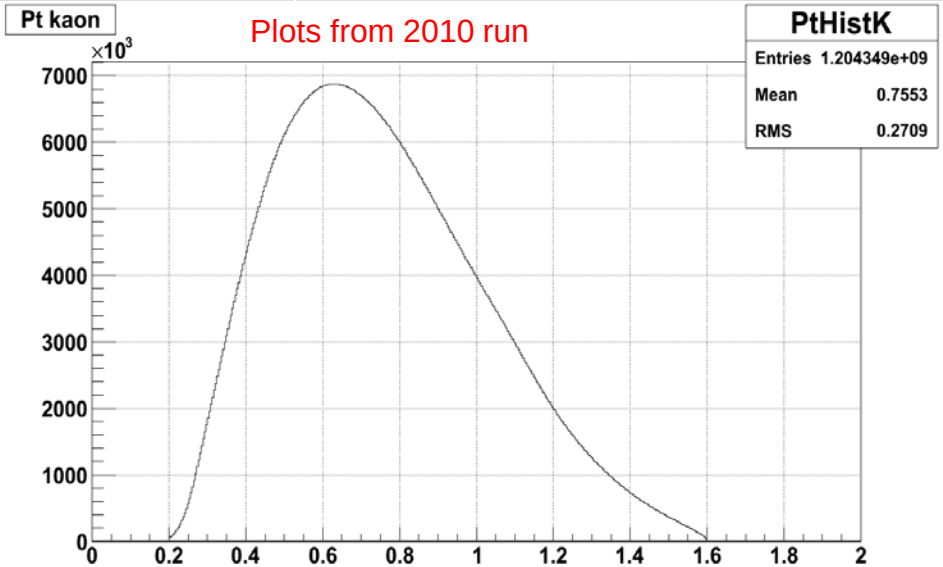
- K-Pi motivation
  - If background is mostly late stage creation of Pi-Pi pairs after the flow has been generated, this background should have a smaller effect in K-Pi correlations
  - Naïve expectation of CME is similar in K-Pi and all-charge combinations

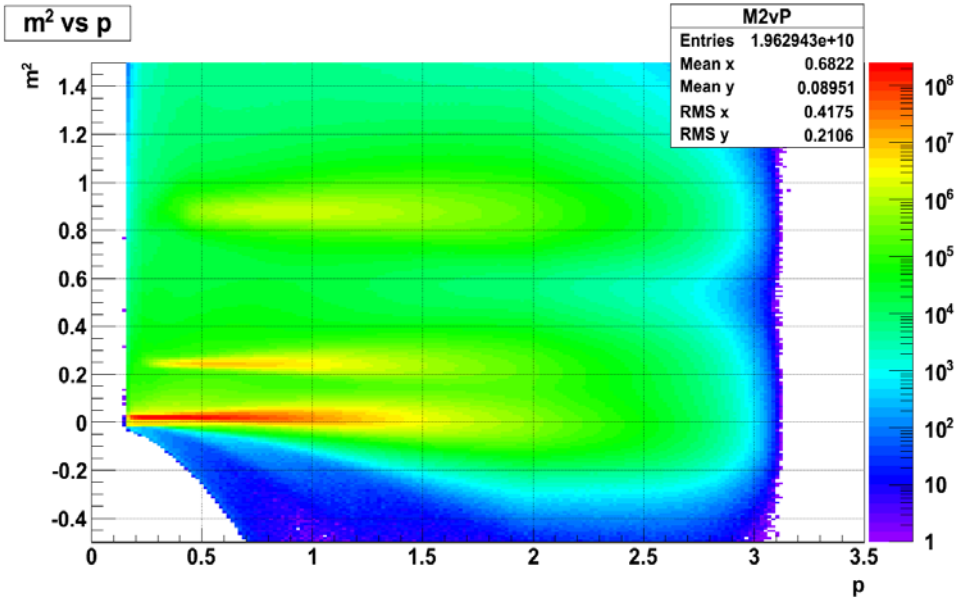
PID Cuts

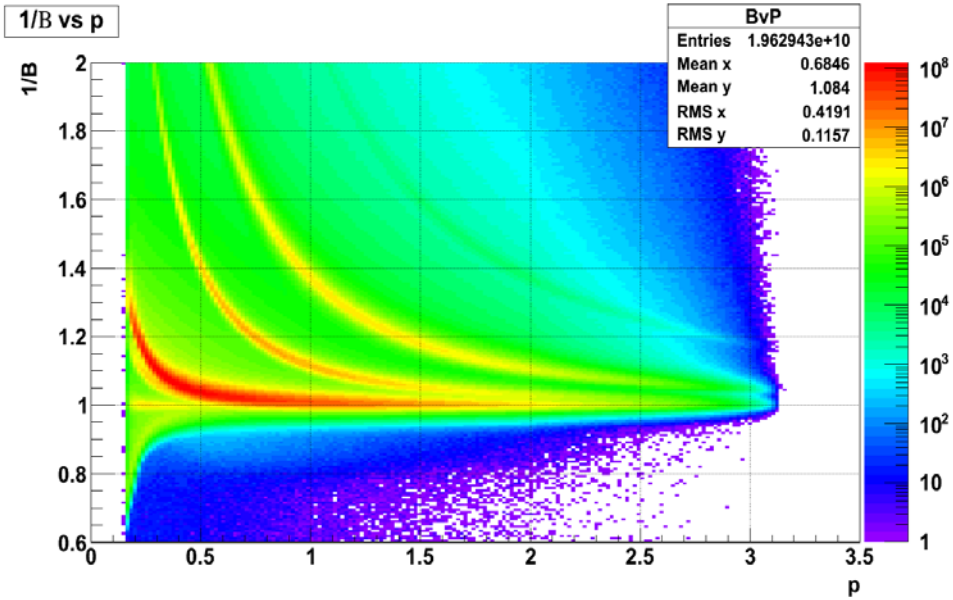
Pion	Kaon
• $0.001 < m^2 < 0.1\ (GeV/c^2)^2$ • $0.2 < p_t, p < 1.6$ GeV/c • $n_{\sigma\pi} < 2$ (after passing TOF m^2 cuts)	• $0.2 < m^2 < 0.3\ (GeV/c^2)^2$ • $0.2 < p_t, p < 1.6$ GeV/c • $n_{\sigma K} < 2$ (after passing TOF m^2 cuts)

Plots from 2010 run

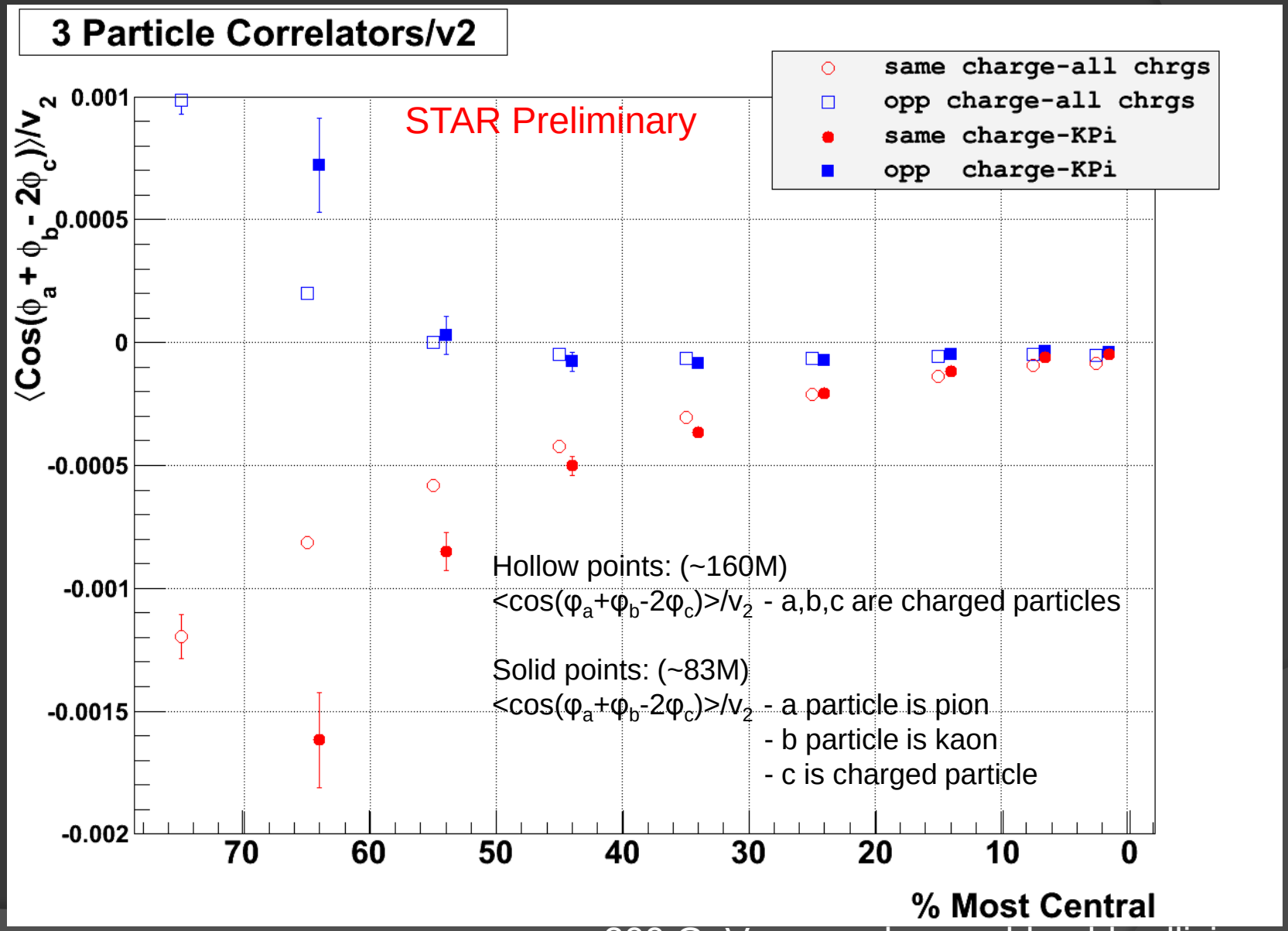








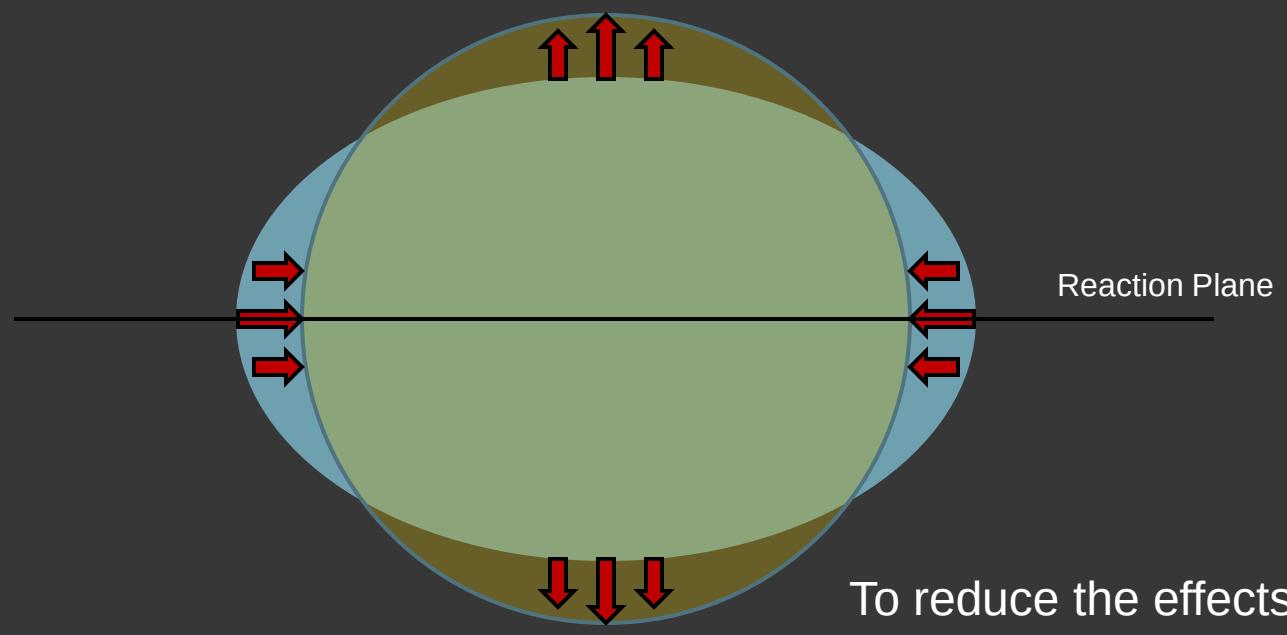
K-Pi correlation vs centrality



200 GeV per nucleon gold-gold collisions

Modifications according to flow

$$\langle\cos(\varphi_a+\varphi_b-2\varphi_c)\rangle/v_2 = \langle\cos(\varphi_a+\varphi_b-2\psi_{RP})\rangle$$
$$= (\langle\cos\Delta\varphi_a\cos\Delta\varphi_b\rangle - \langle\sin\Delta\varphi_a\sin\Delta\varphi_b\rangle)$$



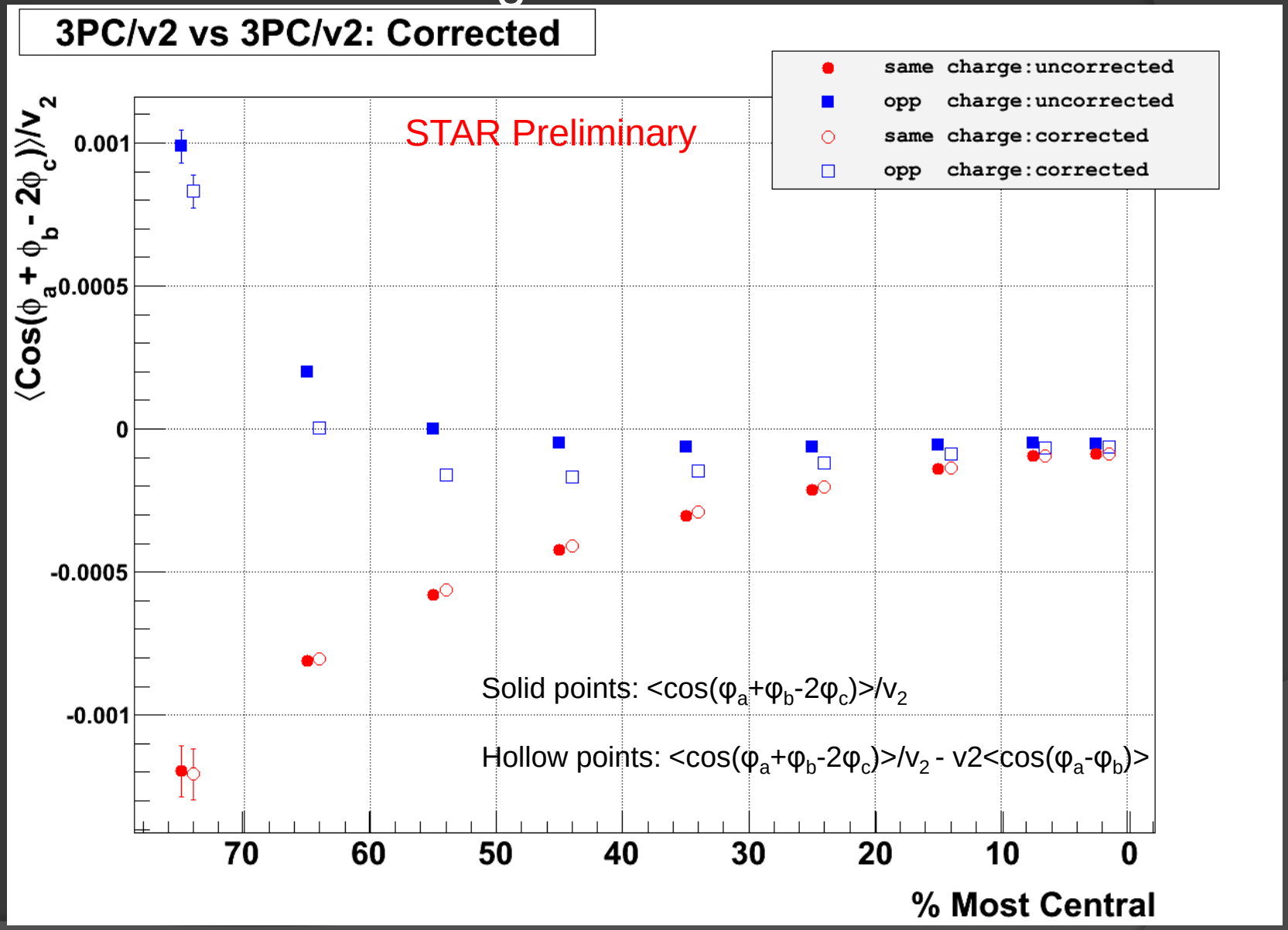
To reduce the effects of the elliptic flow on the correlator

$$(\langle\cos\Delta\varphi_a\cos\Delta\varphi_b\rangle(1-v_2) - \langle\sin\Delta\varphi_a\sin\Delta\varphi_b\rangle(1+v_2)) =$$
$$= (\langle\cos\Delta\varphi_a\cos\Delta\varphi_b\rangle - \langle\sin\Delta\varphi_a\sin\Delta\varphi_b\rangle) - v_2(\langle\cos\Delta\varphi_a\cos\Delta\varphi_b\rangle - \langle\sin\Delta\varphi_a\sin\Delta\varphi_b\rangle)$$
$$= \boxed{(\langle\cos(\varphi_a+\varphi_b-2\varphi_c)\rangle/v_2 - v_2\langle\cos(\varphi_a-\varphi_b)\rangle)}$$

As suggested by S. Pratt (2010)
http://drupal.star.bnl.gov/STAR/system/files/spratt2.pdf

Modifications according to flow

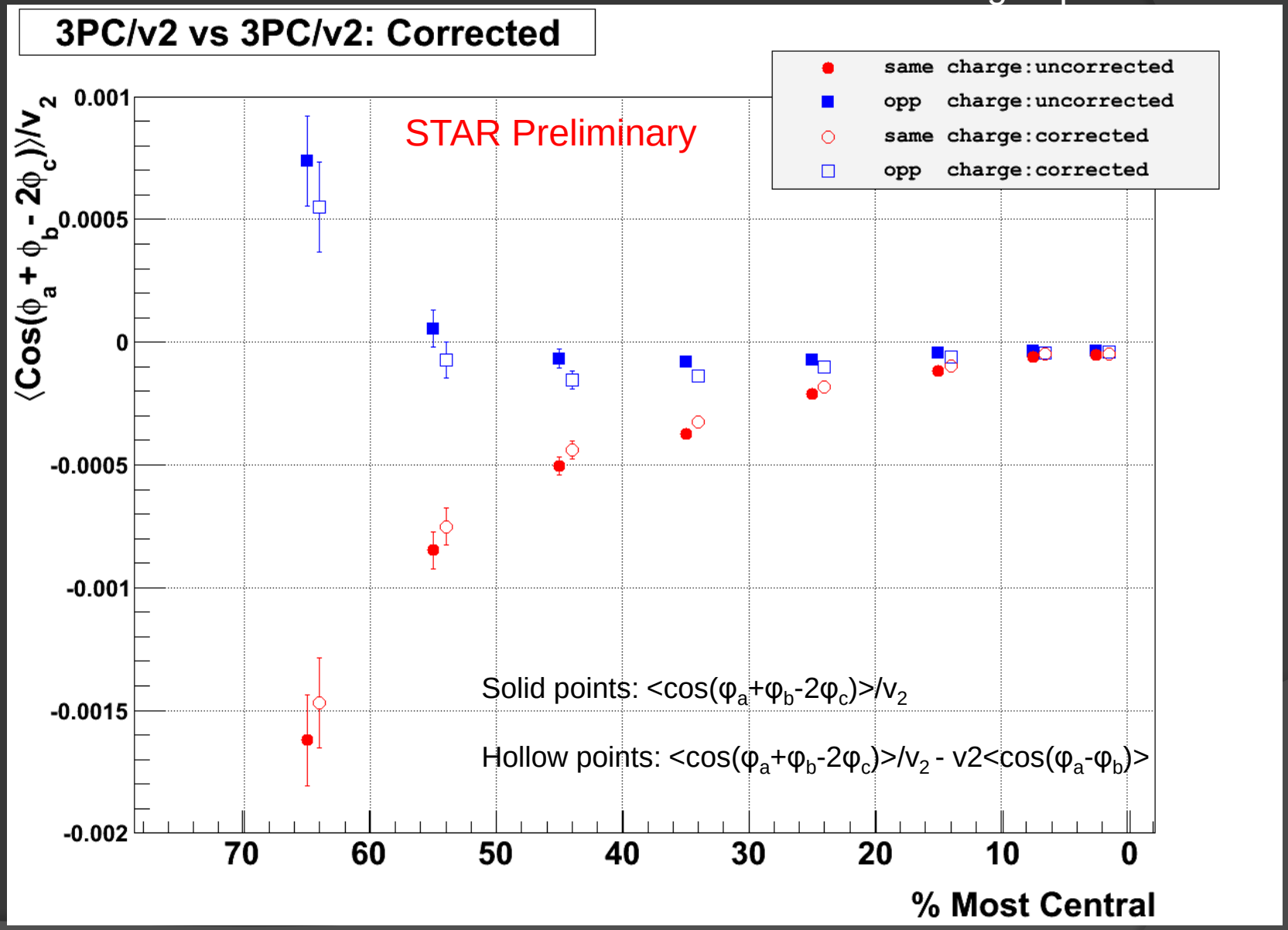
- a,b,c are charged particles



200 GeV per nucleon gold-gold collisions

K-Pi correlation vs modified

- a particle is pion
- b particle is kaon
- c is charged particle

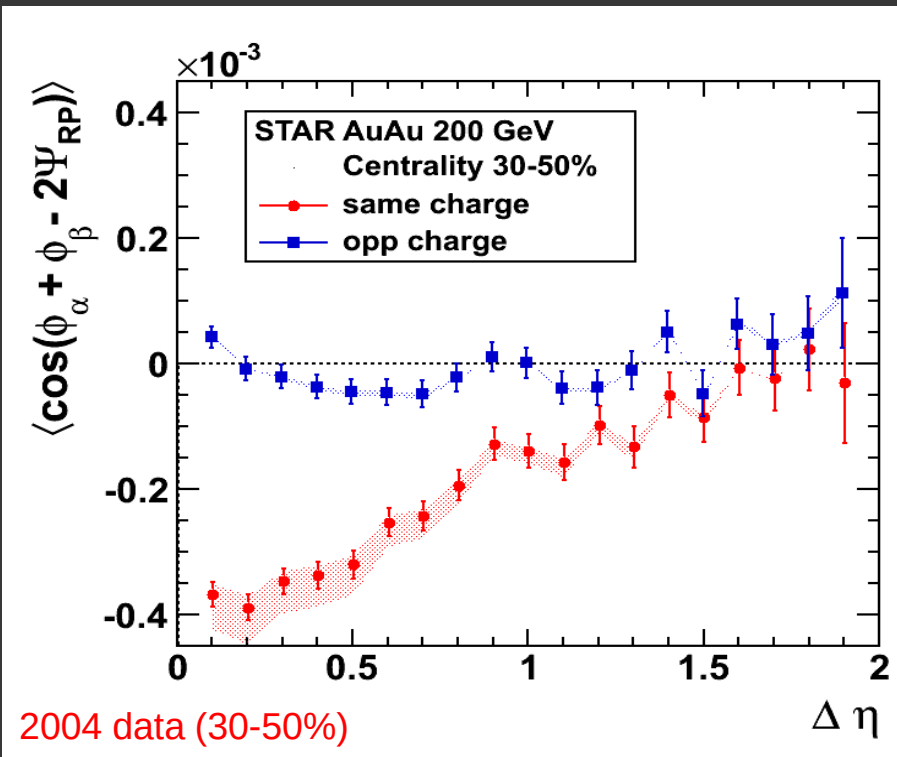


200 GeV per nucleon gold-gold collisions

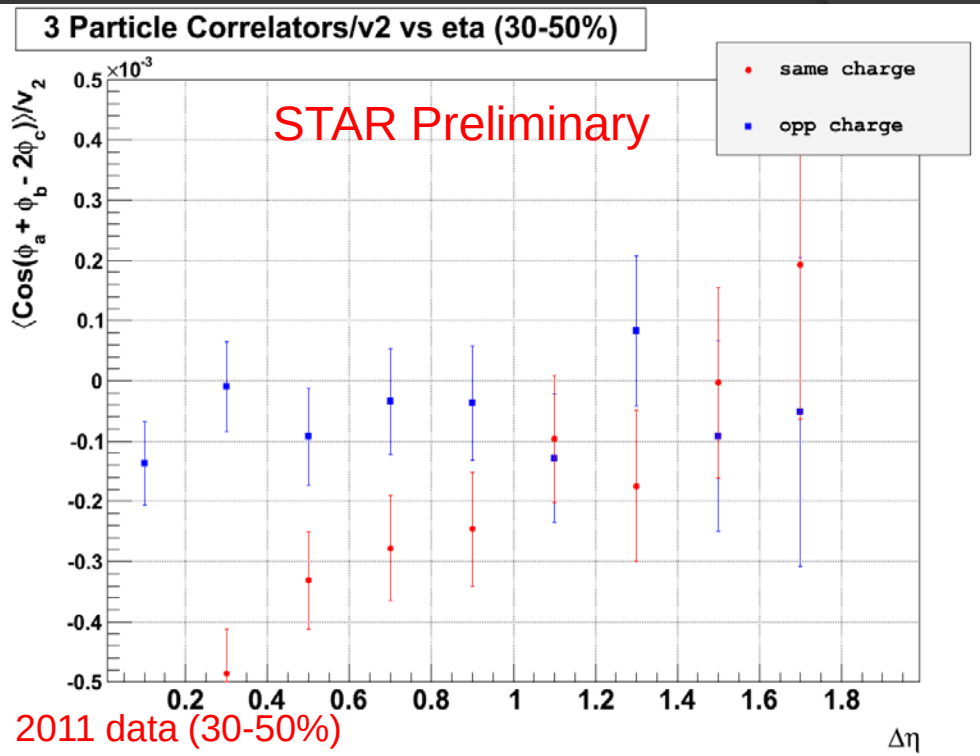
K-Pi correlation vs Δη

- a,b,c are charged particles

- a particle is pion
- b particle is kaon
- c is charged particle



2004 data (30-50%)



2011 data (30-50%)

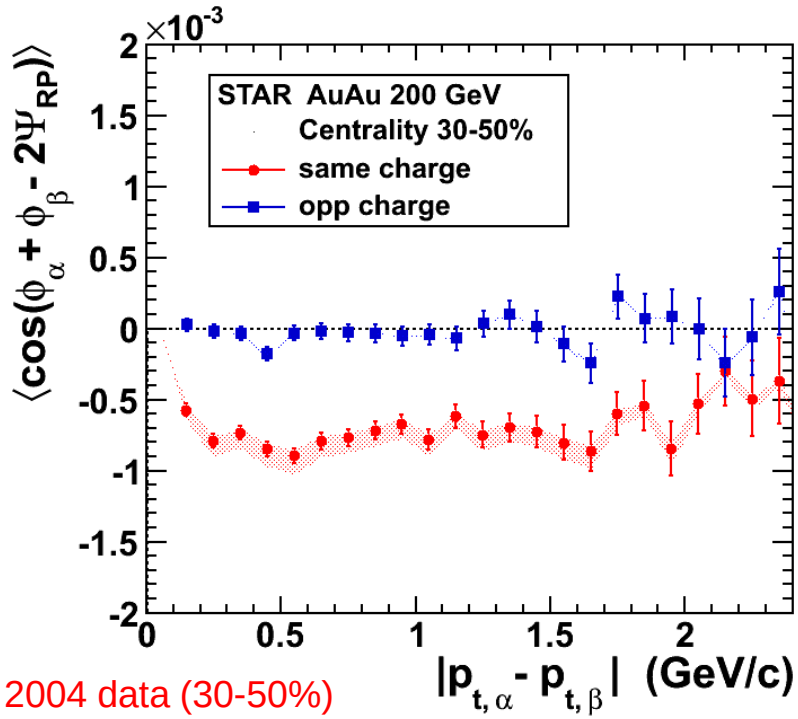
Graph from: STAR collaboration
arXiv:0909.1717v2 [nucl-ex]

200 GeV per nucleon gold-gold collisions

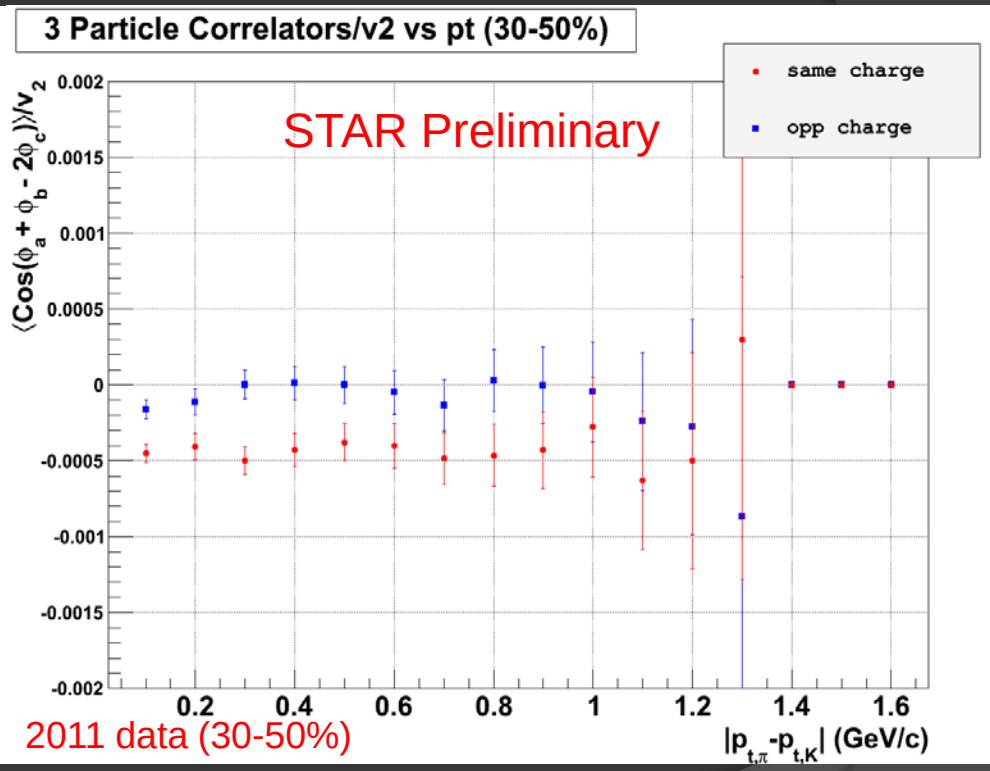
K-Pi correlation vs Δp_T

- a,b,c are charged particles

- a particle is pion
- b particle is kaon
- c is charged particle



2004 data (30-50%)



2011 data (30-50%)

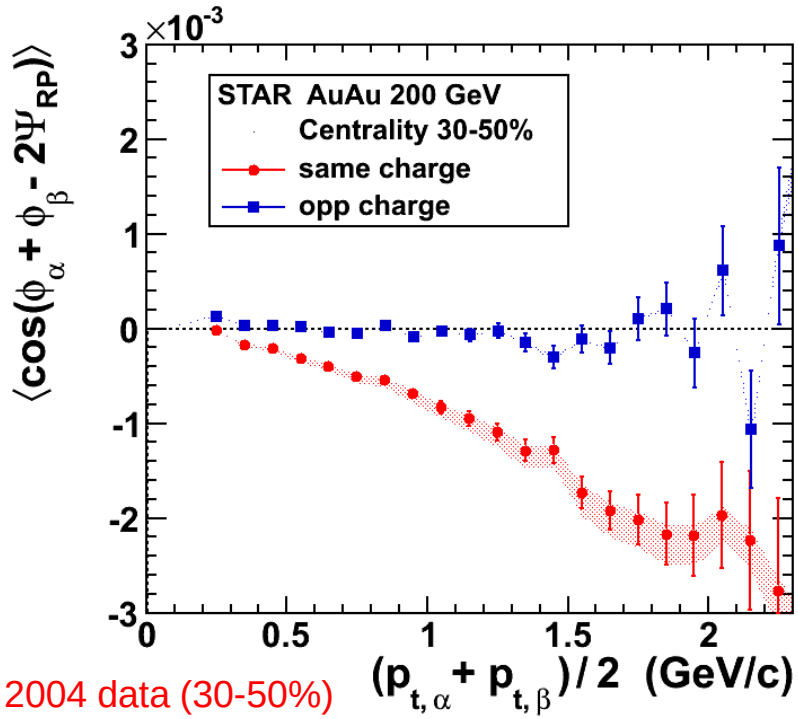
Graph from: STAR collaboration
arXiv:0909.1717v2 [nucl-ex]

200 GeV per nucleon gold-gold collisions

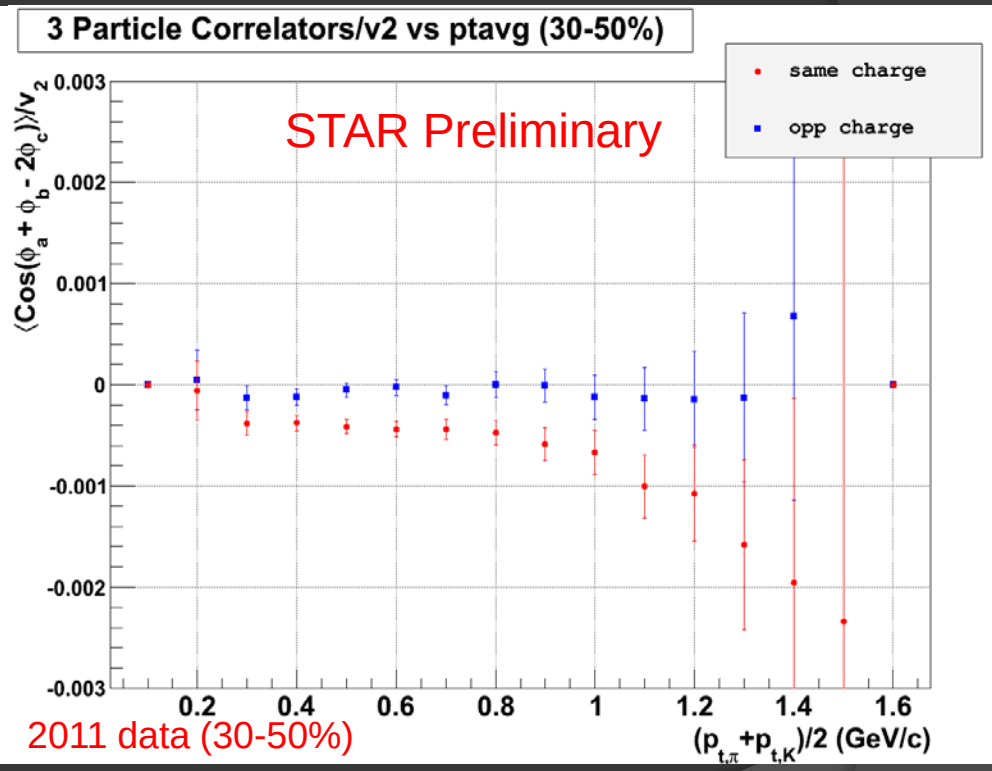
K-Pi correlation vs <p_T>

- a,b,c are charged particles

- a particle is pion
- b particle is kaon
- c is charged particle



2004 data (30-50%)

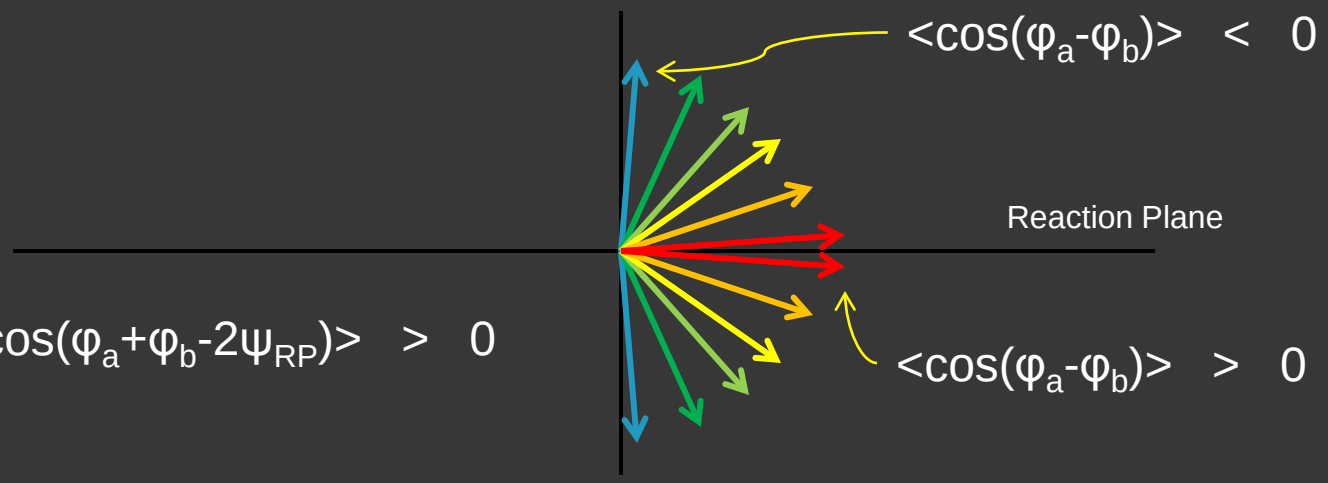


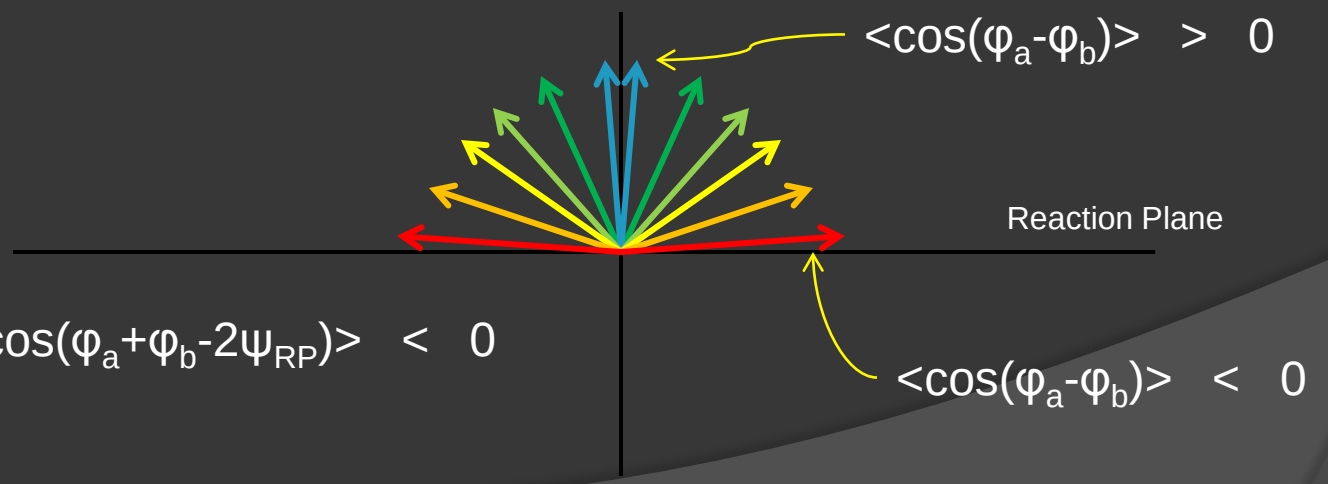
2011 data (30-50%)

Graph from: STAR collaboration
arXiv:0909.1717v2 [nucl-ex]

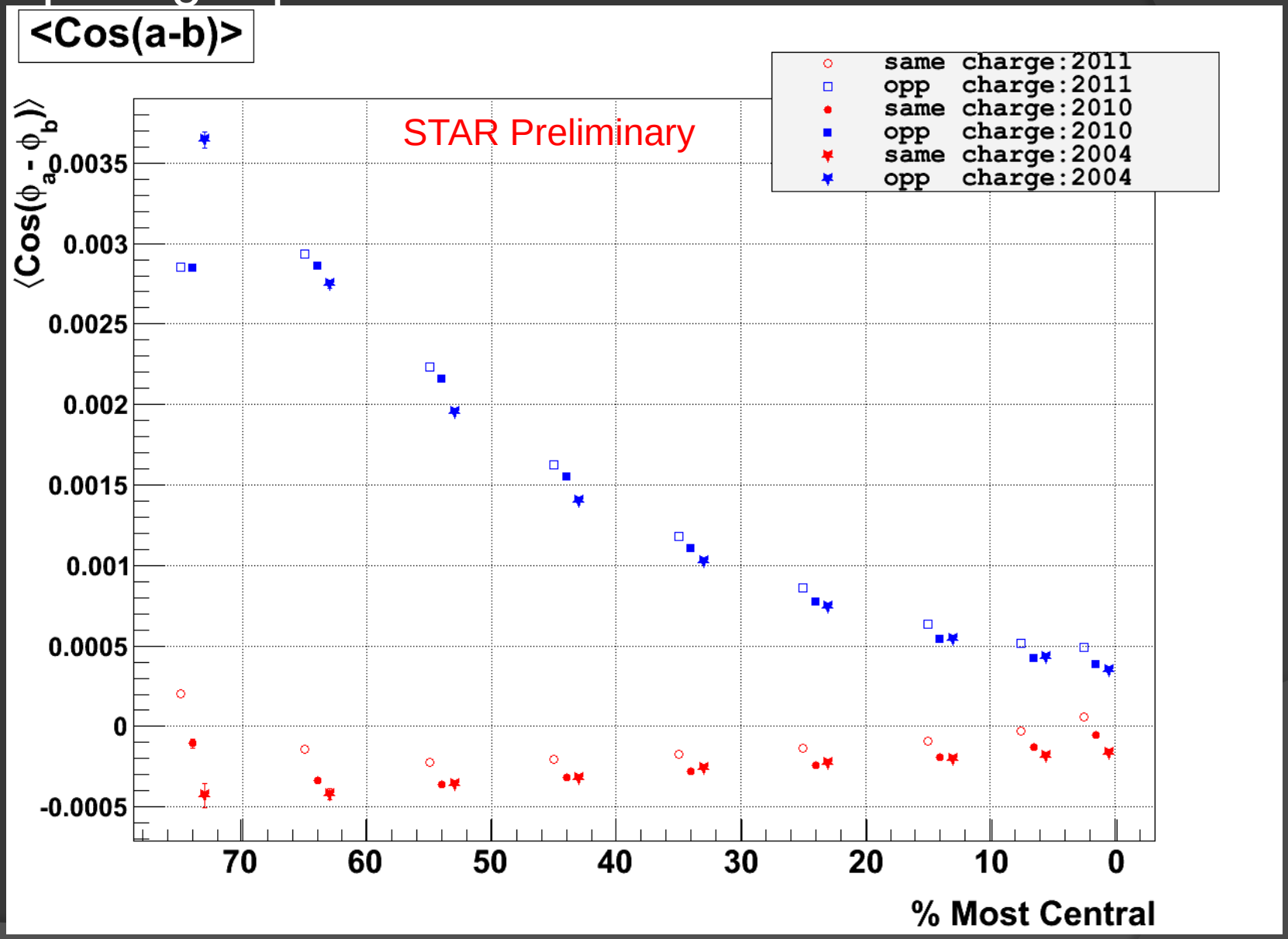
200 GeV per nucleon gold-gold collisions

Measuring the Charge Separation



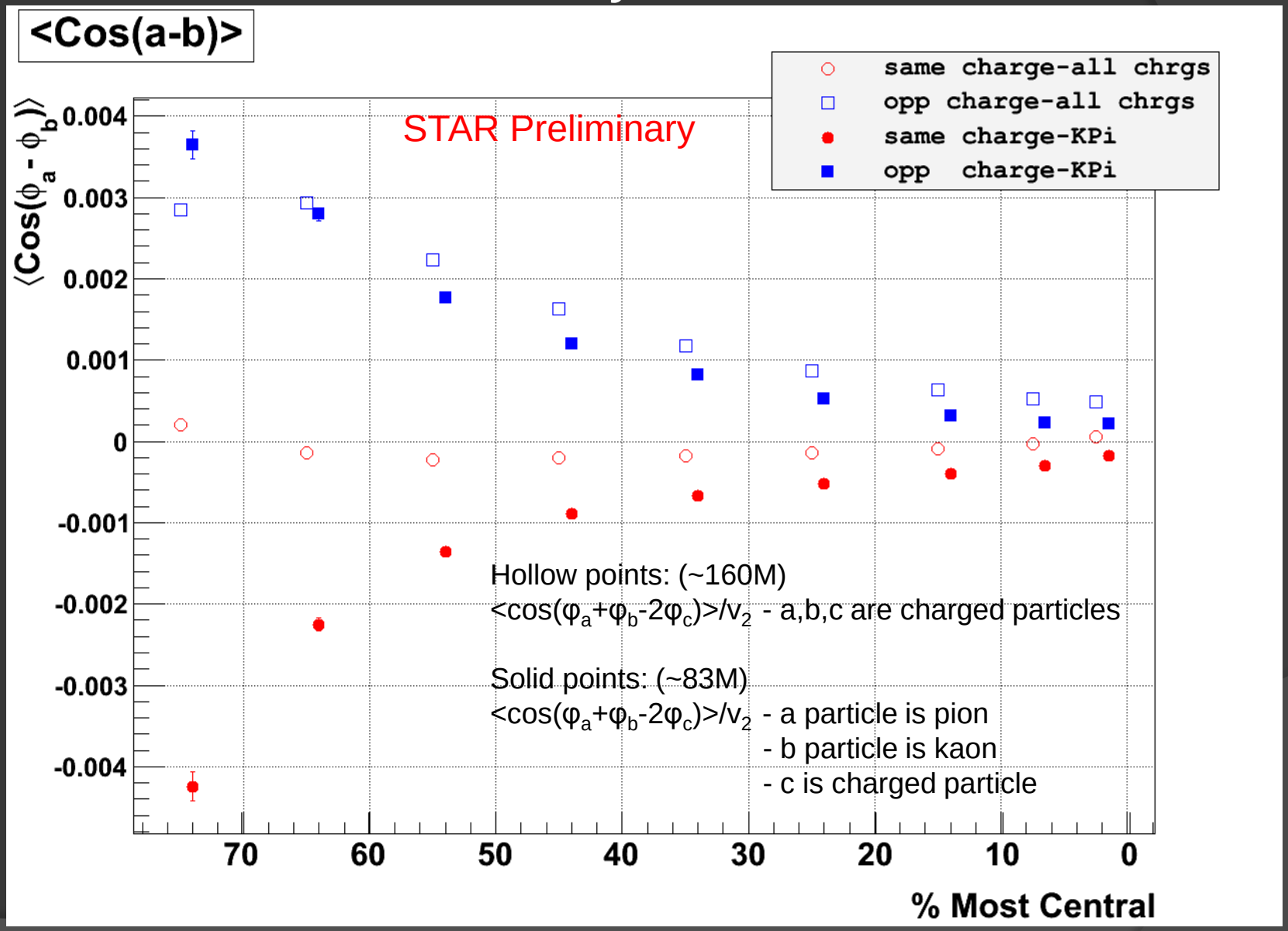


Comparing 2-particle correlations - a,b are charged particles



200 GeV per nucleon gold-gold collisions

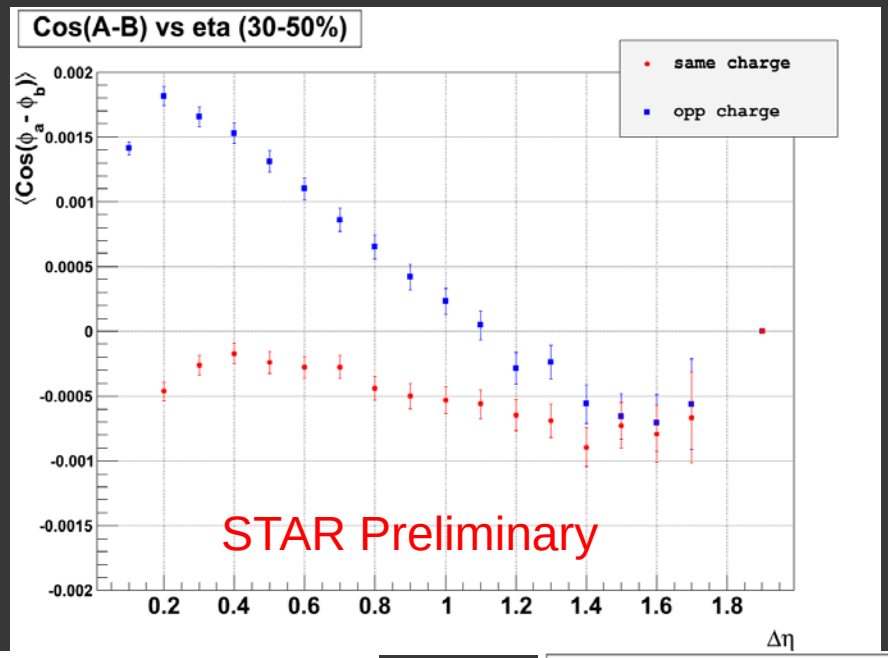
K-Pi correlation vs centrality

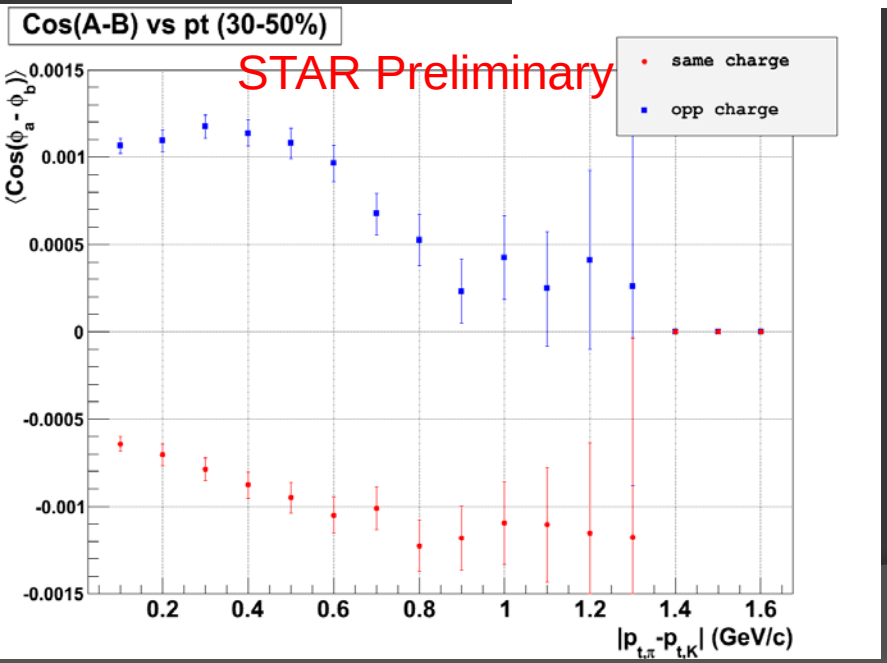


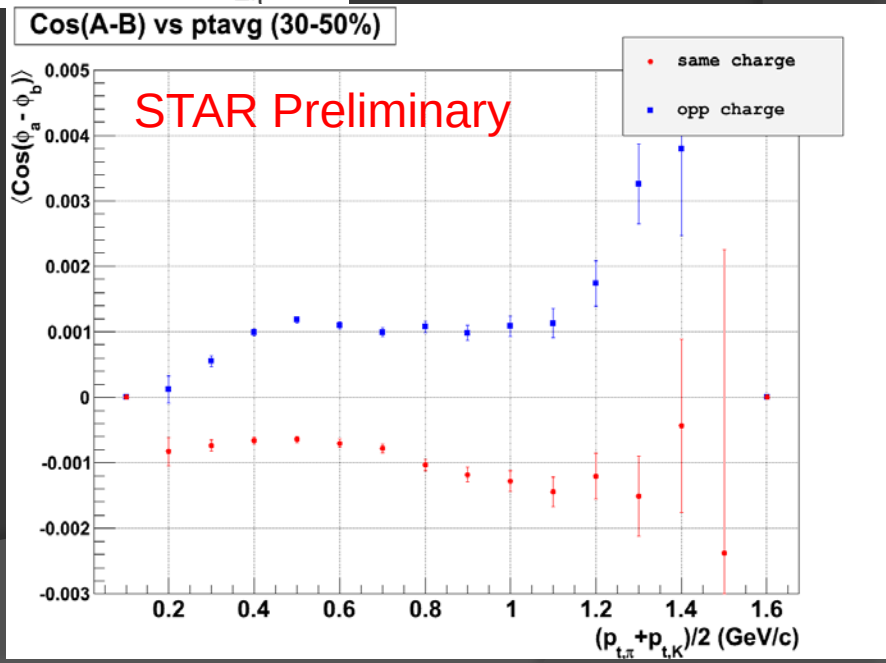
200 GeV per nucleon gold-gold collisions

K-Pi correlation vs $\Delta\eta$, Δp_T ,$<\Delta p_T>$

$<\cos(\varphi_a-\varphi_b)>$ - a particle is pion
- b particle is kaon







Summary

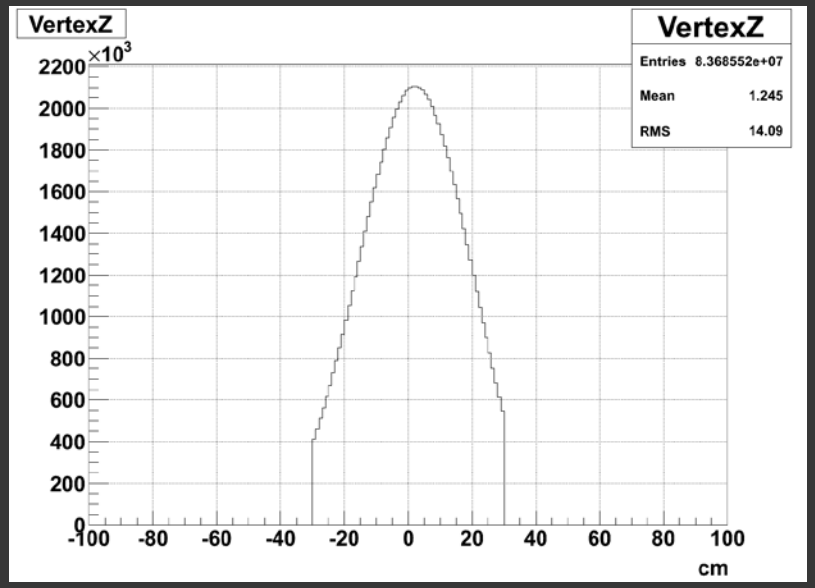
- K-Pi correlations exhibit characteristic LPV signal
- Flow modifications (the $v_2 \langle \cos(\varphi_a - \varphi_b) \rangle$ term) are smaller in K-Pi correlations compared to using all charges in opposite pairs
  - As expected from removing late-stage pion pair production
- To Do:
  - Removing K* resonances – preliminary simulations show no effect
  - Run Pi-Pi correlations
  - K-K correlations – many opp charge pairs produced in early stages of collision
    - statistics

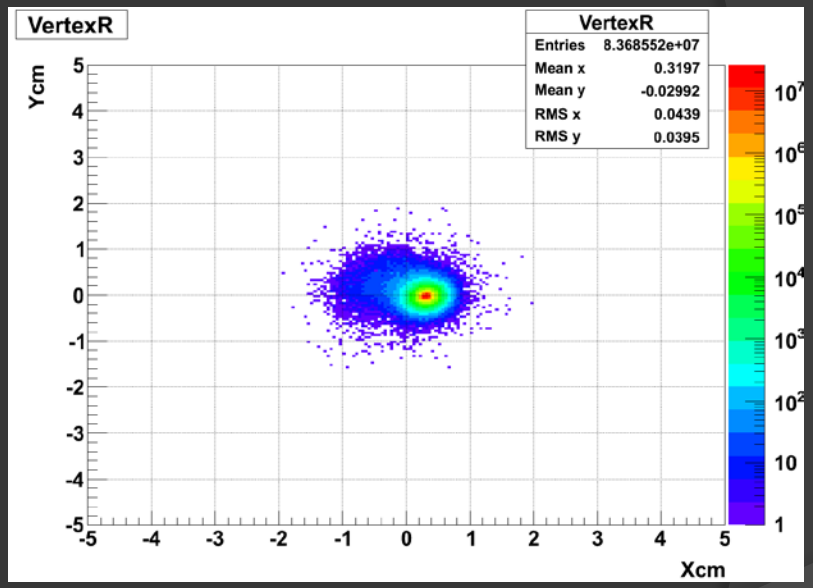
Backup

- Backup
  - Backup
    - Backup
      - Backup
        - Backup
          - Backup
            - Backup
              - Backup
              - Backup

200GeV Dataset and Cuts

Production	Trigger setup	Vertex Cuts	Trigger ID	Events
P11id	AuAu200_production _2011	VertexZ < 30cm VertexR < 2cm VpdZ – VertexZ < 4cm	350043	~160M all charge ~83M K-Pi





Cuts –
Primary tracks
15 < # of hits < 50
0.52 < # of hits / # of possible hits
0.15 < p_t < 2.0 GeV/c
|Eta_a,b,c| < 1
(Using bad runs list in StRefMultCorr)

Centrality bins* (Alex & Hiroshi) – StRefMultCorr

* StRefMultCorr – dependent on RefMult, Z-vertex, Luminosity

v2 – average of 2- and 4-particle cumulant flow

200GeV Dataset and Cuts

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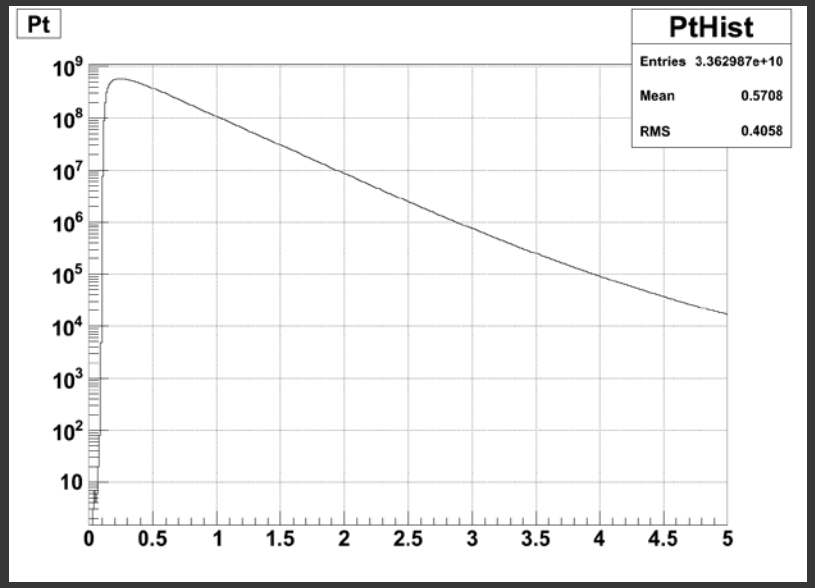

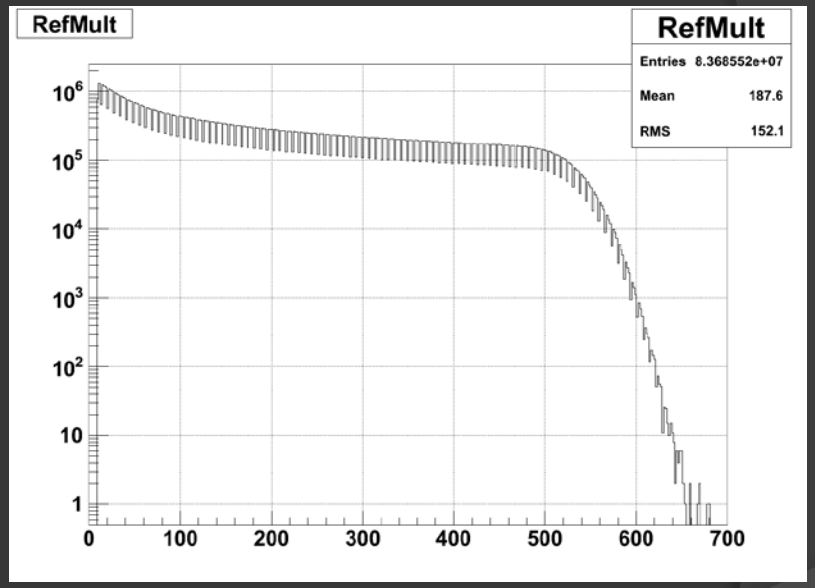

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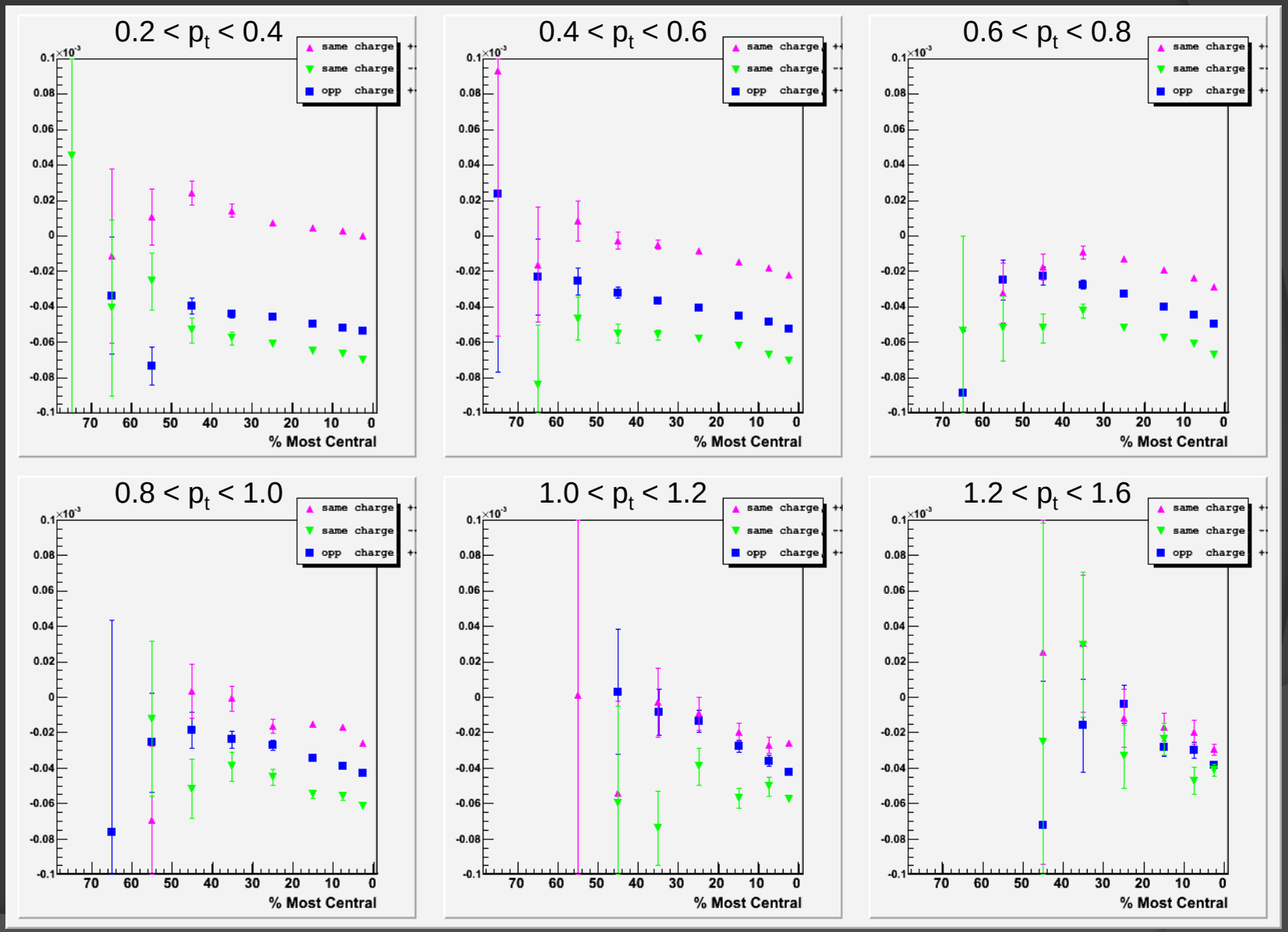
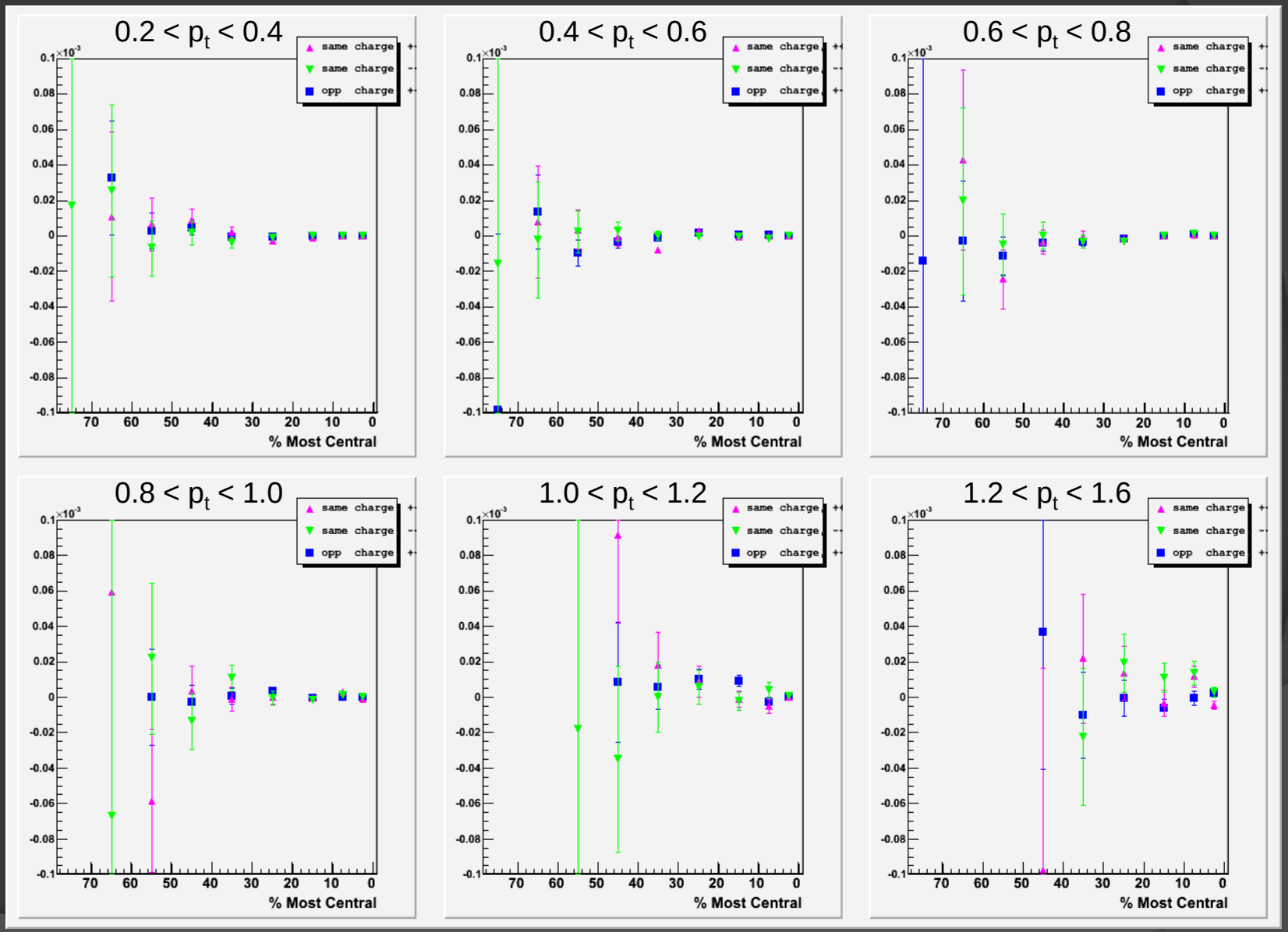
K-Pi simulation (50M events)



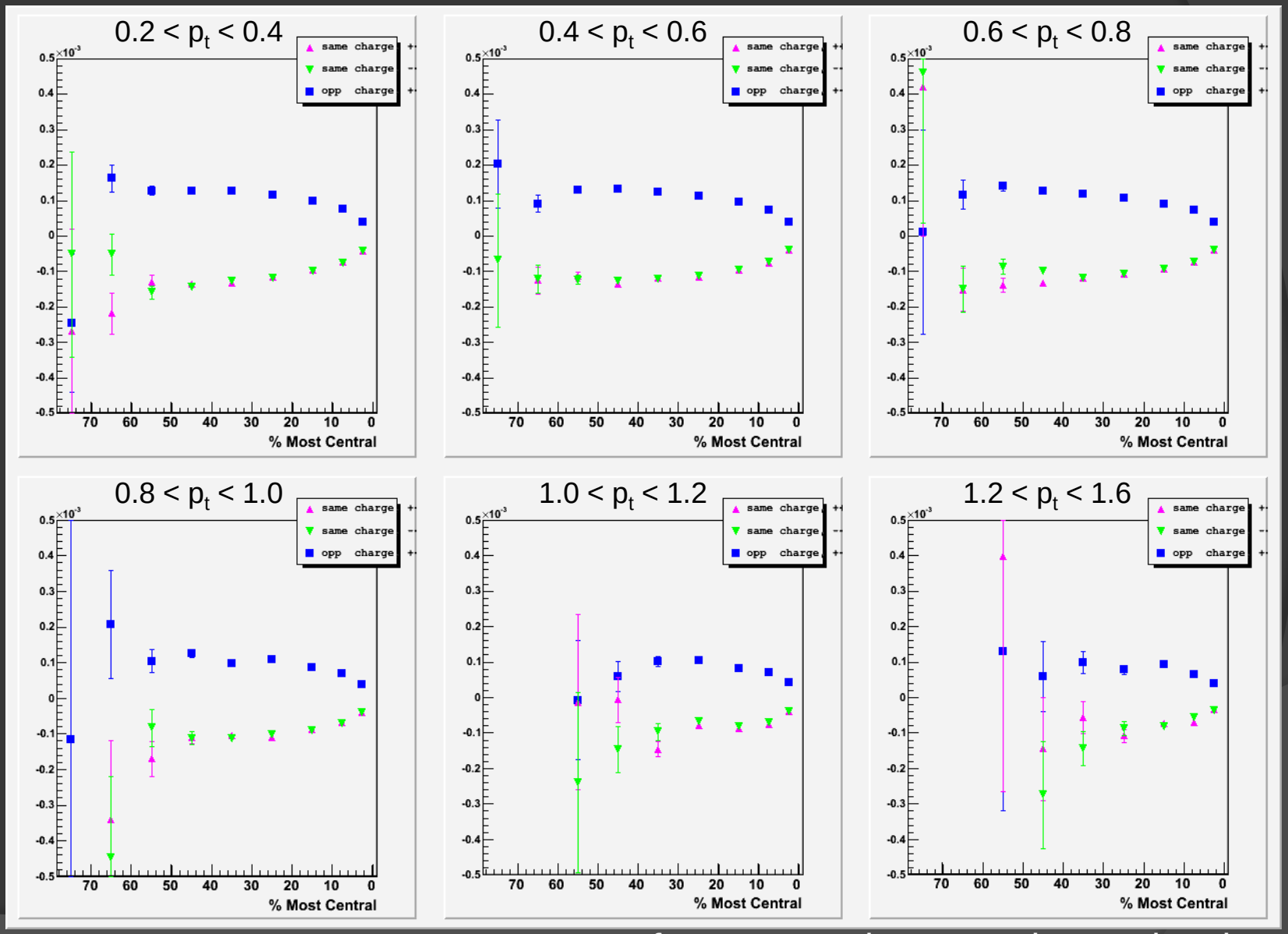
Random particles with elliptic flow

K-Pi simulation (50M events)



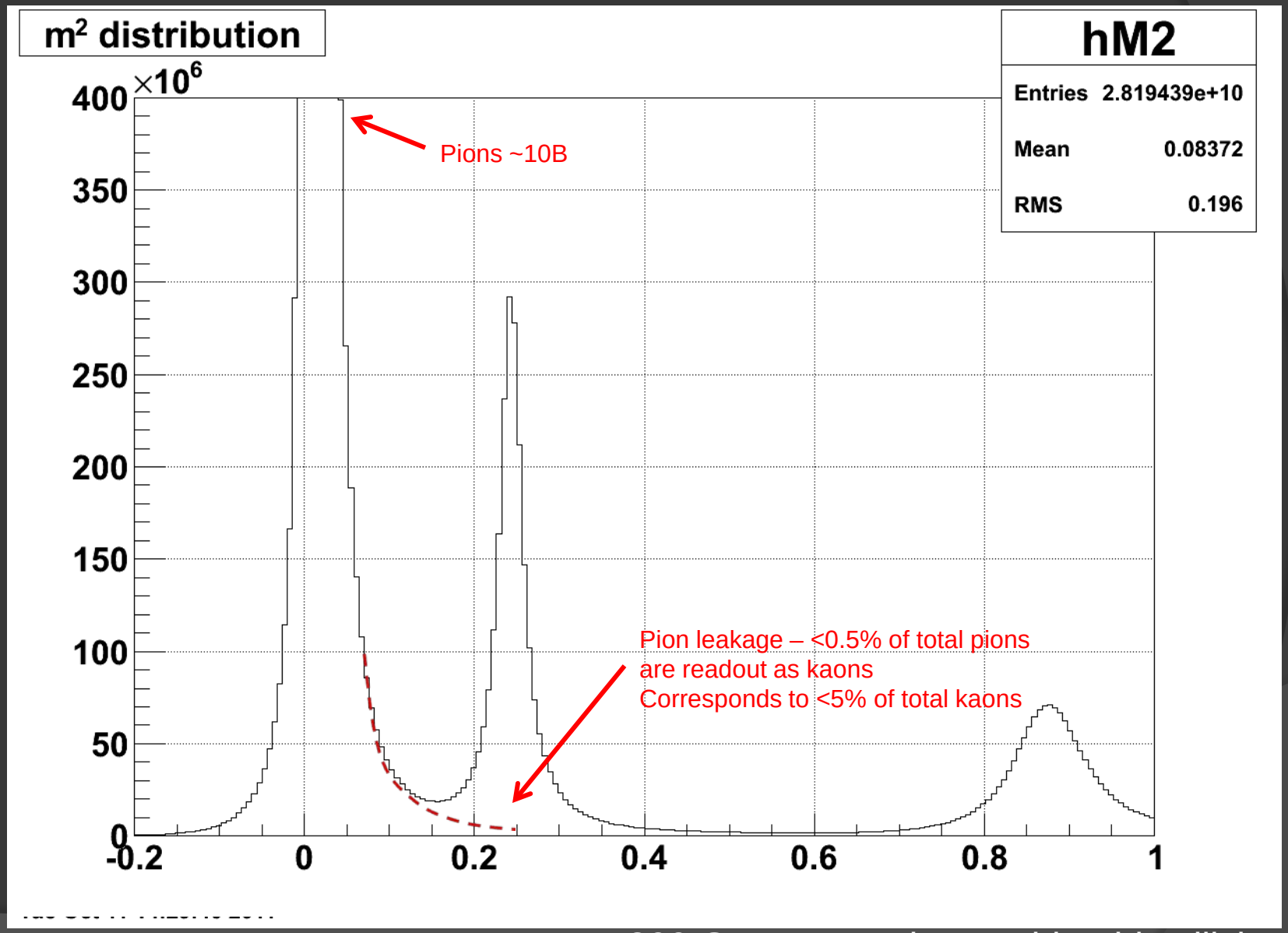
After re-centering procedures

K-Pi simulation (50M events)



After re-centering procedures, signal on

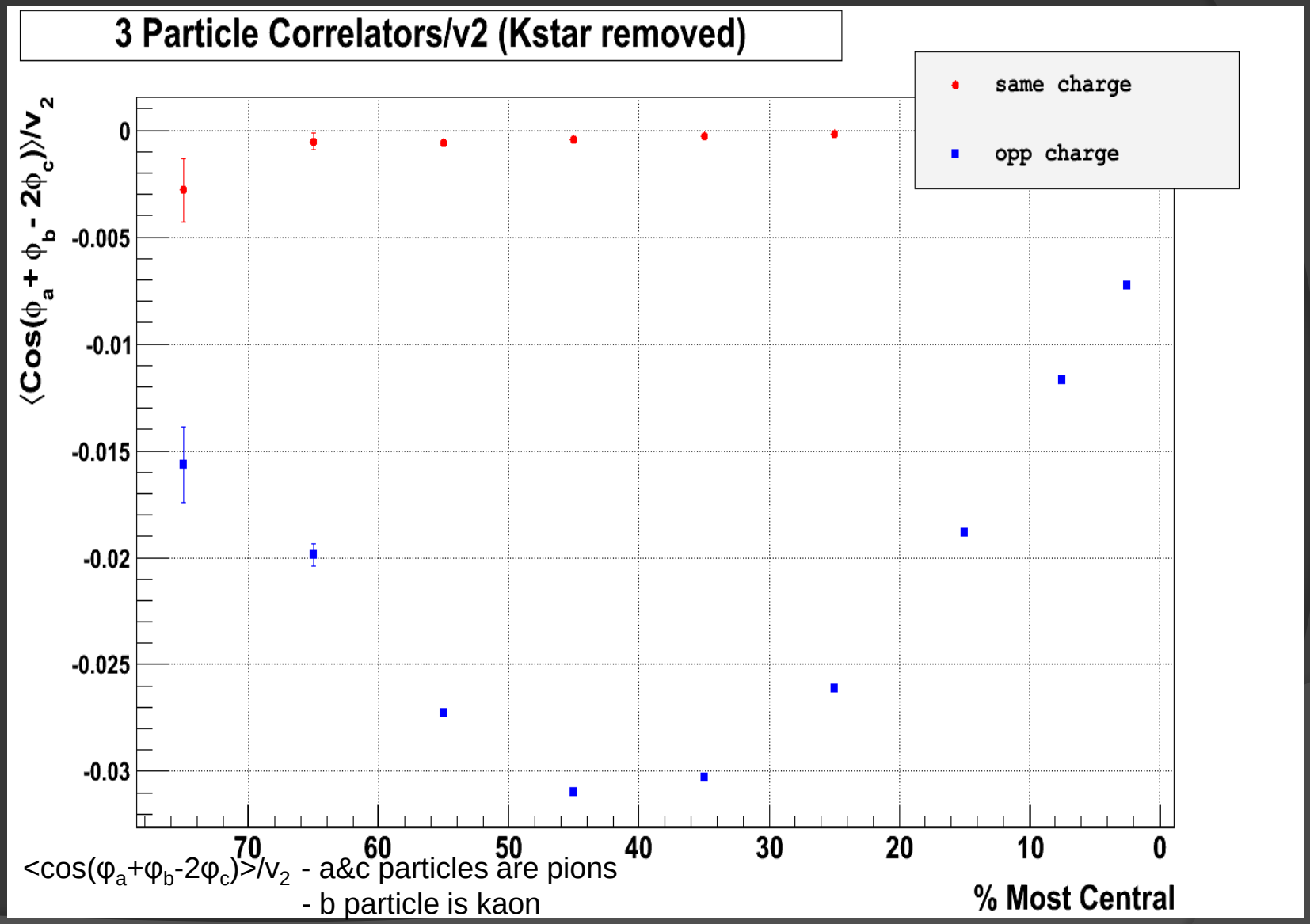
K-Pi correlation



200 GeV per nucleon gold-gold collisions

K-Pi correlation vs centrality

Kaon PID cuts ~97.5% of tracks



$\langle\cos(\varphi_a+\varphi_b-2\varphi_c)\rangle/v_2$ - a&c particles are pions
- b particle is kaon

200 GeV per nucleon gold-gold collisions

K-Pi correlation vs centrality

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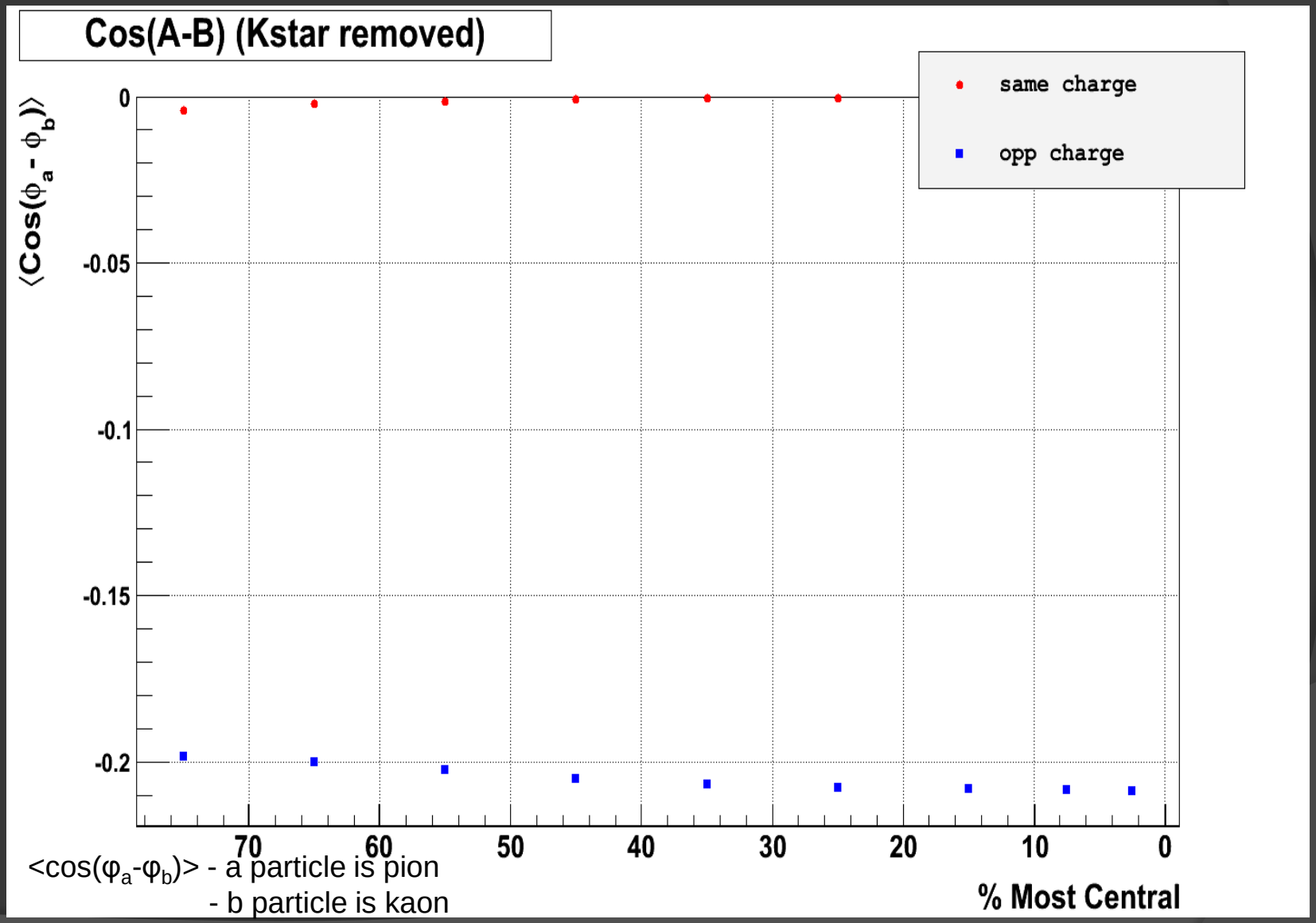


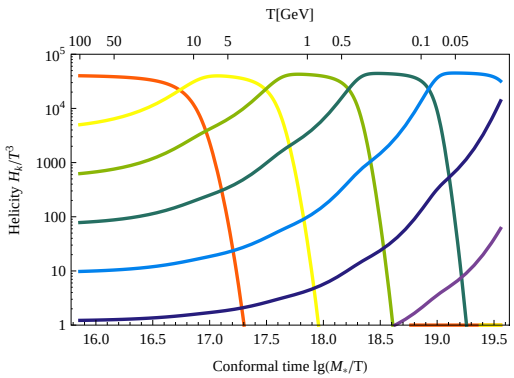
$\langle \cos(\varphi_a - \varphi_b) \rangle$ - a particle is pion
- b particle is kaon

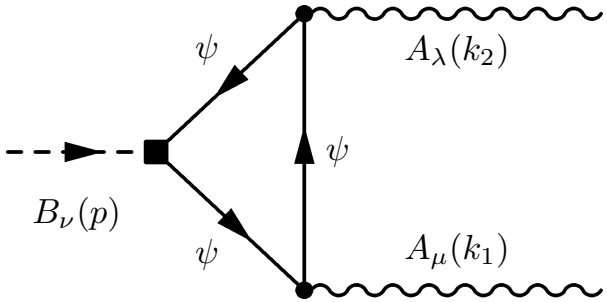
200 GeV per nucleon gold-gold collisions

Chiral anomaly and magnetic fields in the Standard Model plasma

Oleg RUCHAYSKIY

CERN





P- and CP-Odd Effects in Hot and Dense Matter

June 26, 2012

Outline

In this talk I will demonstrate that

- At high temperatures / densities the equilibrium state of the **Standard Model** plasma is not homogeneous and isotropic but should contain large scale magnetic fields

- The description of evolution of helical magnetic fields at high temperatures (by any mechanism) requires **additional "degree of freedom"**

- Both results are based **solely on the Standard Model physics**

Based on the recent works:

– A. Boyarsky, J. Fröhlich, **O.R.**
  Phys. Rev. Lett. (2012) [arXiv:1109.3350]

– A. Boyarsky, **O.R.**, M. Shaposhnikov [arXiv:1204.3604]

Can homogeneous and isotropic plasma **spontaneously break** translational invariance?

[1204.3604]

Reminder: equilibrium plasma

- Properties of the equilibrium system are characterized by its temperature and **the values of conserved charges**

- In the Standard Model at $T < 100$ GeV (when electroweak symmetry is broken) there are **4 conserved charges**:

  – **Baryon number** B
  – Three **flavour lepton numbers** L_e, L_μ, L_τ

  Additionally the plasma is electrically neutral

- Plasma breaks Lorentz invariance down to 3-dimensional symmetry

Static magnetic fields in plasma

- Effective action of the **static** electromagnetic fields has the form

$$\mathcal{F}[A] = \frac{1}{2}\int d^3p\, A_i(\vec{p})\Pi_{ij}(p)A_j(-\vec{p}) + \mathcal{O}(A^3) \qquad (1)$$

(magnetic field $\vec{B} = \nabla \times \vec{A}$)

- **Polarization operator** Π_{ij} should be rotation invariant and gauge invariant (i.e. transversal: $p_i\Pi_{ij} = 0$). The most general form:

$$\Pi_{ij}(\vec{p}) = \underbrace{(p^2\delta_{ij} - p_ip_j)\Pi_1(p^2)}_{\text{parity-even part}} \quad + \quad \underbrace{i\epsilon_{ijk}p^k\Pi_2(p^2)}_{\text{parity-odd part}}$$

  - Π_1 is a renormalization of the electric charge, we will forget about it from now on ($\Pi_1 = 1$)
  - here and below we will speak only about $\Pi_2(0)$ that we denote simply by Π_2

Chern-Simons term

- In coordinate space $\Pi_2 \neq 0$ this leads to a **Chern-Simons term**:

$$\mathcal{F}[A] = \frac{1}{2} \int d^3x \left(\vec{B}^2 + \Pi_2 \vec{A} \cdot \vec{B} \right)$$

- The Chern-Simons term
  - contains less derivatives than $(\nabla \times A)^2$
  - can be both positive and negative

- The matrix Π_{ij} has a negative eigenvalue for

$$p < \Pi_2$$

- **Long-range magnetic fields** with $p < \Pi_2$ will be generated

Maximally helical configuration

- The unstable mode will have a form

$$\vec{A}(\vec{x}) = A_0\Big(\cos(pz), \sin(pz), 0\Big)$$

- The magnetic field

$$\vec{B}(\vec{x}) = -p\vec{A}(\vec{x})$$

— is maximally helical

- On this configuration $\vec{B}^2 = p\vec{A}\cdot\vec{B}$ and are homogeneous

- The effective action:

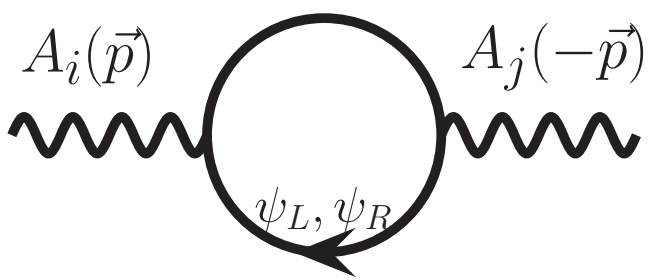
$$\mathcal{F}[A] = \frac{1}{2}\int d^3x \left(p^2 - p\Pi_2\right) A_0^2 < 0$$

for $p < \Pi_2$

Origin of Chern-Simons term

- Chern-Simons terms are usually prohibited by discrete symmetries (P, CP, CPT)
- The origin of this term?
- P, CP, CPT are broken by non-zero chiral charges of chiral fermions (by non-zero chemical potentials μ_L and μ_R)
- If number of **left particles** $\neq$ the number of **right particles** (i.e. they have **different chemical potentials** $\mu_R \neq \mu_L$) then

$$\boxed{\Pi_2 = \frac{\alpha}{\pi}\Delta\mu}$$



Vilenkin (1978)

Redlich & Wijewardhana (1985);

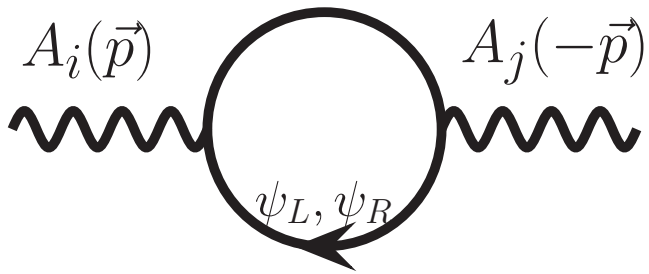
Fröhlich et al. (1998–2001)

Joyce & Shaposhnikov (1997)

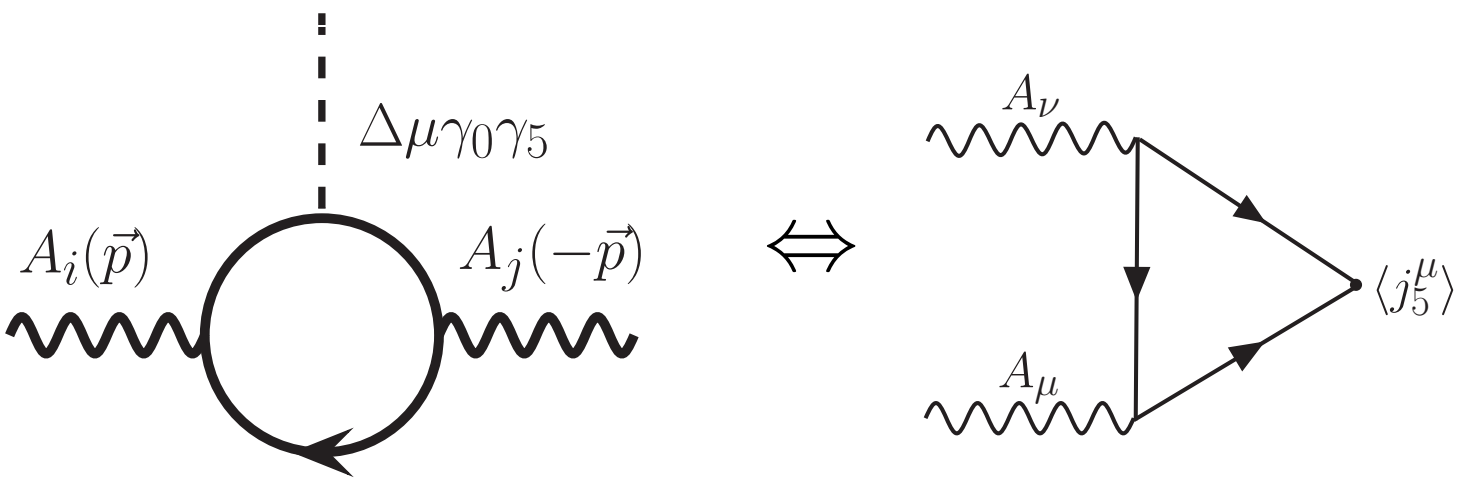
Chern-Simons term and axial anomaly

- In plasma with the different number of left and right particles

$$\Pi_2 = \frac{\alpha}{\pi}\Delta\mu$$



- This diagram is related to axial anomaly



Different number of left and right chiral particles?

$$\frac{1}{\exp\left(\frac{p-\mu_L}{T}\right)+1} \neq \frac{1}{\exp\left(\frac{p-\mu_R}{T}\right)+1} \; ?$$

Left-right equilibration rates

- **Chirality flipping processes** are related to fermion' Yukawa (or mass).

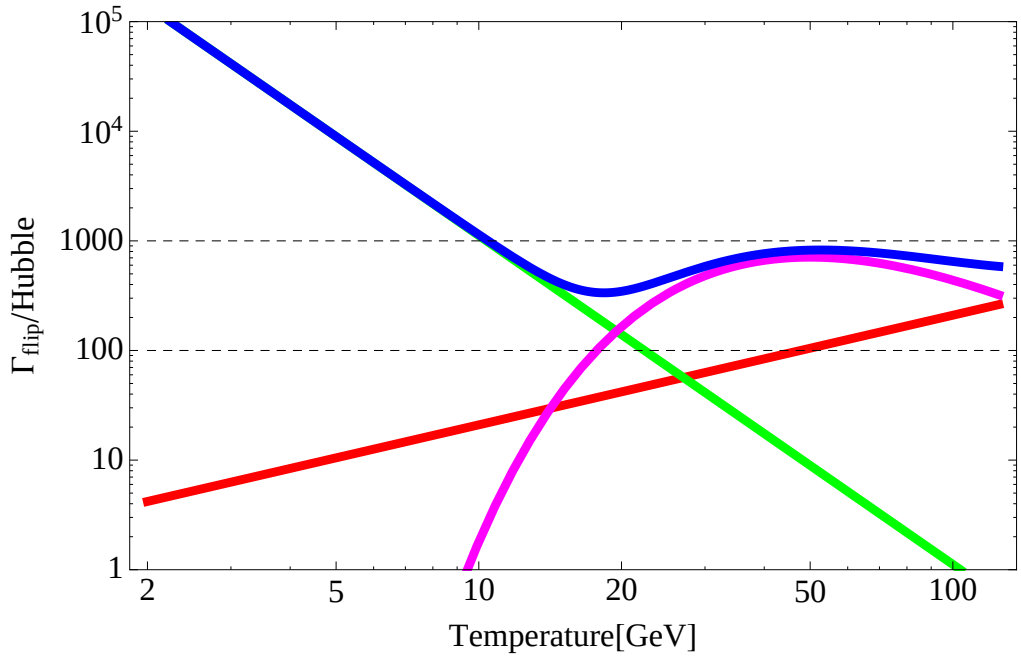
- Although $T \gg m$ and these reactions are suppressed as $(m/T)^2$ as compared to chirality-preserving reactions after long time they will wash out $\Delta\mu$:

$$\frac{\Delta\mu}{dt} = -\Gamma_f \Delta\mu$$

Examples: early Universe

- Starting from $T \sim 80$ TeV chirality flipping processes are in equilibrium ($\Gamma_{\text{flip}}(T) \gg H(T) = T^2/M_*$)

Cambell et al. (1992)



Symmetric phase:

- $\Gamma_{\textbf{high-temp}} \sim \frac{80\ \text{TeV}}{M_*} T$

Broken phase:

- $\Gamma_{\text{EM}} \propto \alpha^2 T \left(\frac{m_e}{3T}\right)^2$
- $\Gamma_{\text{W}} \propto G_F^2 T^5 \left(\frac{m_e}{3T}\right)^2$

Equilibrium vs. non-equilibrium $\Delta\mu$

? Although $\left(\frac{m_e}{80\ \text{TeV}}\right)^2 \sim 10^{-17}$ chirality flipping reactions are in thermal equilibrium for $T < 80$ TeV and drive $\mu_L - \mu_R$ to zero **exponentially fast** (suppression of at least e^{-1000} over one Hubble time)

Joyce & Shaposhnikov

? Only **non-equilibrium relaxation** of initial $\Delta\mu(t)$ is possible? This relaxation can be "slow"...

Laine'05

...

? Equilibrium state is always $\mu_L = \mu_R$?

No!

Boyarsky, **O.R.**, Shaposhnikov [1204.3604]

! It is possible to have **equilibrium** difference of chemical potentials!

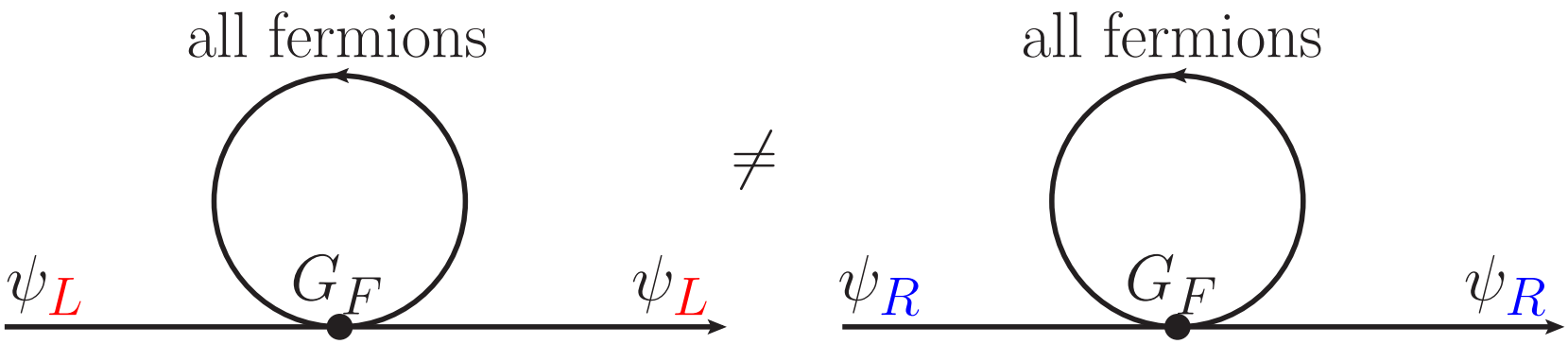
! This **does not require** super-high temperatures (can even happen at zero temperature but finite density!)

Weak corrections

Boyarsky, Shaposhnikov, **O.R.** [1204.3604]

- Weak corrections lead to the **change of dispersion relations** (shift of chemical potentials) of left/right particles

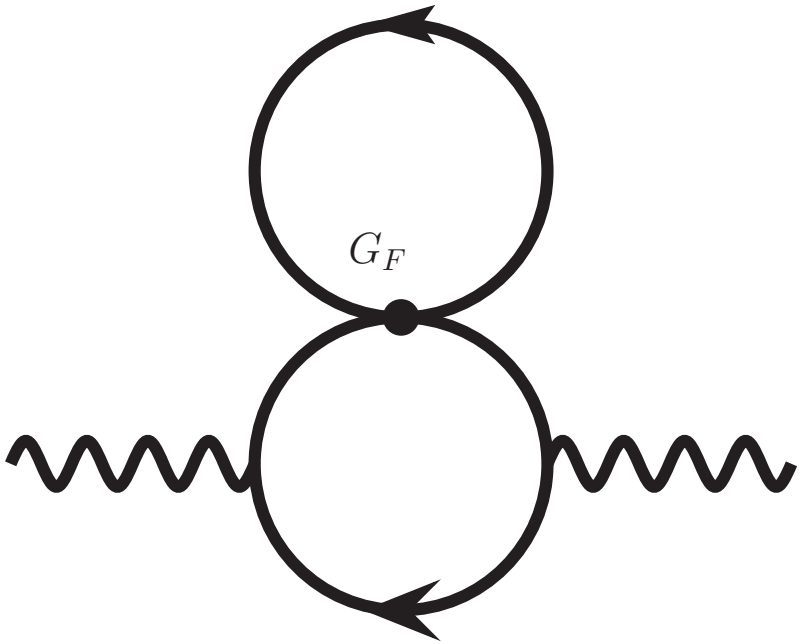
  it is crucial that chirality flipping processes are in equilibrium



- The resulting $\mu_L - \mu_R$ is proportional to the **asymmetry** of all fermions, running in the loops

- Asymmetry $n_\psi - n_{\bar{\psi}} \propto$ global charges(B, L_e, L_μ, L_τ)

Chern-Simons term



Boyarsky,
Shaposhnikov,
O.R.
[1204.3604]

$$\Pi_2 = \frac{\alpha}{2\pi} G_F \times (c_1 \text{ baryon number} + c_2 \text{ lepton numbers}) \neq 0$$

Summary-I

Purely within the Standard Model:

- Chern-Simons term should be added to the Standard Model Lagrangian and finite densities of lepton and/or baryon number
- Homogeneous and isotropic ground state of primordial plasma is **unstable towards developing a long range magnetic fields**
- The origin of this effect is the parity-breaking character of weak interactions and chiral anomaly

From equilibrium to non-equilibrium

- This result was **purely equilibrium** computation $\leftrightarrow$ property of the ground state of the Standard Model plasma at finite temperature and/or density

- How does this effect exhibit itself in the non-equilibrium case? How does a deviation from equilibrium look like? Let us consider the case without non-zero global charges for simplicity

- Reminder: dynamics of the magnetic field is described by the **MHD equations**

$$\frac{\partial B}{\partial t} = \nabla \times (\mathbf{v} \times B) + \frac{1}{\sigma}\nabla^2 B$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot (\nabla \mathbf{v}) = -\nabla p + \nu \nabla^2 \mathbf{v} + (\nabla \times B) \times B$$

(in the absence of $\Delta\mu$!)

Magnetic diffusion

- Ohmic dissipation of magnetic energy — **magnetic diffusion**:

$$\frac{\partial B}{\partial t} = \frac{1}{\sigma}\nabla^2 B$$

- Generically magnetic fields at scales below **magnetic diffusion scale**

$$\ell_\sigma \sim \sqrt{\frac{t}{\sigma}}$$

  are erased **over a characteristic time** t

- **Example: early Universe:** — magnetic diffusion scale today $\sim 10^{13}$ cm. Horizon of EW epoch is stretched to $\sim 10^{15}$cm

What changes in the presence of $\Delta\mu$?

Maxwell equations

- The presence of difference of chemical potential of left and right fermions leads to additional terms in the effective Lagrangian for electromagnetic fields – **Chern-Simons term**

- As a result Maxwell equations contain current, **proportional to $\Delta\mu$** — MHD turns into **chiral MHD**:

$$\begin{aligned} \mathrm{curl}\,\vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \mathrm{curl}\,\vec{B} &= \sigma\vec{E} + \frac{\alpha}{\pi}\Delta\mu\vec{B} \end{aligned}$$

Chiral magnetic effect

Kharzeev'11

Vilenkin (1978)

Redlich & Wijewardhana (1985);

Fröhlich et al. (1998–2001)

Joyce & Shaposhnikov (1997)

New degree of freedom

- **In addition**, $\Delta\mu$ should be allowed to **become dynamical**:
  because $\partial_\mu j_5^\mu \propto E \cdot B$

Boyarsky, Fröhlich, **O.R.**, PRL (2012)

$$\mathrm{curl}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\mathrm{curl}\,\vec{B} = \sigma\vec{E} + \frac{\alpha}{\pi}\Delta\mu\vec{B}$$

Chiral magnetic effect

$$\frac{\partial \Delta\mu}{\partial t} \propto \frac{2\alpha}{\pi}\int d^3x\,\vec{E}\cdot\vec{B} - \Gamma_{\mathrm{flip}}\Delta\mu$$

Chiral anomaly

- **Naively:**
  - Without B chirality flipping reactions drive $\Delta\mu \to 0$
  - Without $\Delta\mu$ finite conductivity drives $B \to 0$

Instability

- Maxwell equations with $\Delta\mu$ are **unstable**:

$$\frac{\partial B}{\partial t} = \frac{1}{\sigma}\nabla^2 B + \frac{\alpha\Delta\mu}{\pi}\,\mathrm{curl}\,B$$

magnetic diffusion

- Exponential growth for $k < \Delta\mu$ (for one of the circular polarizations depending on the sign of $\Delta\mu$) — **generation of helical magnetic fields**

$$B_\pm = B_0 \exp\left(-\frac{k^2}{\sigma}t \pm \frac{\alpha}{\pi}\frac{k\Delta\mu}{\sigma}t\right)$$

- Exponential growth for $k < \frac{\alpha}{\pi}\Delta\mu$

Helical magnetic fields in presence of $\Delta\mu$

- If there are electromagnetic fields in plasma we have

$$\frac{\partial \vec{B}}{\partial t} = \frac{1}{\sigma}\nabla^2 \vec{B} + \frac{\alpha}{\pi}\frac{\Delta\mu}{\sigma}\nabla \times \vec{B}$$

- If $\Delta\mu \neq 0$ – magnetic field grows and

$$\frac{d(\Delta\mu)}{dt} = -(c_\Delta \alpha)\frac{2}{V}\int_V d^3x\, \vec{E}\cdot\vec{B} - \Gamma_f \Delta\mu$$

O.R. with A. Boyarsky, J. Fröhlich PRL (2012)

- One cannot have $\Delta\mu = 0$ if $\int \vec{E}\cdot\vec{B} \neq 0$

Attractor solution

- Consider sharply peaked at k_0 maximally helical field

$$\frac{d\Delta\mu}{dt} = -\rho_B\Big(\Delta\mu - \Delta\mu_{\text{tr}}\Big) - \cancel{\Gamma_{\text{flip}}\Delta\mu}$$

$$\frac{d\rho_B}{dt} = \frac{\rho_B}{t_\sigma}\left(\frac{\Delta\mu}{\Delta\mu_{\text{tr}}} - 1\right)$$

assume $\rho_B \gg \Gamma_{\text{flip}}$

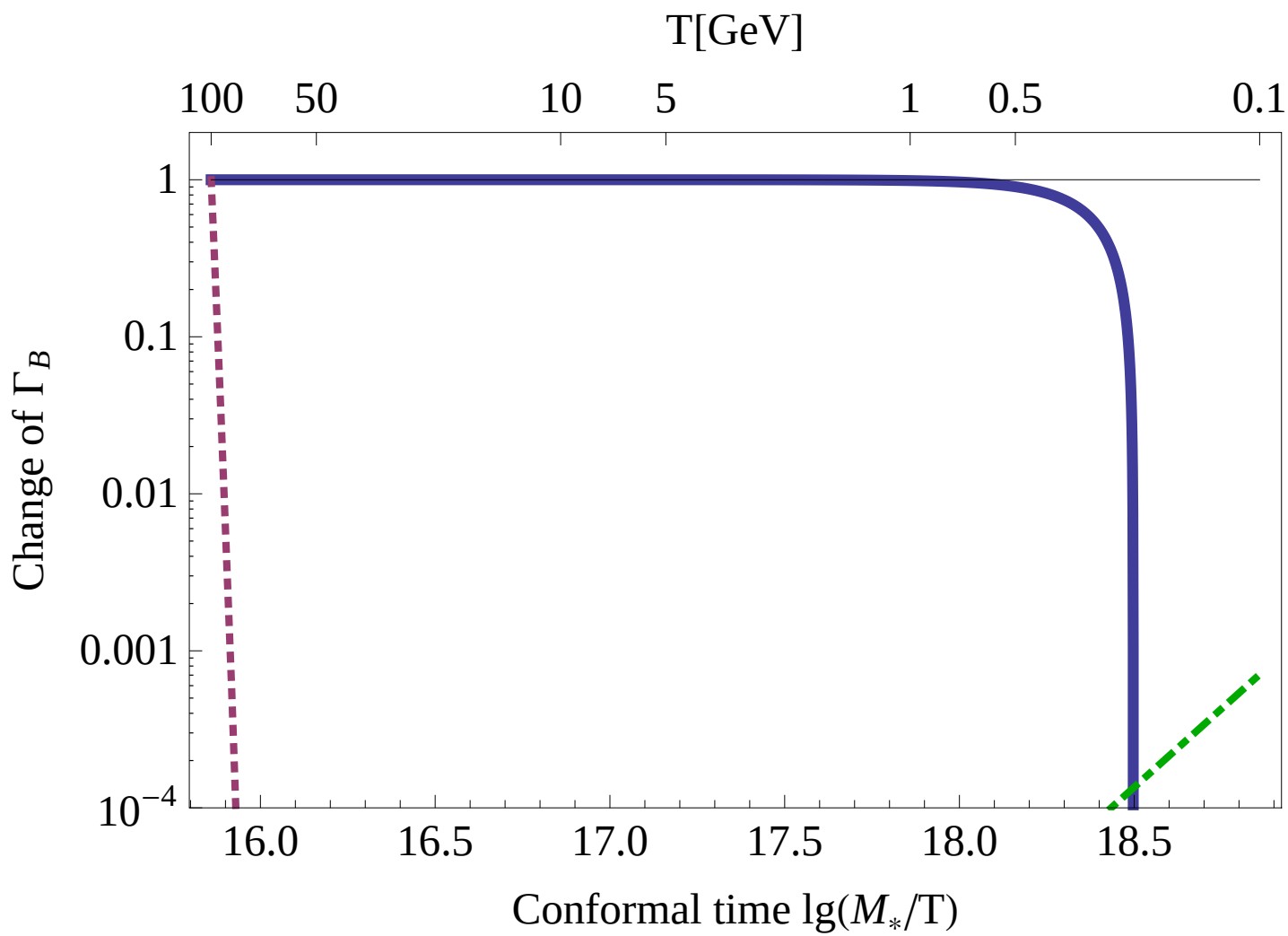
where $\Delta\mu_{\text{tr}} = \dfrac{2\pi k_0}{\alpha}$

- Large ρ_B drives $\Delta\mu$ to an **attractor solution** $\Delta\mu_{\text{tr}}$.

- Electric conductivity of the plasma is **finite** ($t_\sigma = \frac{2\sigma}{k_0^2}$) but **magnetic diffusion is compensated by the presence of** $\Delta\mu$

O.R. with A. Boyarsky, J. Fröhlich PRL 2012 **[1109.3350]**

Evolution of magnetic energy of one mode

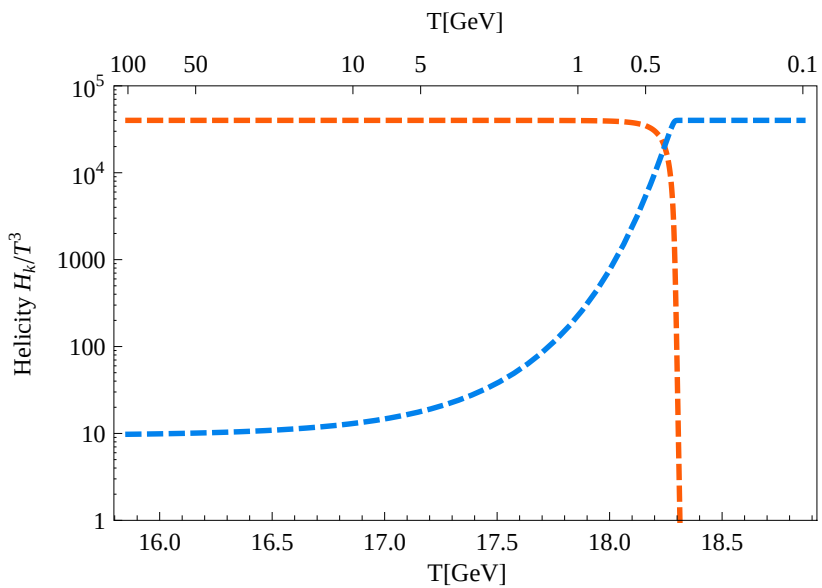


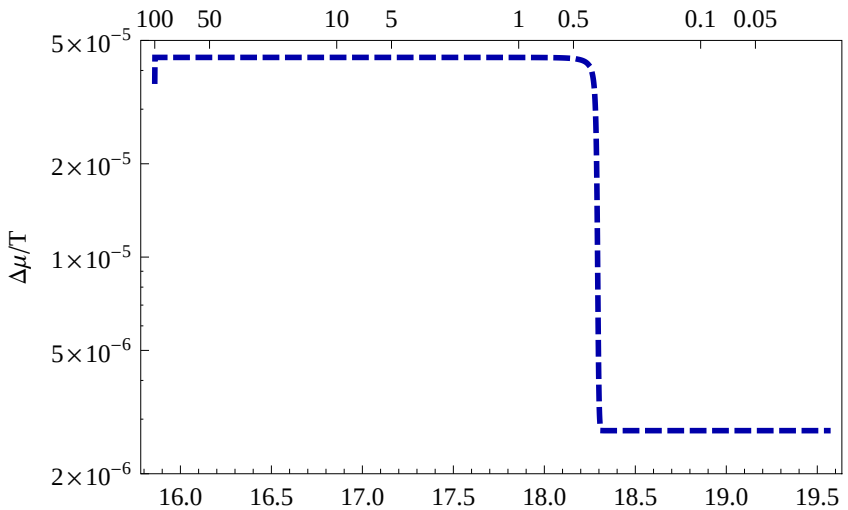
PRL (2012)
[1109.3350]

Relative change of magnetic energy of a single mode in the chiral MHD

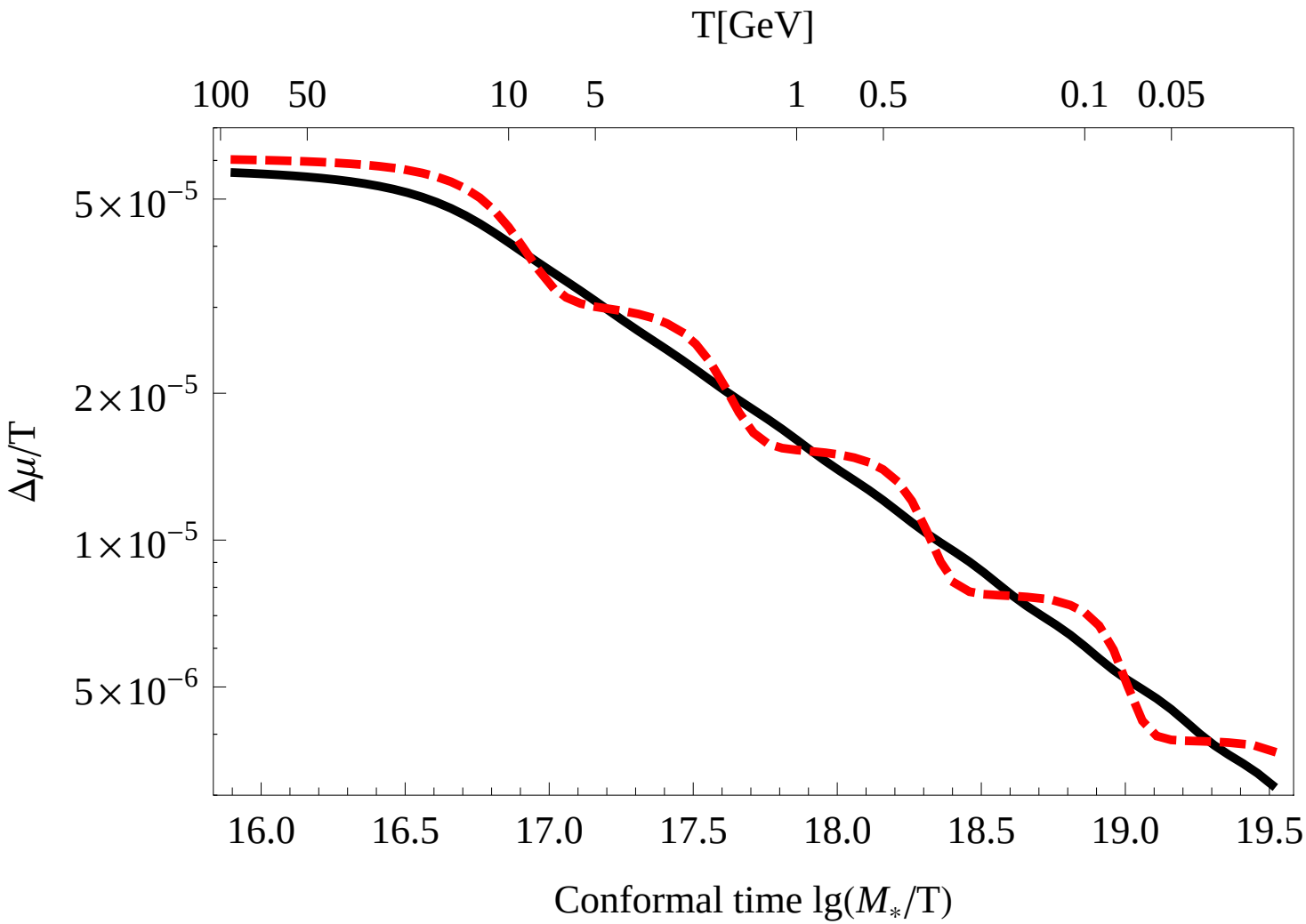
Two modes

- In case of two modes the helicity gets transferred from the shorter one to the longer one
- Chemical potential follows the wave-number of the mode with higher helicity



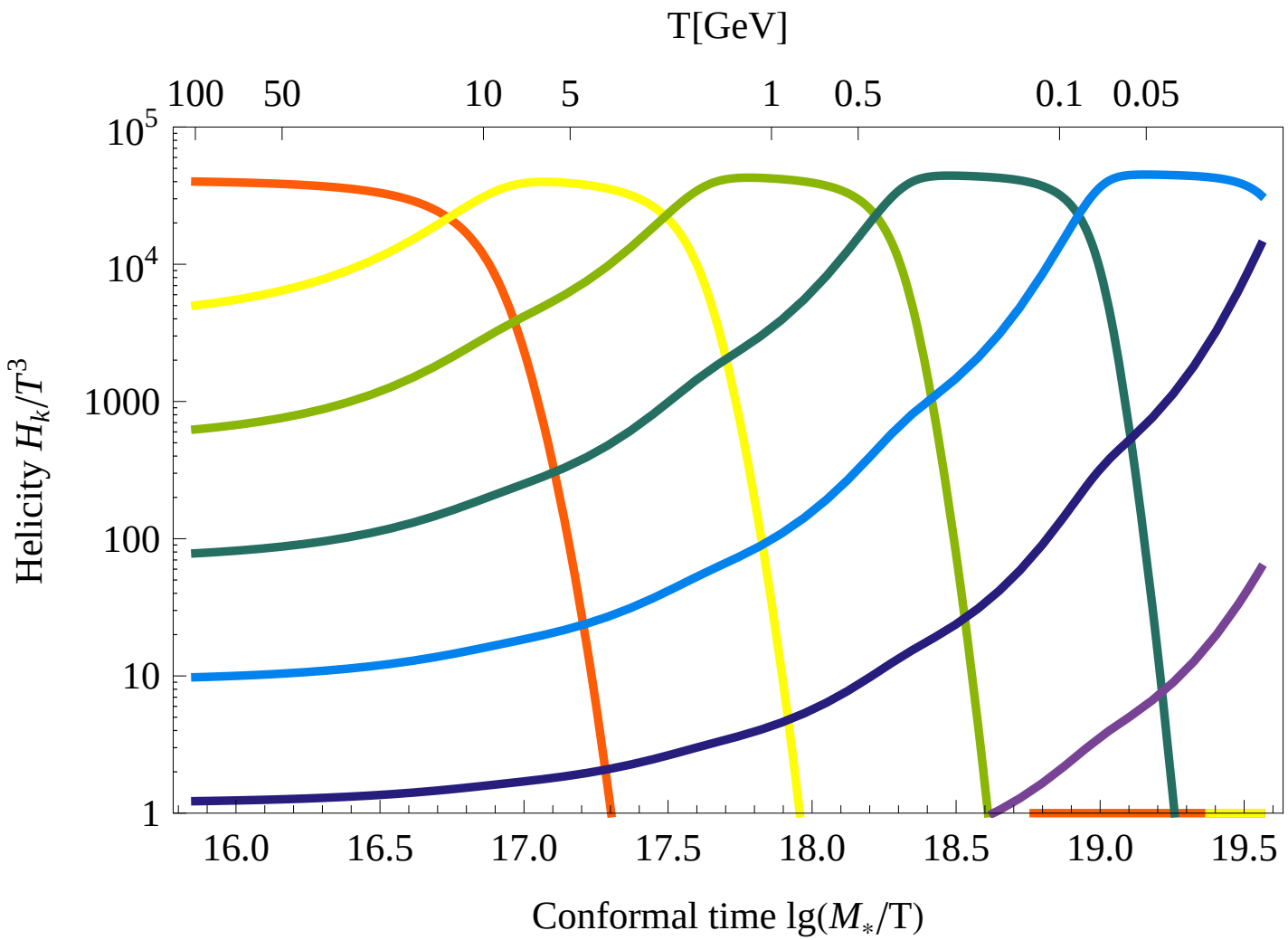


Evolution of chemical potential



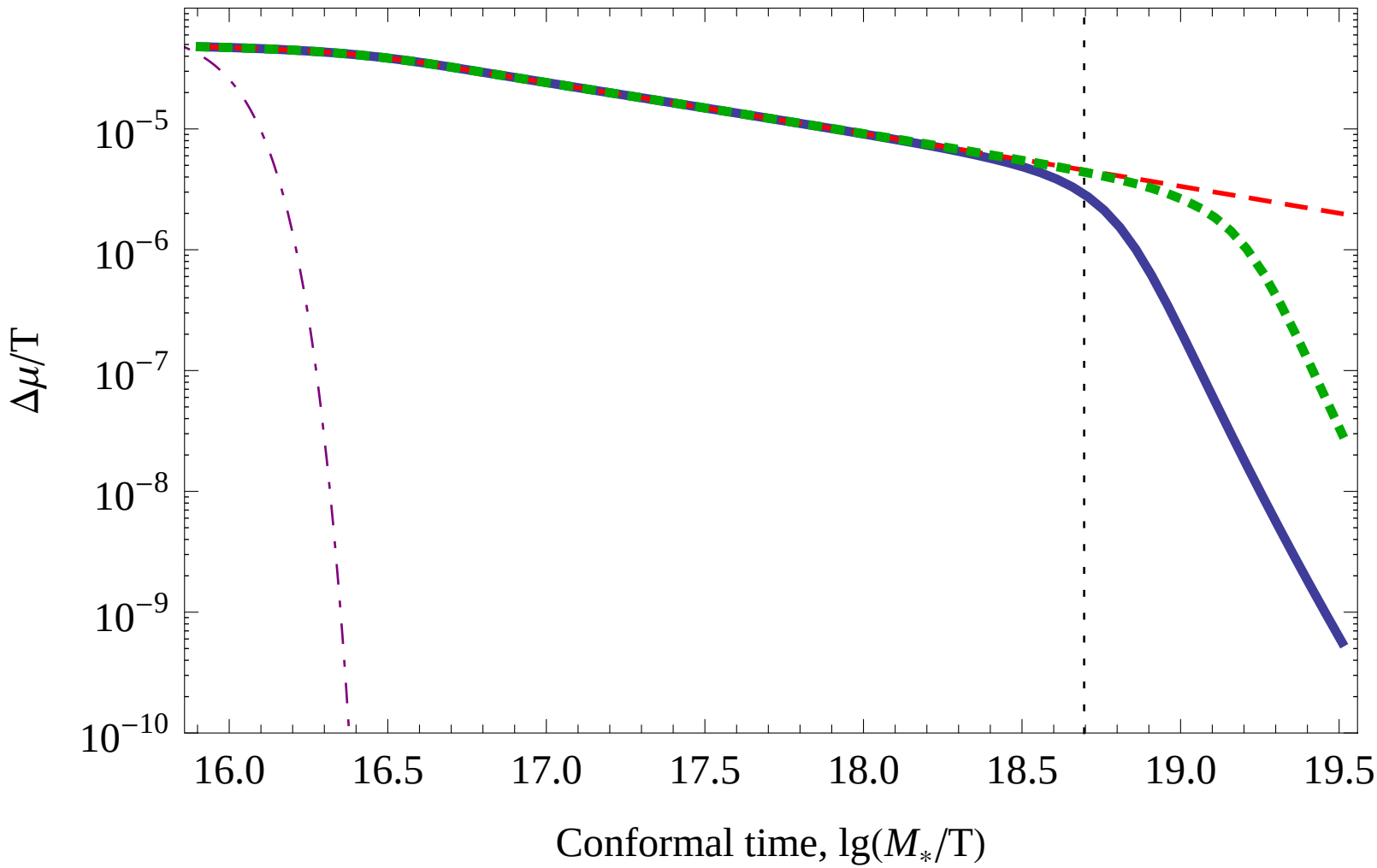
Process continues while $\rho_B \gg \Gamma_{\text{flip}}$ (recall that $\rho_B \propto \rho_B$)

Evolution of helicity spectrum



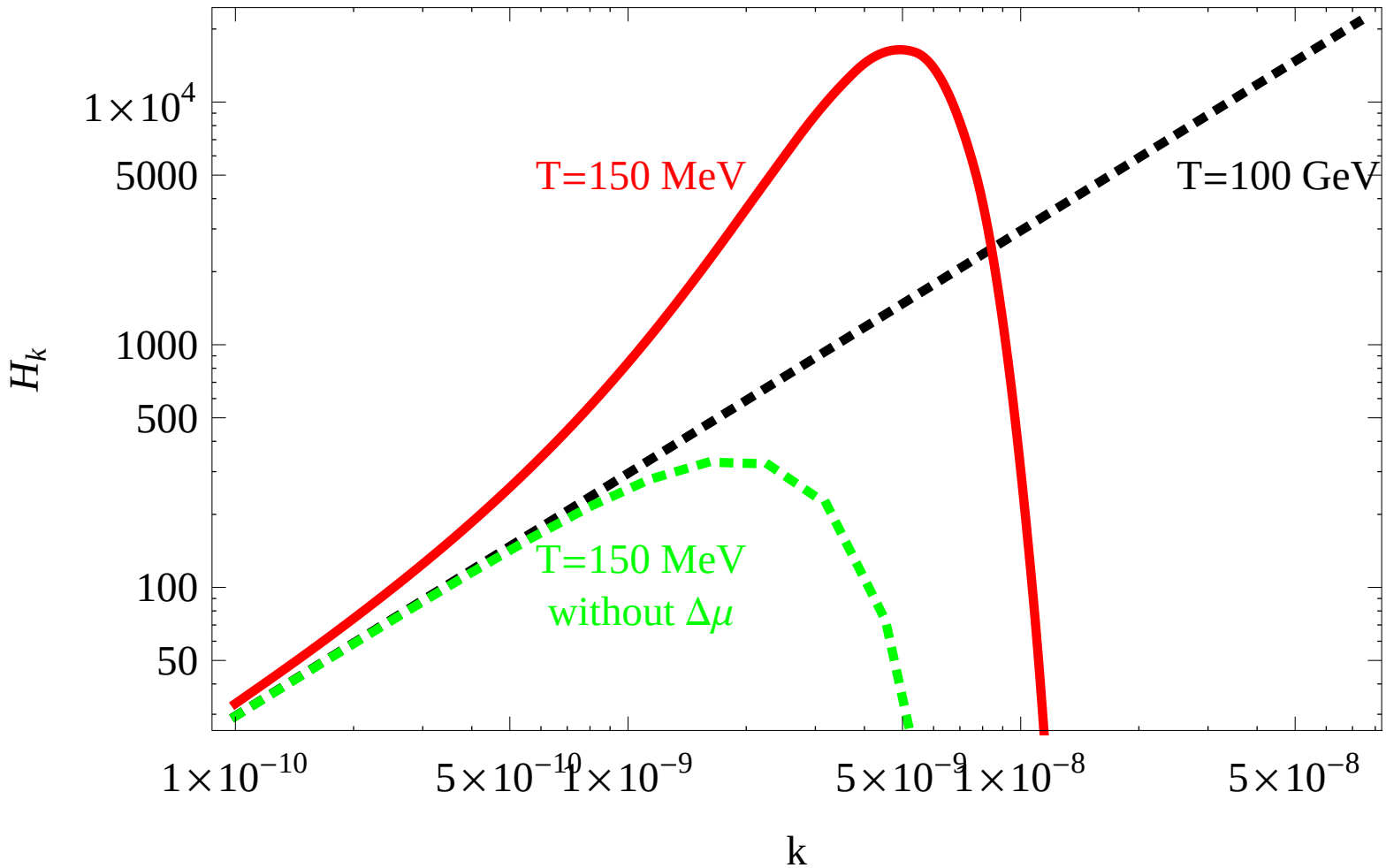
Process continues while $\Gamma_B \gg \Gamma_{\text{flip}}$ (recall that $\Gamma_B \propto \rho_B$)

Evolution of chemical potential



Continuous initial spectrum with $\mathcal{H}_k \propto k$ and fraction of magnetic energy density 5×10^{-5} (blue) or 5×10^{-4} (green). Red – evolution without flip

Evolution of helicity spectrum



Inverse cascade without turbulence!

Summary-I

Purely within the Standard Model:

- Chern-Simons term should be added to the Standard Model Lagrangian and finite densities of lepton and/or baryon number

- Homogeneous and isotropic ground state of primordial plasma is **unstable towards developing a long range magnetic fields**

Summary-II

Purely within the Standard Model:

- Evolution of relativistic plasma with long-wavelength magnetic fields **is not described** by the standard MHD equations (as was previously believed)

- Additional **IR degree of freedom** (the difference of chemical potentials of electrons) should me made dynamical and significantly affects evolution.

- This new degree of freedom:
  - Leads to conservation of helicity of magnetic fields
  - Partially compensates dissipation due to magnetic diffusion. Creates "inverse cascade" in spectrum of B-field (without turbulence)
  - Preserves chemical potential difference while $\rho_B \gg \Gamma_{\text{flip}}$ (e.g. down to $T \sim$ few MeV for electrons)

Consequences for early Universe

- Cosmological magnetic fields of horizon size in the electroweak epoch purely within the Standard Model
- QCD phase transition (can make it first order) Schwarz & Stuke (2009)
- BBN
- Dark matter production (to follow)
- This effect make a survival of helical magnetic fields (generated at and prior to 100 GeV) possible, as the inverse cascade transfers energy at larger and larger scales (e.g. following horizon) Vachaspati, Davidson, Grasso & Riotto, . . .

Thank you for your attention!

Chiral symmetry

- Massless fermions can be left and right-chiral (left and right moving):

$$(i\gamma^\mu\partial_\mu - \cancel{m})\psi = \begin{pmatrix} \cancelto{0}{-m} & i(\partial_t + \vec{\sigma}\cdot\vec{\nabla}) \\ i(\partial_t - \vec{\sigma}\cdot\vec{\nabla}) & \cancelto{0}{-m} \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

where $\gamma_5\psi_{R,L} = \pm\psi_{R,L}$ and $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$

- Number of left $N_L = \int d^3x\, \psi_L^\dagger\psi_L$ and right $N_R = \int d^3x\, \psi_R^\dagger\psi_R$ particles is conserved **independently**

  $N_L + N_R$ and $N_L - N_R$ are conserved independently in the free theory

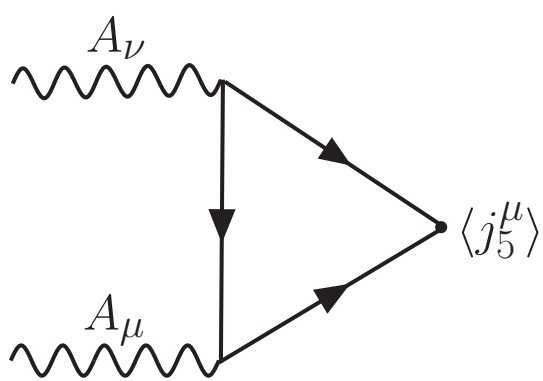
Chiral anomaly

- The situation changes if gauge fields are present. Gauge interactions respects chirality ($D_\mu = \partial_\mu + eA_\mu$)...

$$\begin{pmatrix} 0 & i(D_t + \vec{\sigma}\cdot\vec{D}) \\ i(D_t - \vec{\sigma}\cdot\vec{D}) & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

- ...but the difference of left and right-movers is still **not conserved**:

$$\frac{d(N_L - N_R)}{dt} = \int d^3\vec{x}\Big(\partial_\mu j^5_\mu\Big) = \frac{e^2}{4\pi^2}\int d^3\,\vec{x}\vec{E}\cdot\vec{B}$$



Chiral anomaly at finite fermion densities

- Chiral anomaly for degenerate Fermi gas

$$\delta N_{L,R}(t) = \int dt\, \dot{N}_{L,R}(t) = \mp \int dt \frac{\alpha}{\pi} \int d^3x\, E \cdot B$$

- Fermions occupy levels up to Fermi levels $\mu_{L,R}$.

- The energy change: $\delta\mathcal{E} = \delta N_L \mu_L + \delta N_R \mu_R = \dfrac{\alpha}{2\pi} \Delta\mu \displaystyle\int d^3x\, A \cdot B$

- Free energy density: $\delta\mathcal{F} = \dfrac{\alpha}{2\pi} \Delta\mu A \cdot B$

Nielsen & Ninomiya (1983);

Rubakov (1986)

Andreas Schmitt

Institut für Theoretische Physik
Technische Universität Wien
1040 Vienna, Austria

Inverse magnetic catalysis in dense (holographic) matter

F. Preis, A. Rebhan, A. Schmitt, JHEP 1103, 033 (2011); JPG 39, 054006 (2012)

- **(Chiral) magnetic catalysis**
- **Chiral transition in the Sakai-Sugimoto model**
- ***Inverse* magnetic catalysis at finite chemical potential**
- **Add baryonic matter**

• Magnetic catalysis (page 1/2)

K. G. Klimenko, Theor. Math. Phys. 89, 1161-1168 (1992)

V. P. Gusynin, V. A. Miransky, I. A. Shovkovy, PLB 349, 477-483 (1995)

- (massless) fermions in **Nambu-Jona-Lasinio (NJL)** model

$$\mathcal{L}_{\rm NJL} = \bar{\psi}\, i\gamma^\mu \partial_\mu \psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}\, i\gamma^5\psi)^2\right]$$

- mean-field approximation: $\bar{\psi}\psi = \langle\bar{\psi}\psi\rangle +$ fluctuations

Zero magnetic field:
dynamical fermion mass

$$M \propto \langle\bar{\psi}\psi\rangle \neq 0$$

for coupling $g > g_c = 1$

dimensionless coupling $g \equiv G\Lambda^2/\pi^2$

• Magnetic catalysis (page 1/2)

K. G. Klimenko, Theor. Math. Phys. 89, 1161-1168 (1992)

V. P. Gusynin, V. A. Miransky, I. A. Shovkovy, PLB 349, 477-483 (1995)

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- mean-field approximation: $\bar\psi\psi = \langle\bar\psi\psi\rangle +$ fluctuations

Nonzero magnetic field:

$M \neq 0$ for *arbitrarily small* g,

$$M \propto \sqrt{eB}\, e^{-\text{const.}/eBg}$$

at weak coupling $g \ll 1$

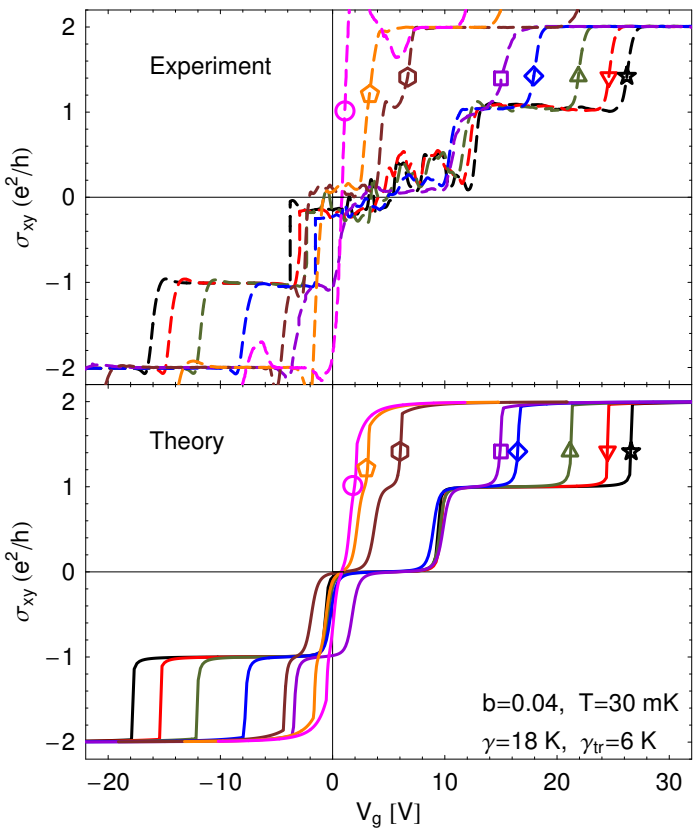
dimensionless coupling $g \equiv G\Lambda^2/\pi^2$

• Magnetic catalysis (page 2/2)

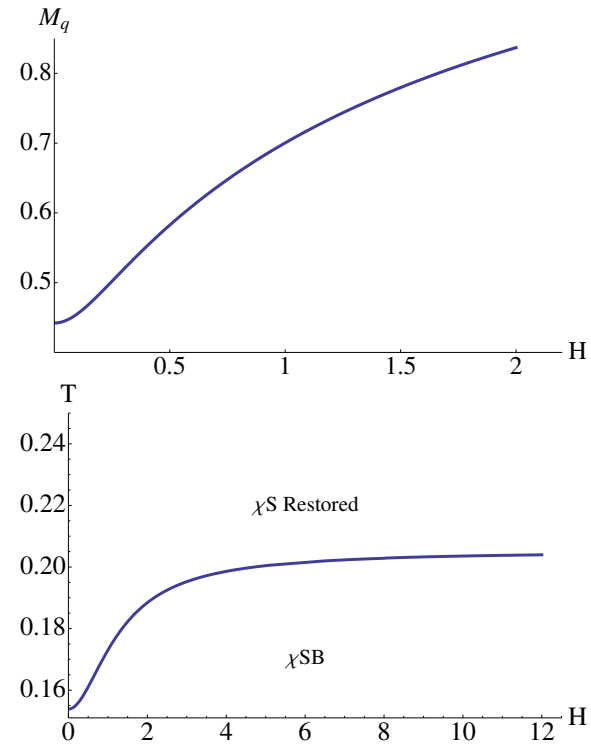
Analogy to BCS Cooper pairing:

BCS superconductor	Magnetic catalysis
Cooper pair condensate $\langle\psi\psi\rangle$	chiral condensate $\langle\overline{\psi}\psi\rangle$
$\Delta \propto \mu\, e^{-\text{const.}/G\nu_F}$ (ν_F: d.o.s. at $E=\mu$ Fermi surface)	$M \propto \sqrt{eB}\, e^{-\text{const.}/G\nu_0}$ (ν_0: d.o.s. at $E=0$ surface)
pairing dynamics effectively (1+1)-dimensional because of Fermi surface	effectively (1+1)-dimensional in lowest Landau level (LLL) because of magn. field
gap equation $\Delta = \frac{\mu^2 G}{2\pi^2}\int_0^\infty dk\, \frac{\Delta}{\sqrt{(k-\mu)^2+\Delta^2}}$	gap equation (LLL) $M = \frac{\lvert q\rvert BG}{2\pi^2}\int_{-\infty}^\infty dk_z\, \frac{M}{\sqrt{k_z^2+M^2}}$

• Magnetic catalysis in the real world and in holography



V.P.Gusynin *et al.*, PRB 74, 195429 (2006)



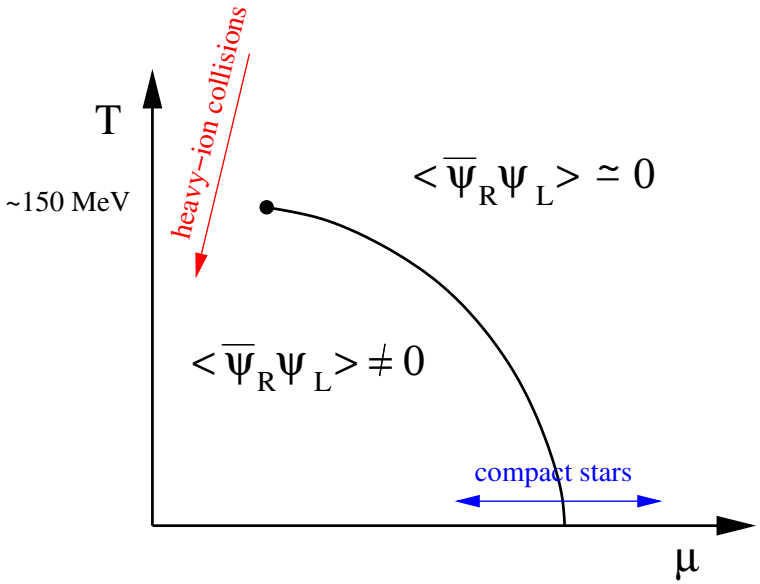
C.V.Johnson, A.Kundu, JHEP 0812, 053 (2008)

- **graphene**: appearance of additional plateaus in strong magnetic fields [$B = 9\,\mathrm{T}$ (pink), $B = 45\,\mathrm{T}$ (black)]

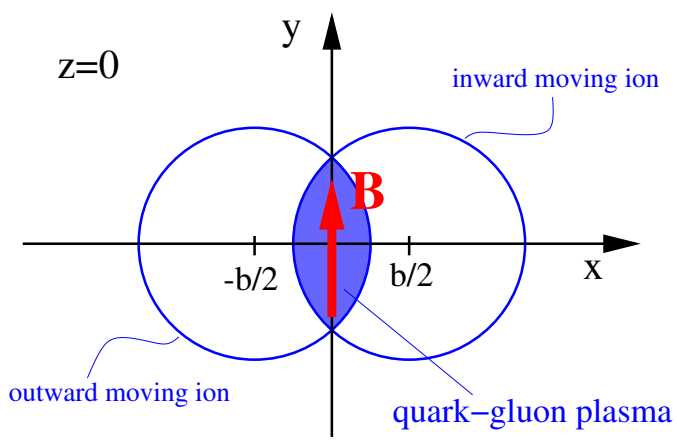
- **Sakai-Sugimoto**: magnetic field enhances dynamical mass M_q and critical temperature T_c

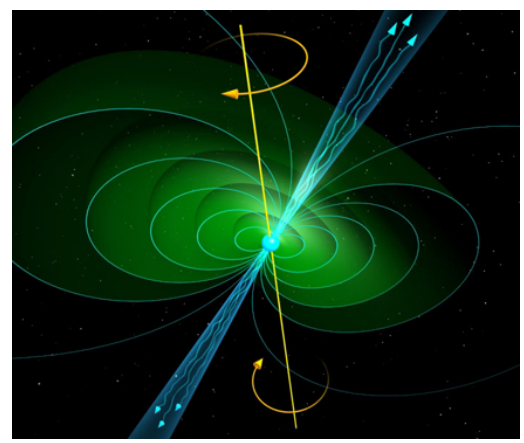
see also: J. Erdmenger *et al.*, JHEP 0712, 091 (2007)
V.G.Filev, R.C.Rashkov, AHEP 473206 (2010)

• (Chiral) magnetic catalysis in QCD?



- chiral transition probed in heavy-ion collisions and compact stars
- in both instances huge magnetic fields are present!



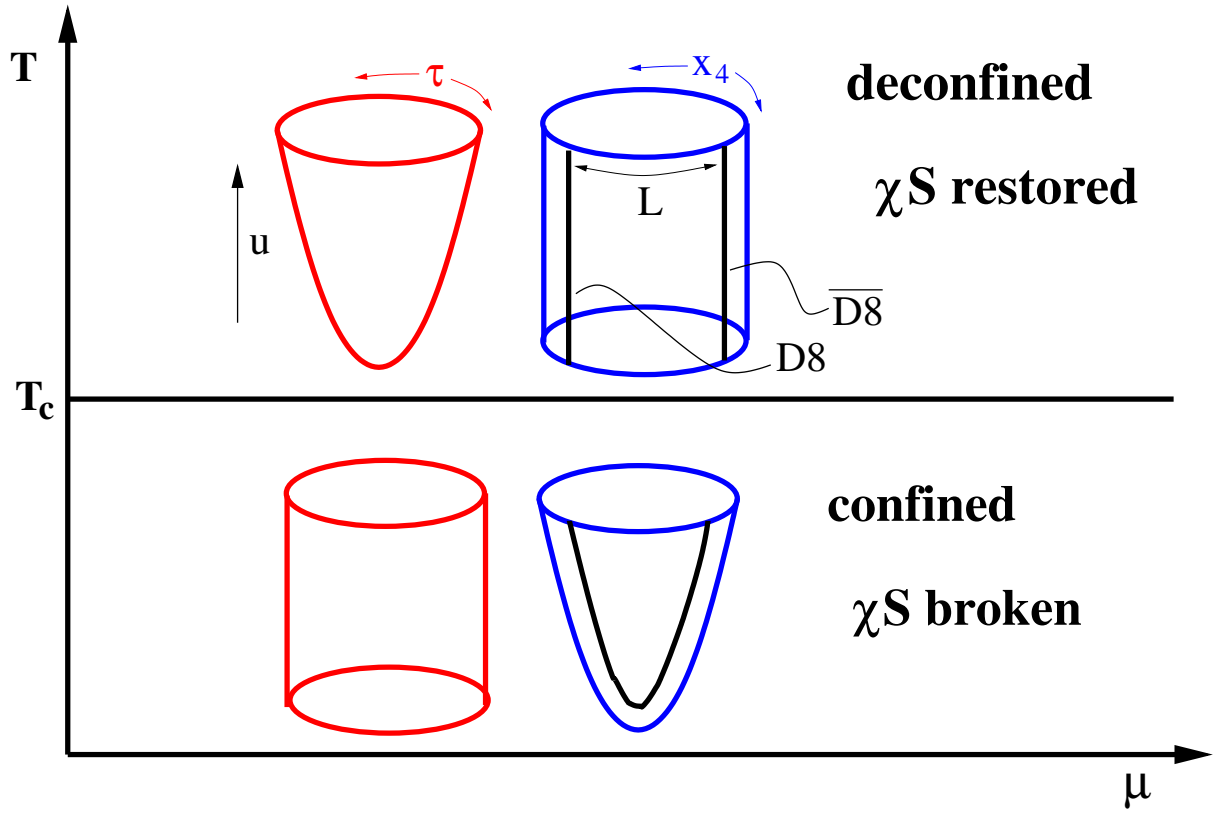


heavy-ion collisions: temporarily
$B \sim 10^{18}$ G

magnetars: at surface
$B \sim 10^{15}$ G

• Chiral transition in the Sakai-Sugimoto model (p. 1/3)

T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843-882 (2005)

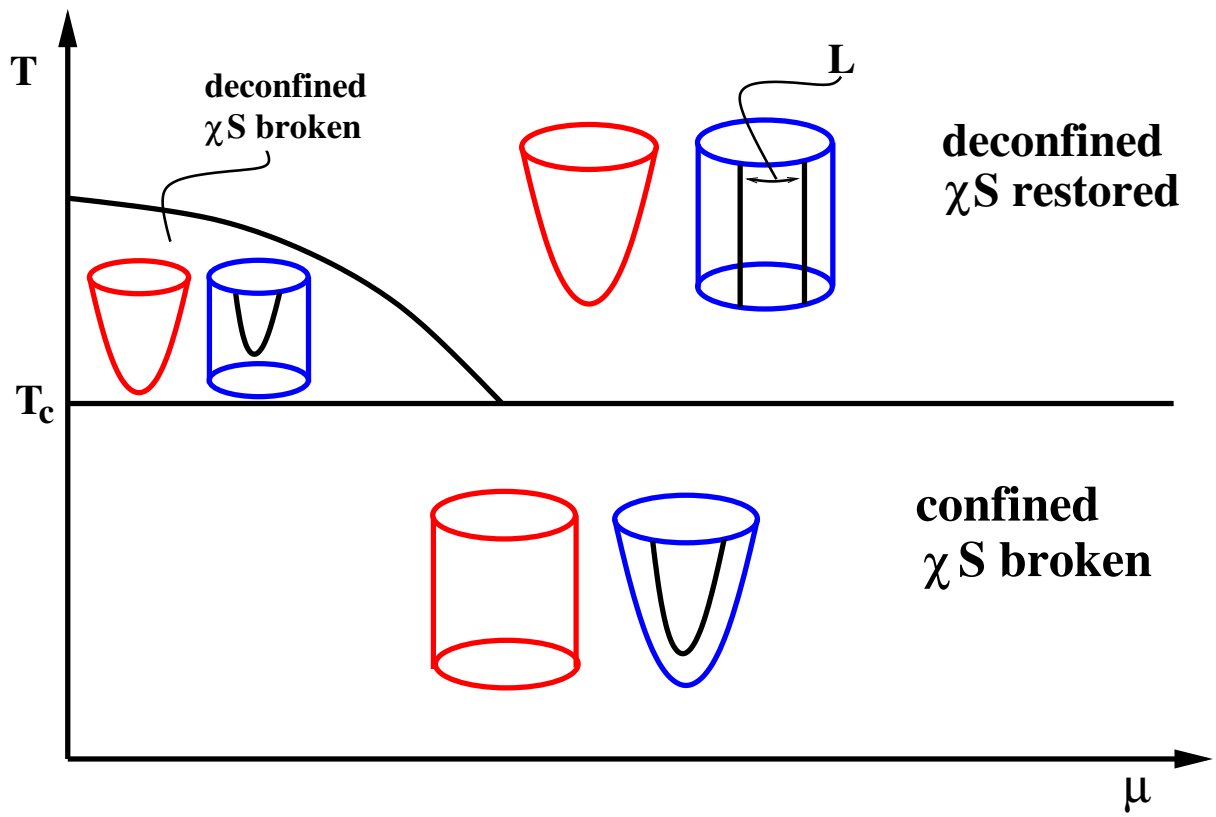


- not unlike expectation from large-N_c QCD
- in probe brane approximation: **chiral transition** unaffected by quantities on flavor branes (μ, B, ...)

• Chiral transition in the Sakai-Sugimoto model (p. 2/3)

- less "rigid" behavior for smaller L
- deconfined, chirally broken phase for $L < 0.3\,\pi/M_{\rm KK}$

O. Aharony, J. Sonnenschein, S. Yankielowicz, Annals Phys. 322, 1420 (2007)
N. Horigome, Y. Tanii, JHEP 0701, 072 (2007)

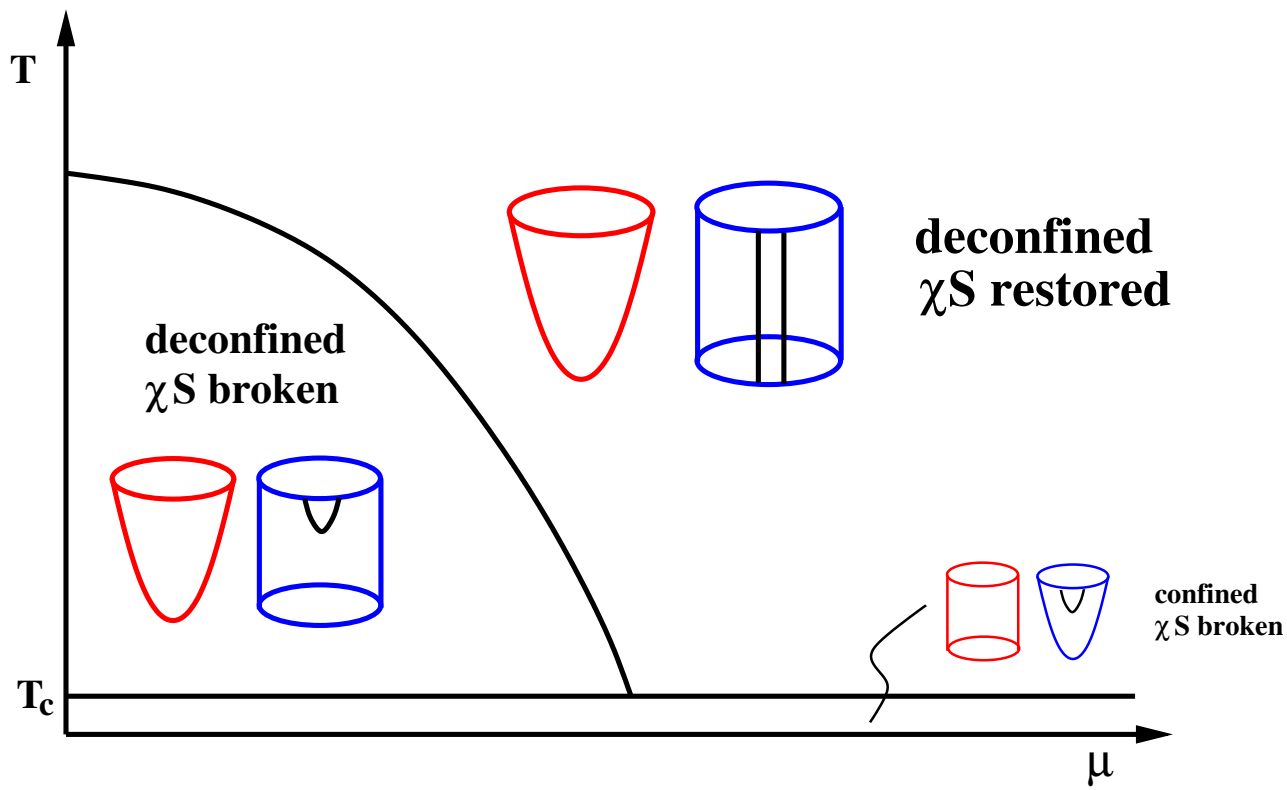


• Chiral transition in the Sakai-Sugimoto model (p. 3/3)

- $L \ll \pi/M_{\mathrm{KK}}$ corresponds to (non-local) NJL model

  E. Antonyan, J. A. Harvey, S. Jensen, D. Kutasov, hep-th/0604017

  J. L. Davis, M. Gutperle, P. Kraus, I. Sachs, JHEP 0710, 049 (2007)



- this limit is considered in the following calculation ...

• Setup of our calculation

- D8-brane action in deconfined geometry for $N_f = 1$
  O. Bergman, G. Lifschytz, M. Lippert, PRD 79, 105024 (2009)

$$S = S_{\rm DBI} + S_{\rm CS}$$
$$= \mathcal{N} \int_{u_T}^{\infty} du \sqrt{u^5 + b^2 u^2} \sqrt{1 + f a_3'^2 - a_0'^2 + u^3 f x_4'^2} + \frac{3\mathcal{N}}{2} b \int_{u_T}^{\infty} du \, (a_3 a_0' - a_0 a_3')$$

- a_0 accounts for μ
- magnetic field $b = F_{12}$
- b, μ induce a_3
  ($\rightarrow$ anisotropic condensate $\nabla \pi^0$)
  E.G.Thompson, D.T.Son, PRD 78, 066007 (2008)
  A. Rebhan *et al.*, JHEP 0905, 084 (2009)
- $x_4(u) = \begin{cases} \text{const.} & \chi S \\ \text{nontrivial} & \chi S b \end{cases}$

- equations of motion:

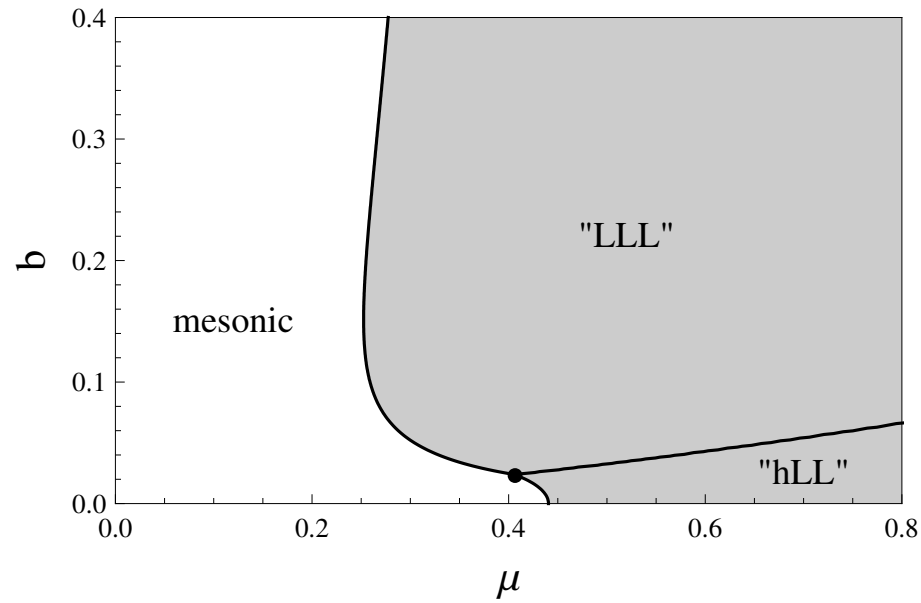
$$\partial_u \left(\frac{a_0' \sqrt{u^5 + b^2 u^2}}{\sqrt{1 + f a_3'^2 - a_0'^2 + u^3 f x_4'^2}} \right) = 3 b a_3'$$

$$\partial_u \left(\frac{f \, a_3' \sqrt{u^5 + b^2 u^2}}{\sqrt{1 + f a_3'^2 - a_0'^2 + u^3 f x_4'^2}} \right) = 3 b a_0'$$

$$\partial_u \left(\frac{u^3 f \, x_4' \sqrt{u^5 + b^2 u^2}}{\sqrt{1 + f a_3'^2 - a_0'^2 + u^3 f x_4'^2}} \right) = 0$$

$\rightarrow$ solve for $a_0(u)$, $a_3(u)$, $x_4(u)$

• $T = 0$ phase diagram



- apparent Landau level transition

  G. Lifschytz, M. Lippert, PRD 80, 066007 (2009)

  (charge density exactly like in LLL for free fermions)

- non-monotonic behavior of critical μ
  (in apparent contrast to magnetic catalysis)

- **Inverse magnetic catalysis**

Why does B *restore* chiral symmetry for certain μ?

- free energy *gain* from $\bar{\psi} - \psi$ pairing increases with B (magnetic catalysis)
- μ induces free energy *cost* for pairing; this cost depends on B!

weak coupling (NJL):

E. V. Gorbar *et al.*, PRC 80, 032801 (2009)

$$\Delta\Omega \propto B[\mu^2 - M(B)^2/2]$$

just like Clogston limit $\delta\mu = \frac{\Delta}{\sqrt{2}}$ in superconductivity

A. Clogston, PRL 9, 266 (1962)

B. Chandrasekhar, APL 1, 7 (1962)

Sakai-Sugimoto:

large B:

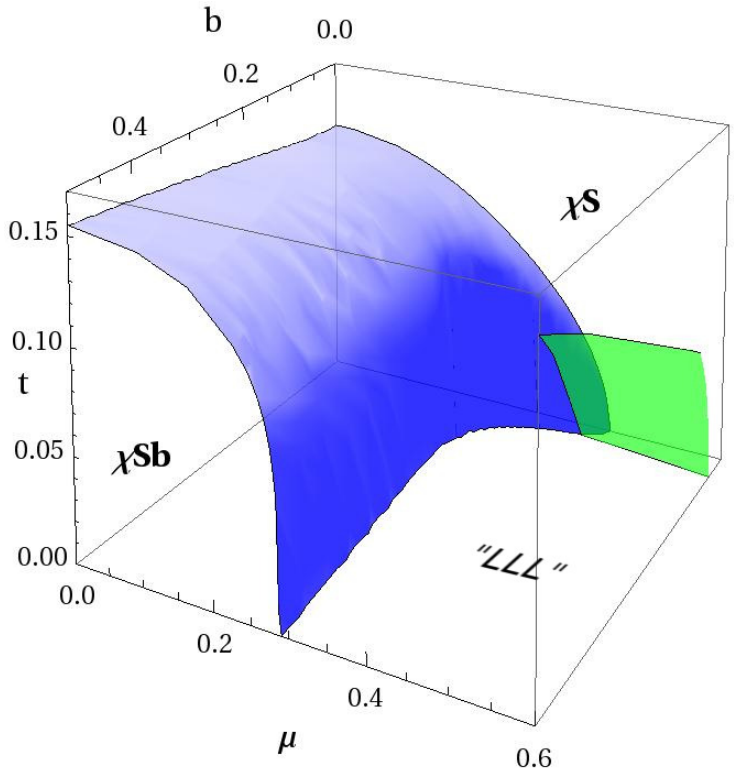
$$\Delta\Omega \propto B[\mu^2 - 0.12\, M(B)^2]$$

small B:

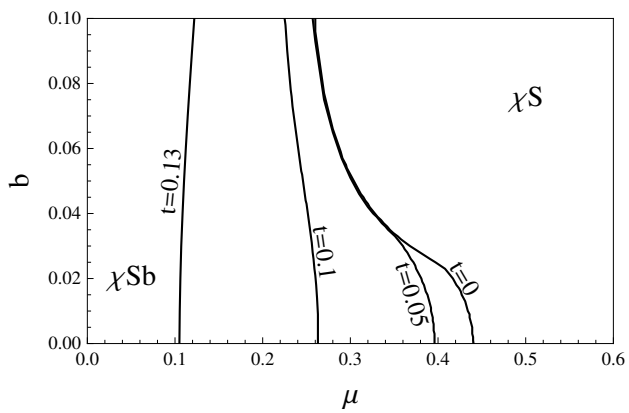
$$\Delta\Omega \propto \mu^2 B - \text{const} \times M(B)^{7/2}$$

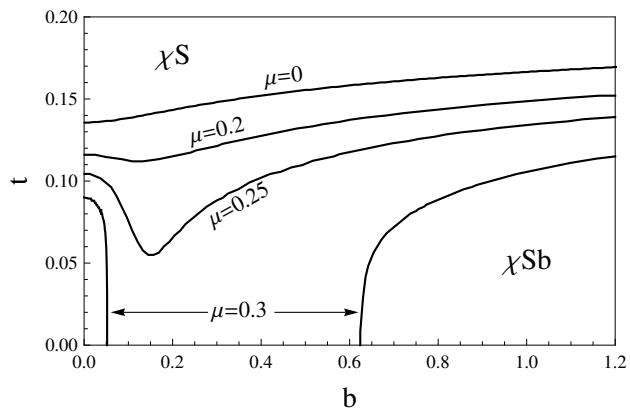
"Inverse magnetic catalysis"

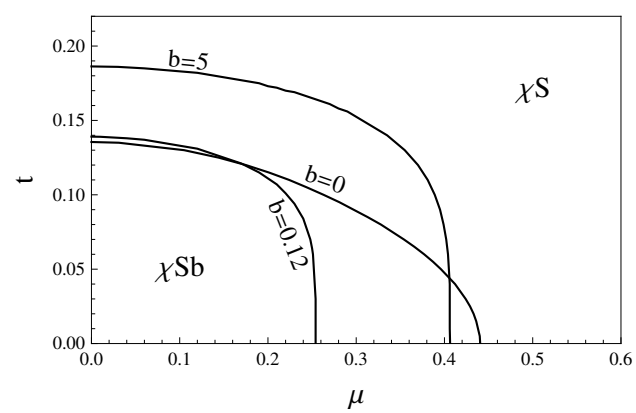
• Phase structure at nonzero temperature



blue: chiral phase transition
green: "LLL" transition



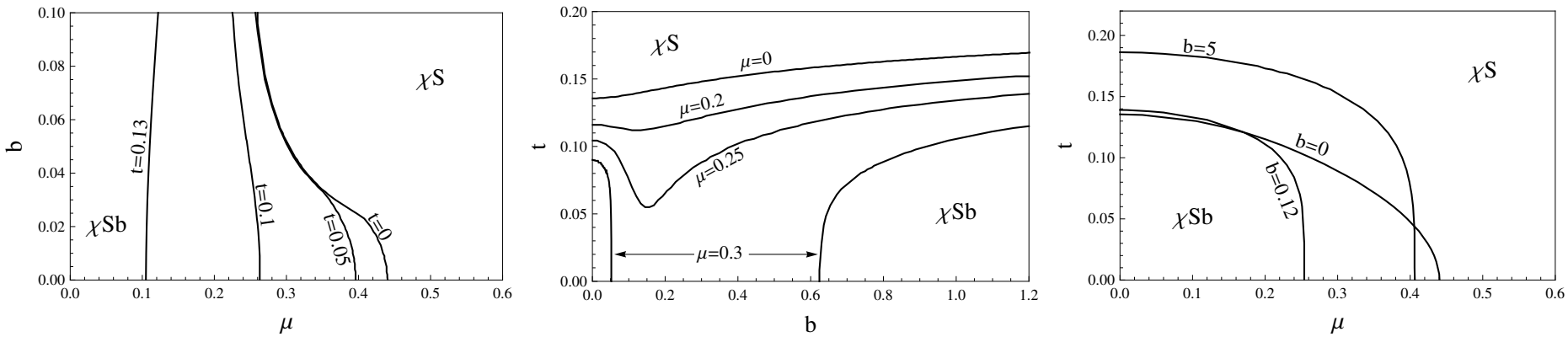




• Agreement with NJL calculation

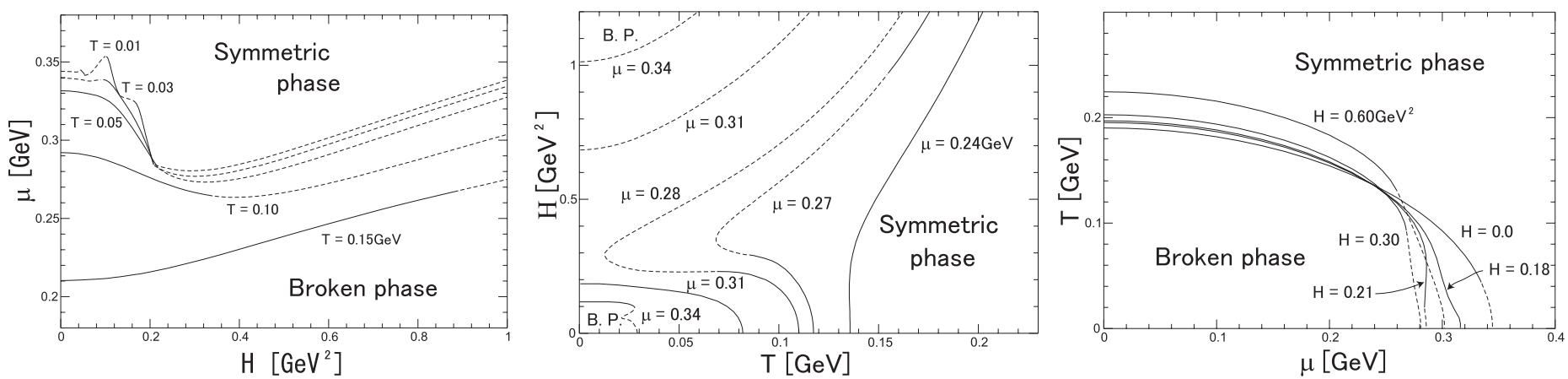
Sakai-Sugimoto:

F. Preis, A. Rebhan and A. Schmitt, JHEP 1103, 033 (2011)



NJL:

T. Inagaki, D. Kimura, T. Murata, Prog. Theor. Phys. 111, 371-386 (2004)



(IMC also in quark-meson model J. O. Andersen and A. Tranberg, arXiv:1204.3360 [hep-ph])

• Homogeneous baryonic matter in Sakai-Sugimoto

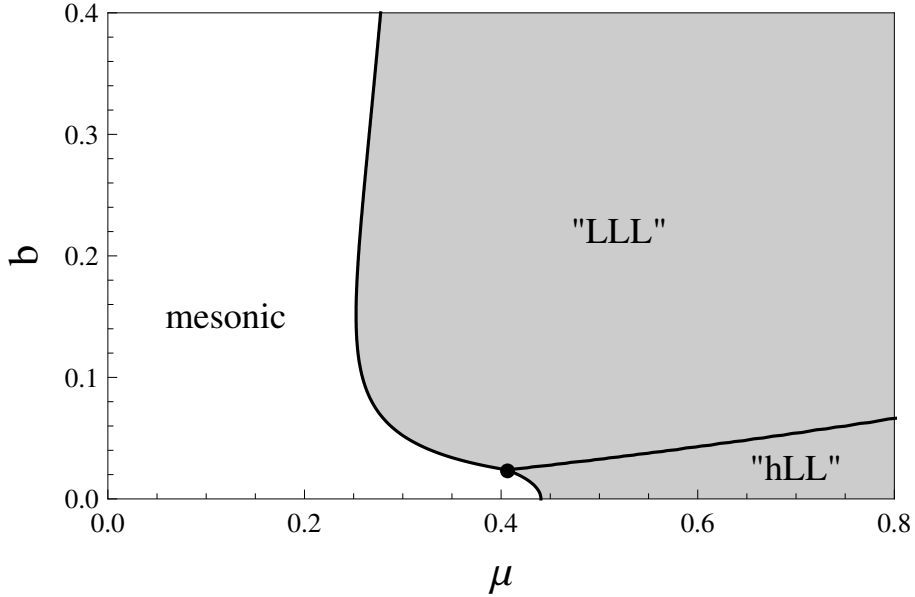
- baryons in AdS/CFT: wrapped D-branes with N_c strings
  E. Witten, JHEP 9807, 006 (1998); D. J. Gross, H. Ooguri, PRD 58, 106002 (1998)
- baryons in Sakai-Sugimoto:
  – D4-branes wrapped on S^4
  – equivalently: instantons on D8-branes ($\rightarrow$ Skyrmions)
  T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843-882 (2005)
  H. Hata, T. Sakai, S. Sugimoto, S. Yamato, Prog. Theor. Phys. 117, 1157 (2007)
- pointlike approximation for $N_f = 1$:
  O. Bergman, G. Lifschytz, M. Lippert, JHEP 0711, 056 (2007)

$$S = S_{\text{from above}} + \underbrace{N_4 T_4 \int d\Omega_4 d\tau\, e^{-\Phi} \sqrt{\det g}}_{\propto n_4 N_c M_q} + \underbrace{\frac{N_c}{8\pi^2} \int_{\mathbb{R}^4 \times \mathcal{U}} A_0 \,\mathrm{Tr}\, F^2}_{\propto n_4 \int A_0(u)\delta(u-u_c)}$$

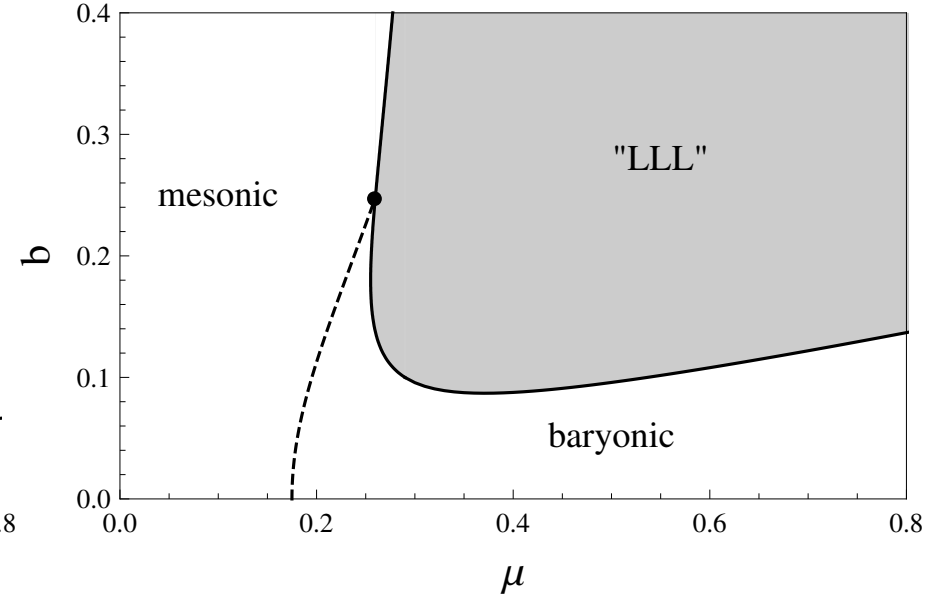
(n_4 baryon density , M_q constituent quark mass , u_c location of D4-branes)

• Effect of baryons on $T = 0$ phase diagram

ignoring baryonic matter



including baryonic matter



- small b: baryonic matter prevents the system from restoring chiral symmetry
- baryon onset (2nd order!) intersects chiral phase transition $\rightarrow$ large b: mesonic matter superseded by quark matter
- with baryonic matter, IMC plays an even more prominent role in the phase diagram

• Summary

- Dense matter in the "NJL limit" of the Sakai-Sugimoto model...
  ... shows a transition reminiscent of LLL
  ... shows a chiral phase transition with
  MC at large B and IMC at small B

- Baryonic matter changes the phase diagram
  dramatically for small B

- Agreement with NJL calculations
  only before including baryonic matter

• Outlook

- Sakai-Sugimoto calculation:
  - understand chiral non-restauration with baryons
    $\rightarrow$ extend to $N_f = 2$
  - more general: problem of large N_c vs. $N_c = 3$!

- inverse catalysis in QCD and NJL models
  - how is IMC affected by the "chiral shift"?
    E. V. Gorbar, V. A. Miransky and I. A. Shovkovy, PRC 80, 032801 (2009)
    F. Preis, A. Rebhan, A. Schmitt, work in progress
  - are recent lattice results for $T_c(B)$ related to IMC?
    G. S. Bali *et al.*, JHEP 1202, 044 (2012)

Workshop On
P- and CP-odd Effects in Hot and Dense Matter (2012)
RIKEN BNL Research Center Workshop
June 25-27, 2012 at Brookhaven National Laboratory

Chiral shift in renormalizable theories in magnetic field

Igor Shovkovy*

Department of Applied Sciences & Mathematics
Arizona State University

***Collaborators:**
E. Gorbar, V. Miransky, and Xinyang Wang

ASU Arizona State University
POLYTECHNIC CAMPUS

Motivation

- Axial vector current in relativistic matter in a magnetic field (3+1 dimensions)

$$\langle j_5^3 \rangle_0 = \frac{-eB}{2\pi^2}\mu_0 \qquad \text{(free theory!)}$$

[Metlitski & Zhitnitsky, Phys Rev D **72**, 045011 (2005)]

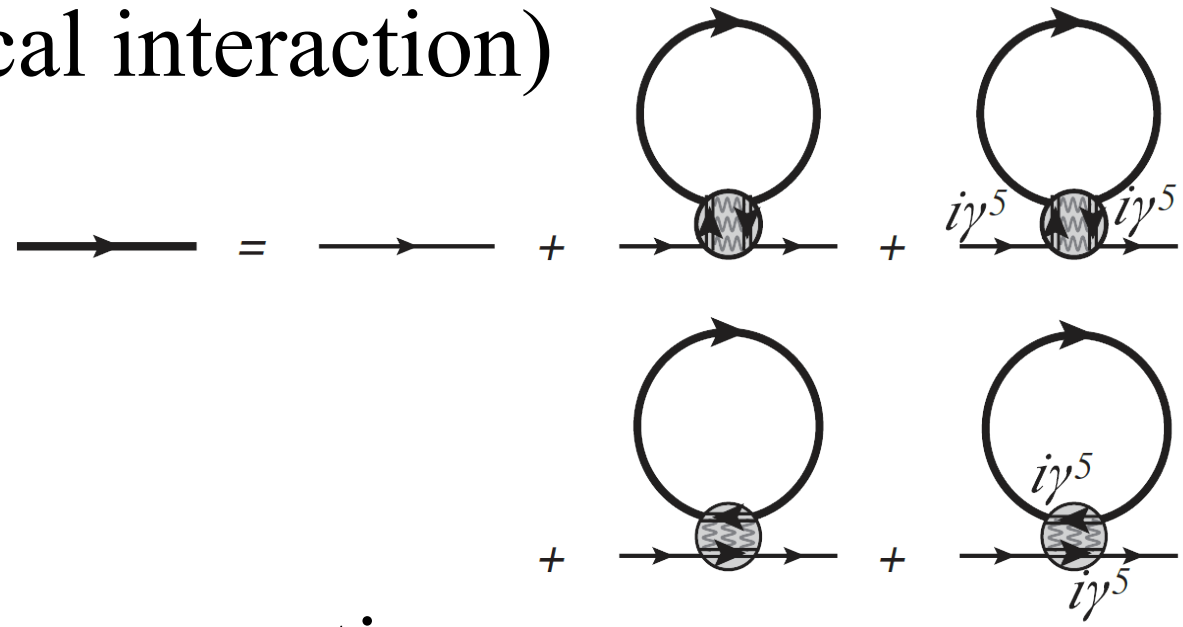
- Is there a dynamical parameter Δ ("chiral shift") associated with this condensate?

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\Delta \quad \text{where} \quad \mathcal{L}_\Delta \simeq \Delta\bar{\psi}\gamma^3\gamma^5\psi$$

- Note: Δ=0 is not protected by any symmetry

Chiral shift in NJL model

- NJL model (local interaction)



- This leads to three equations:

$$\mu = \mu_0 - \frac{1}{2} G_{\rm int} \langle j^0 \rangle$$ ("effective" chemical potential)

$$m = m_0 - G_{\rm int} \langle \bar{\psi}\psi \rangle$$ (dynamical mass)

$$\Delta = -\frac{1}{2} G_{\rm int} \langle j_5^3 \rangle$$ (chiral shift parameter)

Solutions

- Magnetic catalysis solution (vacuum state):

$$m^2 \simeq \frac{|eB|}{\pi} \exp\left(-\frac{4\pi^2}{G_{\rm int}|eB|}\right) \qquad \left(|\mu_0| \lesssim \frac{m}{\sqrt{2}}\right)$$

$$\Delta = 0 \quad \& \quad \mu = \mu_0$$

- State with a chiral shift (nonzero density):

$$m = 0 \quad \& \quad \mu \simeq \frac{\mu_0}{1 + g/(\Lambda l)^2}$$

$$\Delta = \frac{g s_\perp \mu}{(\Lambda l)^2 + \frac{1}{2} g (\Lambda l)^2} \qquad \left(|\mu_0| \gtrsim \frac{m}{\sqrt{2}}\right)$$

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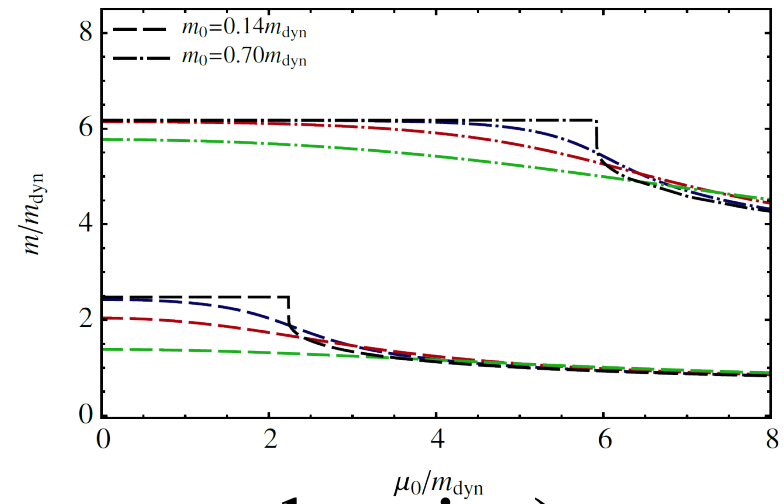

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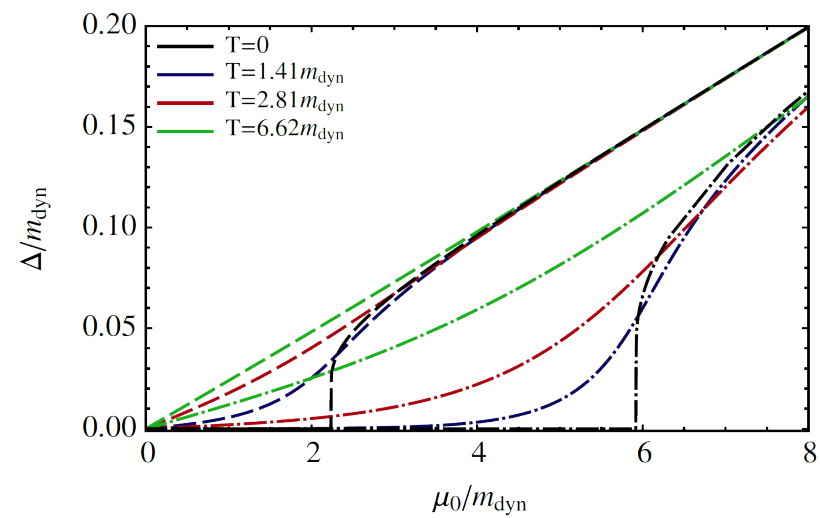


Chiral shift @ Fermi surface

- Chirality is $\approx$ well defined at Fermi surface ($|k^3| \gg m$)
- L-handed Fermi surface:

$$n = 0: \quad k^3 = +\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \quad k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

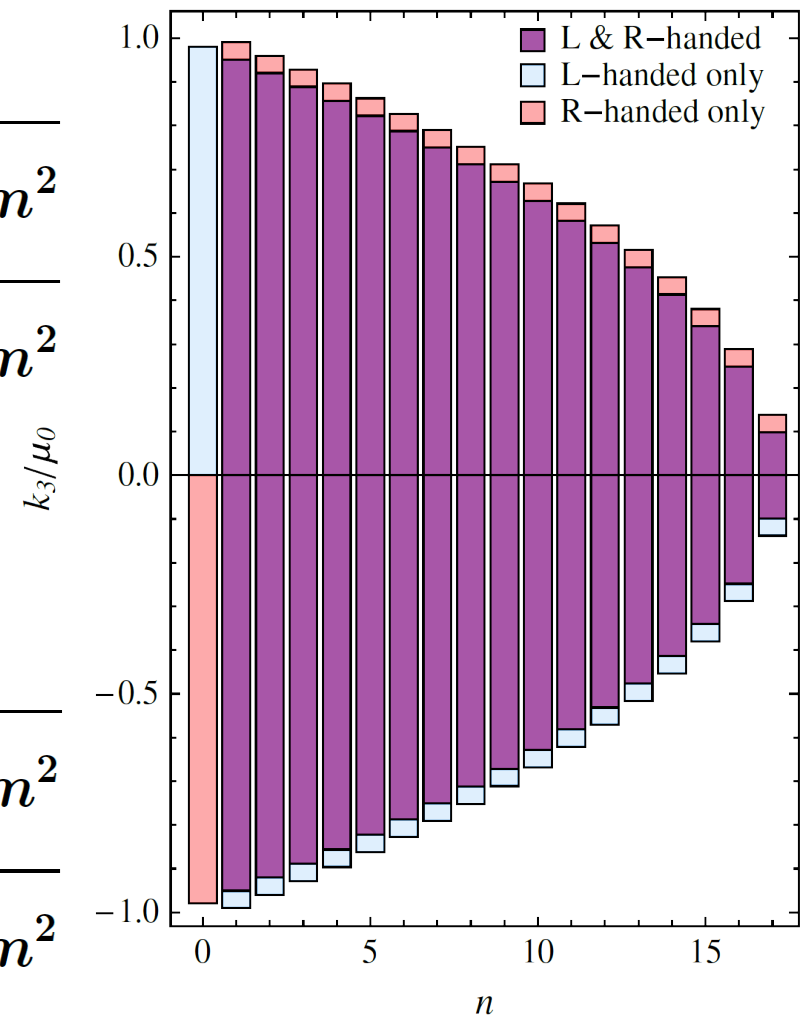
$$k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$

- R-handed Fermi surface:

$$n = 0: \quad k^3 = -\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \quad k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

$$k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$



Chiral shift vs. axial anomaly

- Does the chiral shift modify the axial anomaly relation?

- Using point splitting method, one derives

$$\langle \partial_\mu j_5^\mu(u) \rangle = -\frac{e^2 \epsilon^{\beta\mu\lambda\sigma} F_{\alpha\mu} F_{\lambda\sigma} \epsilon^\alpha \epsilon_\beta}{8\pi^2 \epsilon^2} \left(e^{-is_\perp \Delta\epsilon^3} + e^{is_\perp \Delta\epsilon^3} \right)$$

$$\to -\frac{e^2}{16\pi^2} \epsilon^{\beta\mu\lambda\sigma} F_{\beta\mu} F_{\lambda\sigma} \qquad \text{for} \quad \epsilon \to 0$$

[E. V. Gorbar & V. A. Miransky, I.A.S., Phys. Lett. B 695 (2011) 354]

- Therefore, the chiral shift does **not** affect the conventional axial anomaly relation

Axial current

- Does the chiral shift give any contribution to the axial current?

- In the point splitting method, one has

$$\langle j_5^\mu \rangle_{\text{singular}} = -\frac{\Delta}{2\pi^2\epsilon^2}\delta_3^\mu \sim \frac{\Lambda^2\Delta}{2\pi^2}\delta_3^\mu$$

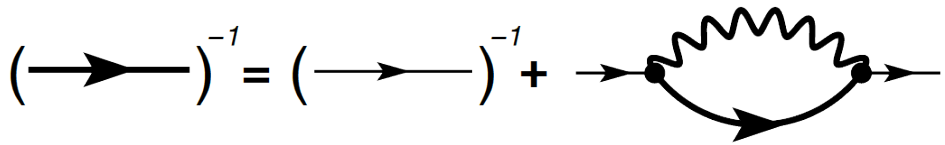
[E. V. Gorbar & V. A. Miransky, I.A.S., Phys. Lett. B 695 (2011) 354]

- This is consistent with the NJL calculations

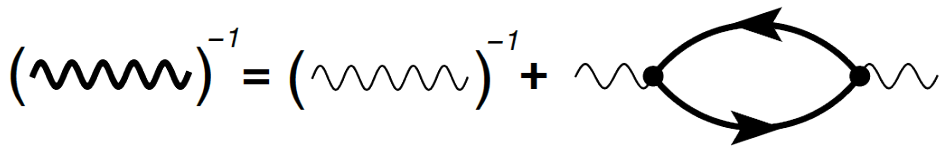
- Since $\Delta \sim g\mu\, eB/\Lambda^2$, the correction to the axial current should be finite

Chiral shift in QED (preliminary)

- Chiral shift is also generated in QED



HDL:



- Fermion propagator

$$G(\boldsymbol{k}_{\parallel}, \mathbf{r}, \mathbf{r}') = e^{i\Phi(\mathbf{r},\mathbf{r}')}\bar{G}(\boldsymbol{k}_{\parallel}, \mathbf{r}-\mathbf{r}')$$

$$\bar{G}(\boldsymbol{k}_{\parallel}, \boldsymbol{k}_{\perp}) = ie^{-k^2\ell^2}\sum_{n=0}^{\infty}(-1)^n D_n(\boldsymbol{k}_{\parallel}, \boldsymbol{k}_{\perp})\,\frac{1}{\mathcal{M}-2n|eB|}$$

A well-defined function of the parameters in the nth Landau level

- Gauge boson propagator

$$D_{\mu\nu}(q) \simeq \frac{|\boldsymbol{q}|}{|\boldsymbol{q}|^3 + \frac{\pi}{4}m_D^2|q_4|}O_{\mu\nu}^{(\mathrm{mag})} + \frac{O_{\mu\nu}^{(\mathrm{el})}}{q_4^2 + |\boldsymbol{q}|^2 + m_D^2}$$

QED: Weak field limit (preliminary)

- Weak field expansion

$$\bar{S}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp) = \bar{S}^{(0)}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp) + \bar{S}^{(1)}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp) + \cdots$$

$$\bar{S}^{(0)}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp) = i\frac{(\omega+\mu_0)\gamma^0 + m_0 - \boldsymbol{k}\cdot\boldsymbol{\gamma}}{(\omega+\mu_0)^2 - m_0^2 - k^2},$$

$$\bar{S}^{(1)}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp) = -\gamma^1\gamma^2 eB\frac{(\omega+\mu_0)\gamma^0 + m_0 - k_3\gamma^3}{\left[(\omega+\mu_0)^2 - m_0^2 - k^2\right]^2}$$

- Perturbative (linear in B) result for chiral shift:

$$\Delta^{(1)} \simeq -\frac{\alpha eB}{8\pi\mu_0}\left[\underbrace{\frac{2}{3}\sin^2\theta_{Bp}\ln\frac{2\mu_0}{|\omega'_E|}}_{\text{magnetic modes}} - \underbrace{\left(\frac{2}{3}\sin^2\theta_{Bp}+1\right)\ln\frac{C_1}{\alpha}}_{\text{electric modes}}\right]$$

- Estimate

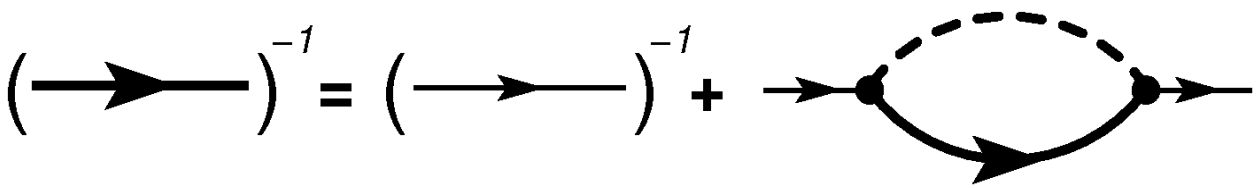
$$\Delta^{(1)} \simeq 0.5\,\text{keV}\left(\frac{400\,\text{MeV}}{\mu_0}\right)\left(\frac{B}{10^{16}\,\text{G}}\right)\ln\frac{\mu_0}{\alpha T} \qquad \text{at} \quad T \simeq 1\,\text{keV}$$

Yukawa model (preliminary)

- One can define a model without IR problems,

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu D_\mu + m_0\right)\psi + \frac{1}{2}\partial_\mu\phi_\sigma\partial^\mu\phi_\sigma + \frac{1}{2}\partial_\mu\phi_\pi\partial^\mu\phi_\pi$$
$$- \frac{1}{2}m_\phi^2\left(\phi_\sigma^2 + \phi_\pi^2\right) + g\bar{\psi}\left(\phi_\sigma + i\gamma^5\phi_\pi\right)\psi.$$

- Chiral shift is also generated,



- The value of the shift will be finite

$$\Delta \propto g^2 \frac{eB}{\mu_0}\ln\left(\frac{\mu_0}{m_\phi}\right)^2$$

QED: Strong field limit (preliminary)

- Lowest Landau level approximation

$$\Delta = \pi\alpha \int \frac{d^2k_\parallel d^2k_\perp}{(2\pi)^4} \mathrm{Tr}\left[\gamma^3\gamma^5\gamma^\mu\, \bar{S}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp)\gamma^\nu\right] D_{\mu\nu}(p-k)$$

where the LLL propagator is used

$$\bar{S}^{(\mathrm{LLL})}(\boldsymbol{k}_\parallel, \boldsymbol{k}_\perp) = 2ie^{-k_\perp^2\ell^2} \frac{(\omega+\mu_0)\gamma^0 + m_0 - k^3\gamma^3}{(\omega+\mu_0)^2 - m_0^2 - (k^3)^2} P_-$$

- The value of the shift:

$$\Delta \simeq \frac{s_\perp \alpha\, \mathrm{sgn}(\mu_0)}{2} \sqrt{|eB|}$$

- Estimate:

$$\Delta \simeq 30\,\mathrm{keV} \sqrt{\frac{B}{10^{16}\,\mathrm{G}}}$$

Brute force approach (preliminary)

- The chiral shift becomes a function of *n*:

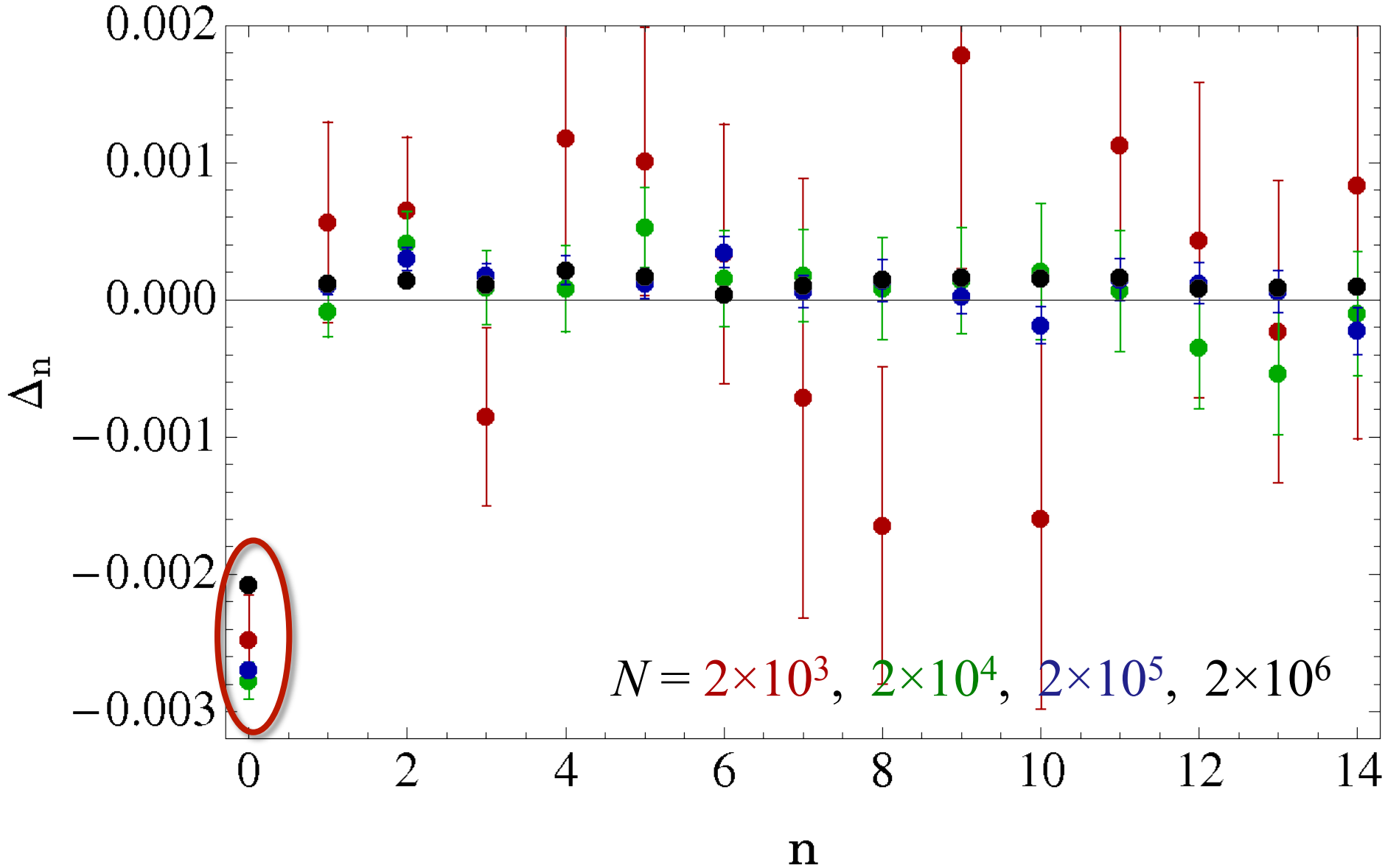
$$\Delta_n = -\mathrm{sgn}(eB)\frac{e^2\ell^2}{\pi}\sum_{n'=0}^{\infty}\int\frac{d\omega_E dk_3 d^2k_\perp d^2p_\perp}{(2\pi)^4}\frac{(-1)^{n+n'}(i\omega_E+\mu_0)e^{-(p_\perp^2+k_\perp^2)\ell^2}}{(\omega_E-i\mu_0)^2+m_0^2+(k^3)^2+2n'|eB|}$$
$$\times\left\{\left[\frac{1}{q_4^2+|\boldsymbol{q}|^2+m_D^2}-\frac{q_\perp^2}{|\boldsymbol{q}|\left(|\boldsymbol{q}|^3+\frac{\pi}{4}m_D^2|q_4|\right)}\right]\left[L_n(2p_\perp^2\ell^2)L_{n'}(2k_\perp^2\ell^2)-L_{n-1}(2p_\perp^2\ell^2)L_{n'-1}(2k_\perp^2\ell^2)\right]\right.$$
$$\left.-\frac{q_\perp^2+2q_3^2}{|\boldsymbol{q}|\left(|\boldsymbol{q}|^3+\frac{\pi}{4}m_D^2|q_4|\right)}\left[L_{n-1}(2p_\perp^2\ell^2)L_{n'}(2k_\perp^2\ell^2)-L_n(2p_\perp^2\ell^2)L_{n'-1}(2k_\perp^2\ell^2)\right]\right\},$$

- There are no (obvious) divergences

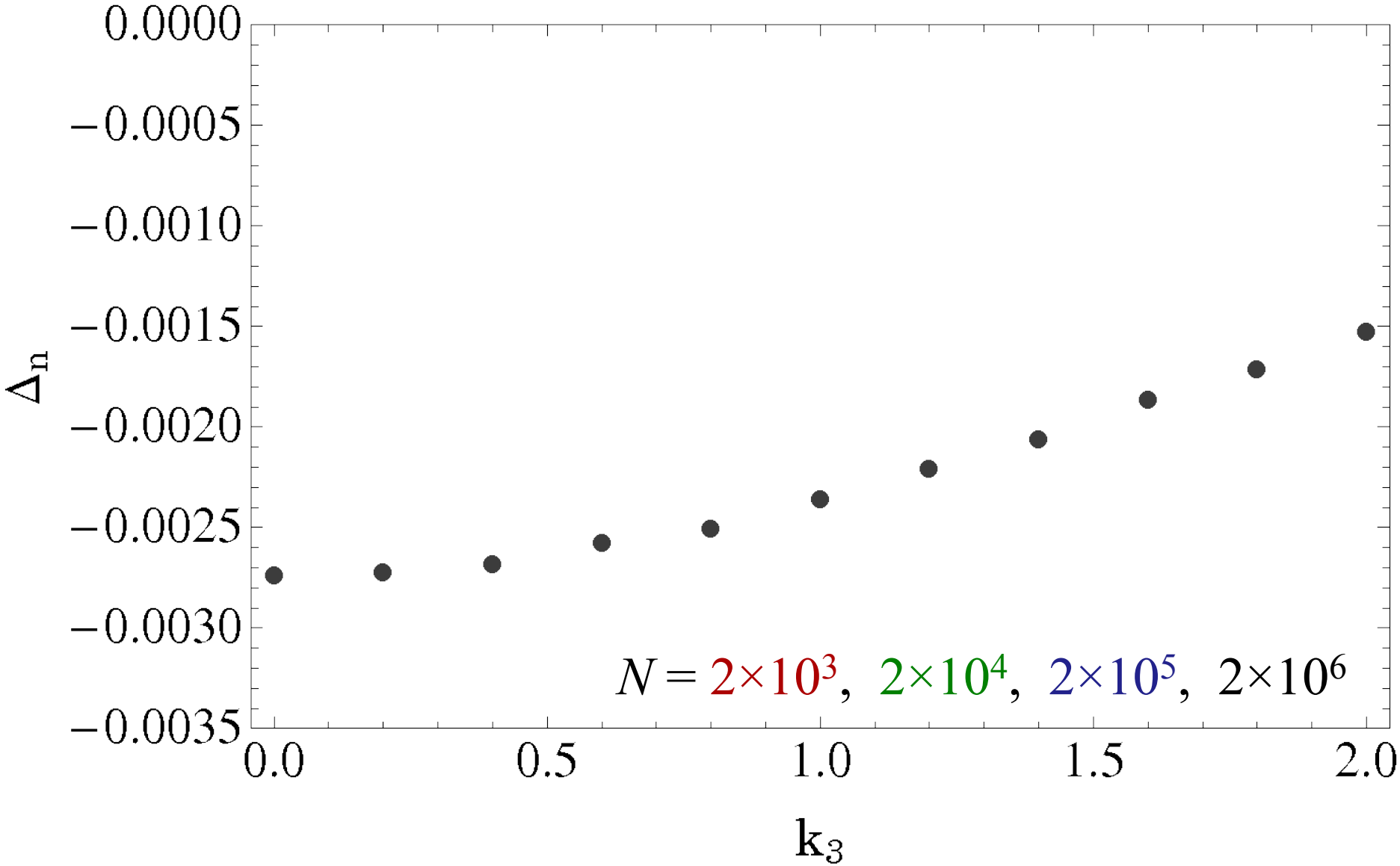
- Two integrals (out of 6) can be done analytically

- The rest (including the sum) has to be done numerically

Δ_n @ Fermi surface (preliminary)



Δ_0 vs. k_3 (preliminary)



Summary (1)

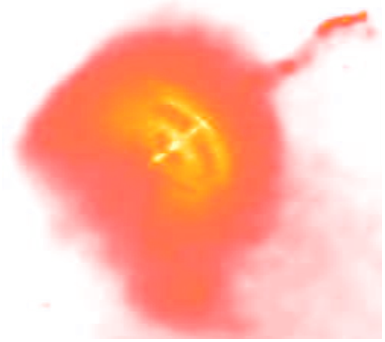
- Chiral shift is generated in magnetized matter (evidence from renormalizable models)
- The magnitude of chiral shift scales as

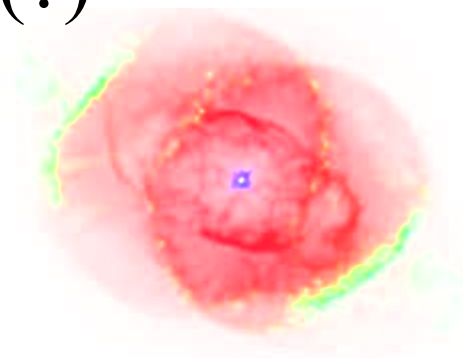
$$\Delta \propto \alpha \frac{eB}{\mu_0} \quad \text{or} \quad \Delta \propto \alpha \sqrt{eB}$$

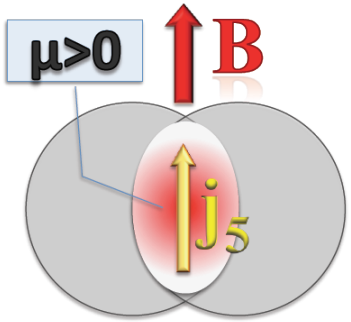
- Chiral shift induces a chiral asymmetry at the Fermi surface
- Chiral shift may potentially contribute to the axial current density

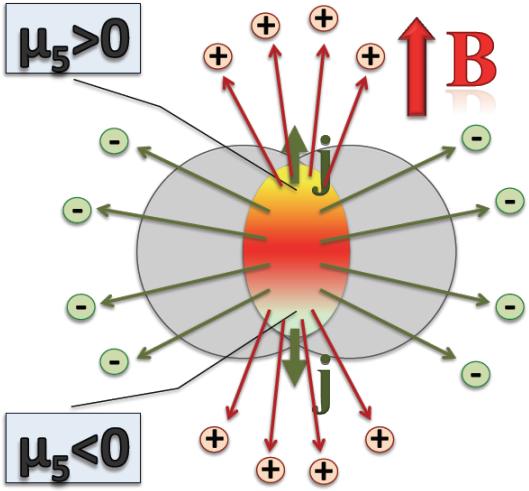
Summary (2)

- Potential applications:
  - Pulsar kicks (?)
    - Quark/hybrid stars
  - Facilitation of supernova explosions (?)
  - Axial current in hot QGP
    - Quadrupole CME









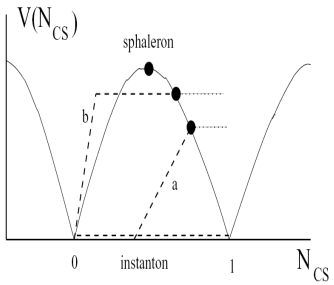
Can the Standard Model explain the Cosmological Baryogenesis?

CPODD 2012

Edward Shuryak
Stony Brook

main collaborators:
Y.Burnier, G.Carter,
V.Flambaum,D.Ostrovsky

A short answer: Yes,
but in a non-standard Cosmology



Let us start at the beginning

- The energy of Yang-Mills field versus the Chern-Simons number $N_{CS} = \int d^3x K_0$ is a periodic function, with zeros at integer points.

- The *instanton* (the lowest dashed line) describes tunneling between vacua. It is a path at E=0, it starts and ends at no field strength
- If energy is deposited, the paths (the dashed lines) goes up and emerge from ''under the barrier'' into real (Minkowskian) world at the turning points, where momenta (in the $A_0 = 0$ gauge) "$\vec{p}$" $= \frac{d\vec{A}}{dt} = \vec{E} = 0$ => the field is magnetic
- Real time motion outside the barrier (shown by horizontal dotted lines) => explosions
- The maximal cross section corresponds to the top of the barrier, called the sphaleron =''ready to fall'' in Greek, according to Klinkhammer and Manton

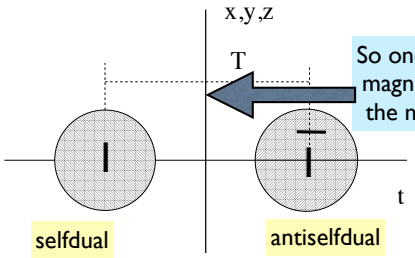
F.R.Klinkhamer and N.S.Manton,
Phys. Rev. D 30, 2212–2220 (1984)

Dashen et al, 1974 suggested such soliton first

arXiv:hep-ph/0204224v2 25 Apr 2002

Forced Tunneling and Turning State Explosion in Pure Yang-Mills Theory

D. M. Ostrovsky[1], G. W. Carter[2], and E. V. Shuryak[1]
[1] *Department of Physics and Astronomy, State University of New York, Stony Brook, NY 11794-3800*
[2] *Department of Physics, Box 351560, University of Washington, Seattle, WA 98195-1560*
(18 April 2002)



So one finds a pure magnetic object in the mid-3d plane!

One can see that, in the simplest case of identical sizes and orientations for the I and $\bar{I}$, time reflection symmetry $t \to -t$ of the problem is indeed manifest, so that

$$\mathcal{A}_0^a(\vec{r}, t=0) = 0\,, \quad \mathcal{E}_m^a(\vec{x}, t=0) = 0\,. \tag{21}$$

I tried various trial functions but only Yung ansatz worked

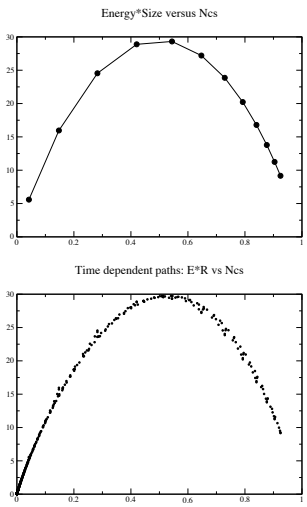


FIG. 6. The normalized energy, ER, versus the Chern-Simons number for the Yung ansatz. Plot (a) shows the positions of the turning states for various T, while (b) combines many points along the path ($t \neq 0$); their small spread means that Yung ansatz is nearly going directly uphill, thus passing via the same points for different T.

Electroweak sphalerons: scale v=246 GeV =>mass of about 12-14 TeV, numerical solution (Klinkhammer-Manton etc)

Carter-Ostrovsky,ES: QCD sphalerons

- What is the minimal potential energy of static Yang-Mills field, consistent with the constraints:

(i) the given value of (corrected) Chern-Simons number.

(ii) the given value of the r.m.s. size $< r^2 >= \int d^3x r^2 \mathcal{B}^2 / \int d^3x \mathcal{B}^2$

- Solution (found by D.Ostrovsky) is a ball made of three magnetic gluon fields (out of 8 in SU(3)) rotated around x,y,z axes

$B^2/2 = 24(1-\kappa^2)^2\rho^4/(r^2+\rho^2)^4$

$E_{stat} = 3\pi^2(1-\kappa^2)^2/(g^2\rho)$ $\tilde{N}_{CS} = \mathrm{sign}(\kappa)(1-|\kappa|)^2(2+|\kappa|)/4.$

Eliminating κ one gets the topological potential energy, $\kappa = 0$ gives the sphaleron

$$\mathcal{A}^a_j = A(r,t)\Theta^a_j + B(r,t)\Pi^a_j + C(r,t)\Sigma^a_j \qquad \mathcal{A}^a_0 = D(r,t)\frac{x^a}{r}$$

$$\Theta^a_j = \frac{\epsilon_{jam}x^m}{r}\,, \quad \Pi^a_j = \delta_{aj} - \frac{x_a x_j}{r^2}\,, \quad \Sigma^a_j = \frac{x_a x_j}{r^2}\,.$$

$$A = \frac{1+\phi\sin\alpha}{r}, \quad B = \frac{\phi\cos\alpha}{r}, \quad C = A_1, \quad D = A_0.$$

$$S = \frac{1}{4g^2}\int d^3x dt\left[\left(\mathcal{B}^a_j\right)^2 - \left(\mathcal{E}^a_j\right)^2\right] = 4\pi\int drdt\left[\left(\partial_\mu\phi\right)^2 + \phi^2\left(\partial_\mu\alpha - A_\mu\right)^2 + \frac{(1-\phi^2)^2}{2r^2} - \frac{r^2}{2}\left(\partial_0 A_1 - \partial_1 A_0\right)^2\right]$$

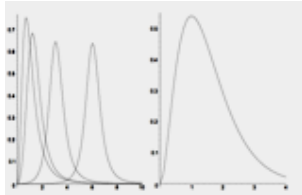
standard Witten ansatz

our solution for any kappa and the sphaleron

$$\phi(r) = \left(1 - (1-\kappa^2)\frac{4\rho^2 r^2}{(r^2+\rho^2)^2}\right)^{1/2} \qquad \partial_r\alpha(r) = -2\kappa\frac{1-\phi^2}{\phi^2}\frac{\rho}{r^2+\rho^2}\,.$$

$$B^2 = 24\rho^4\frac{(1-\kappa^2)^2}{(r^2+\rho^2)^4} \qquad \tilde{N}_{CS} = \frac{1}{4}\mathrm{sign}(\kappa)(2+|\kappa|)(1-|\kappa|)^2$$

$$\phi(r) = \frac{|r^2-\rho^2|}{r^2+\rho^2}\,, \quad \alpha(r) = \pi\theta(r-\rho).$$



Explosion of the Turning States

Left: the snapshots of the r^2 energy
Right: the spectrum of the produced fermions

- Solved both numerically (G.Carter) and analytically (by D.Ostrovsky based on work by Luescher and Schehter from 1977 which can also be via conformal transformation -Zahed)
- Sphalerons at $t = 0 \Rightarrow$ (at large t) into a spherical transverse wave $4\pi e(r,t) = \frac{8\pi}{g^2\rho^2}(1-\kappa^2)^2\left(\frac{\rho^2}{\rho^2+(r-t)^2}\right)^3$
- ES+Zahed: new solution to the Dirac equation in exploding backgroun obtained by inversion of the fermionic O(4) zero mode of O(4) symmetric solution.

It explicitly shows how the quark acceleration occurs, starting from zero energy at t=0 to the final spectrum

- The sphalerons produce one level crossing N_F LR quarks, and the antisphaleron-like clusters the chirality opposite.

Baryogenesis

- Sakharov (1967) had formulated 3 conditions => **B-violation, CP-violation, non-equilibrium**
- All 3 are there in the Standard Model (SM)
- And yet we do not know how $n_B/n_\gamma = 6*10^{-10}$ has been obtained… as way too small numbers are obtained
- beyond the SM? (very popular)
  or beyond the standard cosmology instead?

Hybrid (cold) scenario

combines establisment of the EW broken phase with the end of inflation
J. Garcia-Bellido, D.Y. Grigoriev, A. Kusenko and M.E. Shaposhnikov, Phys. Rev. D 60 (1999) 123504 [hep-ph/9902449]. L.M. Krauss and M. Trodden, PRL 83 (1999) 1502 [hep-ph/9902420].

J. Garcia-Bellido, M. Garcia-Perez and A. Gonzalez-Arroyo, Phys. Rev. D **69**, 023504 (2004) [arXiv:hep-ph/0304285].
S. Tranberg and J. Smit, JHEP **0311**, 016 (2003) [arXiv:hep-ph/0310342].

- Real-time lattice simulations
- Only bosons: 2 scalars (Higgs and inflaton) and the gauge fields (photons, W,Z)
- Mostly focused on generation of primordial magnetic field (not to be discussed in this talk)
- diffusion in the Chern-Simons number => the rate of the **B violation**

Hybrid (cold) scenario (cont)

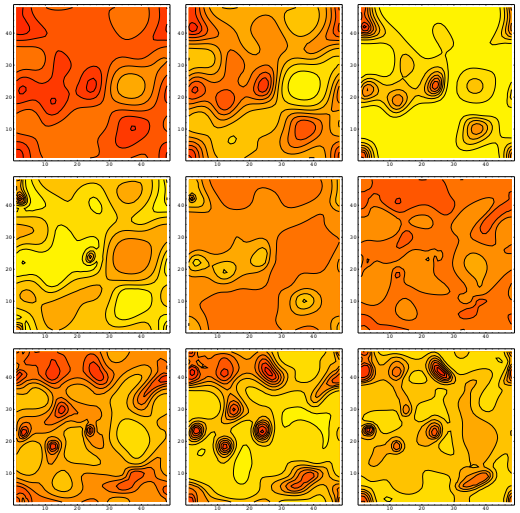
- Subsequent time snapshots of the **Higgs VEV modulus** shows appearance of the ``**hot spots"**

(red) which have near-zero Higgs VEV

They remain till time

t m about 20

(m is the mass scale, in this simulation it is 240 GeV)



Hybrid (cold) scenario (cont)

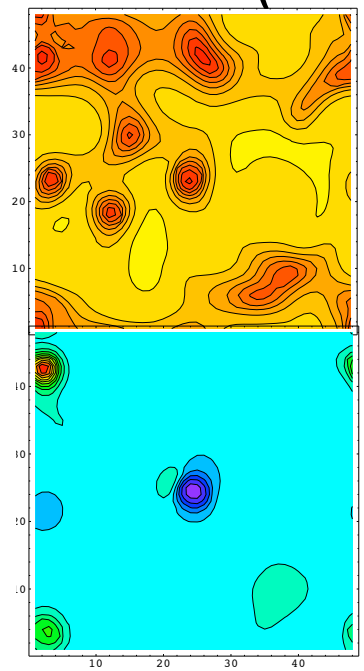
Topological charge
Q = GG_{dual} is also localized

The topological transitions happen **only inside (some of) the "hot spots"**

Hot spots take volume fraction of few percents, sphalerons in them also have P of few percents

=>Γ/T^4 about 10^{-4},

Integrated in time 10^{-3}



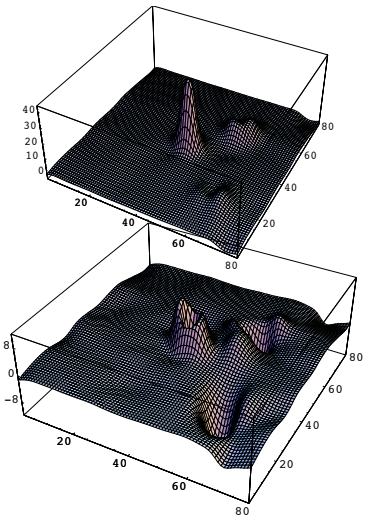
$|\phi|$

T m=19
The same
Time and
place

Q(x)

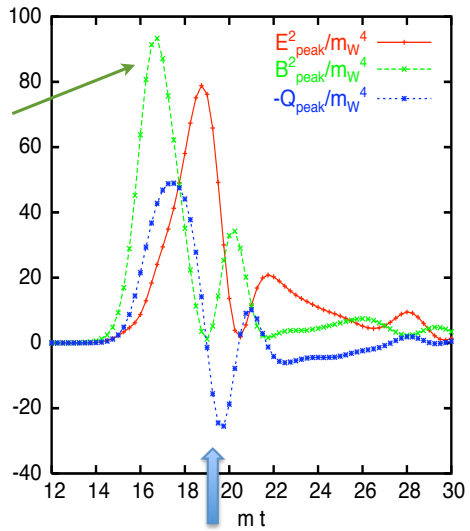
Hybrid (cold) scenario (cont)

- Topological charge explodes after a peak
- Then spherical explosion with an empty shell



Hybrid (cold) scenario (cont)

- Further details:
- Magnetic field B first
- Then electric E
- Note that topology has sign inversion, before it comes back



Numerically observed topological fluctuations can be described by the COS sphalerons (Flambaum, ES, PRD82 (2010) 073019 arXiv:1006.0249 [hep-ph]

Does the magnetic field fits well to COS sphaleron solution? The profile of the the magnetic field in COS configuration is given by the following simple expression

$$B^2(r) = \frac{48\rho^4}{g^2(r^2+\rho^2)^4} \tag{19}$$

which (unlike the KM one) is just spherically symmetric. This form does indeed fit well the observed shape of the B^2 at the sphaleron moment. The maximum of B^2/g^2 can be related with its radius, yielding $m\rho \approx 3.9$, which is very close to optimal size we got above. This value corresponds to the total energy of the COS sphaleron (5)

$$E_{tot} = 3\pi^2/g^2\rho \approx 2\ TeV \tag{20}$$

As we already mentioned in the INtroduciton, it is 7 times less than the KM sphaleron mass, and for the temperatures we are dealing with $T_{in} = 200 - 100\,GeV$ it makes a huge difference for he rate.

$$\Gamma_{sph}(\rho) \sim exp\left[-\frac{\Delta F}{T_{in}}\left(\frac{4\pi\rho^3}{3}\right) - \frac{3\pi^2}{g^2\rho T_{in}}\right]$$

$$\rho_* = \left(\frac{3\pi}{4g^2\Delta E}\right)^{1/4}$$

The rate and size found on the lattice are both reproduced:

the rate is determined by the thermal conditions at the time

sphalerons badly need the tops! (top recycling)

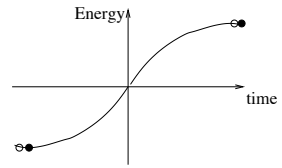
The well known Adler-Bell-Jackiw anomaly require that a change in gauge field topology by $\Delta Q \pm 1$ must be accompanied by a corresponding change in baryon and lepton numbers, B and L. More specifically, such topologically nontrivial fluctuation can thus be viewed as a "t'Hooft operator" with 12 fermionic legs. Particular fermions depend on orientation of the gauge fields in the electroweak SU(2): since we are interested in utilization of top quarks, we will assume it to be "up". In such case the produced set contains $t_r t_b t_g c_r c_b c_g u_r u_b u_g \tau \mu, e$, where r, b, g are quark colors. to which we refer below as the $0 \to 12$ reaction. Of course, in matter with a nonzero fermion density many more reactions of the type $n \to (12-n)$ are allowed, with n (anti)fermions captured from the initial state.

The $3 \to 9$ fermion process saves a lot of energy, as in it the initial top quark energy can be completely transferred from the "sphaleron doorway state" to the gauge field during the compression stage. In the example we discuss, with $m\rho = 3.9$, this mean energy of 3 top quarks is

$$\Delta E = 3 * 3/\rho \approx 600\, GeV \tag{39}$$

An estimate with exponential accuracy we find its enhancement by the factor of about

$$F_{recycling} \sim \exp(\Delta E/T) \sim 20 \tag{40}$$



Let us show by simple estimates that under conditions we are discussing the $0 \to 12$ fermion production process is actually impossible. The final kinetic energy for fermion produced by COS sphaleron is $E_q = 3/\rho$ which is $\approx 200\, GeV$ for the $m\rho = 3.9$ example displayed as typical in Fig.3. Multiplied by 12 fermionic species produced it would require $2.4\, TeV$ of energy which apparently exceeds the total available energy of the gauge fields in the topological fluctuations observed numerically.

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e-Print: arXiv:1006.0645

The W-Z-Top Bags

Marcos P. Crichigno[1], Victor V.Flambaum[2], Michael Yu.Kuchiev[2] and Edward Shuryak[1]

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(Dated: June 1, 2010)

We discuss a new family of multi-quanta bound states in the Standard Model, which exist due to the mutual Higgs-based attraction of the heaviest members of the SM, namely, gauge quanta W, Z and (anti)top quarks, $\bar{t}, t$. We use a self-consistent mean-field approximation, up to a rather large particle number N. In this paper we do not focus on weakly-bound, non-relativistic bound states, but rather on "bags" in which the Higgs VEV is significantly modified/depleted. The minimal number N above which such states appear strongly depends on the ratio of the Higgs mass to the masses of $W, Z, \bar{t}, t$: For a light Higgs mass $m_H \sim 50\,GeV$ bound states start from $N \sim O(10)$, but for a "realistic" Higgs mass, $m_H \sim 100\,GeV$, one finds metastable/bound W, Z bags only for $N \sim O(1000)$. We also found that in the latter case pure top bags disappear for all N, although top quarks can still be well bound to the W-bags. Anticipating cosmological applications (discussed in a companion paper) of these bags as "doorway states" for baryosynthesis , we also consider the existence of such metastable bags at finite temperatures, when SM parameters such as Higgs, gauge and top masses are significantly modified.

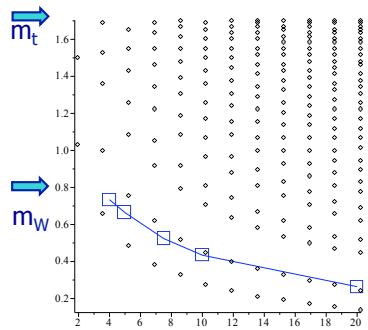
The nontopological solitons made of W-Z-tops can be identified with the ``hot spots" found in numerical simulations

The lifetime of the top bags: weak decays

t=> b W in vacuum is possible,
(m_t=172 GeV > m_W=80 GeV)
But is it so in the bag?

Not always!

The reason is fermions prefer to have surface bags
And they have zero mode
While the Ws do not!



From Flambaum,ES,
"Ws in the top bags"

The levels vs the bag size.
Large boxes – W levels,
small circles – the t levels

CP violation in the SM

- Jarlskog factor J appears (for 4 Ws) which gives the area of unitary triangles of CKM
- If any two masses of upper or lower quarks coinside, **no CP =>** ?

No summation

$$\mathrm{Im}\left[V_{ab}V_{bc}^{\dagger}V_{cd}V_{da}^{\dagger}\right] = J\sum_{e,f}\epsilon_{ace}\epsilon_{bdf},$$

$$J = s_1^2 s_2 s_3 c_1 c_2 c_3 \sin(\delta):$$

CKM part

$$\Delta_{CP} = v^{-12}\mathrm{Im\ Det}\left[m_u m_u^{\dagger}, m_d m_d^{\dagger}\right]$$
$$= J\, v^{-12}\prod_{i<j}(\tilde{m}_{u,i} - \tilde{m}_{u,j}^2)\prod_{i<j}(\tilde{m}_{d,i}^2 - \tilde{m}_{d,j}^2) \simeq 10^{-19},$$

$$m_d m_d^{\dagger} = D\tilde{m}_d^2 D^{\dagger}, \quad m_u m_u^{\dagger} = U\tilde{m}_u^2 U^{\dagger}.$$

If so, forget the SM Baryogenesis!

Not true in general, only if **the scale is above all masses**, in denominators
Not so for K and B decays!
Loops at intermediate scales have other way to vanish => e.g. log(m_i/m_j)=0
for equal masses as well, no need to have a difference as a factor…

Can the Standard Model CP violation near the W-bags explain the cosmological baryonic asymmetry?

Yannis Burnier* and Edward Shuryak†
Department of Physics, State University of New York, Stony Brook, NY 11794, USA
(Dated: July 21, 2011)

arXiv:1107.4060v1 [hep-ph] 20 Jul 2011

Consider a process, 4-th order in weak interaction, describing interferences between various paths by which t quark (or anti-top) leaves the W bag.

CP violation => top-antitop population difference in the bag

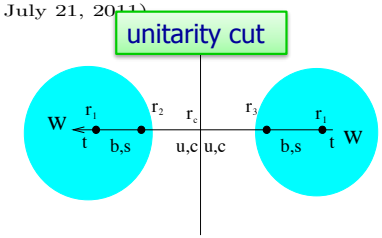


FIG. 1: (Color online) Schematic shape of the fourth order process involving only quarks of the 2nd and 3rd generations. The shaded objects on the left and right represent the Higgs bag with strong gauge fields (indicated by W in the figure) inside. The vertical line is the unitarity cut. The four black dots indicate the four points $r_i, i = 1..4$ where the W quanta are interacting with the quark, changing it from up to down component.

The flavor trace part of the CP Violation

Flavor-independent part of S cancels out,
small flavor-dependent phases remain

$$M_t - M_{\bar{t}} = 2iJ(S^u_{23} - S^c_{23})$$
$$(-S^s_{34} * S^b_{12} - S^d_{34} * S^s_{12} + S^d_{34} * S^b_{12}$$
$$+S^d_{12} * S^s_{34} - S^d_{12} * S^b_{34} + S^s_{12} * S^b_{34})$$

$$exp(i \int dx \sqrt{E^2 - m_i^2}) \approx exp[i(E - m_i^2/2E) r_{ij}]$$

similar phase to neutrino oscillations

$$M_t - M_{\bar{t}} \propto J \frac{m_b^4 m_c^4 m_s^2 r_{23}^2 r_{12} r_{31} (r_{12} + r_{31})}{64 E^5}.$$

Everything cancels till the 5th order in phases,
the remaining expression has **definite sign which is correct! More t leave, more bar-t in the bag**

use **E as small as possible** <= plasma mass $g_w T$ or m_b scale of few GeV

All **r_{ij} are as large as possible** <= limited by phases not larger than 1,

One point in the bag, one as far possible consistent with exponential fall of with W mass

If so, the CP difference in t and anti-t is

$$\delta_{CP} = \tau J \frac{g^4 N_w^2 m_c}{16E} \frac{m_b^4}{E^4} \frac{m_c^3}{m_w^3} \frac{m_s^2}{E_w^2} \sim 10^{-11} \left(\frac{N_w}{1000}\right)^2 .$$

it require less energy. The probability to find 3 antitops is actually proportional to $(1+\delta_{CP})^3 \approx 1+3\delta_{CP}$, while it is $(1-3\delta_{CP})$ for tops: it gives factor 3. Another factor 3 appears because of the fact that each sphaleron event creates 3 units of baryon number, not one. Together with baryon asymmetry (time integrated) sphaleron rates of 10^{-2} and $3 \times 3 \times \delta_{CP}$ we arrive to our final estimate

$$\frac{\Delta B}{B+\bar{B}} \sim 10^{-12\pm1}$$

No numbers tuned, just
As in numerical works

Final comments on sphalerons in pp:

- Double-diffractive cluster production in STAR+"Roman pots" => proposal pending
- Sphaleron produces 6 units of chirality!
- Expected exclusive channels which are well observed like KKπ etc (but not $\pi\pi\pi$) like it was in η_c decays (Bjorken,Schafer)
- Because pp has magnetic field => charge asymmetry in final states (Kharzeev)

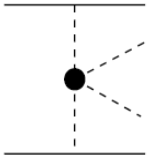
summary

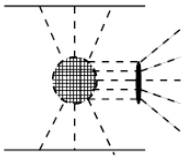
- KM sphalerons in the broken phase are too heavy (high barrier) to get any effect. The rate before Tc is too high, it erases the asymmetries. So SM+standard cosmology does not work
- Non-standard Big Bang, with inflation going right into electroweak scale. **Electroweak sphalerons found numerically:** they provide rate 10^-3 in cold cosmological model, can be described by **COS sphalerons.**
- Top recycling idea 3=>9: one order gained and direct relation to CP violation established. Top decay in the bags naturally lead to diffusion of lighter quarks away and CP violation **of the right order and sign.** This opens the possibility to explain baryogenesis inside the SM
- COS sphalerons in pure gauge => QCD clusters: not yet observed but may be found at RHIC in doubly-diffractive pp events (Zahed+ES) or in AA collisions by CP-odd fluctuations (see most other talks)

Pomeron from instantons

- Soft Pomeron with ''instanton ladder'' provides reasonable intercept and slope
- No Odderon SU(3) allows for a colorless combination of 3 gluons.

Perturbatively, the odderon/pomeron ratio is $O(\alpha_s)$ and not as suppressed as the data shows. Instantons are SU(2) beasts and do not allow it.





- Instanton is the only nontrivial field for which straight Wilson lines can be easily analytically calculated
$\int \tau^a A^a_\mu dx_\mu \sim (\tau^a \eta^a_{\mu\nu} e^t_\nu e^l_\mu) \int F(x^2) dt$
- The trick with rotating cross section to Minkowski seem to work perturbatively and in general (Meggiolaro): we tried it for instanton

QCD sphalerons and their explosion => diffractive clusters at RHIC

Semiclassical Theory of High Energy Collisions based on Instantons and Sphalerons

- ''pomeron from instantons'':

ES, Zahed PRD62:085014,2000 hep-ph/0005152
D. E. Kharzeev, Y. V. Kovchegov and E. Levin Nucl. Phys. A **690**, 621 (2001) [hep-ph/0007182].
M. A. Nowak, ES and Zahed, PRD 64, 034008 (2001) [hep-ph/0012232].
G.W.Carter,D.Ostrovsky and ES, Phys. Rev. D **65**, 074034 (2002) [hep-ph

- the turning states and their explosion

D.Ostrovsky, G.W.Carter and ES, hep-ph/0204224,PRD

- Landau method for cross section - rescaled YM sphalerons are produced

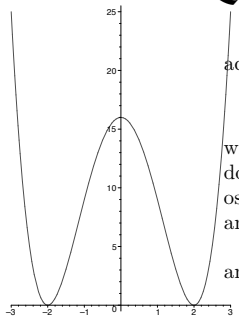
D. Diakonov and V. Petrov, Phys. Rev. D **50**, 266 (1994) R. A. Janik, ES and Zahed, hep-ph/0206005.

- Explicit solution of the Dirac eqn in the exploding field background: an end of ''the fermion puzzle''?

ES and Zahed, hep-ph/0206022.

- Gluonic cluster production in double-Pomeron processes (e.g. $pp->pp\eta'; pp f_0(1600), pp+$cluster) compared to data; ES and Zahed,2002
- Instanton-induced Double DIS $\gamma^*\gamma^*$)ES and Zahed,2003

significance of the sphalerons: a QM example (ES,hep-ph/0205031)



To answer those questions, let us introduce an external periodic perturbation acting on the system

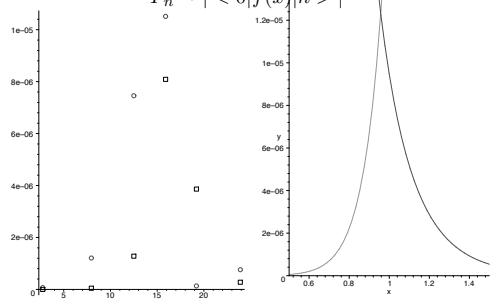
$$\delta V(x,t) = f(x) exp[-it\omega] \tag{5}$$

with $f(x)$ well localized under the barrier, at $x \sim 0$. The specific shape of $f(x)$ does not matter as soon as it does not extends to the wells, where the ordinary oscillation quanta (analogs of gluons) can be excited. I have used several of them and will show results for $f = exp(-4x^2)$ and $f = 1, |x| < .2; f = 0, |x| > .2$.

The time dependence can be tuned to excite the n-th level $\omega = (E_n - E_0)$, and then the excitation probability

$$P_n \sim | < 0|f(x)|n > |^2 \tag{6}$$

The systems jumps at the lowest physical state available for the same coordinate

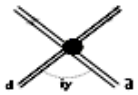


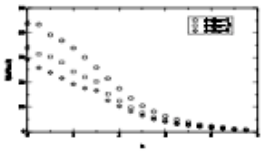
Example of the cleanest process: the double DIS, no hadrons, just excitation of one cluster (sphaleron)

Double-DIS $\gamma^*\gamma^*$ (Next linear collider?)

About the same small gluonic spot without any nucleon

ES, I.Zahed hep-ph/0307103, PRD in press
Two dipoles cross the instanton, the angle is changed to iy at the end of the calculation
$\sigma \sim d\vec{r}_1 d\vec{r}_2$ times the dimensionless function of the three 2-d vector variables $I(\vec{d1}/\rho, \vec{d2}/\rho, \vec{b}/\rho)$ explicitly given and tabulated: see b-profile





Semiclassical Double-Pomeron Production of Glueballs, η' and clusters

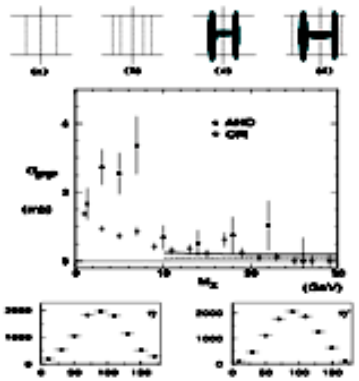
ES and I.Zahed,Phys.Rev. D68 (2003) 034001

Pomeron-Pomeron into cluster, cross section from UA8 collaboration: heavy gluonic clusters with isotropic decay. What are they?

Note: a cross section that is an order of magnitude larger than the one predicted by Pomeron factorization

WA102 collaboration at CERN, pp Double-Pomeron into identified central hadron: strong dependence of the cross section on the azimuthal angle ϕ (between two kicks to two protons), not expected from standard Pomeron phenomenology.

we get $cos^2(\phi)$ for P=+ and $sin^2(\phi)$ for P=-1.



Chiral kinetic theory

M. Stephanov

U. of Illinois at Chicago

with Yi Yin

Motivation

- Interesting applications of chiral magnetic/vortical effect involve highly non-equilibrium conditions – such as heavy-ion collisions.

- Derivations of chiral magnetic effect are usually done in equilibrium.

- Can we generalize this description to non-equilibrium conditions?

- Kinetic description is a solution.

- There are limitations: Classical description Weakly coupled

But it would be an important step for our understanding of the anomalous effects.

- Condensed matter literature:

Sundaram-Niu (1999), Wong-Tserkovnyak (2011), Son-Yamamoto, Son-Spivak (2012). In the context of CME, CVE - Loganayagam-Surowka.

Approach

- Kinetic equation describes the motion of particles in the regime where collisions are rare enough that motion between collisions is classical.

In terms of the distribution function $f(t, \boldsymbol{x}, \boldsymbol{p})$ the equation is

$$\frac{df}{dt} \equiv \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \boldsymbol{x}} \dot{\boldsymbol{x}} + \frac{\partial f}{\partial \boldsymbol{p}} \dot{\boldsymbol{p}} = C[f].$$

We think of a “cloud” of particles each of which follows classical trajectory $\boldsymbol{x}(t)$, $\boldsymbol{p}(t)$. As a result the distribution evolves with time. But if you follow a local volume along the trajectory, the number of particles in it can only be changed by collisions.

Ignore collisions for now.

- Clearly, the number of particles in the phase space <u>cannot change</u>. The phase-space density obeys conservation equation.

- How can a kinetic equation account for anomalous particle creation?

Also, how can <u>classical</u> equation account for <u>quantum</u> anomaly?

Two words:

Approach

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- How can a kinetic equation account for anomalous particle creation?

Also, how can <u>classical</u> equation account for <u>quantum</u> anomaly?

Two words: **Berry monopole**

Path integral

- Consider a Weyl particle: $H = \boldsymbol{\sigma} \cdot \boldsymbol{p}$

For each momentum $\boldsymbol{p}$ it is a two-state system. With gap $2|\boldsymbol{p}|$.

- Transition amplitude. Insert complete sets at every time slice:

$$\langle f|e^{-iH(t_f-t_i)}|i\rangle = \ldots \int_{\boldsymbol{p}_1,\boldsymbol{x}_1} \sum_{\lambda_1,s_1} \ldots \langle \boldsymbol{x}_2, s_2|\boldsymbol{p}_1,\lambda_1\rangle\langle \boldsymbol{p}_1,\lambda_1|e^{-iH\Delta t}|\boldsymbol{x}_1,s_1\rangle\langle \boldsymbol{x}_1,s_1|\ldots$$

$$= \left[\int \mathcal{D}x\mathcal{D}p\,\mathcal{P}\exp\left\{i\int_{t_i}^{t_f}(\boldsymbol{p}\cdot\dot{\boldsymbol{x}} - \boldsymbol{\sigma}\cdot\boldsymbol{p})dt\right\}\right]_{fi}$$

is path-ordered product of $\exp\{-i\boldsymbol{\sigma}\cdot\boldsymbol{p}\Delta t\}$ over each phase space path $\boldsymbol{x}(t)$, $\boldsymbol{p}(t)$.

Classical limit

- To take the classical limit we diagonalize $V_{\boldsymbol{p}}^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{p} V_{\boldsymbol{p}} = |\boldsymbol{p}|\sigma_3$ at each point:

$$\ldots V_{\boldsymbol{p}_2} V_{\boldsymbol{p}_2}^\dagger \exp\{-i\boldsymbol{\sigma}\cdot\boldsymbol{p_2}\Delta t\} V_{\boldsymbol{p}_2} V_{\boldsymbol{p}_2}^\dagger V_{\boldsymbol{p}_1} V_{\boldsymbol{p}_1}^\dagger \exp\{-i\boldsymbol{\sigma}\cdot\boldsymbol{p_1}\Delta t\} V_{\boldsymbol{p}_1} V_{\boldsymbol{p}_1}^\dagger \ldots$$
$$= \ldots V_{\boldsymbol{p}_2} \exp\{-i|\boldsymbol{p}_2|\sigma_3\Delta t\} \underbrace{V_{\boldsymbol{p}_2}^\dagger V_{\boldsymbol{p}_1}}_{\text{extra rotation}} \exp\{-i|\boldsymbol{p}_1|\sigma_3\Delta t\} V_{\boldsymbol{p}_1}^\dagger \ldots$$

$$V_{\boldsymbol{p}_2}^\dagger V_{\boldsymbol{p}_1} \approx \exp(-i\hat{\boldsymbol{a}}_{\boldsymbol{p}}\cdot\Delta\boldsymbol{p}) \quad \text{where} \quad \hat{\boldsymbol{a}}_{\boldsymbol{p}} = -iV_{\boldsymbol{p}}^\dagger \boldsymbol{\nabla}_{\boldsymbol{p}} V_{\boldsymbol{p}}.$$

- If we did not insist on diagonalizing $\boldsymbol{\sigma}\cdot\boldsymbol{p}$, we could choose arbitrary $U(2)$ rotation, say $V_{\boldsymbol{p}} \to V_{\boldsymbol{p}} U_{\boldsymbol{p}}$.

This results in a “gauge transformation” of the “action” such that

$$-|\boldsymbol{p}|\sigma_3 \to -|\boldsymbol{p}| U_{\boldsymbol{p}}^\dagger \sigma_3 U_{\boldsymbol{p}} \quad \text{and} \quad \hat{\boldsymbol{a}}_{\boldsymbol{p}} \to U_{\boldsymbol{p}}^\dagger \hat{\boldsymbol{a}}_{\boldsymbol{p}} U_{\boldsymbol{p}} + iU_{\boldsymbol{p}}^\dagger \boldsymbol{\nabla}_{\boldsymbol{p}} U_{\boldsymbol{p}}$$

- Corresponds to the free choice of the phase and spin quantization direction for the momentum states: $|\boldsymbol{p}, s\rangle \to U_{\boldsymbol{p}}|\boldsymbol{p}, s\rangle$.

We chose helicity basis at each $\boldsymbol{p}$ to diagonalize $\boldsymbol{\sigma}\cdot\boldsymbol{p}$.

Abelian projection and the monopole

$$\langle f|e^{-iH(t_f-t_i)}|i\rangle = \left[V_{\boldsymbol{p}_f}\int \mathcal{D}x\mathcal{D}p\,\mathcal{P}\exp\left\{i\int_{t_i}^{t_f}(\boldsymbol{p}\cdot\dot{\boldsymbol{x}}-|\boldsymbol{p}|\sigma_3-\hat{\boldsymbol{a}}_{\boldsymbol{p}}\cdot\dot{\boldsymbol{p}})dt\right\}V^\dagger_{\boldsymbol{p}_i}\right]_{fi}$$

- To describe classical motion of a particle of a given helicity we need to be able to neglect off-diagonal transitions, which are caused by the off-diagonal components of $\hat{\boldsymbol{a}}_{\boldsymbol{p}}$. I.e., $|\hat{\boldsymbol{a}}_{\boldsymbol{p}}\cdot\dot{\boldsymbol{p}}| \ll 2|\boldsymbol{p}|$.

Forces ($\dot{\boldsymbol{p}}$) should not be too strong ($B \ll |\boldsymbol{p}|^2$).

- However, we cannot neglect the diagonal components – Berry phases!

- Although non-abelian $\hat{\boldsymbol{a}}_{\boldsymbol{p}}$ is pure gauge, abelian component $[\hat{\boldsymbol{a}}_{\boldsymbol{p}}]_{11} \equiv \boldsymbol{a}_{\boldsymbol{p}}$ is non-trivial: the field of a "monopole" at $|\boldsymbol{p}| = 0$ (as in 't Hooft, Polyakov):

$$\boldsymbol{b} \equiv \boldsymbol{\nabla}_{\boldsymbol{p}} \times \boldsymbol{a}_{\boldsymbol{p}} = \frac{\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|^2}.$$

Singularity is due to level crossing at $\boldsymbol{p} = 0$, where classical description breaks down.

Classical action and equations

In the external ordinary magnetic field $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$ the clasical action is then

$$I = \int_{t_i}^{t_f} (\boldsymbol{p} \cdot \dot{\boldsymbol{x}} + \boldsymbol{A} \cdot \dot{\boldsymbol{x}} - |\boldsymbol{p}| - \boldsymbol{a_p} \cdot \dot{\boldsymbol{p}}) dt$$

The equations of motion can be obtained by variations:

$$\dot{\boldsymbol{x}} - \hat{\boldsymbol{p}} + \boldsymbol{b} \times \dot{\boldsymbol{p}} = 0;$$
$$\dot{\boldsymbol{p}} + \boldsymbol{B} \times \dot{\boldsymbol{x}} = 0$$

No effect from $\boldsymbol{b}$ without $\boldsymbol{B}$ ($\dot{\boldsymbol{p}} = 0$).

Solving for $\dot{\boldsymbol{x}}$ and $\dot{\boldsymbol{p}}$ we can then write the desired equation:

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \boldsymbol{x}} \dot{\boldsymbol{x}} + \frac{\partial f}{\partial \boldsymbol{p}} \dot{\boldsymbol{p}} = 0.$$

NB: the invariant measure on the phase space is $\sqrt{G}\, \dfrac{d^3\boldsymbol{x} d^3\boldsymbol{p}}{(2\pi)^3}$,

where $G = (1 + \boldsymbol{b} \cdot \boldsymbol{B})^2$ is the det of the 6x6 matrix $G_{AB} = \dfrac{\partial^2 L}{\partial \xi_A \partial \dot{\xi}_B} - \dfrac{\partial^2 L}{\partial \xi_B \partial \dot{\xi}_A}$,

where $\xi_A = (\boldsymbol{x}, \boldsymbol{p})$.

Current and CME

The current density can be calculated as

$$\boldsymbol{j} = \int_{\boldsymbol{p}} \sqrt{G} f \dot{\boldsymbol{x}}$$

were $\sqrt{G}\,\dot{\boldsymbol{x}} = \hat{\boldsymbol{p}} + \boldsymbol{B}(\hat{\boldsymbol{p}} \cdot \boldsymbol{b})$ and $\int_{\boldsymbol{p}} \equiv \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}$.

Thus

$$\boldsymbol{j} = \underbrace{\int_{\boldsymbol{p}} f \hat{\boldsymbol{p}}}_{\text{regular current}} + \underbrace{\boldsymbol{B} \int_{\boldsymbol{p}} f(\hat{\boldsymbol{p}} \cdot \boldsymbol{b})}_{\text{CME}}$$

Using $\boldsymbol{b} = \frac{\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|^2}$, $E \equiv |\boldsymbol{p}|$:

$$\boldsymbol{j}_{\text{CME}} = \boldsymbol{B} \frac{1}{4\pi^2} \int_0^\infty \overline{f(E, \hat{\boldsymbol{p}})} dE$$

(cf. Loganayagam-Surowka in isotropic case) and is $= \frac{1}{4\pi^2} \mu \boldsymbol{B}$ for FD distribution.

EM anomaly

Turn on electric field:

$$\dot{\boldsymbol{x}} = \hat{\boldsymbol{p}} + \dot{\boldsymbol{p}} \times \boldsymbol{b};$$
$$\dot{\boldsymbol{p}} = \boldsymbol{E} + \dot{\boldsymbol{x}} \times \boldsymbol{B}.$$

Solve for $\dot{\boldsymbol{x}}$ and $\dot{\boldsymbol{p}}$:

$$\sqrt{G}\,\dot{\boldsymbol{x}} = \hat{\boldsymbol{p}} + \boldsymbol{E} \times \boldsymbol{b} + \boldsymbol{B}(\hat{\boldsymbol{p}} \cdot \boldsymbol{b}).$$
$$\sqrt{G}\dot{\boldsymbol{p}} = \boldsymbol{E} + \hat{\boldsymbol{p}} \times \boldsymbol{B} + \boldsymbol{b}(\boldsymbol{E} \cdot \boldsymbol{B}).$$

Integrate kinetic equation $\partial_t f + \dot{\boldsymbol{x}} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}} f + \dot{\boldsymbol{p}} \cdot \boldsymbol{\nabla}_{\boldsymbol{p}} f = 0$ over $\int_p \sqrt{G}$:

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{j} = (\boldsymbol{E} \cdot \boldsymbol{B}) \int_{\boldsymbol{p}} f \boldsymbol{\nabla}_{\boldsymbol{p}} \boldsymbol{b} = \frac{1}{4\pi^2} \boldsymbol{E} \cdot \boldsymbol{B}\, f_{\boldsymbol{0}}.$$

where $\boldsymbol{\nabla}_{\boldsymbol{p}} \boldsymbol{b} = 2\pi\delta^3(\boldsymbol{p})$ and $f_{\boldsymbol{0}} \equiv f|_{\boldsymbol{p}=0}$.

$$\boldsymbol{j} = \int_{\boldsymbol{p}} \sqrt{G} f \dot{\boldsymbol{x}} = \underbrace{\underbrace{\int_{\boldsymbol{p}} f \hat{\boldsymbol{p}}}_{\text{regular current}} + \underbrace{\boldsymbol{E} \times \int_{\boldsymbol{p}} f \boldsymbol{b}}_{\text{anom. Hall current}}}_{\text{=0 in equilibrium}} + \underbrace{\boldsymbol{B} \int_{\boldsymbol{p}} f(\hat{\boldsymbol{p}} \cdot \boldsymbol{b})}_{\text{CME}}$$

Strictly speaking ...

... we cannot treat region near $\boldsymbol{p} = 0$ classically.

- Define a phase space current density with 6+1 components: $(\rho, \rho\dot{\boldsymbol{x}}, \rho\dot{\boldsymbol{p}})$, where $\rho = \sqrt{G} f$. It obeys continuity equaton:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho\dot{\boldsymbol{x}})}{\partial \boldsymbol{x}} + \frac{\partial(\rho\dot{\boldsymbol{p}})}{\partial \boldsymbol{p}} = 0.$$

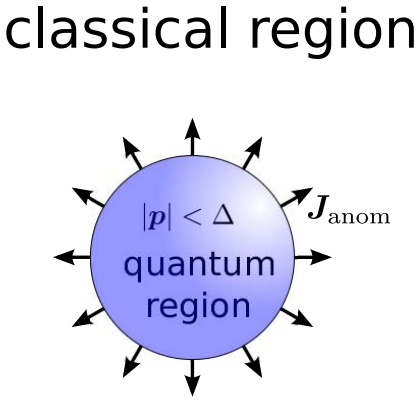
- Exclude non-classical $|\boldsymbol{p}| < \Delta$, where $\Delta \sim \sqrt{B}$, define $(n_\Delta, \boldsymbol{j}_\Delta) = \int_{|\boldsymbol{p}|>\Delta} \rho(1, \dot{\boldsymbol{x}})$. Then up to $\mathcal{O}(\Delta/|\boldsymbol{p}|)^2$

$$\frac{\partial n_\Delta}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{j}_\Delta = \int d\boldsymbol{S}_\Delta \cdot \frac{\rho\dot{\boldsymbol{p}}}{(2\pi)^3}.$$

- Anomaly is matched by the net flow into the classical region of phase space

$$\boldsymbol{J}_{\text{anom}} = \frac{\rho\dot{\boldsymbol{p}}}{(2\pi)^3} = \frac{\boldsymbol{E}\cdot\boldsymbol{B}}{4\pi^2} f \frac{\hat{\boldsymbol{p}}}{4\pi|\boldsymbol{p}|^2}.$$

through the cutoff surface S_Δ.



CVE

- Replace Lorentz force with Coriolis force:

$$\dot{\boldsymbol{p}} = 2|\boldsymbol{p}|\boldsymbol{\omega} \times \dot{\boldsymbol{x}}$$

I.e., $\boxed{\boldsymbol{B} \to 2|\boldsymbol{p}|\boldsymbol{\omega}}$.

- The non-equilibrium CVE is then

$$\boldsymbol{j}_{\text{CVE}} = \boldsymbol{\omega} \int_{\boldsymbol{p}} 2|\boldsymbol{p}| f(\hat{\boldsymbol{p}} \cdot \boldsymbol{b})$$

which is

$$\boldsymbol{j}_{\text{CVE}} = \boldsymbol{\omega} \frac{1}{4\pi^2} \int_0^\infty \overline{f(E, \hat{\boldsymbol{p}})}\, 2E dE$$

(cf. Loganayagam-Surowka in isotropic case) and is $= \dfrac{1}{4\pi^2}\mu^2\boldsymbol{\omega}$ for FD distribution.

NB: $\boldsymbol{B}$ is <u>not</u> "$\sim$" $\mu\boldsymbol{\omega}$.

Conclusions and discussion

- Kinetic description of CME, CVE and anomaly:

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \boldsymbol{x}}\dot{\boldsymbol{x}} + \frac{\partial f}{\partial \boldsymbol{p}}\dot{\boldsymbol{p}} = C[f]$$

with

$$\dot{\boldsymbol{x}} = \hat{\boldsymbol{p}} + \dot{\boldsymbol{p}} \times \boldsymbol{b};$$
$$\dot{\boldsymbol{p}} = \boldsymbol{E} + \dot{\boldsymbol{x}} \times \boldsymbol{B}$$

and the Berry monopole

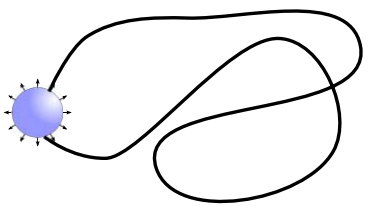
$$\boldsymbol{b} = \frac{\boldsymbol{p}}{2|\boldsymbol{p}|^3}.$$

- Out-of-equilibrium CME, CVE:

$$\boldsymbol{j}_{\text{CME}} = \boldsymbol{B}\int_{\boldsymbol{p}} f(\hat{\boldsymbol{p}}\cdot\boldsymbol{b}); \quad \boldsymbol{j}_{\text{CVE}} = 2\boldsymbol{\omega}\int_{\boldsymbol{p}} f(\hat{\boldsymbol{p}}\cdot\boldsymbol{b})|\boldsymbol{p}|.$$

- The description is limited by $|\boldsymbol{p}| > \Delta \gg \sqrt{B}$.

- The anomaly "works" inside $|\boldsymbol{p}| < \Delta$. In the classical region, without collisions, particles cannot be created or destroyed. They can enter or exit through the boundary at $|\boldsymbol{p}| = \Delta$. The net flux is $\frac{1}{4\pi^2}\boldsymbol{E}\cdot\boldsymbol{B}\, f_0$



Chiral Symmetry and the Thermodynamics of the Dyonic Vacuum

Tin Sulejmanpašić[†]

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Universität Regensburg[†]

Stony Brook University[*]

June 27, 2012

Introduction

History of Calorons with non-trivial holonomy

- 1998 Kraan, van Baal and separately Lee and Lu - Discovery of calorons with non-trivial holonomy
- 2004 D. Diakonov, N. Gromov, V. Petrov, S. Slizovskiy - Quantum weight of dyons and of instantons with nontrivial holonomy
- 2007 D. Diakonov and V. Petrov argue that the so called "confining holonomy" is preferred
- 2009 V. G. Bornyakov, E.-M. Ilgenfritz, B. V. Martemyanov, M. Muller-Preussker - Quenched lattice studies of the effect of dyons
- 2011 F. Bruckmann, S. Dinter, E. M. Ilgenfritz, B. Maier, M. Muller-Preussker and M. Wag- ner, arXiv:1111.3158 [hep-ph] - Random dyon gas is confining

What are (SU(2)) instanton Dyons?

- Dyons are generalizations of instantons at finite T (caloron)
- In adition to instanton parameters (size, position, and color orientation), they posses so-called holonomy

$$\mathcal{P} e^{\oint_0^\beta A_4 dt} = e^{iv\beta \frac{\tau^3}{2}}$$

- Once $v \neq 0$ the instanton can split into two stationary dyons
- One says that it is trivial if $v\beta = 0$ and nontrivial if $v\beta = \pi$ (sometimes referred to as confining and non-confining)
- Hence one can view nontrivial holonomy as an appearance of a "A_4 condensate"

What are (SU(2)) instanton Dyons?

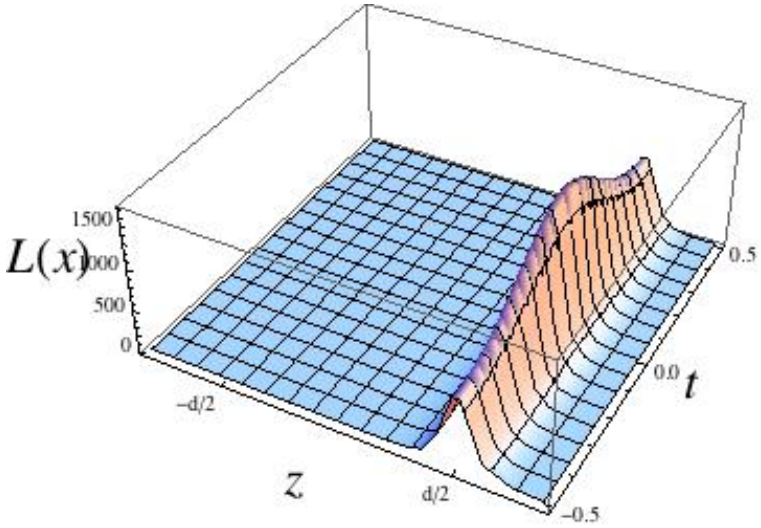
$$F^2 \propto [A_4^a, A_i^b]^2 \rightarrow v^2 A_i^2 \epsilon^{abc} \tau_c$$

Gluons which have different color direction then A_4 condensate obtain an effective mass

Monopole Solution!

However since we can always change $v \rightarrow v - 2\pi T$ by an anti-periodic gauge transformation (which keeps gauge fields periodic), there are actually two monopole solutions! It turns out that the second (twister) monopole has its action proportional to $\bar{v} = 2\pi T - v$.

What are (SU(2)) Instanton Dyons?



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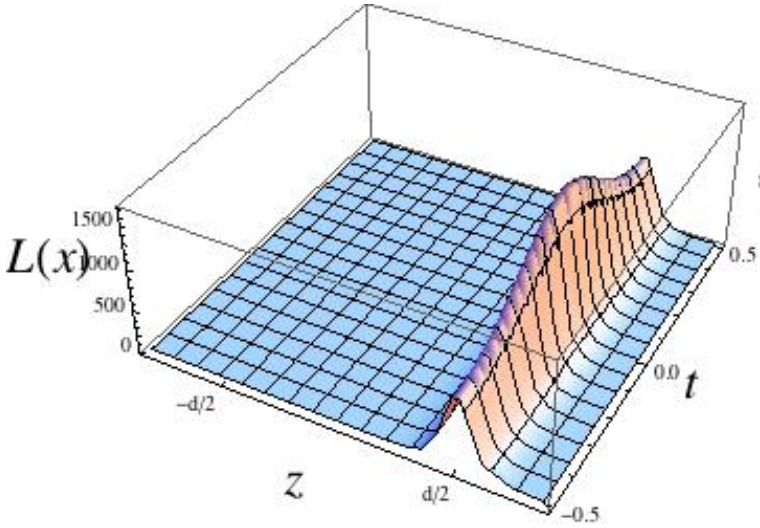

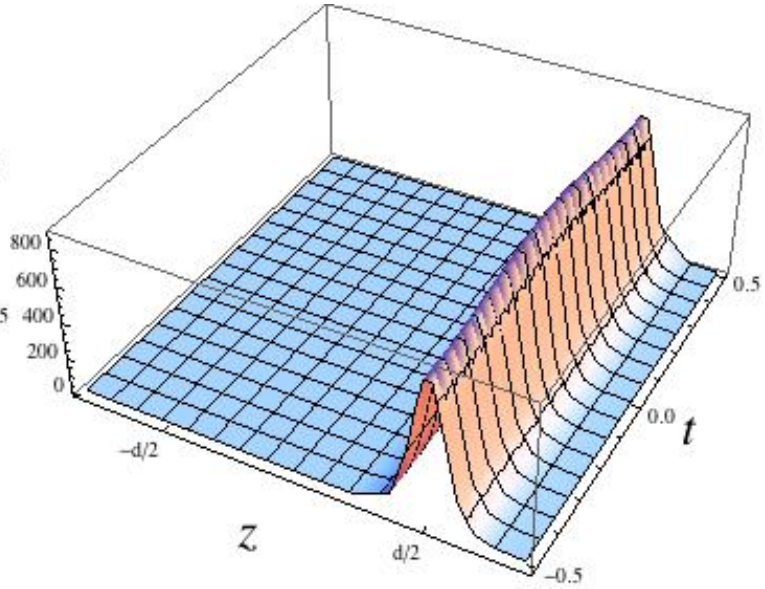

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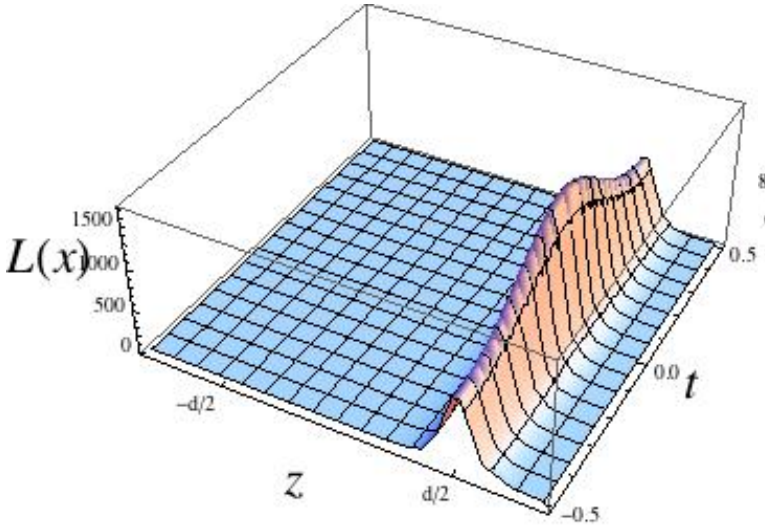

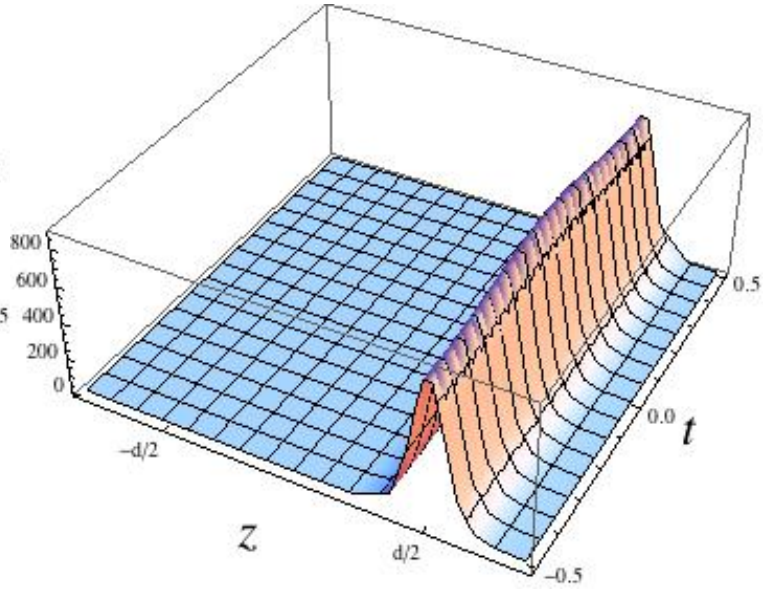

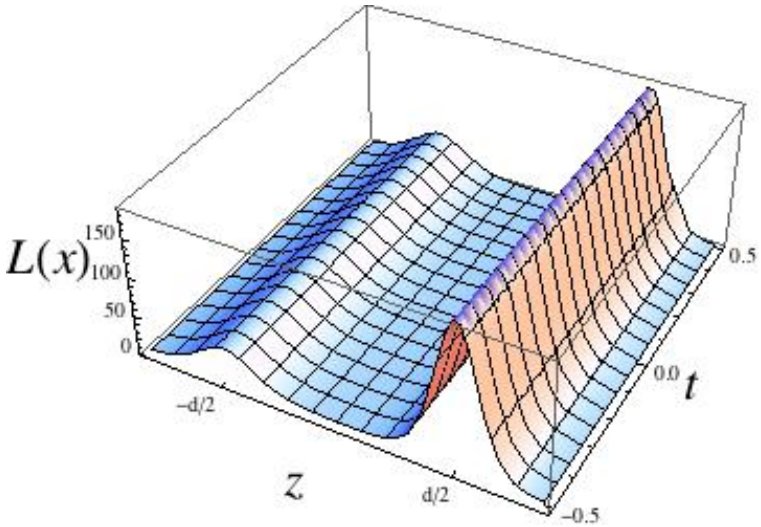

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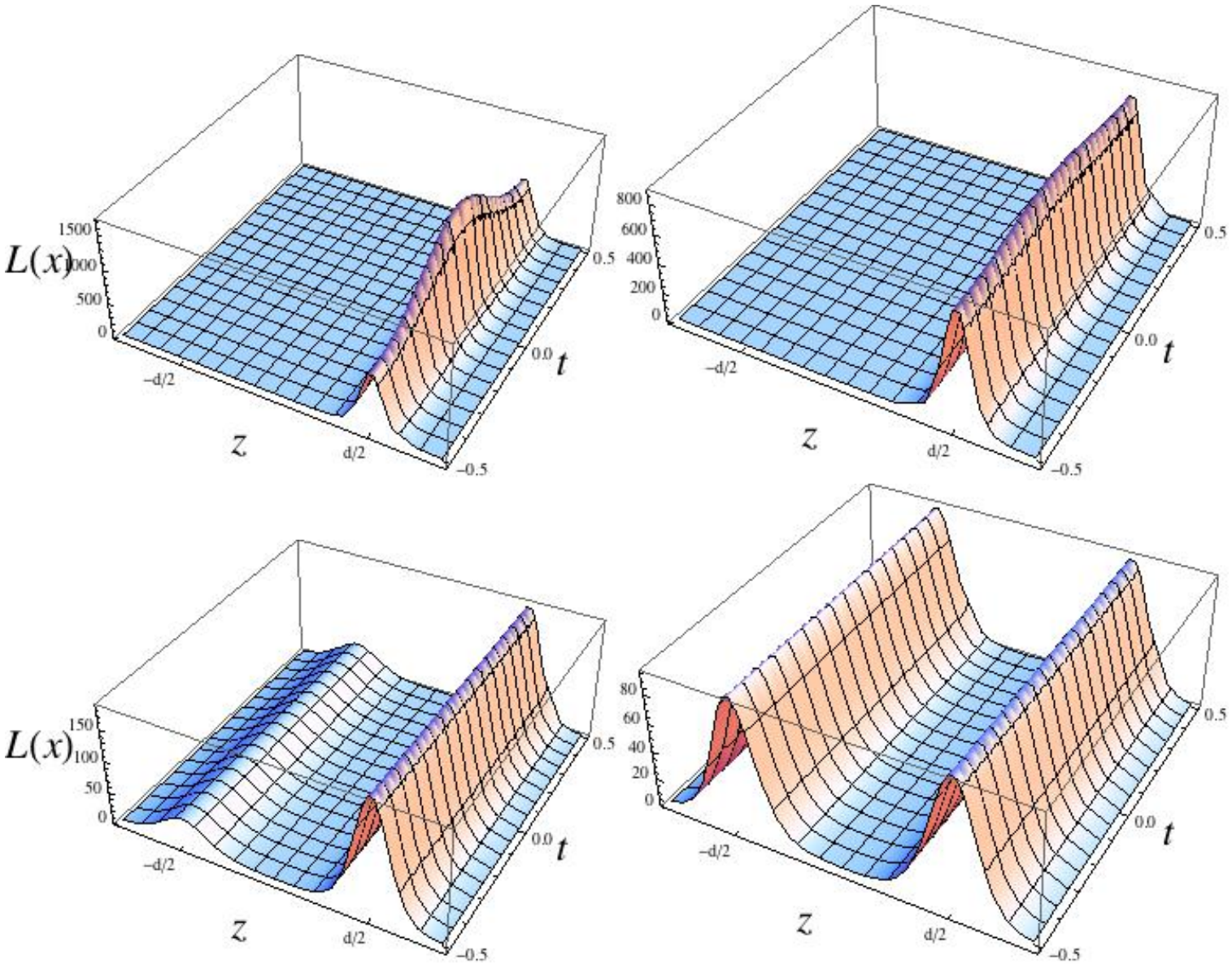
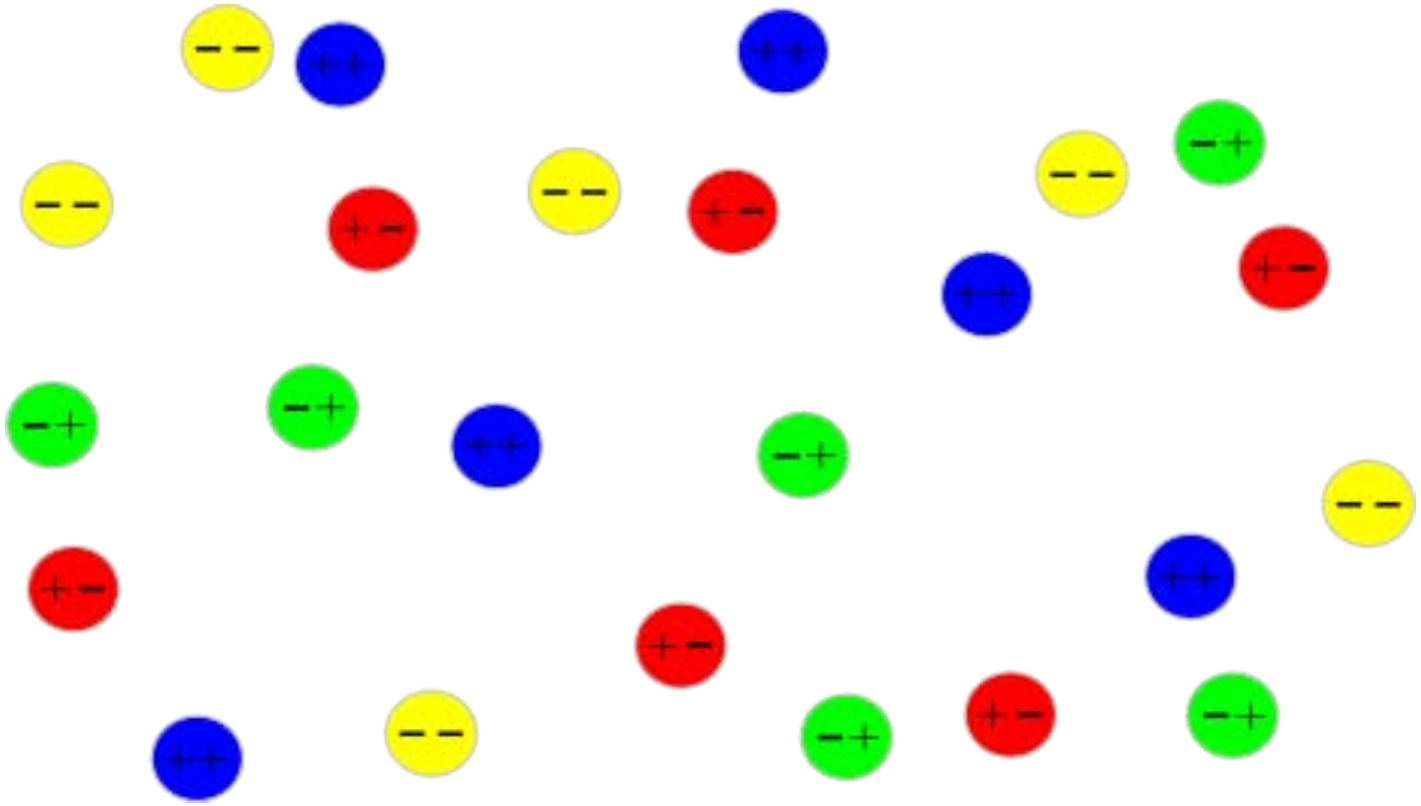


Dyons in Vacuum

The Dyons in Vacuum



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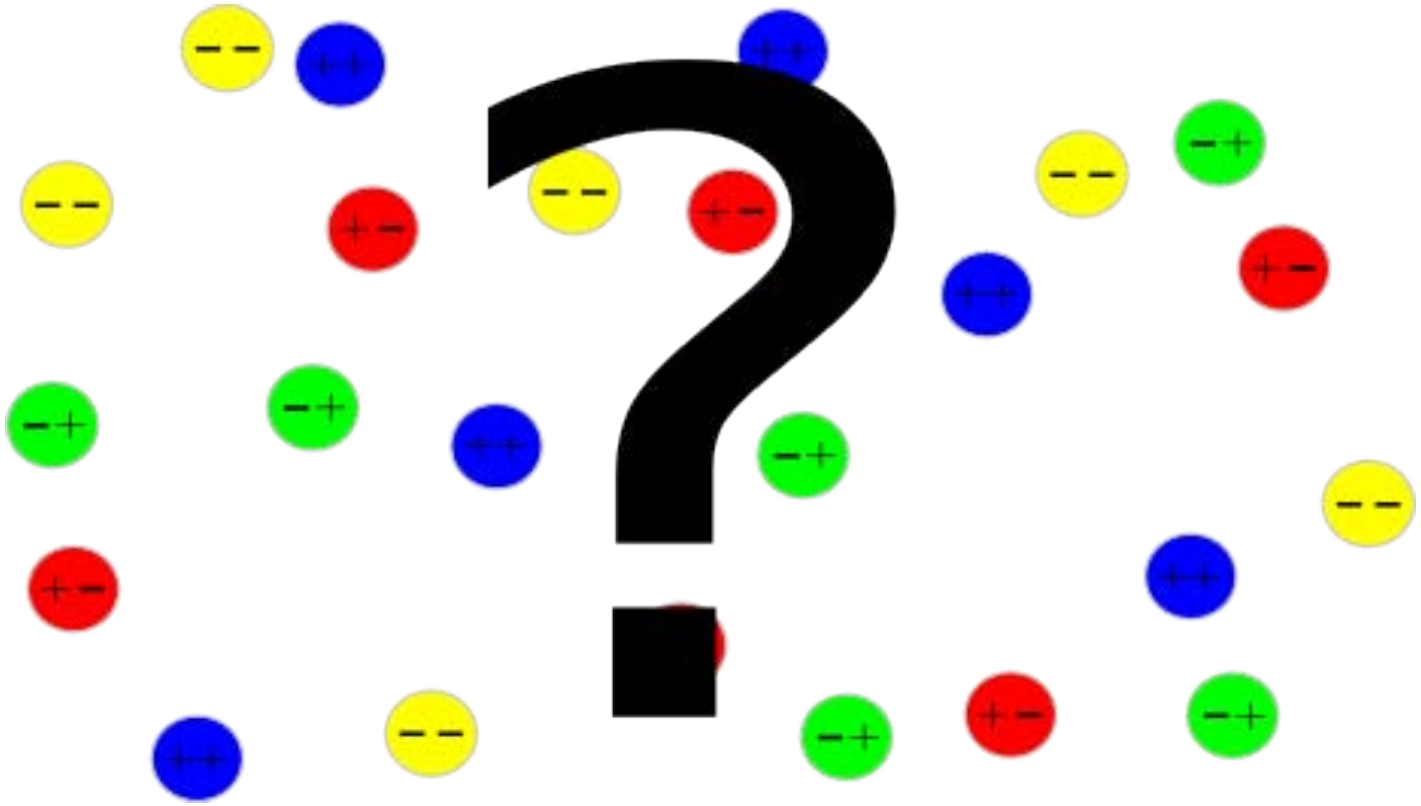

Thermodynamics of Dyons

- We study the thermodynamics of instantons with nonzero holonomy, i.e. thermodynamics of dyons

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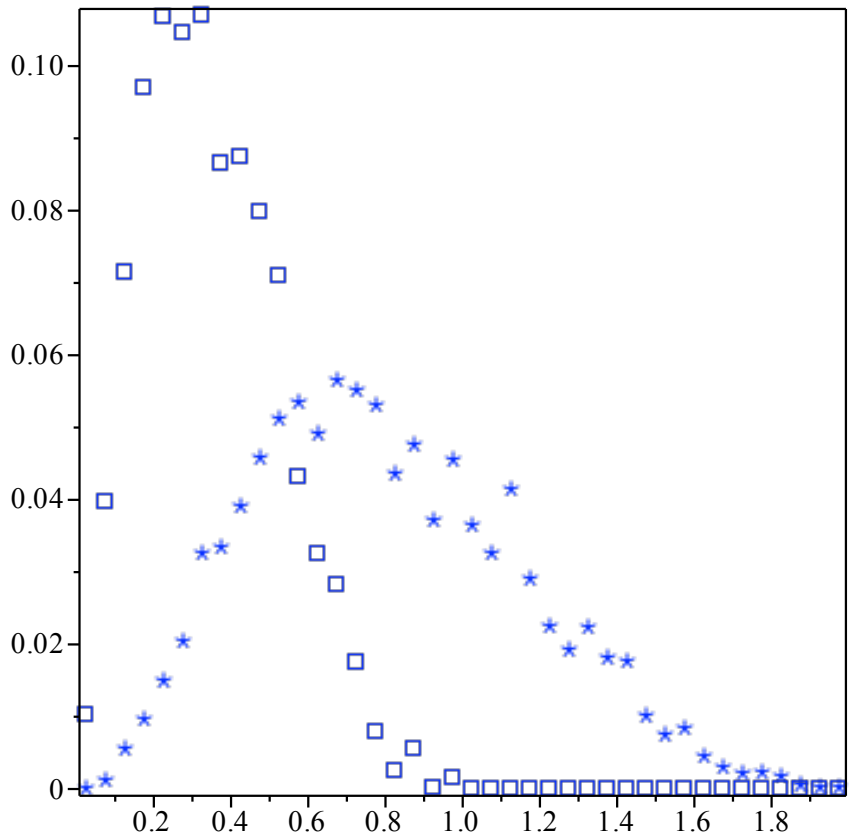
- Essential to this task is to understand the interactions between these objects
  - Classical
  - One loop gluon - perturbative
  - Fermion
  - Moduli space interaction

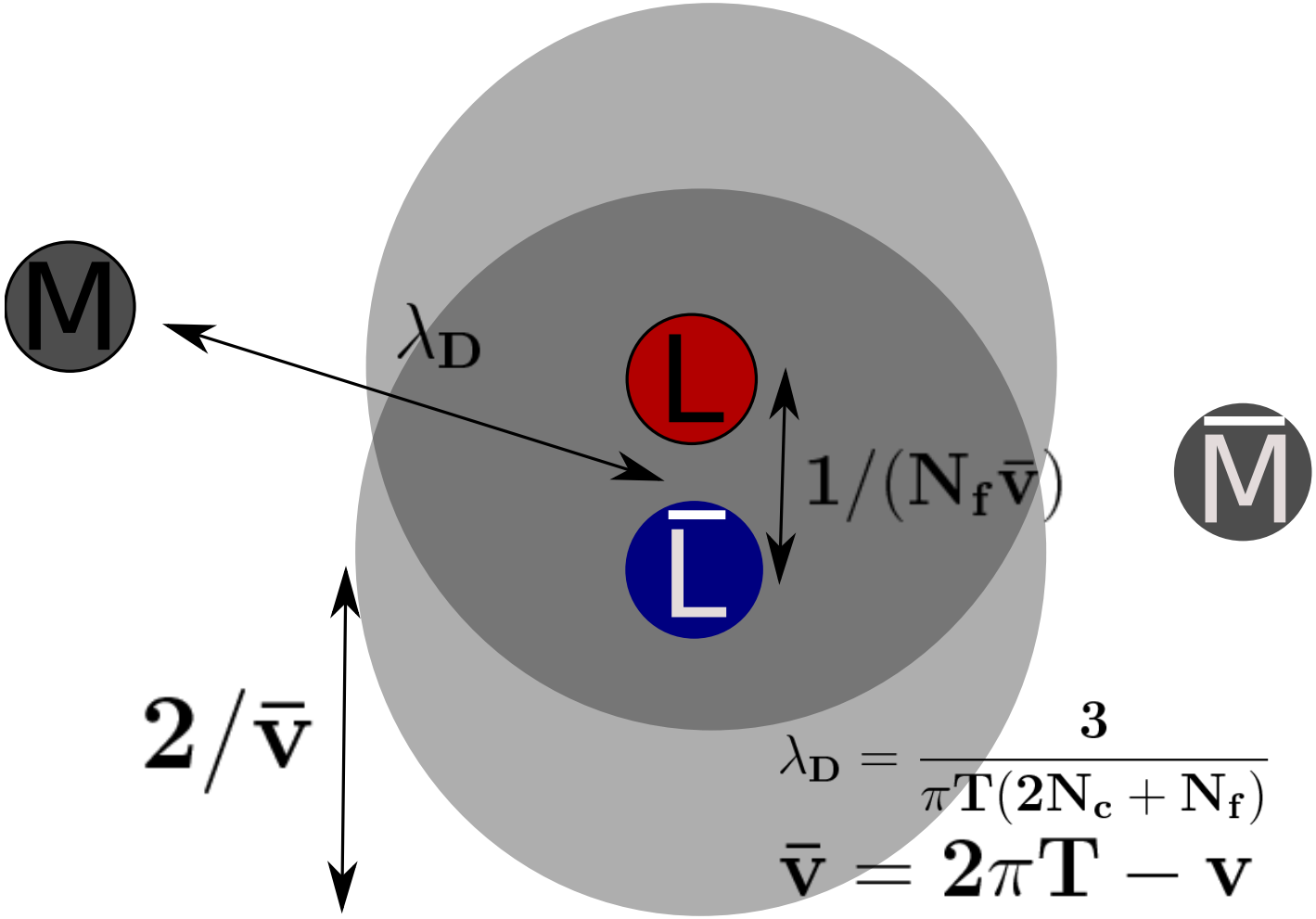
The partition function for a single neutral molecule

Partition Function for a Dyon Molecule

$$dZ_{mol} = dZ_{LM} dZ_{\bar{L}\bar{M}} \left[\frac{m_f^2 + |T(r_{L\bar{L}})|^2}{\Lambda^2} \right]^{N_f} C(N_f) \left(\frac{\pi^2 r_{LM} r_{\bar{L}\bar{M}} \Lambda^4}{T^2} \right)^{N_f/6} e^{-V_{scr} - V_{L\bar{L}}}$$



Dyon Molecule



$$\lambda_D = \frac{3}{\pi T(2N_c + N_f)}$$

$$\bar{v} = 2\pi T - v$$

Three Molecular Models

Three Molecular Models

1. Random Dyon Gas Model
2. Random Molecular Model
3. Reweighed Molecular Model

The Random Dyon Gas Model

Random Dyon Gas Model

- Take N dyons and antidyons which carry zeromodes (i.e. $L, \bar{L}$ dyons for anti-periodic fermions)
- Randomly select their positions in a box of size L so that density $N/V = N/L^3 = 1$
- Taking that $\not{D} \approx T_{I\bar{J}}$, where $T_{I\bar{J}}$ is the Dirac operator in the basis of quasi-zeromodes, i.e.

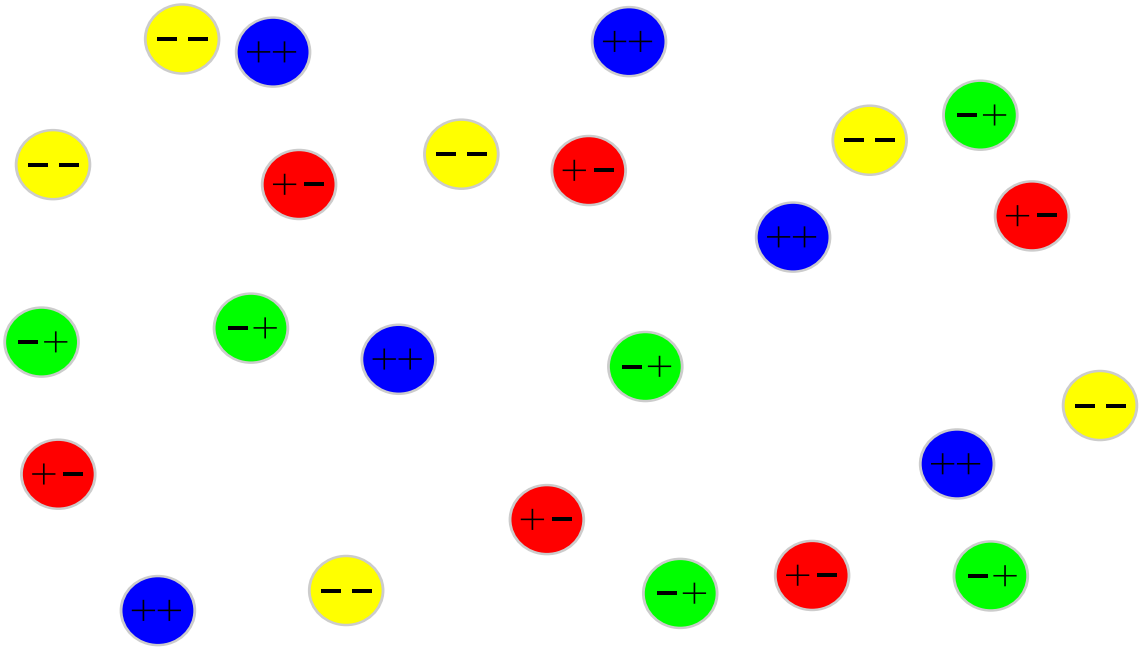
$$T_{I\bar{J}} = \int d^3x \psi_I^\dagger \not{D} \psi_{\bar{J}} \approx \int d^3x \psi_I^\dagger \not{\partial} \psi_{\bar{J}}$$

- Zeromodes $\psi_{I,\bar{I}} \propto \frac{e^{-\frac{\bar{v}r}{2}}}{\sqrt{r}}$ at large r therefore we take that

$$T_{I\bar{J}} = c \frac{e^{-Mr}}{\sqrt{1 + rM}}$$

where M is connected with holonomy (i.e. $M = v/2$ or $M = \bar{v}/2$).

The Random Molecular Gas Model



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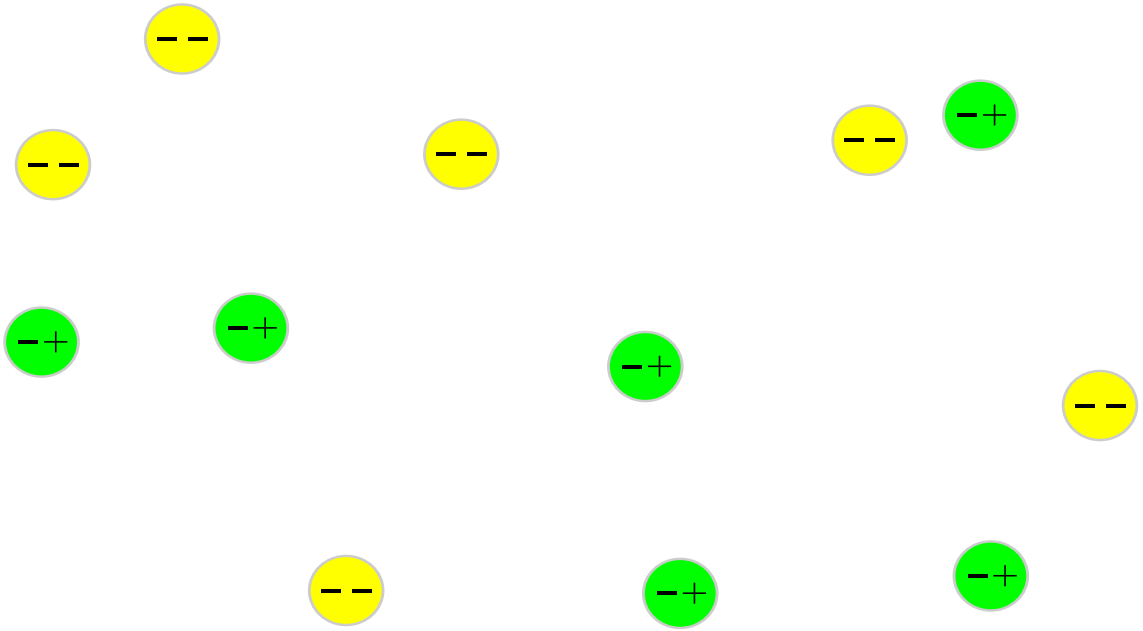

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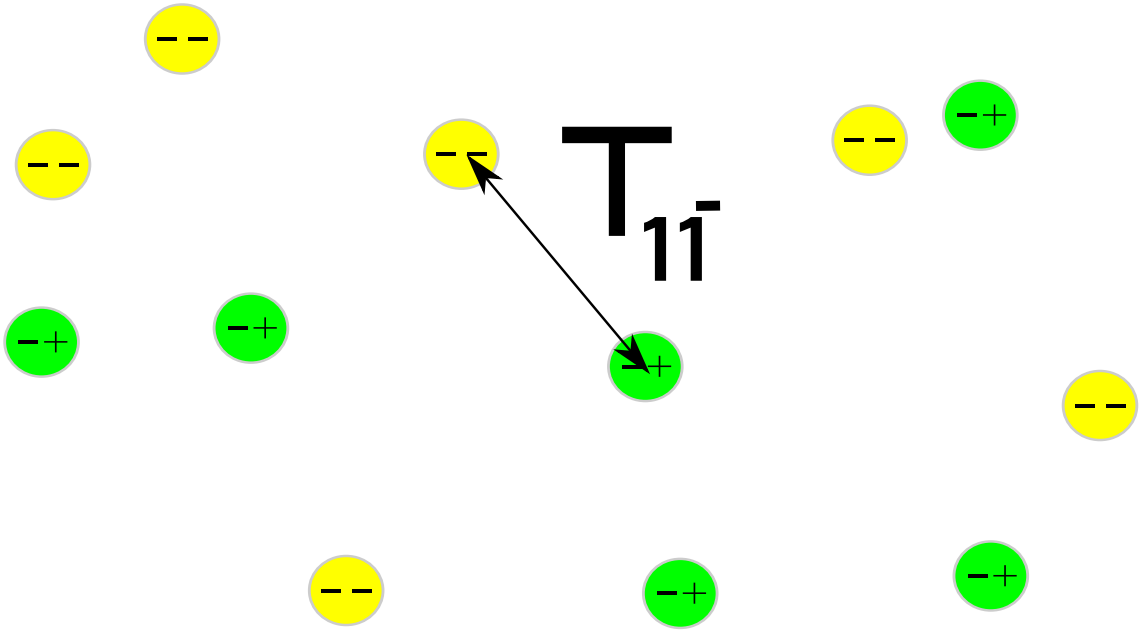

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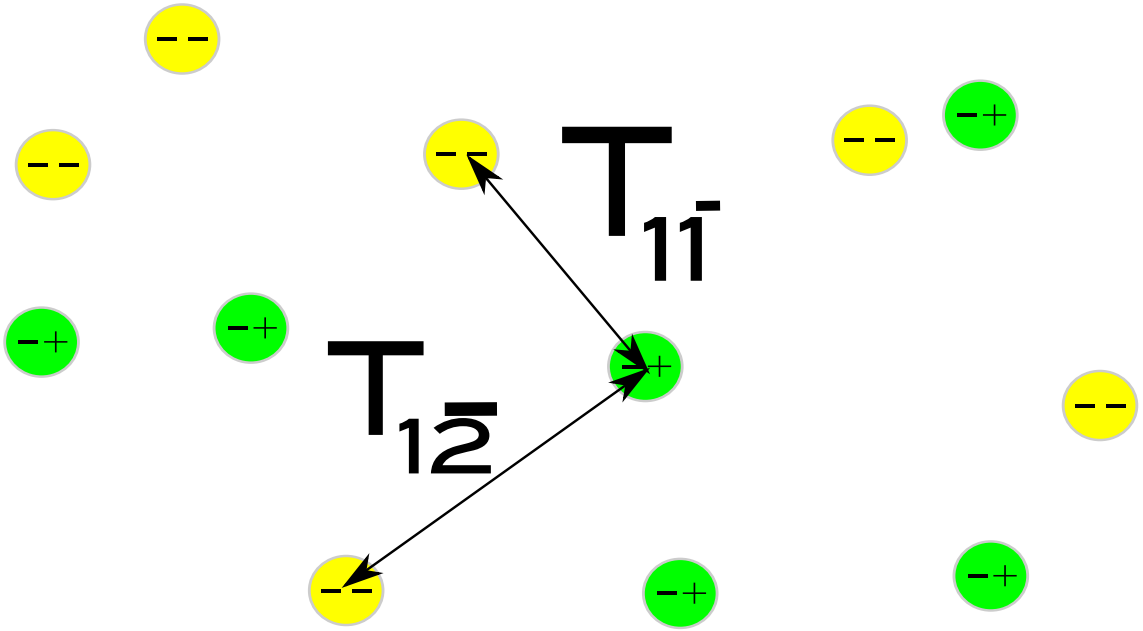
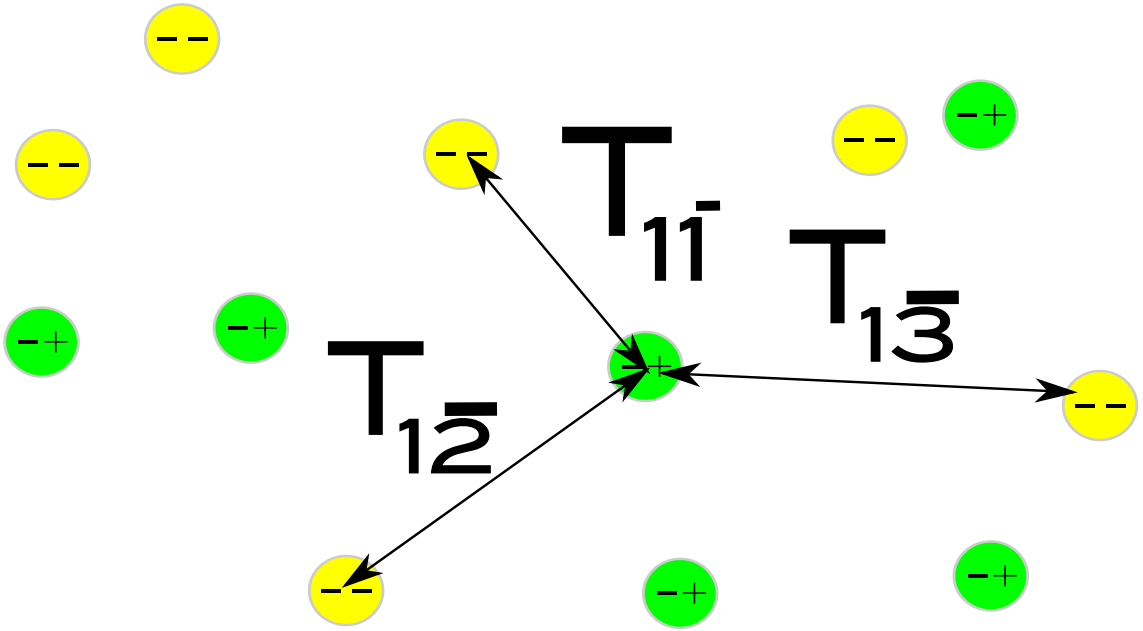
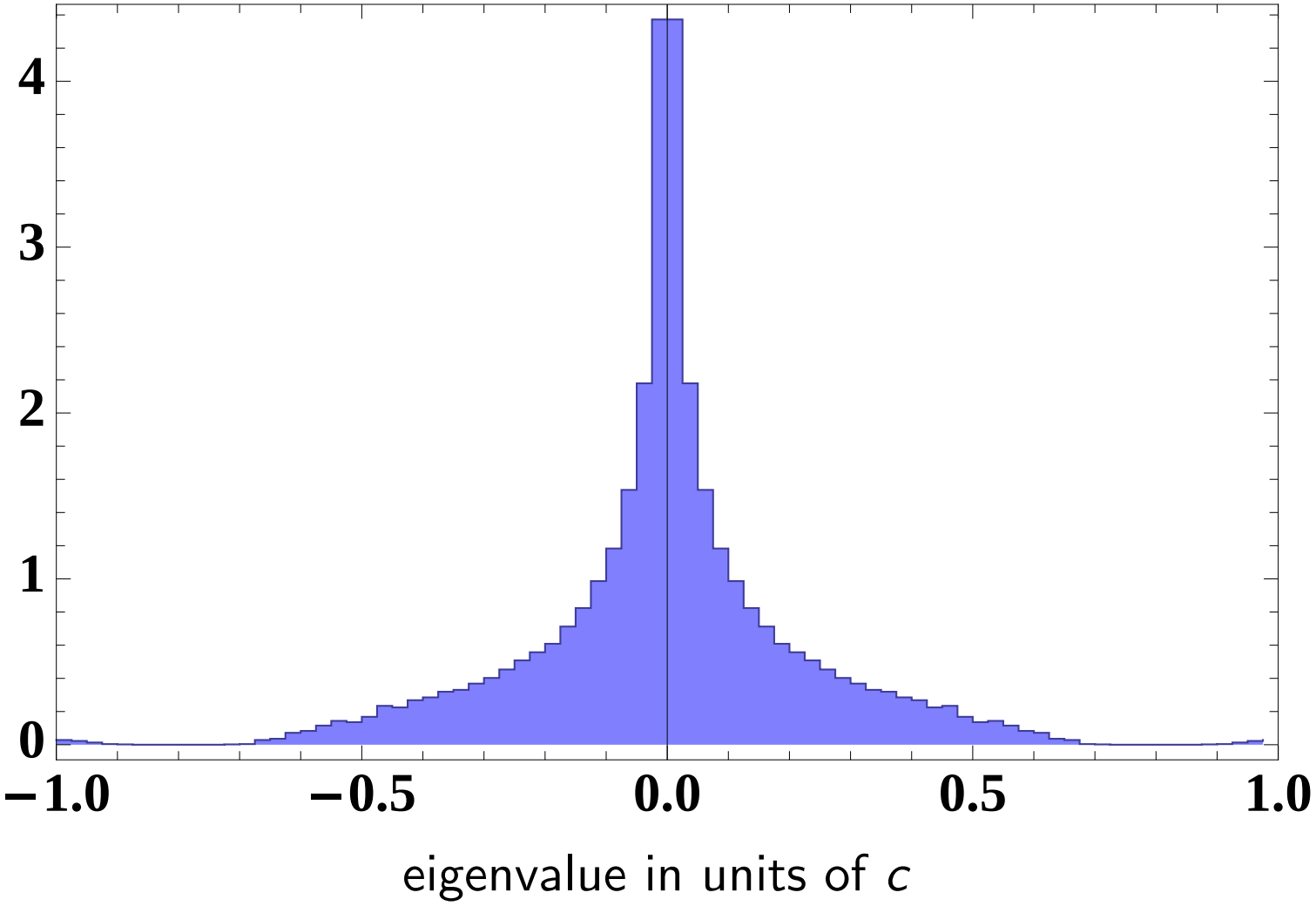


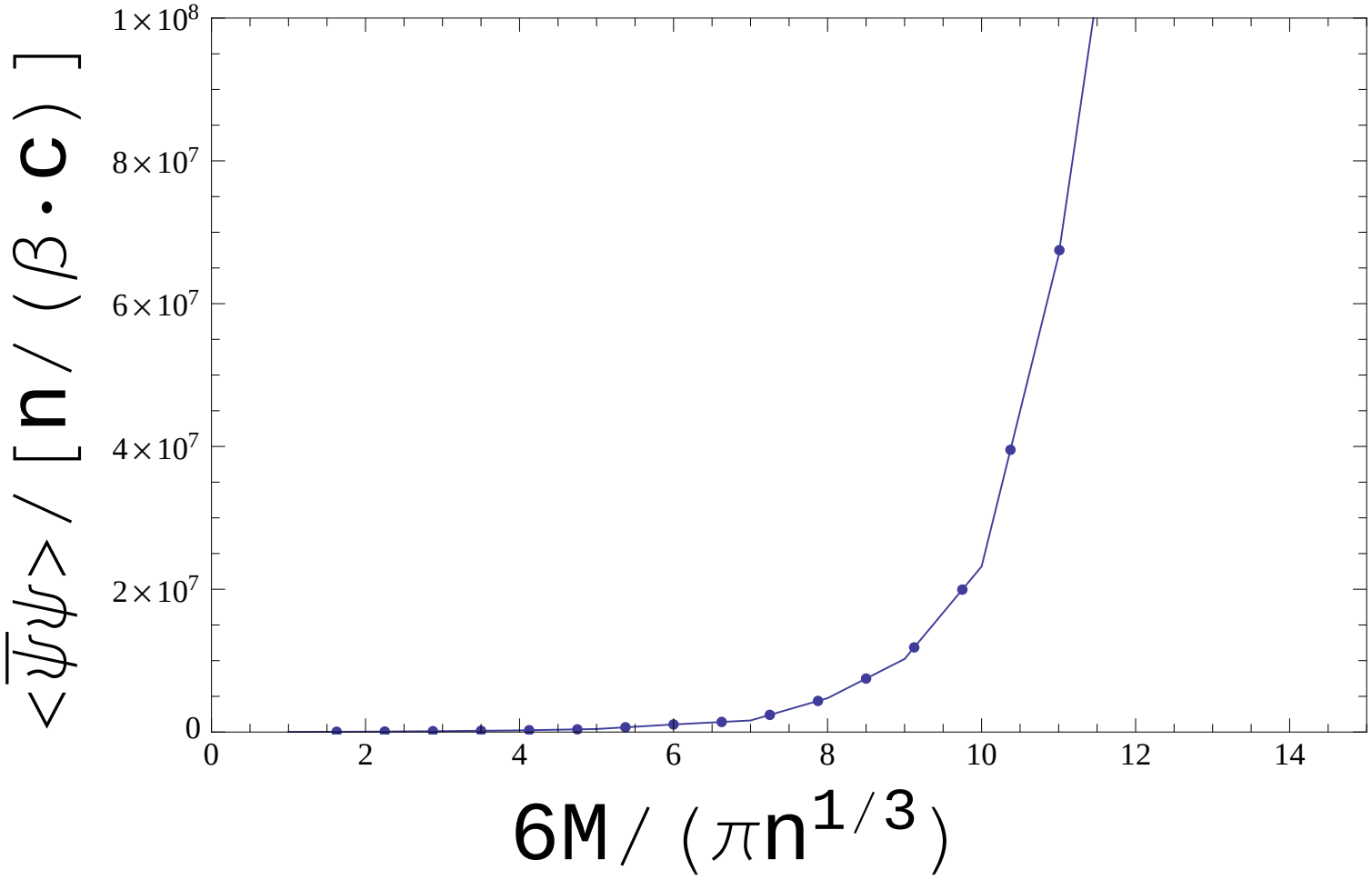
The Random Molecular Gas Model



The Spectrum



Chiral Condensate



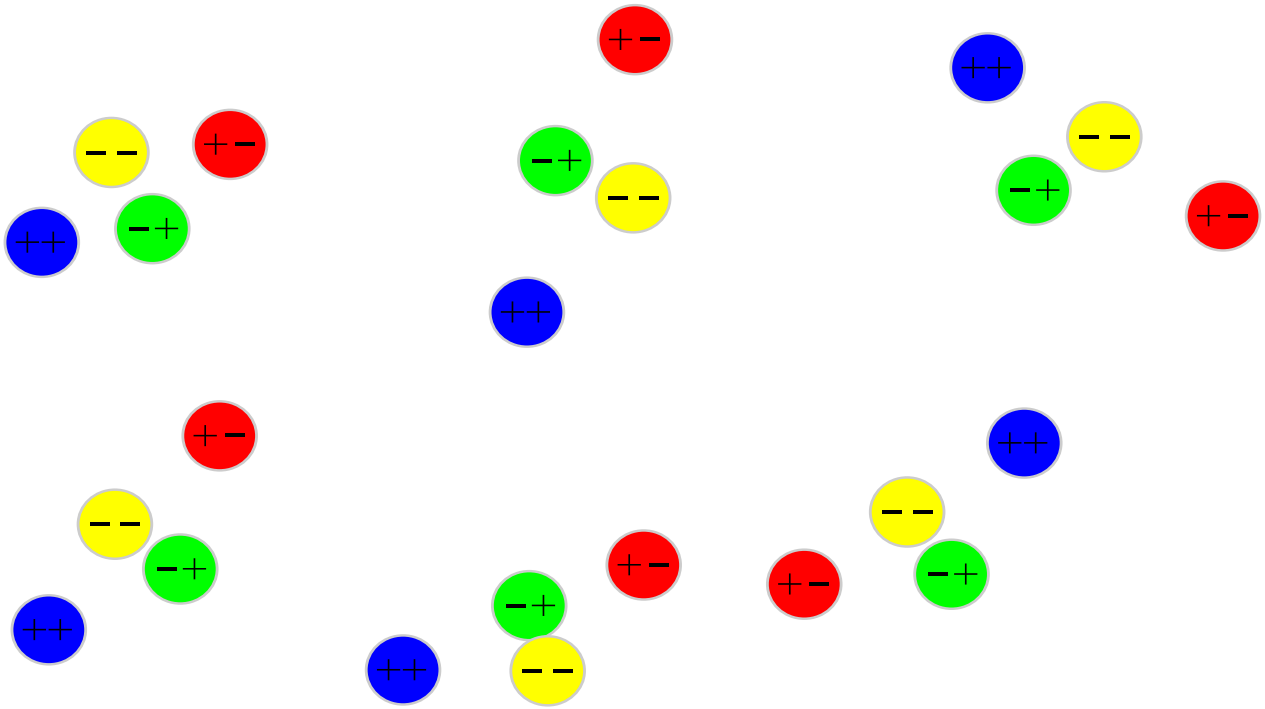
The Random Molecular Gas Model

The Random Molecular Gas Model

- Take N *pairs* of dyons and antidyons which carry zeromodes (i.e. $L, \bar{L}$ dyons for anti-periodic fermions)
- Randomly select the position of these pairs positions in a box of size L so that density $N/V = N/L^3 = 1$
- The distribution of the relative distance between the members of the pair is given by

$$\text{distribution} = r^2 \left(\frac{e^{-Mr}}{\sqrt{1+Mr}} \right)^{2N_f}$$

The Random Molecular Gas Model



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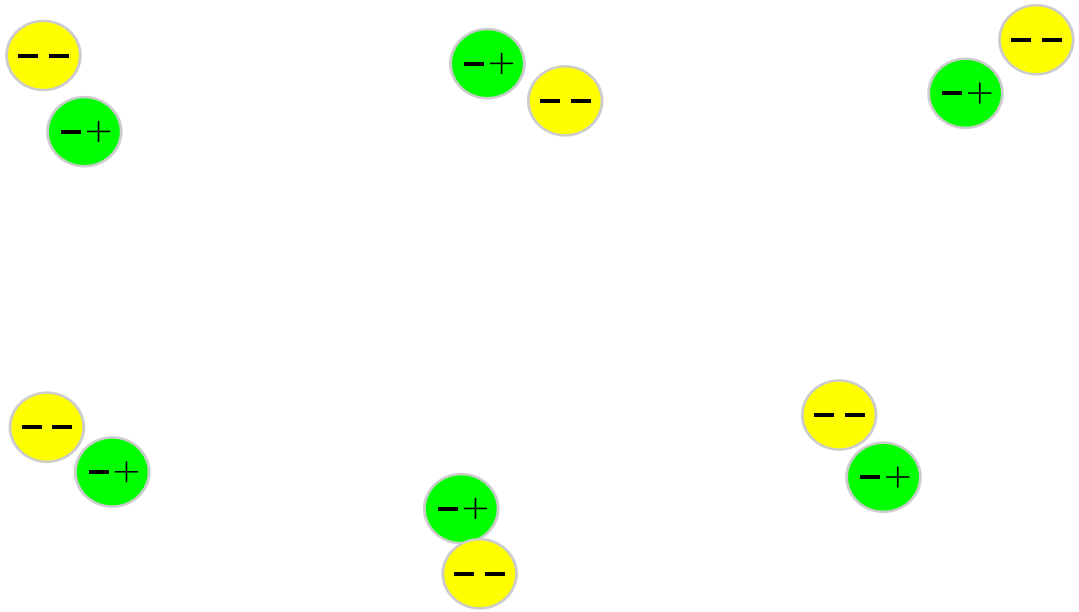

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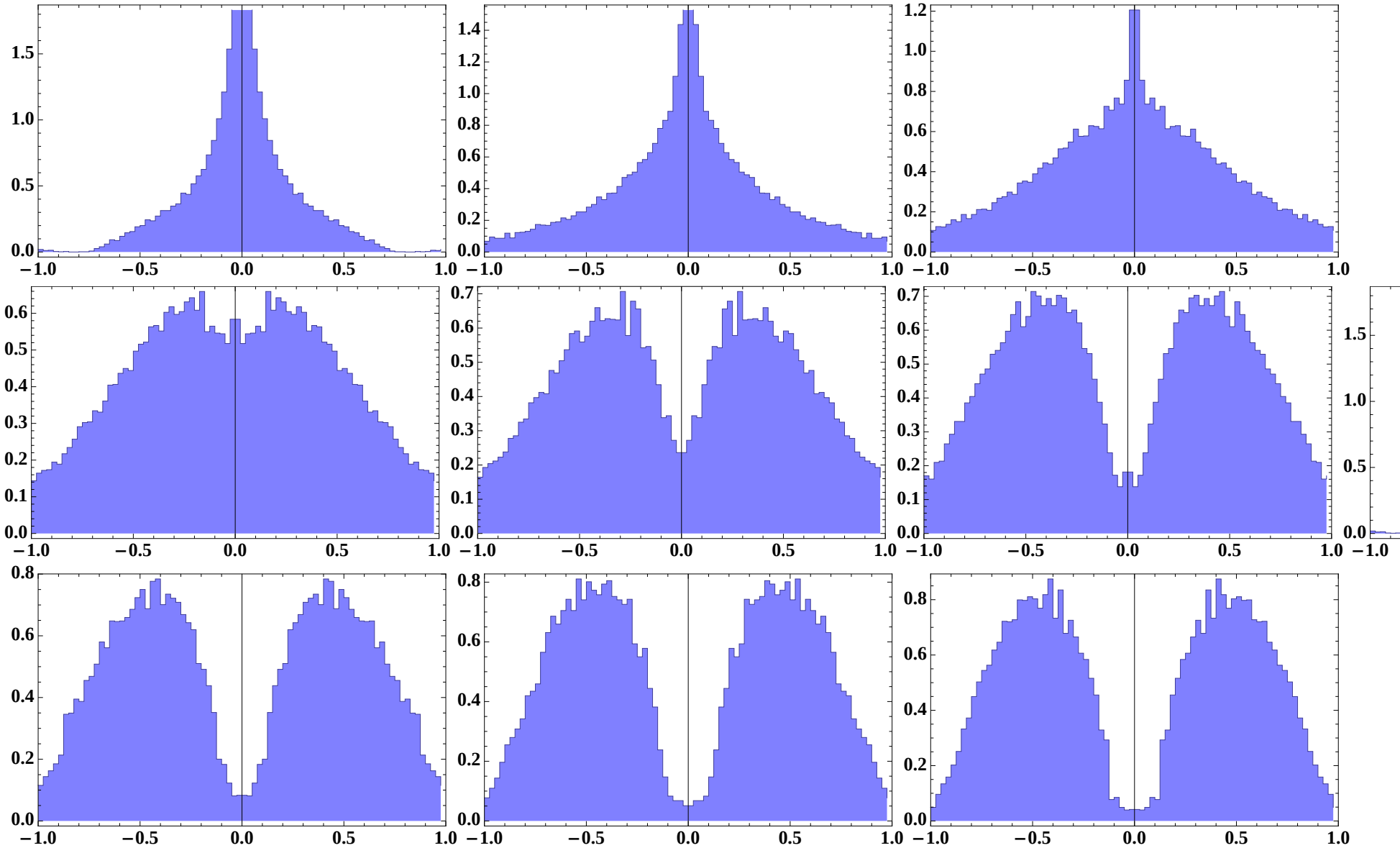

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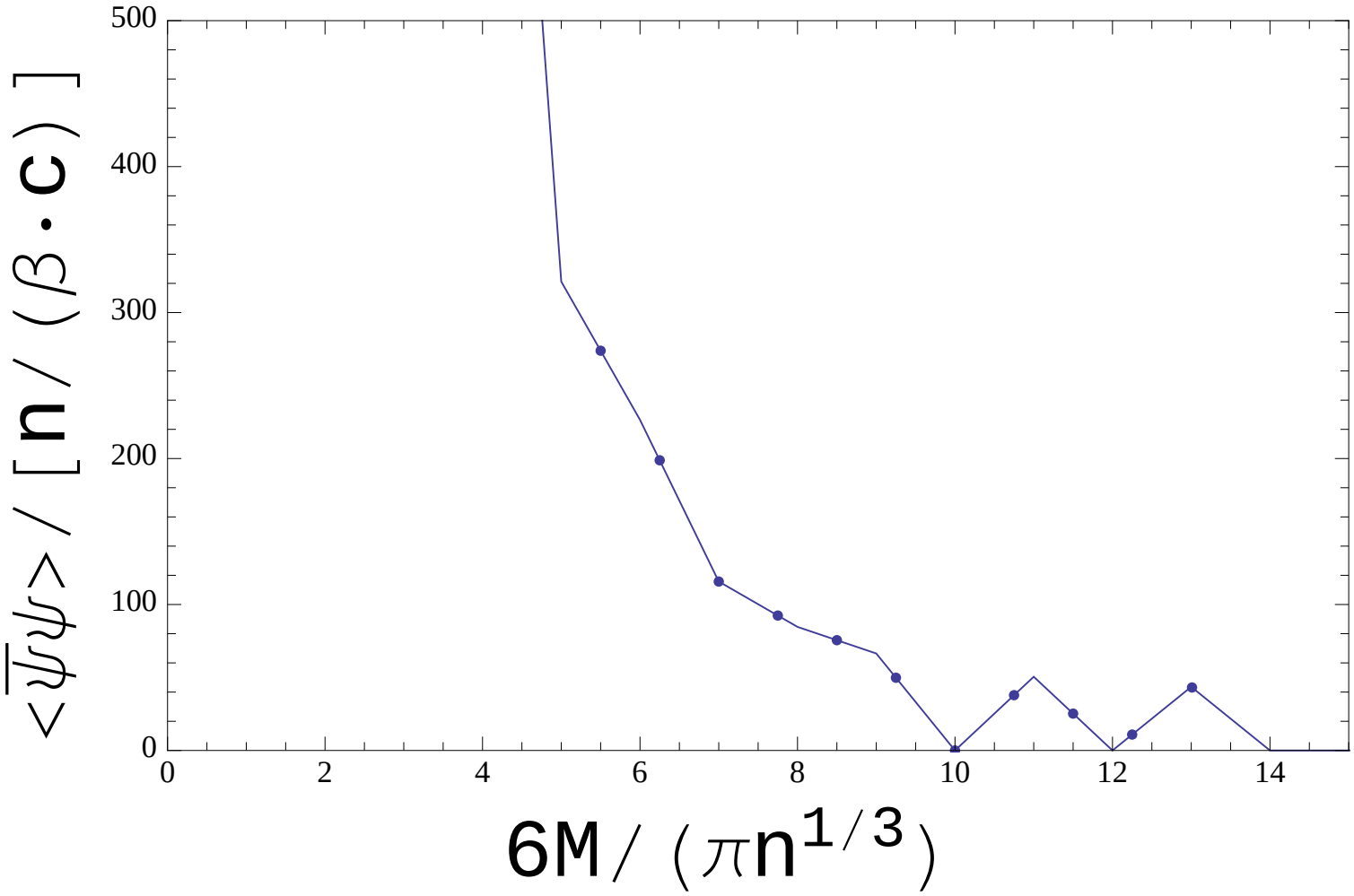
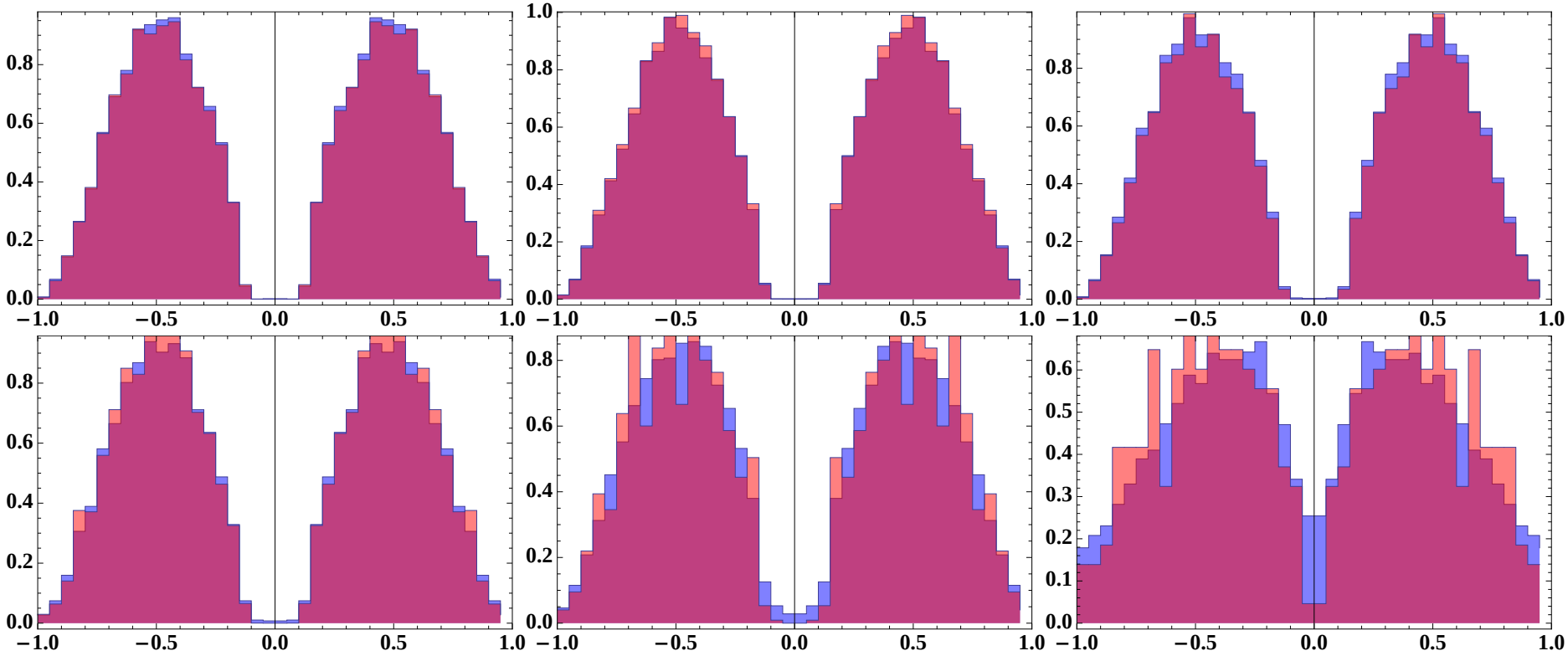
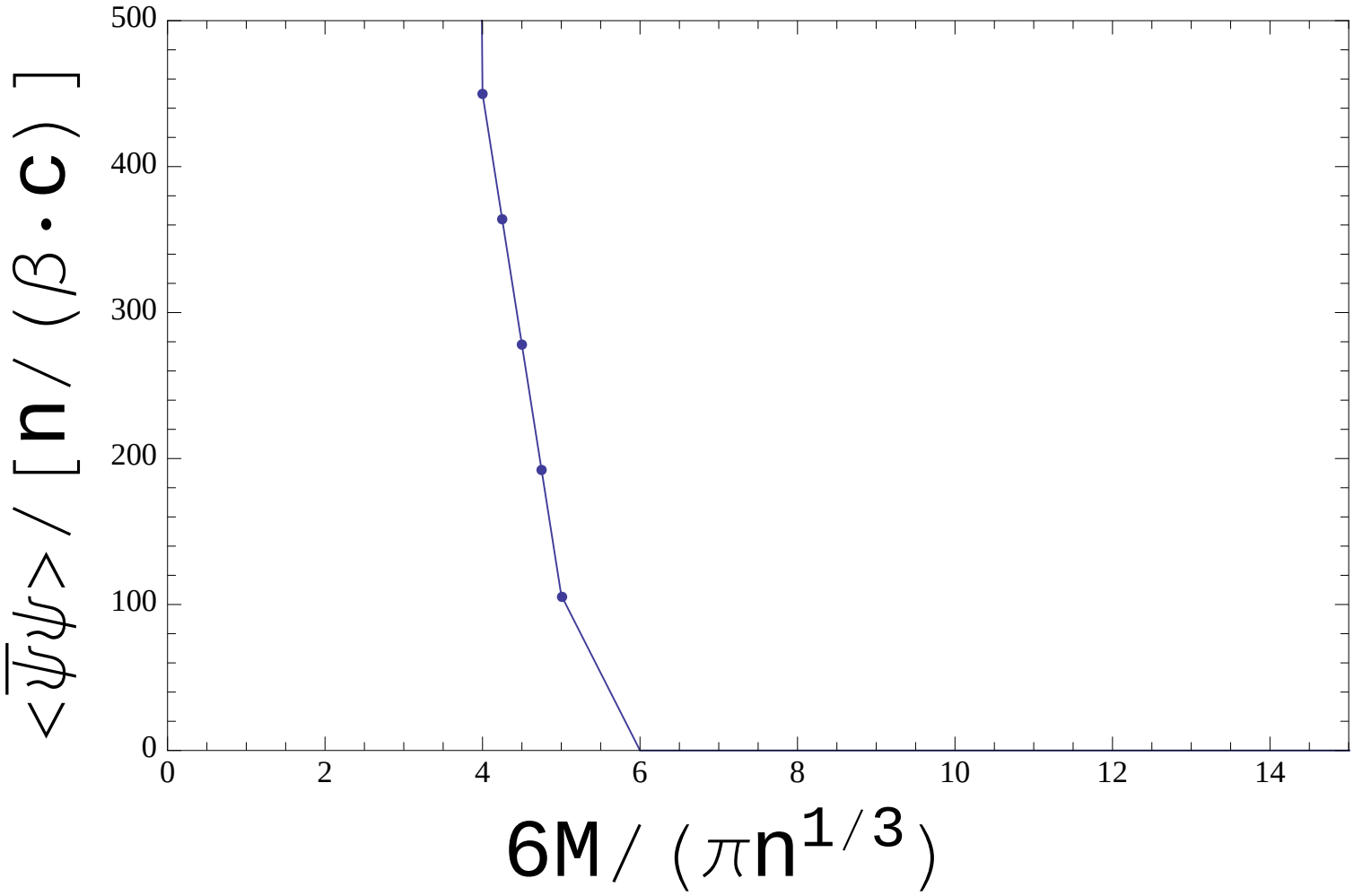


Reweighted Random Molecular Gas Model

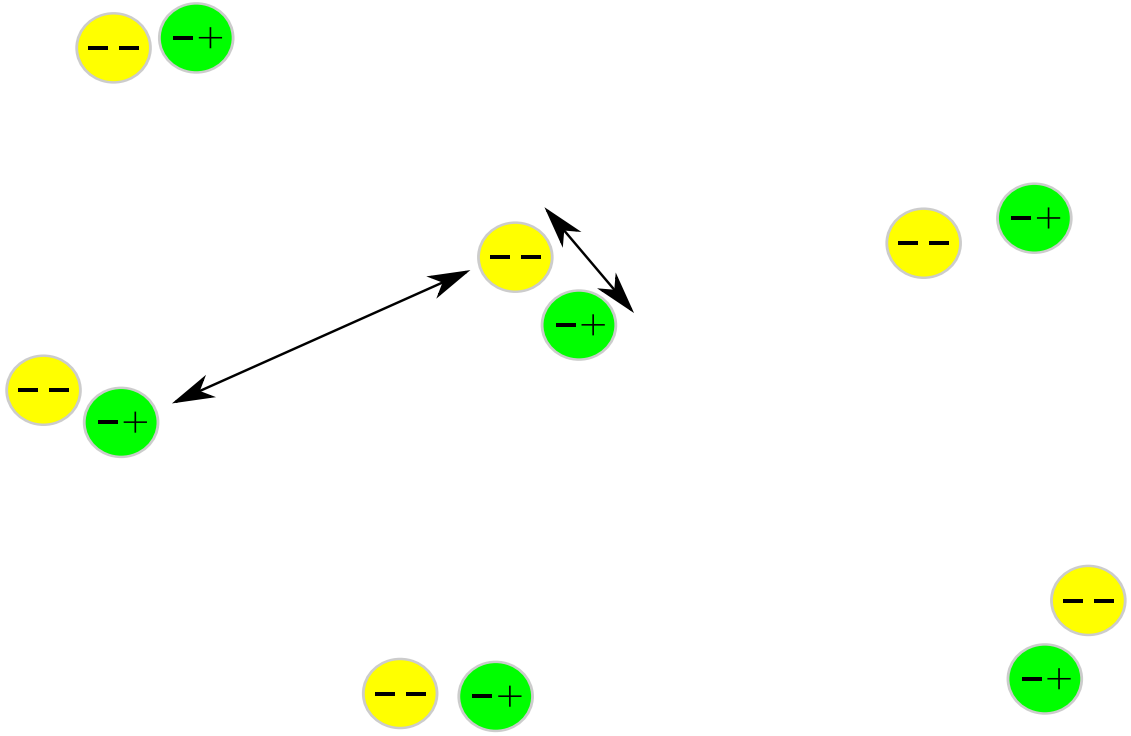
The Spectrum



Chiral Condensate



What Exactly Happens?



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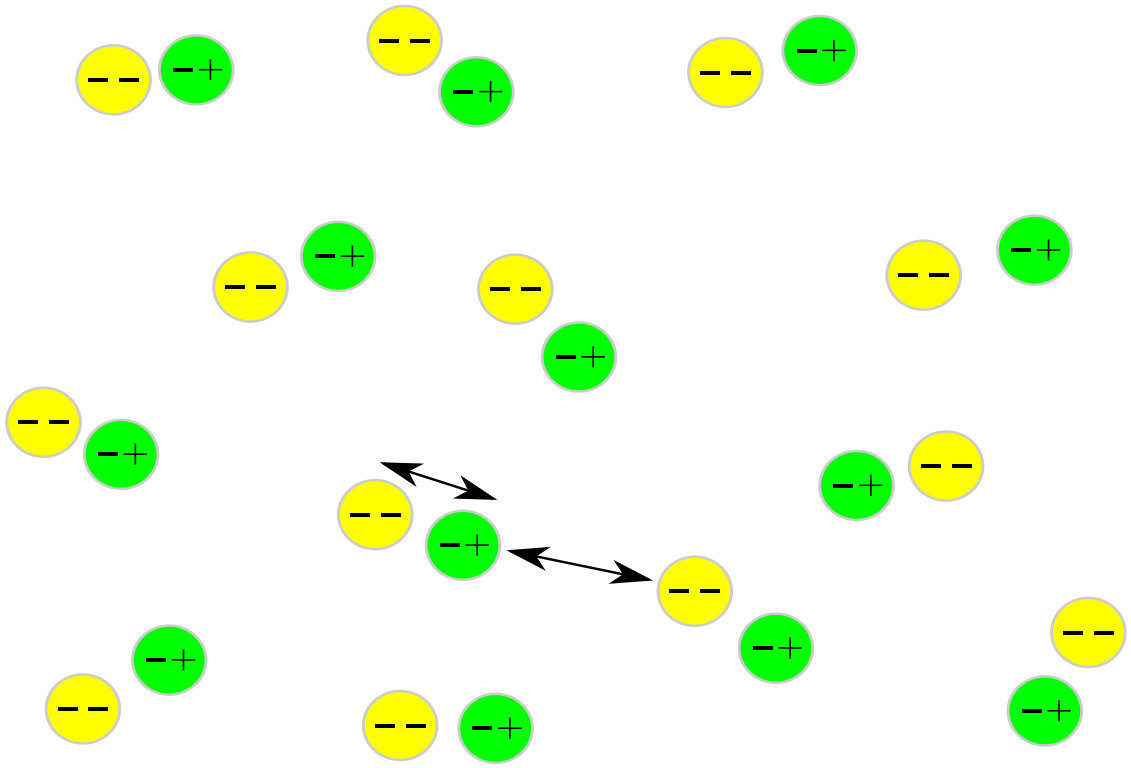


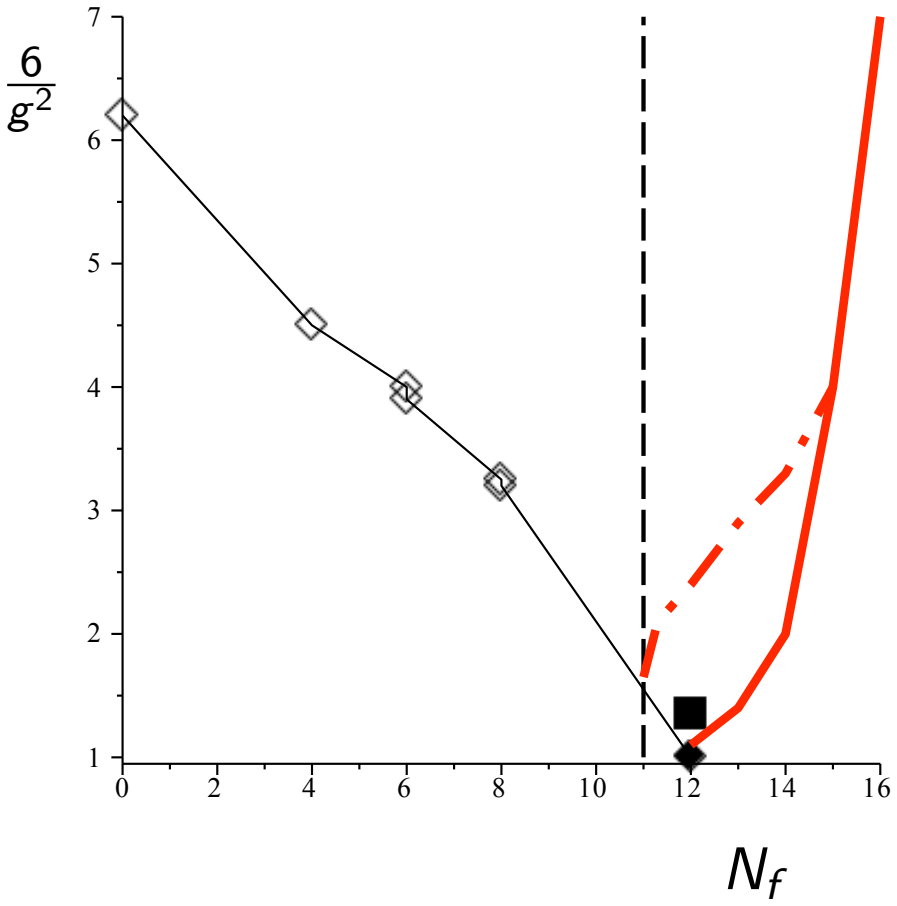
Prediction!

$$nN_f^3 = \text{const}$$

$$C_1 e^{-\frac{8\pi^2}{g_1^2}} {N_f^1}^3 - C_2 e^{-\frac{8\pi^2}{g_2^2}} {N_f^2}^3 = 0$$

$$\frac{8\pi^2}{g_2^2} = \frac{8\pi^2}{g_1^2} - 3\log\frac{N_f^2}{N_f^1} - \log\frac{C_2}{C_1}$$

Lattice results

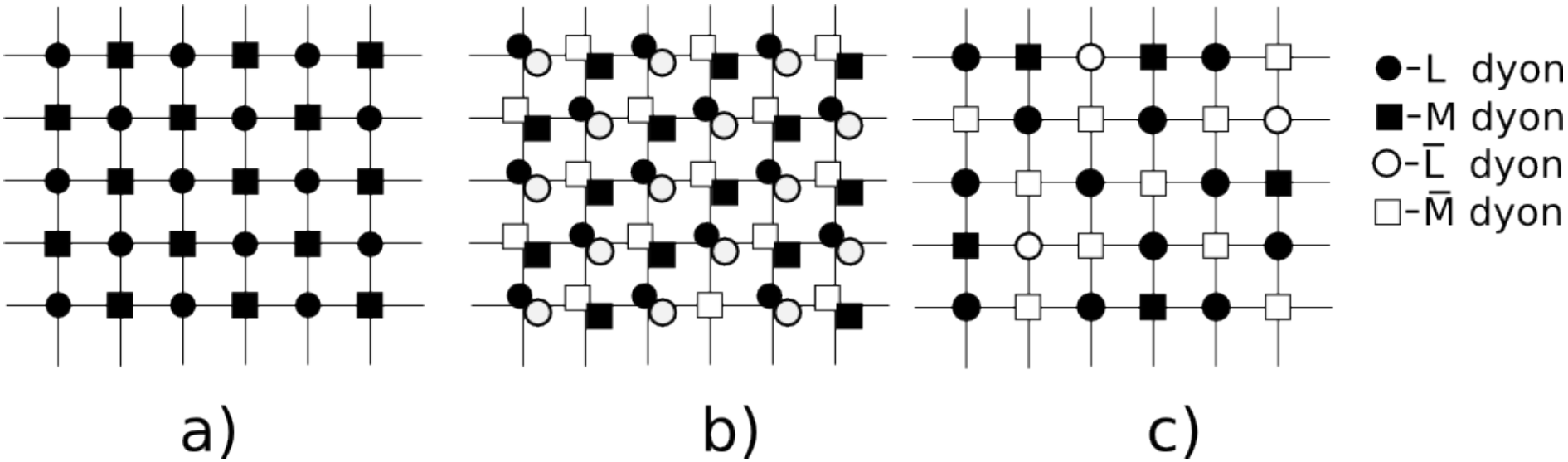


data from K. Miura, M. P. Lombardo and E. Pallante, arXiv:1110.3152

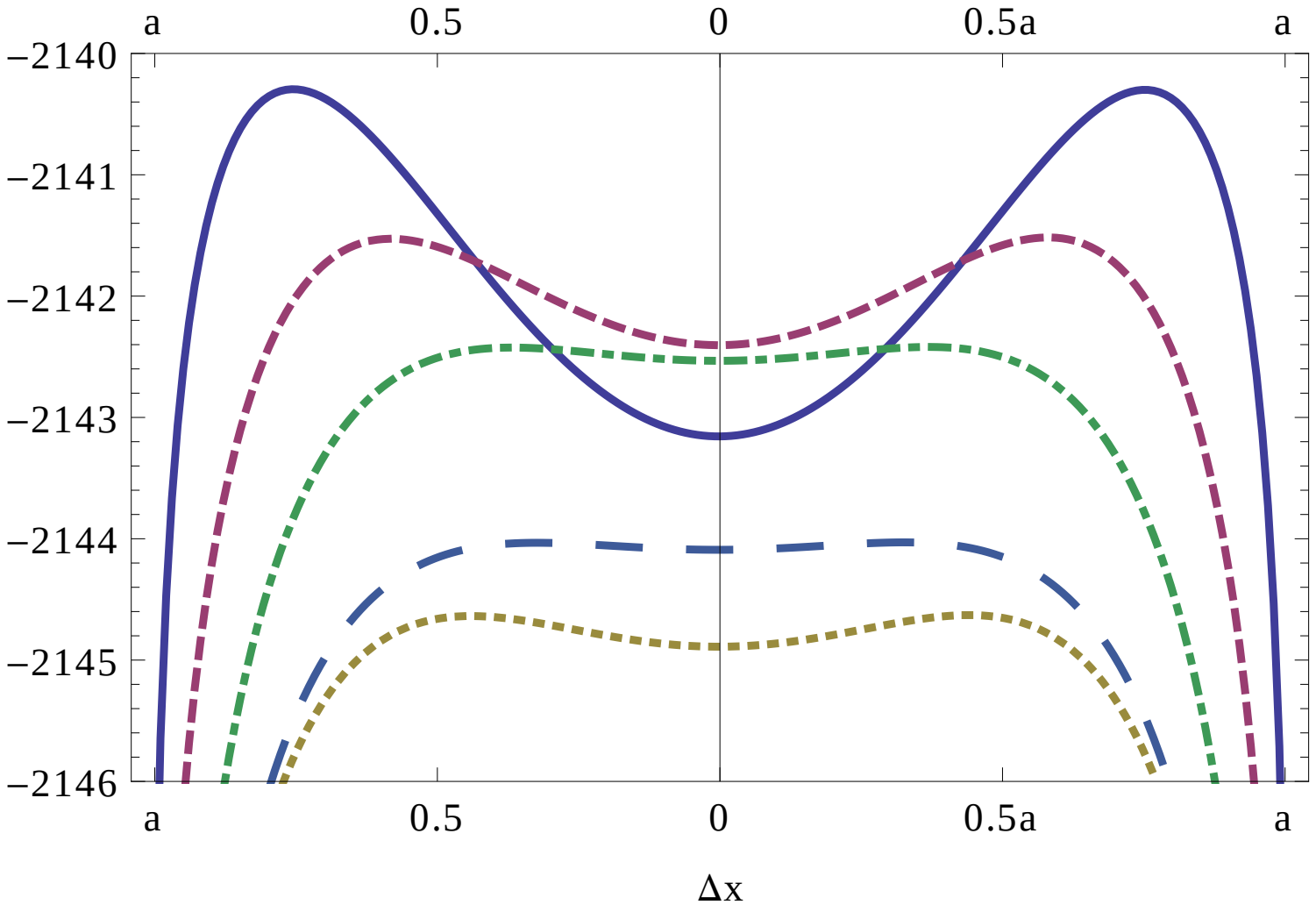
Diakonov Moduli Space Metric

- Diakonov, Petrov proposed a guess for the self-dual metric for dyons
- It consists of the determinant of attractive Coulombic terms between $L - M$ dyons and repulsive terms between $L - L$ and $M - M$ dyons
- This suggests that dyons may "condense" into a crystal under certain circumstances

Diakonov Moduli Space Metric



Stability of the Crystal



Summary of Predictions

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- Looking for the "dyonic molecule" at $T > T_c$ by
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  - Topological susceptibility measurements?
  - Counting fermionic zeromodes
- Crystal-like correlations
  - Diakonov crystal-like next-neighbor correlations below T_c
  - Hexagonal-densly packed "crystal" correlations for $N_f \neq 0$ between $L - \bar{L}$

Outlook

- Do more rigorous simulations

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- Do more rigorous simulations
- Explore the case of adjoint fermions more closely and explain the difference between T_c for adjoint and fundamental fermions more quantitatively
- Compute free energy dependence on holonomy parameter (adjoint and fundamental)

Towards kinetic theory with anomalies

(Based on PS and R. Loganayagam arXiv:1201.2812)

Piotr Surówka
Theoretische Natuurkunde, Vrije Universiteit Brussel
and the International Solvay Institutes

P and CP odd effects in hot dense matter

BNL June 26, 2012

Outline

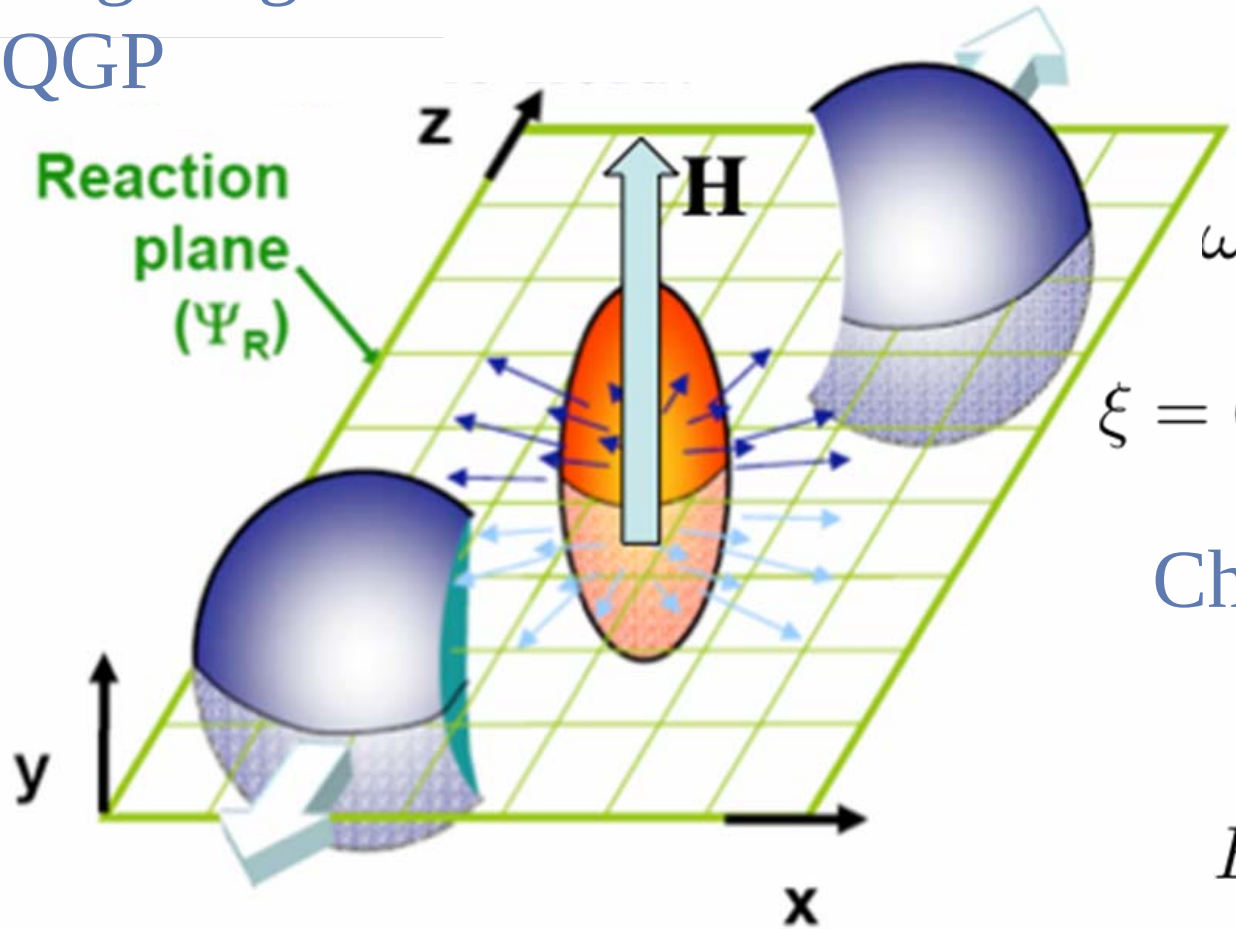
- Motivation
- Hydrodynamics and kinetic theory
- Anomaly and transport in 2d Weyl gas
- Generalization to arbitrary dimensions
- Berry phase effects in non-relativistic systems
- Summary and outlook

Motivation

- Hydrodynamics is a very universal effective field theory used to describe heavy-ion collisions and condensed matter systems
- Violation of P and CP symmetry in QGP
- The right description of parity-odd hydrodynamics requires inclusion of gauge and gravitational anomalies
- Kinetic theory is a complementary semi-classical description of weakly coupled hydrodynamics
- Where do anomalies appear in kinetic theory?

Nucleus-Nucleus collision and hydro

Strong magnetic field and vortices in QGP



Chiral vortical effect

$$j^\mu = \xi \omega^\mu$$

$$\omega^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho$$

$$\xi = C_{anom}\left(\mu^2 - \frac{2}{3}\frac{n\mu^3}{\epsilon + p}\right)$$

Chiral magnetic effect

$$j^\mu = \xi_B B^\mu$$

$$B^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} u_\nu F_{\lambda\rho}$$

$$\xi_B = C_{anom}\left(\mu - \frac{1}{2}\frac{n\mu^2}{\epsilon + p}\right)$$

Kinetic theory

Kinetic theory treats the evolution of the one-particle distribution function, which can be associated with the number of on-shell particles per unit phase space

$$f(\vec{p}, \vec{x}; t) = \frac{dN}{d^3p d^3x}$$

If collisions between particles can be neglected and there is no Berry phase effects, the evolution of $f(\vec{p}, \vec{x}; t)$ follows from Liouville's theorem

Given this interpreation the particle number density should be proportional to

$$\int d^3p f(\vec{p}, \vec{x}; t)$$

Summing instead with a weight of particle energy, one expects a result proportional to the product of number density and energy, or energy density, which is a part of the energy-momentum tensor.

Hydrodynamics $\Leftrightarrow$ *Kinetic theory*

We can derive hydrodynamic quantities from kinetic theory e.g.

$$T^{\mu\nu} \equiv \int \frac{d^4p}{(2\pi)^3} p^\mu p^\nu \delta(p^\mu p_\mu - m^2) 2\theta(p^0) f(p,x)$$

If we take the distribution function in equilibrium we recover energy-momentum tensor of a perfect fluid. One can derive the correspondence between kinetic theory out of equilibrium and viscous hydrodynamics by considering small departures from equilibrium where

$$f(p^\mu, x^\mu) = f_{\rm eq}\left(\frac{p^\mu u_\mu}{T}\right)[1 + \delta f(p^\mu, x^\mu)]$$

This procedure allows one to study dissipative effects (first order in the derivatives of fields). Performing the integral one gets perfect fluid contribution plus shear tensor

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + \int \frac{d^4p}{(2\pi)^3} p^\mu p^\nu f_{\rm eq} \delta f = T^{\mu\nu}_{(0)} + \pi^{\mu\nu}$$

Weyl fermion in 2d

What is the hydrodynamic description of such an ideal gas? Naively we have

$$T^{\mu\nu}_{\text{naive}} = \varepsilon u^\mu u^\nu + p\left(g^{\mu\nu} + u^\mu u^\nu\right)$$
$$J^\mu_{\text{naive}} = n u^\mu$$
$$J^\mu_{S,\text{naive}} = s u^\mu$$

However, a free Weyl fermion is a holomorphic 2d CFT and hence only the holomorphic components of the currents can be non-zero. The above relations are in clear contradiction with holomorphy - the charge/entropy currents are time-like rather than null as would be predicted by holomorphy.

We need correct the above expressions to recover the required properties of 2d CFT. We do that by populating solutions of the Dirac equation by means of kinetic theory

$$\left[u^\mu + \chi_{d=2}\epsilon^{\mu\nu}u_\nu\right]\partial_\mu\psi = 0$$

Anomalous part

Solving the Weyl equation we obtain

$$\psi = \int_0^\infty \frac{dE_p}{2\pi} \frac{1}{\sqrt{2E_p}} \left[a_p e^{ip.x} + b_p^\dagger e^{-ip.x}\right]_{p^\mu = E_p\left[u^\mu + \chi_{d=2}\epsilon^{\mu\nu}u_\nu\right]}$$

Populating these states leads to anomalous correction to hydrodynamics

$$T^{\mu\nu} = \sum_{species} \int_0^\infty \frac{dE_p}{2\pi}(f_q + f_{-q})E_p \left[u^\mu + \chi_{d=2}\epsilon^{\mu\alpha}u_\alpha\right]\left[u^\nu + \chi_{d=2}\epsilon^{\nu\lambda}u_\lambda\right]$$
$$= \varepsilon u^\mu u^\nu + p(g^{\mu\nu} + u^\mu u^\nu) + q^\mu_{anom}u^\nu + q^\nu_{anom}u^\mu$$

$$J^\mu = \sum_{species} \int_0^\infty \frac{dE_p}{2\pi}(qf_q - qf_{-q})\left[u^\mu + \chi_{d=2}\epsilon^{\mu\alpha}u_\alpha\right]$$
$$= nu^\mu + J^\mu_{anom}$$

$$J^\mu_S = -\sum_{species} \int_0^\infty \frac{dE_p}{2\pi}(\mathcal{H}_q + \mathcal{H}_{-q})\left[u^\mu + \chi_{d=2}\epsilon^{\mu\alpha}u_\alpha\right]$$
$$= su^\mu + J^\mu_{S,anom}$$

Gibbs current

The above anomalous quantities can be generated from

$$\bar{\mathcal{G}}_{anom} = \sum_F \int_0^\infty \frac{dE_p}{2\pi} g_q \; \chi_{d=2} u$$
$$\bar{J}_{anom} = -\frac{\partial \bar{\mathcal{G}}_{anom}}{\partial \mu} \quad , \quad \bar{J}_{S,anom} = -\frac{\partial \bar{\mathcal{G}}_{anom}}{\partial T}$$
$$\bar{q}_{anom} = \bar{\mathcal{G}}_{anom} + T\bar{J}_{S,anom} + \mu \bar{J}_{anom}$$

where $g_q \equiv -\frac{1}{\beta} \ln\left[1 + e^{-\beta(E_p - q\mu)}\right]$ and we used Hodge duals for simplicity.

We have to evaluate one thermal integral to get

$$\bar{\mathcal{G}}_{anom} = -2\pi \left[\frac{\mu^2}{2!(2\pi)^2} \left(\sum_{species} \chi_{d=2} q^2 \right) + \frac{T^2}{4!} \left(\sum_{species} \chi_{d=2} \right) \right] u$$

Crucial observation : the anomalous contribution is completely proportional to the U(1) anomaly coefficient $\sum_{species} \chi_{d=2} q^2$ and the Lorentz anomaly coefficient $\sum_{species} \chi_{d=2}$

Anomaly polynomials

The anomaly coefficients of a system are summarized by a polynomial in gauge field strength and spacetime curvature:

$$\mathcal{P}_{anom}(F,\mathfrak{R}) \equiv -2\pi \left[\frac{F^2}{2!(2\pi)^2} \left(\sum_{species} \chi_{d=2} q^2 \right) - \frac{p_1(\mathfrak{R})}{4!} \left(\sum_{species} \chi_{d=2} \right) \right]_{2d}$$

Using this we can write a rule to get from the anomaly polynomial to the anomaly induced Gibbs current

$$\bar{\mathcal{G}}_{anom} = u\, \mathcal{P}_{anom} \left(F \mapsto \mu \ , \ p_1(\mathfrak{R}) \mapsto -T^2 \right)$$

Motivated by this result we can generalize the Gibbs current to higher dimensions introducing concept of chiral spectral current, repeat the analysis and match to hydrodynamics

$$\mathcal{G}^{\mu}_{anom} = \sum_F \int_0^\infty dE_p \mathcal{J}^{\mu}_q \ g_q$$

Chiral spectral current

To determine chiral spectral current we will use adiabaticity in the position and energy space

$$\nabla_\mu \mathcal{J}_q^\mu + \frac{\partial}{\partial E_p} \mathcal{J}_q^E = 0$$

The Current in the energy direction is the sum of the electric force and pseudo force

$$\mathcal{J}_q^E = E_\mu q \mathcal{J}_q^\mu - a_\mu E_p \mathcal{J}_q^\mu$$

Using insight from adiabaticity in hydrodynamics and thermodynamic relations we can solve the above equation. The Hodge dual reads

$$\bar{\mathcal{J}}_q = \frac{\chi_{d=2n}}{2\pi} \left(\frac{qB + 2\omega E_p}{2\pi} \right)^{n-1} \wedge \frac{u}{(n-1)!}$$

Berry phase

Consider a physical system described by a Hamiltonian that depends on time through a set of parameters $R = (R_1, R_2, \dots)$

$$|\Psi_n(t)> = e^{i\gamma_n(t)} e^{-\frac{i}{\hbar}\int_0^t dt' \varepsilon_n(R(t'))} |n(R(t))>$$

Insering the above expression to the Schrodinger equation and multiplying by bra one finds that the phase factor can be expressed as a path integral in the parameter space

$$\gamma_n = \int_{\mathcal{C}} dR (i < n(R)| \frac{\partial}{\partial R} |n(R)>) \equiv \int_{\mathcal{C}} dR \cdot \mathcal{A}_n(R)$$

where we have defined the Berry connection. We see that in addition to a dynamical phase quantum state will acquire an additional phase during the adiabatic evolution along closed contour.

Gas of fermions

Consider a gas of non-relativistic fermions with a Berry curvature on the fermi surface in the presence of electromagnetic field. The lagrangian of such system is given by:

$$\mathcal{L} = \hbar \mathbf{k} \cdot \dot{\mathbf{r}} - \varepsilon_M(\mathbf{k}) + e\phi(\mathbf{r}) - e\dot{\mathbf{r}} \cdot A(\mathbf{r}, t) + \hbar \dot{\mathbf{k}} \cdot \mathcal{A}_n(\mathbf{k})$$

We can derive the EOMs

$$\dot{\mathbf{r}} = \frac{\partial \varepsilon_M(\mathbf{k})}{\hbar \partial \mathbf{k}} - \dot{\mathbf{k}} \times \mathbf{\Omega}(\mathbf{k})$$

$$\hbar \dot{\mathbf{k}} = -e\mathbf{E} - e\dot{\mathbf{r}} \times \mathbf{B}$$

where we have defined the Berry curvature

$$\mathbf{\Omega}^n_{\mu\nu}(R) = \frac{\partial}{\partial R^\mu} \mathcal{A}^n_\nu(R) - \frac{\partial}{\partial R_\nu} \mathcal{A}^n_\mu(R)$$

We see the so-called anomalous Karplus-Luttinger contribution to velocity

Density of states

Liouvile theorem guarantees constatnt density of states in the classical systems. This is not the case in the presence of the Berry phase and magnetic field. Let us calculate the dynamics of the volume element

$$\frac{1}{\Delta V}\frac{\partial \Delta V}{\partial t} = \nabla_{\mathbf{r}} \cdot \dot{\mathbf{r}} + \nabla_{\mathbf{k}} \cdot \dot{\mathbf{k}}$$

We can solve the above equation

$$\Delta V = \frac{V_0}{(1+(e/\hbar)\mathbf{B}\cdot\mathbf{\Omega})}$$

The fact that the Berry curvature is generally momentum dependent and the magnetic field is position dependent implies that the phase-space volume changes during time evolution. We can introduce a modified density of states

$$D(\mathbf{r},\mathbf{k}) = \frac{1}{(2\pi)^d}(1 + \frac{e}{\hbar}\mathbf{B}\cdot\mathbf{\Omega})$$

such that $O = \int d\mathbf{k}\, D(\mathbf{k}) f(\mathbf{k}) O(\mathbf{k})$

Summary

- QFT anomalies lead to hydrodynamic transport
- New transport coefficients calculated by linear responce theory
- Anomalous transport leads to modified density of states and the emergence of chiral spectral current linked to a Dirac monopole in the momentum space. Possible relation with Berry phase proven for magnetic field part in Landau's Fermi Liquid by Son and Yamamoto
- Calculation of Gibbs free energy which was expressed in terms of anomaly polynomials

Future goals

- Boltzmann operator with anomalies
- Berry phase derivation of chiral vortical effect
- Check of the prescribed conjecture for anomaly polynomials by means of AdS/CFT and Kubo formulae
- Wigner formulation of QM with anomalies
- Applictions to CMT systems such as Weyl-semi metals

Sphalerons, Turbulence and Magnetic Fields

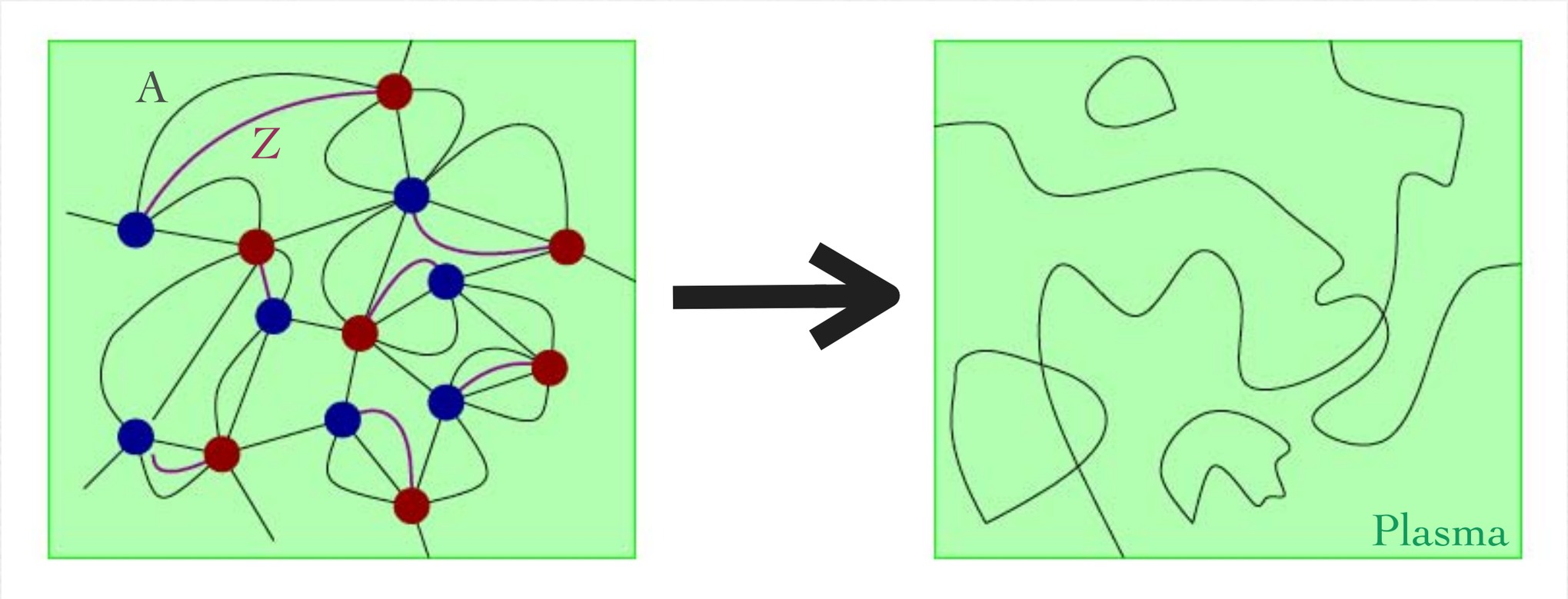
Tanmay Vachaspati

Arizona State University

P & CP
Brookhaven National Laboratory
25 June 2012

Magnetic fields from the electroweak phase transition

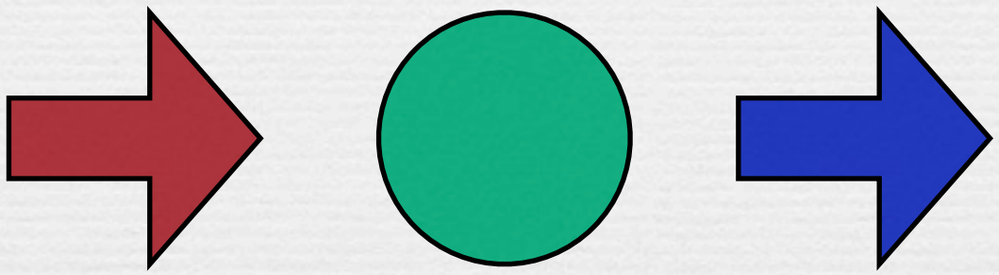
TV, 1991, 1994



Magnetic fields with large coherence scale are frozen-in.

How many monopoles? How much magnetic field?
On what scales?

baryon #=0



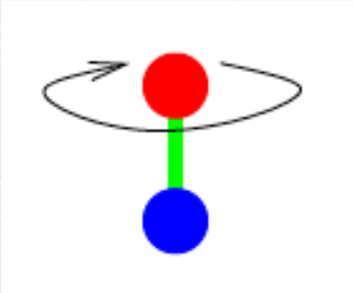
baryon number≠0

~Sphaleron

Sphaleron

= monopole-antimonopole bound state solution

Taubes
Manton
Manton & Klinkhamer
TV & Field
James & Hindmarsh



$$\mu_{\text{sphaleron}} \approx 0.314 \text{ GeV}^{-1} \quad \mu_W = \frac{e}{m_W} \approx 0.0038 \text{ GeV}^{-1}$$

Sphaleron decay

Copi, Ferrer, TV & Achucarro
Y-Z. Chu, J. Dent & TV

We know a decay path for the sphaleron.

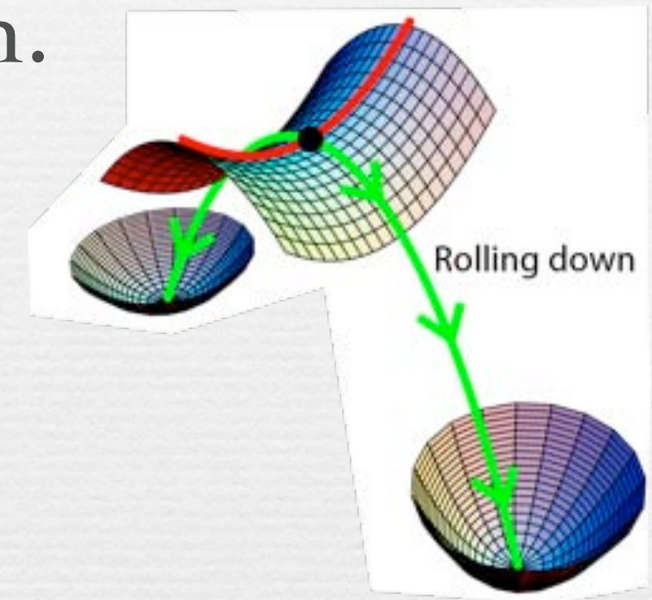


Image- http://spie.org/x31524.xml?ArticleID=x31524

Calculate currents along the decay path and then solve Maxwell eq.:

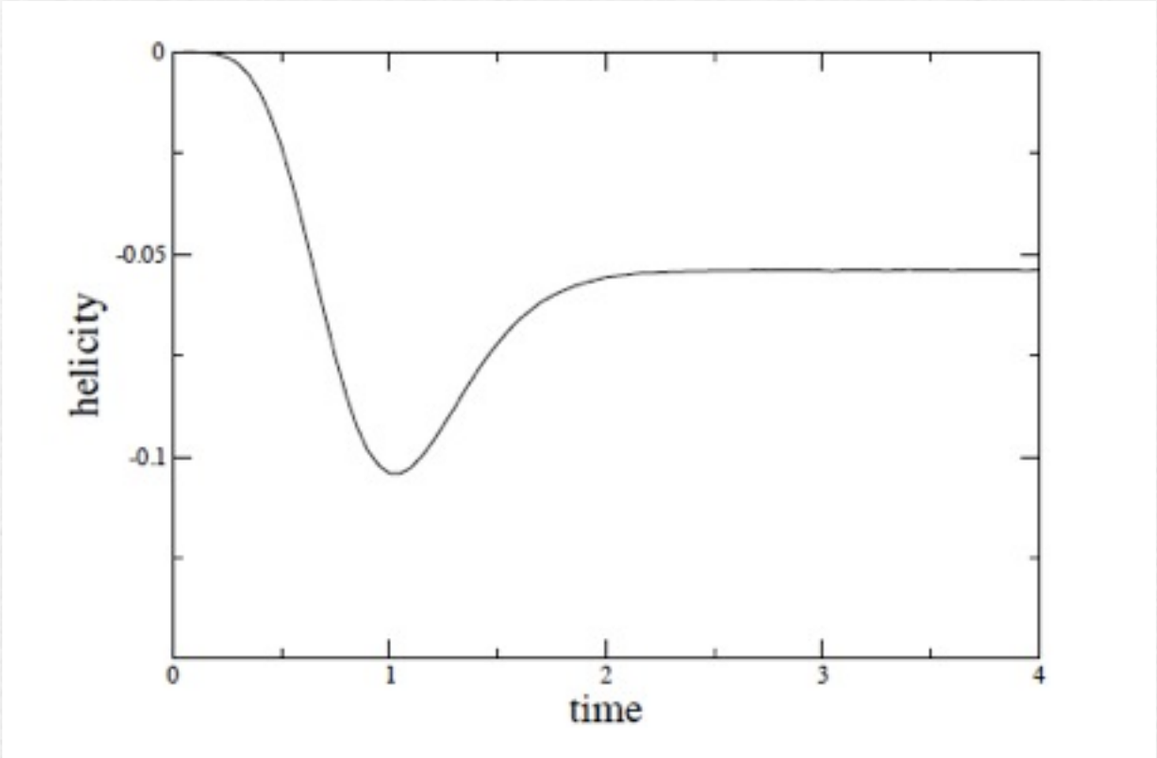
$$(\partial_t^2 - \nabla^2) A^\mu = J^\mu$$

Evaluate magnetic helicity:

$$\mathcal{H}(t) = \int d^3x \; \mathbf{A} \cdot \mathbf{B}$$

Asymptotic helicity is independent of flow velocity.

Magnetic helicity



Helicity is conserved at late times. $\mathcal{H}(\infty) \sim -\frac{\sin^2\theta_w}{g^2}$

Baryon production implies left-handed helicity.

Cosmological magnetic helicity

Every $\Delta B \implies \Delta\mathcal{H}$

J. Cornwall
TV

$$\implies \quad \boxed{h \approx -\#\frac{n_b}{\alpha}}$$

$\# \approx 0.01$

Copi, Ferrer, TV & Achucarro
Y._Z. Chu, J. Dent & TV

Independent of details of electroweak baryogenesis scenario.

Baryons vs Sphalerons

Copi, Ferrer, TV & Achucarro

Magnetic helicity depends on baryon number density.

Magnetic energy depends on number of sphalerons.

The two are related by a CP factor.

Therefore initial magnetic energy is much larger than what one would estimate using helicity alone.

Other intriguing scenarios...

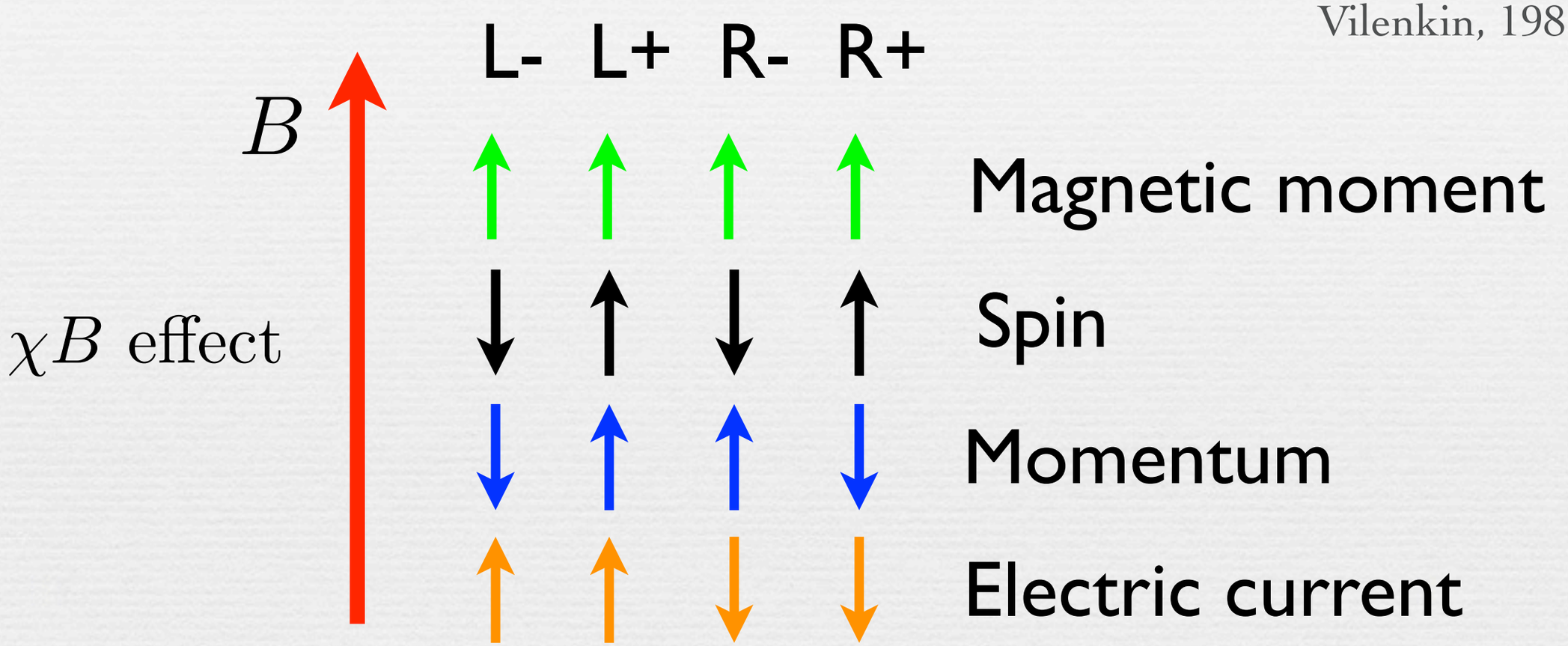
What if cosmological medium is chiral?

Joyce & Shaposhnikov
Boyarsky, Frohlich & Ruchayskiy

- Chiral-Magnetic Effect — Vilenkin, 1980
- Chiral-Vorticity Effect — Vilenkin, 1979

Chiral Magnetic Effect

Vilenkin, 1980



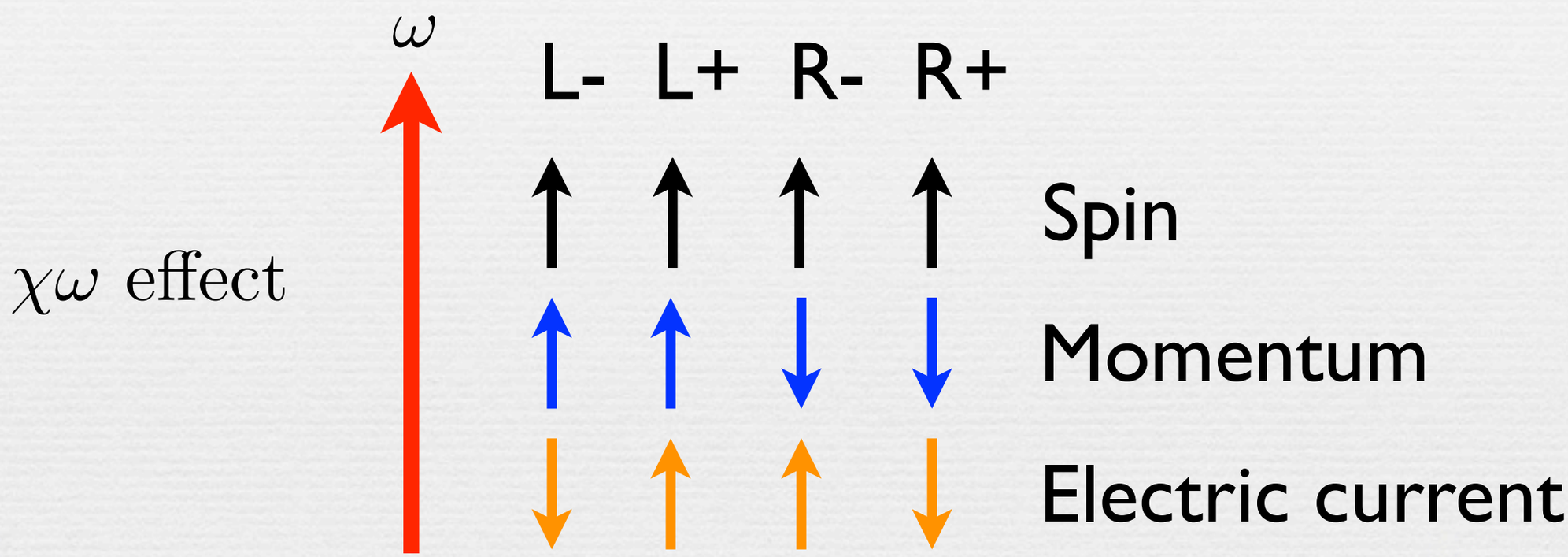
$$J_{\chi B} \propto [n(e_L^-) - n(e_R^+)] - [n(e_R^-) - n(e_L^+)]$$

$$\mathbf{J}_{\chi B} = \frac{e^2}{2\pi^2} \Delta\mu \, \mathbf{B}$$

For massive particles, flipping between chiral states can damp the current.

Chiral-vorticity Effect

Vilenkin, 1979



$$J_{\chi\omega} \propto [n(e_L^-) + n(e_R^+)] - [n(e_R^-) + n(e_L^+)]$$

$$\mathbf{J}_{\chi\omega} = \frac{e}{4\pi^2}\Delta\mu^2\ \omega$$

Chiral Effects on B

Hiroyuki Tashiro, TV & Alex Vilenkin

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \gamma_D \nabla^2 \mathbf{B} + \gamma_\omega \nabla \times \omega + \gamma_B \nabla \times \mathbf{B}$$

"advection" "diffusion" "$\chi\omega$" "χB"

$$\gamma_D = \frac{1}{4\pi\sigma}\ , \quad \gamma_\omega = \frac{e\Delta\mu^2}{4\pi^2\sigma}\ , \quad \gamma_B = \frac{e^2\Delta\mu}{2\pi^2\sigma}$$

Chiral anomaly eq.: $$\frac{d(\Delta\mu)}{dt} = -\frac{c_\Delta \alpha}{T^2}\frac{dh}{dt} - \Gamma_F \Delta\mu$$

Chirality flipping unimportant above 80 TeV temperature.

Campbell et al

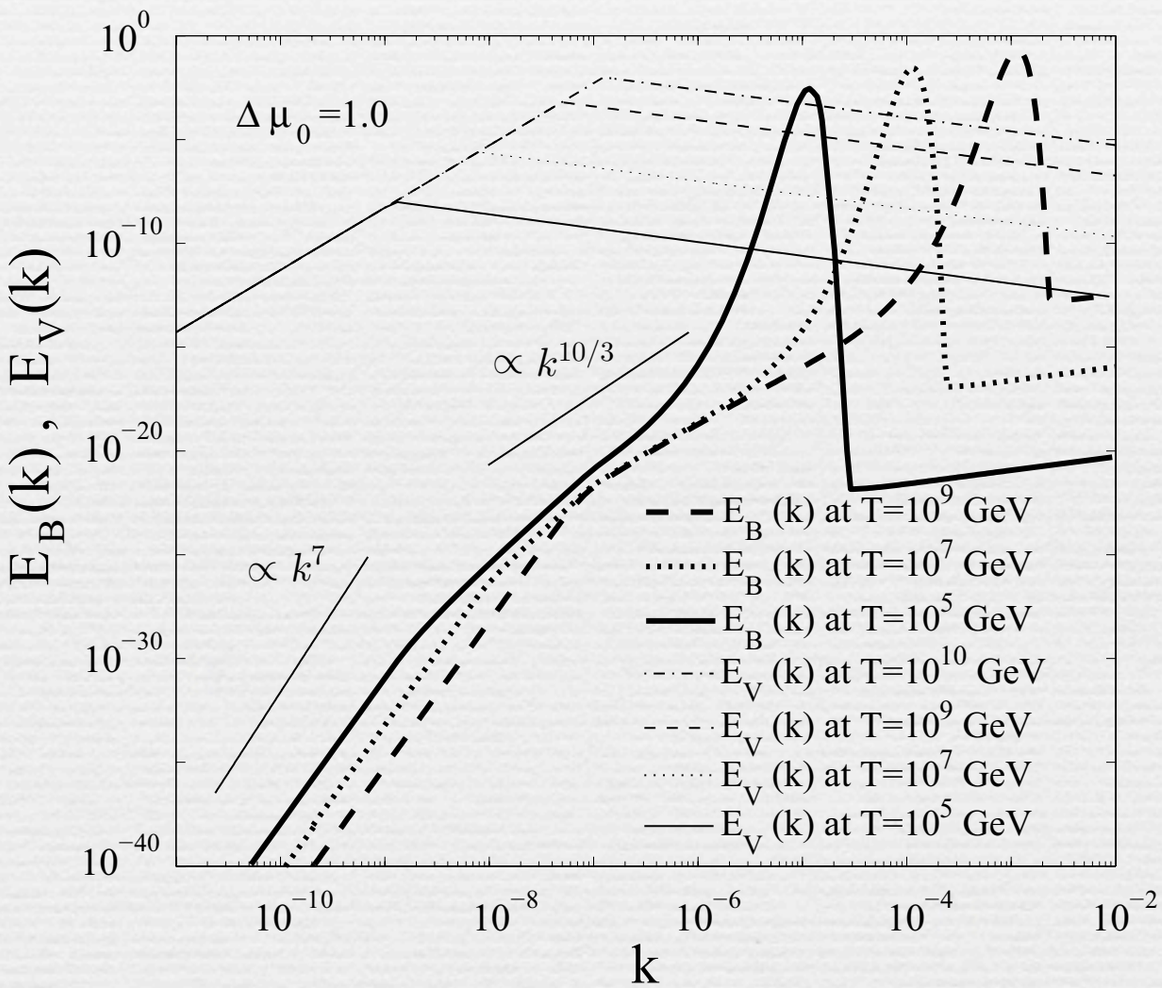
Initial Conditions

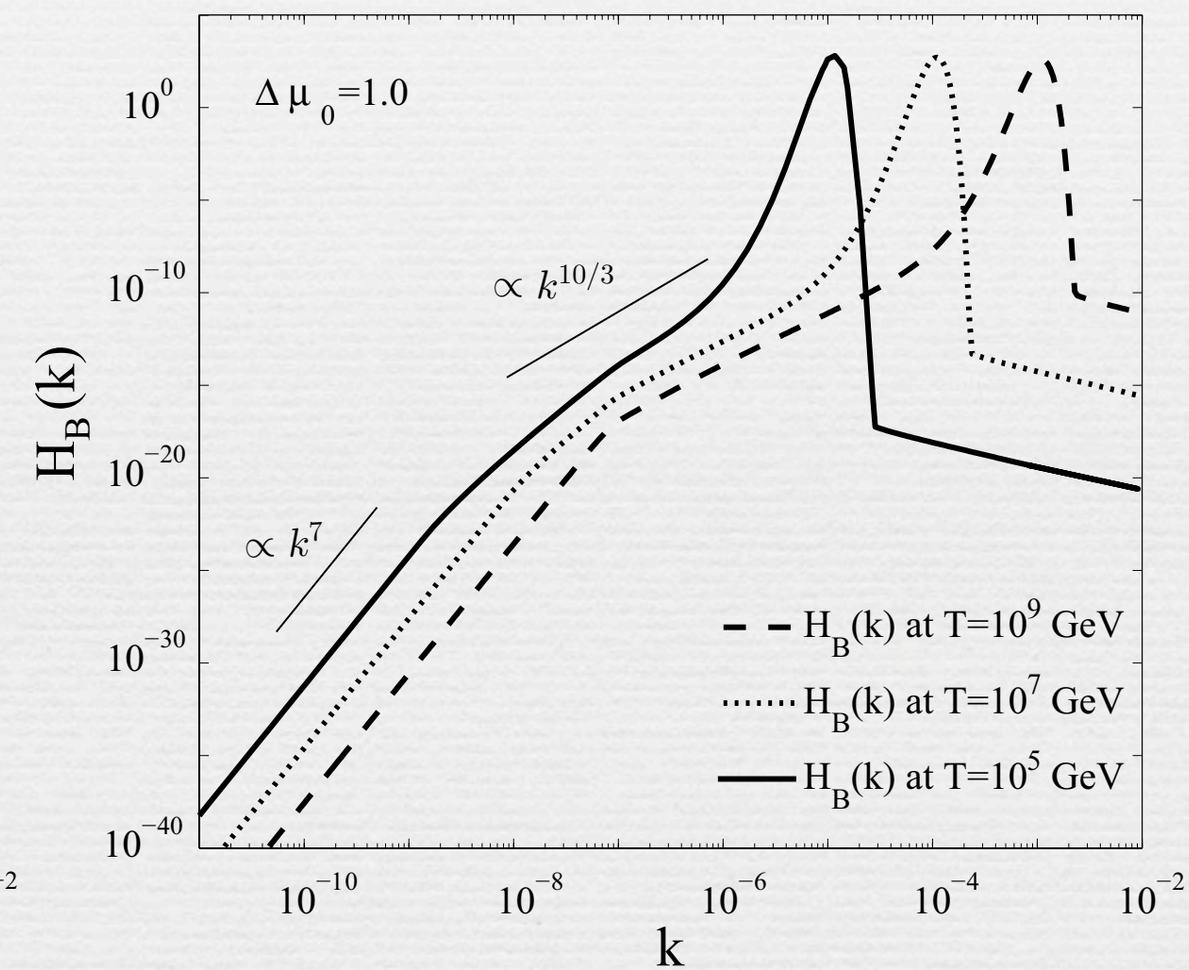
Initial conditions: chirality& turbulence, B=0.

Contrast:
Joyce & Shaposhnikov -- chiral medium & initial non-zero B but v=0;
Boyarsky et al -- helical B (e.g. due to baryogenesis) but non-chiral medium and v=0.

Velocity field: white noise at small k, Kolmogorov at large k.

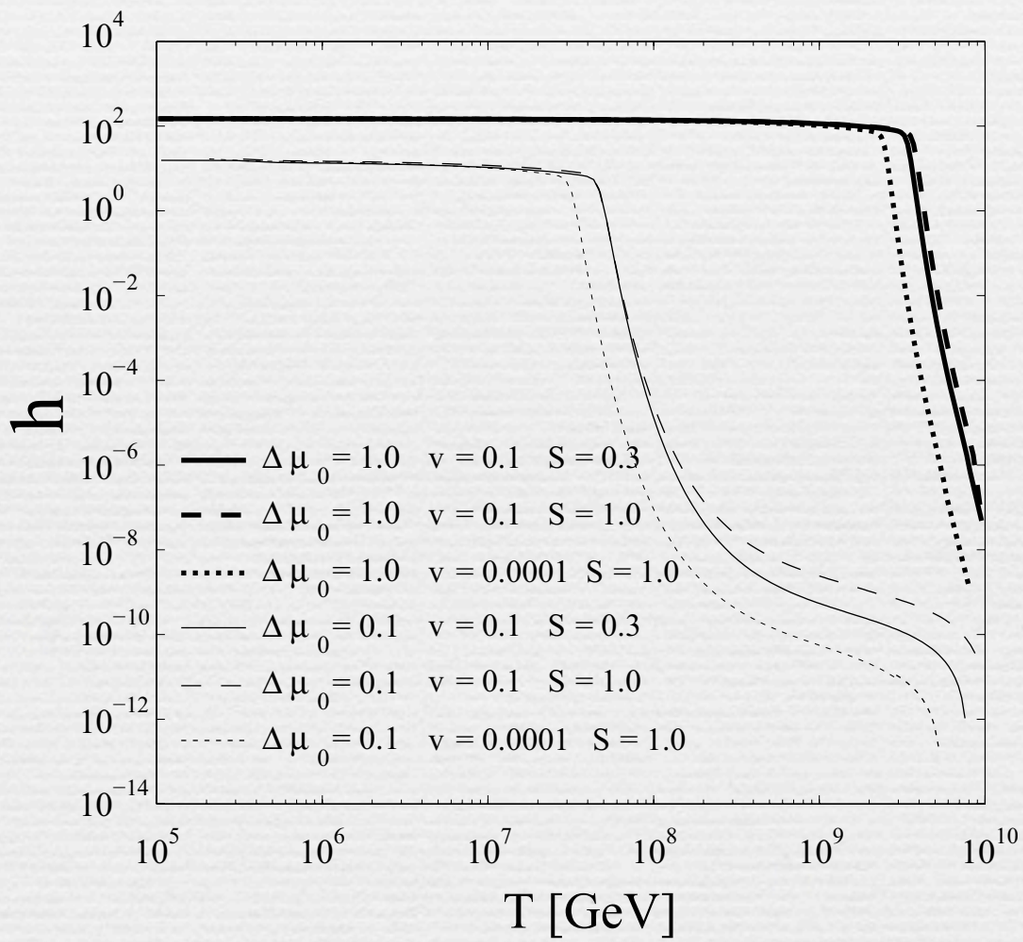
Evolution (without advection)

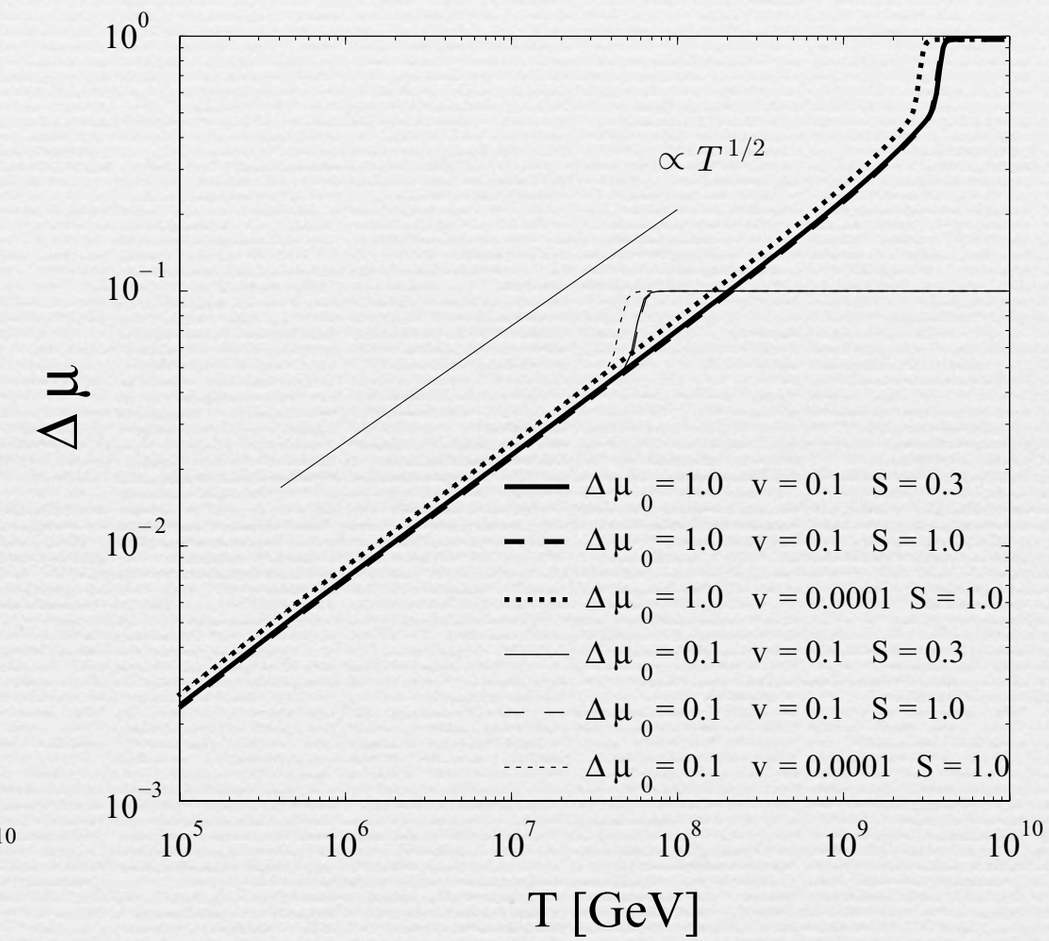




Magnetic field is produced with a peak that shifts to larger length scales with time.

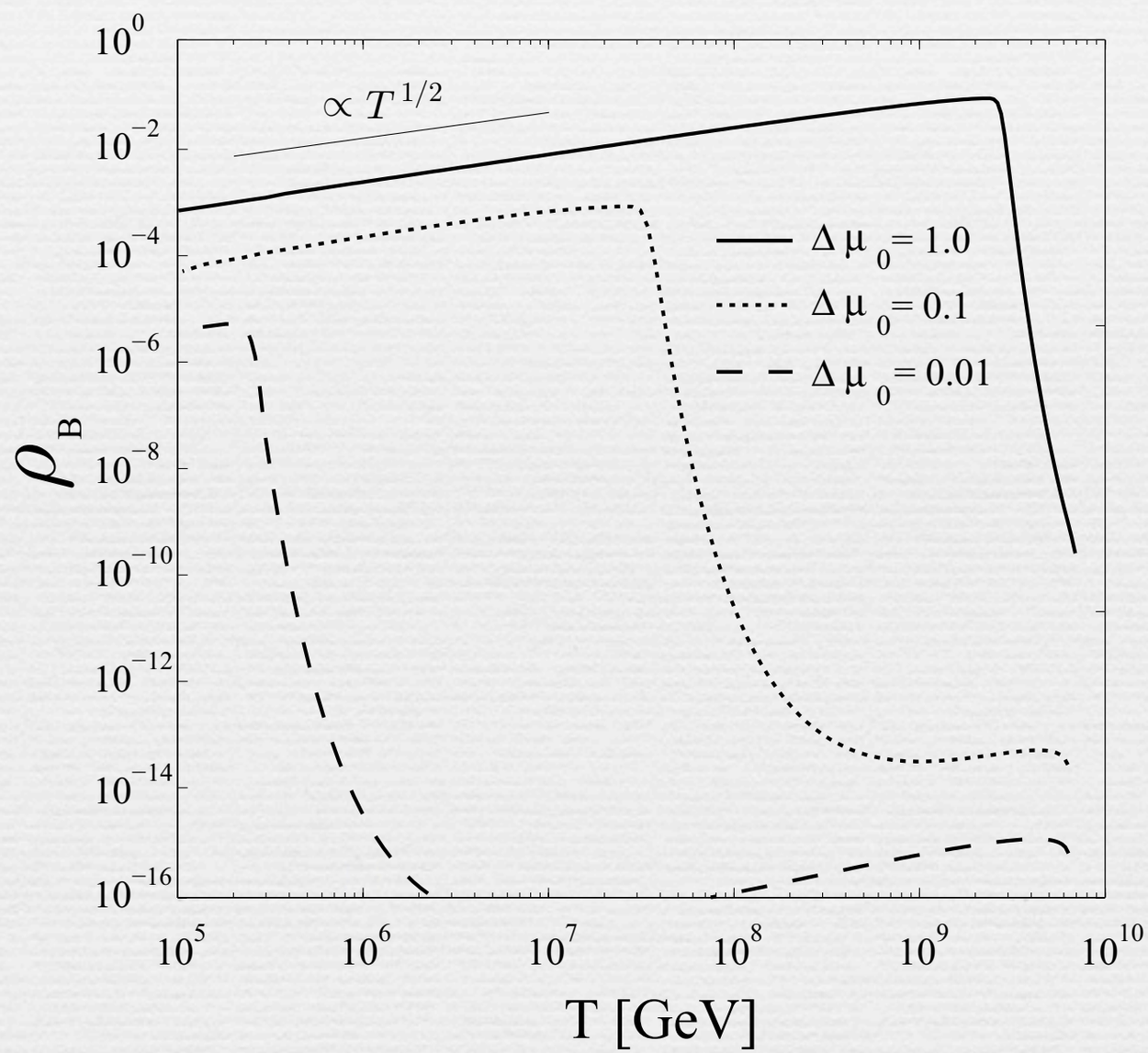
Evolution of helicity & chirality





Attractor solution!

Evolution of energy density



Analytical Results

Mode decomposition e.g.

$$\mathbf{B}(\eta, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} \left[\tilde{B}^+(\eta, \mathbf{k}) \mathbf{q}^+(\mathbf{k}) + \tilde{B}^-(\eta, \mathbf{k}) \mathbf{q}^-(\mathbf{k}) \right]$$

Turbulence with chiral-omega effect produces seed magnetic field.
After a brief inital phase, the equations are:

$$\partial_\eta |B^\pm| = -\gamma_D k (k \mp k_p) |B^\pm|$$

$$\frac{d\Delta\mu}{d\eta} = -c_\Delta \alpha \int \frac{k dk}{2\pi^2} \partial_\eta [|B^+|^2 - |B^-|^2] - \Gamma_F \Delta\mu$$

$$k_p \equiv \frac{\gamma_B}{\gamma_D} = \frac{e^2 \Delta\mu}{2\pi^2}$$

Solution

$$|B^{\pm}(\eta,k)| = |B_0^{\pm}| \exp(\gamma_D K_p^2 \eta) \exp\left[-\gamma_D \left(k \mp K_p\right)^2 \eta\right]$$

seed B — *exponential amplification* — *Gaussian spectrum*

Spectral peak at:

$$K_p \equiv \frac{1}{2\eta} \int_0^{\eta} d\eta k_{\ p}$$

Only one helicity mode (+ or -) survives and the magnetic field quickly becomes maximally helical.

Gaussian peak gets sharper with time: $\Delta k = \dfrac{1}{\sqrt{2\gamma_D \eta}}$

Attractor

$$K_{p*} \equiv \frac{1}{2\eta}\int_0^\eta d\eta k_{\ p*} = k_{p*}$$

$$K_{p*} \approx k_{p*} = \frac{e^2\Delta\mu}{2\pi^2} \approx CT^{1/2}$$

$$C \approx (2\gamma_D M_*)^{-1/2}(\ln X)^{1/2} \approx 5\times 10^{-8}$$

(sensitive only to electrical conductivity)

$$\text{Asymptotic helicity} = h_* \approx \frac{\Delta\mu_0}{\alpha}$$

$$\text{Maximal helicity} \implies \rho_B \approx K_p h$$

Estimates

In the attractor solution: $K_p \approx 5 \times 10^{-8} \eta^{-1/2}$

Once flipping is important: $K_p \propto \frac{1}{\eta}$

With this scaling:

$$\text{Coherence scale today} \approx 1 \text{ kpc}$$

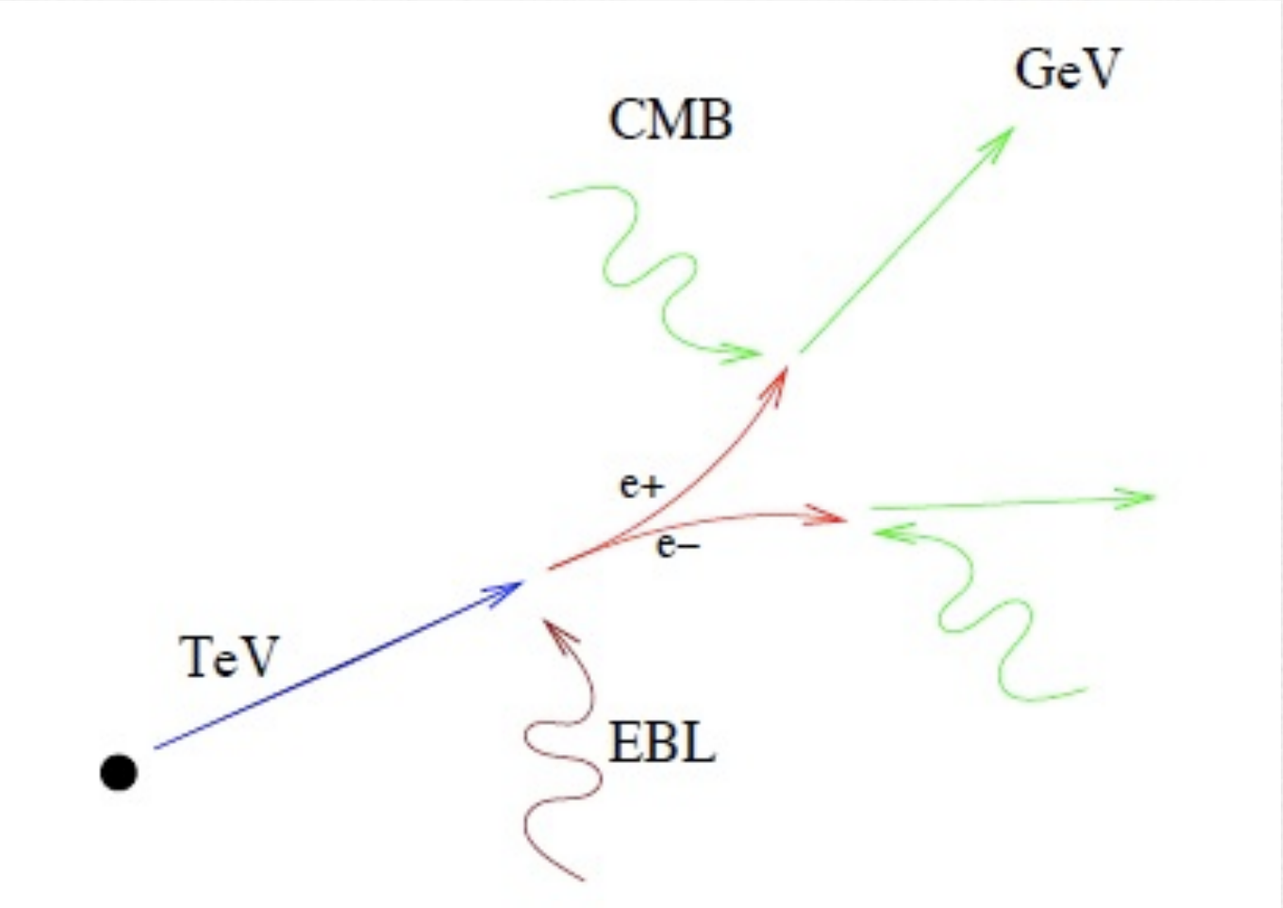
Assuming helicity conservation:

$$\text{Energy density} \approx K_p h_*$$

$$B \sim 10^{-16} (\Delta\mu_0)^{1/2} \, \text{G}$$

Observations

TeV gamma ray sources should have GeV halos in the absence of magnetic fields. Absence of GeV halos indicates a *lower* bound on intergalactic magnetic fields.



Claim:
FERMI data gives *lower* bound of ~10^{-16} Gauss magnetic field.

Neronov & Vovk, 2010

Claim:
Measured magnetic field at ~3x10^{-16} Gauss.

Ando & Kusenko, 2010

Counterpoint: PSF of telescope.

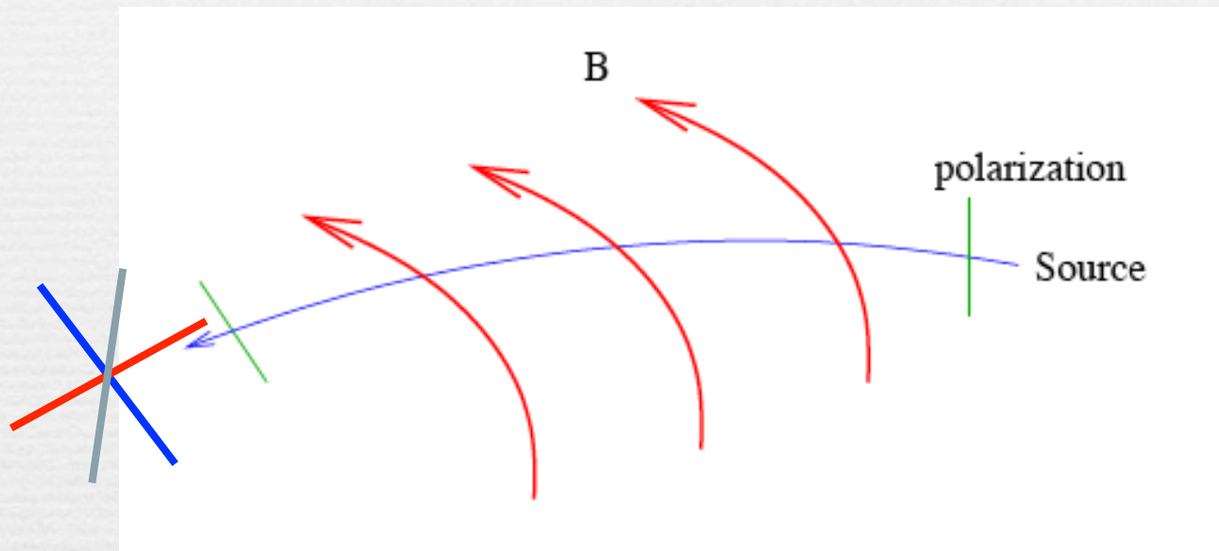
Neronov, Semikoz, Tinyakov & Tkachev, 2010

Claim:
MHD instabilities can give rise to absence of GeV gamma rays.

Broderick, Chang & Pfrommer, 2011

B in CMB

(Earlier literature mostly focused on very coherent B.)



Pogosian, Yadav, Ng & TV

$$\Delta\theta = \lambda^2 \ \mathrm{RM} = \frac{3\lambda^2}{2\pi e} \int \dot{\tau}(\mathbf{x})\mathbf{B} \cdot d\mathbf{l}$$

Faraday rotation of CMB polarization converts E-modes to B-modes.

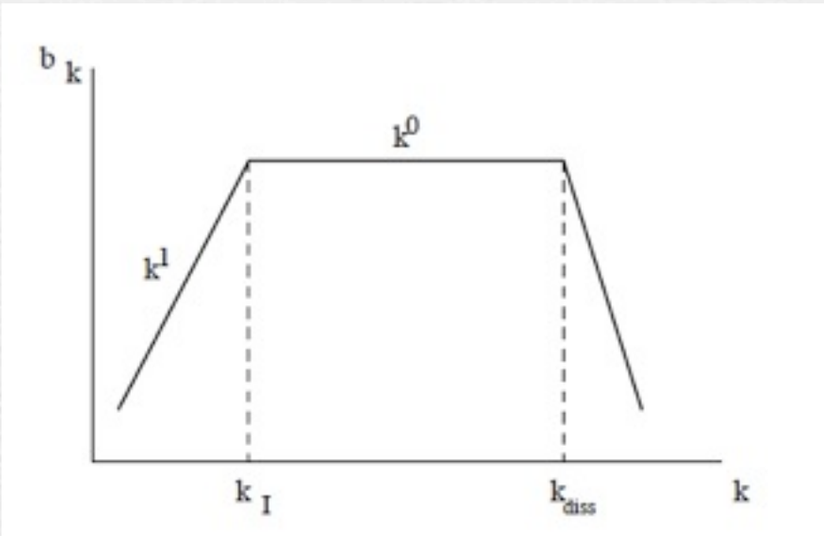
Frequency dependent.

Correlated with temperature fluctuations.

Grows at small angular scales.

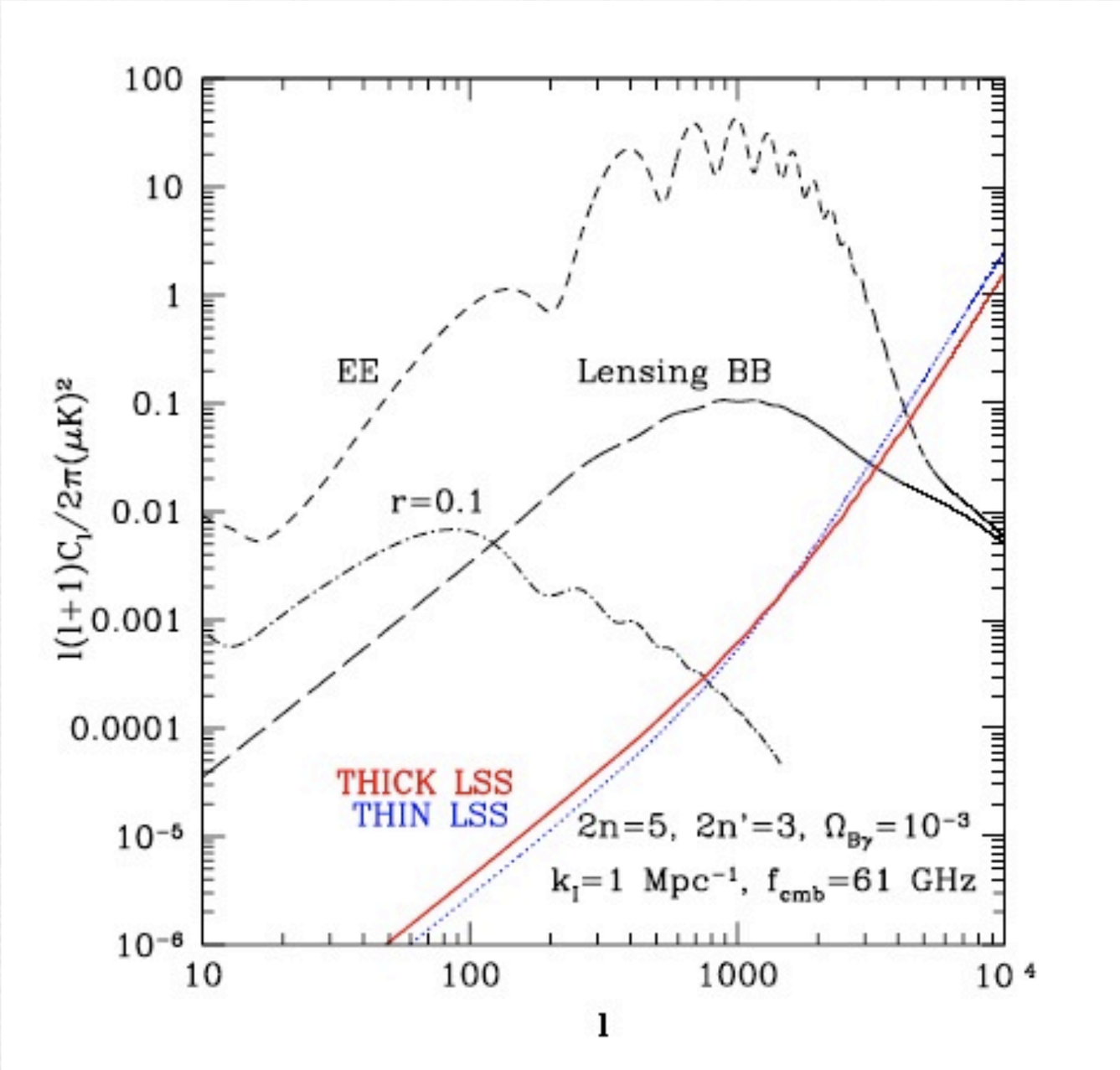
C_l^{BB}

Pogosian, Yadav, Ng & TV



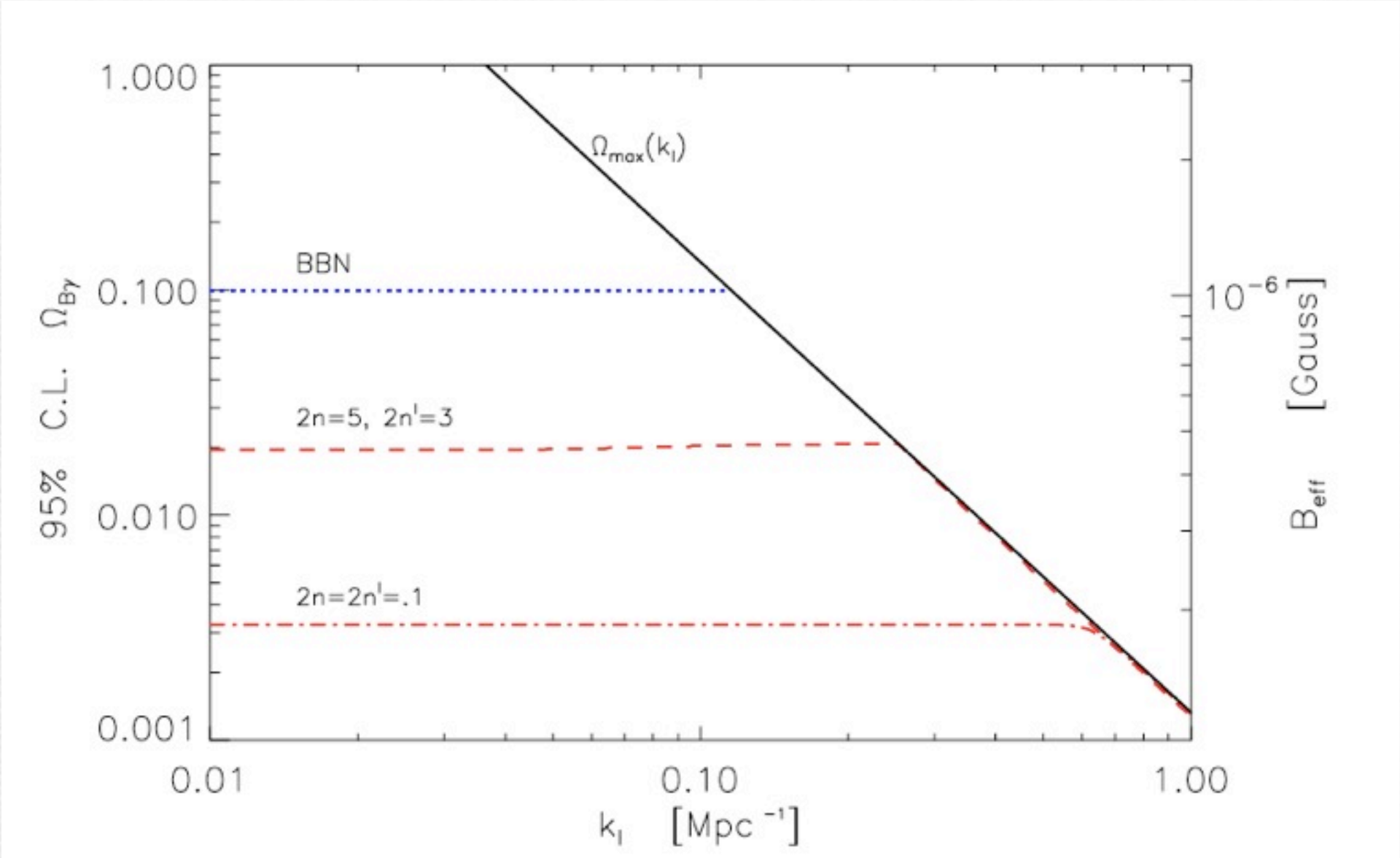
Durrer & Caprini

Jedamzik & Sigl



Current Constraints

Pogosian, Yadav, Ng & TV



Omega_{max} motivated by Alfven wave dissipation.

Jedamzik, Katalinic & Olinto

Kahniashvili, Tevzadze, Sethi, Pandey & Ratra

Conclusions

Baryogenesis gives helical magnetogenesis.

Non-helical turbulence plus chirality gives non-helical magnetogenesis.

Chirality amplifies magnetic field and helicity.

Resulting magnetic field is maximally helical, has a sharply peaked spectrum, and evolves along an attractor.

Field strength is fixed by number of sphaleron events (not by baryon number density) or by initial chirality.

Naive estimates:
Coherence scale ~ 1 kpc; field strength $\sim 10^{-16}(\Delta\mu_0)^{1/2}$ G.

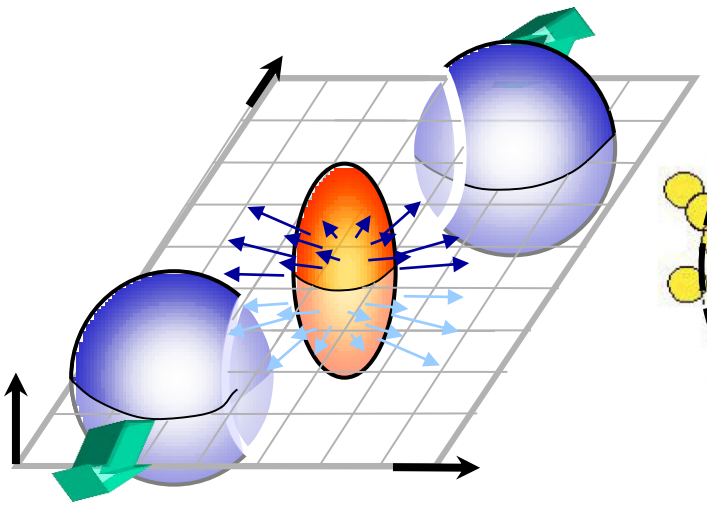
Di-hadron correlations measurements from CMS

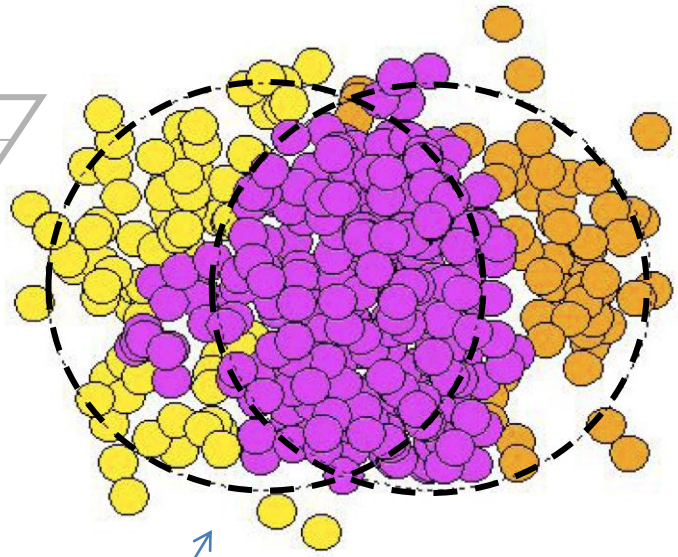
Dragos Velicanu

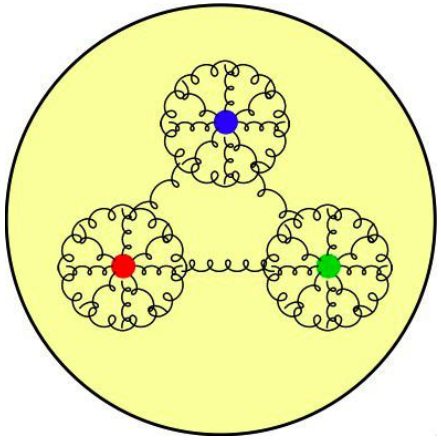
Workshop on P- and CP-odd Effects in Hot and Dense Matter (2012)

Understanding Initial Conditions

Goal: To understand initial conditions of PbPb and pp collisions through long range correlations of produced charged particles





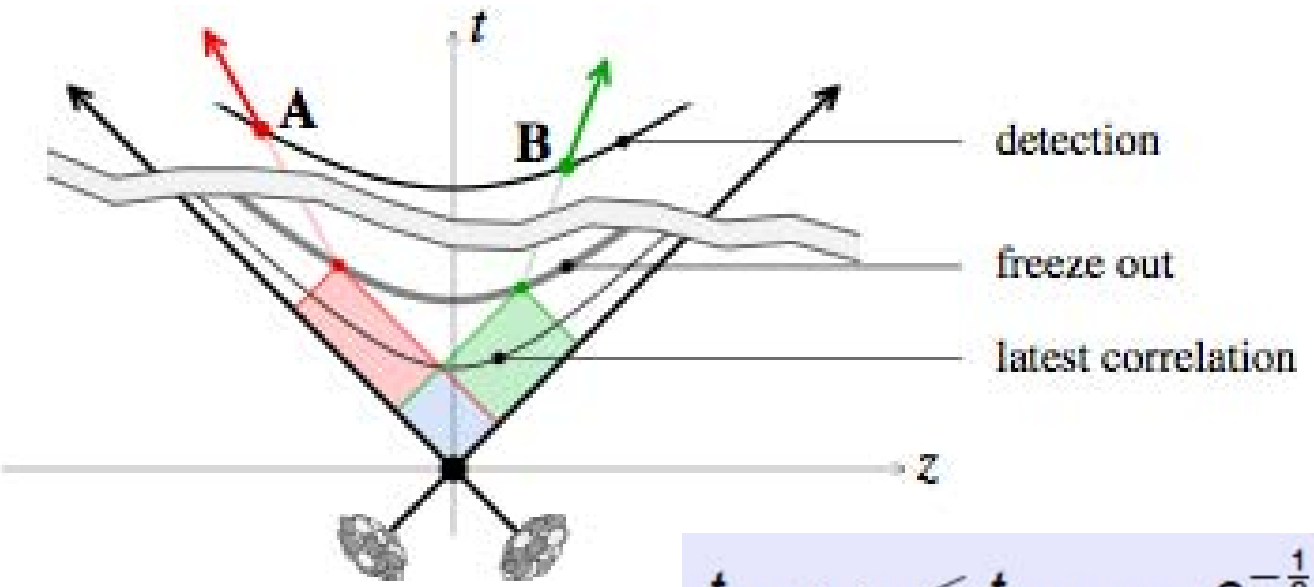


Two views of possible PbPb initial conditions

Probe the initial condition of proton structure

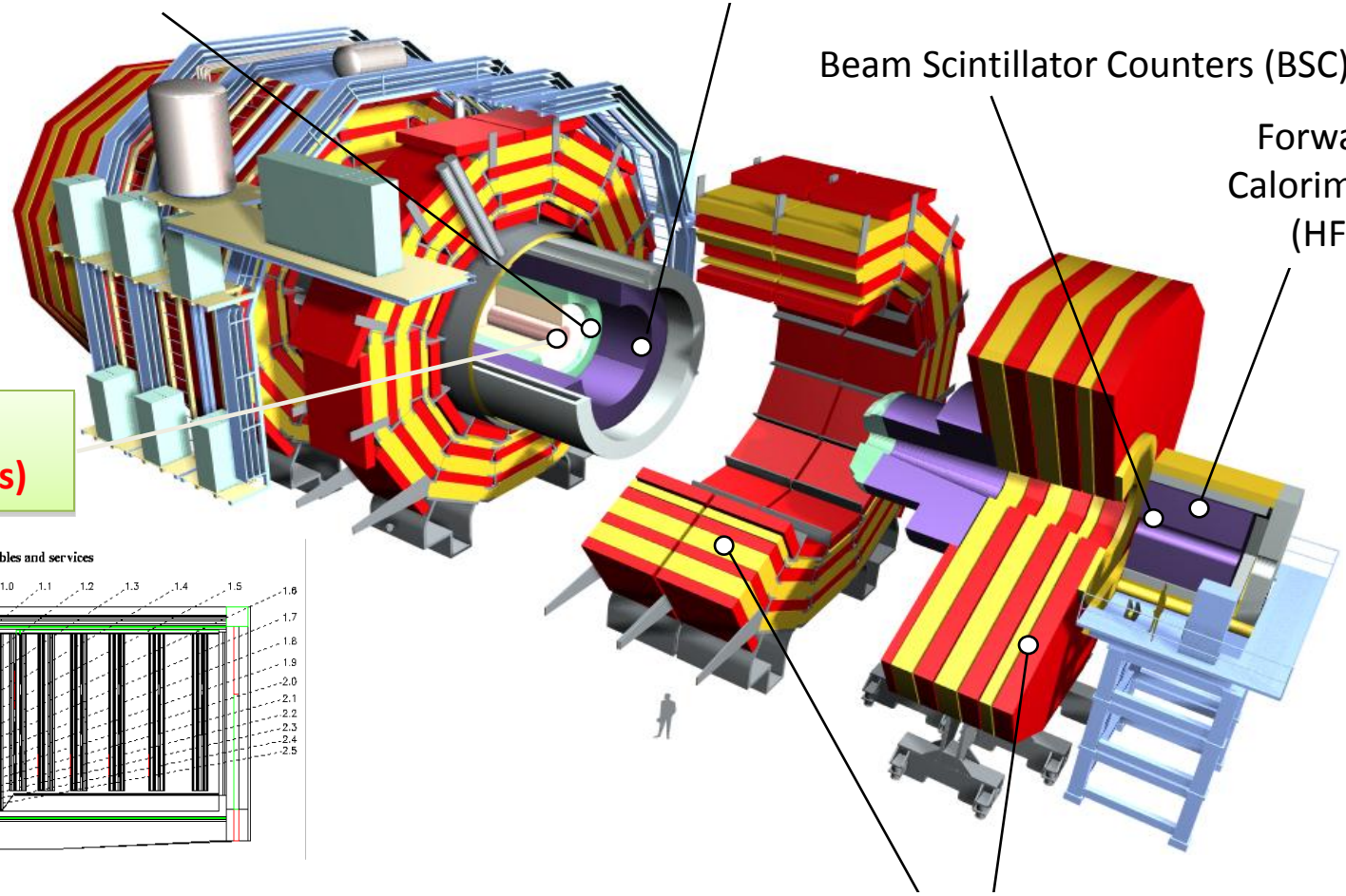
Understanding Initial Conditions

Goal: To understand initial conditions of PbPb and pp collisions through <u>long range</u> correlations of produced charged particles

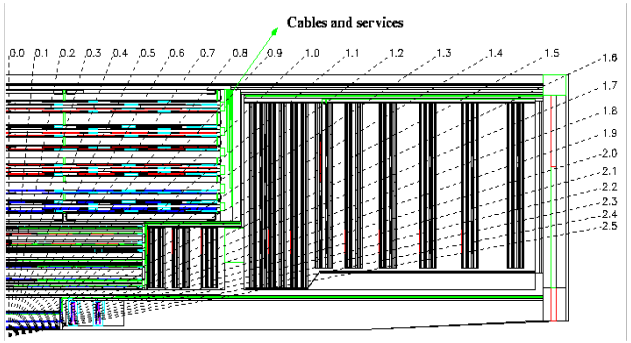


$$t_{\text{correlation}} \leq t_{\text{freeze out}} \; e^{-\frac{1}{2}|\eta_A - \eta_B|}$$

CMS Detector

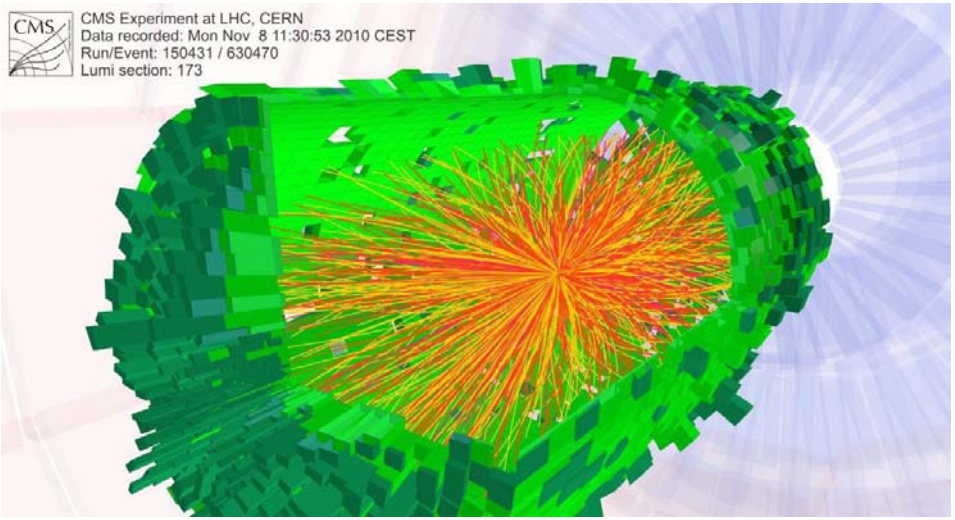


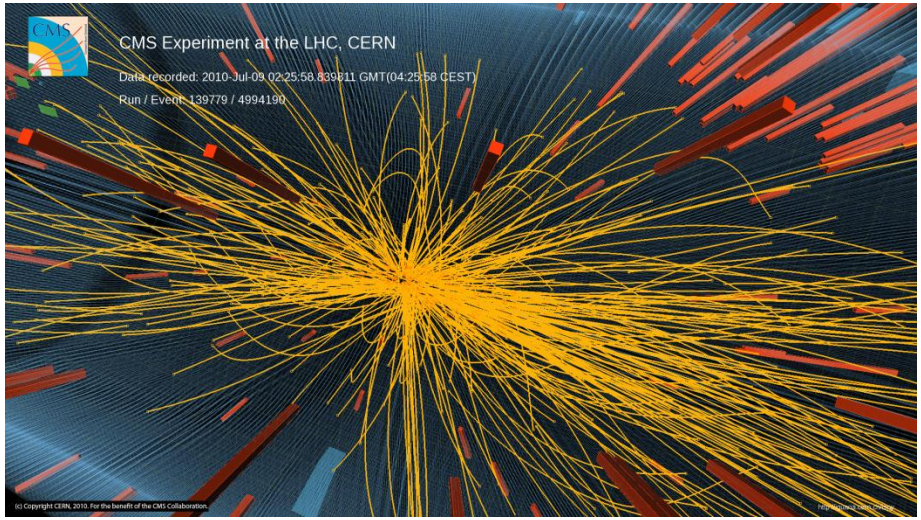
TRACKER (Pixels and Strips)



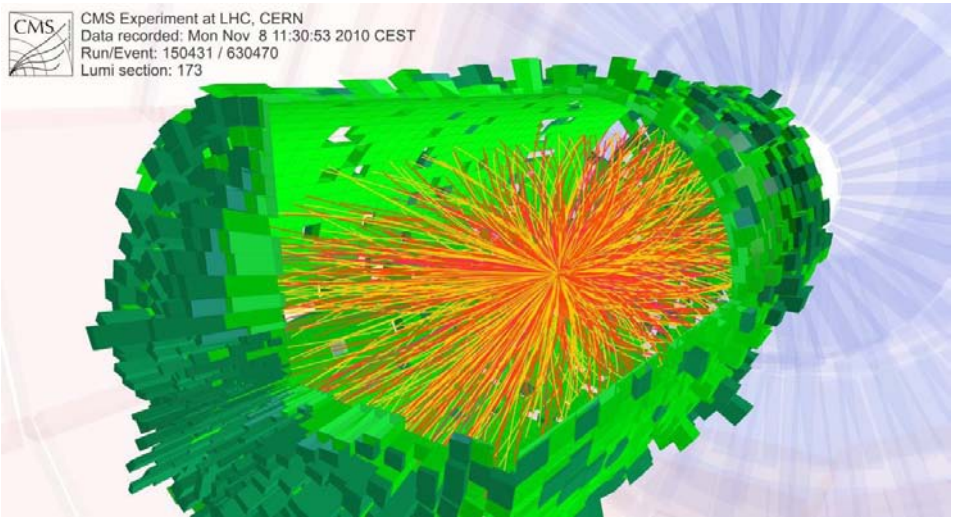
Very large coverage for tracking ($|\Delta\eta|$ up to 5.0)!

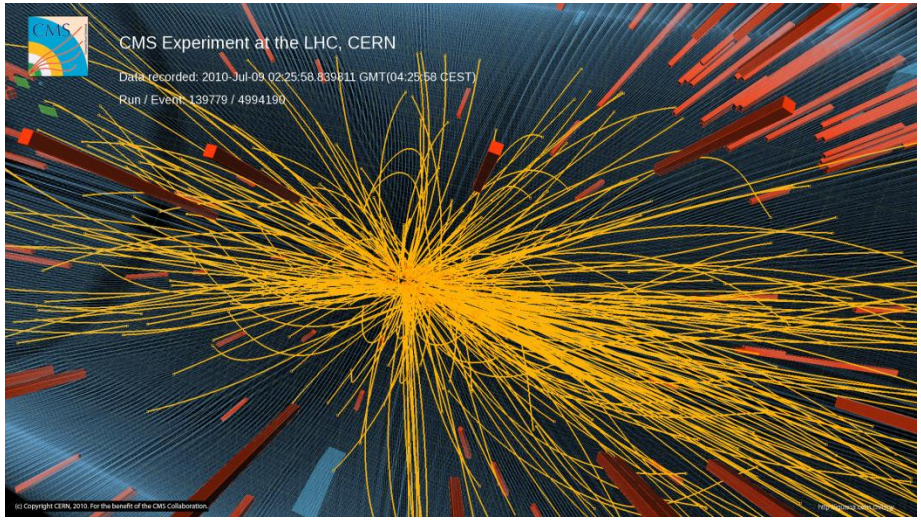
PbPb and pp collisions at the LHC





Correlations of charged tracks

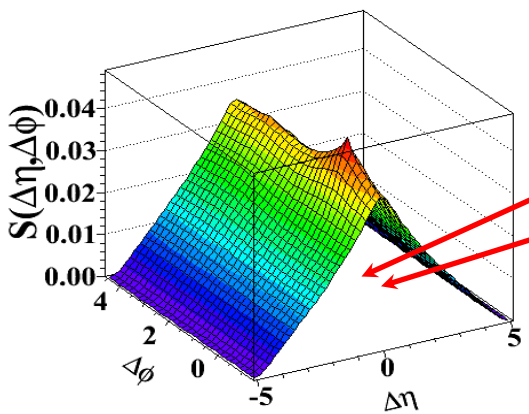


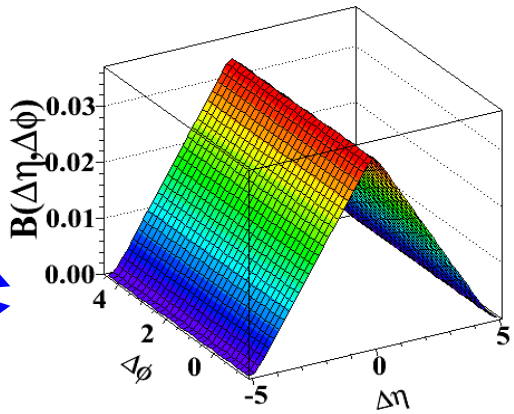


Signal pair distribution:

Background pair distribution:

same event pairs





mixed event pairs

$$S(\Delta\eta, \Delta\phi) = \frac{1}{N_{trig}} \frac{d^2 N^{same}}{d\Delta\eta d\Delta\phi}$$

$$B(\Delta\eta, \Delta\phi) = \frac{1}{N_{trig}} \frac{d^2 N^{mix}}{d\Delta\eta d\Delta\phi}$$

Correlations of charged tracks

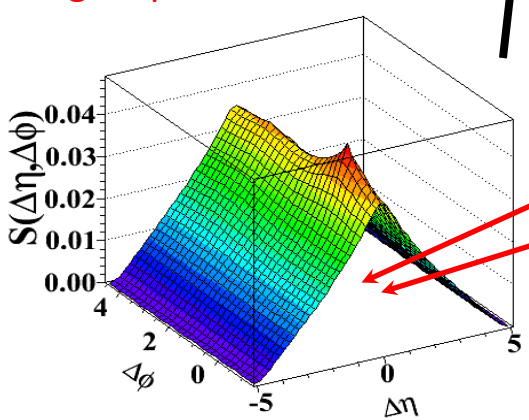
Divide Signal by Background

Associated hadron yield per trigger:

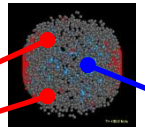
$$\frac{1}{N_{trig}}\frac{d^2N^{pair}}{d\Delta\eta d\Delta\phi} = B(0,0) \times \frac{S(\Delta\eta,\Delta\phi)}{B(\Delta\eta,\Delta\phi)}$$

Signal pair distribution:

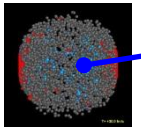
same event pairs



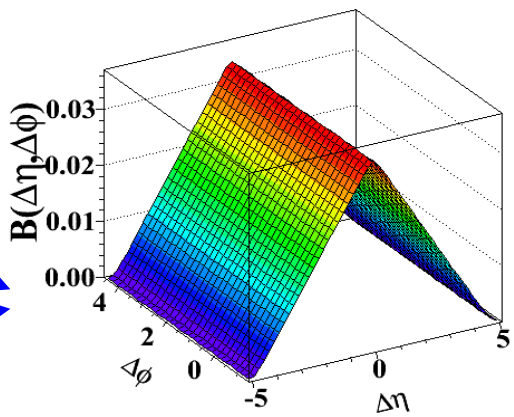
Event 1



Event 2



Background pair distribution:



mixed event pairs

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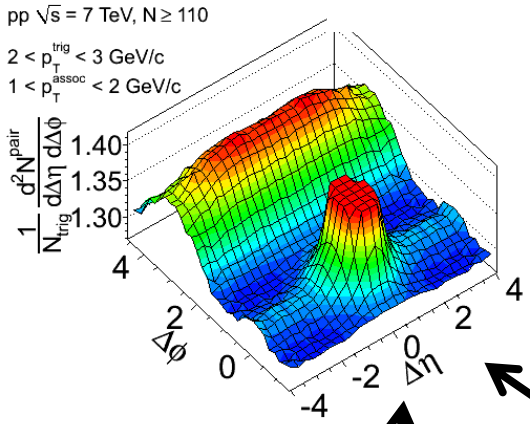
$$B(\Delta\eta,\Delta\phi) = \frac{1}{N_{trig}}\frac{d^2N^{mix}}{d\Delta\eta d\Delta\phi}$$

Correlation function

High multiplicity pp (N>110) $\sqrt{s} = 7\ TeV$



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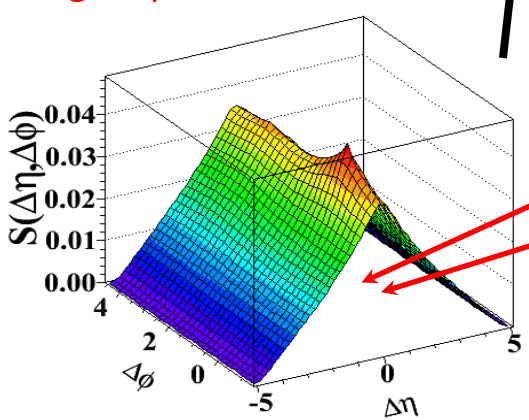
Associated hadron yield per trigger:

High multiplicity pp (N>110) $\sqrt{s} = 7\ TeV$

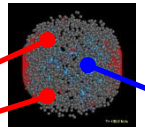
Signal pair distribution:

Background pair distribution:

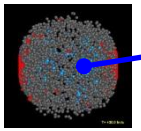
same event pairs

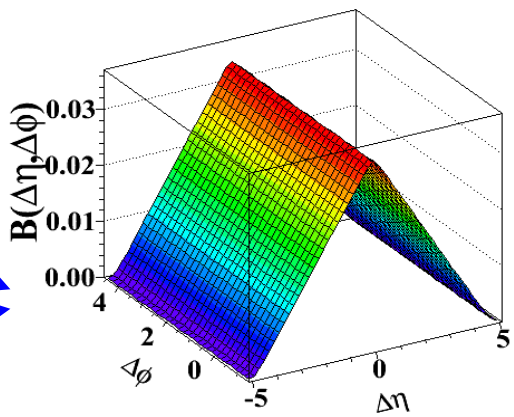


Event 1



Event 2





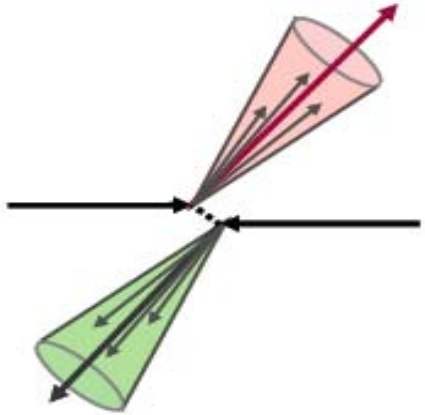
mixed event pairs

$$S(\Delta\eta, \Delta\phi) = \frac{1}{N_{trig}} \frac{d^2 N^{same}}{d\Delta\eta d\Delta\phi}$$

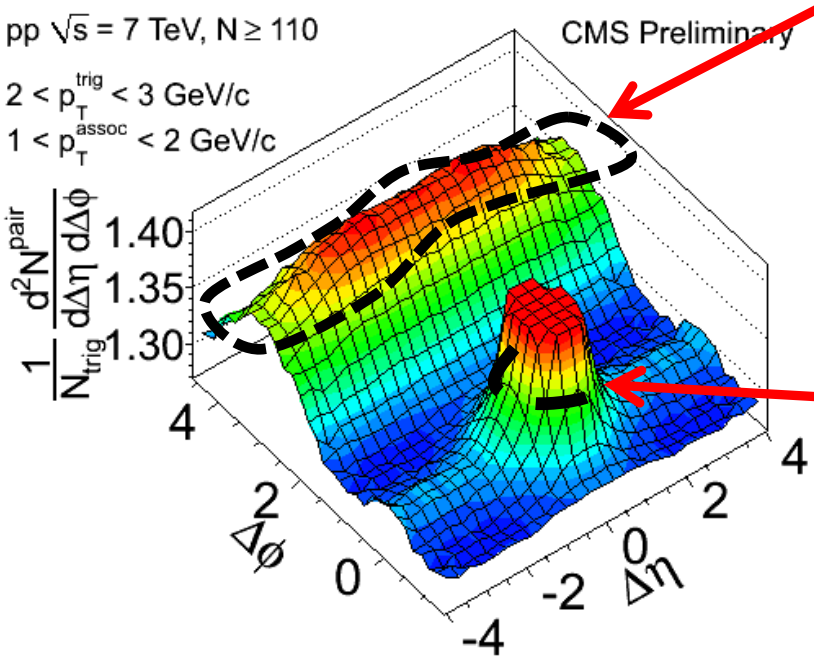
$$B(\Delta\eta, \Delta\phi) = \frac{1}{N_{trig}} \frac{d^2 N^{mix}}{d\Delta\eta d\Delta\phi}$$

Understanding the pp correlations

High multiplicity pp: more than 8 times more particles are produced from a single collision than average, 7e-5 selection



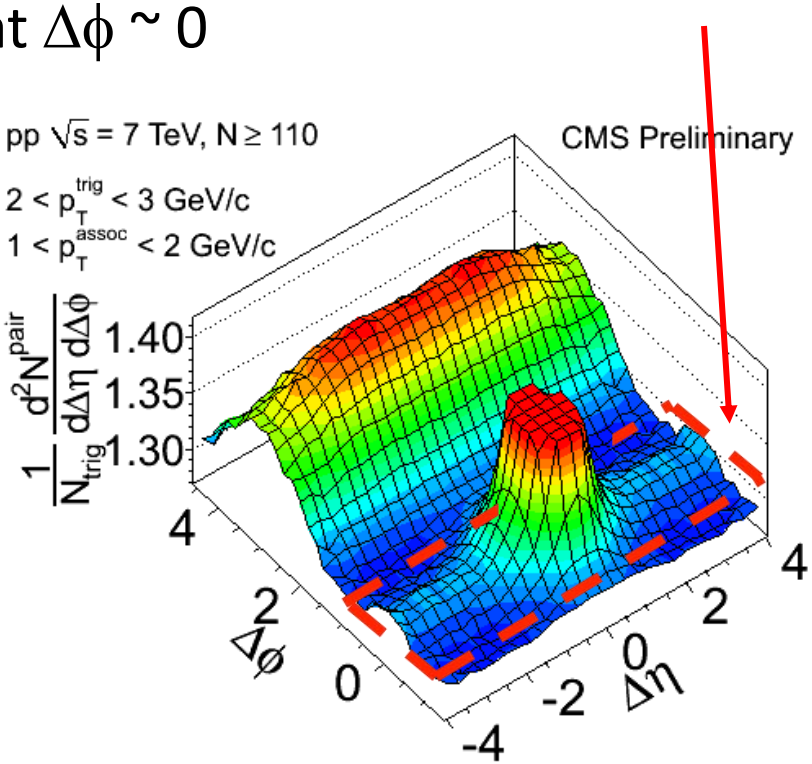
"Away-side" ($\Delta\phi \sim \pi$)
back-to-back jet correlations



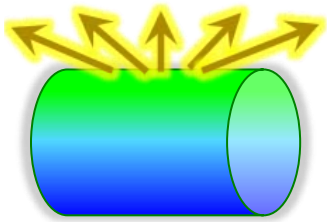
"Near-side" ($\Delta\phi$, $\Delta\eta \sim 0$)
correlations from single jets

Understanding the pp correlations

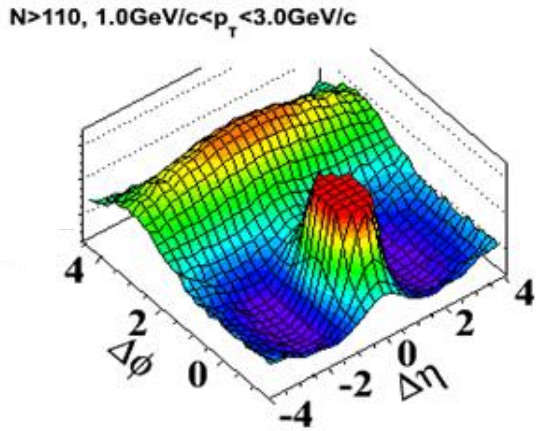
Striking **"ridge-like"** structure extending over $\Delta\eta$ at $\Delta\phi \sim 0$



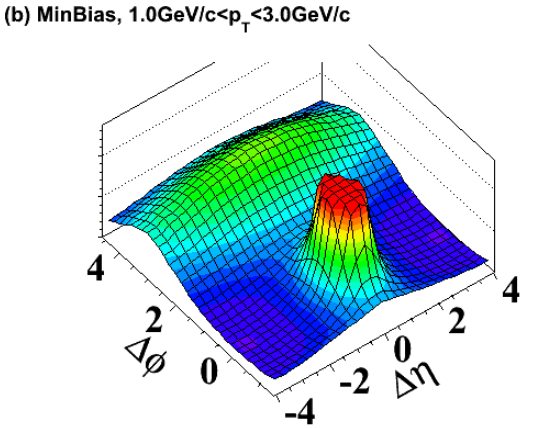
Not seen before in high multiplicity MC nor MinBias pp!



High multiplicity MC

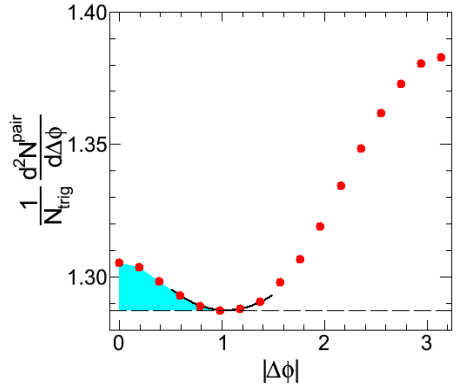


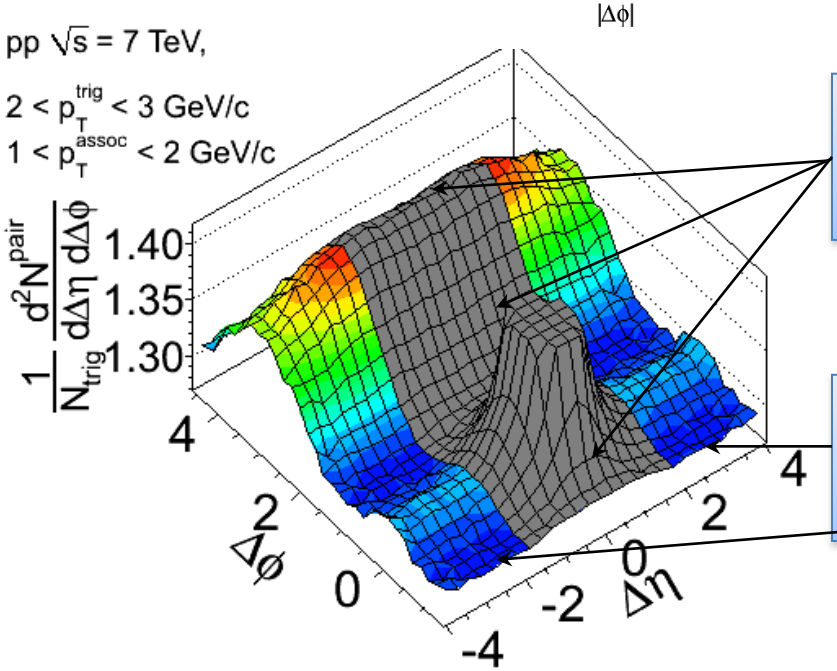
Minbias pp



Quantitative ridge Measurements

Zero-Yield-At-Minimum (ZYAM)



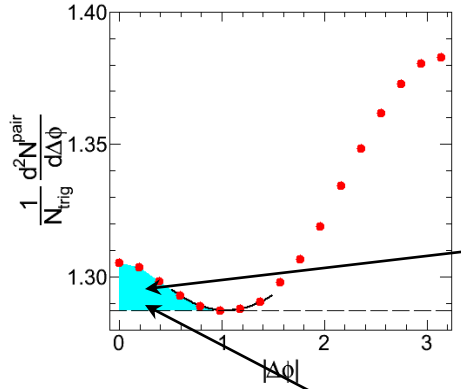


Jet region:
$|\Delta\eta| < 1$

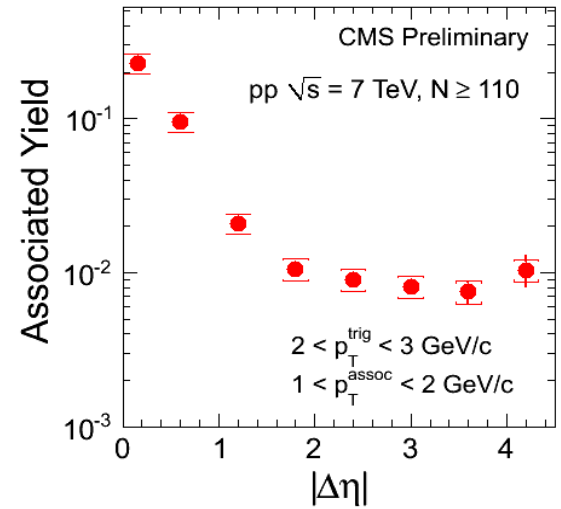
Ridge region:
$2<|\Delta\eta|<4$

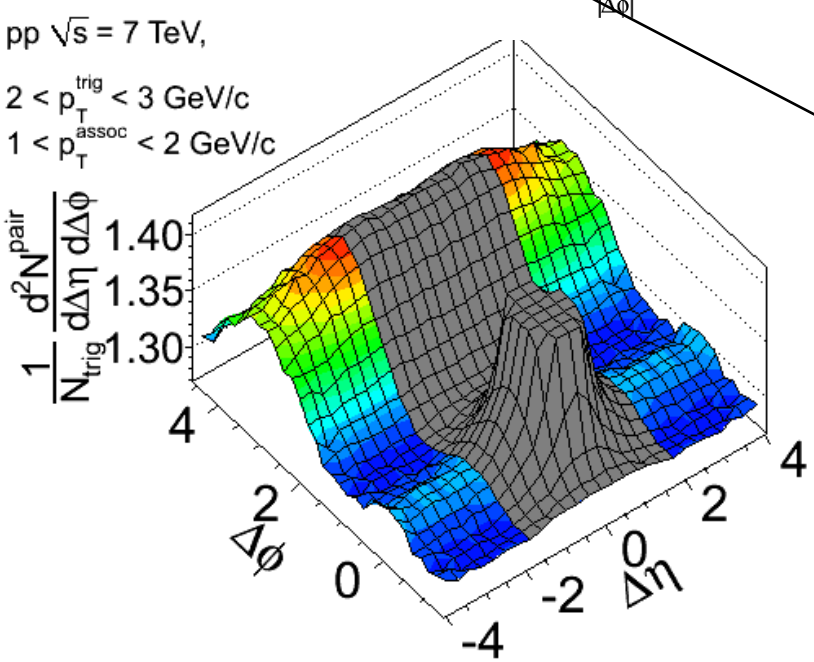
Ridge |Δη| and multiplicity dependence

Zero-Yield-At-Minimum (ZYAM)

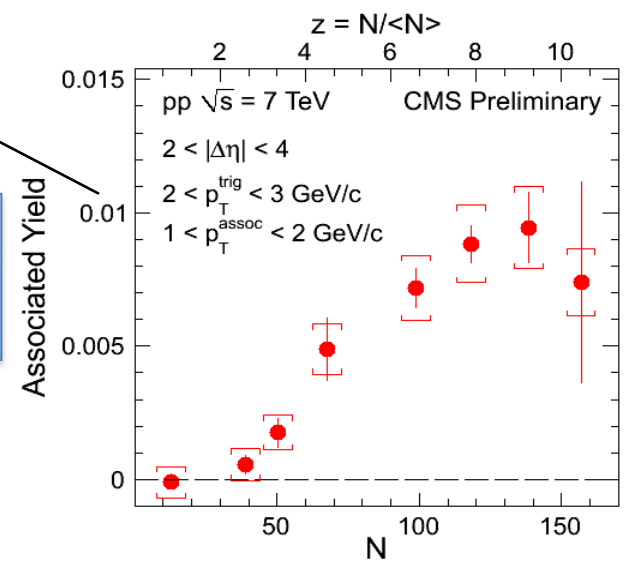


Ridge is flat in |Δη|

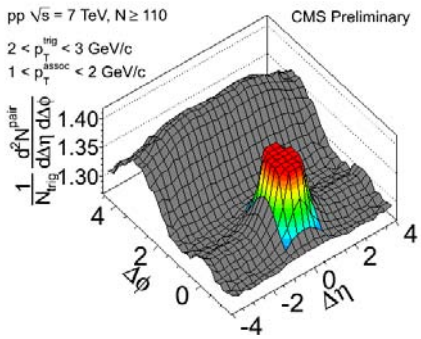


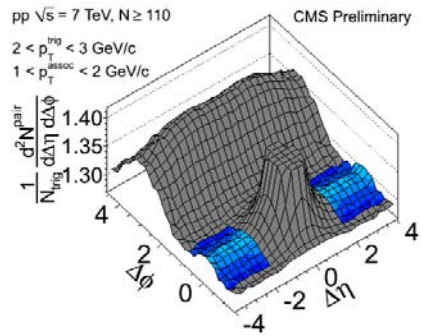


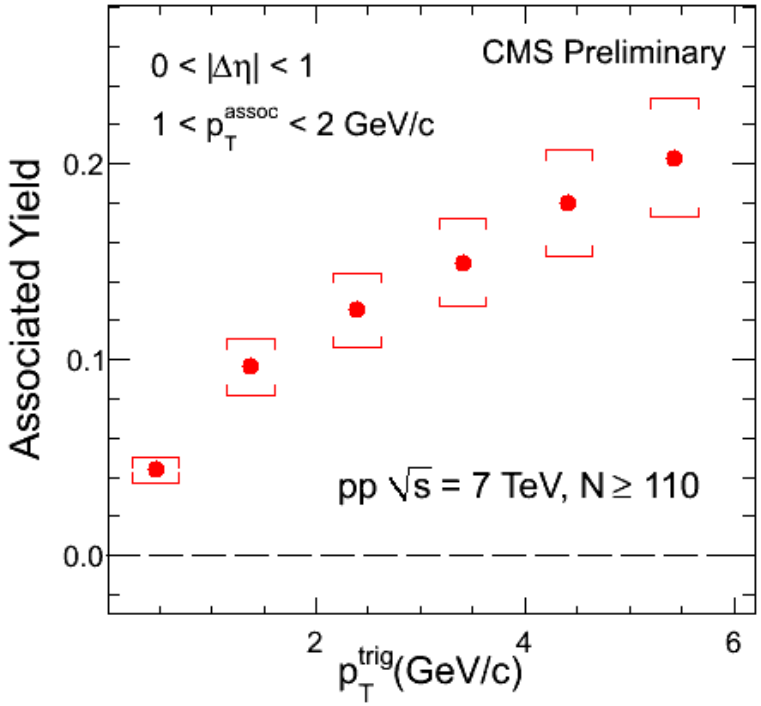
Ridge region: 2<|Δη|<4

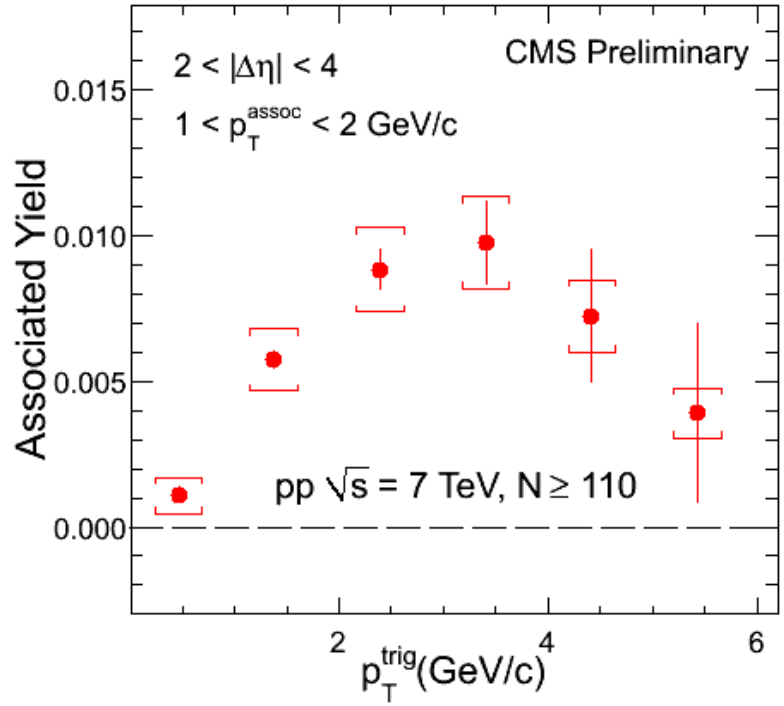


p_T dependence of the pp ridge





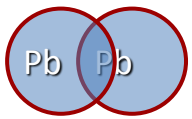




Jet region $|\Delta\eta|<1$

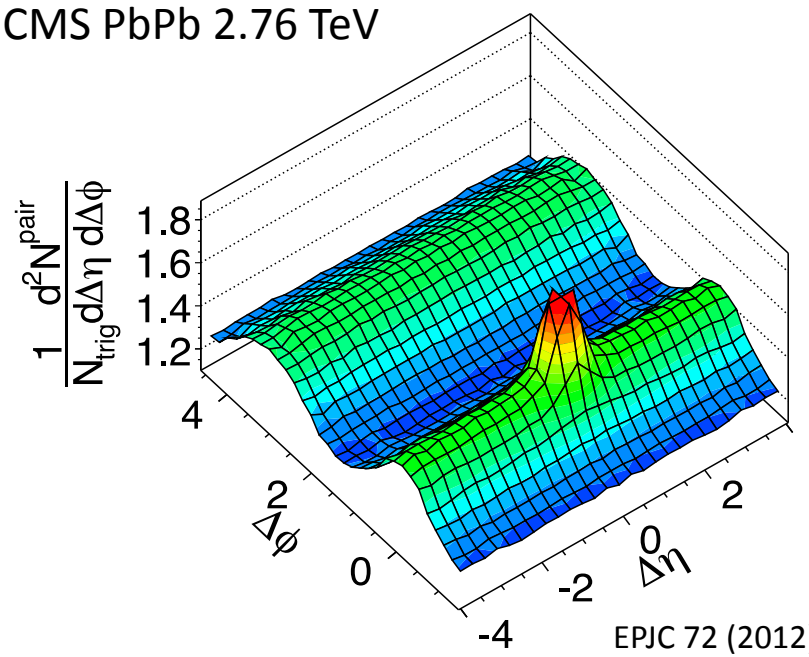
Ridge region: $2<|\Delta\eta|<4$

PbPb correlations



35-40% centrality

CMS PbPb 2.76 TeV



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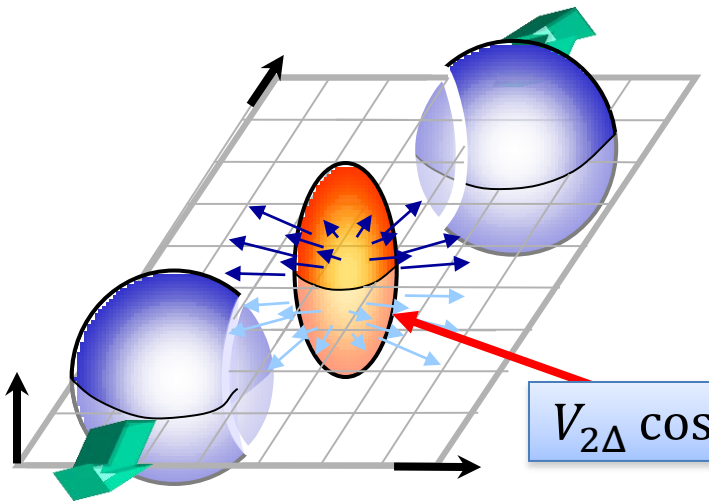
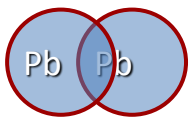
p_T^{trig}: 4–6 GeV/c

p_T^{assoc}: 2–4 GeV/c

PbPb correlations: Geometry

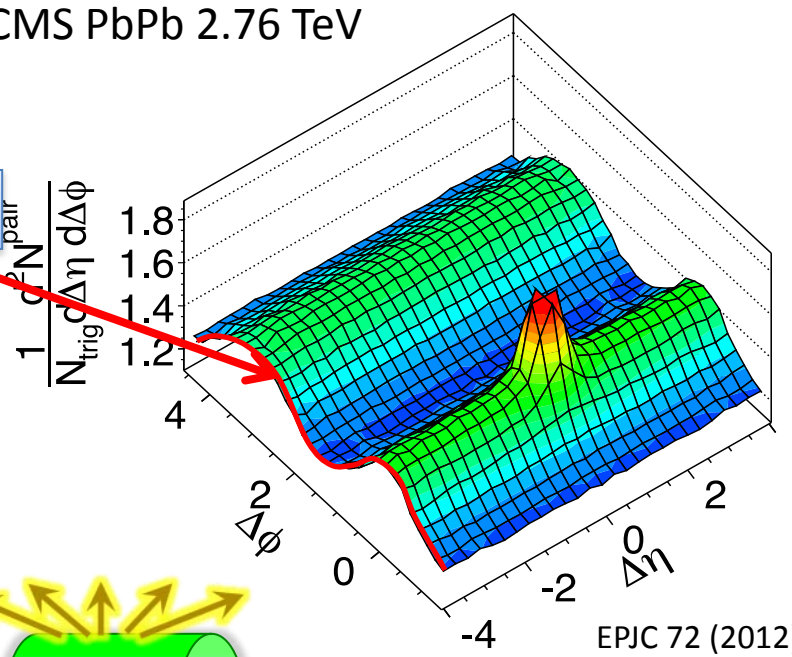


$V_{2\Delta}\cos(2\,\Delta\phi)$



35-40% centrality

CMS PbPb 2.76 TeV



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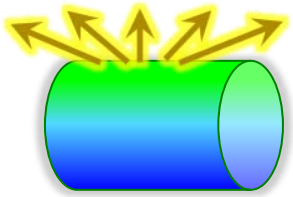

p_T^{trig}: 4–6 GeV/c

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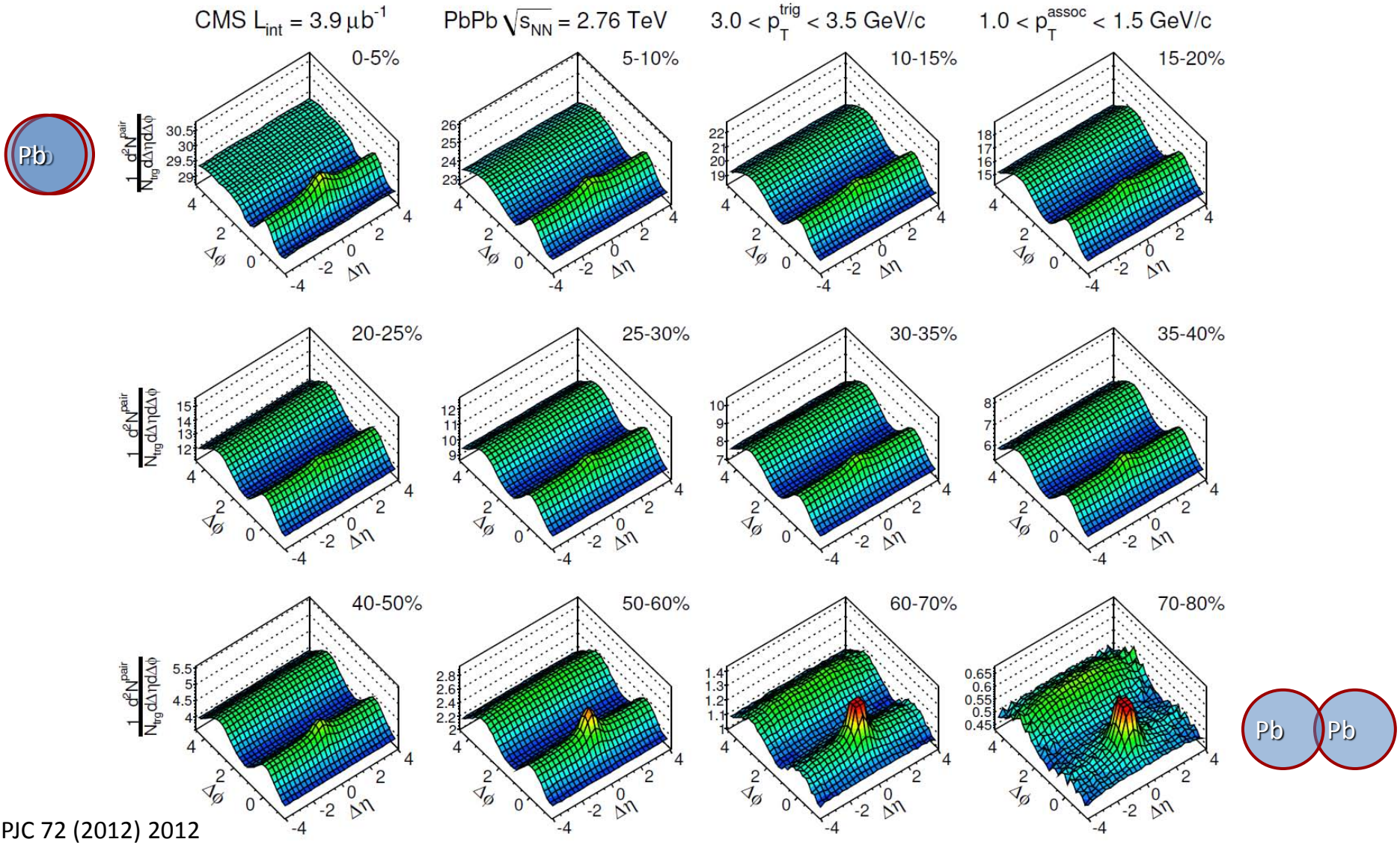
PbPb correlations: Geometry



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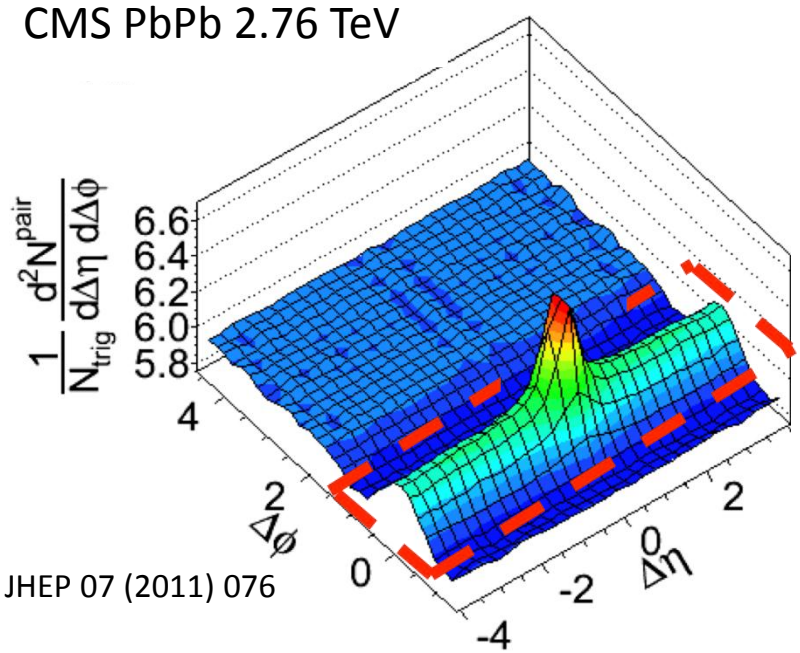
PbPb correlations: Geometry?

0-5% centrality

CMS PbPb 2.76 TeV



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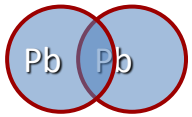
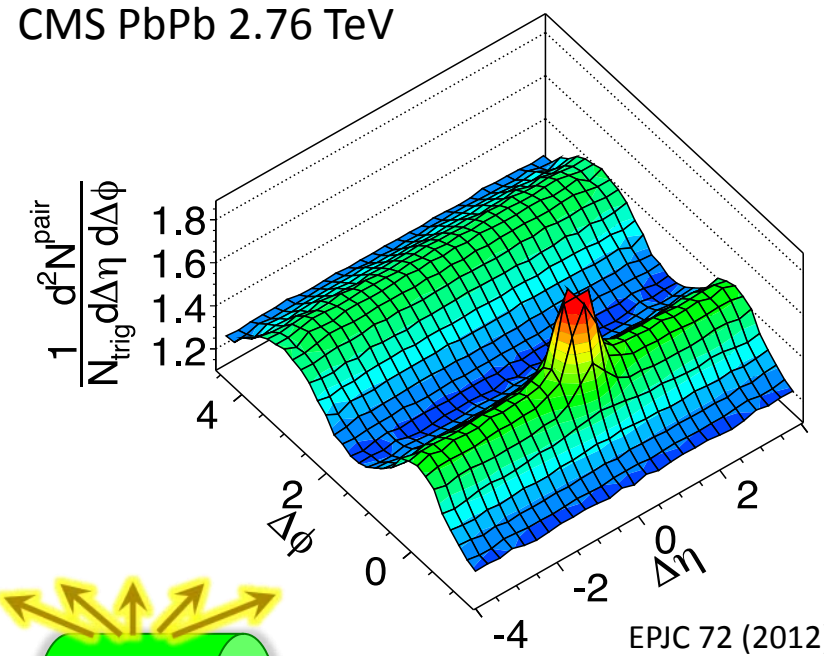
elliptic flow cos(2Δφ) term not prominent



35-40% centrality

CMS PbPb 2.76 TeV



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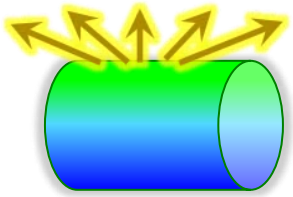
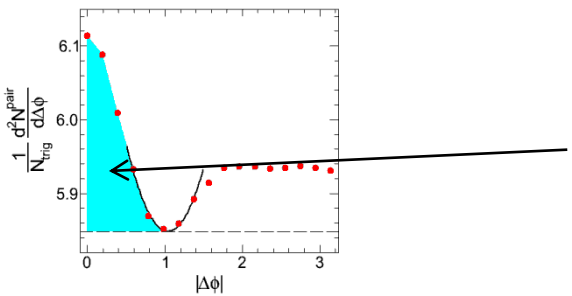


p_T^{trig}: 4–6 GeV/c

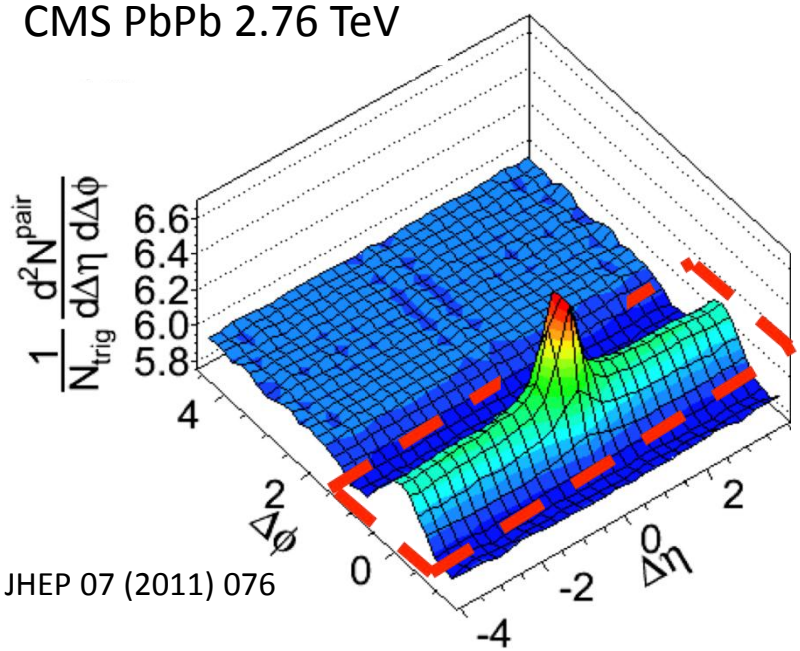
p_T^{assoc}: 2–4 GeV/c

PbPb correlations: Ridge Yield



Associated yield

CMS PbPb 2.76 TeV



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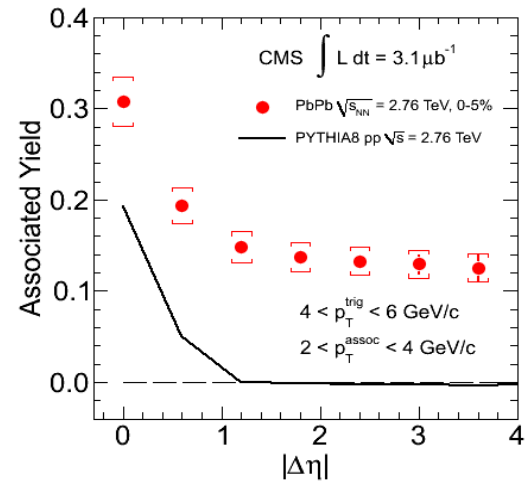
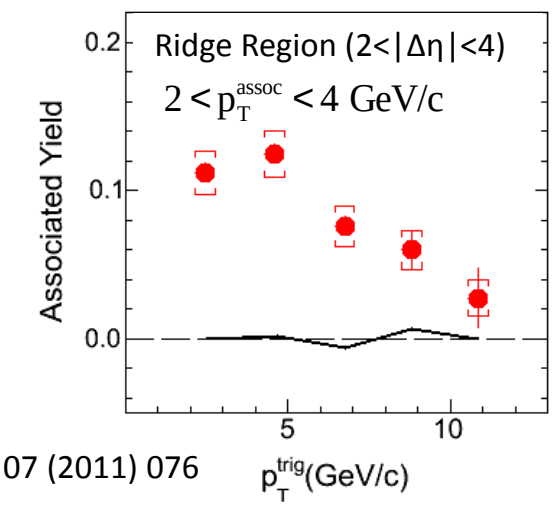
elliptic flow cos(2Δφ) term not prominent

Ridge is flat in Δη



Ridge yield goes down at high p_T



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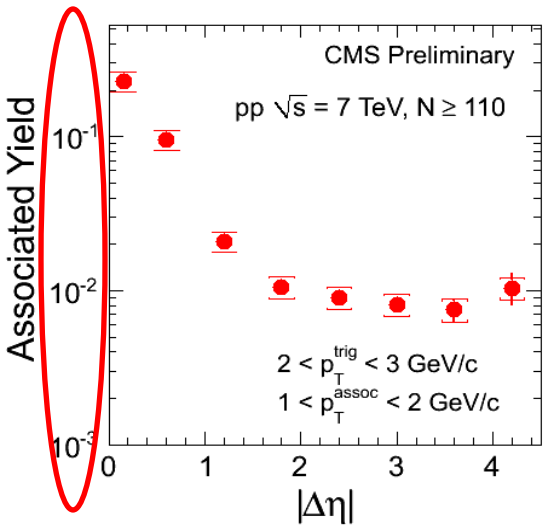
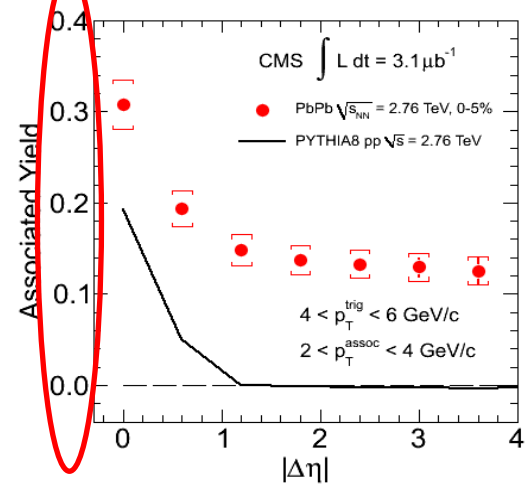
Comparison of ridge in pp and PbPb

Ridge is flat in Δη



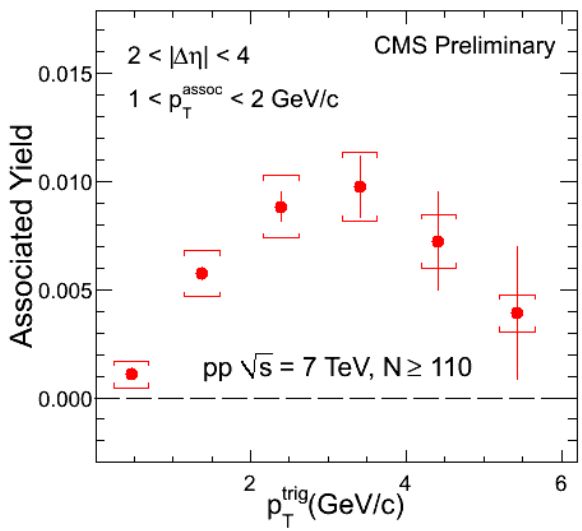
pp ⟷ PbPb

Ridge is flat in Δη



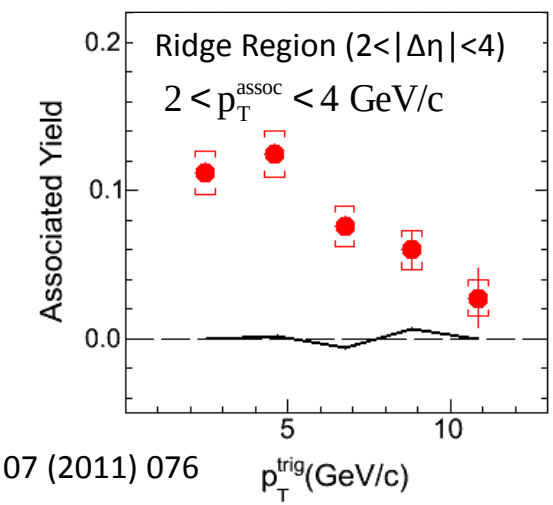
Qualitatively similar properties but not quantitatively

Ridge yield goes down at high p_T



pp ⟷ PbPb

Ridge yield goes down at high p_T

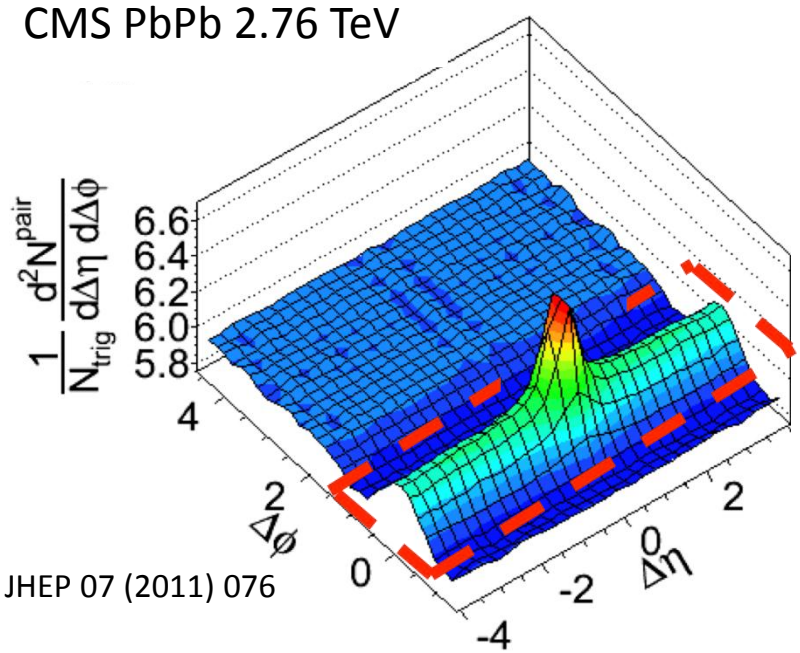


JHEP 07 (2011) 076

PbPb correlations: Geometry?

0-5% centrality

CMS PbPb 2.76 TeV



JHEP 07 (2011) 076

elliptic flow cos(2Δφ) term not prominent

PbPb correlations: Fluctuations

Elliptic flow Triangular flow

$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\left(\phi - \psi_2\right) + 2v_3 \cos 3\left(\phi - \psi_3\right) + \ldots$$

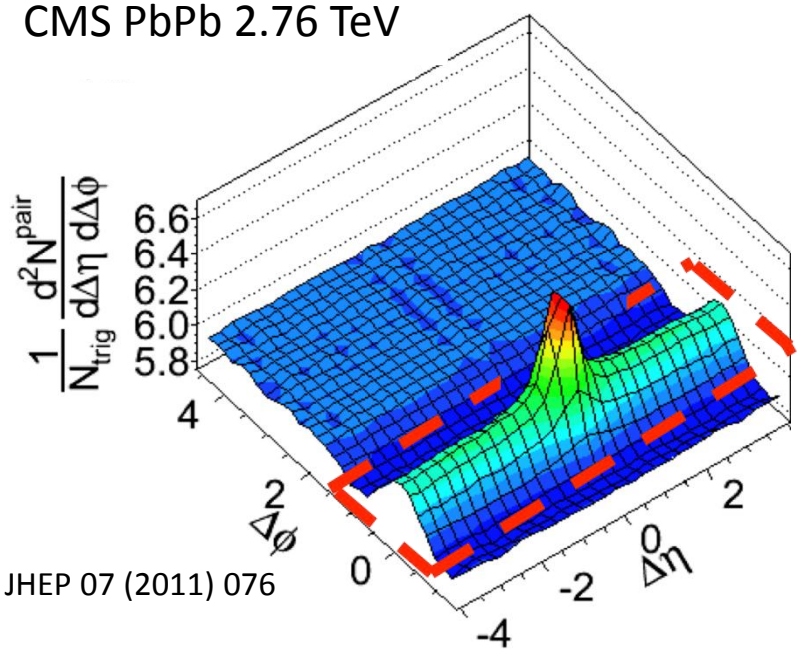
$$\frac{dN^{pair}}{d\Delta\phi} \propto 1 + 2v_2^2 \cos 2\Delta\phi + 2v_3^2 \cos 3\Delta\phi + \Longleftarrow$$

Two-particle correlation

0-5% centrality

n=2 + n=3 + n=4 + n=5

CMS PbPb 2.76 TeV



JHEP 07 (2011) 076

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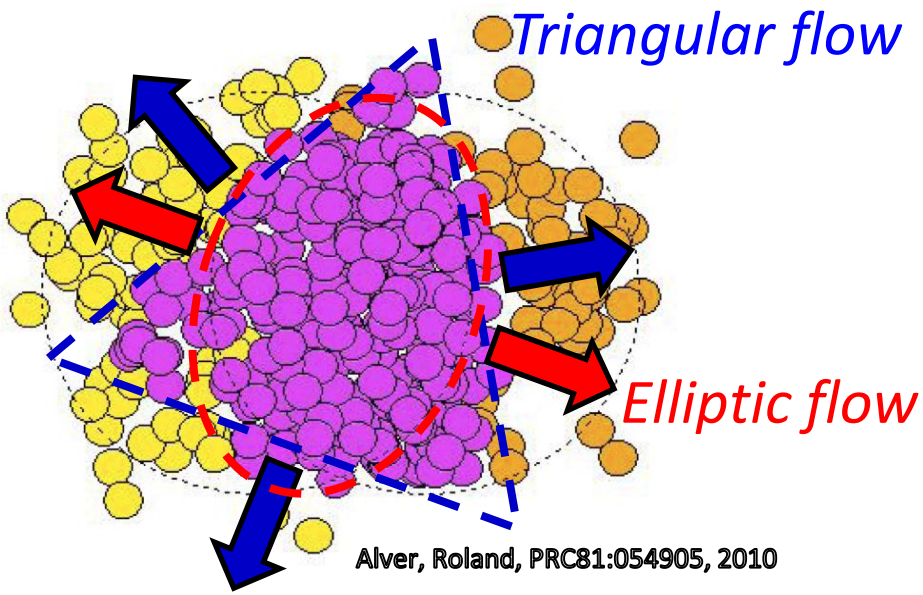
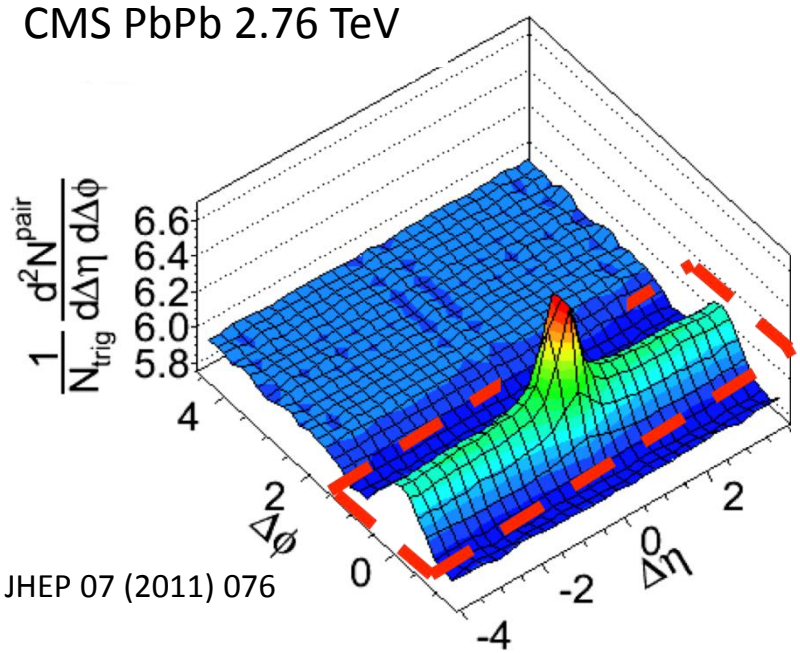


Alver, Roland, PRC81:054905, 2010

PbPb correlations: Fluctuations

0-5% centrality

CMS PbPb 2.76 TeV



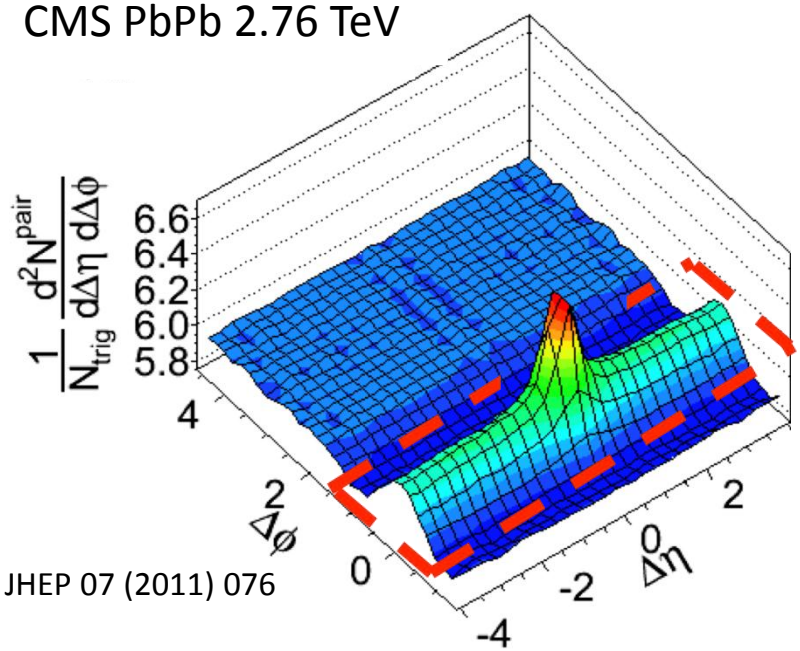
JHEP 07 (2011) 076

elliptic flow cos(2Δφ) term not prominent

PbPb correlations: Fourier Analysis

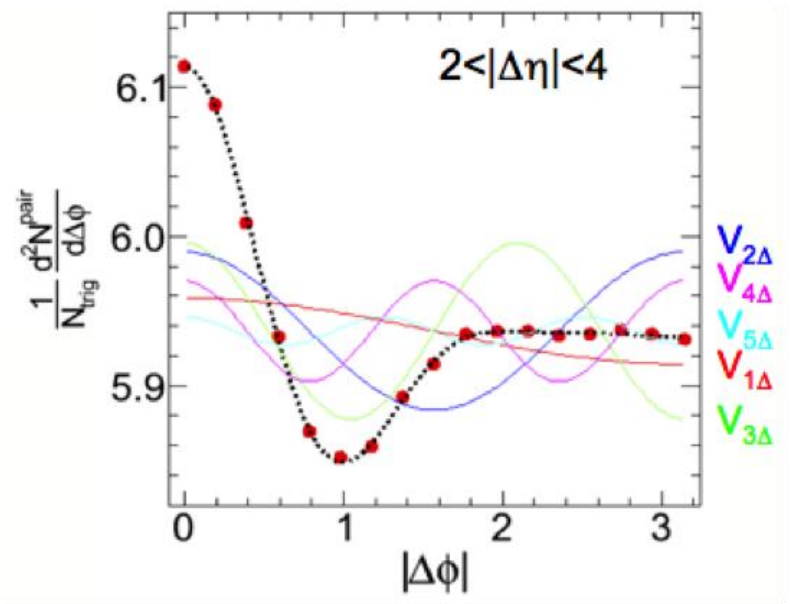
0-5% centrality

CMS PbPb 2.76 TeV



JHEP 07 (2011) 076

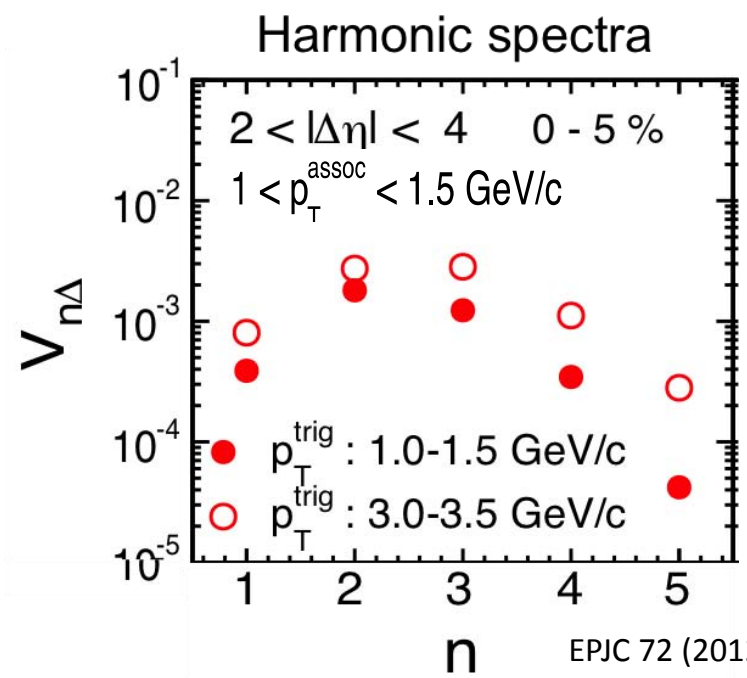
elliptic flow cos(2Δφ) term not prominent



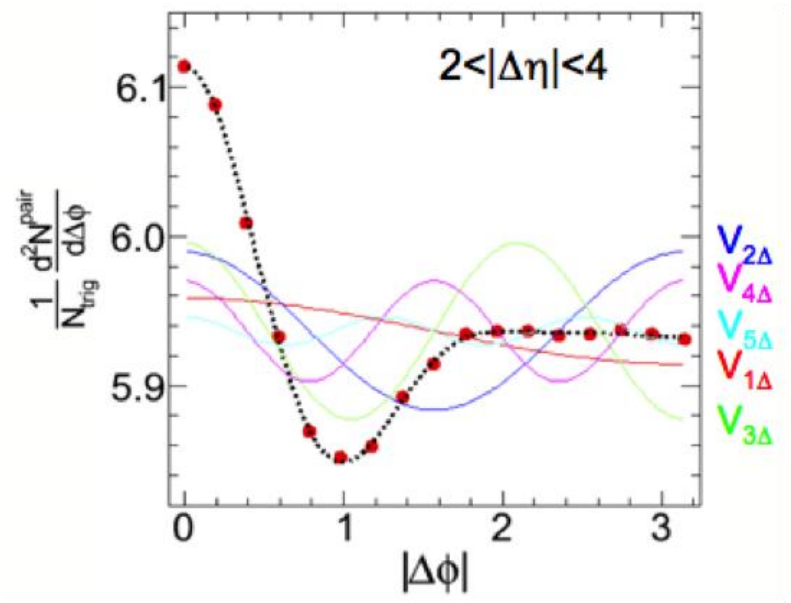
Fourier decomposition:

$$\frac{dN^{pair}}{d\Delta\phi} \propto 1 + \sum_{n=1} V_{n\Delta} \cos(n\Delta\phi)$$

PbPb correlations: Fourier Analysis



EPJC 72 (2012) 2012



Fourier decomposition:

$$\frac{dN^{pair}}{d\Delta\phi} \propto 1 + \sum_{n=1} V_{n\Delta} \cos(n\Delta\phi)$$

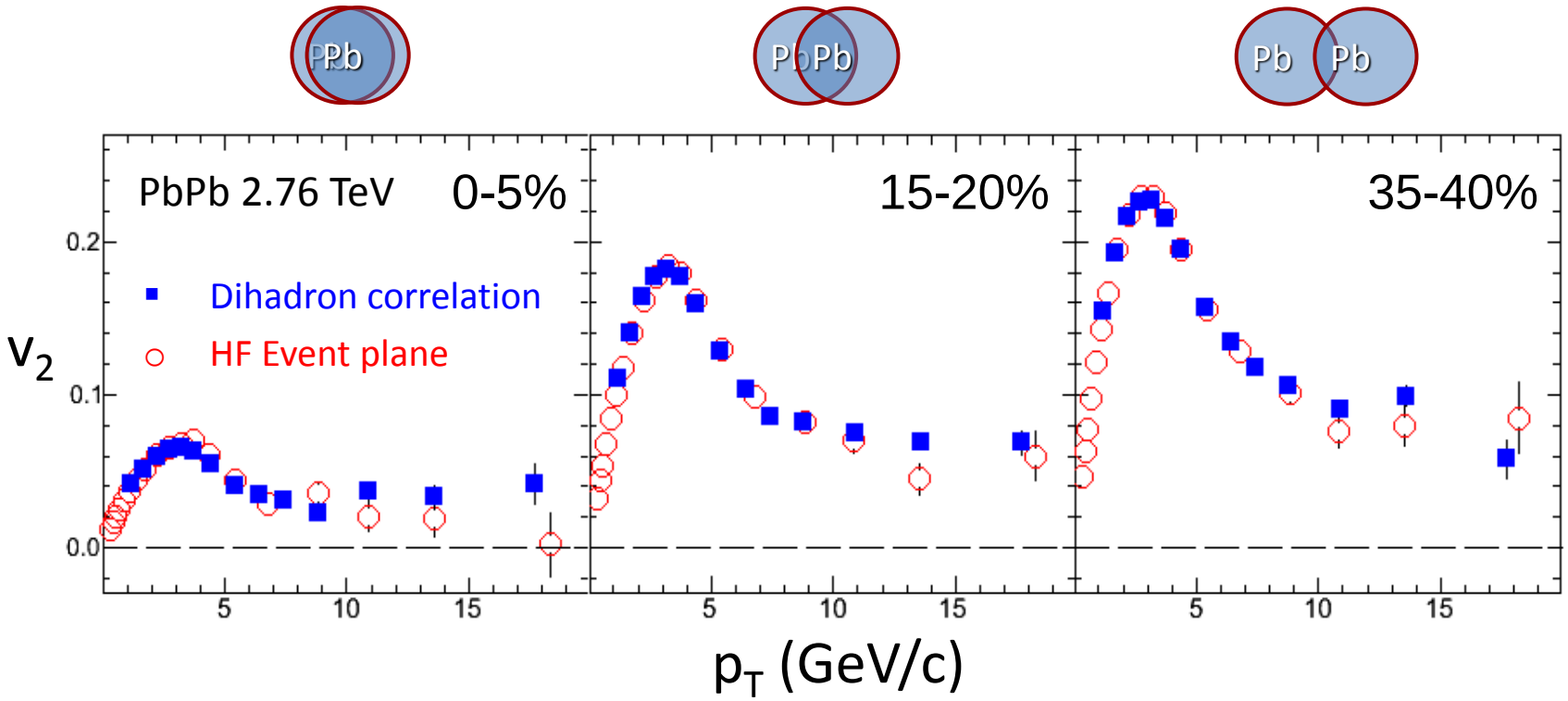
For flow driven correlations:

$$V_{n\Delta} = v_n\left(p_T^{trig}\right) \times v_n\left(p_T^{assoc}\right)$$ —— factorization

However,

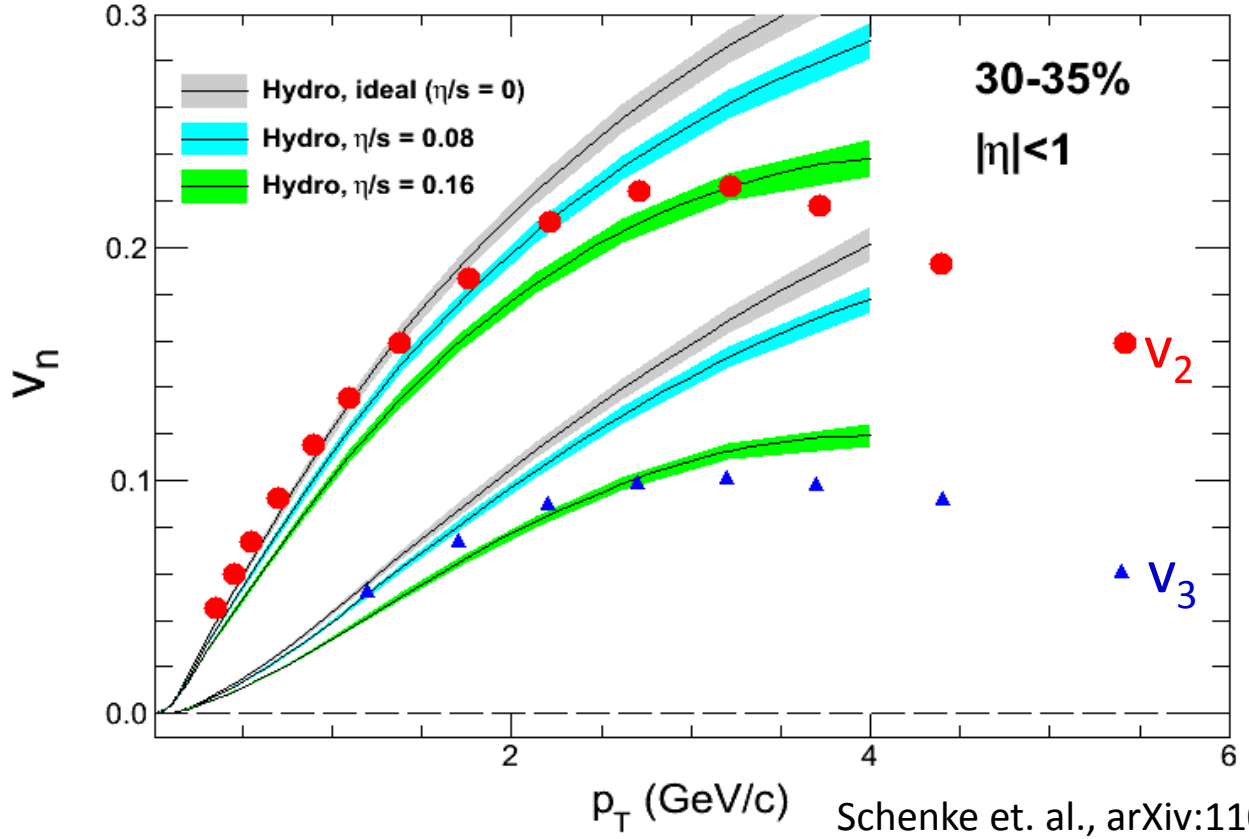
- p_T dependent flow fluctuation could break the factorization
- Non-flow could also factorize

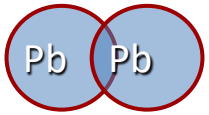
“Factorization” seems to work



$v_2(p_T)$ from correlation method derived using fixed p_T^{assoc} of 1-1.5 GeV/c agrees well with EP method

More Constraints from Higher v_n





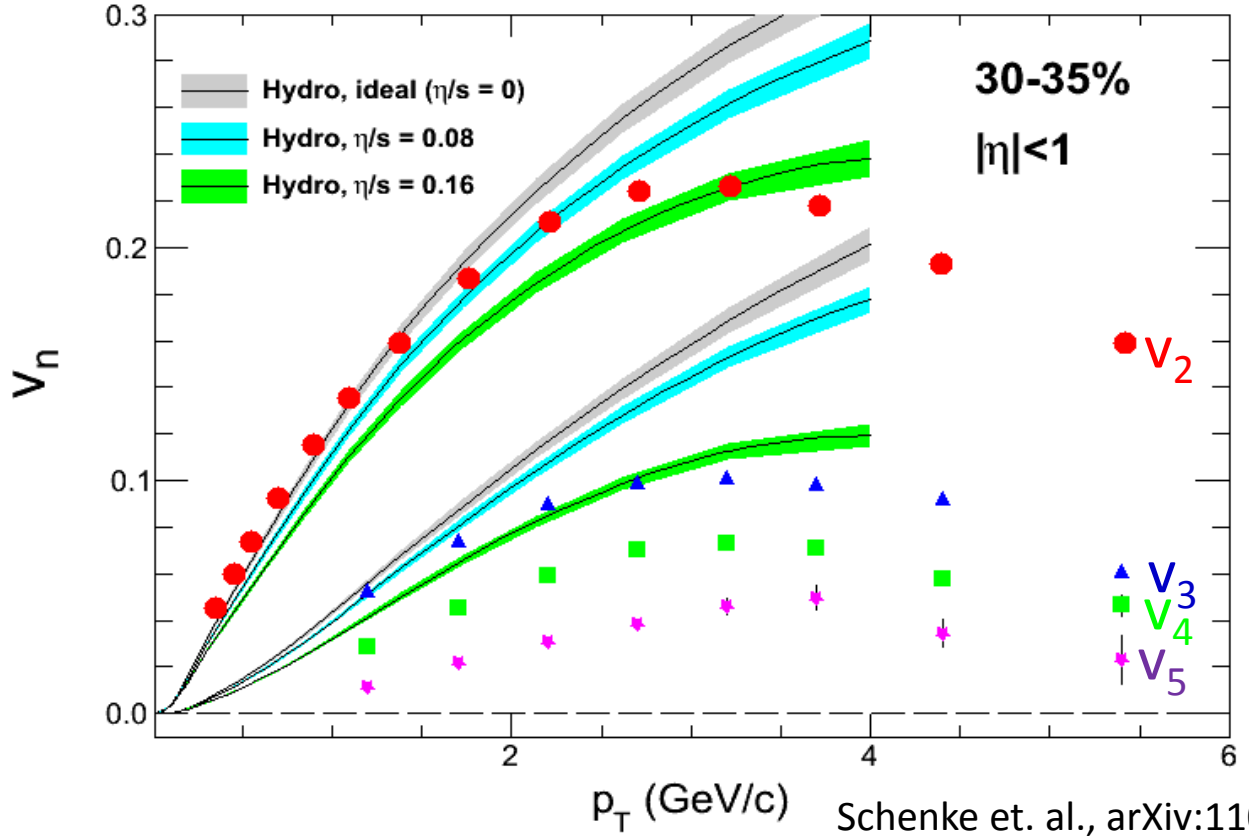
Schenke et. al., arXiv:1109.6289

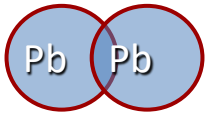
Measure all v_n to over-constrain the hydro calculations

✧ η/s

✧ Initial conditions (Glauber vs CGC)

More Constraints from Higher v_n





Schenke et. al., arXiv:1109.6289

Measure all v_n to over-constrain the hydro calculations

✧ η/s

✧ Initial conditions (Glauber vs CGC)

Summary

- pp-ridge extends out to at least $|\Delta\eta| \approx 4$, tends to flat in $\Delta\eta$ - long range correlation
- pp-ridge first appears at 3-4× average multiplicity and seems to saturate at 8-10×
- pp-ridge yield peaks at intermediate $p_T^{trig} \approx 3$ GeV/c and decreases at high p_T^{trig}
- PbPb also exhibits long range correlations that rise and fall with p_T^{trig} and centrality
- Precise measurements of higher v_n will better nail down the PbPb initial conditions and η/s

Backup

Factorization of Fourier Harmonics

Test of factorization:

$$V_{n\Delta}\left(p_T^{trig} p_T^{assoc}\right) \longleftrightarrow v_n\left(p_T^{trig}\right) \times v_n\left(p_T^{assoc}\right)$$

p_T^{assoc}

$V_{n\Delta}(m,1)$	$V_{n\Delta}(m,2)$	$V_{n\Delta}(m,3)$		$V_{n\Delta}(m,m)$
⋮	⋮	⋮	⋰	
$V_{n\Delta}(3,1)$	$V_{n\Delta}(3,2)$	$V_{n\Delta}(3,3)$	⋯	$V_{n\Delta}(3,m)$
$V_{n\Delta}(2,1)$	$V_{n\Delta}(2,2)$	$V_{n\Delta}(2,3)$	⋯	$V_{n\Delta}(2,m)$
$V_{n\Delta}(1,1)$	$V_{n\Delta}(1,2)$	$V_{n\Delta}(1,3)$	⋯	$V_{n\Delta}(1,m)$

p_T^{trig}

Assuming factorization

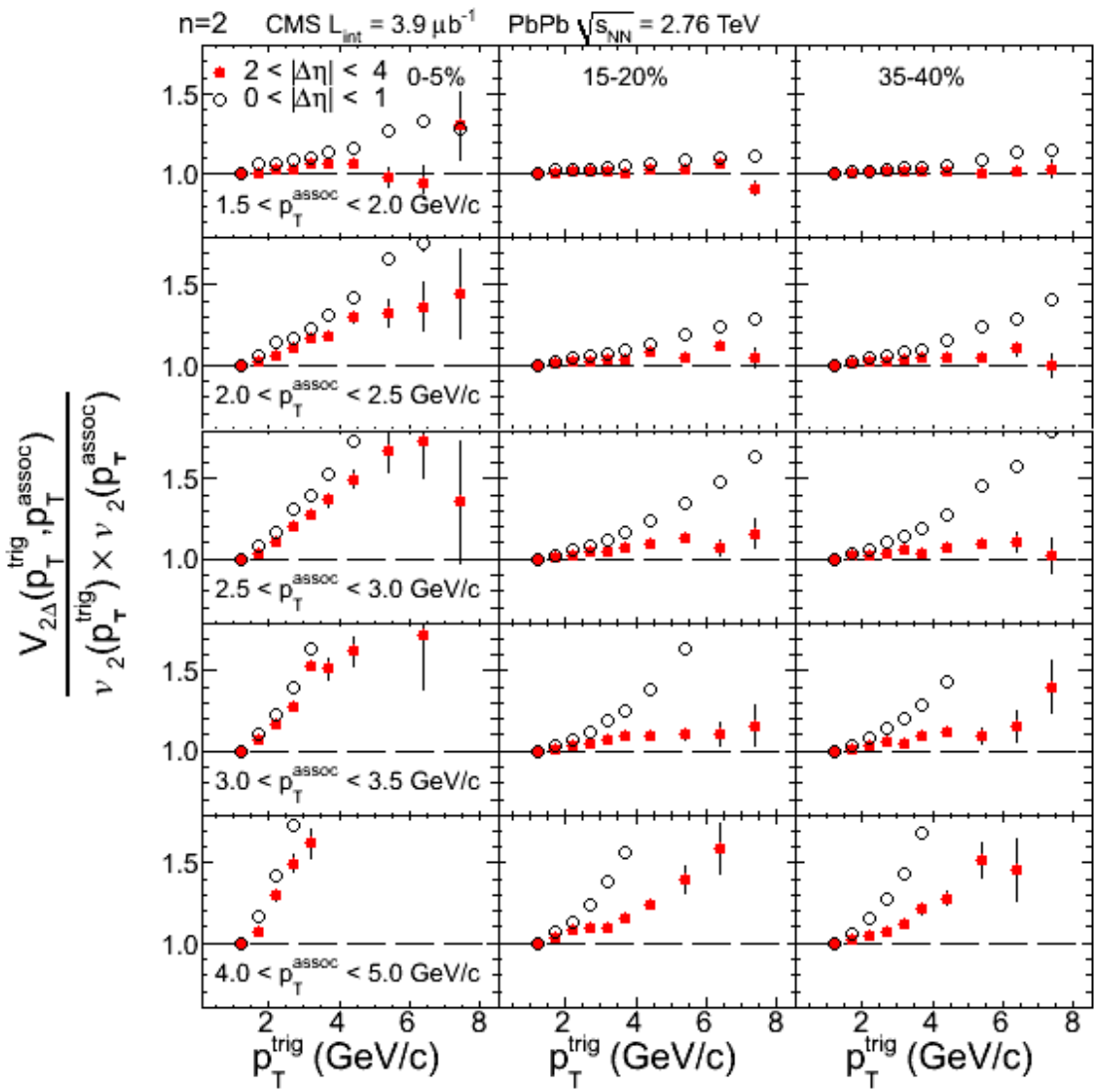
$$v_n\left(p_T\right) = \frac{V_{n\Delta}(p_T,1)}{\sqrt{V_{n\Delta}(1,1)}}$$

Take low reference p_T bin (1-1.5 GeV/c)

Check if $v_n(p_T)$ can derive rest of matrix:

$$\frac{V_{n\Delta}\left(p_T^{trig},p_T^{assoc}\right)}{v_n\left(p_T^{trig}\right)\times v_n\left(p_T^{assoc}\right)} \stackrel{?}{=} 1$$

Factorization of Fourier Harmonics



EPJC 72 (2012) 2012

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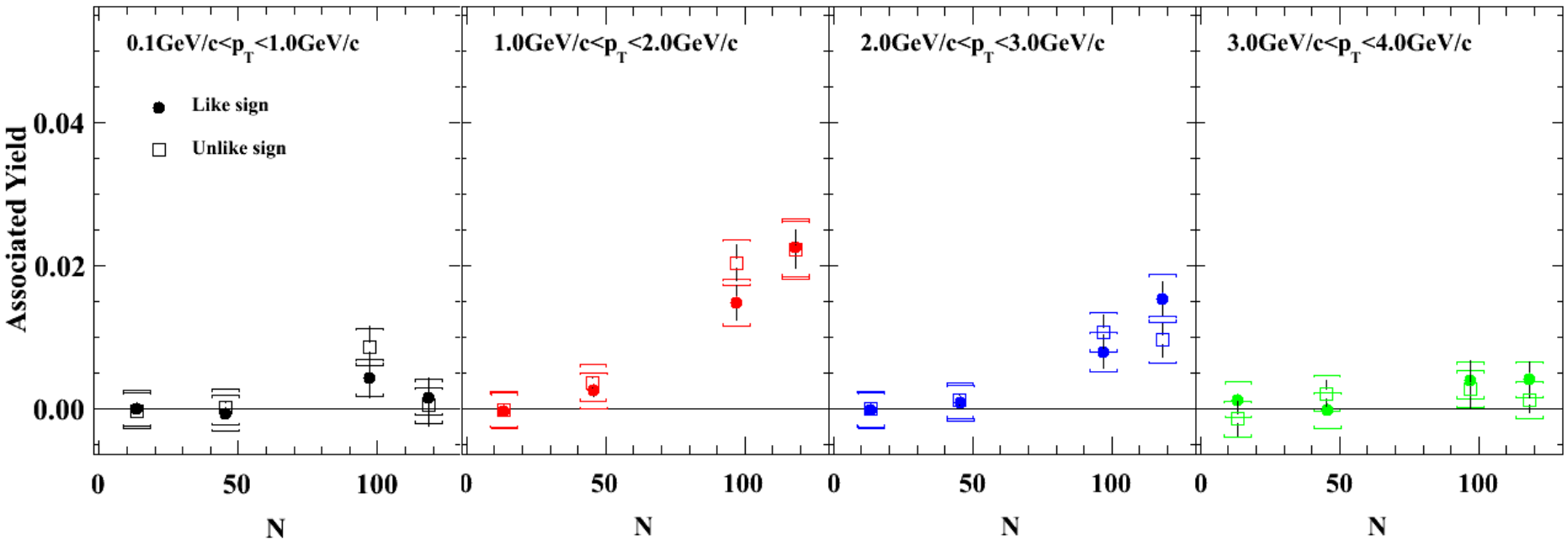
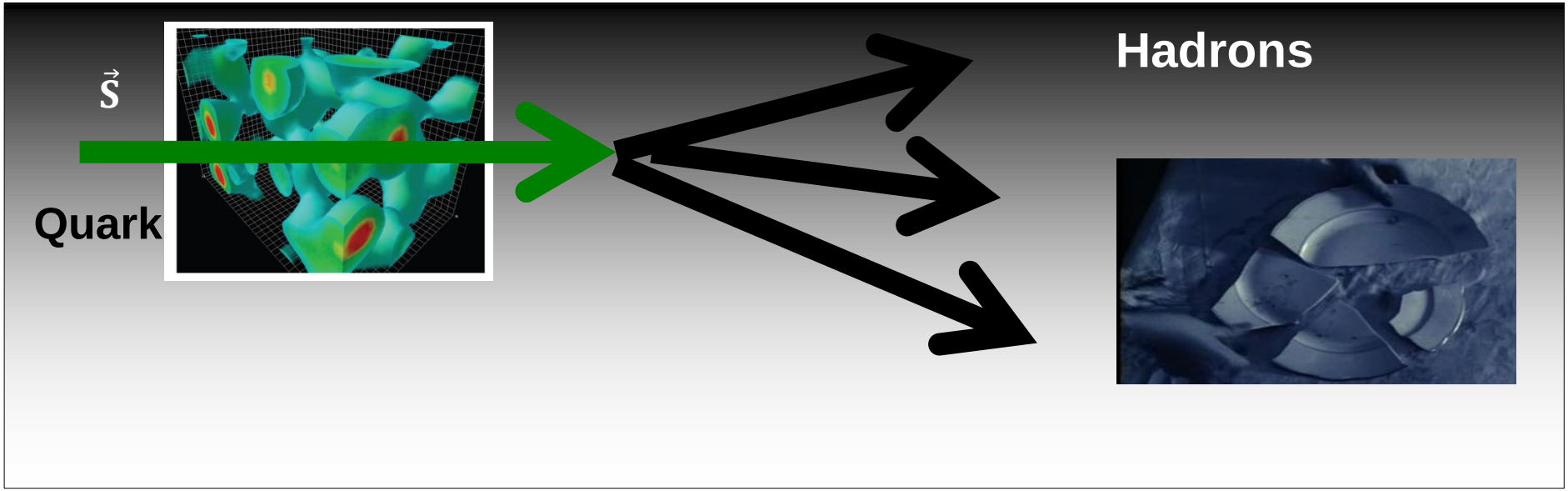
Like-Sign vs Unlike-Sign pp ridge



No dependence on relative charge sign

Prospects for Detecting Local Parity Violating Effects in Quark Fragmentation at

BELLE and Belle II



Anselm Vossen
INDIANA UNIVERSITY

CP-Violation in quark Fragmentation

CERN–TH/95–139

CP-Violating Effect of QCD Vacuum in Quark Fragmentation

A. Efremov[a,1] and D. Kharzeev[b,c,2]

[a]) *Laboratory of Theoretical Physics*
Joint Institute for Nuclear Research
Dubna, 141980 Russia

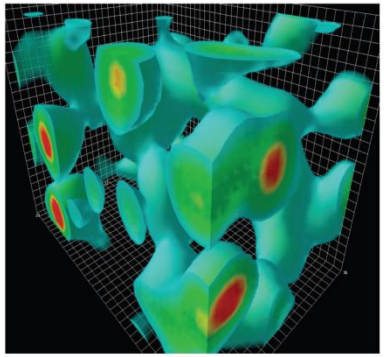
[b]) *Theory Division*
CERN
Geneva, Switzerland

[c]) *Fakultät für Physik*
Universität Bielefeld
33615 Bielefeld, Germany

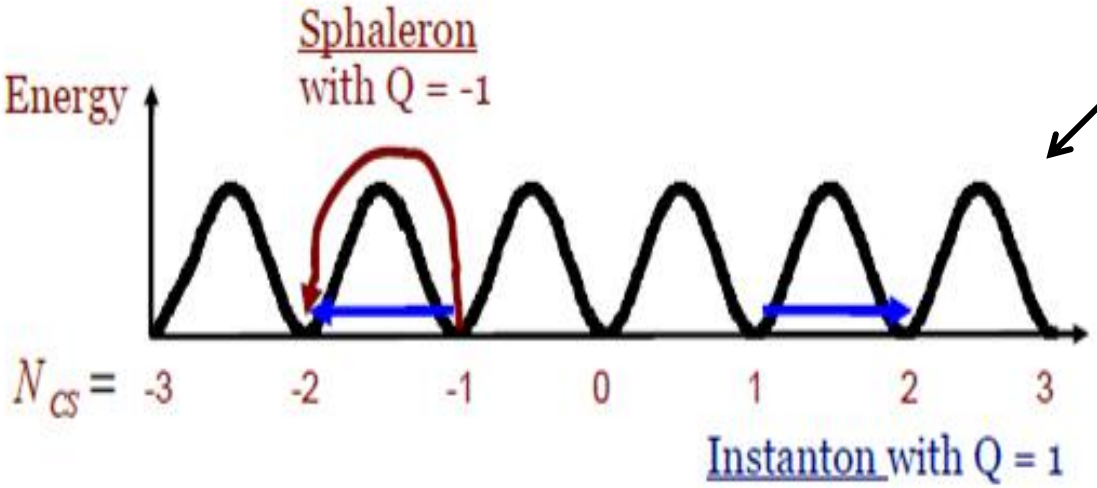
Abstract

We demonstrate that the non-trivial CP-violating structure of QCD vacuum can lead to an observable effect in $e^+e^- \to q\bar{q} \to$ 2-jet annihilation. We find that the sign of the jet handedness correlation can be opposite to that predicted by factorization of $q\ \bar{q}$ fragmentation and CP-conjugation of the two jets. A simple model estimation of the handedness correlation magnitude is given.

Transitions in the QCD vacuum carry net chirality



The QCD Vacuum

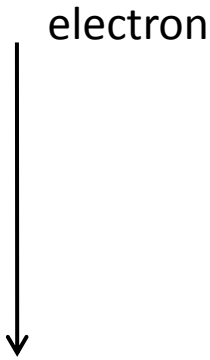


Difference in winding number:
Net chirality carried by
Instanton/Sphaleron

- Vacuum states are characterized by “winding number”
- Transition amplitudes: Gluon configurations, carry net chirality
- e.g. quarks: net spin momentum alignment

Picture: H. Warringa,

Event Topology in e+ e-



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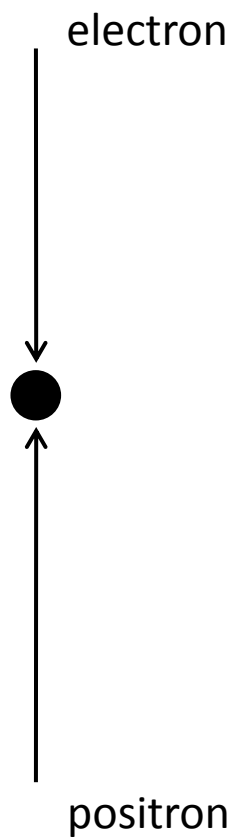

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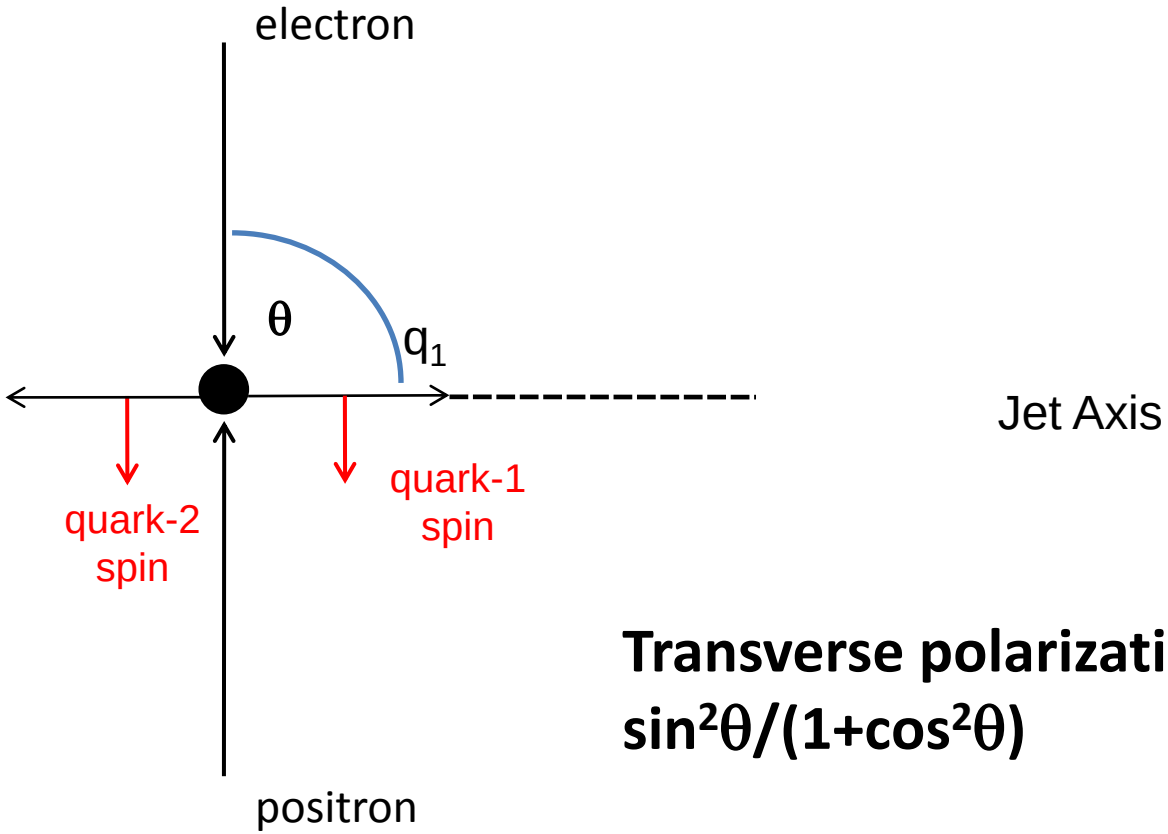
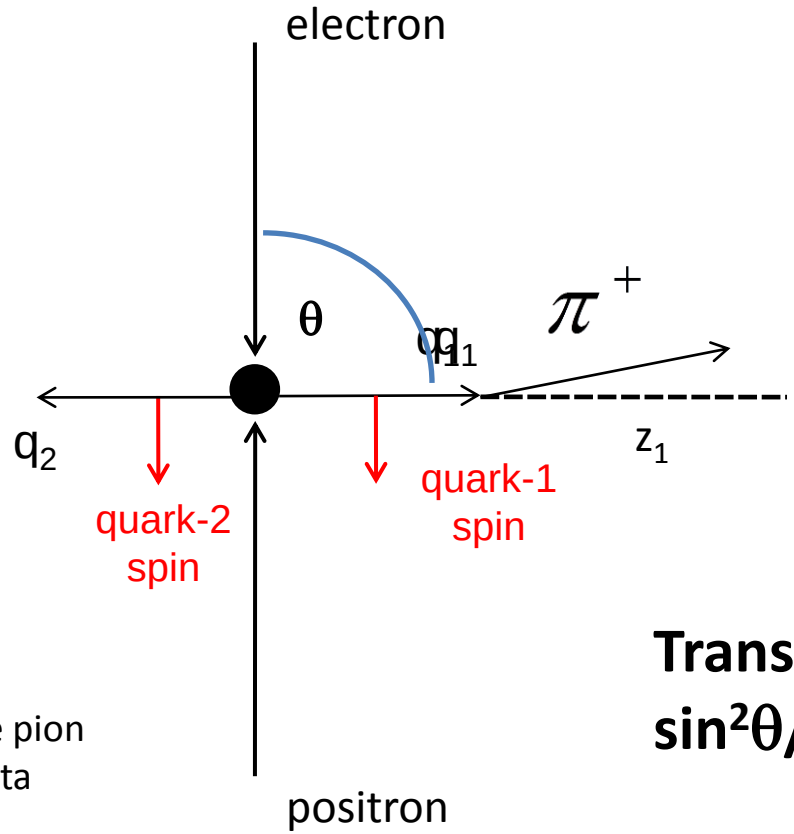


Transverse polarization:
$\sin^2\theta/(1+\cos^2\theta)$

Event Topology in e+ e-

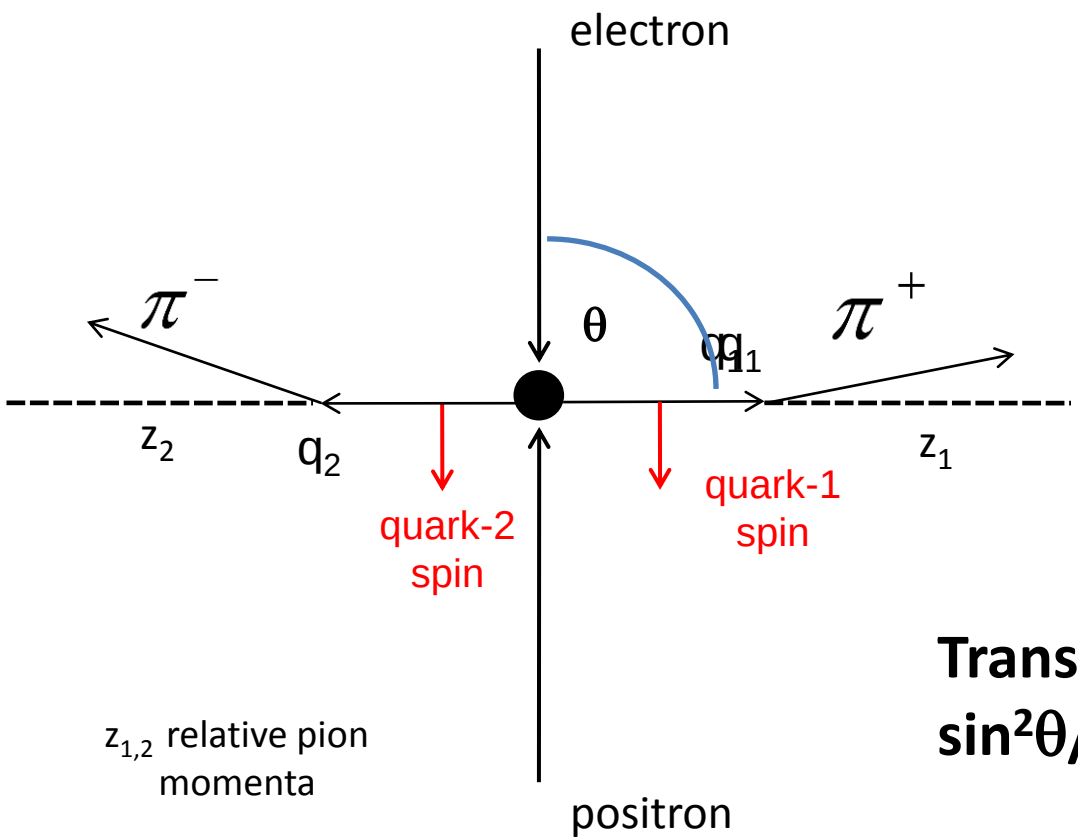


Jet Axis

$z_{1,2}$ relative pion momenta

Transverse polarization:
$\sin^2\theta/(1+\cos^2\theta)$

Event Topology in e+ e-



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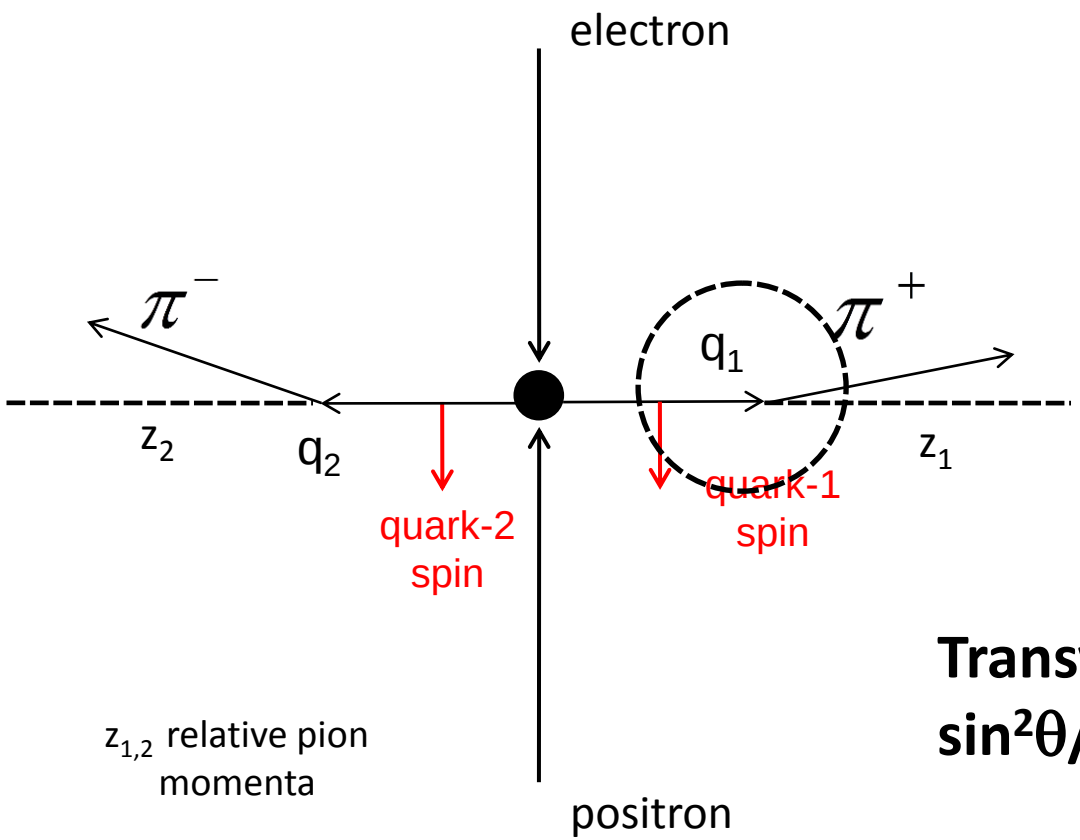

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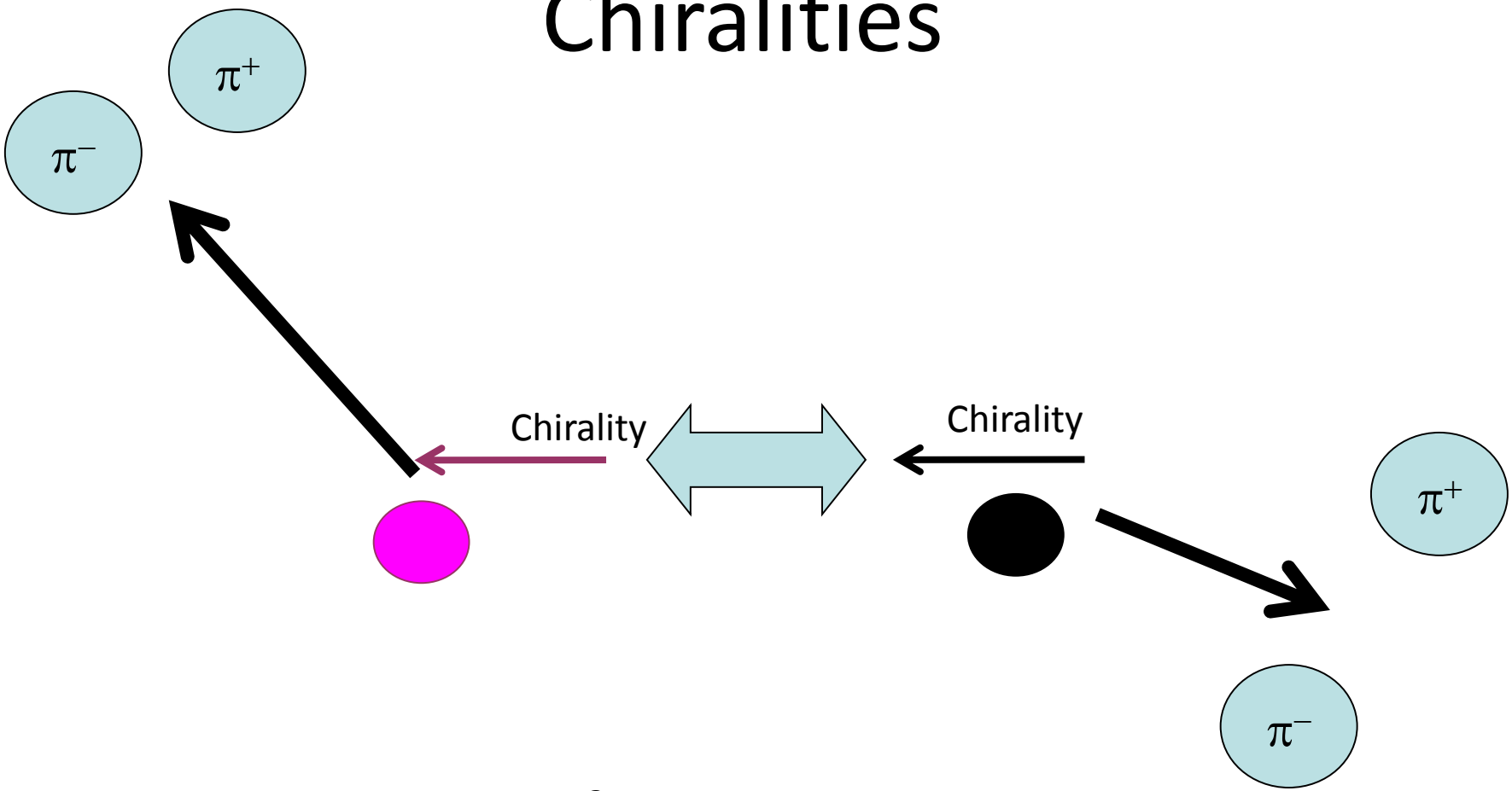
Jet Handedness Sensitive to quark Chiralities

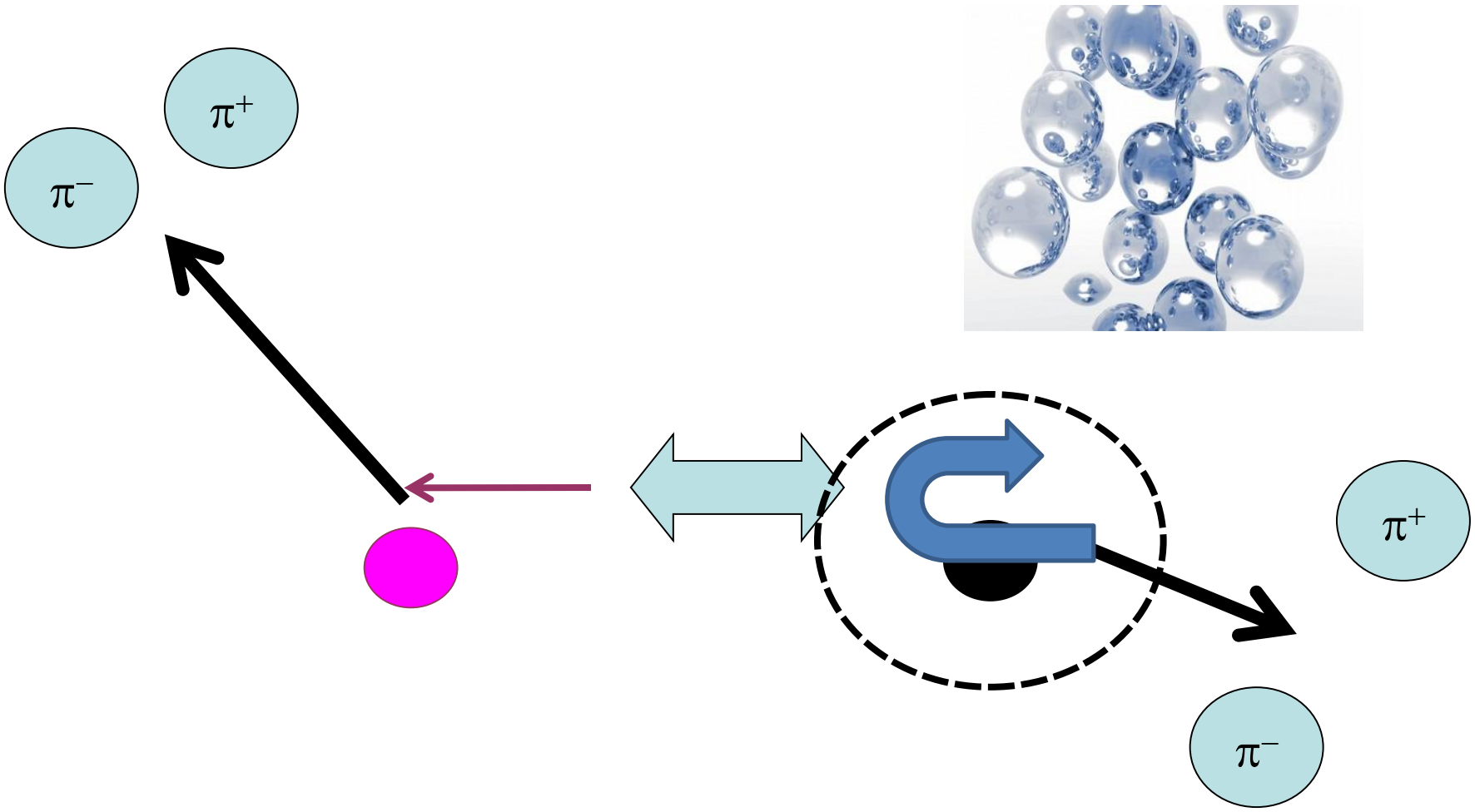


Handedness: $\frac{(\overrightarrow{k_+} \times \overrightarrow{k_-}) \cdot \vec{t}}{|\, k_+ \,||\, k_- \,|} = \sin\Phi \overset{?}{>} 0$ ⇨ L/R

Jet *handedness:* $\frac{N_R - N_L}{N_R + N_L}$

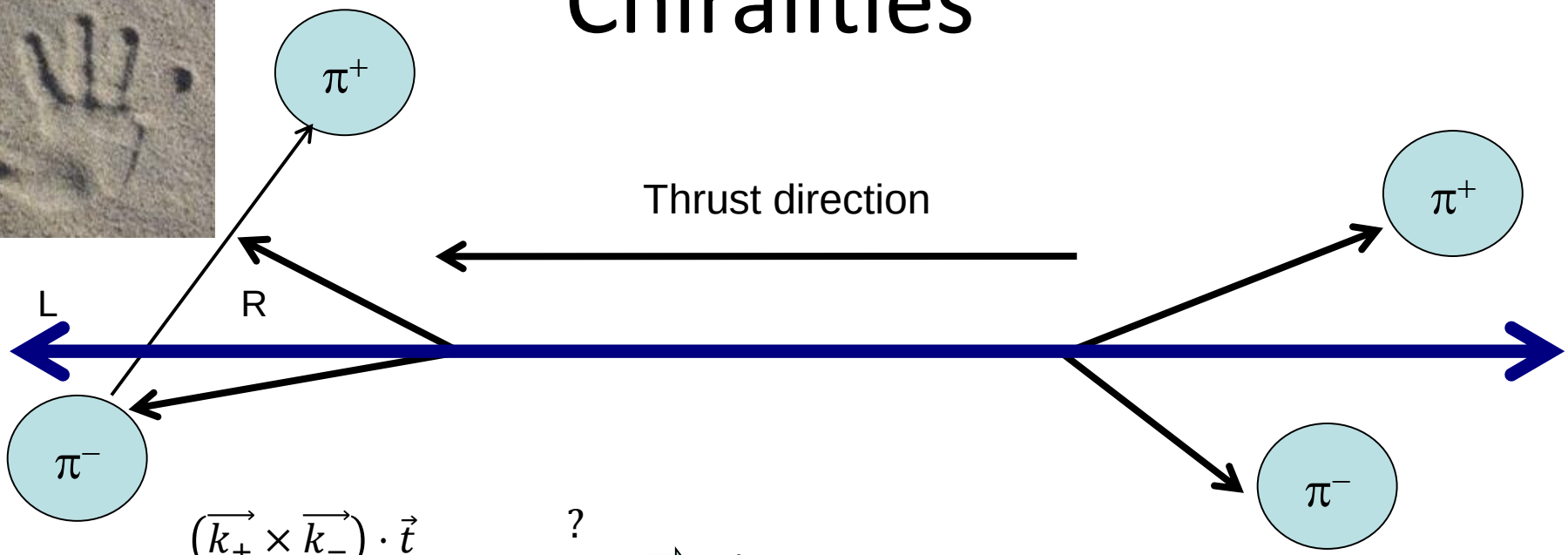
Efremov, Khazeev, Phys. Lett. B366, 311

Fragmentation in the Big Bubble



Efremov, Khazeev, Phys. Lett. B366, 311

Jet Handedness Sensitive to quark Chiralities



Handedness: $\frac{(\overrightarrow{k_+} \times \overrightarrow{k_-}) \cdot \vec{t}}{|\, k_+ \,||\, k_- \,|} = \sin\Phi \overset{?}{>} 0$ ⇨ L/R

Jet *handedness:* $\frac{N_R - N_L}{N_R + N_L}$

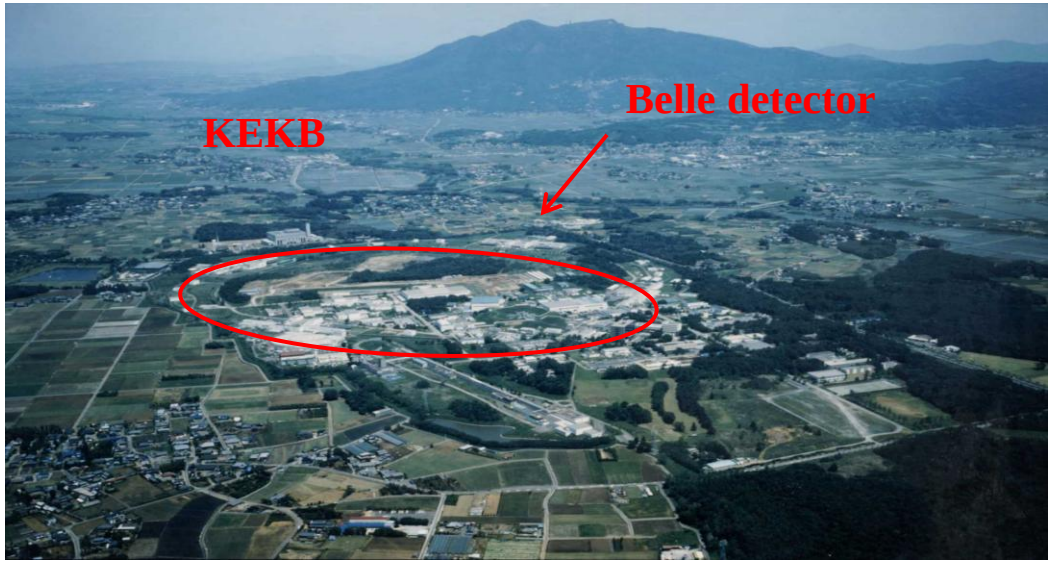
C: $\frac{N_{RL} + N_{LR} - N_{RR} - N_{LL}}{N_{RL} + N_{LR} + N_{RR} + N_{LL}}$

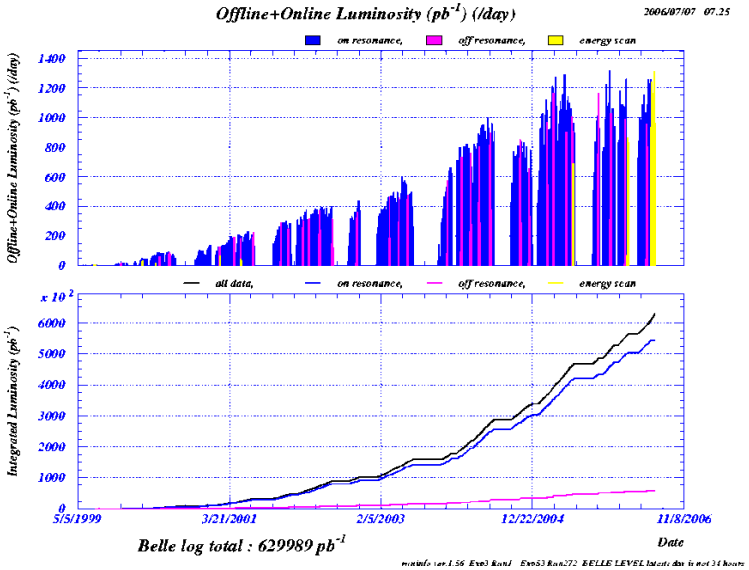
Expect positive correlation for P-odd effect

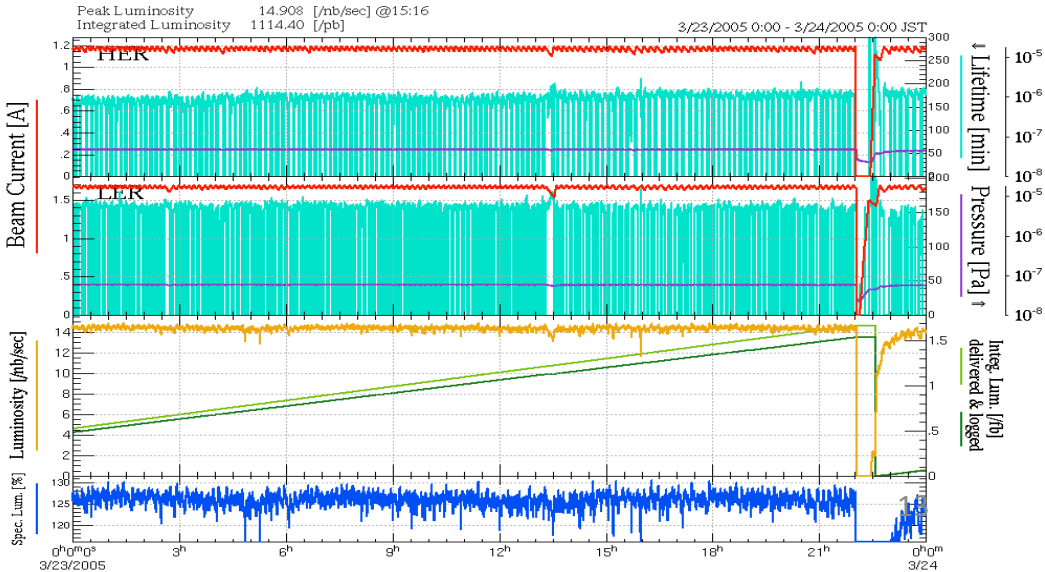
(A.E., D.K: 0.5%, Delphi Prelim: ~3%, naively very high instanton rate (every second quark?))

Measurement of Fragmentation Functions @ (till June 30th 2010)

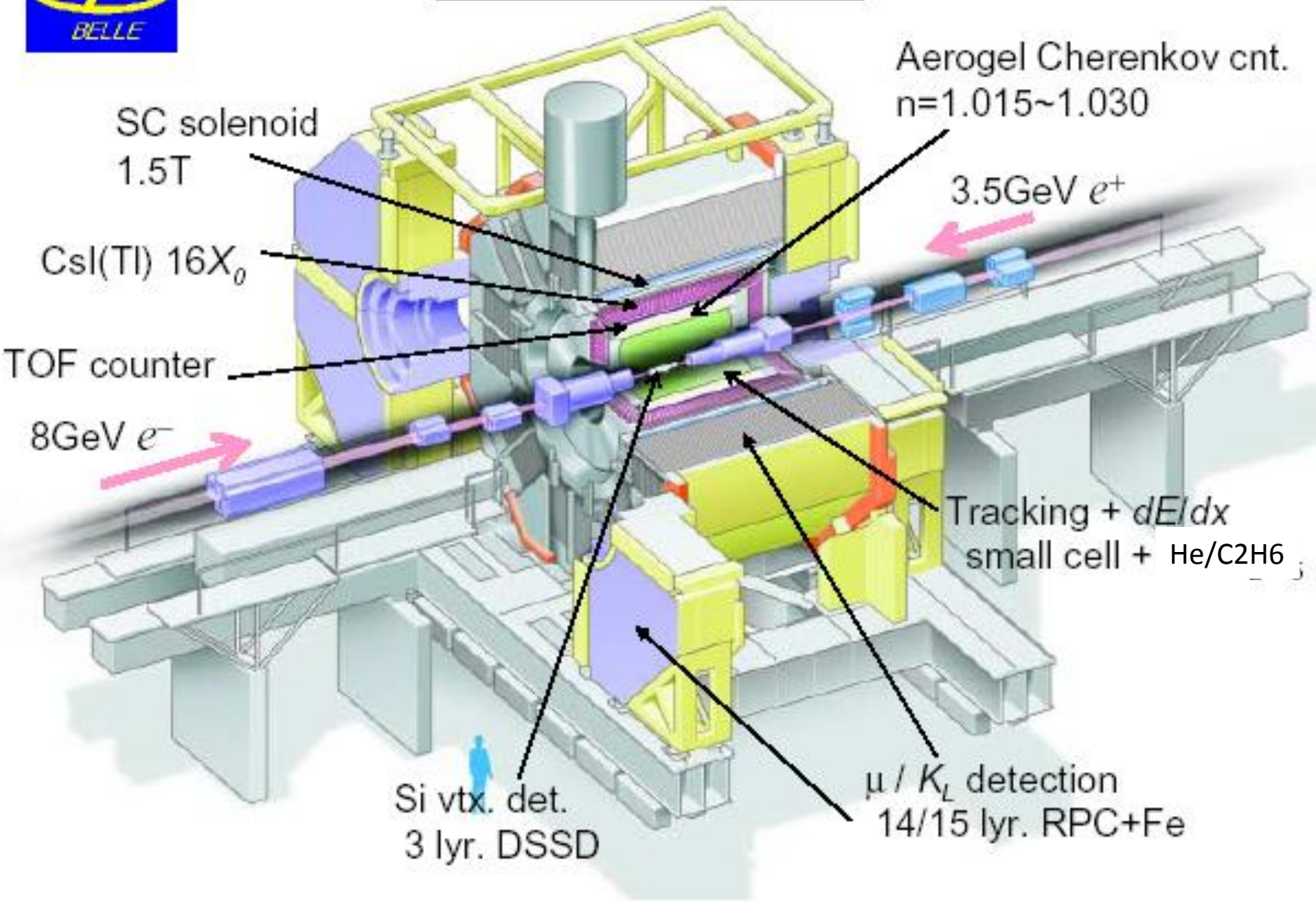
- KEKB: L>2.11 x 10^{34}cm^{-2}s^{-1}
- Asymmetric collider:
- 8GeV e^- + 3.5 GeV e^+
- $\sqrt{s}$=10.58 GeV (□(4S))
- e^+e^-□□(4S)□BB
- Integrated Luminosity: > 1000 fb^{-1}
- Continuum production: 10.52 GeV
- e^+e^-□(u, d, s, c)
- >70 fb^{-1} => continuum



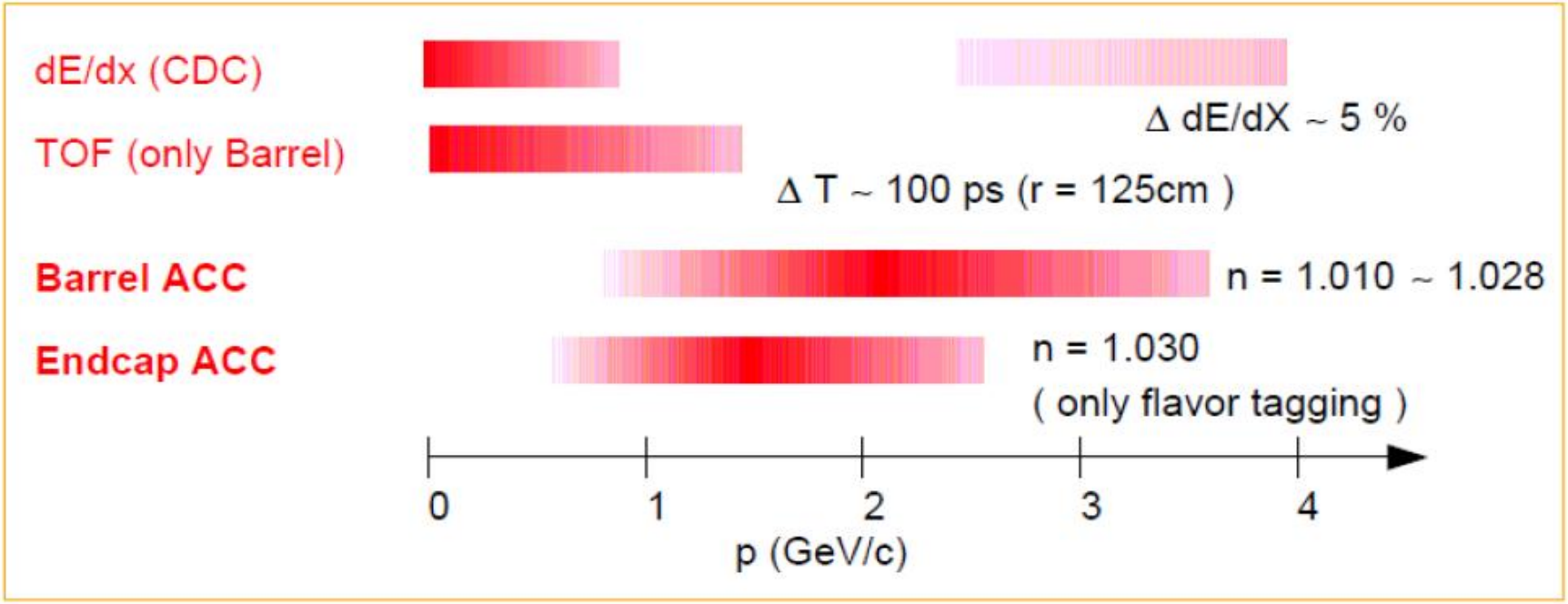


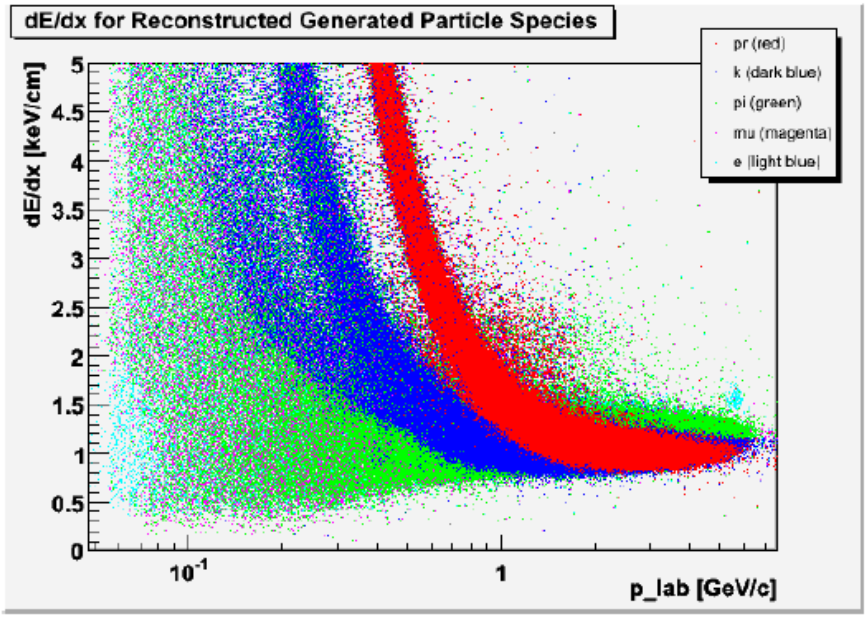


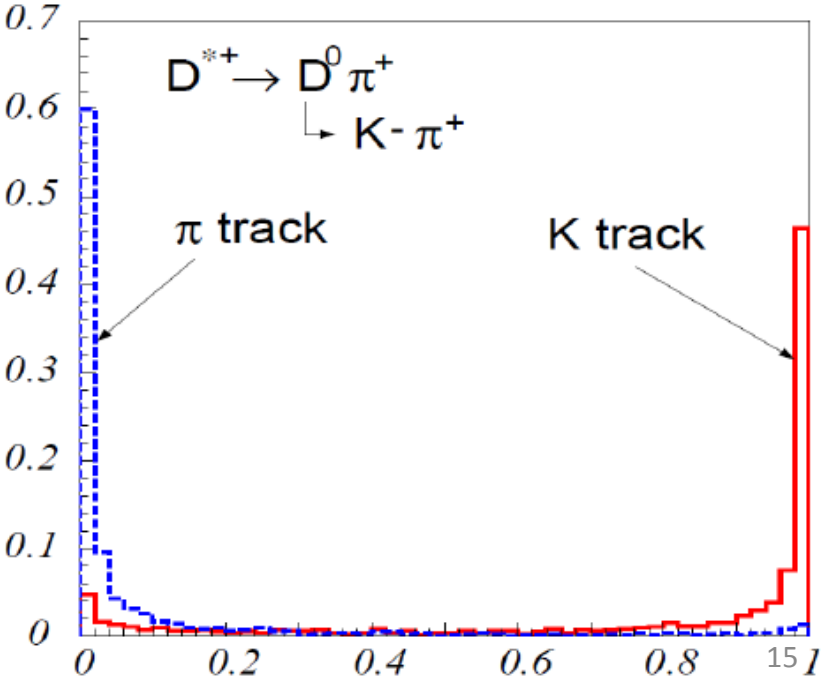
Belle Detector



Large acceptance, good tracking and particle identification!

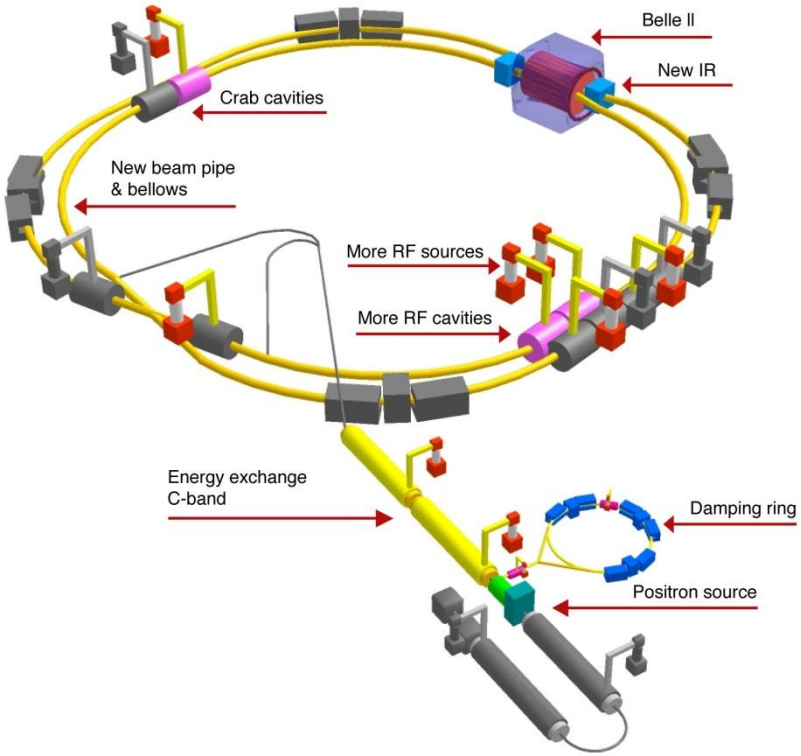

dE/dx (CDC)
Δ dE/dX ~ 5 %
TOF (only Barrel)
Δ T ~ 100 ps (r = 125cm)
Barrel ACC
n = 1.010 ~ 1.028
Endcap ACC
n = 1.030
(only flavor tagging)
0
1
2
3
4
p (GeV/c)

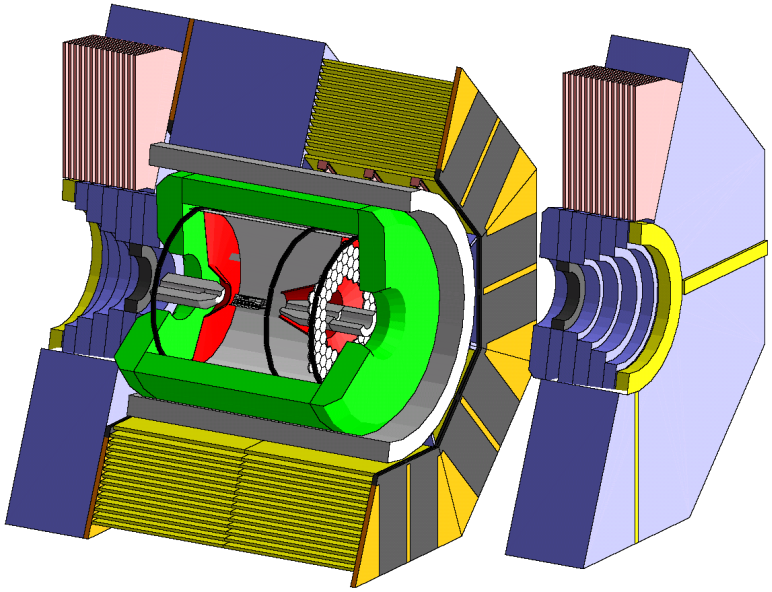

dE/dx for Reconstructed Generated Particle Species
pr (red)
k (dark blue)
pi (green)
mu (magenta)
e (light blue)
dE/dx [keV/cm]
p_lab [GeV/c]


$D^{*+} \rightarrow D^0 \pi^+$
$\hookrightarrow K^- \pi^+$
π track
K track

KEKB/Belle upgrade (2010–2015)

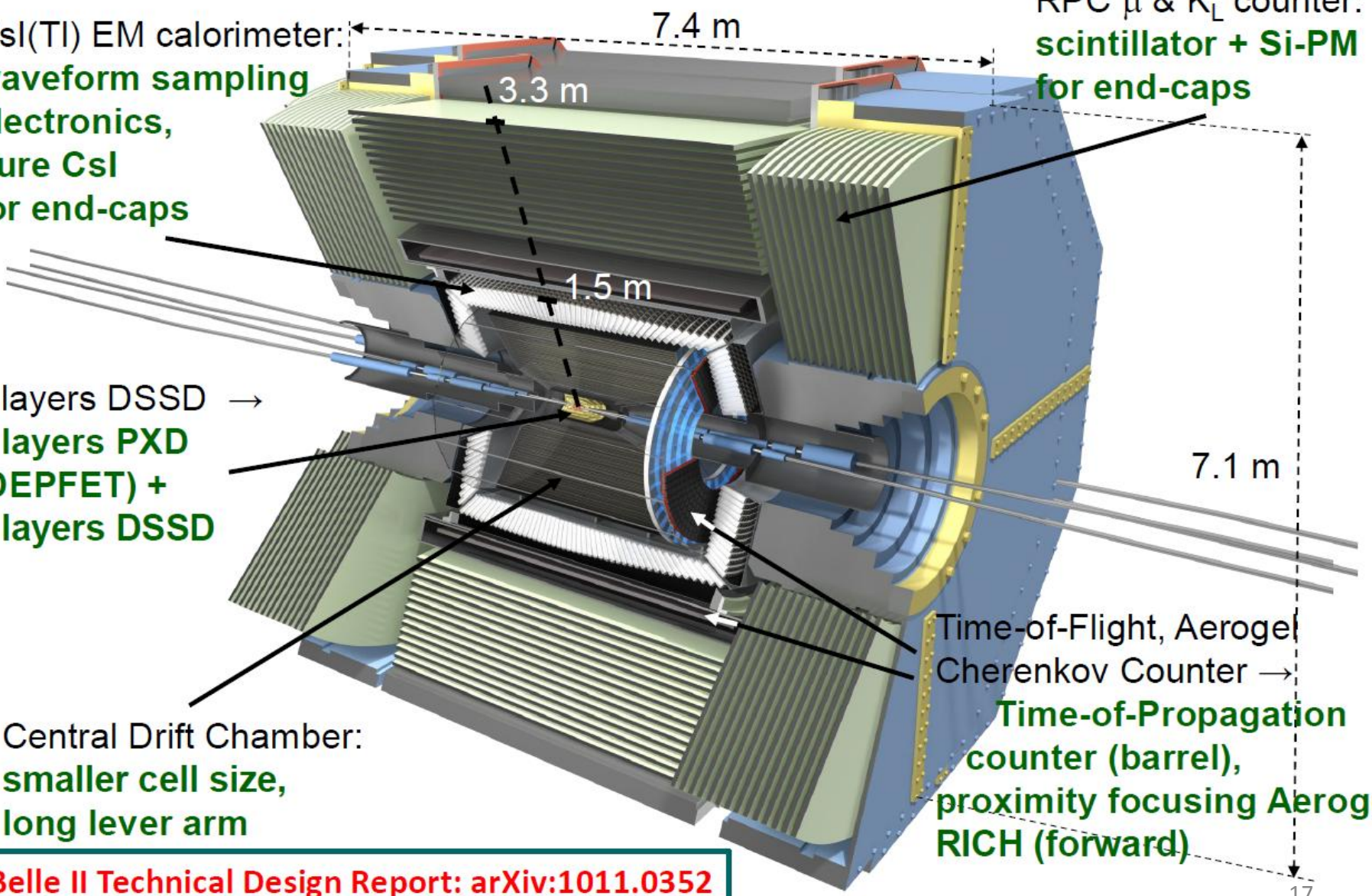
- Aim: super-high luminosity ~10^{36} $cm^{-2}s^{-1}$
- Upgrades of Accelerator (Microbeams + Higher Currents) and Detector (Vtx,PID, higher rates, modern DAQ)
- Significant US contribution





http://belle2.kek.jp

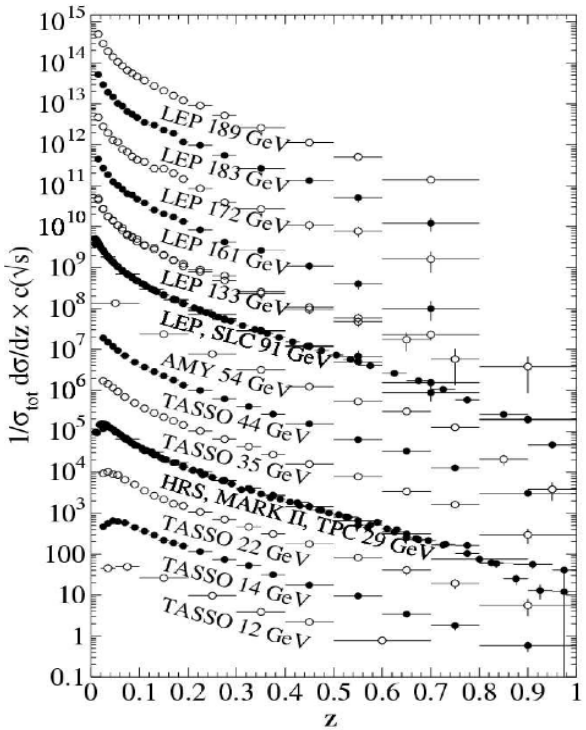
The Belle II Detector

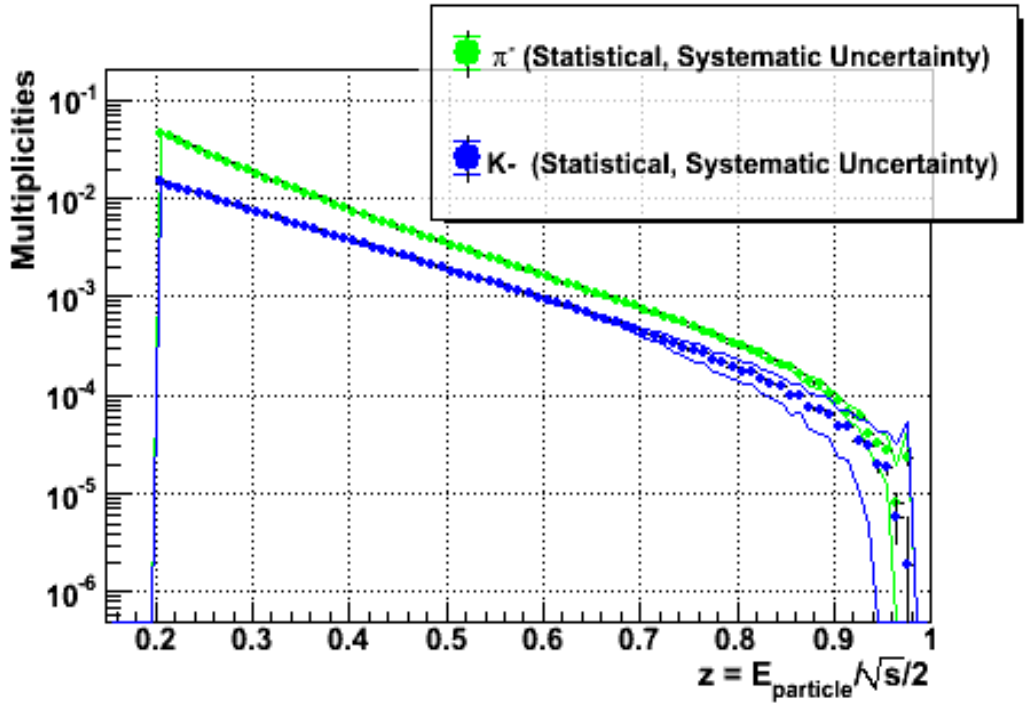


Belle II Technical Design Report: arXiv:1011.0352

Belle: Vast improvement in Fragmentation Function Precision

- Pion and Kaon Yields

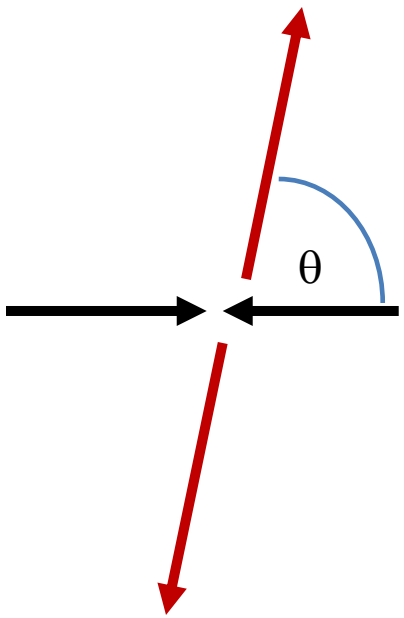


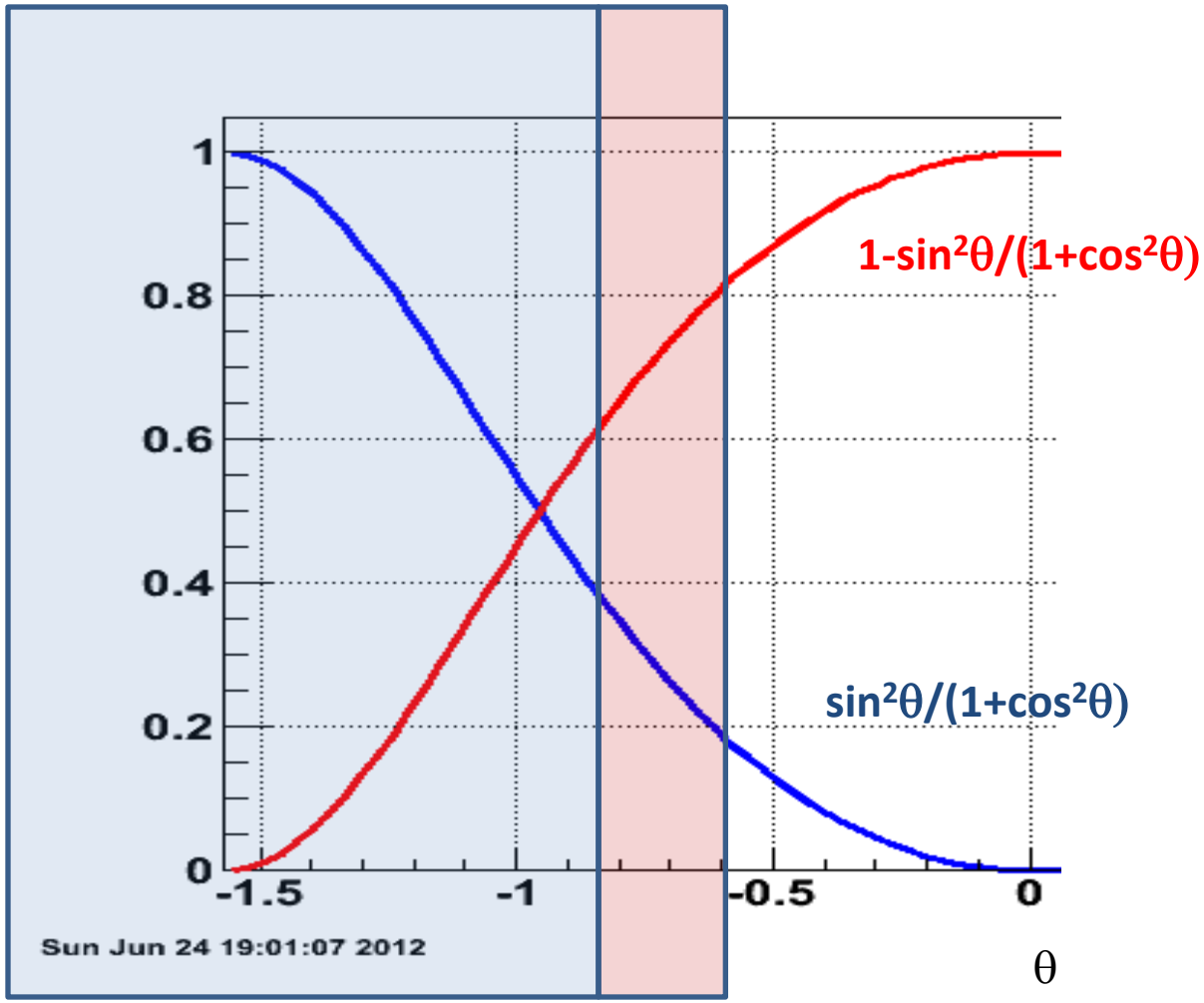


Lower CME:

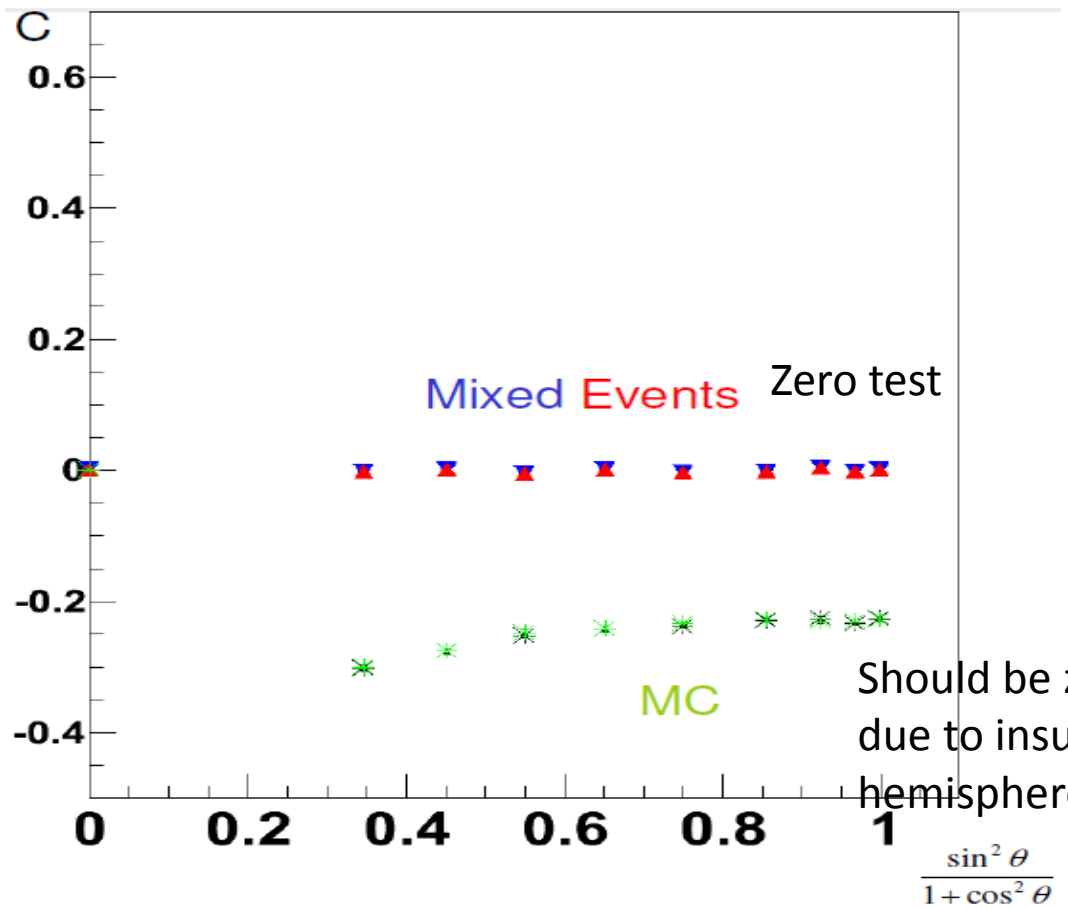
- Advantage: longer formation time of mesons (larger effects of Chromo B field on quark)
- Disadvantage: Lower Multiplicities

Spin Projection in Barrel and Endcap





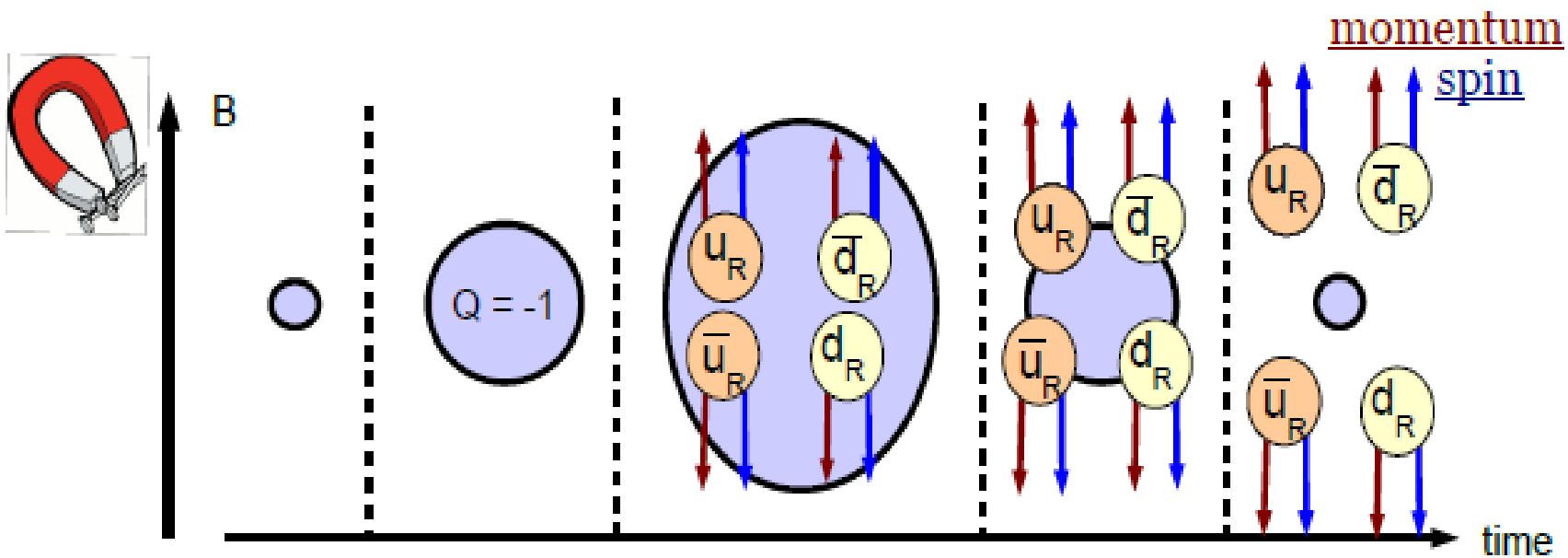
First Look at Handedness Correlation



Should be zero. Effect probably due to insufficient separation of hemispheres

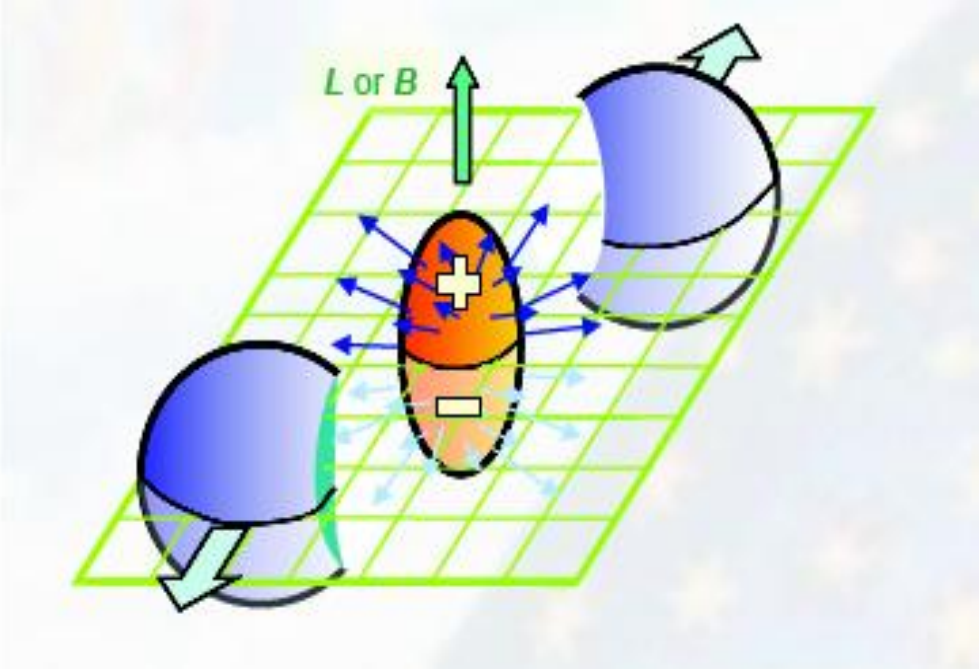
Work in Progress….

Chiral Magnetic Effect leads to Charge Separation

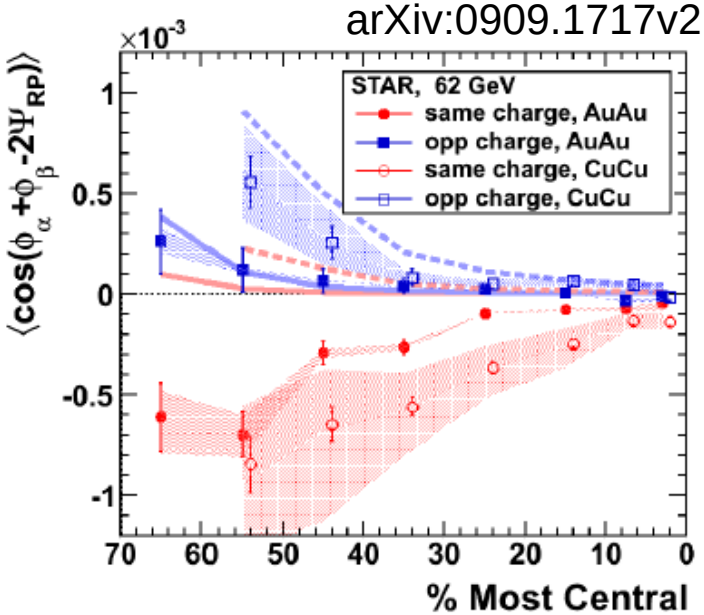


Kharzeev, McLerran and Warringa, arXiv:0711.0950,
Fukushima, Kharzeev and Warringa, arXiv:0808.3382

Chiral Magnetic Effect leads to Charge Separation

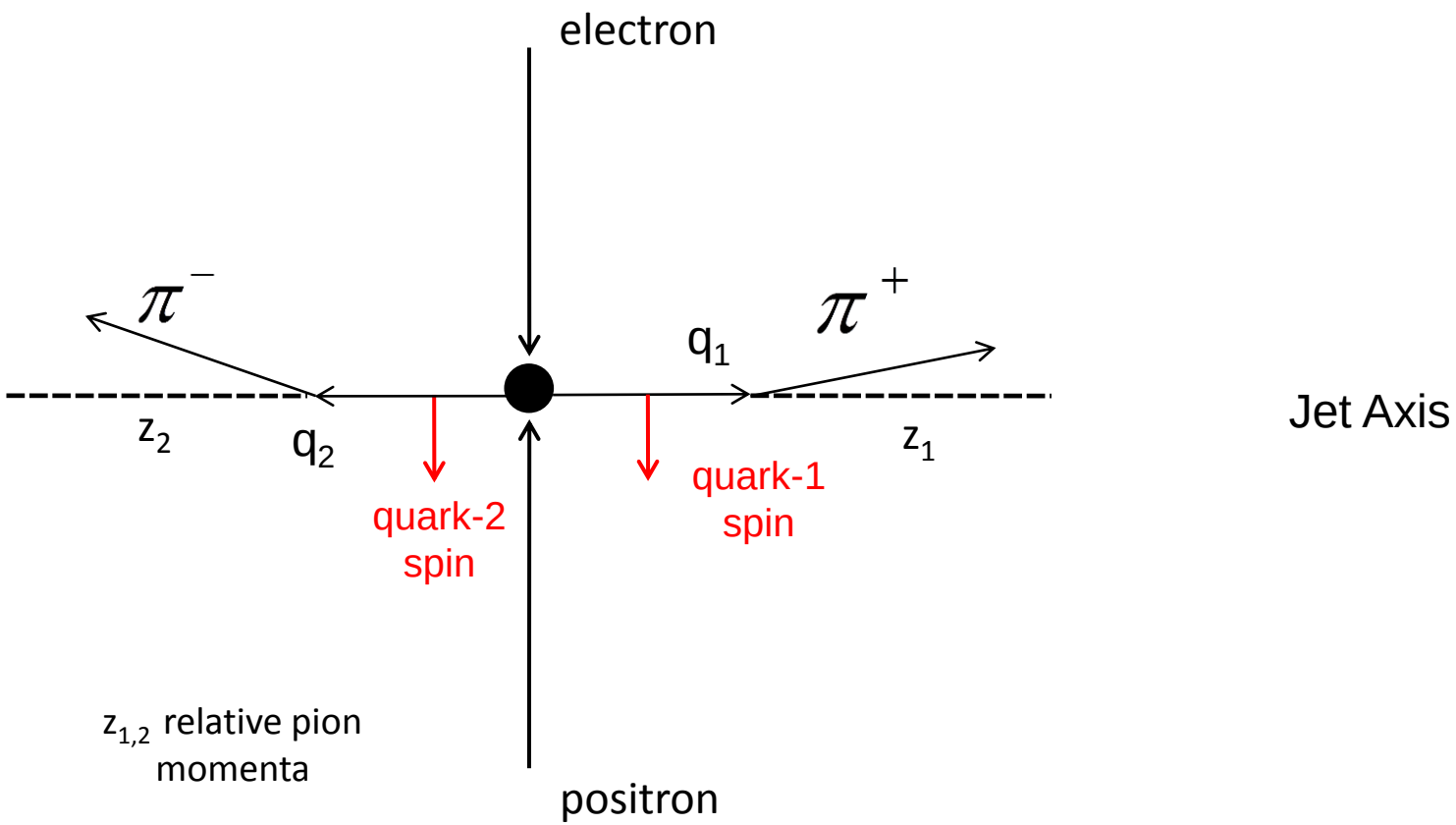


arXiv:0909.1717v2 [

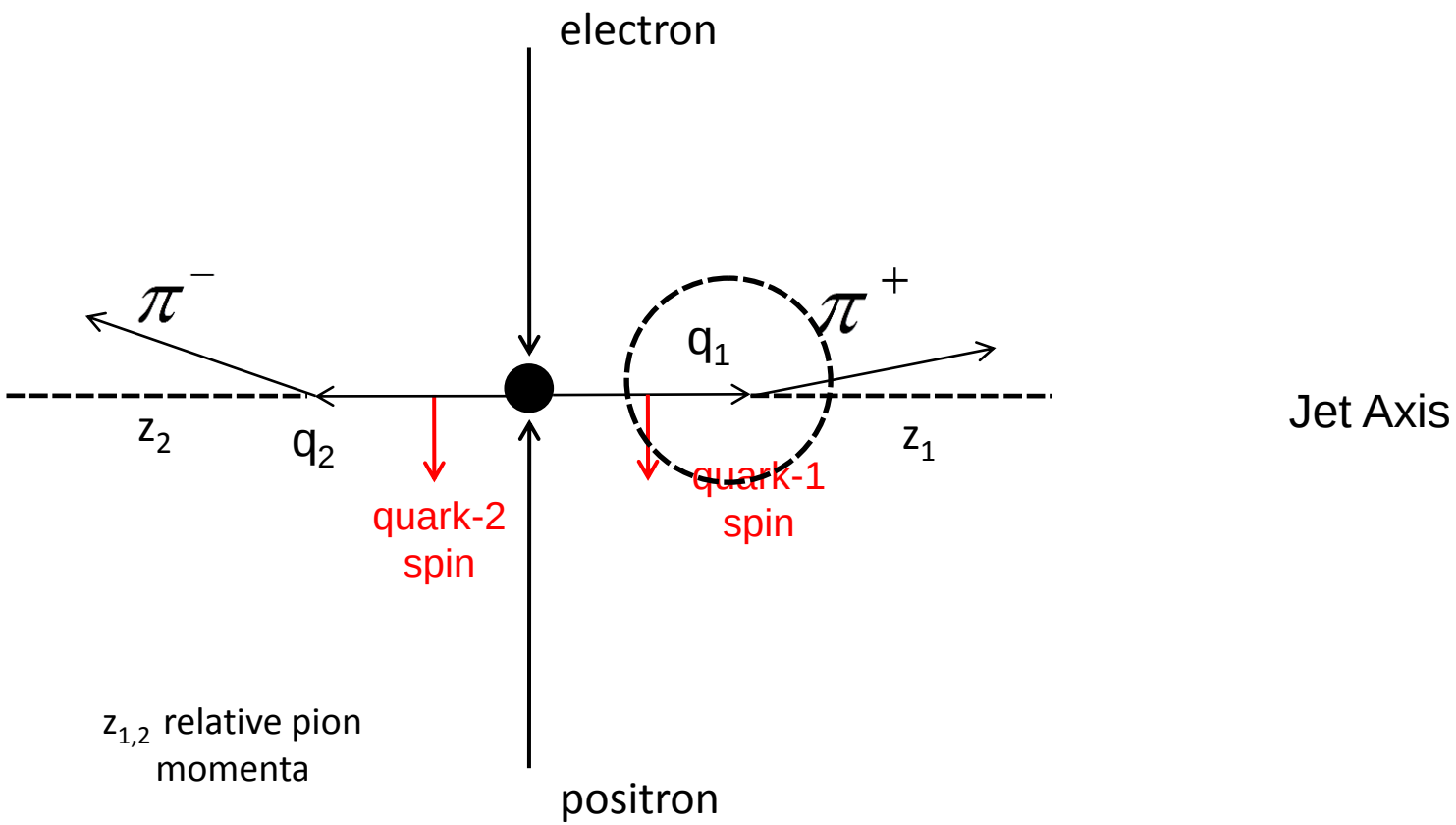


- In Heavy Ion Collisions charged particle correlations agree with expectations from p-odd bubbles
- But: can also be explained by other dynamical effects in the quark gluon plasma
- B approx 10^{18} G

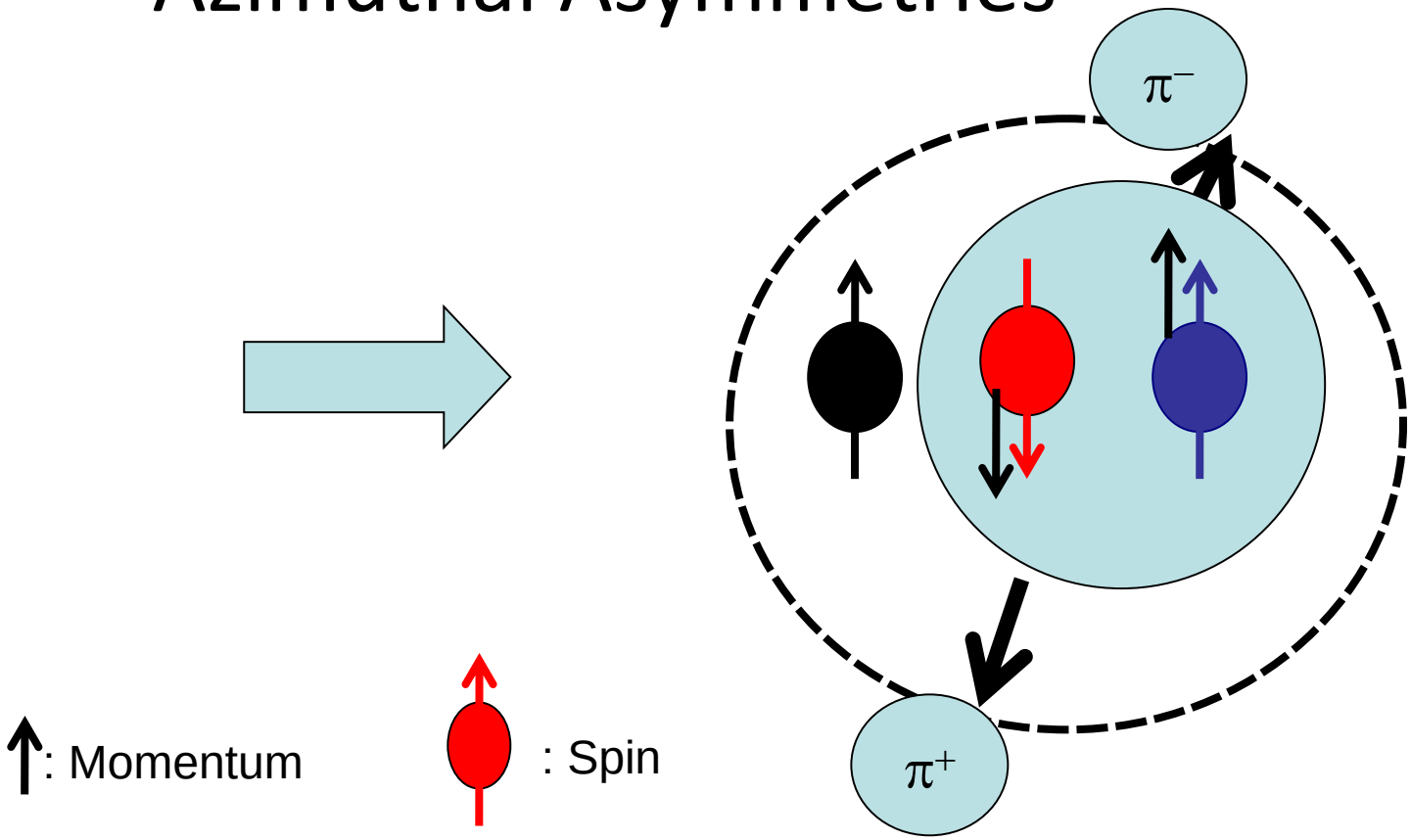
Event Topology in e+ e-



Event Topology in e+ e-

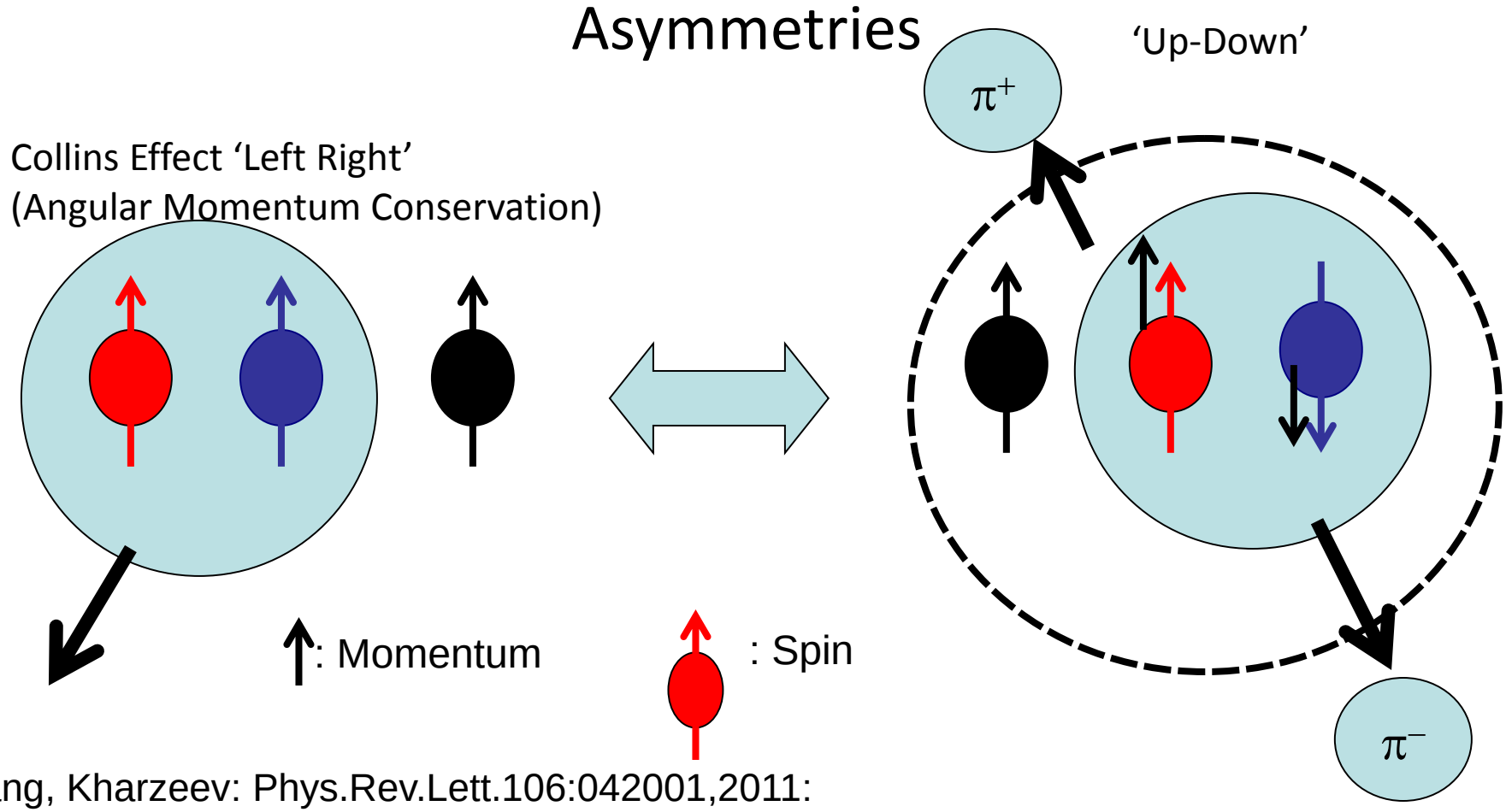


Fragmentation in P odd Bubbles leads to Azimuthal Asymmetries



- Fragmentation in P-odd bubble leads spin-momentum correlation
- Difference in 'Winding number' gives effective increment in chirality
- Spin alignment via chromomagnetic-electric effect
- Azimuthal event by event modulation
- Measurement: Extract width of distribution of first moments

Mix of P-odd FF with Collins FF leads to Event by Event Asymmetries



Kang, Kharzeev: Phys.Rev.Lett.106:042001,2011:

Coupling to Collins FF leads to $\sin(\phi_1+\phi_2)$ asymmetry

Compare to Collins x Collins ~ (P-odd FF) x (P-odd FF): $\cos(\phi_1+\phi_2)$

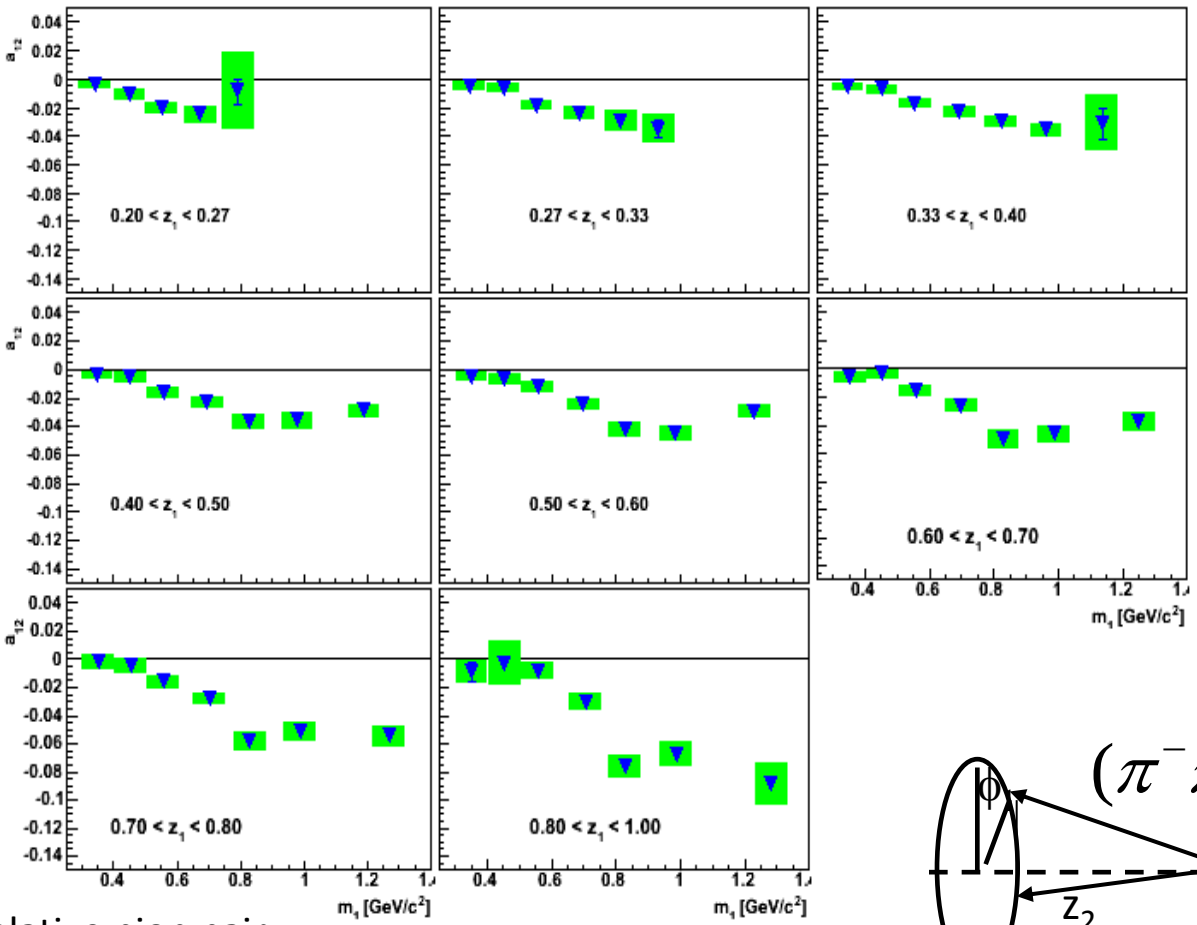
Averages out Event by Event since we do not know if quark or antiquark is in p-odd bubble

What about flavors or spin 1?

ρ^++ x π^+ compared with π^- x π^+ Collins is sensitive to spin

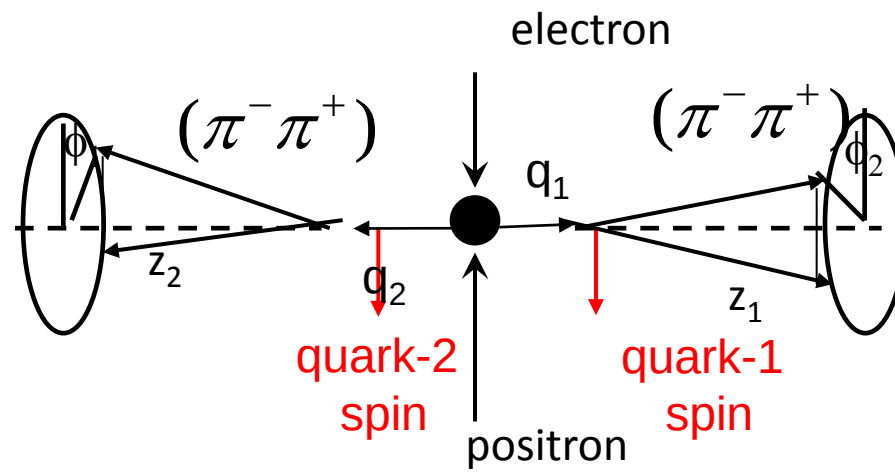
Expect sign change for Collins but not for P-Odd FF (sensitive to quark flavor)

Transverse Spin Dependent FF measurements



Di-hadron Advantage:
No contribution by
Gluon radiation

$z_{1,2}$ relative pion pair momenta

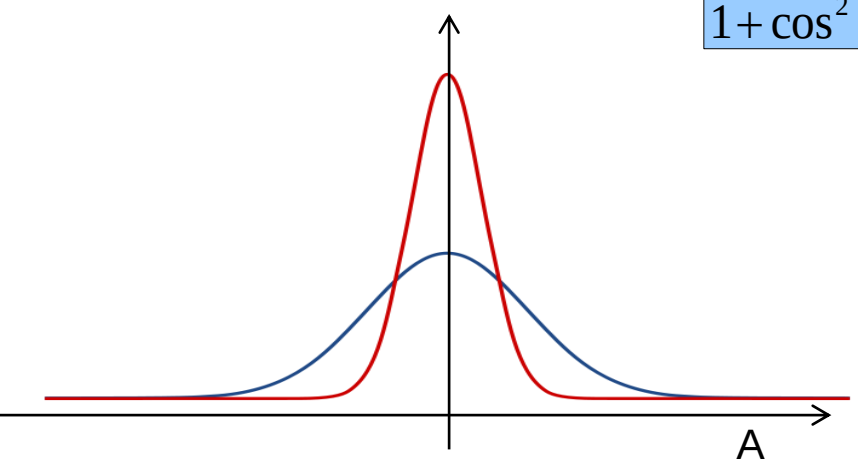


$$A \propto H_1^{\angle}(z_1, m_1)\overline{H}_1^{\angle}(z_2, m_2)\cos(\varphi_1 + \varphi_2)$$

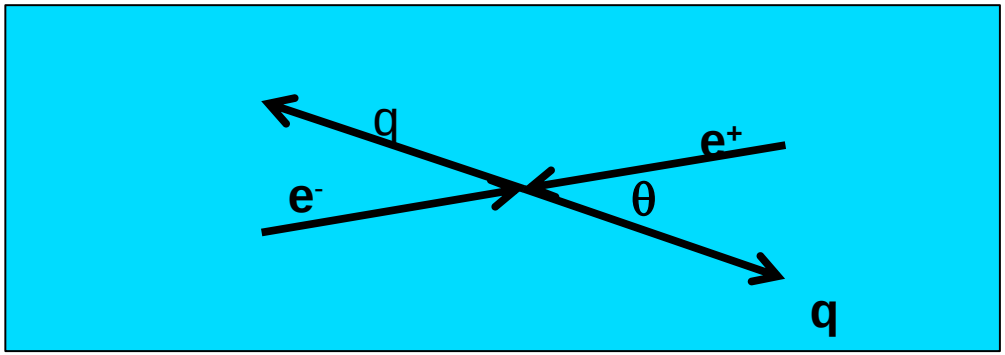
A.V. et. al, PRL **107,** 072004(2011)

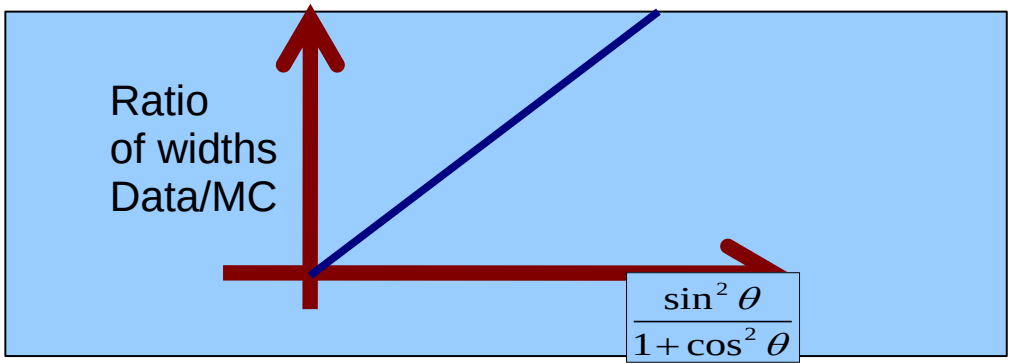
First try….

- Use unbinned maximum likelihood fit for each event ->extract asymmetry A
- Width of distribution of A is indication of effect
- Compare with simulation
- Physical effect has to have linear dependence on giving the transverse spin projection

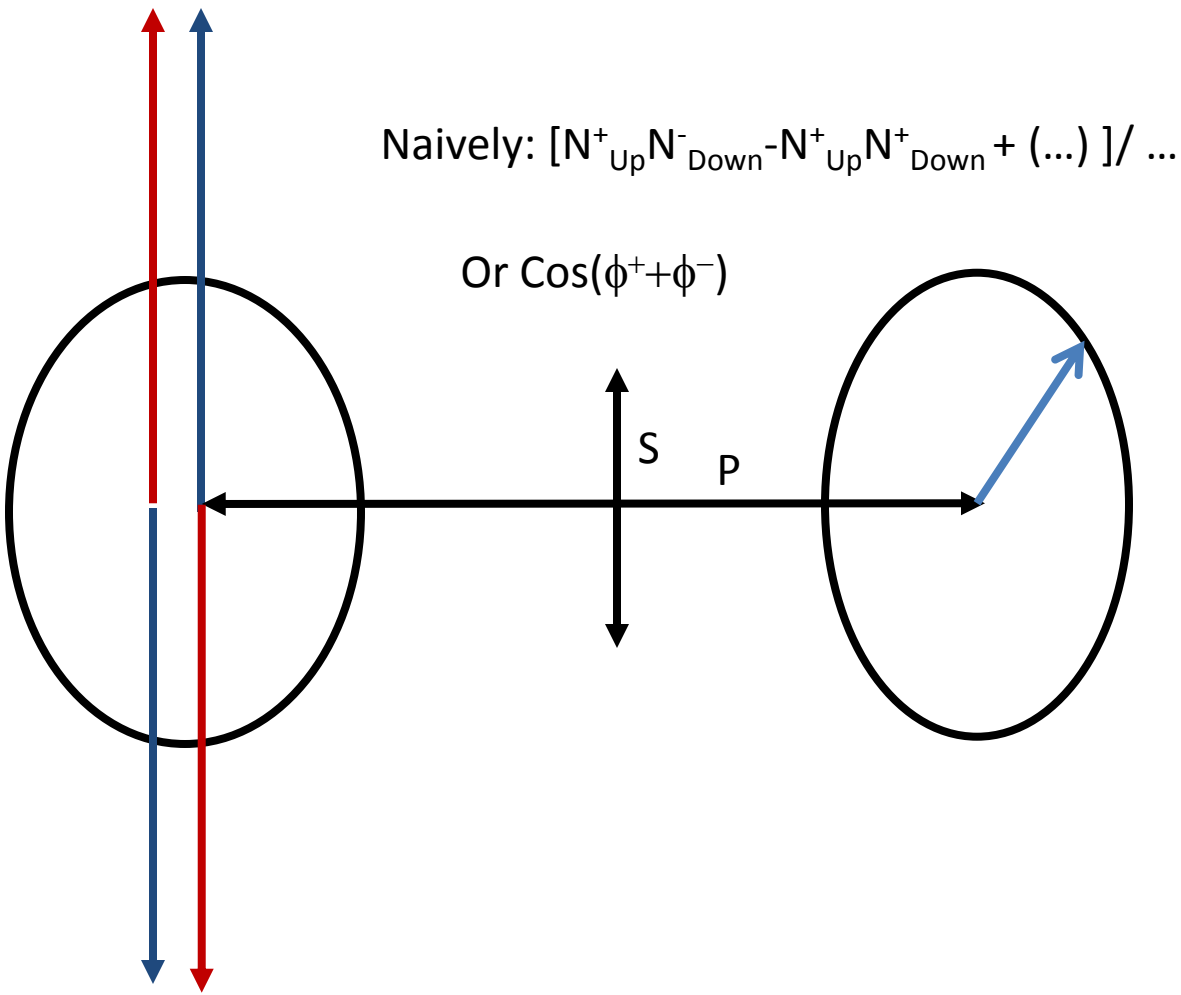


$$\frac{\sin^2\theta}{1+\cos^2\theta}$$





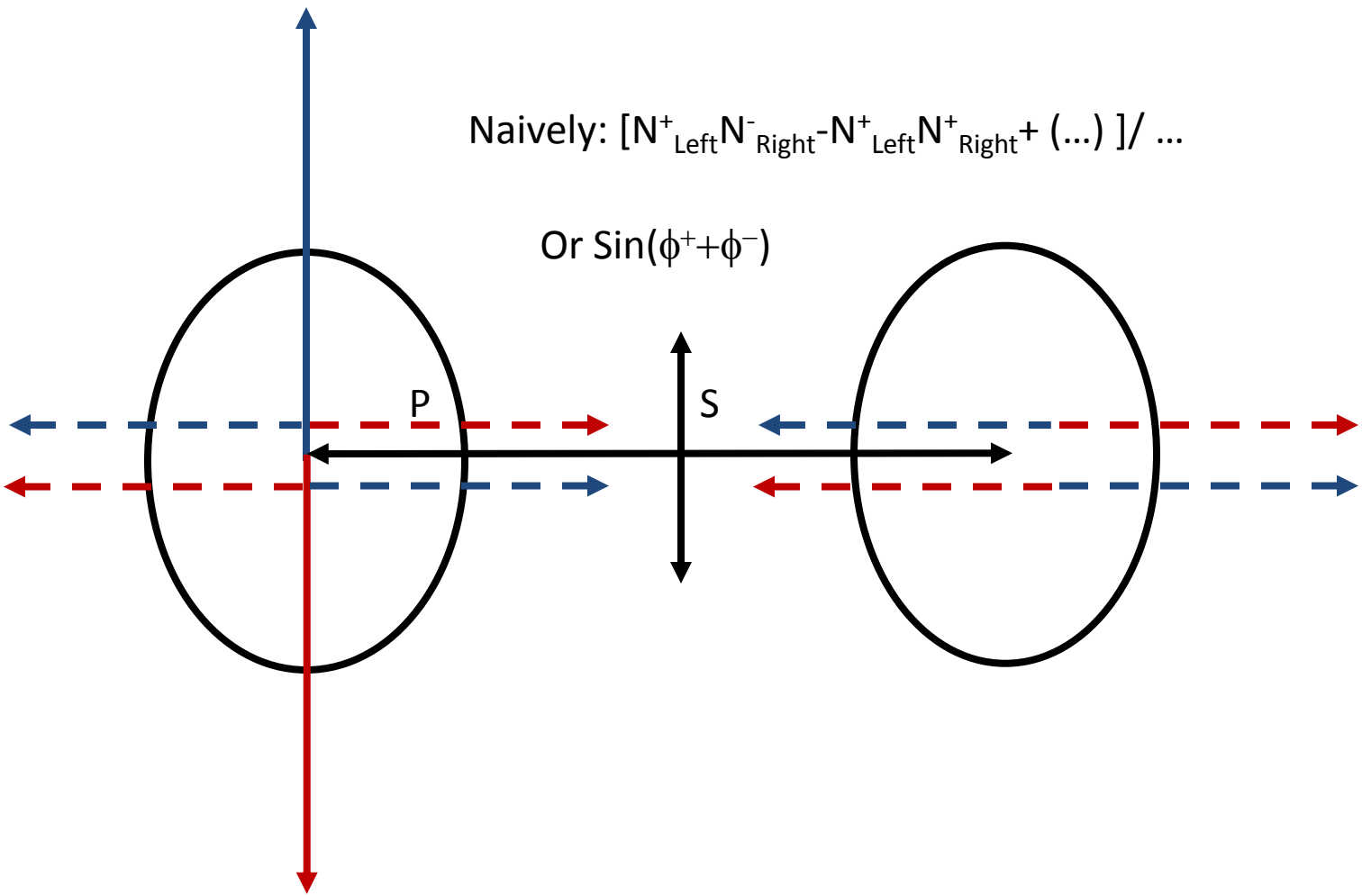
Development of a more robust Correlator



Can compare with Collins

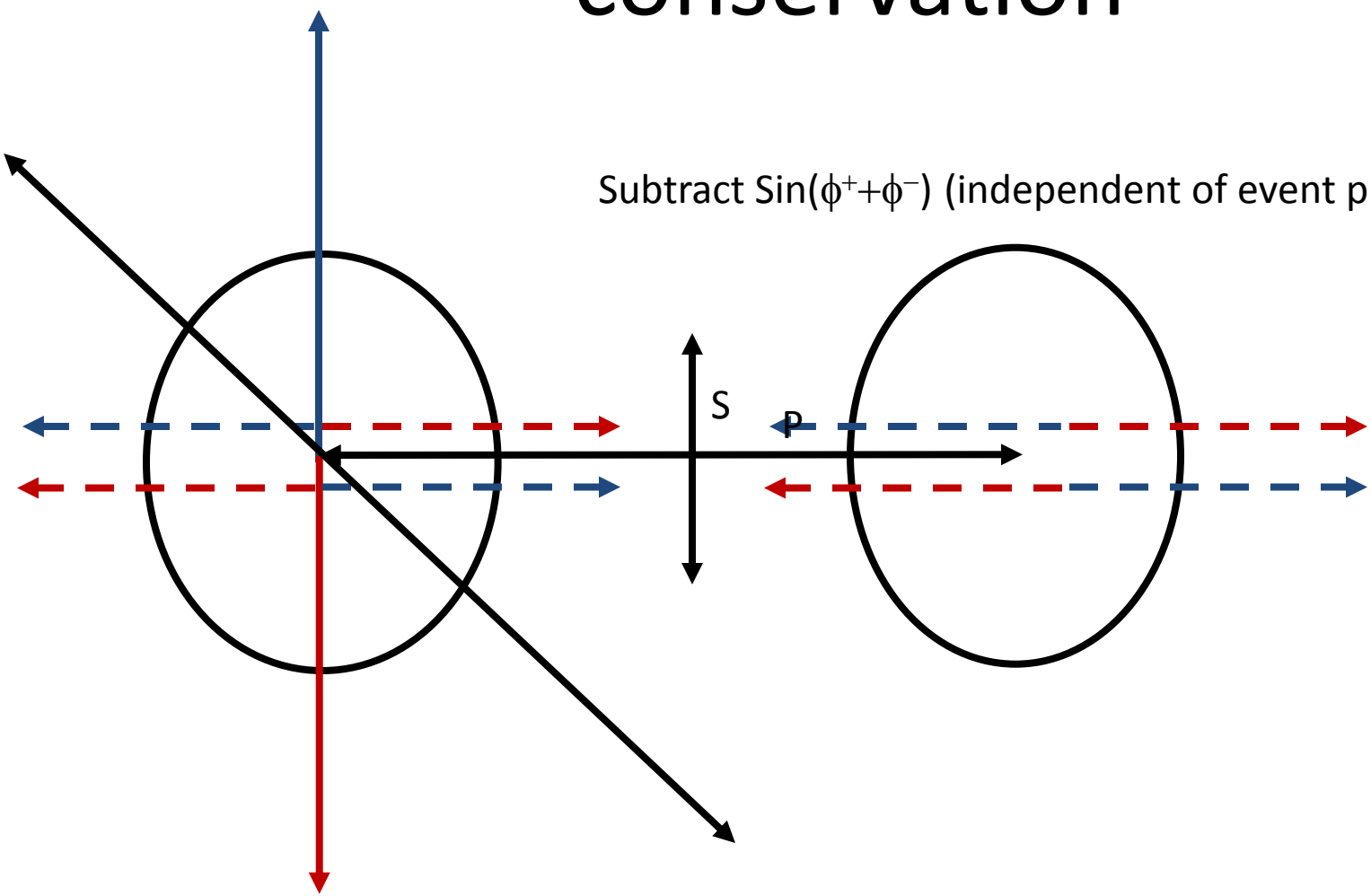
Naively: $[N^{+}_{Left}N^{-}_{Right}-N^{+}_{Left}N^{+}_{Right}+ (\ldots)]/ \ldots$

Or $\mathrm{Sin}(\phi^{+}+\phi^{-})$

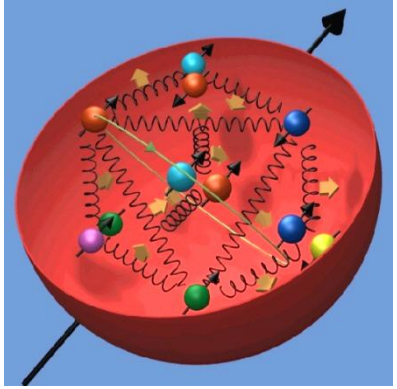


However: Multiplicity Small, same effect from momentum conservation

Subtract Sin($\phi^+ + \phi^-$) (independent of event plane)?



Summary & Outlook

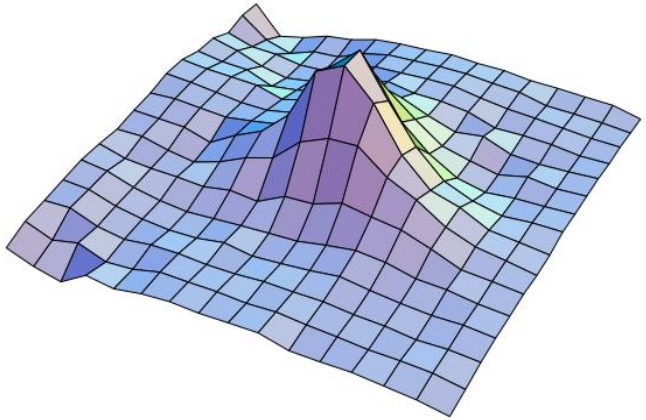


- Probing local parity violating effects in quark fragmentation independent probe
- Belle & Belle II allow high precision measurement
- Low Multiplicity per event is challenging
- Need ‘simple’ correlators
  - Jet Handedness seems appropriate: Signal magnitude?
  - Transverse Spin effects advantageous for Belle acceptance (~factor 2 higher signal)
  - Use Vector Mesons?

THANK
YOU
THANK

Significance

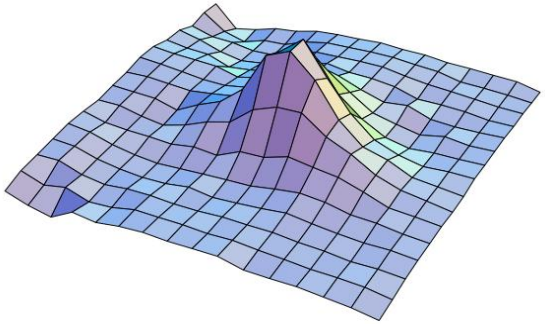
- Access to QCD vacuum structure
- First observation of Sphaleron/Instanton induced processes: non-perturbative topological objects
- In EW sector similar transitions are needed for Baryogenesis

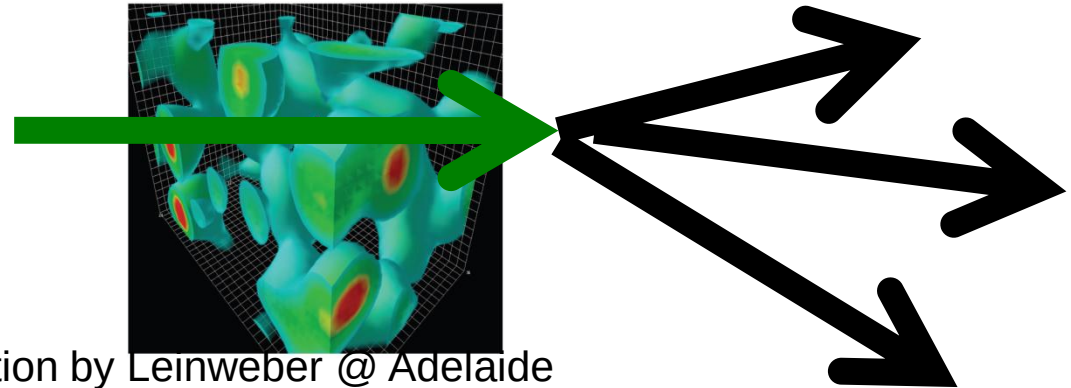


- Need independent probe!

Accessing QCD vacuum fluctuations in Quark Fragmentation

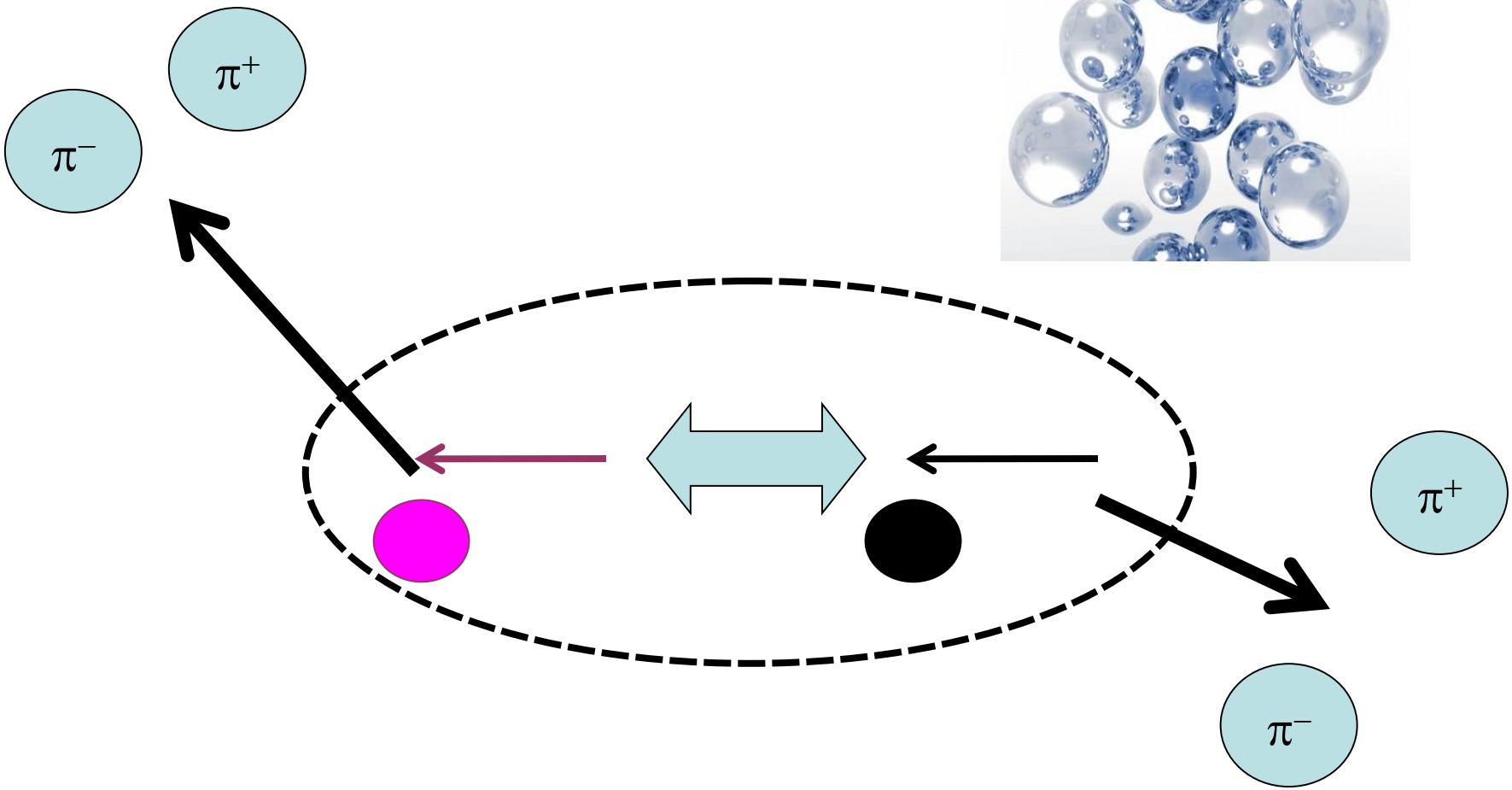
- Transitions between QCD vacuum ground states by non-perturbative gluon configurations:
- On microscopic scale quarks coupling to these leads to P-odd effects
  - First results at STAR and PHENIX
  - Planned measurements in Belle: needed as a 'tie breaker'
  - Model Calculations predict 2% effect
- Access to nonperturpative properties of QCD



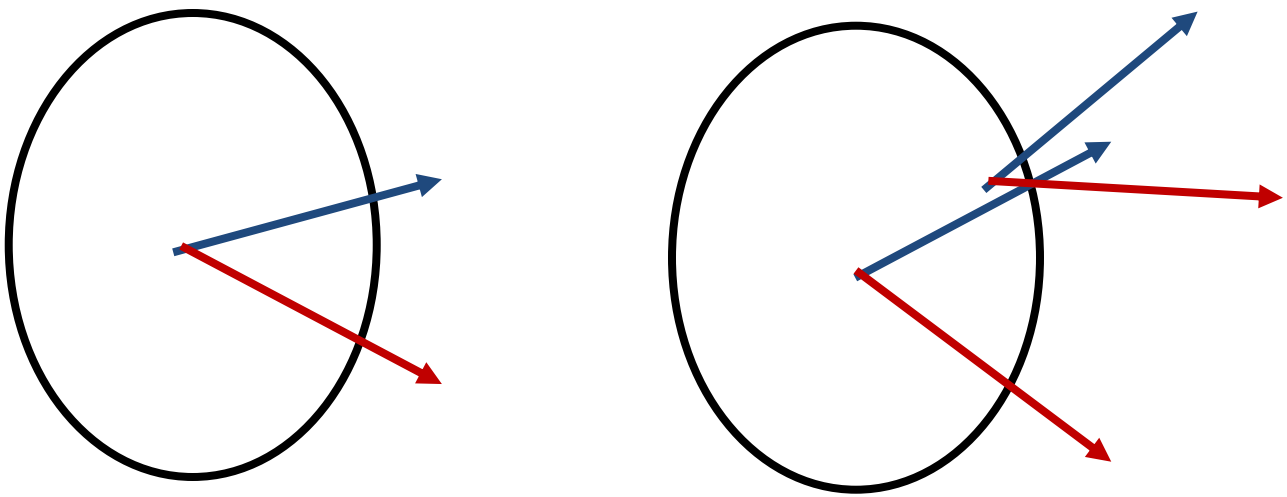


Simulation by Leinweber @ Adelaide

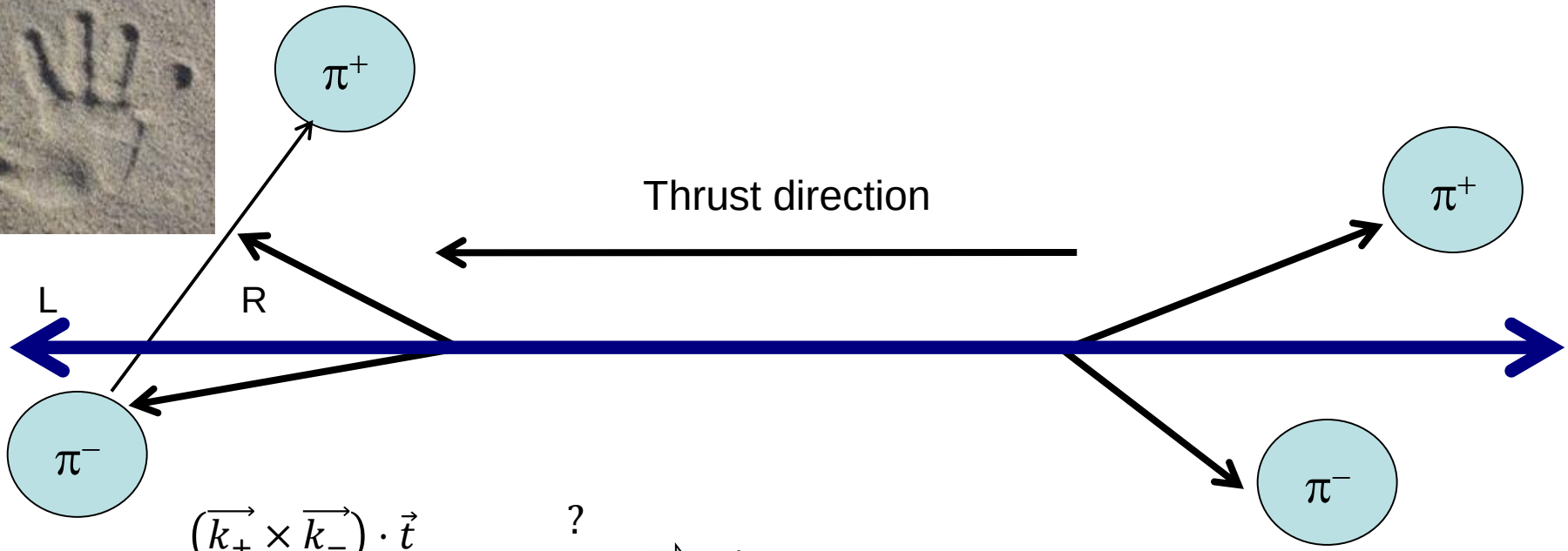
Fragmentation in the Big Bubble



Efremov, Khazeev, Phys. Lett. B366, 311



Handedness



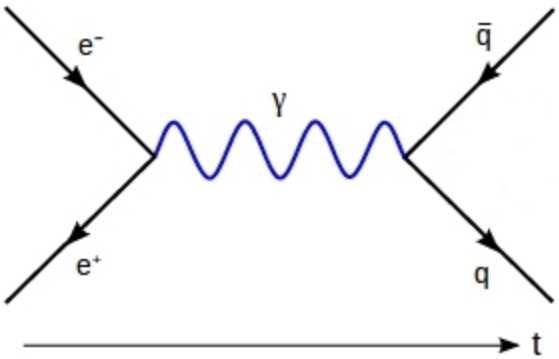
Handedness: $\dfrac{(\overrightarrow{k_+} \times \overrightarrow{k_-}) \cdot \vec{t}}{|\, k_+ \,||\, k_- \,|} = \sin\Phi \overset{?}{>} 0$ ⇨ L/R

Jet *handedness:* $\dfrac{N_R - N_L}{N_R + N_L}$

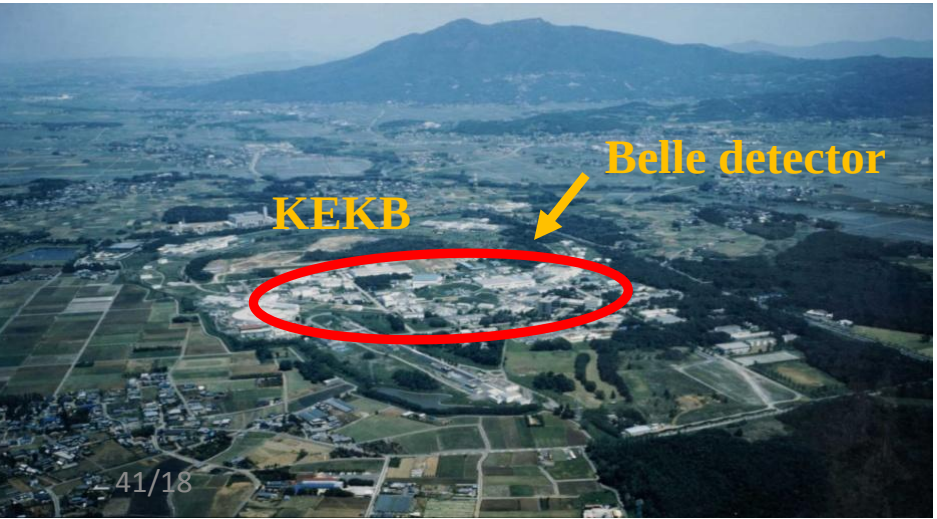
C: $\dfrac{N_{RL} + N_{LR} - N_{RR} - N_{LL}}{N_{RL} + N_{LR} + N_{RR} + N_{LL}}$

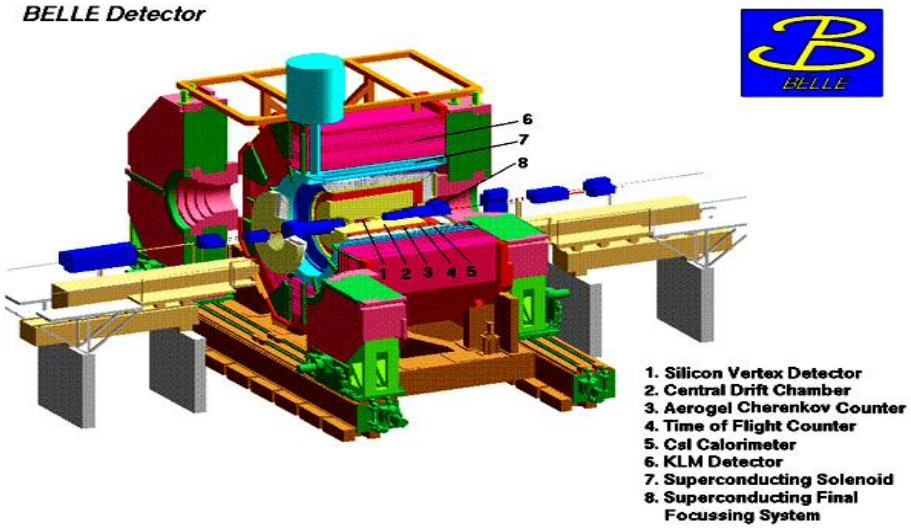
Expect positive correlation for p-odd effect

Measurements of Fragmentation Functions in e+e- at Belle

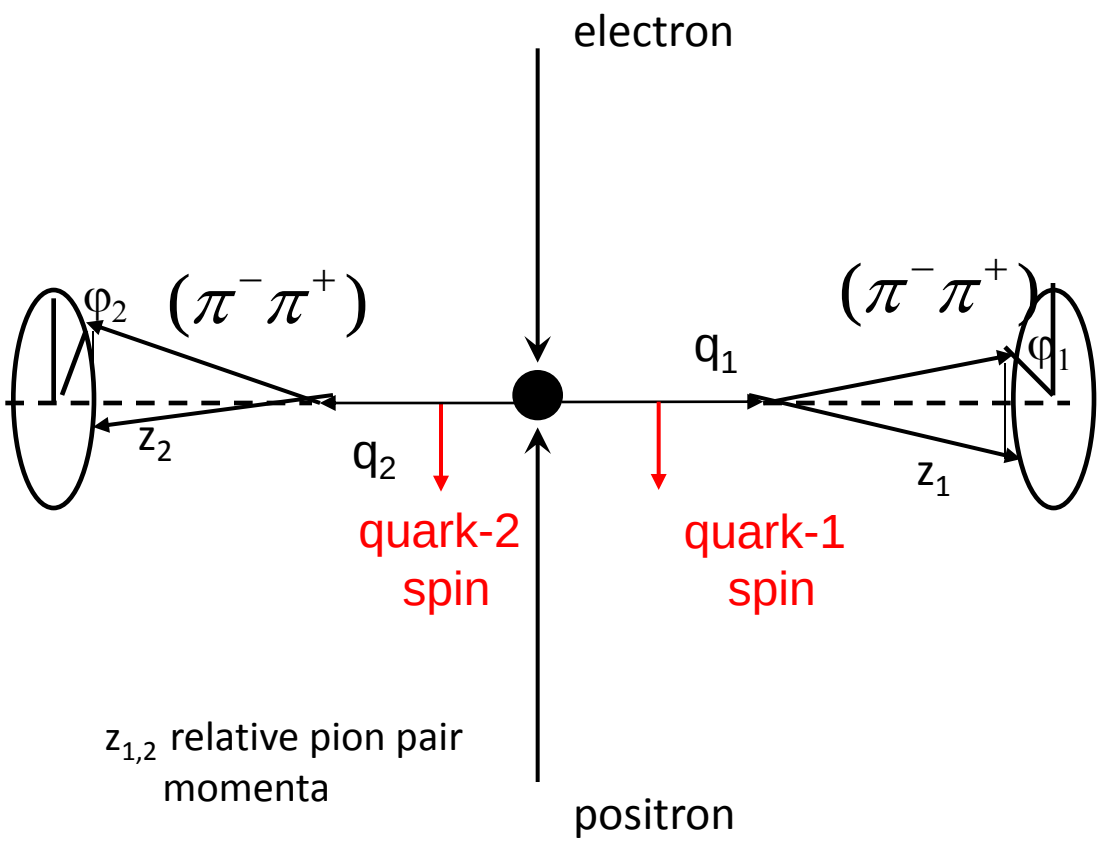


- KEK-B: asymmetric e^+ (3.5 GeV) e^- (8 GeV) collider:
  - **$\sqrt{s}$ = 10.58 GeV,** $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\ \bar{B}$
  - **$\sqrt{s}$ =** 10.52 GeV, e+e- → qqbar (u,d,s,c) 'continuum'
- ideal detector for high precision measurements:
  - tracking acceptance θ [17 °;150°]: Azimuthally symmetric
  - particle identification (PID): dE/dx, Cherenkov, ToF, EMcal, MuID
- Available data:
  - ~1.8 *10^9 events at 10.58 GeV,
  - ~220 *10^6 events at 10.52 GeV





Measuring transverse spin dependent di-Hadron Correlations In unpolarized e^+e^- Annihilation into Quarks



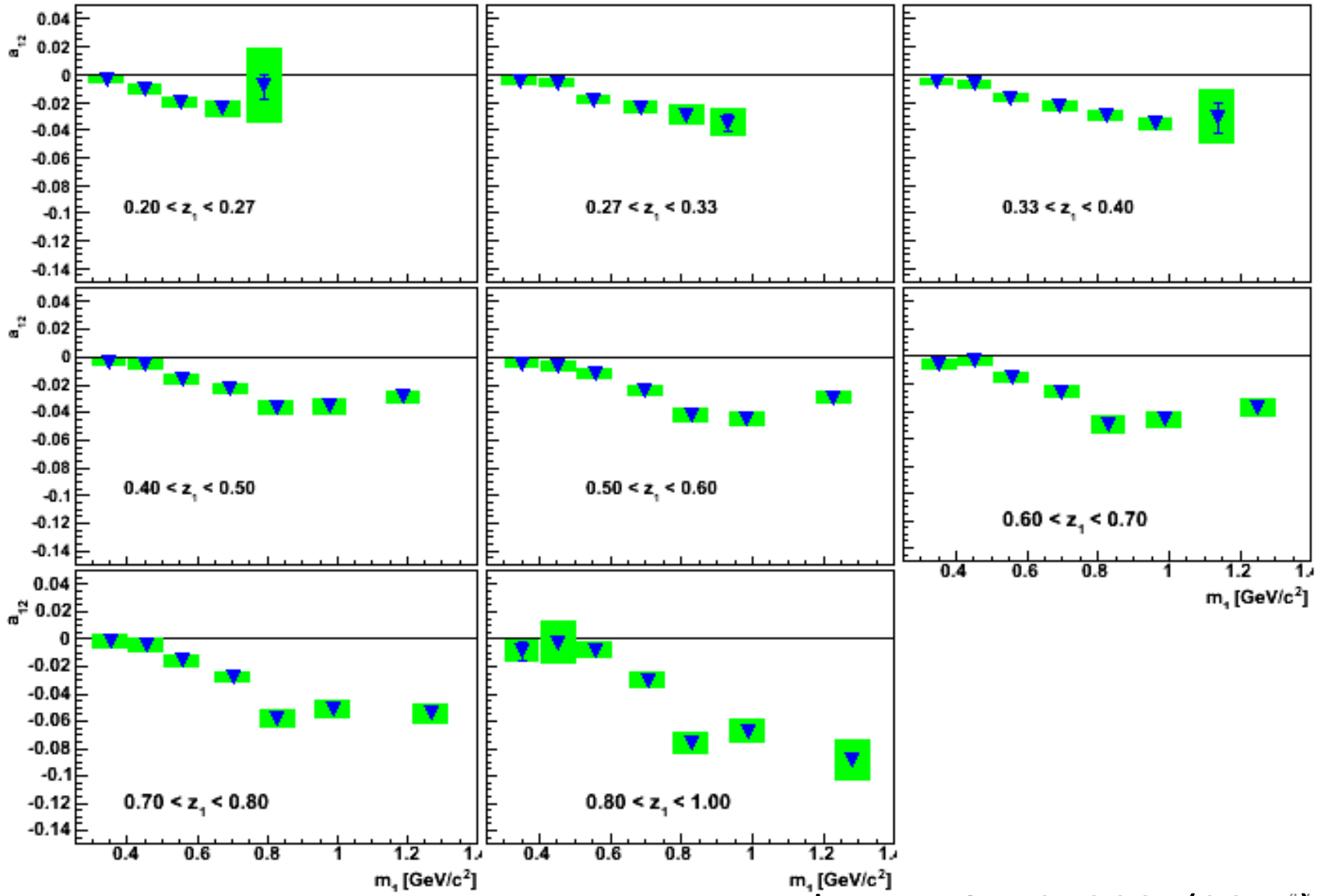
Interference effect in e^+e^- quark fragmentation will lead to azimuthal asymmetries in di-hadron correlation measurements!

Experimental requirements:

- Small asymmetries ➔ very large data sample!
- Good particle ID to high momenta.
- Hermetic detector

$$A \propto H_1^{\angle}(z_1, m_1)\overline{H}_1^{\angle}(z_2, m_2)\cos(\varphi_1 + \varphi_2)$$

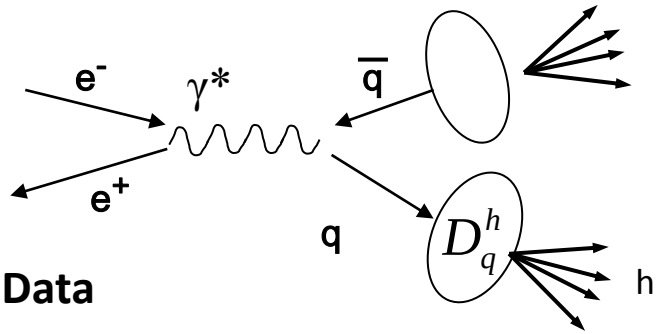
Results or IFF at BELLE (z_1x m_1) Binning



AV et. al, PRL **107,** 072004(2011)

Spin-Averaged FF from Pion and Kaon Multiplicities

- In LO: FF D_i^h describes probability for a parton ***i*** to fragment into a hadron ***h***



- **Extraction from Experimental Data**
  - FF at different energy scales relatable by DGLAP evolution equations
  - FFs D_i^h can be extracted from e+e- data in pQCD analysis:

$$N^h(z,Q^2) = \frac{1}{\sigma_{\text{tot had}}} \frac{d\sigma(e^+e^- \to hX)}{dz} \qquad z \equiv \frac{E_h^{cms}}{\sqrt{s}/2}$$

measured: hadron multiplicity

$$\overset{\text{NLO QCD}}{=} \sum_{i=q,\bar{q},g} C_i^{NLO}(z,\alpha_s) \otimes D_i^h(z,Q^2)$$

pQCD fit

extracted: FFs

Extraction from Experimental Data

- **recent extractions** of unpolarized FFs D_i^h propagating experimental uncertainties:

First FF extraction including uncertainties (e^+e^-):

Hirai, Kumano, Nagai, Sudoh (KEK)

Phys. Rev. D **75**, 094009 (2007)

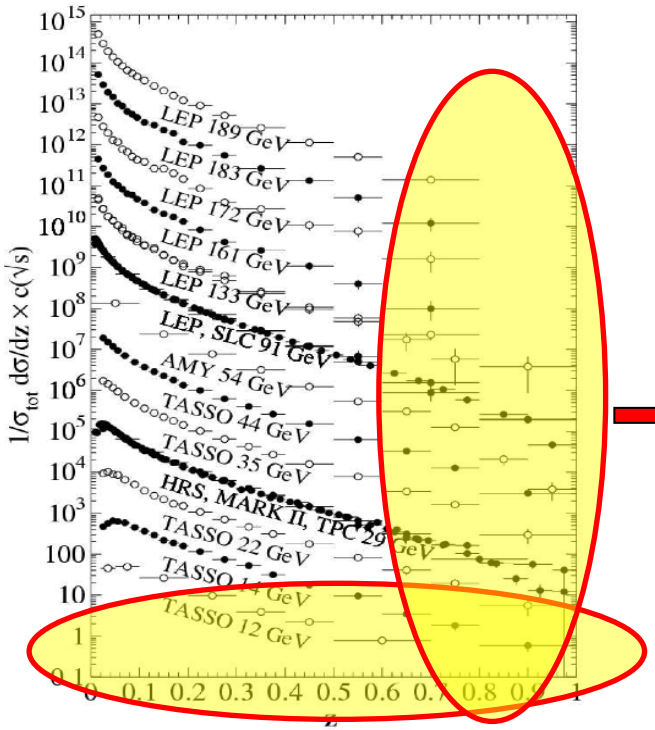
'Global' Analyses (e^+e^-, SIDIS, pp):

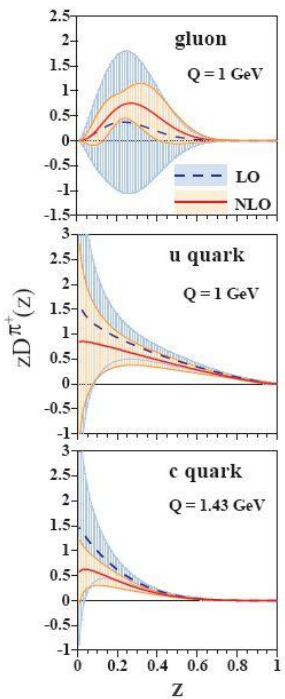
de Florian, Sassot, Stratmann

Phys. Rev. D **75**, 114010 (2007)

and

Phys. Rev. D **76**, 074033 (2007)





$D^{\pi+}_i$

large uncertainties (esp. gluon FF) due to:

- Lack of **precise data at low energy scales** (far from LEP)

- Lack **of precise data at high z**

- **Improve knowledge of FF** via **high precision** hadron measurement **at low Q^2**

Systematic Corrections-Particle Misidentification/PID Calibration

- Particle misidentification expected to be largest uncertainty:

  particle identification probabilities $p_{(i \to j)}$:
  probability that particle of species ***i*** PID-selected as particle of species ***j***.

Physical particle

π → Belle PID likelihood information from: Drift Chamber (dE/dx), Cherenkov, ToF, Calorimeter, Muon Detector →

Reconstructed particle

- $p_{(\pi \to e)}$ → e
- $p_{(\pi \to \mu)}$ → μ
- $p_{(\pi \to \pi)}$ → π
- $p_{(\pi \to K)}$ → K
- $p_{(\pi \to p)}$ → p

$$[P]_{ij} = \begin{pmatrix} p_{(e \to e)} & p_{(\mu \to e)} & p_{(\pi \to e)} & p_{(K \to e)} & p_{(p \to e)} \\ p_{(e \to \mu)} & p_{(\mu \to \mu)} & p_{(\pi \to \mu)} & p_{(K \to \mu)} & p_{(p \to \mu)} \\ p_{(e \to \pi)} & p_{(\mu \to \pi)} & p_{(\pi \to \pi)} & p_{(K \to \pi)} & p_{(p \to \pi)} \\ p_{(e \to K)} & p_{(\mu \to K)} & p_{(\pi \to K)} & p_{(K \to K)} & p_{(p \to K)} \\ p_{(e \to p)} & p_{(\mu \to p)} & p_{(\pi \to p)} & p_{(K \to p)} & p_{(p \to p)} \end{pmatrix}$$

$$\vec{N}_x = \begin{pmatrix} N_e \\ N_\mu \\ N_\pi \\ N_K \\ N_p \end{pmatrix}$$

$$\vec{\tilde{N}}_j = \hat{P}\,\vec{N}_i$$

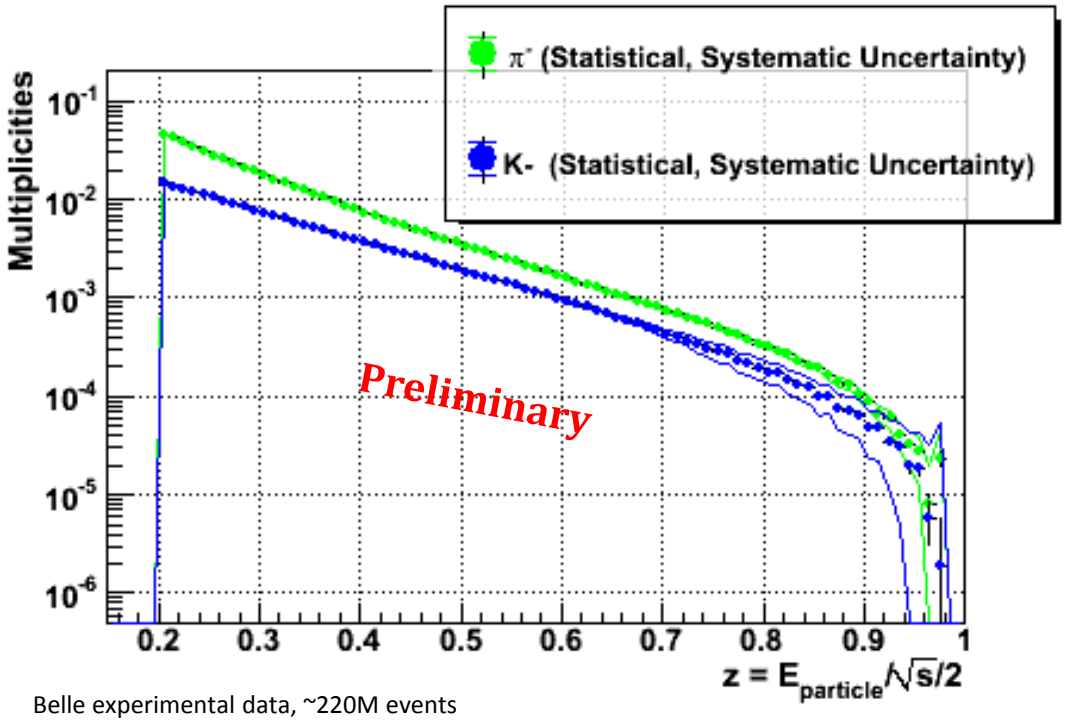
$$\vec{N}_i = \hat{P}^{-1}\,\vec{\tilde{N}}_j :$$

correction through inversion of matrix.

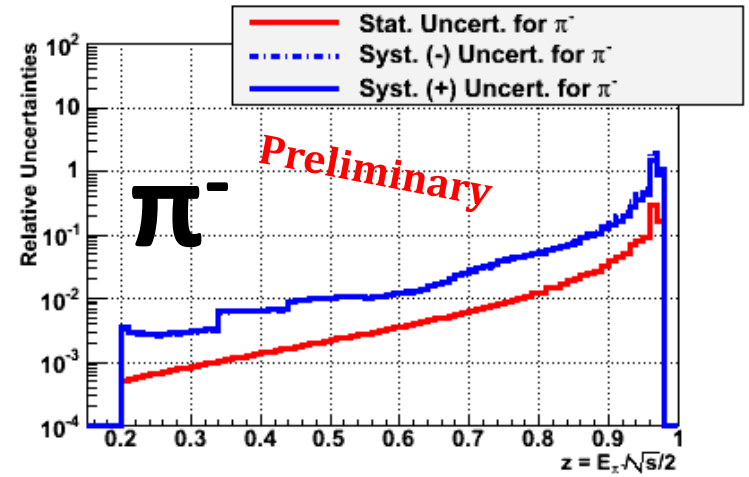
Pion and Kaon Multiplicities

Preliminary Results

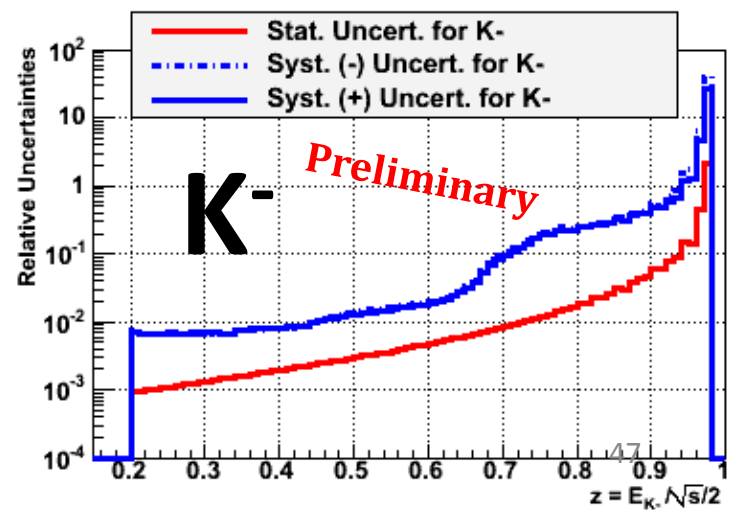
- Binning in z: width = 0.01; yields normalized to hadronic cross section
- Systematic uncertainties: z ~0.6: 1% (2%) for π (K);
  z ~0.9: 14% (50%) for π (K)



Belle experimental data, ~220M events



Additional normalization uncertainty of 1.4% not shown.

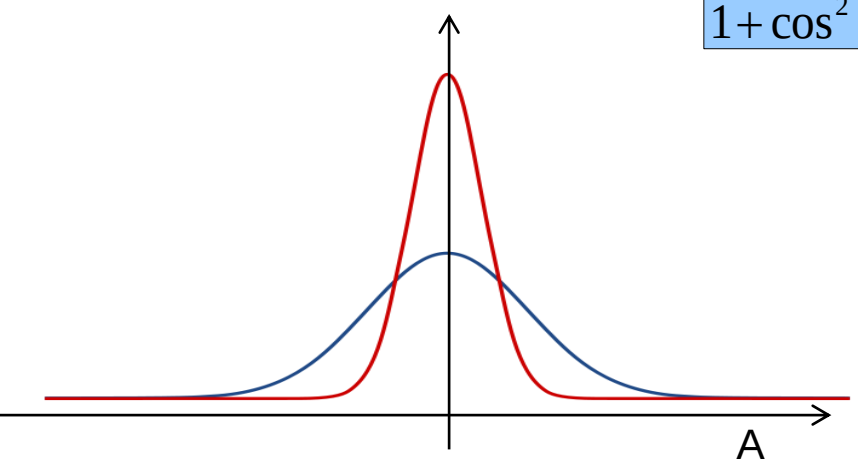


Summary and Outlook

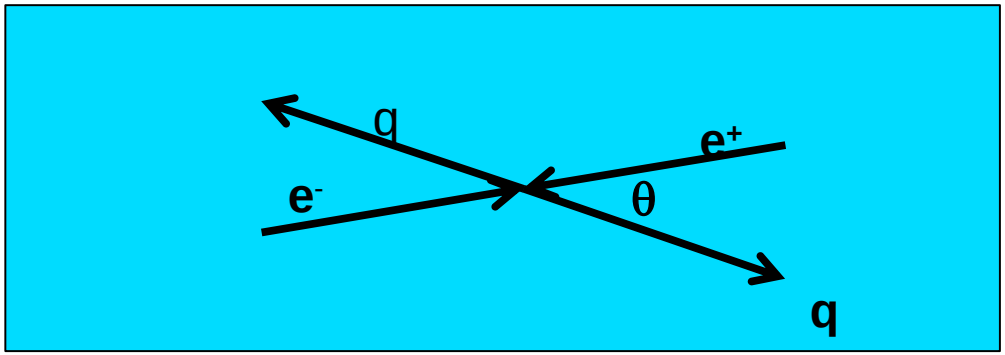
- RHIC collected data in polarized p+p from $\sqrt{s}$=62.4 GeV – $\sqrt{s}$=500 GeV
- Non-zero signals for correlation measurements in the central region single TSA in forward region
- Data taken this year will be able to probe p_t dependence of A_N, access transversity in di-hadron and Collins asymmetries
- Belle measured
  - unpolarized yield of pion and Kaons
  - Transverse spin dependent single and di-hadron FFs

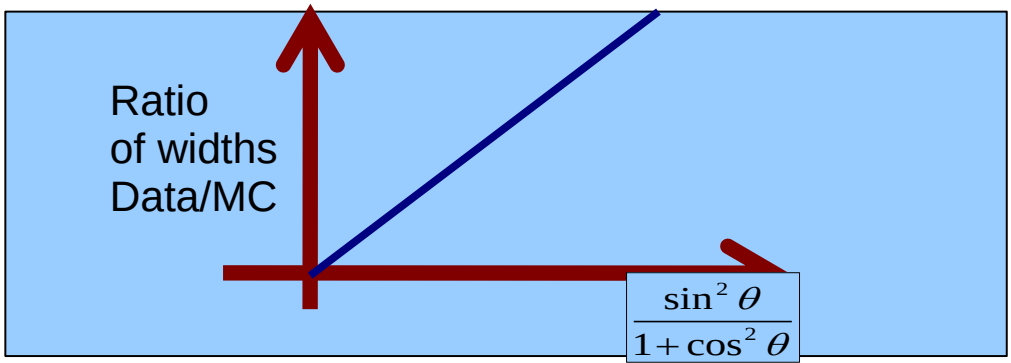
Current Analysis (not public yet)

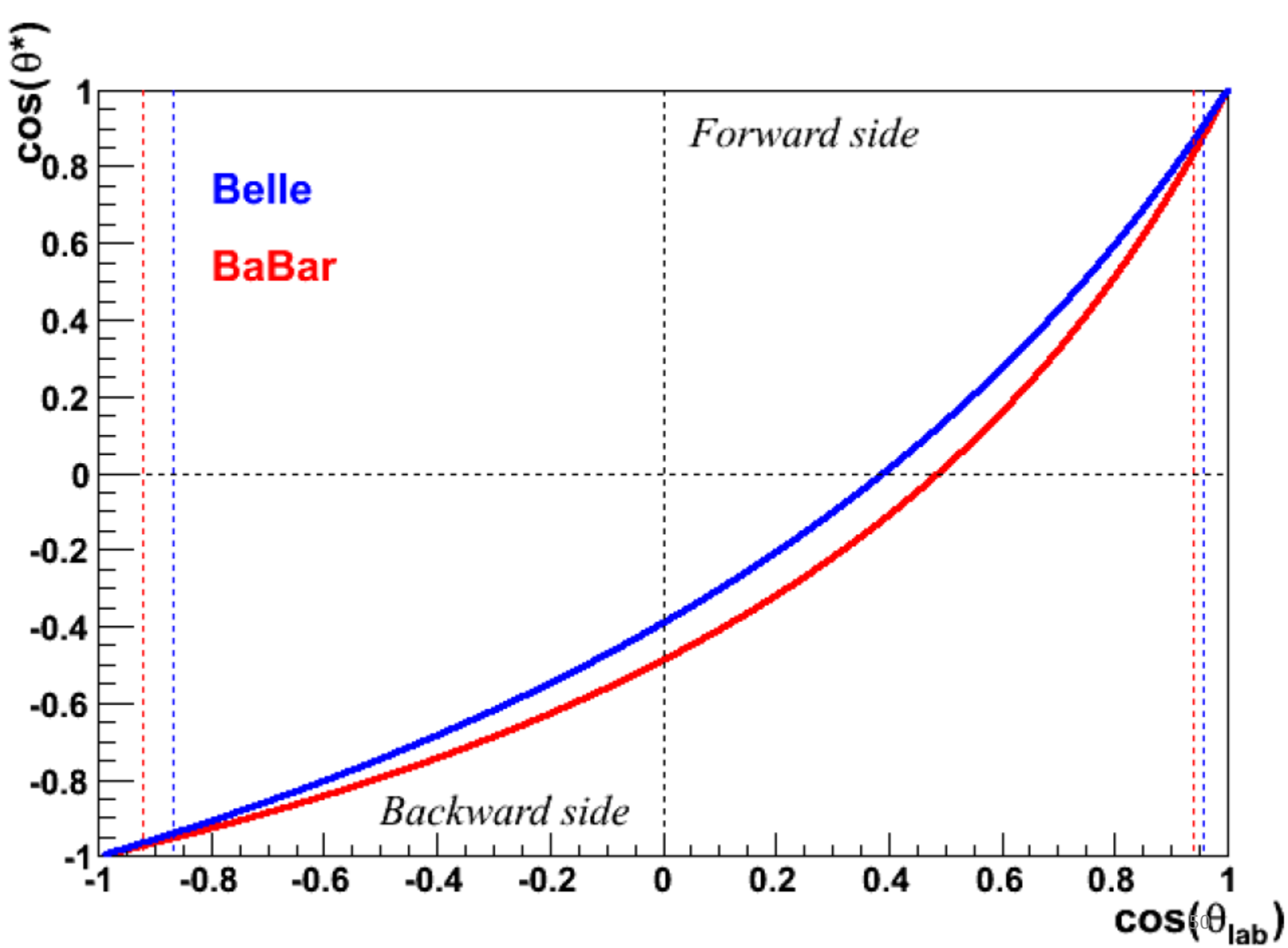
- Use unbinned maximum likelihood fit for each event ->extract asymmetry A
- Width of distribution of A is indication of effect
- Compare with simulation
- Physical effect has to have linear dependence on giving the transverse spin projection



$$\frac{\sin^2\theta}{1+\cos^2\theta}$$



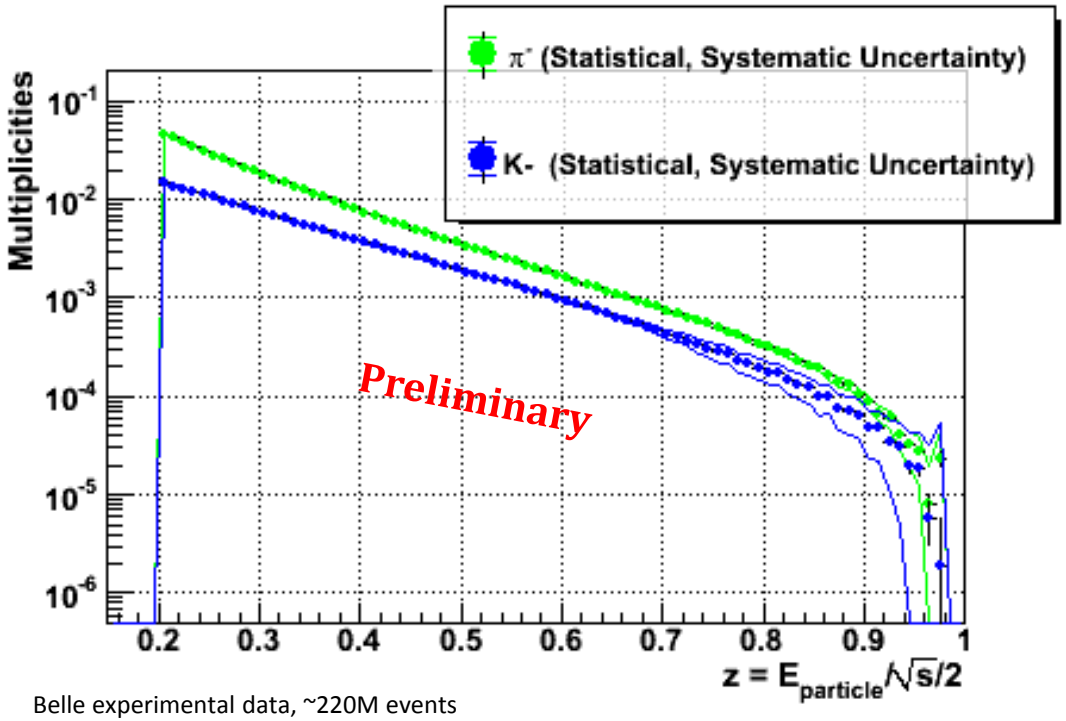



cos(θ*)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
Belle
BaBar
Forward side
Backward side
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
cos(θlab)

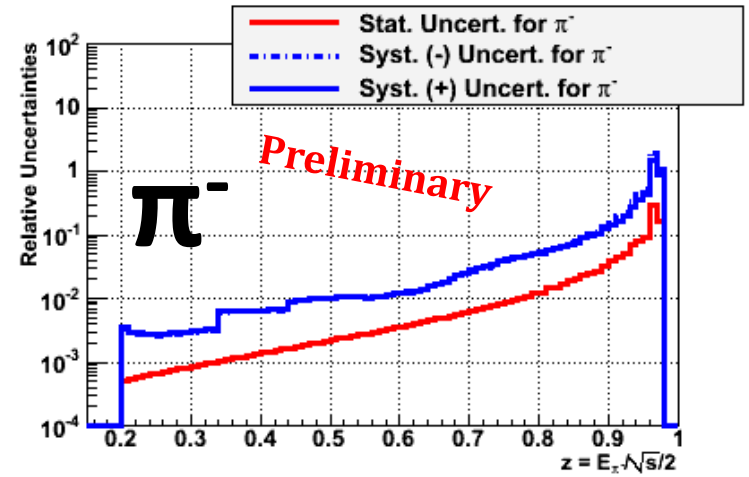
Pion and Kaon Multiplicities

Preliminary Results

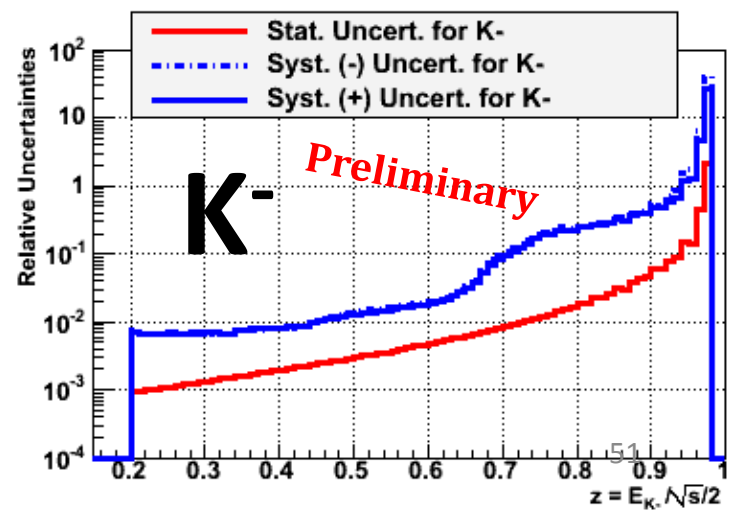
- Binning in z: width = 0.01; yields normalized to hadronic cross section
- Systematic uncertainties: z ~0.6: 1% (2%) for π (K);
  z ~0.9: 14% (50%) for π (K)



Belle experimental data, ~220M events



Additional normalization uncertainty of 1.4% not shown.



Workshop On
P- and CP-odd Effects in Hot and Dense Matter (2012)
RIKEN BNL Research Center Workshop
June 25-27, 2012 at Brookhaven National Laboratory

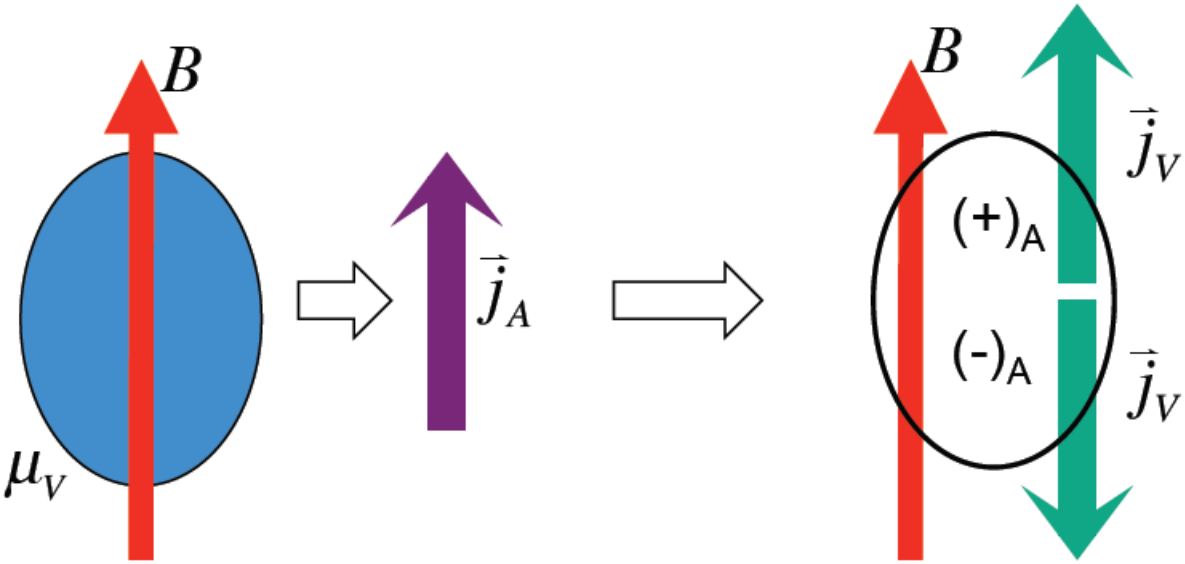
Search for effects related to Chiral Magnetic Wave at STAR

Gang Wang (UCLA)
for STAR Collaboration

Motivation

CSE + CME □ Chiral Magnetic Wave:

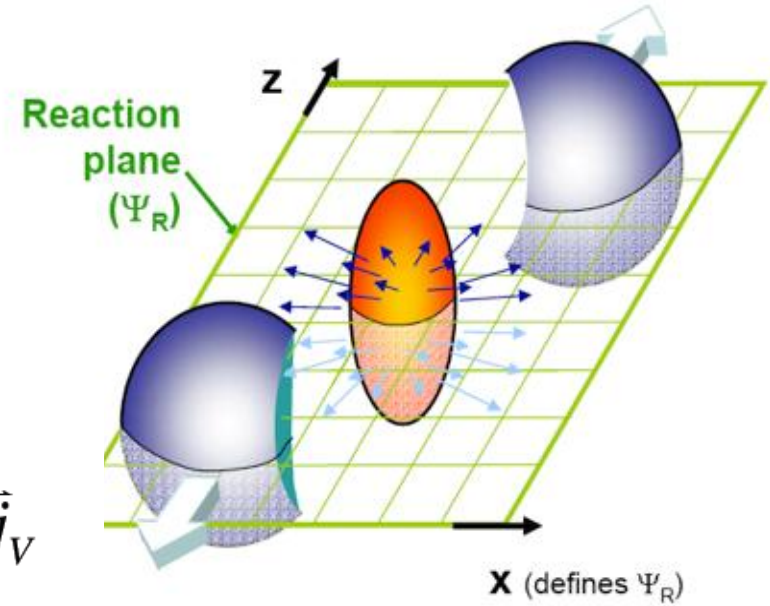
- gapless collective excitation
- signature of Chiral Symmetry Restoration



$$j_A = \frac{N_c e}{2\pi^2}\mu_V B \qquad j_V = \frac{N_c e}{2\pi^2}\mu_A B$$

Chiral Separation Effect

Chiral Magnetic Effect

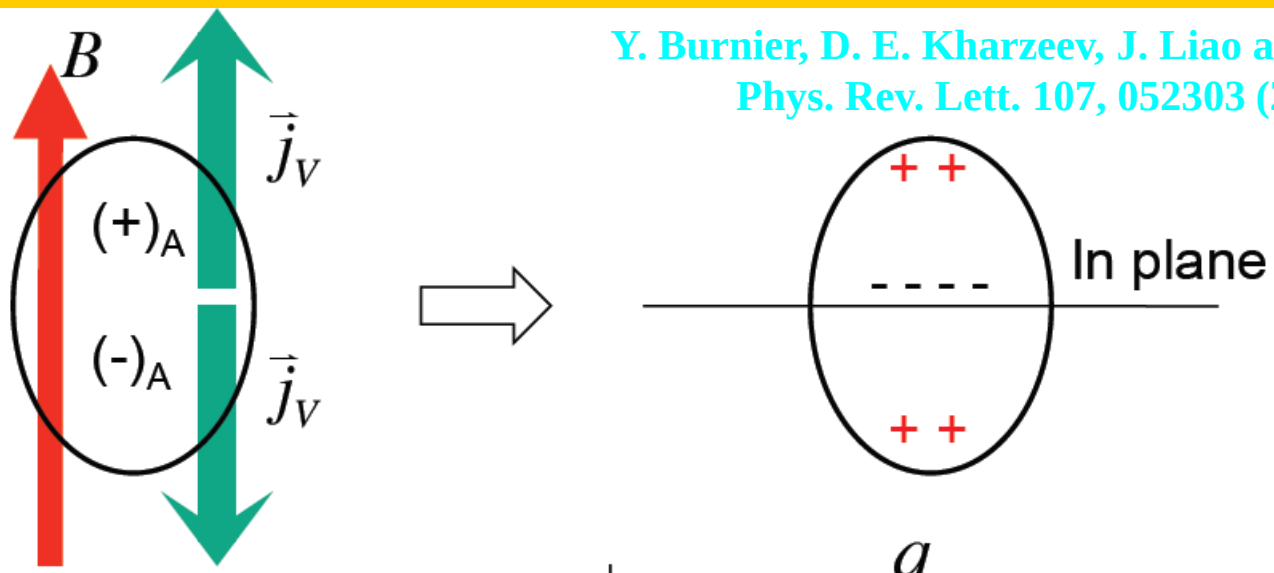


Peak magnetic field ~ **10^{15} Tesla !**

(Kharzeev et al. NPA 803 (2008) 227)

Observables

Y. Burnier, D. E. Kharzeev, J. Liao and H-U Yee, Phys. Rev. Lett. 107, 052303 (2011)

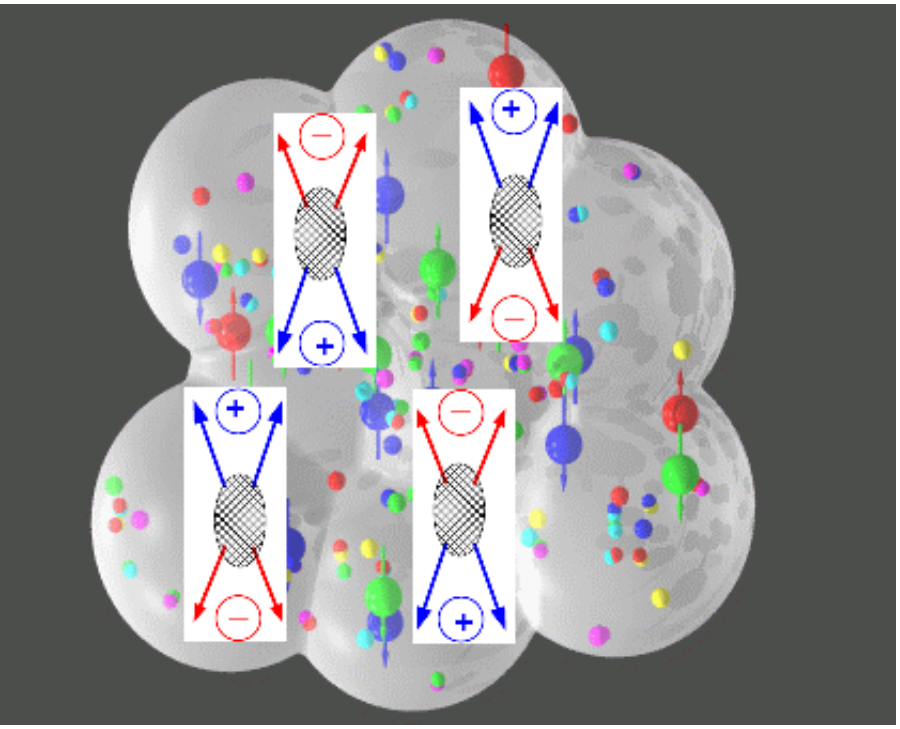


Formation of electric quadrupole: $v_2^{\pm} = v_2 \mp (\frac{q_e}{\bar{\rho}_e}) A_{\pm}$,

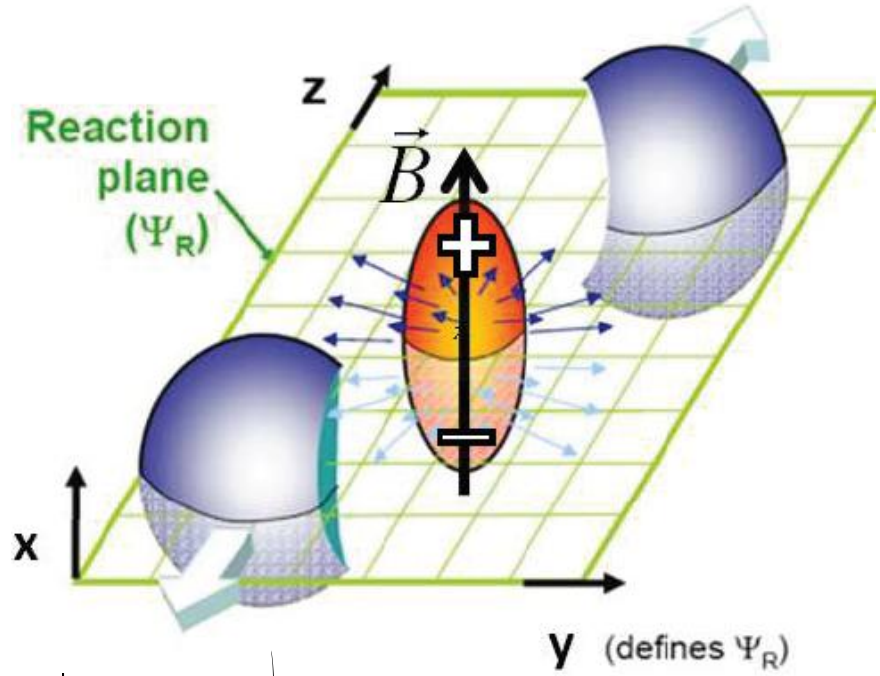
where charge asymmetry is defined as $A_{\pm} = \frac{\bar{N}_+ - \bar{N}_-}{\bar{N}_+ + \bar{N}_-}$.

Then π^- v_2 should have a positive slope,
and π^+ v_2 should have a negative slope with the same magnitude.
The integrated v_2 of π^- is not necessarily bigger than π^+: (other physics)
only the $A_{\pm}$ dependency matters for CMW testing.

Observables



CMW + Parity-odd domain,
=> charge separation across RP

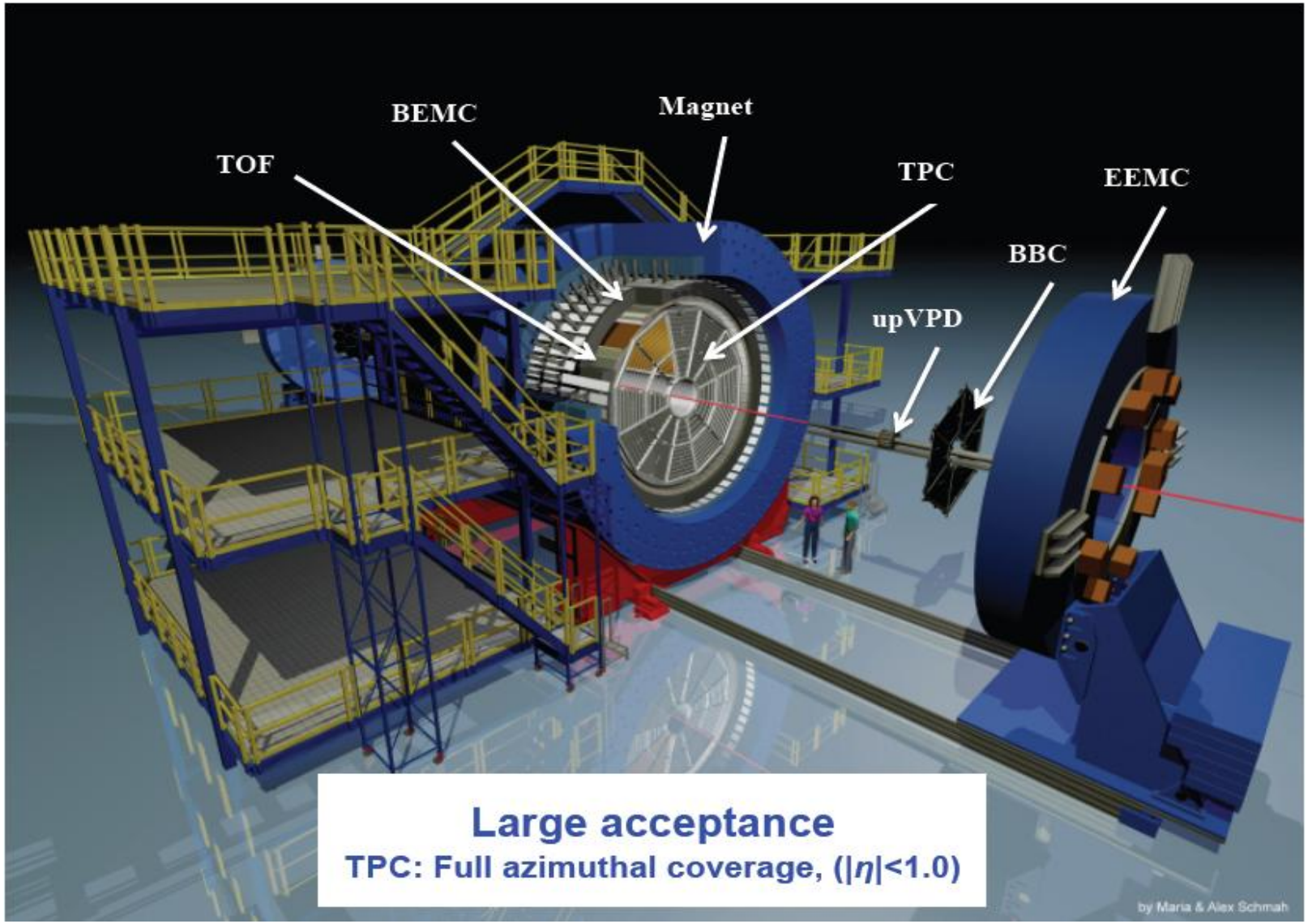


$$\frac{dN_{\pm}}{d\phi} \propto 1 + 2a_{\pm} \cdot \sin\left(\phi^{\pm} - \Psi_{RP}\right)$$

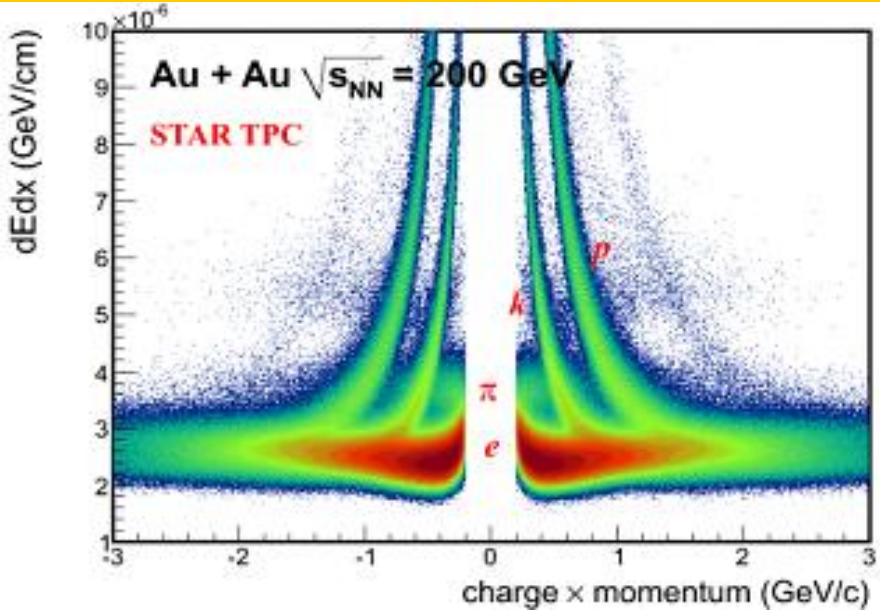
This charge separation effect needs to be beyond statistical fluctuation and conventional physics background.

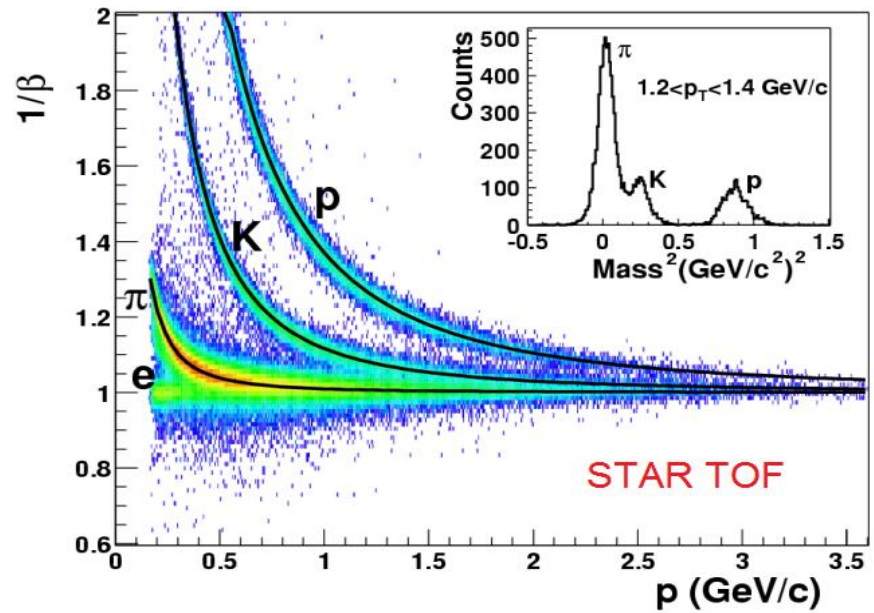
Kharzeev, PLB633:260 (2006)
Kharzeev, McLerran, Warringa, NPA803:227 (2008)

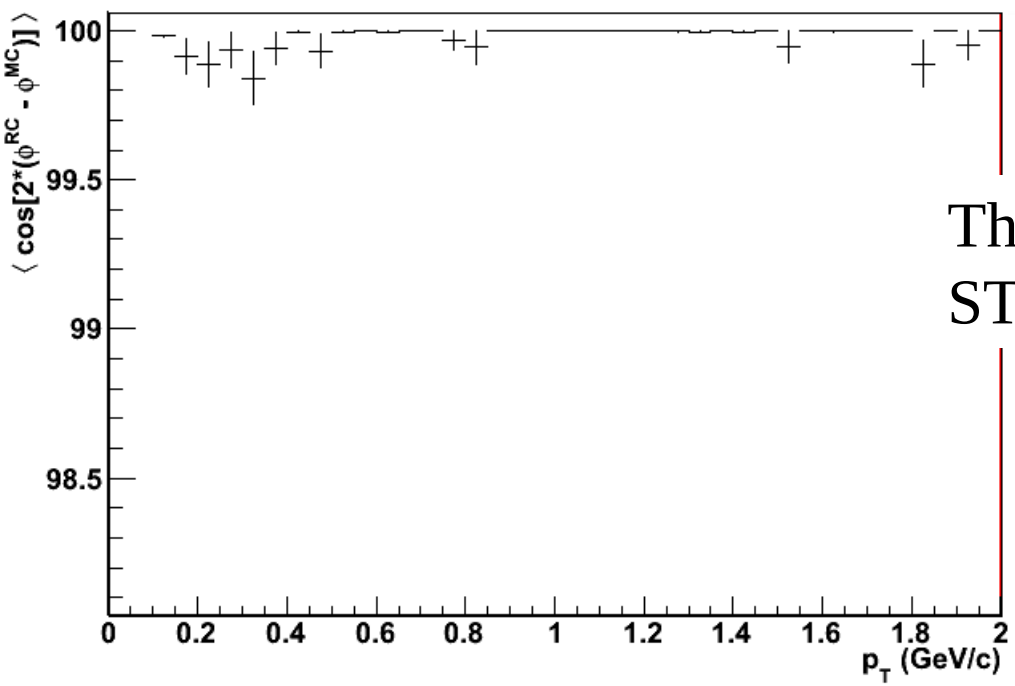
STAR


BEMC
Magnet
TOF
TPC
EEMC
BBC
upVPD
Large acceptance
TPC: Full azimuthal coverage, (|η|<1.0)
by Maria & Alex Schmah

STAR: excellent PID and tracking

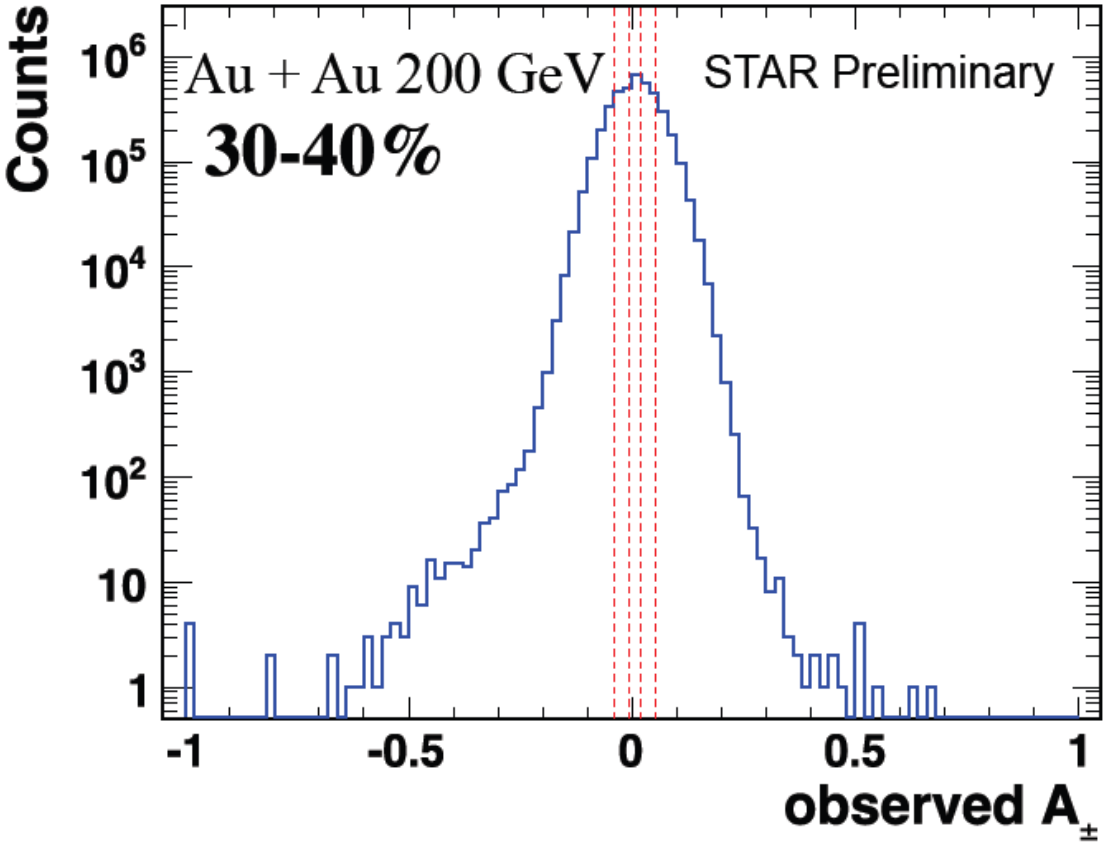






The correlation measurements at STAR are accurate to relative 0.1%.

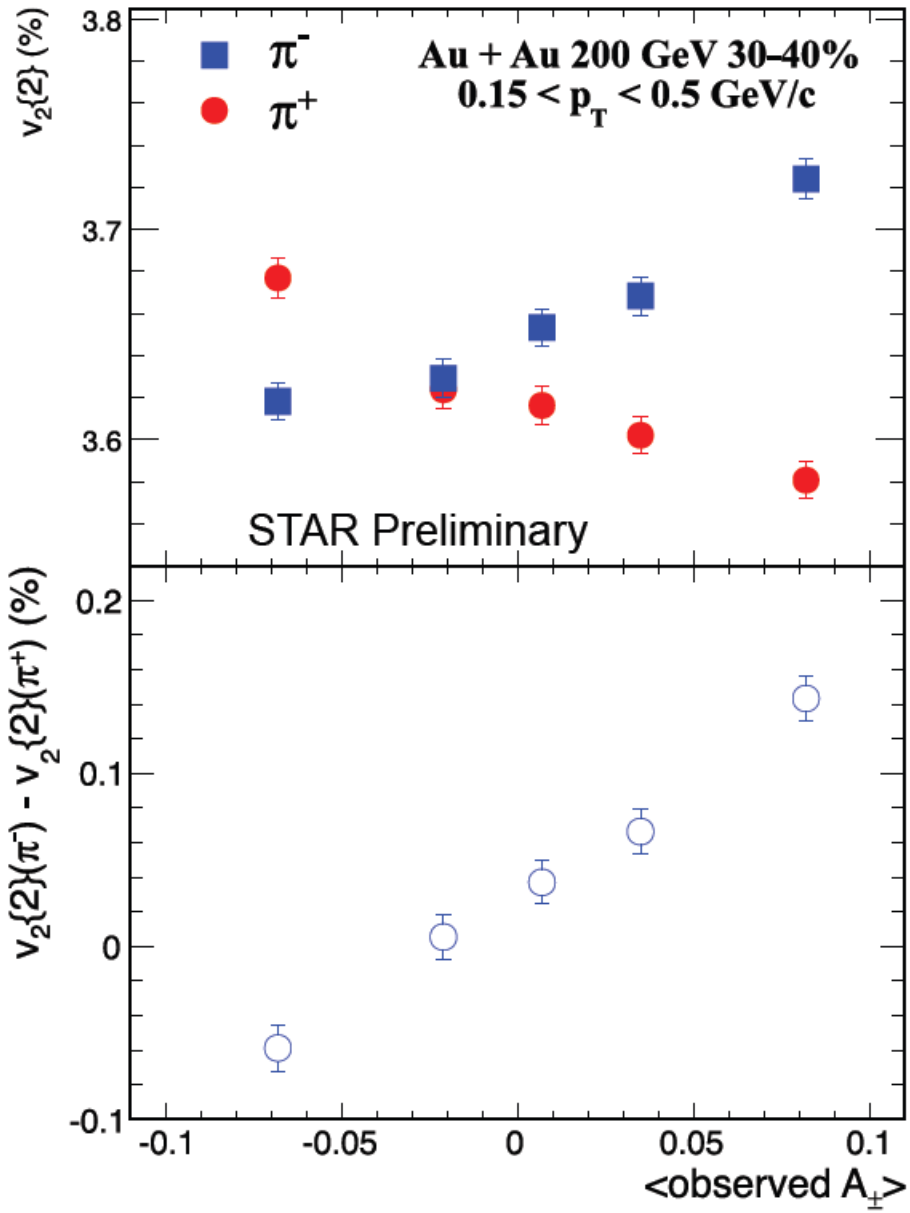
Observed charge asymmetry



$$A_\pm = \frac{\overline{N}_+ - \overline{N}_-}{\overline{N}_+ + \overline{N}_-}$$

- N^+ (N^-) is the number of positive (negative) particles within $|\eta|<1$.
- The distribution was divided into 5 bins, with roughly equal counts.
- Tracking efficiency was corrected later.

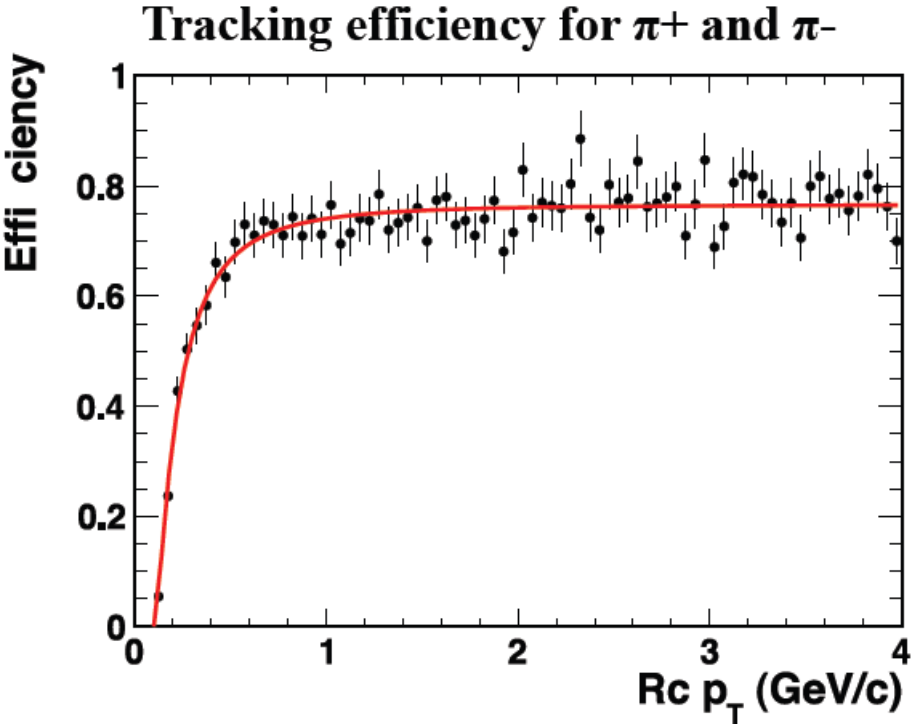
Charge asymmetry dependency

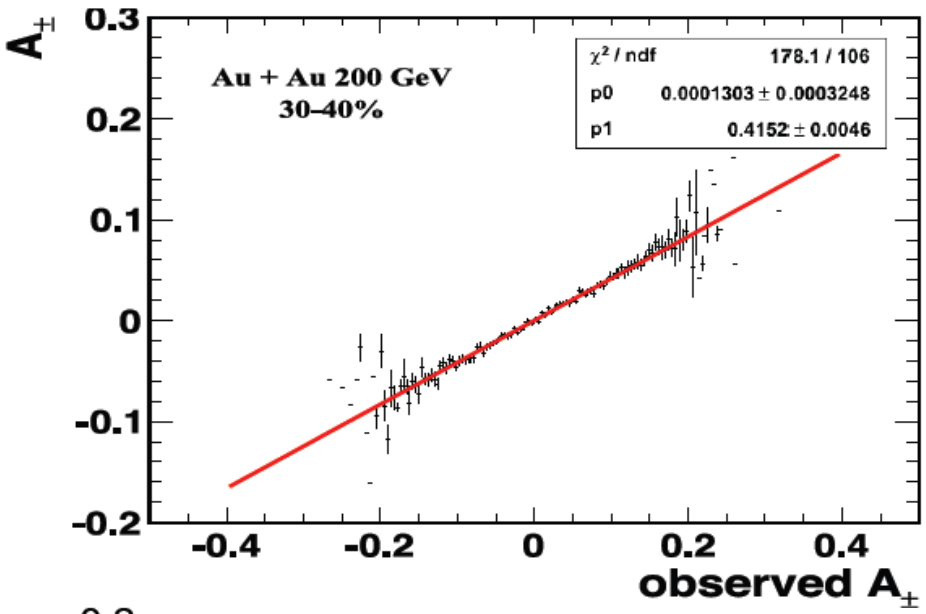


- v_2 was measured with the Q-cumulant method.
- Clear $A_\pm$ dependency
- $v_2(A_\pm)$ slopes for $\pi^\pm$:
  - opposite sign
  - similar magnitude
- v_2 difference vs $A^\pm$ may have a non-zero intercept: other physics?

$$v_2^\pm = v_2 \mp \left(\frac{q_e}{\bar{\rho}_e}\right) A_\pm$$

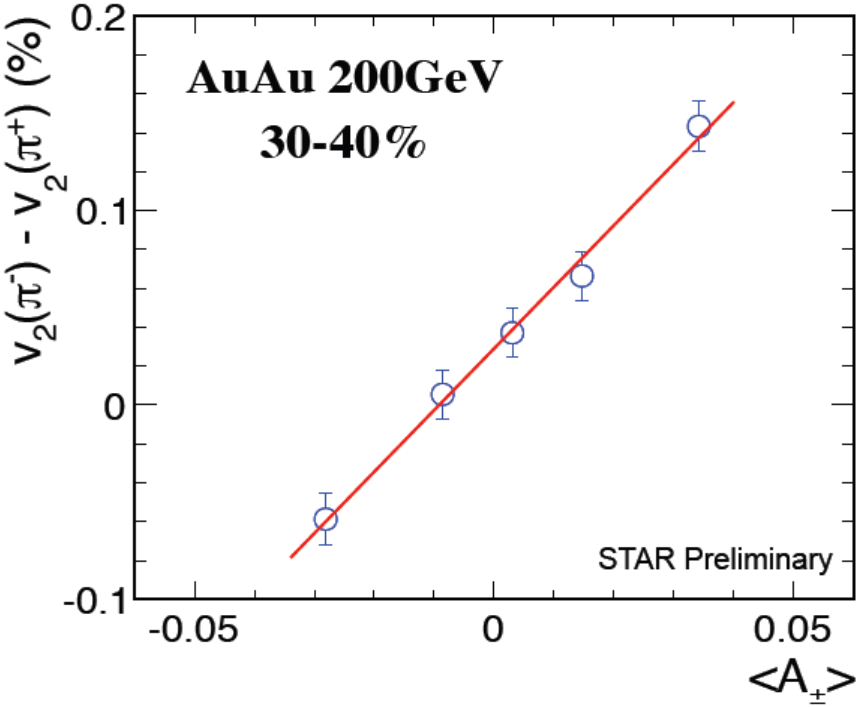
Correction for tracking efficiency



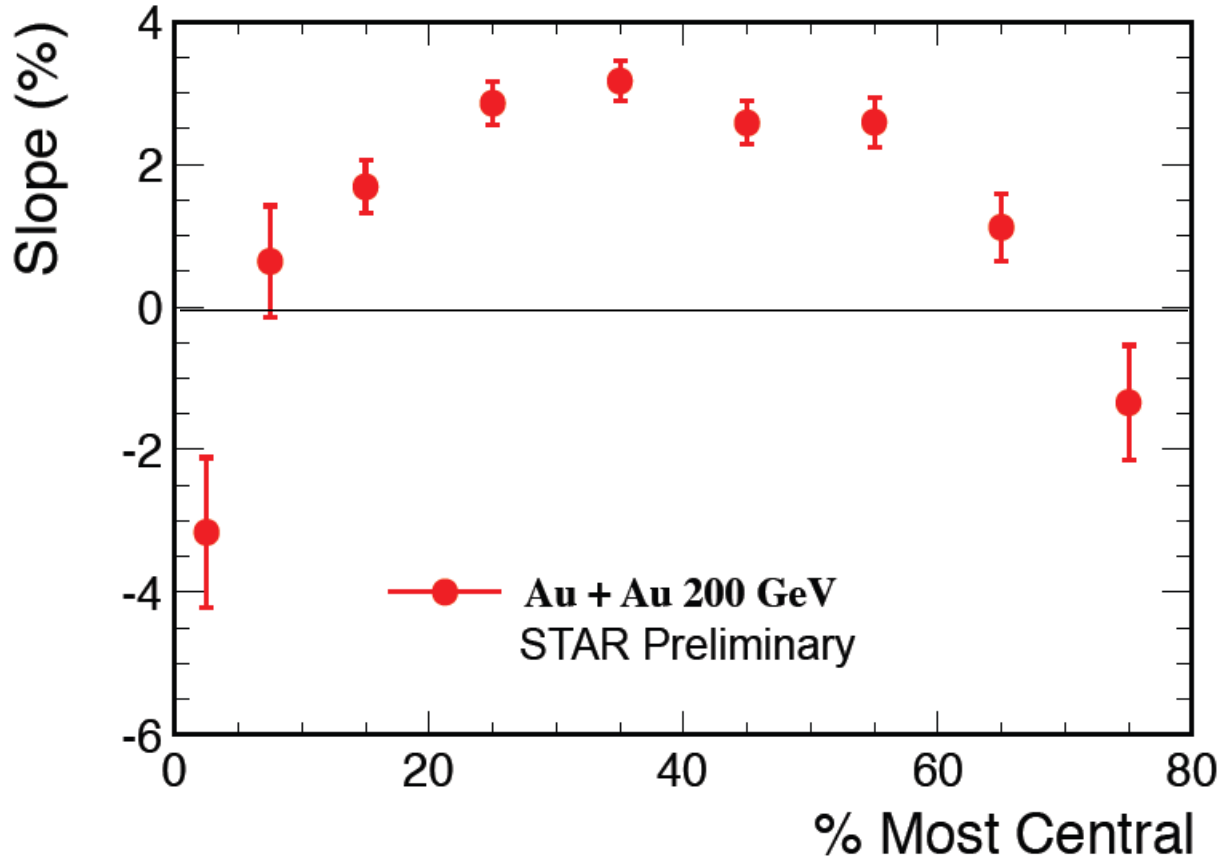


$$v_2^- - v_2^+ = 2\left(\frac{q_e}{\bar{\rho}_e}\right) A_\pm$$

- Fit with a straight line to extract the slope $r = 2\frac{q_e}{\bar{\rho}_e}$.
- Do the same for all centralities



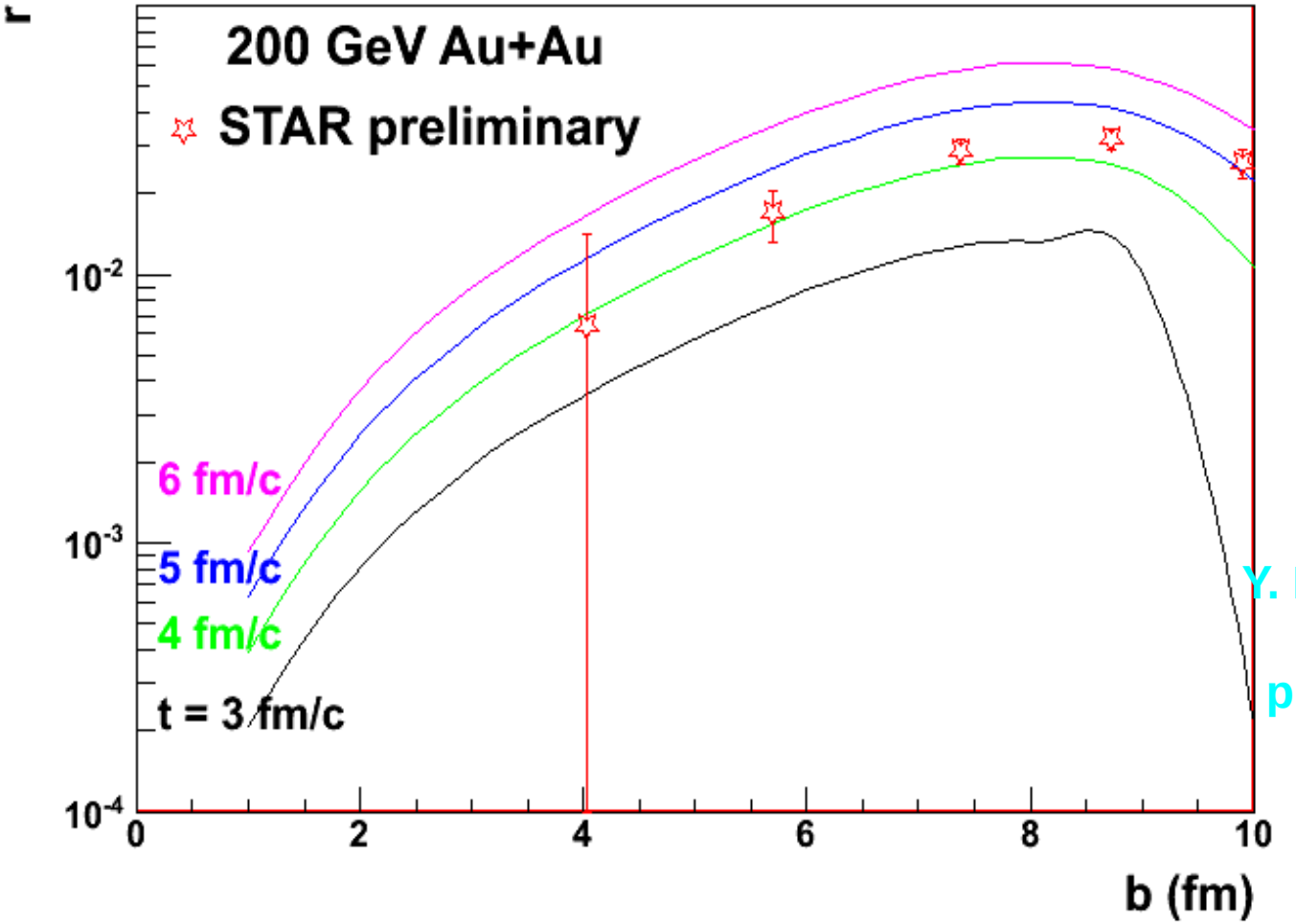
Slope vs centrality



- Statistical errors only
- Smooth trend
- Positive and significant most centrality bins
- Systematic uncertainty:
  - weak decays (Λ et al.)
  - tracking efficiency
  - $\langle A_{\pm} \rangle$ bin center effect
  - different v_2 methods

$$r = 2\frac{q_e}{\overline{\rho}_e}$$

Theoretical calculations

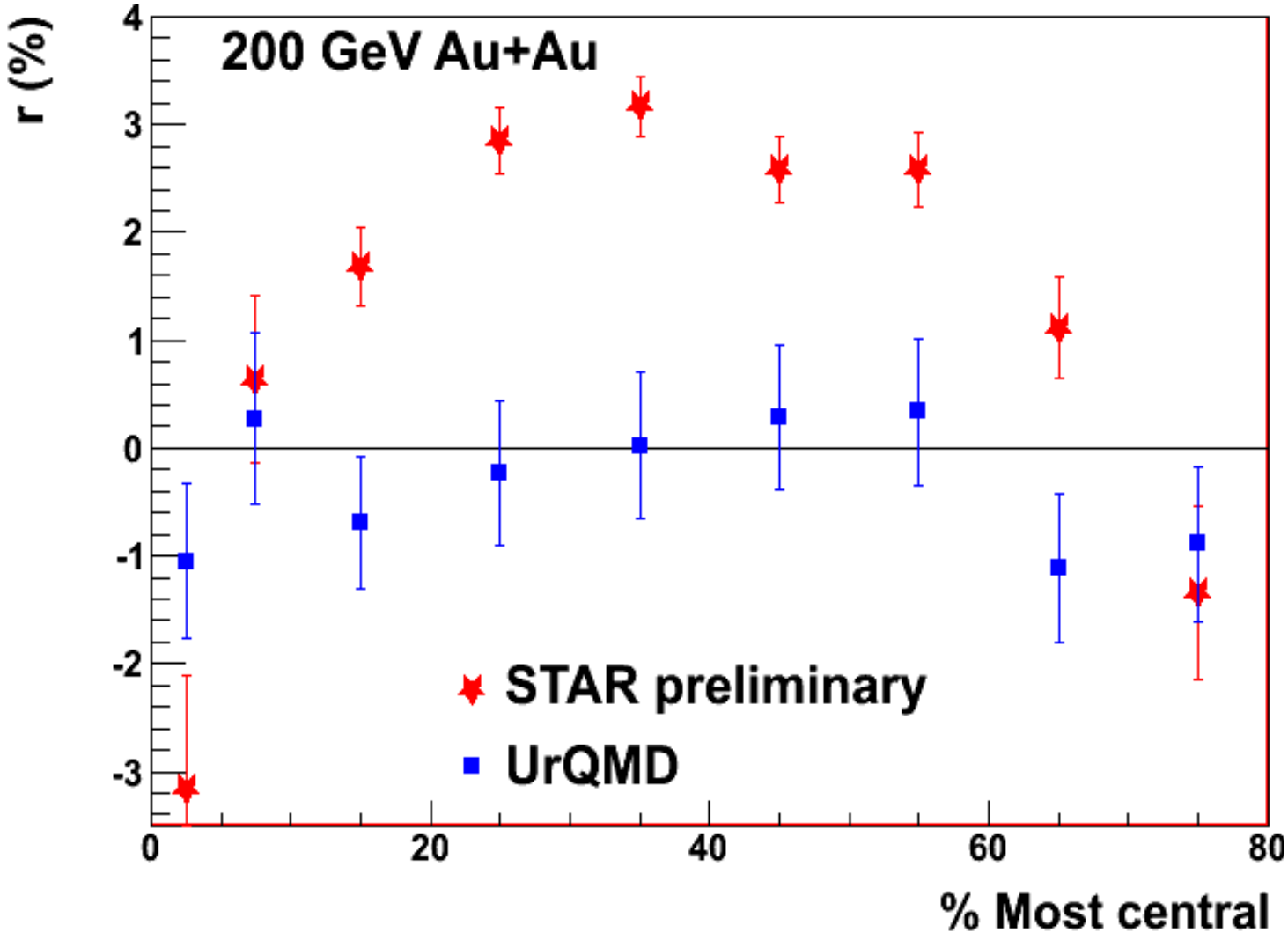


$$r = 2\frac{q_e}{\overline{\rho}_e}$$

Y. Burnier, D. E. Kharzeev, J. Liao and H-U Yee, private communication.

- Very similar trend between data and theoretical calculations
- To compare the magnitude, the acceptance effects need investigation.

UrQMD



$$r = 2\frac{q_e}{\overline{\rho}_e}$$

Hongwei Ke

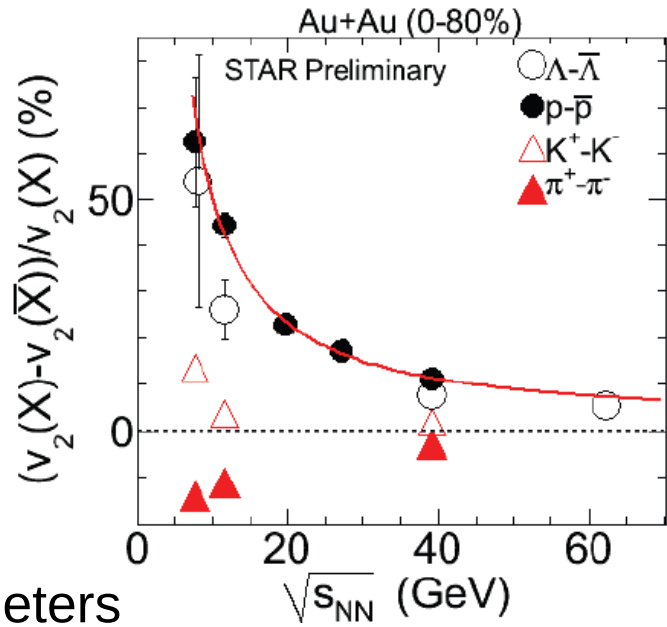
The slopes from UrQMD are consistent with 0 using the same approach.

Outlook

- Further studies of the weak decay contributions
  - DCA (Distance of Closest Approach) cut on particles
  - simulation such as UrQMD and AMPT

- Beam energy scan (39 GeV, 27 GeV and 19.6 GeV)
  - different charge asymmetry distribution
  - different magnetic field?

- Kaon v_2
  - opposite v_2 ordering than pion ($v_2(K^+) > v_2(K^-)$)
  - what about $A_\pm$ dependency?

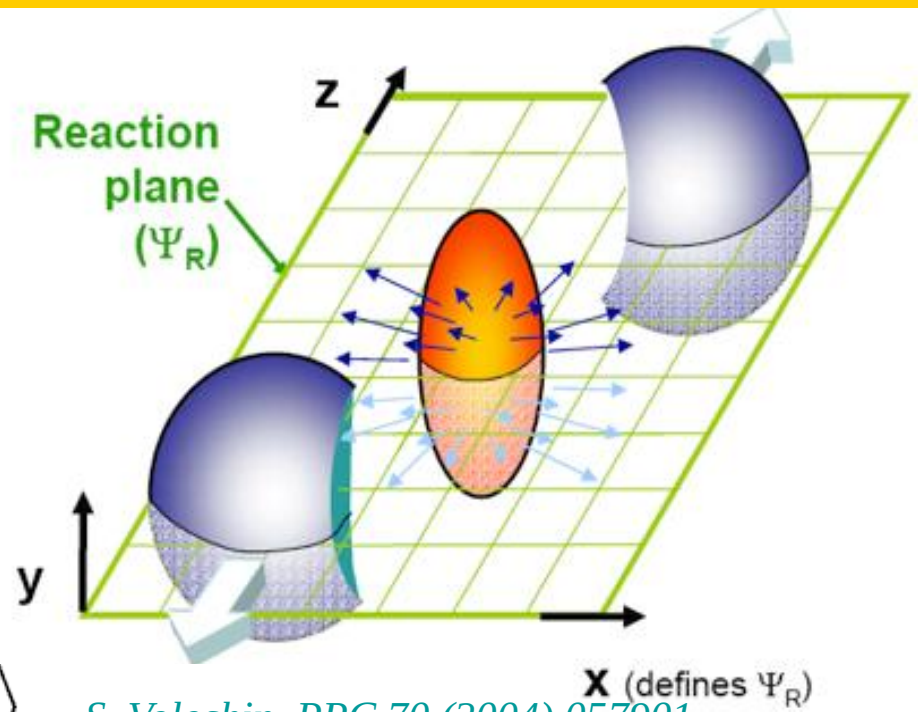


- Handle on the magnetic field
  - trigger on spectators with Zero Degree Calorimeters
  - is the signal still there when we turn off B?

CMW + Local Parity Violation

$$\frac{dN_{\pm}}{d\phi} \propto 1 + 2a_{\pm} \cdot \sin\left(\phi^{\pm} - \Psi_{RP}\right)$$

A direct measurement of the *P*-odd quantity *"a"* should yield *zero*.



$$\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle$$

S. Voloshin, PRC 70 (2004) 057901

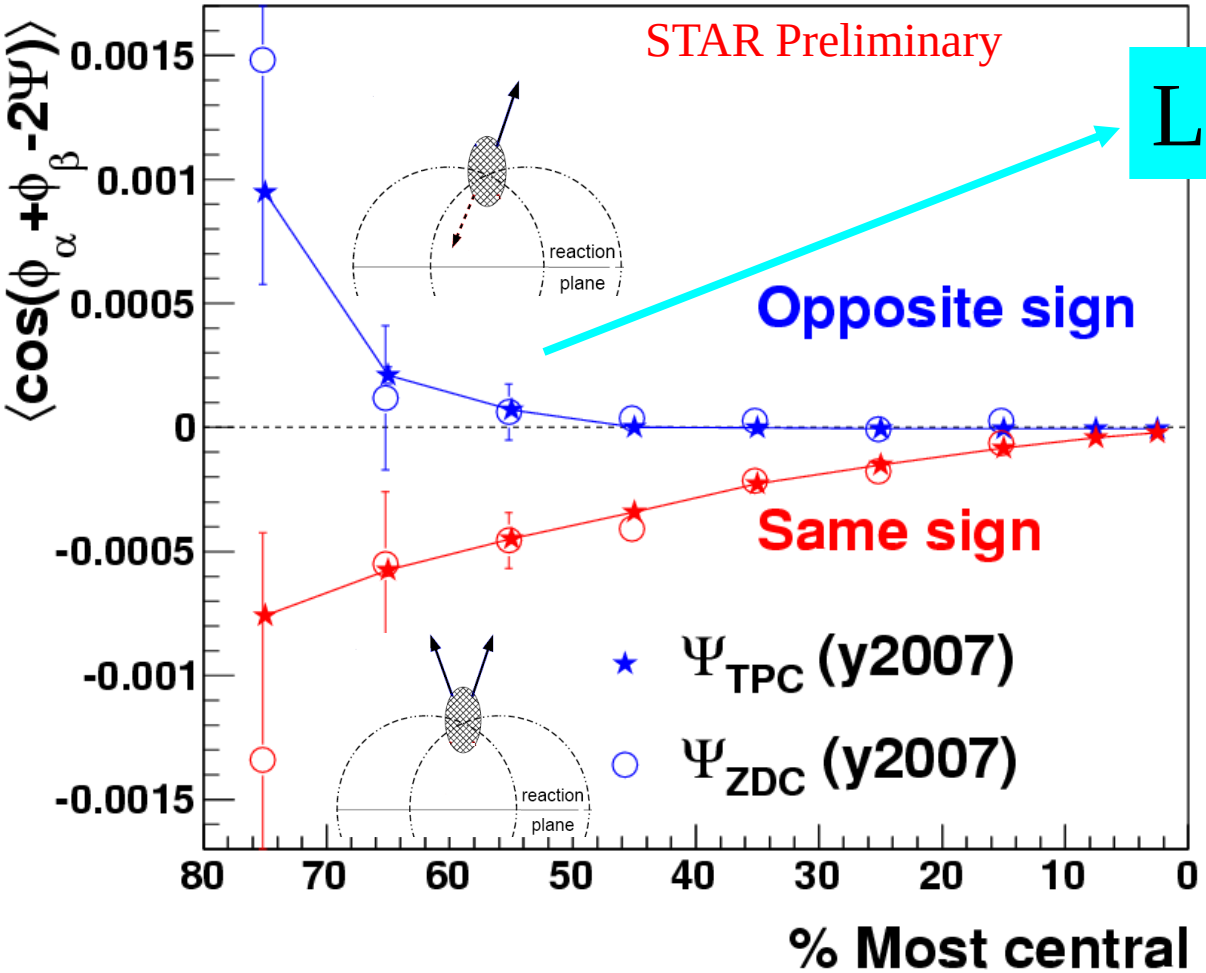
$$= [\langle v_{1,\alpha} v_{1,\beta} \rangle + B_{in}] - [\langle a_\alpha a_\beta \rangle + B_{out}]$$

Non-flow/non-parity effects: largely cancel out

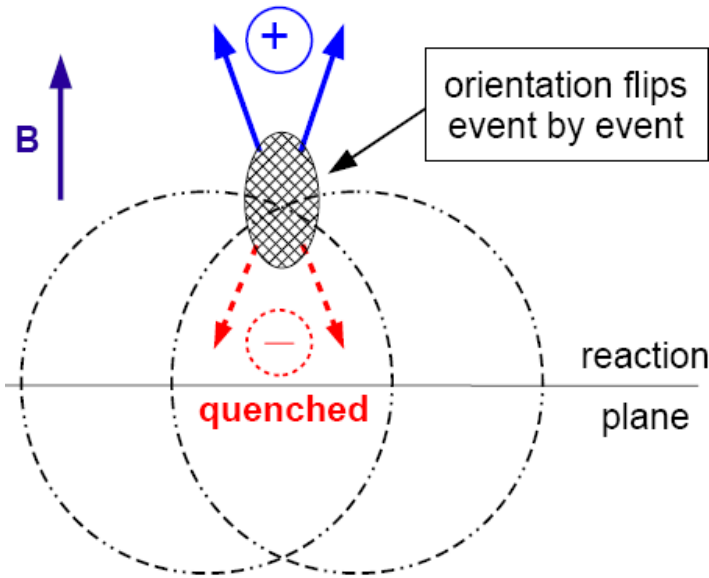
Directed flow: vanishes if measured in a symmetric rapidity range

P-even quantity: still sensitive to charge separation

Results with different EPs

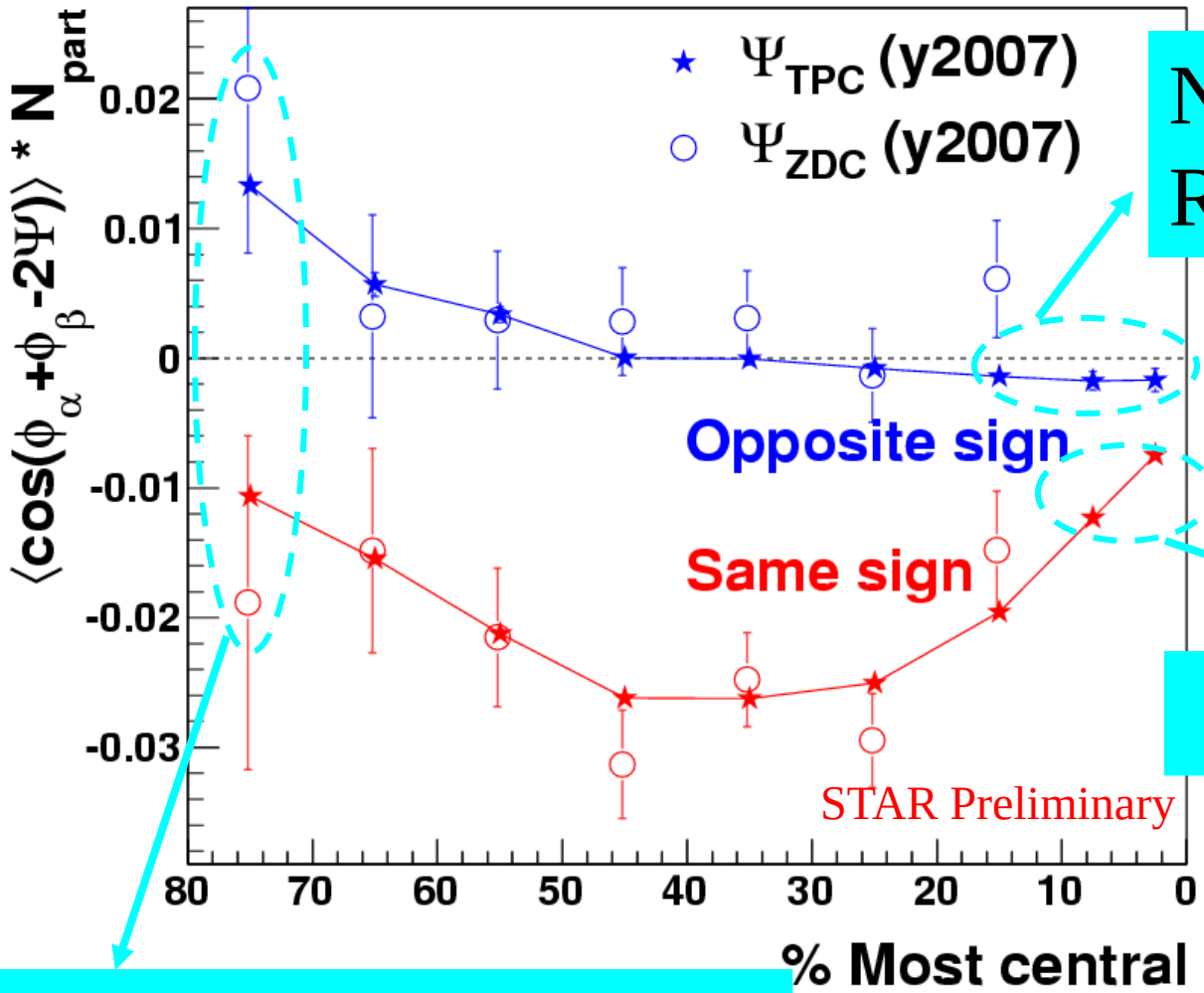


Lost in the medium?



The correlators using TPC/ZDC event planes are consistent with each other.

Dilution effect



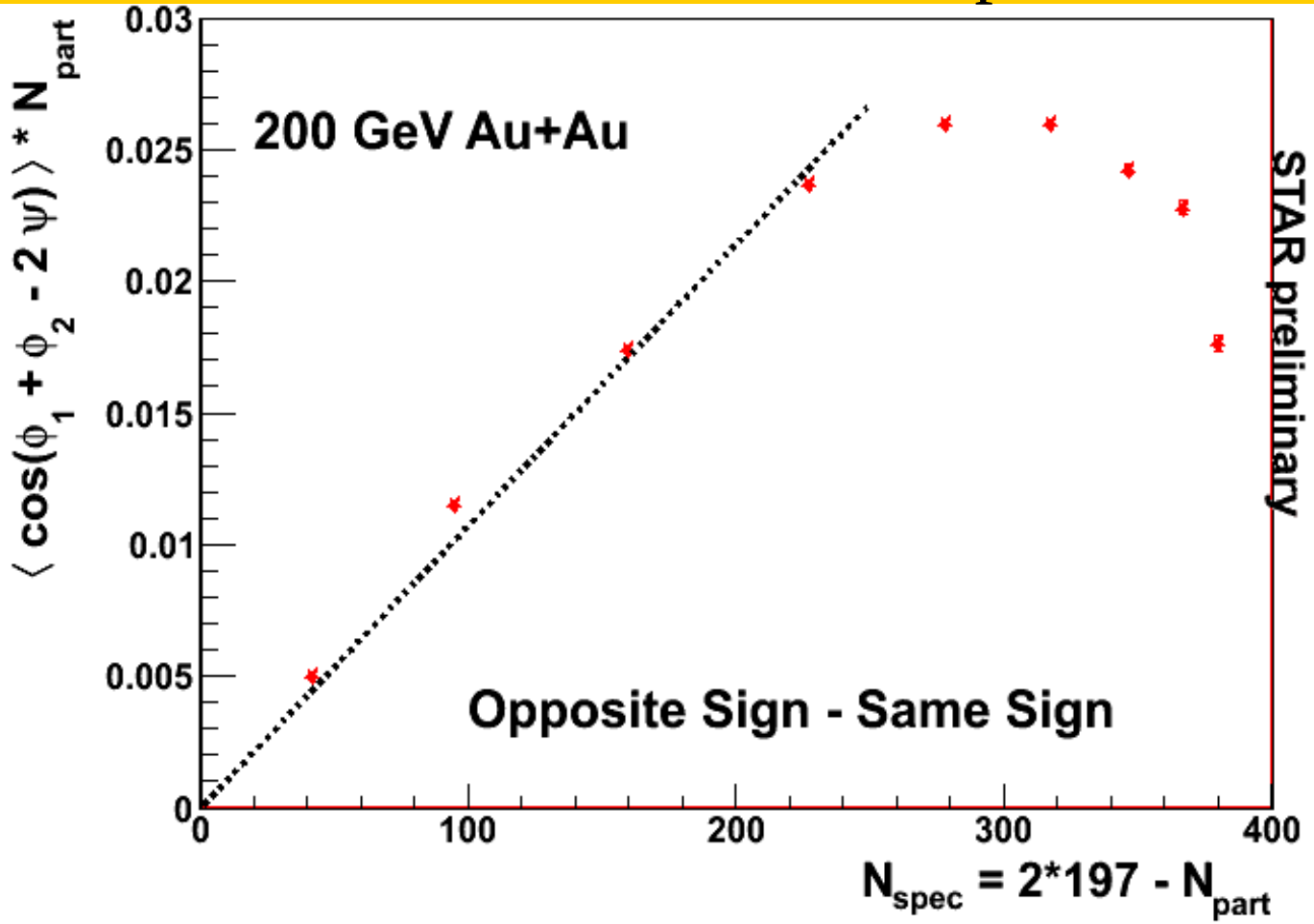
Non-zero $B_{in} - B_{out}$
Radial flow?

Weaker B field

Momentum conservation?

The factor N_{part} is used to compensate for dilution effect

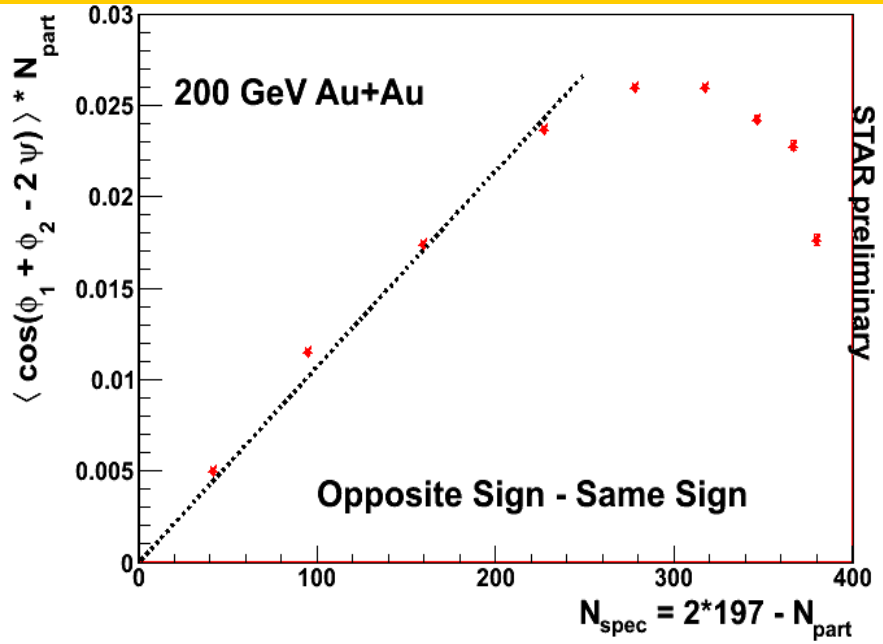
(OS - SS)*N_{part}

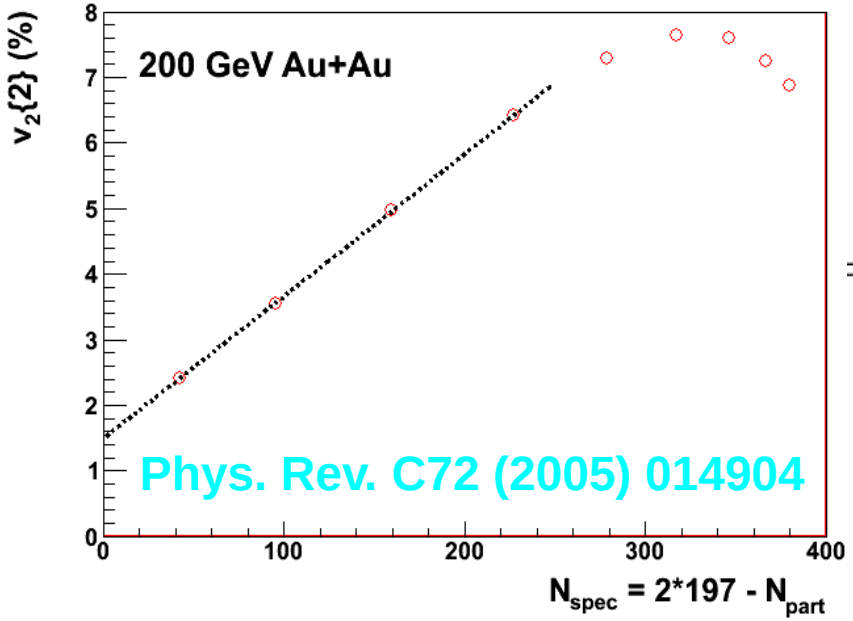


If N_{spec} is a measure for B, the signal (OS-SS)*N_{part} is roughly proportional to the magnetic field in central/mid-central collisions.

Experimentally, we may trigger on N_{spec} to control B.

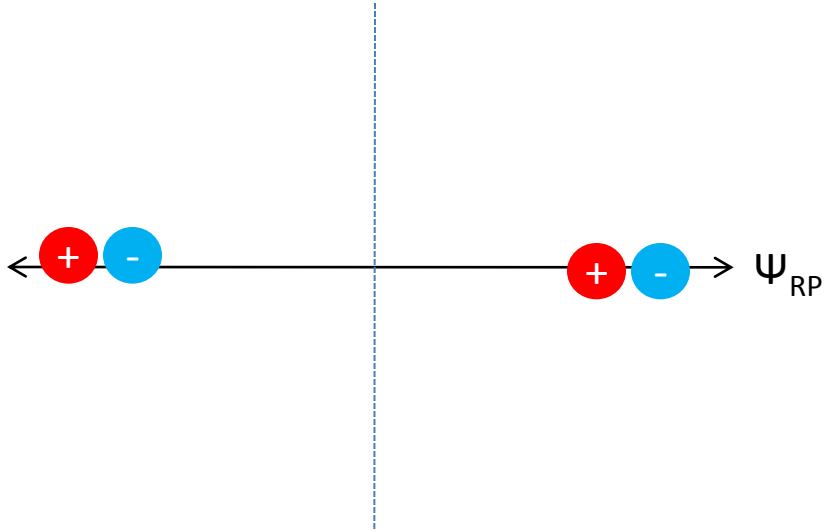
Possible physics background





Physics background for LPV:

charge conservation/cluster + v_2

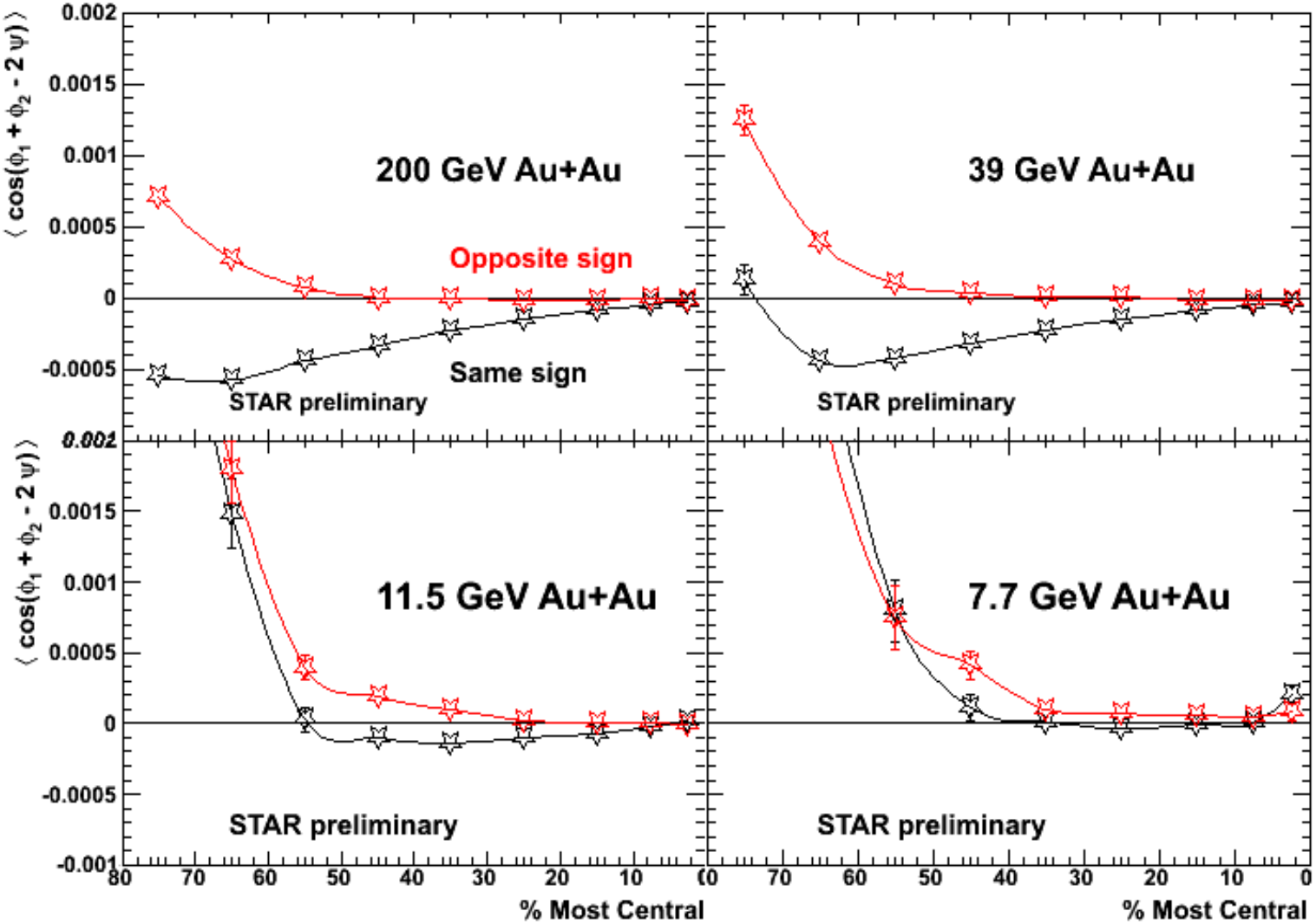


$$\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle$$
$$= \langle \cos((\phi_\alpha + \phi_\beta - 2\phi_{res}) + 2(\phi_{res} - \Psi_{RP})) \rangle$$
$$\approx \frac{f_{res} \langle \cos(\phi_\alpha + \phi_\beta - 2\phi_{res}) \rangle \, v_{2,res}}{N_{ch}}$$

Qualitatively similar trend!

Need to trigger on very small N_{spec}.

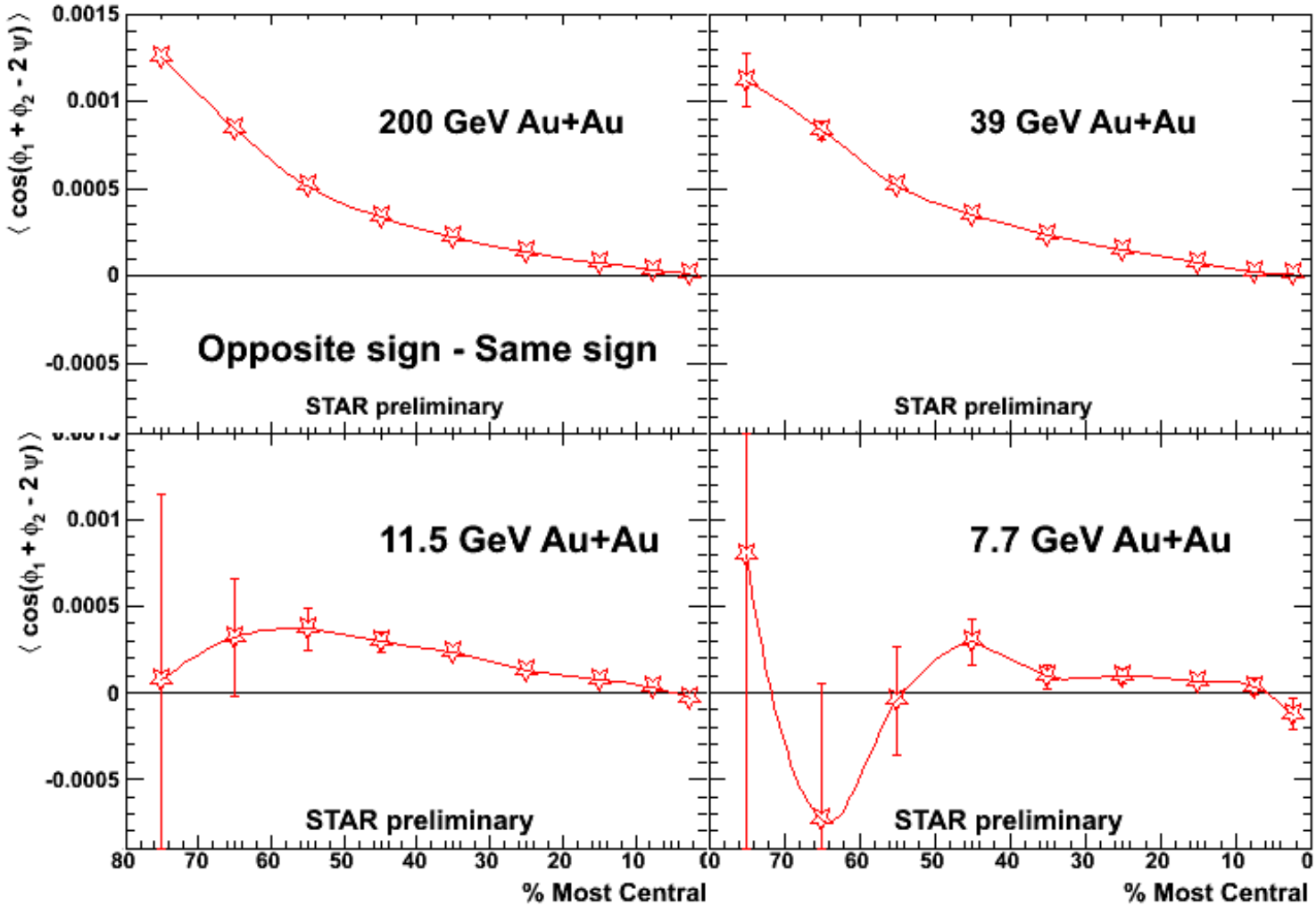
Beam energy scan



As we lower the beam energy, changes start to show from the peripheral collisions.

27 GeV and 19.6 GeV coming soon...

If we consider OS-SS to be signal...



The signal seems to be disappearing at 7.7 GeV, but the statistical errors are large.

Outlook

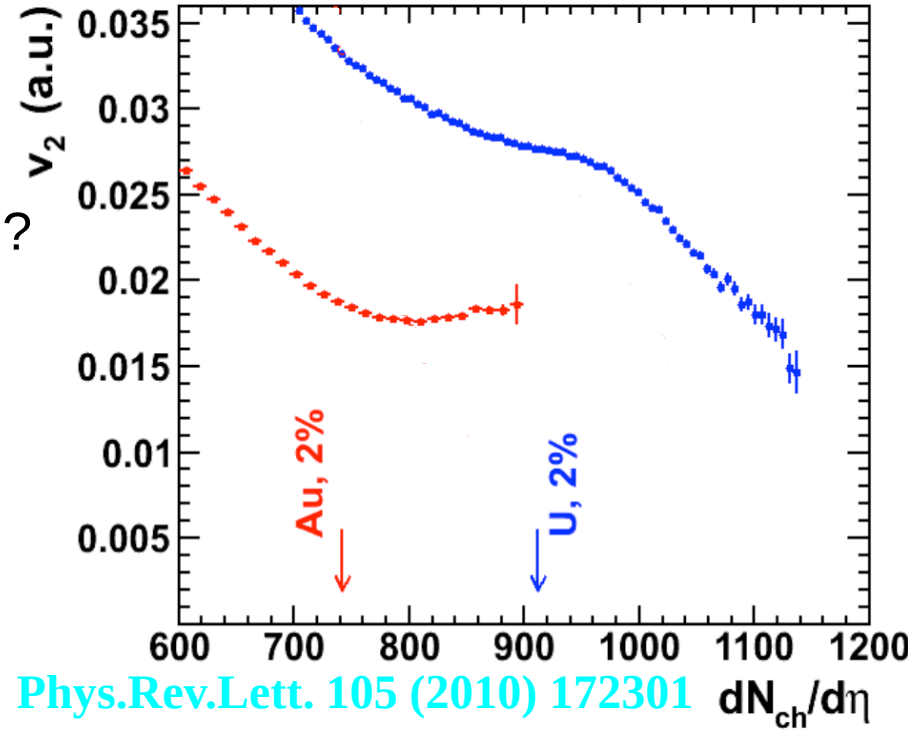
- Further studies of identified particles
  - Kπ correlation
  - pπ, Λπ...
  - ΛΛ correlation: vorticity

See also Kent Riley's talk!

- Beam energy scan (27 GeV and 19.6 GeV)
  - Is there a smooth transition from 200 GeV to 7.7 GeV?
  - different magnetic field?

- Handle on the magnetic field
  - trigger on spectators with ZDCs
  - is there still signal when we turn off B?

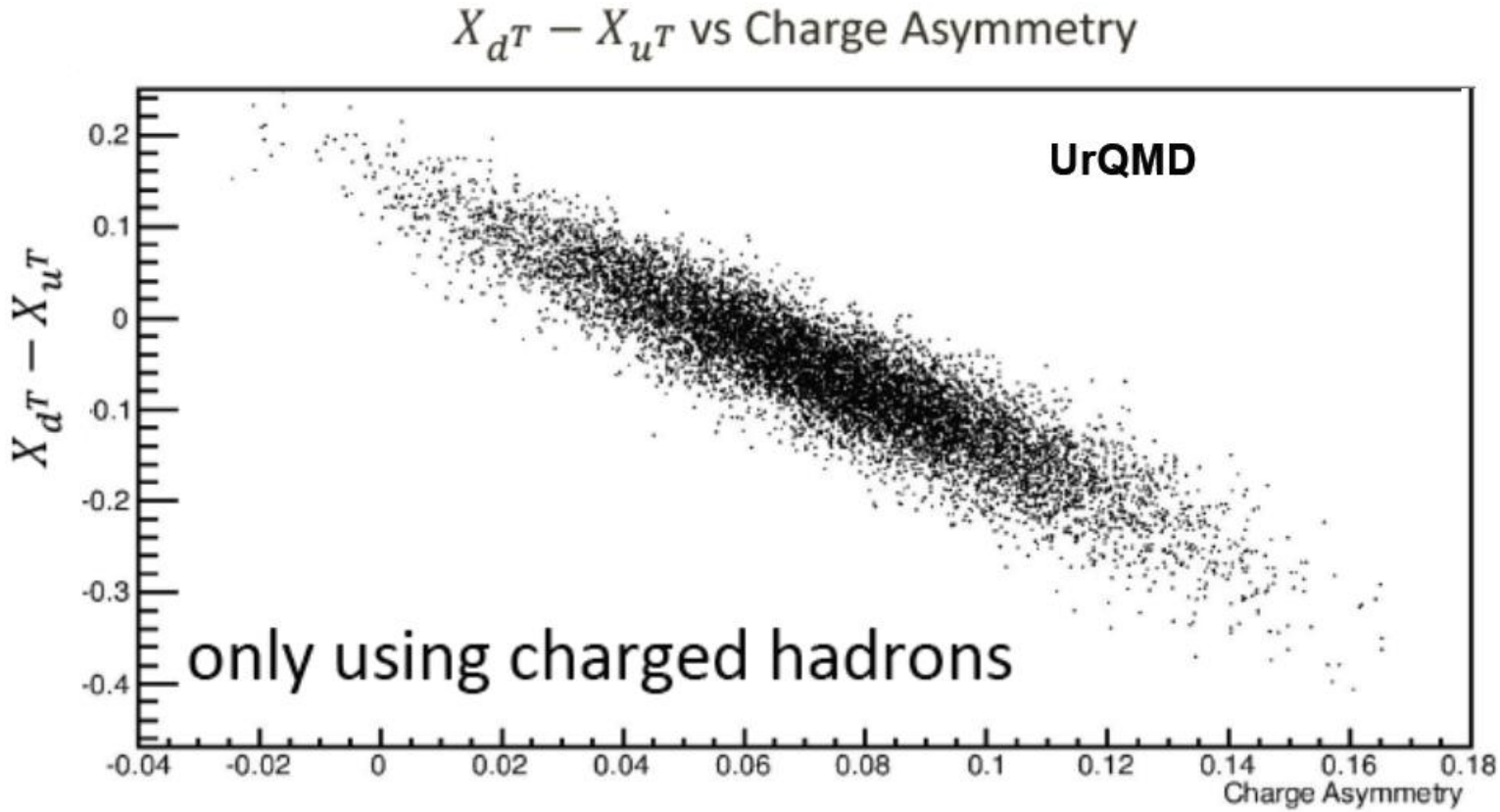
- U+U collisions
  - different v_2 trend from Au+Au?



Phys.Rev.Lett. 105 (2010) 172301

Backup slides

Multi-component Coalescence (MCC) + Quark Transport

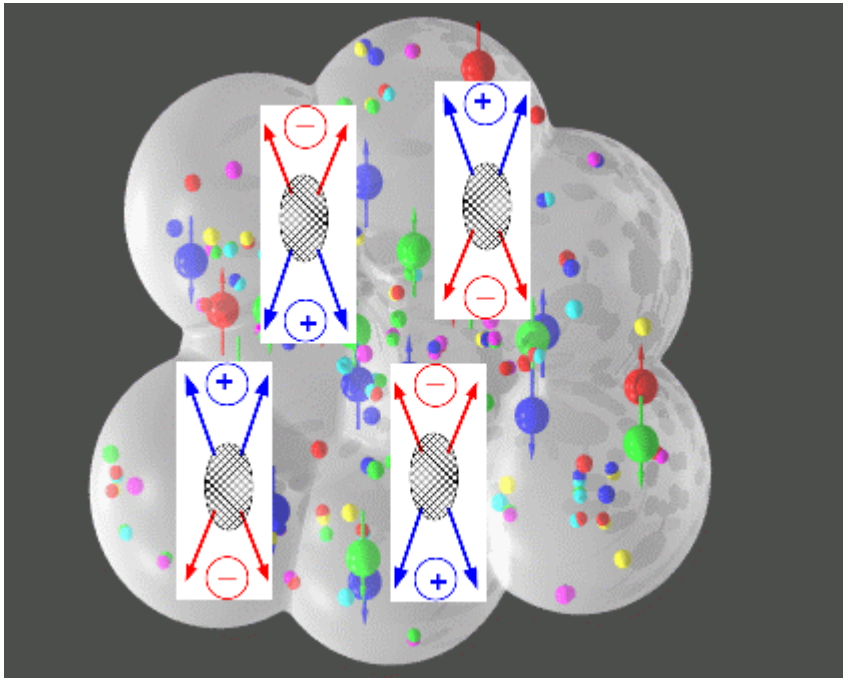


Pearson coefficient : -0.92 → Strong negative correlation

$$\Delta v_2{}^{\pi} \equiv v_2{}^{\pi^-} - v_2{}^{\pi^+} = \boxed{(X_{d^T} - X_{u^T})}(v_2{}^{T} - v_2{}^{P})$$

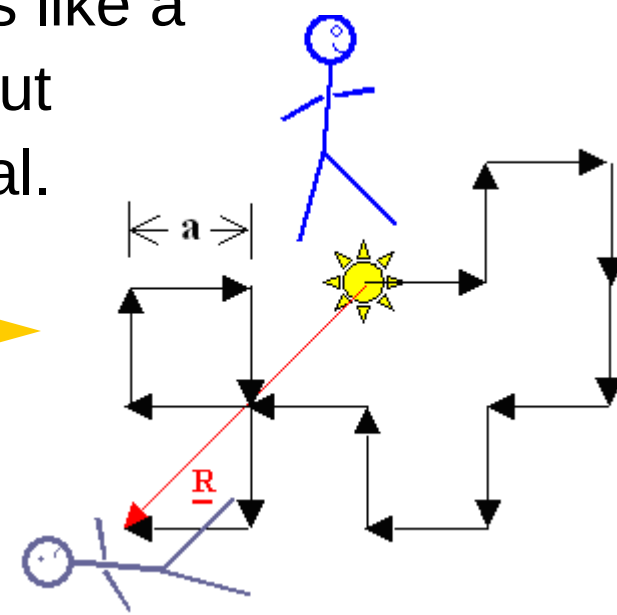
John Campbell & Mike Lisa, preliminary study, publication in preparation.

Dilution effect



In the quark-gluon medium, there could be multiple *P*-odd domains.

The net effect is like a *random walk*, but one-dimensional.



What do we know about the position R_n after n steps?

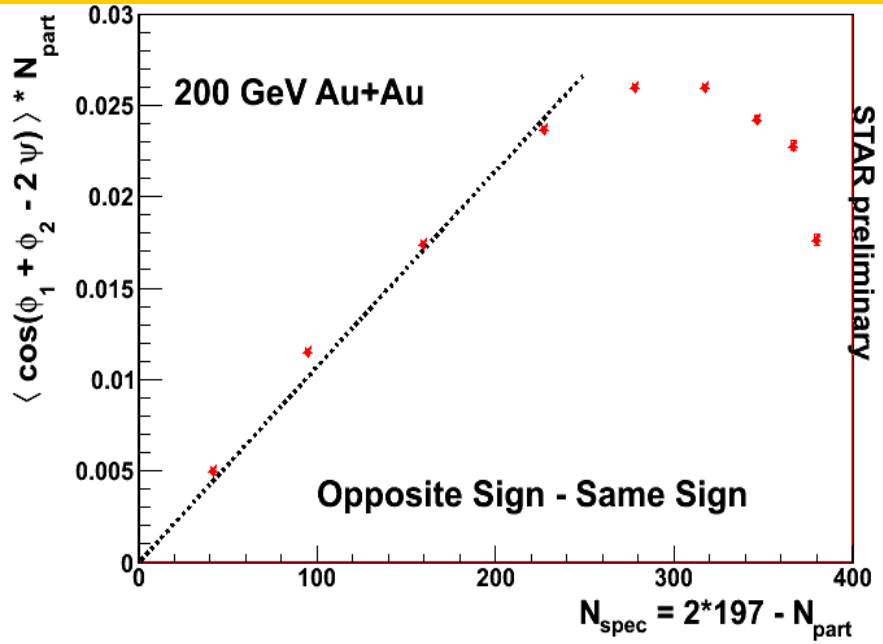
R_n follows a Gaussian distribution: *mean* = 0, and *rms* = $\sqrt{n}$

Our measurement of PV is like R_n^2, expected to be n.

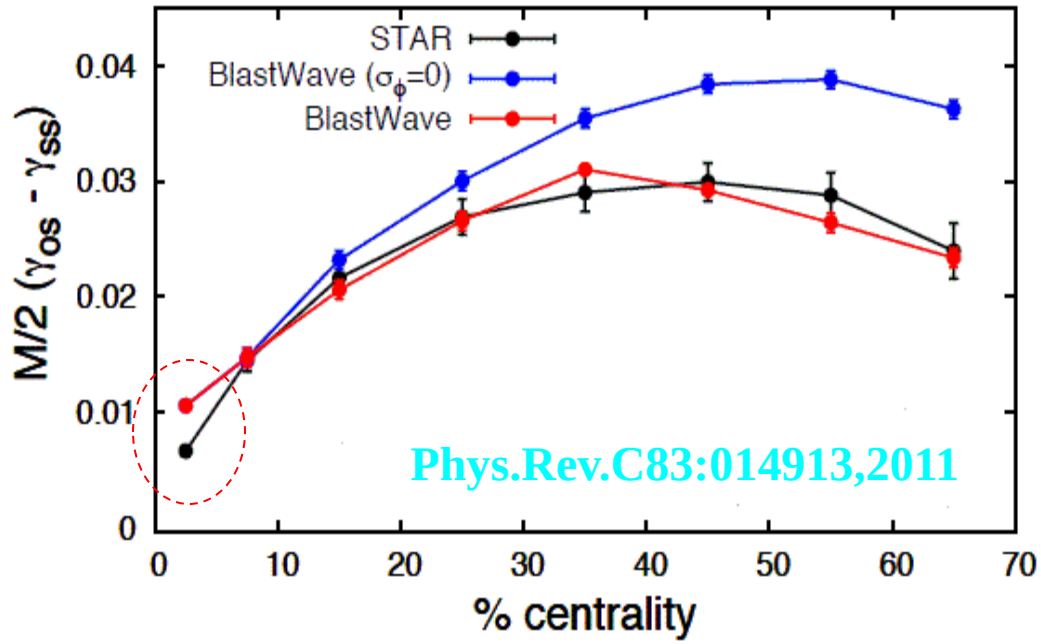
Compared with going in one fixed direction, where $R_n^2 = n^2$,

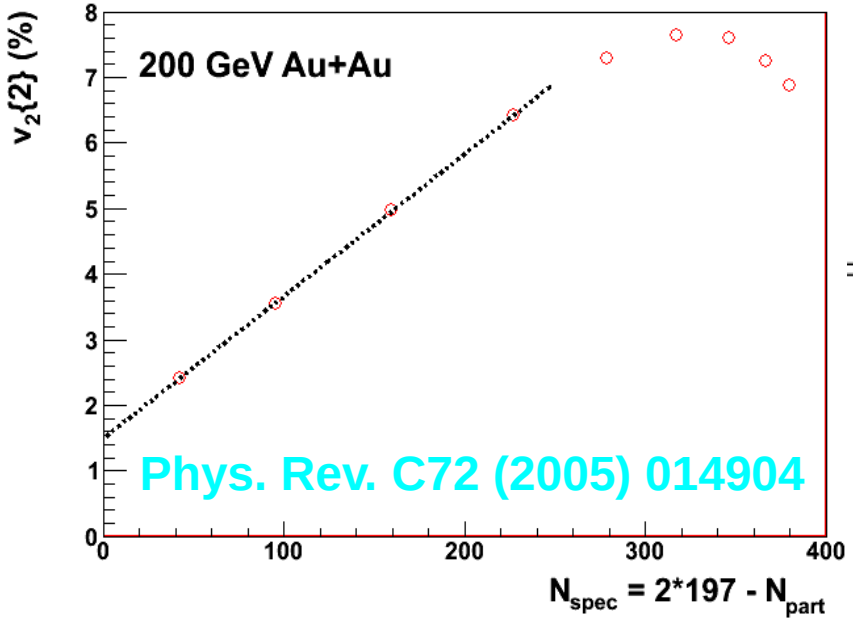
the "random-walk" measurement is diluted by a factor $\sim n \sim N_{ch}$.

Possible physics background



charge conservation/cluster + v_2





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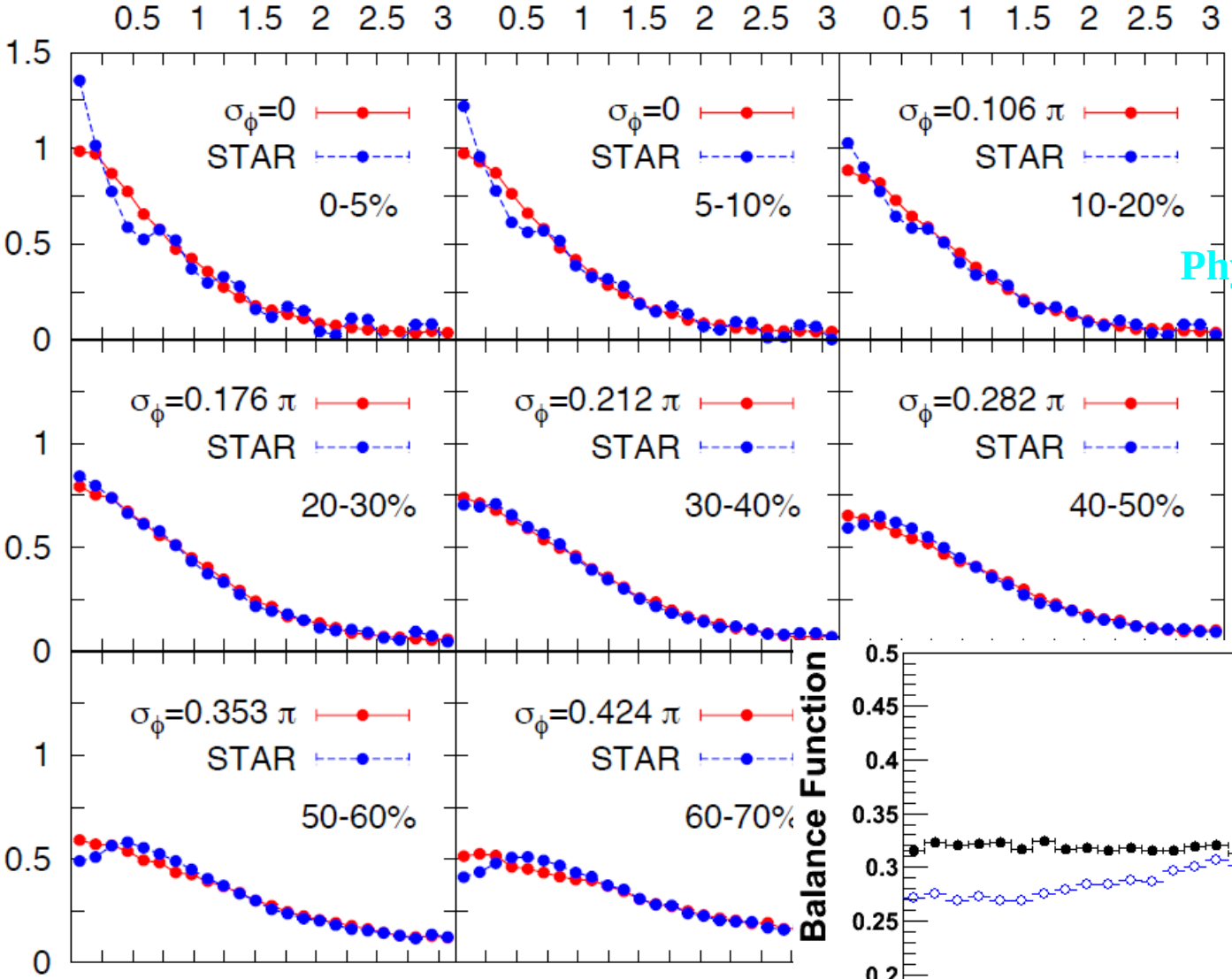
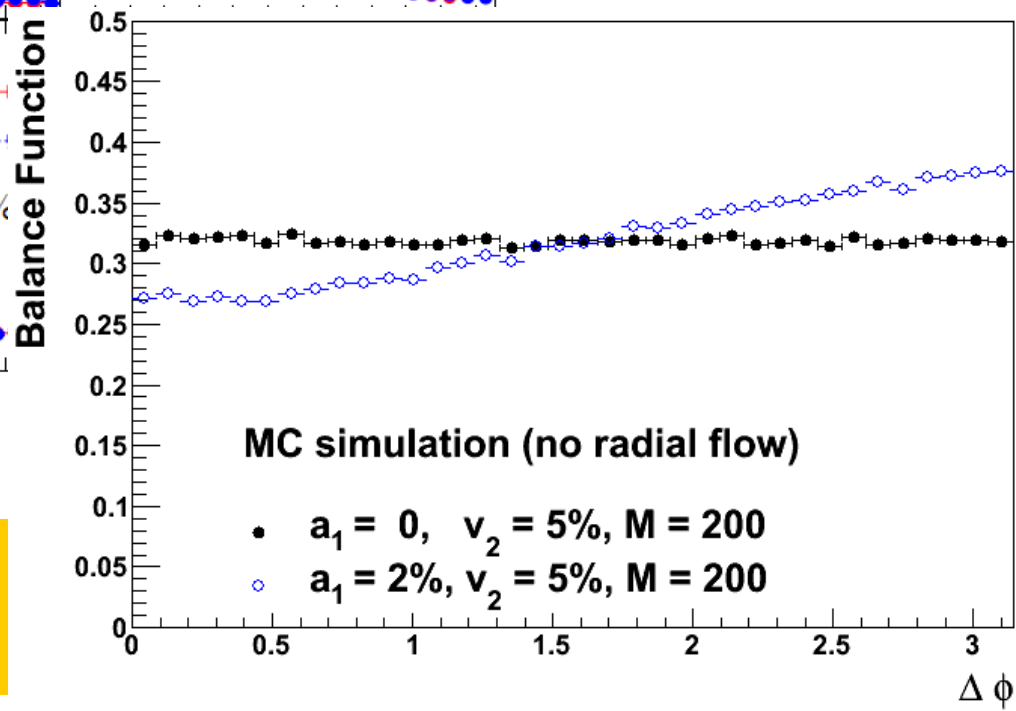
$$= \langle \cos((\phi_\alpha + \phi_\beta - 2\phi_{res}) + 2(\phi_{res} - \Psi_{RP})) \rangle$$
$$\approx \frac{f_{res} \langle \cos(\phi_\alpha + \phi_\beta - 2\phi_{res}) \rangle\ v_{2,res}}{N_{ch}}$$

Qualitatively similar trend!

Need to trigger on very small N_{spec}.



Phys.Rev.C83:014913,2011



Balance function

Hydrodynamics and the entropy current

Amos Yarom

Together with: K. Jensen, M. Kaminski, P. Kovtun, R. Meyer, A. Ritz

Hydrodynamics with anomalies from thermodynamics on cones

Amos Yarom

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Hydrodynamics with anomalies from thermodynamics on cones

Amos Yarom

Together with: K. Jensen, M. Kaminski, P. Kovtun, R. Meyer, A. Ritz

(See also: Banerjee, Bhattacharya, Bhattacharyya, Jain, Minwalla, Sharma)

Thermodynamics

Thermodynamics

Thermodynamic behavior is characterized by:

$$Z = Tr(e^{-\beta H + \beta \mu Q})$$

Thermodynamics

Thermodynamic behavior is characterized by:

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$$S = \int L dt$$

Thermodynamics

Thermodynamic behavior is characterized by:

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$$S = \int L dt$$

$\mathbf{R}^{d,1}$

Thermodynamics

Thermodynamic behavior is characterized by:

$$Z = Tr(e^{-\beta H + \beta \mu Q})$$

$$S = \int L dt$$

$$S_E = i \int L d\tau$$

$$\mathbf{R}^{d,1}$$

Thermodynamics

Thermodynamic behavior is characterized by:

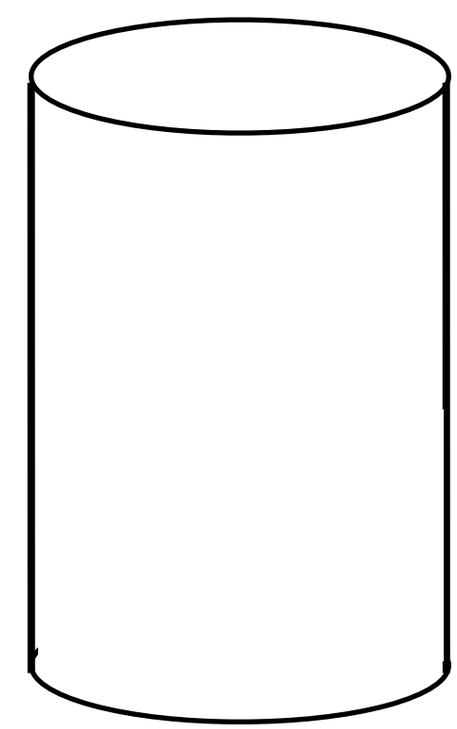
$$Z = Tr(e^{-\beta H + \beta \mu Q})$$

$$S = \int L dt$$

$$S_E = i \int L d\tau$$

$$\mathbf{R}^{d,1}$$

$$\mathbf{R}^{d} \times S^{1}$$



Thermodynamics

Thermodynamic behavior is characterized by:

$$Z = Tr(e^{-\beta H + \beta \mu Q})$$

$$S = \int L dt$$

$$S_E = i \int L d\tau$$

$\mathbf{R}^{d,1}$

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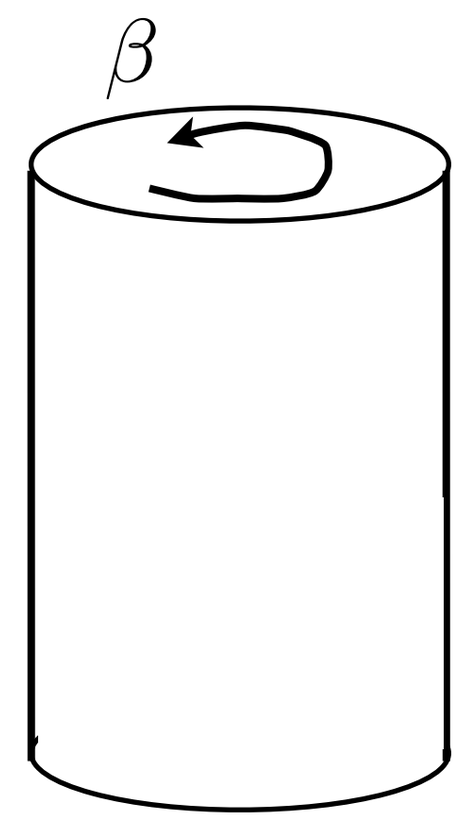

Thermodynamics

Thermodynamic behavior is characterized by:

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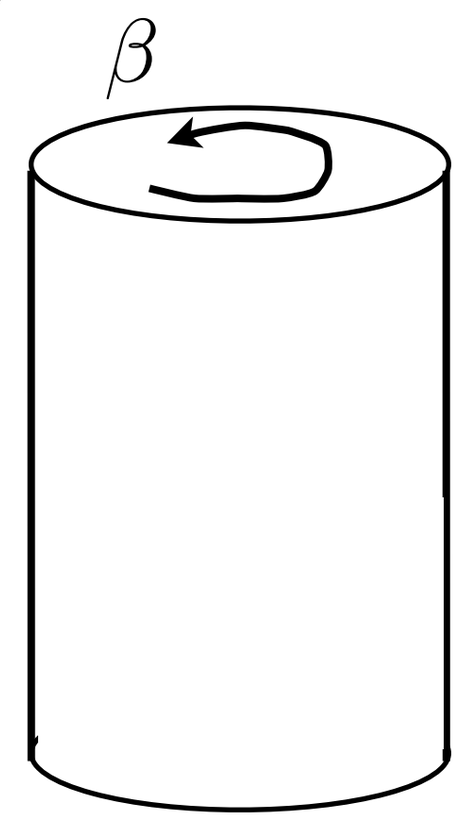

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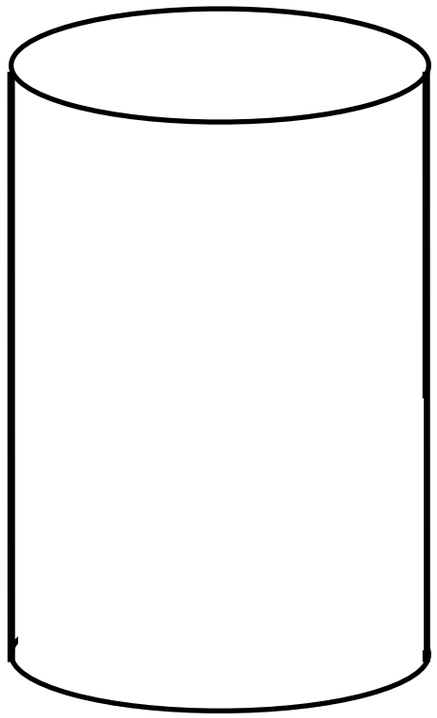

Thermodynamics

Thermodynamic behavior is characterized by:

$$Z = Tr(e^{-\beta H + \beta \mu Q})$$

$$S_E = i \int_0^\beta L d\tau$$

$$W = \ln Z = -iS_E$$

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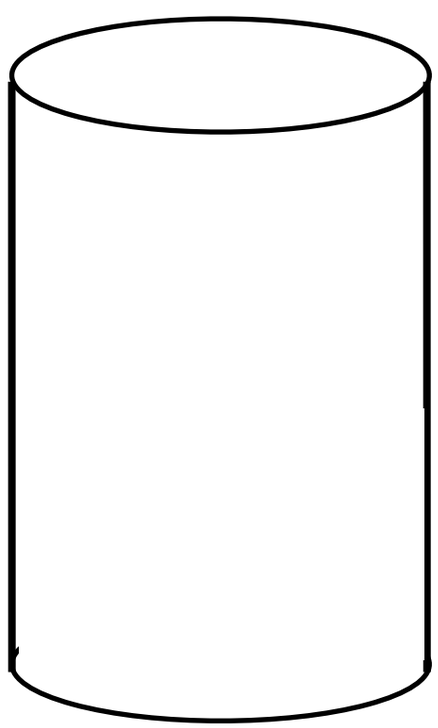

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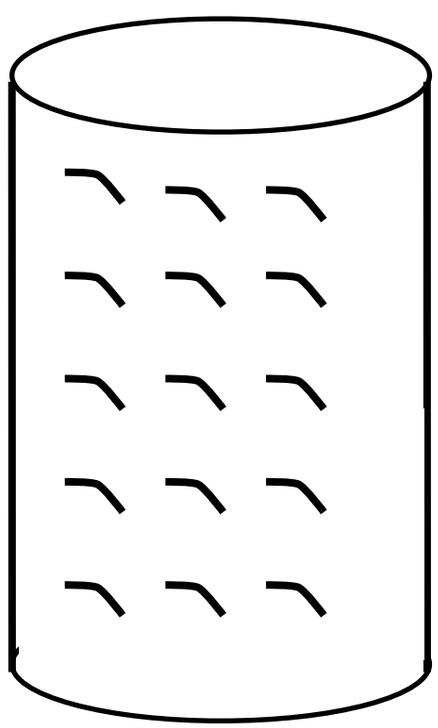

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

Thermodynamics

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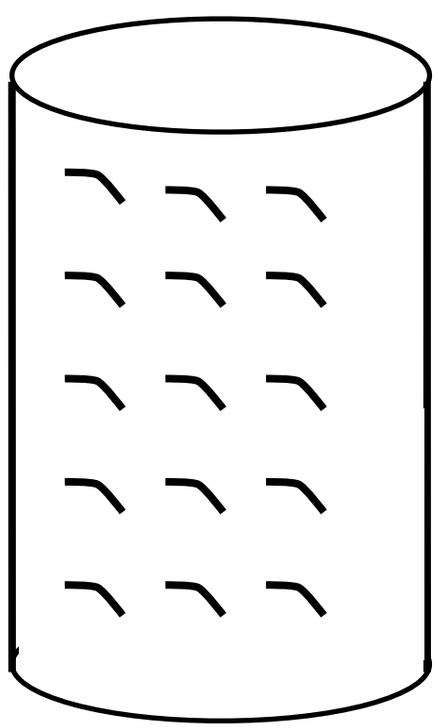
$$Z = Tr(e^{-\beta H + \beta \mu Q})$$

$$S_E = i \int_0^\beta L d\tau$$

$$W = \ln Z = -iS_E$$

$$Tr\left(\rho T^{\mu_1 \nu_1}(\vec{x}_1) \ldots T^{\mu_n \nu_n}(\vec{x}_n)\right)$$

$$\mathbf{R}^d \times S^1$$



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$$S_E = i \int_0^\beta L d\tau$$

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$$Tr\left(\rho\, T^{\mu_1 \nu_1}(\vec{x}_1) \ldots T^{\mu_n \nu_n}(\vec{x}_n)\right)$$

$$= \frac{2}{\sqrt{g}} \frac{\delta^n W}{\delta h_{\mu_1 \nu_1}(\vec{x_1}) \ldots \delta\, h_{\mu_n \nu_n}(\vec{x}_n)}$$

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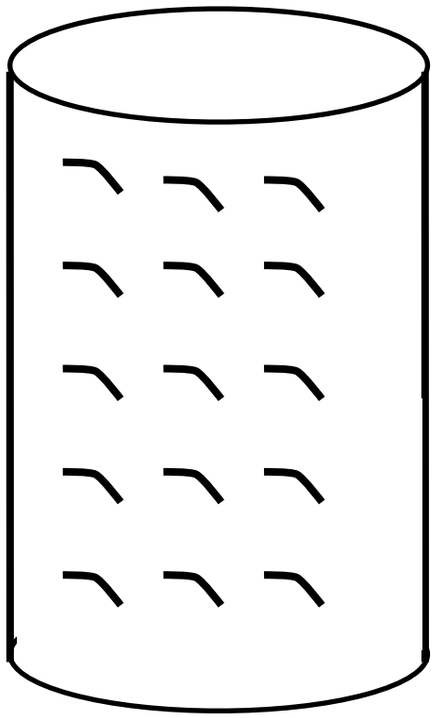

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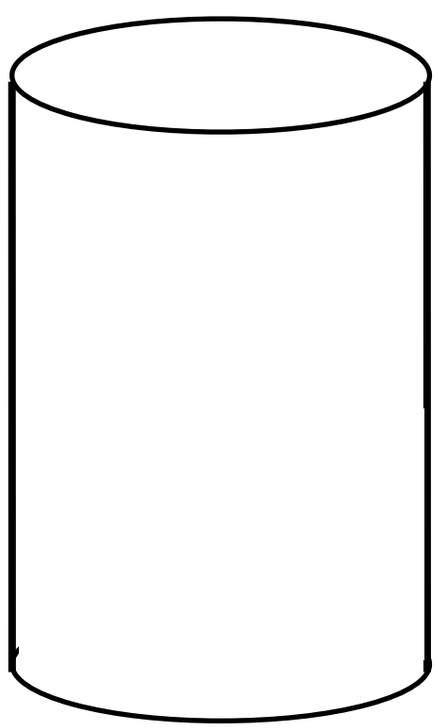

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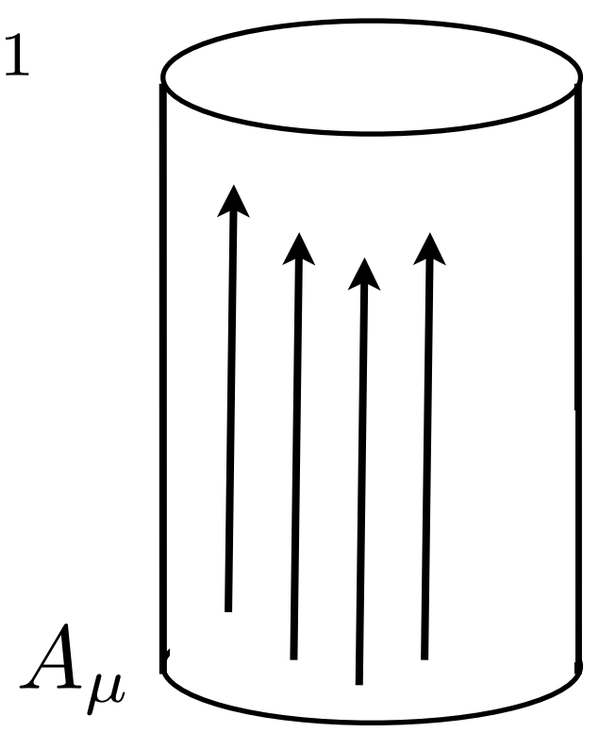

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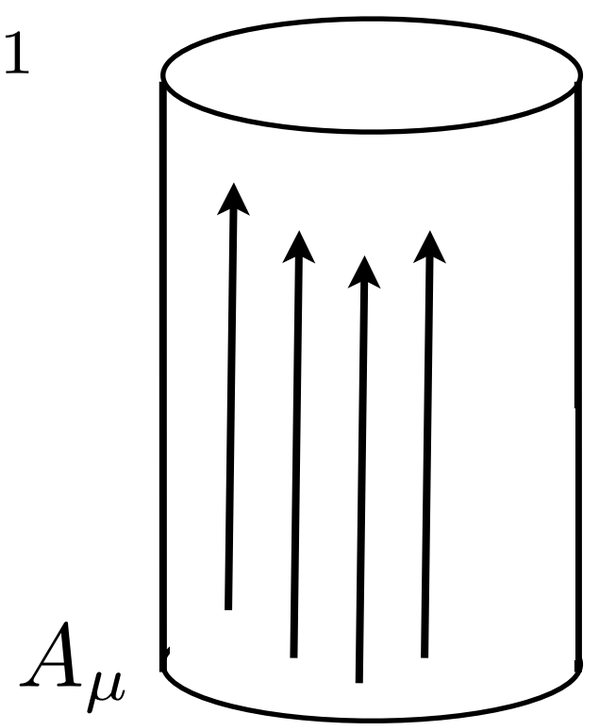

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$$Tr(\rho J^{\mu_1}(\vec{x}_1) \ldots J^{\mu_n}(\vec{x}_n))$$
$$= \frac{1}{\sqrt{g}} \frac{\delta^n W}{\delta A_{\mu_1}(\vec{x}_1) \ldots \delta A_{\mu_n}(\vec{x}_n)}$$

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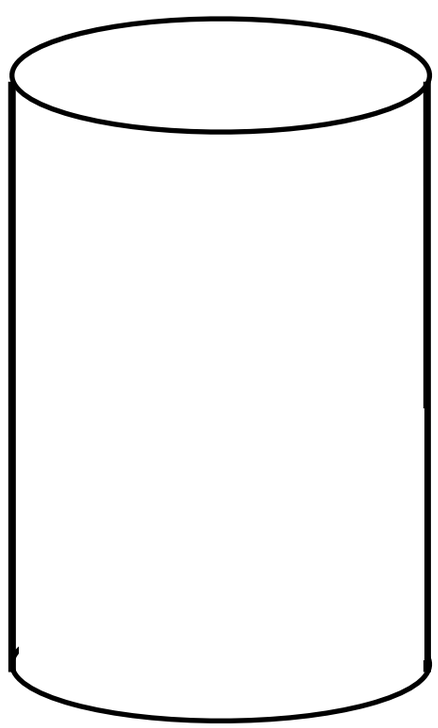

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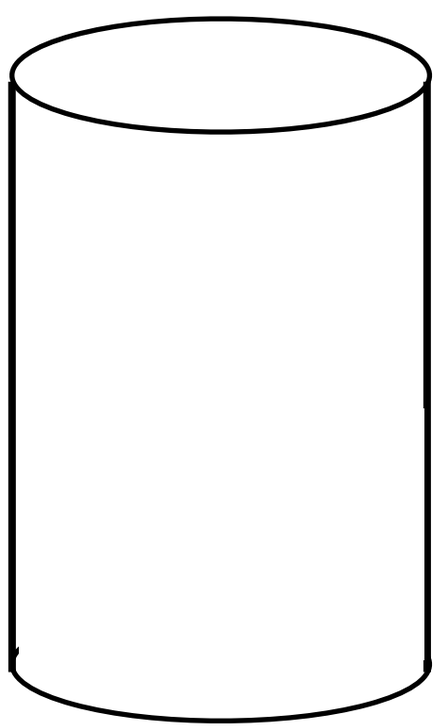

$$\frac{1}{\sqrt{g}} \frac{\delta^2 W}{\delta A_{\mu_1} \delta A_{\mu_2}} \xrightarrow[r \to \infty]{} 0$$

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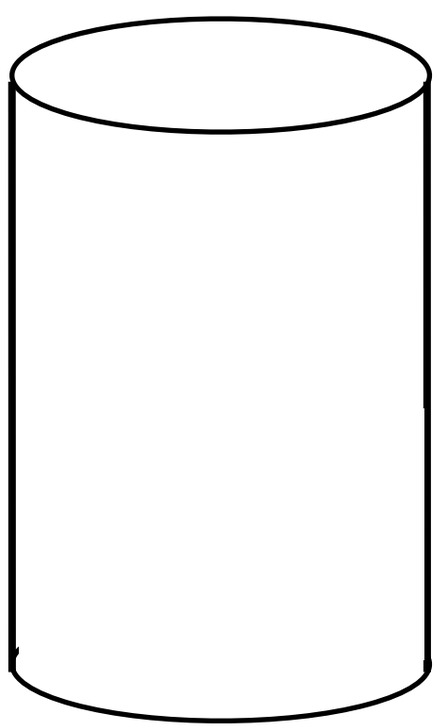

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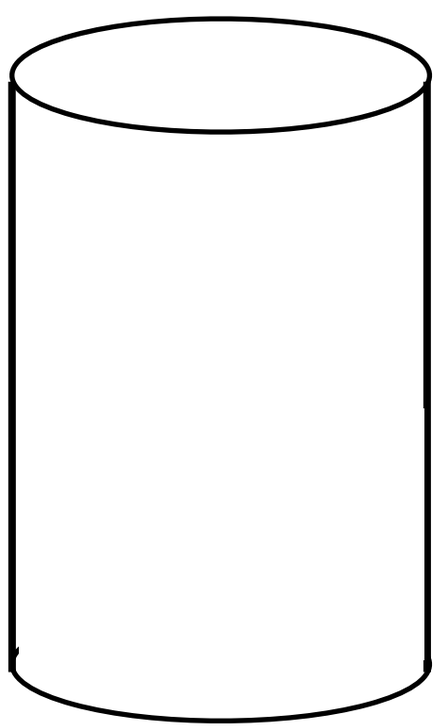

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$$W = \int \mathcal{L}[sources] d^{d+1}x$$

Thermodynamics

$$W = \ln Z = -iS_E$$

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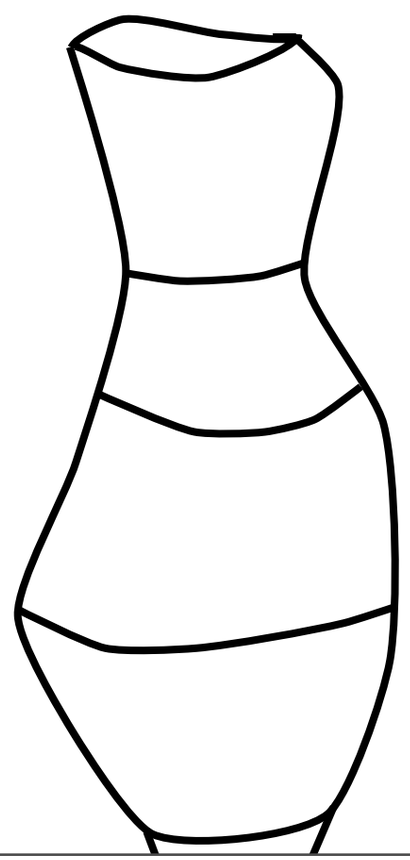

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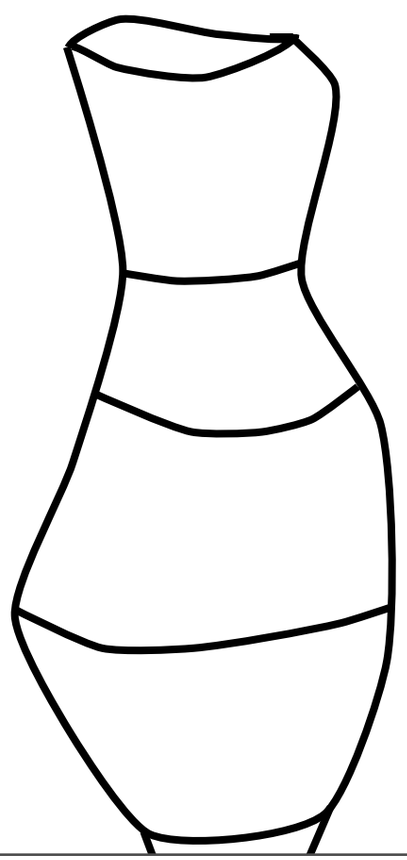

$$V^\mu = (1, 0)$$

Thermodynamics

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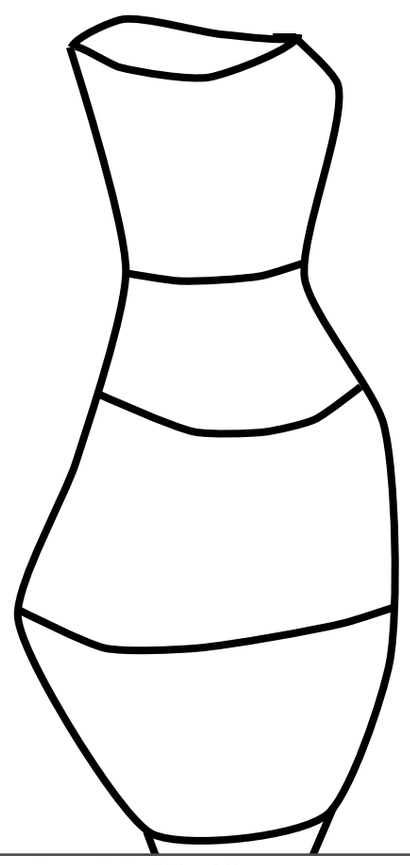
$$\mathcal{L}[g_{\mu\nu}, A_\mu]$$



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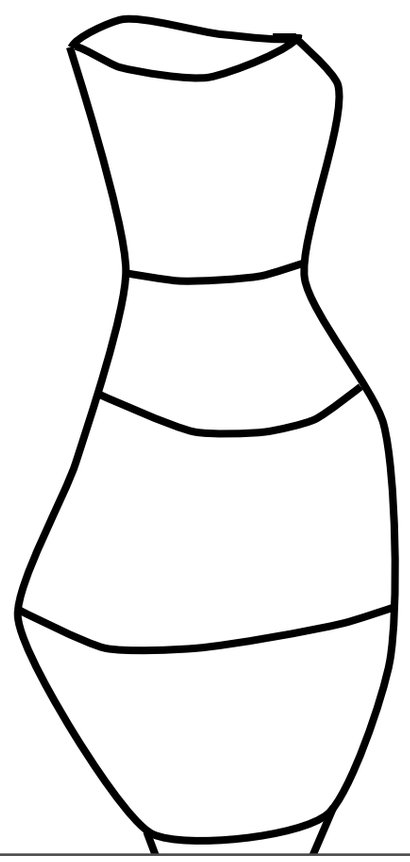

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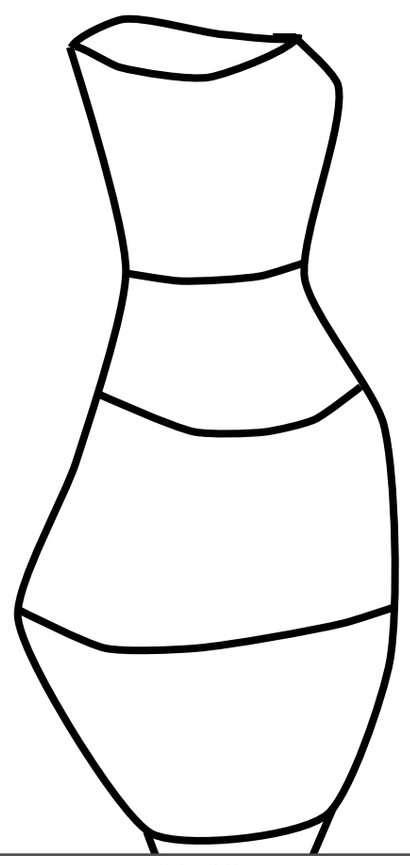

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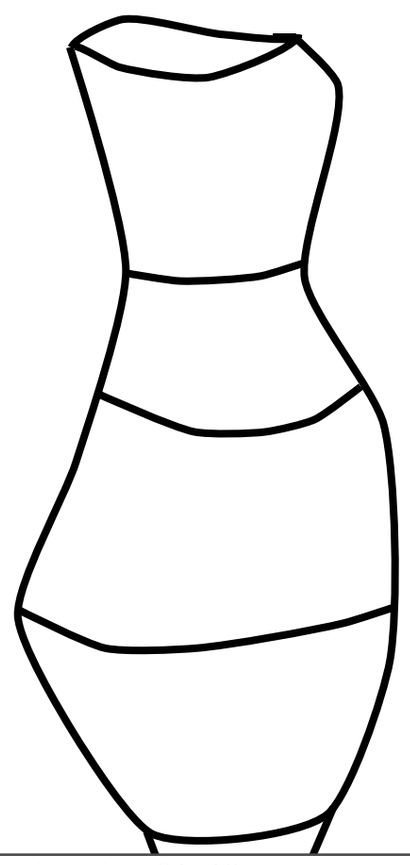

$$V^\mu = (1, 0)$$

$$T^{-1} = \int_0^\beta \sqrt{V^2} d\tau$$

Thermodynamics

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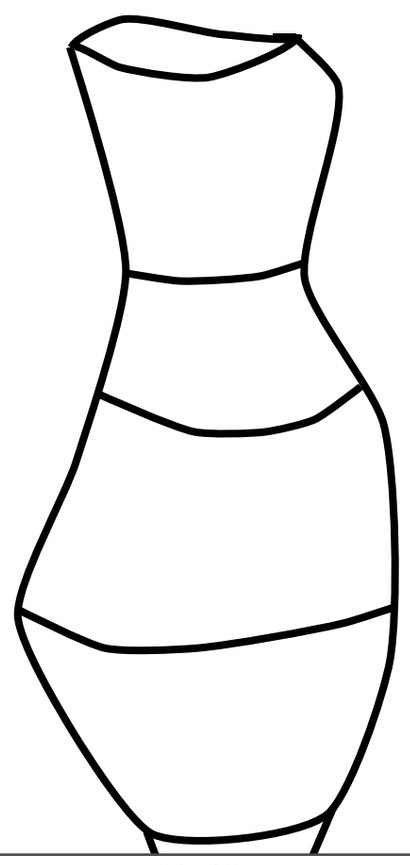

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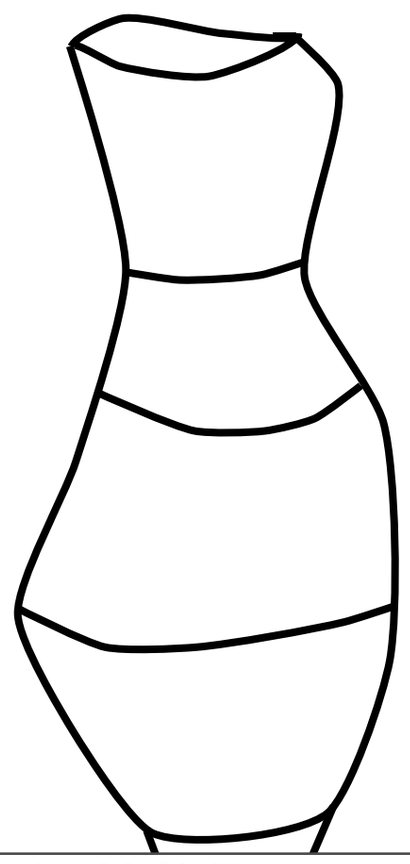
$$\mu = T \ln P_A$$

Thermodynamics

$$W = \ln Z = -iS_E$$

$$W = \int \mathcal{L}[sources] d^{d+1}x$$

$$\mathcal{L}[g_{\mu\nu}, A_\mu, u^\mu, T, \mu]$$



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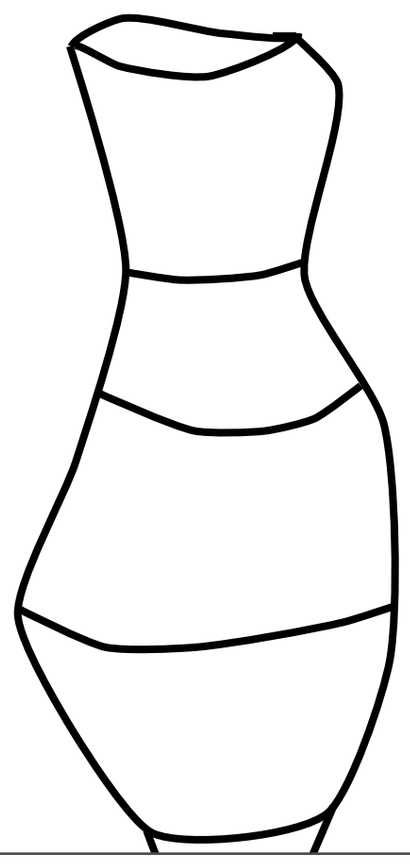
$$W = \int \mathcal{L}[sources] d^{d+1}x$$

$$\mathcal{L}[g_{\mu\nu}, A_\mu, u^\mu, T, \mu] = ?$$



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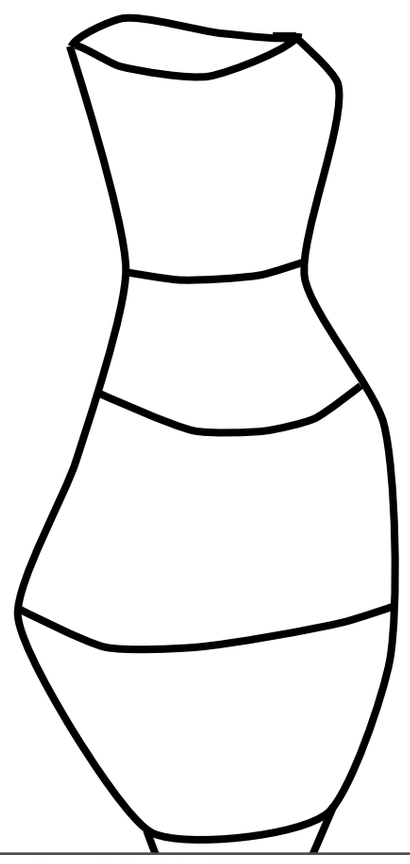

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$$T^{\mu\nu} = \frac{2}{\sqrt{g}} \frac{\delta W}{\delta g_{\mu\nu}}$$

Thermodynamics

$$W = \int \sqrt{g} P(T,\mu) + \mathcal{O}(\partial) d^{d+1}x$$

$$T^{\mu\nu} = \frac{2}{\sqrt{g}} \frac{\delta W}{\delta g_{\mu\nu}}$$

$$\xrightarrow[t_E \to t]{} \left(T \frac{\partial P}{\partial T} + \mu \frac{\partial P}{\partial \mu} - P \right) u^\mu u^\nu + \left(g^{\mu\nu} + u^\mu u^\nu \right) P$$

Thermodynamics

$$W = \int \sqrt{g} P(T,\mu) + \mathcal{O}(\partial) d^{d+1}x$$

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$$= \epsilon u^\mu u^\nu + \left(g^{\mu\nu} + u^\mu u^\nu \right) P$$

Thermodynamics

$$W = \int \sqrt{g} P(T, \mu) + \mathcal{O}(\partial) d^{d+1}x$$

$$T^{\mu\nu}$$

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$$W = \int \sqrt{g} P(T, \mu) + \mathcal{O}(\partial) d^{d+1}x$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + \left(g^{\mu\nu} + u^\mu u^\nu\right) P$$

$$J^\mu = \frac{\partial P}{\partial \mu} u^\mu = \rho u^\mu$$

Thermodynamics

$$W = \int \sqrt{g} P(T, \mu) + \mathcal{O}(\partial) d^{d+1}x$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + \left(g^{\mu\nu} + u^\mu u^\nu\right) P$$

$$J^\mu = \rho u^\mu$$

Thermodynamics

$$W = \int \sqrt{g} P(T, \mu) + \mathcal{O}(\partial) d^{d+1}x$$

What do we expect to $\mathcal{O}(\partial^2)$?

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + \left(g^{\mu\nu} + u^\mu u^\nu\right) P$$

$$J^\mu = \rho u^\mu$$

Thermodynamics

$$D_\mu u^\mu$$

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$$D_\mu u^\mu$$

$$u^\alpha D_\alpha T$$

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Thermodynamics

$$W = \int \sqrt{g} P(T, \mu) + \mathcal{O}(\partial) d^{d+1}x$$

$$D_\mu u^\mu \sim \partial_t \sqrt{g} u^t$$

$$u^\alpha D_\alpha T$$

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$$D_\mu u^\mu \sim \partial_t \sqrt{g} u^t = 0$$

$$u^\alpha D_\alpha T \sim \partial_t T = 0$$

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What do we expect to $\mathcal{O}(\partial^2)$?

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + \left(g^{\mu\nu} + u^\mu u^\nu\right) P$$

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Conclusion:

$$\tau_1 = \tau_2 = \chi_1 = \chi_2 = 0$$

in equilibirum

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Out of equilibrium?

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Out of equilibrium?

Reintroduce: $\partial_\mu u^\mu \qquad u^\mu \partial_\mu T \qquad u^\mu \partial_\mu \frac{\mu}{T}$

$$a_\mu T + \partial_\mu T \qquad \partial_\mu \frac{\mu}{T} - F_\mu{}^\nu u_\nu$$

$$\partial_{\langle\mu} u_{\nu\rangle}$$

Hydrodynamics

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$$\partial_\mu T^{\mu\nu} = F^{\nu\alpha} J_\alpha$$

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Hydrodynamics

$\partial_\mu u^\mu \qquad u^\mu \cancel{\partial_\mu} T \qquad u^\mu \cancel{\partial_\mu} \frac{\mu}{T}$

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$$u^{\mu\,\prime} = u^\mu + \delta u^\mu$$

Hydrodynamics

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$$T^{\mu\nu} = \epsilon u^\mu u^\nu + \left(g^{\mu\nu} + u^\mu u^\nu\right) P - \zeta \partial_\alpha u^\alpha P^{\mu\nu}$$

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$$J^\mu = \rho u^\mu - \kappa \left(T P^{\mu\nu} \partial_\nu \frac{\mu}{T} - F^\nu_\mu u_\nu \right)$$

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$$\zeta + \frac{4}{3}\eta = - \lim_{\omega \to 0} \frac{i}{\omega} \langle T_{xx}(\omega) T_{xx}(0) \rangle$$

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + \left(g^{\mu\nu} + u^\mu u^\nu\right) P - \zeta \partial_\alpha u^\alpha P^{\mu\nu} - \eta \partial_{\langle\mu} u_{\nu\rangle}$$

$$J^\mu = \rho u^\mu - \kappa \left(T P^{\mu\nu} \partial_\nu \frac{\mu}{T} - F^\nu_\mu u_\nu \right)$$

$$\eta = -\lim_{\omega\to 0} \frac{i}{\omega} \langle T_{xy}(\omega) T_{xy}(0) \rangle$$

$$\zeta + \frac{4}{3}\eta = -\lim_{\omega\to 0} \frac{i}{\omega} \langle T_{xx}(\omega) T_{xx}(0) \rangle$$

$$\kappa \sim -\lim_{\omega\to 0} \frac{i}{\omega} G^{xx}(\omega)$$

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + \left(g^{\mu\nu} + u^\mu u^\nu\right) P - \zeta \partial_\alpha u^\alpha P^{\mu\nu} - \eta \partial_{\langle\mu} u_{\nu\rangle}$$

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$$\eta = -\lim_{\omega\to 0} \frac{i}{\omega} \langle T_{xy}(\omega) T_{xy}(0) \rangle = \rho(T_{xx})$$

$$\zeta + \frac{4}{3}\eta = -\lim_{\omega\to 0} \frac{i}{\omega} \langle T_{xx}(\omega) T_{xx}(0) \rangle$$

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Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + \left(g^{\mu\nu} + u^\mu u^\nu\right) P - \zeta \partial_\alpha u^\alpha P^{\mu\nu} - \eta \partial_{\langle\mu} u_{\nu\rangle}$$

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$$\eta = -\lim_{\omega\to 0} \frac{i}{\omega} \langle T_{xy}(\omega) T_{xy}(0) \rangle = \rho(T_{xx}) > 0$$

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Hydrodynamics

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Hydrodynamics

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$$\zeta + \frac{4}{3}\eta = -\lim_{\omega\to 0} \frac{i}{\omega} \langle T_{xx}(\omega) T_{xx}(0) \rangle = \rho(T_{xy}) > 0$$

$$\kappa \sim -\lim_{\omega\to 0} \frac{i}{\omega} G^{xx}(\omega) = \rho(J_x) > 0$$

Summary

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + \left(g^{\mu\nu} + u^\mu u^\nu\right) P - \zeta \partial_\alpha u^\alpha P^{\mu\nu} - \eta \partial_{\langle \mu} u_{\nu \rangle}$$

$$J^\mu = \rho u^\mu - \kappa \left(T P^{\mu\nu} \partial_\nu \frac{\mu}{T} - F^\nu_\mu u_\nu \right)$$

Summary

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + \left(g^{\mu\nu} + u^\mu u^\nu\right) P - \zeta \partial_\alpha u^\alpha P^{\mu\nu} - \eta \partial_{\langle\mu} u_{\nu\rangle}$$

$$J^\mu = \rho u^\mu - \kappa \left(T P^{\mu\nu} \partial_\nu \frac{\mu}{T} - F^\nu_\mu u_\nu \right) + \chi_1 P^{\mu\nu} \partial_\nu T + \chi_2 P^{\mu\nu} \partial_\nu \frac{\mu}{T}$$

Summary

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Equilibrium physics: $\chi = 0$

Summary

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + \left(g^{\mu\nu} + u^\mu u^\nu\right) P - \zeta \partial_\alpha u^\alpha P^{\mu\nu} - \eta \partial_{\langle\mu} u_{\nu\rangle}$$

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Equilibrium physics: $\chi = 0$

Unitarity: $\eta \geq 0$ $\zeta \geq 0$ $\kappa \geq 0$

$\not{P}$ in hydrodynamics

$$J^\mu = \rho u^\mu - \kappa T P^{\mu\nu} \partial_\nu \frac{\mu}{T}$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + \left(g^{\mu\nu} + u^\mu u^\nu\right) P - \zeta \partial_\alpha u^\alpha P^{\mu\nu} - \eta \partial_{\langle\mu} u_{\nu\rangle}$$

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$$J^\mu = \rho u^\mu - \kappa T P^{\mu\nu} \partial_\nu \frac{\mu}{T}$$

$$W = \int \sqrt{g} P(T, \mu) d^4 x$$

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$$J^\mu = \rho u^\mu - \kappa T P^{\mu\nu} \partial_\nu \frac{\mu}{T}$$

$$W = \int \sqrt{g} \left(P(T,\mu) + k_1 T^2 \epsilon^{\mu\nu\rho\sigma} A_\mu u_\nu \partial_\rho u_\sigma \right) d^4x$$

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$$J^\mu = \rho u^\mu - \kappa T P^{\mu\nu}\partial_\nu \frac{\mu}{T} + k_1 T^2 \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$$

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$$J^\mu = \rho u^\mu - \kappa T P^{\mu\nu}\partial_\nu \frac{\mu}{T} + k_1 T^2 \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma - 2c\mu^2 \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$$

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Vilenkin (1980)

Erdmenger, Haack, Kaminski, AY (2008)

Banerjee, Bhattacharya, Bhattacharyya, Dutta, Loganyagam, Surowka (2008)

Son, Surowka (2009)

Neiman, Oz (2010)

Bhattacharya, Bhattacharyya, Minwalla, AY (2011)

$\not{P}$ in hydrodynamics

$$J^\mu = \rho u^\mu - \kappa T P^{\mu\nu}\partial_\nu \frac{\mu}{T} + k_1 T^2 \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma - 2c\mu^2 \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$$

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$$J^\mu = \rho u^\mu - \kappa T P^{\mu\nu}\partial_\nu \frac{\mu}{T} + (k_1 T^2 - 2c\mu^2)\epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$$

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?

$$W = \int \sqrt{g}\Big(P(T,\mu) + k_1 T^2 \epsilon^{\mu\nu\rho\sigma} A_\mu u_\nu \partial_\rho u_\sigma - \frac{c}{3} A_\mu u_\nu \epsilon^{\mu\nu\rho\sigma}(8\mu F_{\rho\sigma} + 4\mu^2 \partial_\rho u_\sigma)\Big) d^4x$$

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?

Landsteiner, Megias, Pena-Benitez (2011)
Loganayagam (2011)
Loganayagam, Surowka (2011)

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$$k_1 = c_m$$

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?

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Loganayagam (2011)
Loganayagam, Surowka (2011)

$$k_1 = c_m$$

$$\partial_\mu J^\mu = c\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + c_m \epsilon^{\mu\nu\rho\sigma} R^\alpha_{\beta\mu\nu} R^\beta_{\alpha\rho\sigma}$$

$\not{P}$ in hydrodynamics

$\not{P}$ in hydrodynamics

$$W = \int \sqrt{g} \left(P(T,\mu) + \left(\begin{array}{c} \text{scalars and} \\ \text{pseudoscalars} \end{array} \right) \right) d^2x$$

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$$+ \int \sqrt{g}\left(-\frac{k}{2} A_\mu \epsilon^{\mu\nu} u_\nu - c g u_\alpha u^\beta \epsilon^{\mu\nu} \partial_\mu \Gamma^\alpha_{\beta\nu}\right) d^2x$$

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$$W = \int \sqrt{g}\left(k_1 A_\mu \epsilon^{\mu\nu} u_\nu - c_g u_\alpha u^\beta \epsilon^{\mu\nu} \Gamma^\alpha_{\beta\nu}\right) + \ldots d^2x$$

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$$ds^2 = dr^2 + r^2 d\phi^2$$

$\not{P}$ in hydrodynamics

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$$\phi \sim \phi + 2\pi\delta$$

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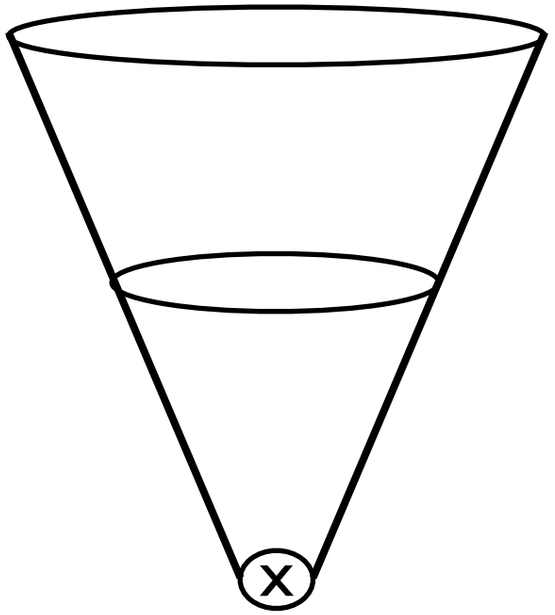


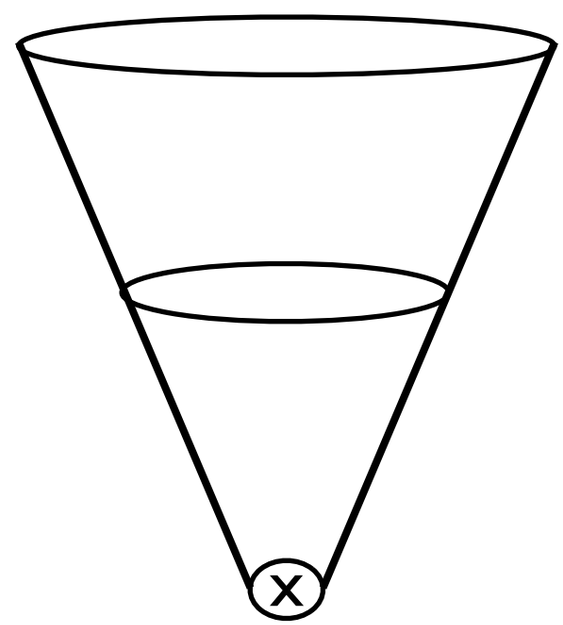
$\not{P}$ in hydrodynamics

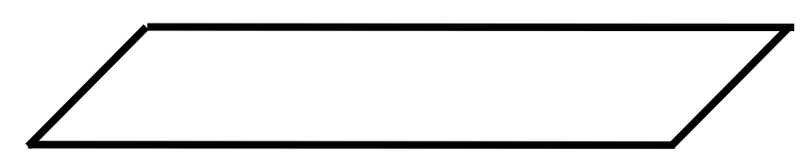
$$W = \int \sqrt{g}\left(k_1 A_\mu \epsilon^{\mu\nu} u_\nu - c_g u_\alpha u^\beta \epsilon^{\mu\nu} \Gamma^\alpha_{\beta\nu}\right) + \ldots d^2x$$

$$ds^2 = dr^2 + r^2 d\phi^2$$

$$\phi \sim \phi + 2\pi\delta$$

$$\delta = 1$$





$\not{P}$ in hydrodynamics

$$W = \int \sqrt{g}\left(k_1 A_\mu \epsilon^{\mu\nu} u_\nu - c_g u_\alpha u^\beta \epsilon^{\mu\nu} \Gamma^\alpha_{\beta\nu}\right) + \ldots d^2x$$

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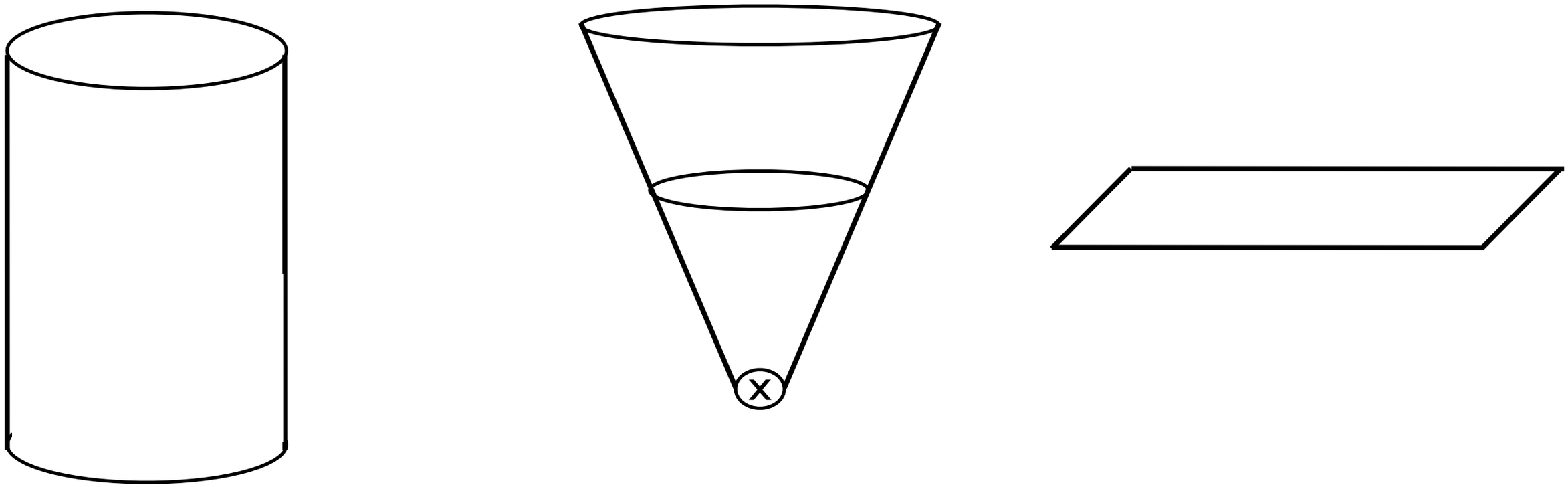
$\delta = 0$

$\delta = 1$



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$$T^{01} = k_1 + 8\pi^2 \delta^2 c_g$$

$\not{P}$ in hydrodynamics

$$W = \int \sqrt{g}\left(k_1 A_\mu \epsilon^{\mu\nu} u_\nu - c_g u_\alpha u^\beta \epsilon^{\mu\nu} \Gamma^\alpha_{\beta\nu}\right) + \ldots d^2x$$

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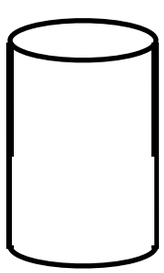

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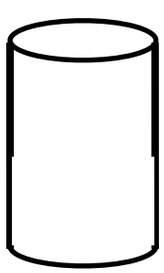


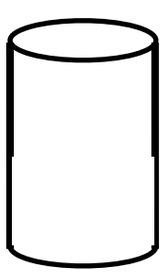
$\delta = 1$

$\not{P}$ in hydrodynamics

$$W = \int \sqrt{g}\left(k_1 A_\mu \epsilon^{\mu\nu} u_\nu - c_g u_\alpha u^\beta \epsilon^{\mu\nu} \Gamma^\alpha_{\beta\nu}\right) + \ldots d^2x$$

$$T^{01} = k_1 + 8\pi^2\delta^2 c_g \xrightarrow[\delta\to 1]{} 0$$

$\delta = 0$



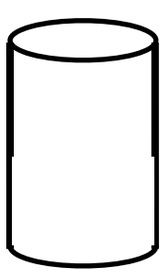
$\delta = 1$

$\not{P}$ in hydrodynamics

$$W = \int \sqrt{g}\left(k_1 A_\mu \epsilon^{\mu\nu} u_\nu - c_g u_\alpha u^\beta \epsilon^{\mu\nu} \Gamma^\alpha_{\beta\nu}\right) + \ldots d^2x$$

$$T^{01} = k_1 + 8\pi^2\delta^2 c_g \xrightarrow[\delta\to 1]{} 0$$

$\delta = 0$



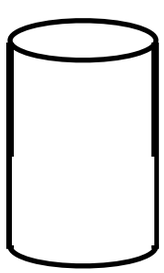
$$k_1 = -8\pi^2 c_g$$

$\delta = 1$

$\not{P}$ in hydrodynamics

$$W = \int \sqrt{g}\left(k_1 A_\mu \epsilon^{\mu\nu} u_\nu - c_g u_\alpha u^\beta \epsilon^{\mu\nu} \Gamma^\alpha_{\beta\nu}\right) + \ldots d^2x$$

$\delta = 0$

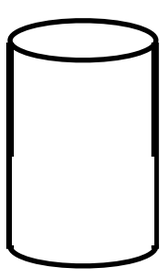


$\delta = 1$

$\not{P}$ in hydrodynamics

$$W = \int \sqrt{g}\left(p_0 T^2 + k_1 \epsilon^{0\nu} u_\nu + c(u^\alpha \partial_\alpha u_\mu)^2 - c_g u_\alpha u^\beta \epsilon^{\mu\nu} \Gamma^\alpha_{\beta\nu}\right) d^2x$$

$\delta = 0$



$\delta = 1$

$\not{P}$ in hydrodynamics

$$W = \int \sqrt{g}\left(p_0 T^2 + k_1 \epsilon^{0\nu} u_\nu + c(u^\alpha \partial_\alpha u_\mu)^2 - c_g u_\alpha u^\beta \epsilon^{\mu\nu} \Gamma^\alpha_{\beta\nu}\right) d^2x$$

$$T^{\mu\nu} = \begin{pmatrix} p_0 - 4c\pi^2\delta^2 & k_1 + 8\pi^2\delta^2 c_g \\ k_1 + 8\pi^2\delta^2 c_g & p_0 - 4c\pi^2\delta^2 \end{pmatrix} T^2$$

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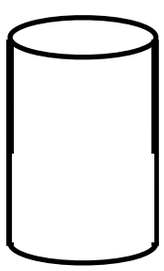

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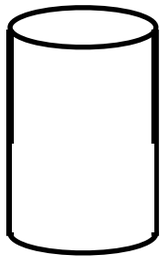

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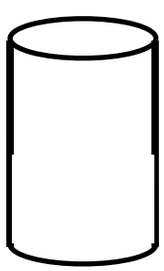

$$p_0 = 4\pi^2 c$$

$$k_1 = -8\pi^2 c_g$$

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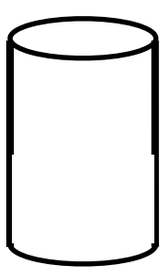

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$$k_1 = -8\pi^2 c_g$$

Blote, Cardy, Nightingale (1986)

Affleck (1986)

$\delta = 1$

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$$J^\mu = \rho u^\mu - \kappa T P^{\mu\nu}\partial_\nu \frac{\mu}{T} + (k_1 T^2 - 2c\mu^2)\epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma + c_m j_m^\mu$$

$$k_1 = c_m$$??

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$$k_1 = c_m \quad \checkmark$$

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Summary

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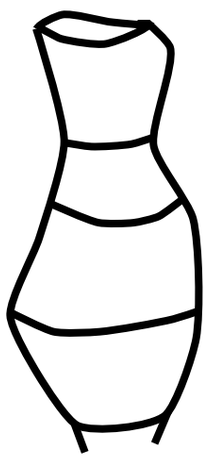
$$W = \int \sqrt{g} P(T, \mu) + \mathcal{O}(\partial^2) d^{d+1}x$$



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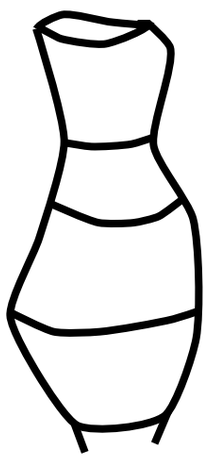

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + \left(g^{\mu\nu} + u^\mu u^\nu\right) P - \zeta \partial_\alpha u^\alpha P^{\mu\nu} - \eta \partial_{\langle\mu} u_{\nu\rangle}$$

$$J^\mu = \rho u^\mu - \kappa \left(T P^{\mu\nu} \partial_\nu \frac{\mu}{T} - F^\nu_\mu u_\nu \right) + \chi_1 P^{\mu\nu} \partial_\nu T + \chi_2 P^{\mu\nu} \partial_\nu \frac{\mu}{T}$$

$$\eta \geq 0 \quad \zeta \geq 0 \quad \kappa \geq 0 \quad \chi = 0$$

Summary

$$J^\mu = \rho u^\mu \; -\kappa \left(T P^{\mu\nu} \partial_\nu \frac{\mu}{T} - F^\nu_\mu u_\nu \right) -2c\mu^2 \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$$

$$+k_1 T^2 \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma + \ldots$$

$$\partial_\mu J^\mu = c\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + c_M \epsilon^{\mu\nu\rho\sigma} R^\alpha_{\beta\mu\nu} R^\beta_{\alpha\sigma\tau}$$

Summary

$$J^\mu = \rho u^\mu \; -\kappa\left(TP^{\mu\nu}\partial_\nu \frac{\mu}{T} - F^\nu_\mu u_\nu\right) -2c\mu^2\epsilon^{\mu\nu\rho\sigma}u_\nu\partial_\rho u_\sigma$$

$$-k_1 T^2\epsilon^{\mu\nu\rho\sigma}u_\nu\partial_\rho u_\sigma + \ldots$$

$$k_1 = c_M$$

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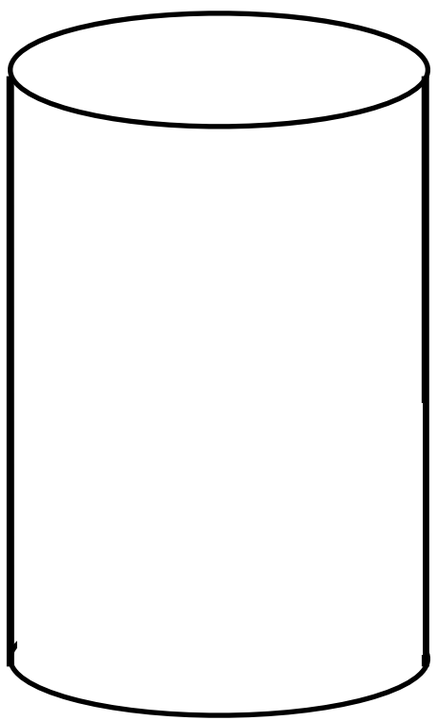
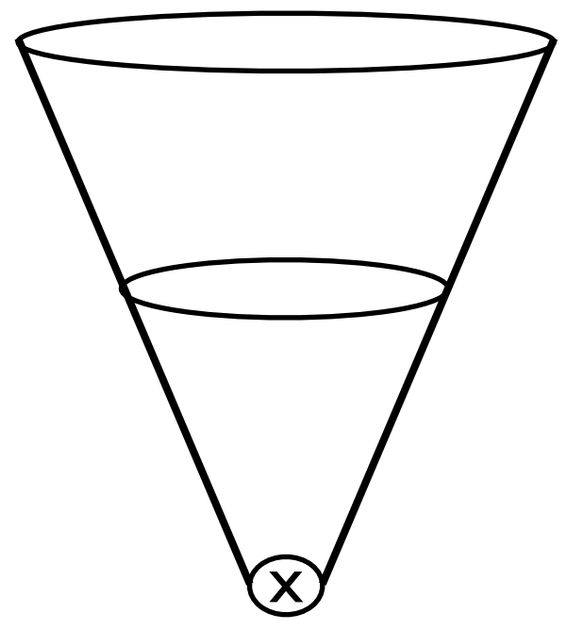
Summary

$\delta = 0$



$\phi \sim \phi + 2\pi\delta$



$\delta = 1$

Thank you

$\not{P}$ in hydrodynamics

$$J^\mu = \rho u^\mu - \kappa T P^{\mu\nu}\partial_\nu \frac{\mu}{T} + (k_1 T^2 - 2c\mu^2)\epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma + c_m j_m^\mu$$

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$$k_1 = c_m \quad ?$$

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P-odd fluctuations and long range order in heavy ion collisions and cosmology.

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University
of British Columbia,
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CP-ODD -2012 *June 25-27*

1. Motivation.

- The main goal of this talk is to argue that two, naively unrelated, phenomena:

- 1.Local P, CP violating fluctuations in QCD as observed at RHIC and LHC

- 2. Dark Energy with the scale $\rho_{DE} \simeq (10^{-3}\mathrm{eV})^4$

- are in fact tightly related, as they describe different sides of the same fundamental physics related to quantum anomalies and <u>long range order</u> as a result of nontrivial topological properties of strongly coupled QCD

2. Charge separation effect. CME.

- The $\theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu a}$ term is a key player in strongly interacting QCD

- $\theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu a} = \theta \partial_\mu K^\mu$ is total derivative, does not change the equation of motion. Still, it leads to the physically observable effects: dipole moment, $\eta' \to 2\pi$

- $\theta < 10^{-9}$ must be small (now) as it violates P, CP invariance in strong interactions.

- Still, a large domain with effective $\theta_{ind} \neq 0$ may be induced. It may result in local P odd fluctuations which are observed at RHIC, LHC. This <u>long range order</u> is a crucial element of this talk.

- For the uniform magnetic field the electric field will be induced along B in the presence of large domain θ (assuming a large coherent effect)

$$L^2 E_z^{ind} = -\left(\frac{e\,\theta}{2\pi}\right) l, \quad \text{where} \quad l = \frac{e}{2\pi}\int d^2x_\perp {B_z}^{ext}$$

- the induced electric field will lead to the induced currents and to the separation of charges along B (CME) on macroscopical scale:

$$\vec{J} = (\mu_L - \mu_R)\frac{e\vec{B}}{2\pi^2}, \quad \text{where} \quad (\mu_L - \mu_R) = \dot{\theta}$$

- A similar phenomenon happens when the system is rotating (it effectively replaces the magnetic field B). Therefore: an upper hemisphere can thus have either excess of quarks over anti-quarks or vice-versa on scale L.

$$[Q(z=+L) - Q(z=-L)] \sim \left(\frac{e\,\theta}{2\pi}\right) l$$

- <u>Crucial Question</u>: why the size L of $\mathcal{P}$ odd domain with $\theta_{ind} \neq 0$ is large, much larger than conventional $L \gg \Lambda_{QCD}^{-1}$ scale ? This is a required feature to have coherent charge separation effect.

- New scale L (size of the system): Where does it come from in a gapped QCD theory?

- Understanding of this new scale L is the key point which will be addressed in this talk.

- We argue in favour of a Casimir suppression L^{-1} for P-odd asymmetries which is in contrast with naively expected $\exp(-L)$. It will be the crucial element relating heavy ion collisions with $(\Lambda_{QCD}L)^{-1} \sim 0.1$ and cosmological Dark Energy with $L \sim 10\ Gyr$ leading to the observed DE density $\rho_{DE} \sim \Lambda_{QCD}^3/L \sim (10^{-3}\mathrm{eV})^4$

3. The old story: θ dependence in QCD

- The θ dependence in QCD determines the η' mass (Witten,Veneziano, 1979)

$$L = \frac{1}{2}\partial_\mu \eta' \partial^\mu \eta' - \frac{1}{\chi} Q^2 - \left(\theta - \frac{\eta'}{f_{\eta'}}\right) Q + N_f m_q | < \bar{q}q > | \cos\left[\frac{\eta'}{f_{\eta'}}\right]$$

$$Q \equiv \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu a}, \quad \chi = -\frac{\partial^2 \epsilon_{vac}(\theta)}{\partial \theta^2} = i \int d^4x < T\{Q(x), Q(0)\} >$$

- The topological susceptibility $\chi \neq 0$ does not vanish in spite of the fact that operator Q is total derivative, the so-called Witten's contact term.

- Sign of χ is <u>opposite</u> to what one should expect from a physical degree of freedom.

- Integrating out Q field produces η' mass

$$\chi_{dispersive} \sim \lim_{k\to 0} \sum_n \frac{\langle 0|q|n\rangle\langle n|q|0\rangle}{-k^2 - m_n^2} < 0,$$

- conventional physical degrees of freedom always contribute with sign (-) while we need sign (+) to satisfy WI and resolve the U(1) problem

$$\chi_{Eucl} = \int dx \, \langle 0|T\left\{\frac{\alpha_s}{8\pi} G\tilde{G}(x) \, \frac{\alpha_s}{8\pi} G\tilde{G}(0)\right\}|0\rangle = \frac{1}{N_c^2}|E_{\rm vac}(\theta)| \quad > 0.$$

- Contact term with "wrong" sign (+) is originated from the <u>degenerate topological sectors of the theory</u>. It can be computed in many simplified field theories as well as in holographic QCD.

- Crucial point: the contact (non-dispersive) term can not be restored from conventional dispersion relations. Therefore, it may, in principle, exhibit a power like scaling L^{-p} rather than $\exp(-\Lambda_{QCD} L)$.

4. Long range order in the equilibrium. Lessons from the lattice studies

- a.The recent Monte Carlo studies have revealed a laminar structure in the vacuum consisting of extended, thin, coherent, locally low-dimensional sheets of topological charge embedded in 4d space, with opposite sign sheets interleaved (Horvath et al 2003, Ilgenfritz et al 2007 +many more).

- b.The tension of these objects vanishes below the critical temperature and they percolate through the vacuum, forming a kind of a vacuum condensate

- c.They do not percolate through the whole 4d volume, but rather lie on 2d surfaces

- d. The total area of the surfaces is dominated by a single percolating cluster

- e. The contribution of the percolating surfaces to conformal WI is opposite in sign in comparison to its total value

$$\int dx \, \langle 0|T\left\{\frac{b\alpha_s}{8\pi}G^2(x)\,\frac{b\alpha_s}{8\pi}G^2(0)\right\}|0\rangle = -4\langle 0|\frac{b\alpha_s}{8\pi}G^2|0\rangle \quad < 0,$$

- f. The contribution of the percolating surfaces to chiral WI (topological susceptibility) has the same sign compared to its total value

$$\chi_{Eucl} = \int dx \, \langle 0|T\left\{\frac{\alpha_s}{8\pi}G\tilde{G}(x)\,\frac{\alpha_s}{8\pi}G\tilde{G}(0)\right\}|0\rangle = \frac{1}{N_c^2}|E_{\rm vac}(\theta)| \quad > 0.$$

- g. The width of the percolating objects apparently vanishes in the continuum limit.

- h. These coherent low dimensional objects have precisely required properties to saturate the "wrong signs" in WI with singular behaviour (vanishing width) of non-dispersive terms.

5. Interpretation: Tunnelling

- We interpret these <u>coherent objects</u> in Euclidean space as tunnelling events in Minkowski space.

- This statement becomes precise in "deformed QCD" where all computations are under complete theoretical control.

- the "wrong sign" contribution in topological susceptibility emerges as a result of tunnelling between topologically distinct (but physically equivalent) sectors $\mathcal{T}|n>=|n+1>, \quad [\mathcal{T}, H]=0$

- This concludes the old story when the system is in the equilibrium.

- What happens to the contact term if the system receives a small jolt ?

- The contact term is saturated by long range coherent objects. How does it vary with L ?

- The non-dispersive term changes when the background varies. These coherent topological fluctuations carry P-odd quantum numbers.

6. Applications to collisions. The Basic idea

- The tunnelling processes in vacuum do not lead to any emission or absorption of real particles, similar to the persistent tunnelling events in Bloch's case in condensed matter physics.

- These tunnelling events simply select an appropriate ground state of the system which is a specific superposition of $|n\rangle$ states.

- While each gauge configuration has definite sign of the topological charge density, the opposite sign sheets alternate. This delicate cancellation between the opposite sign sheets leads to the known P-even results.

- When some external sources are present in the system a <u>delicate cancellation</u> between the opposite sign extended sheets can not be maintained.

- How does vacuum energy scales with the size of the distorted system size ~L ? Normally, deviation in vacuum energy density is small, $\Delta E \sim \exp(-mL)$. However, in gauge theories the sensitivity to large distances is much stronger, $\Delta E \sim L^{-p}$.

- We call this "topological Casimir effect" when no massless fields are preset in the system. Still, it exhibits the <u>Casimir like</u> scaling in a <u>gapped theory</u>.

- Idea has been tested in a number of simplified QFTs as well as in holographic QCD when the massless axion field living in extra dimensions gives $\Delta E \sim L^{-p}$ (*AZ, 2011+ Gokce Basar, Dmitri Kharzeev, Ho-Ung Yee, Ariel Zhitnitsky, 2012, to appear*)

- Main assumption is that high energy collisions <u>do not</u> completely destroy this coherent structure studied on lattice in vacuum. These long range correlations always existed in vacuum before the collision occurs. The collision slightly disturbs this coherent structure.

- A disturbed long range structure exhibits $\mathcal{P}$ violation in the system on scales ~L. Indeed, the vacuum extended configurations responsible for the tunnelling carry the quantum number of $G\tilde{G}$. This structure is coherent on large scales $\gg \Lambda_{QCD}^{-1}$

- This picture is very different from conventional transitions due to instantons/sphalerons with a typical size Λ_{QCD}^{-1} which are $\mathcal{P}$ violating effects, but can not lead to any large coherent phenomena as they are suppressed $\exp -(\Lambda_{QCD} L)$

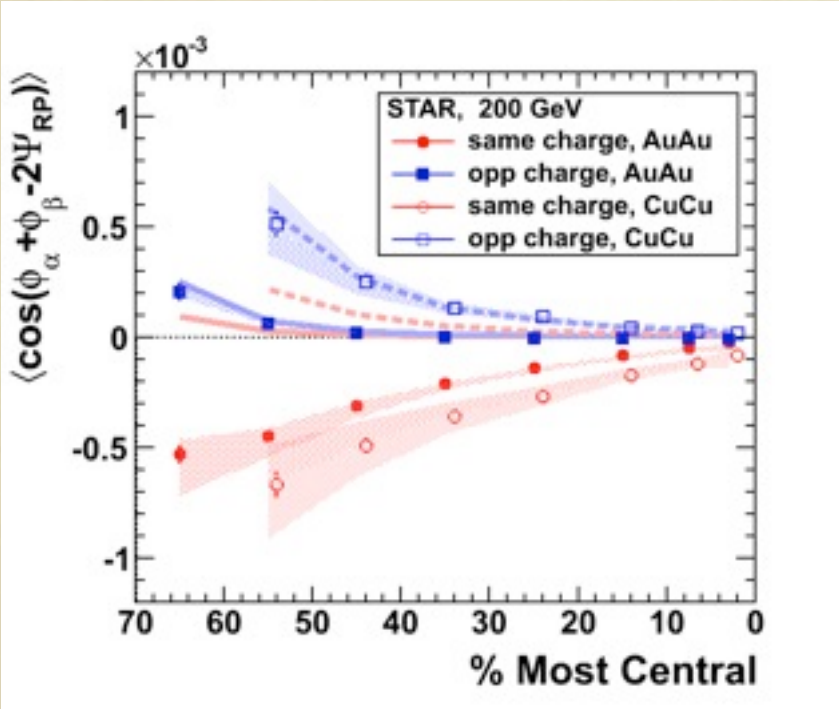
7. Comparison with observations

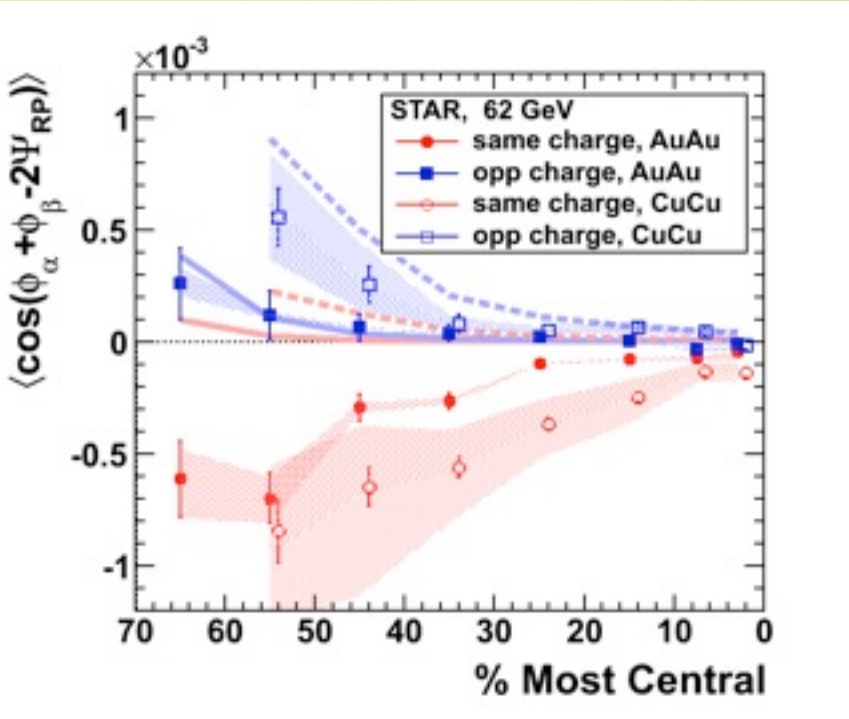
- Few consequences of this picture which are model-independent and not sensitive to any specific details of the framework:

- a). For finite system size L the P-odd correlations show the Casimir like scaling L^{-p} rather than naively expected exponential suppression $\exp(-L)$ when a mass gap is present in then system.

- b). Some suppression of the measured correlations with increasing size of the system L apparently indeed have been observed. The effect for Au+Au with A=197 is suppressed in comparison with Cu +Cu with A=64

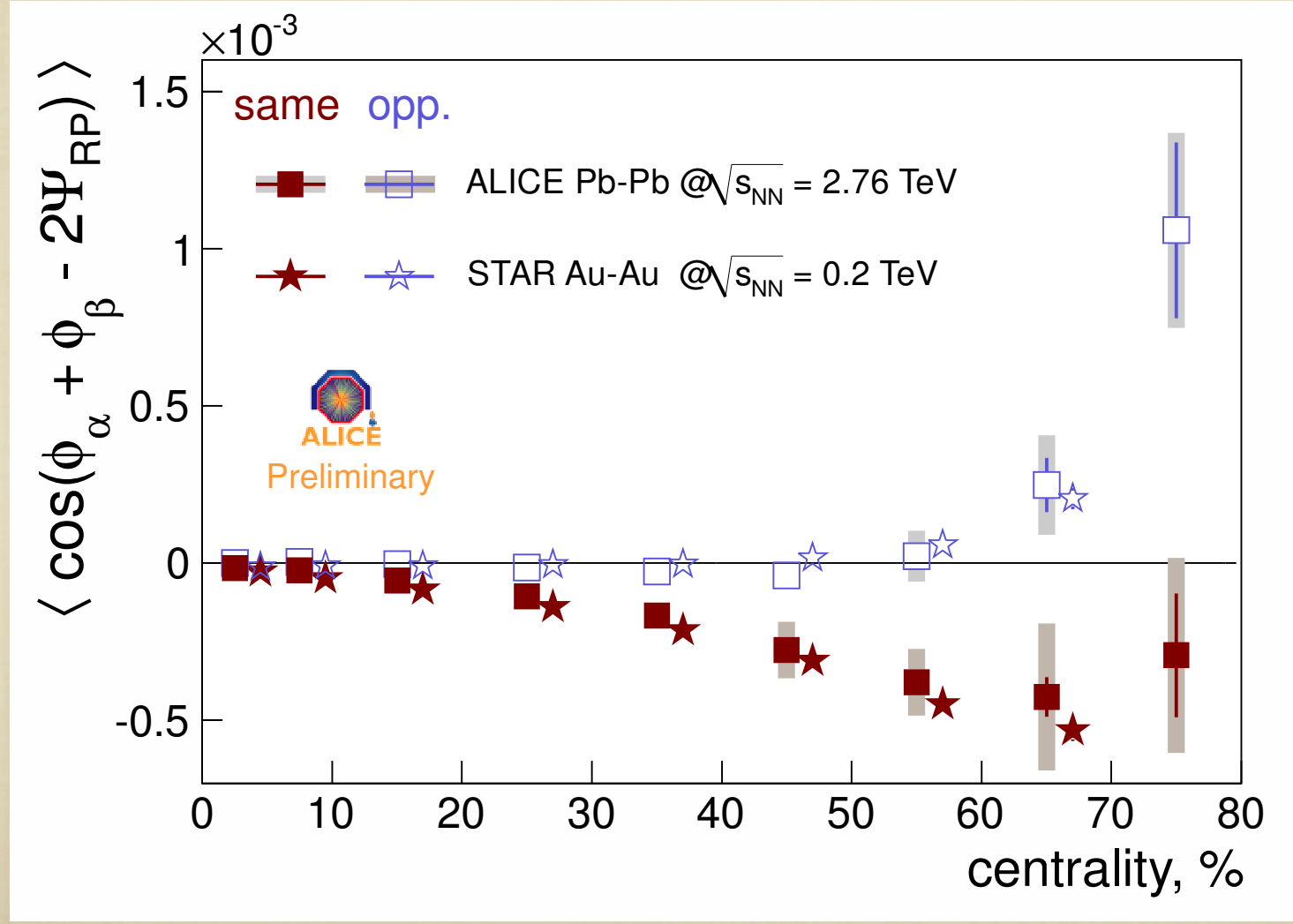
- c).The correlations demonstrate the universal behaviour similar to the universality observed in all high energy collisions. In particular, the correlations do not depend on energy/charge of colliding ions. Indeed, all effects are due to disturbed long range vacuum structure, rather than a result of direct impact of colliding ions.





- We interpret the difference between Au+Au and Cu+Cu as manifestation of a coherent L^{-p} scaling

- d). We expect the same tendency to continue for the LHC energies. Preliminary results for lead (Pb+Pb collisions from ALICE at 2.76 TeV) support this expectation. *In fact such a prediction had been made before the ALICE results were posted on arxiv*



8. Relevant time scales of the problem

- All our estimates are based on assumption of small deviations from vacuum solution such that all expectation values are small (derivative expansion):

$$|\langle \partial_\mu a(x)\rangle| \sim |\langle \partial_\mu \eta(x)\rangle| \sim L^{-1} \sim \mathbf{a} \ll \Lambda_{QCD}$$

- In reality this parameter may not be small. To observe the asymmetry the condition $\Delta t \leq \mathbf{a}^{-1}$ must be satisfied, where Δt is a typical emission time from region $L \sim \mathbf{a}^{-1}$.

- It is similar to P-violation in π -meson condensate background with $\lambda_\pi^{cond} \gg \Lambda_{QCD}^{-1}$ when all physical modes are formed in the P-odd background.

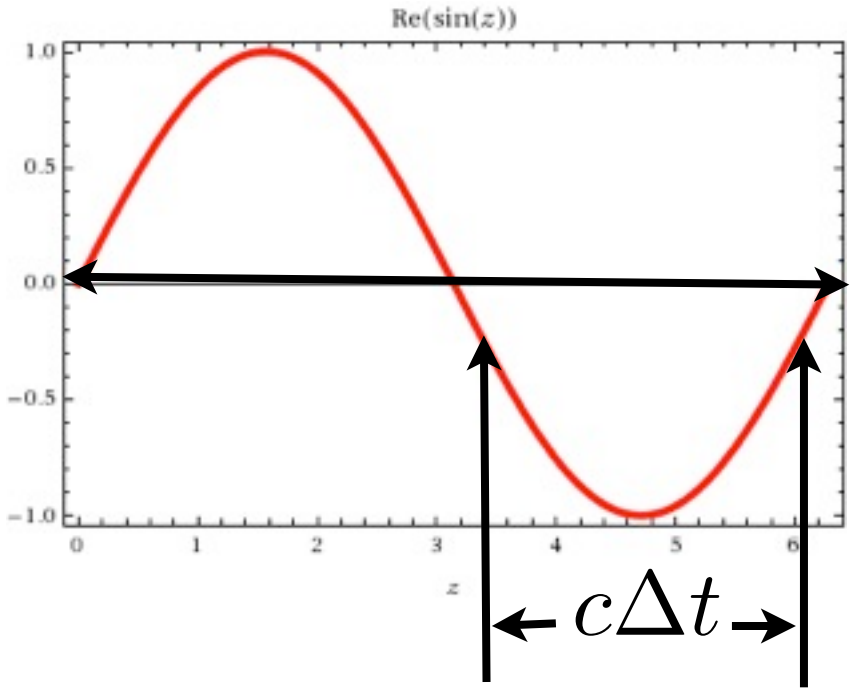
- Peripheral collisions correspond to a small disturbance, small $\mathbf{a}$, while $\Delta t \sim \Lambda_{QCD}^{-1}$ ~ const. Therefore, the condition $\Delta t \leq \mathbf{a}^{-1}$ might be satisfied.

- We parametrize the relevant time scales by dimensionless $\gamma \equiv \mathbf{a}\Delta t$: which enters formula for asymmetry through $f(\gamma)$

$$\sim e \cdot \kappa \cdot \frac{f(\gamma)}{L\Lambda_{\rm QCD}} \sim 5 \cdot 10^{-4}, \quad \gamma \equiv \mathbf{a}\Delta t, \quad f(\gamma \ll 1) \simeq const, \quad f(\gamma \gg 1) \simeq 0,$$

- Evaluation of $f(\gamma)$ is a hard problem. It requires model dependent assumptions, while items a)-d) formulated above are model -independent consequences which are based exclusively on key principles of this framework. Parameter $\kappa \sim N_c^{-2}$ is a suppression of the θ related physics (can be computed in the Veneziano ghost model).

- The condition $\Delta t \leq \mathbf{a}^{-1}$ is less likely to be satisfied for centralities (0%-5%) in heavy ion collisions as well as in e^+e^-, pp and $p\bar{p}$ collisions. In this case the asymmetry likely will be washed out as $\gamma \equiv \mathbf{a}\Delta t \sim 1$.



$\lambda \sim \dfrac{2\pi}{\mathbf{a}}$ is typical size of a domain

$\Delta t \sim \Lambda_{QCD}^{-1}$ is a typical emission time from region λ

- If $\Delta t \leq \mathbf{a}^{-1}$ the P-violating correlations can be observed as emissions effectively occur in P odd environment.

9. Heavy Ion collisions and Dark Energy

- We adapt the paradigm that the relevant energy which enters the Einstein equation is the difference $\Delta E \equiv (E - E_{\rm Mink})$ between energy in expanding universe and Minkowski space, similar to Casimir.

- This is a standard subtraction procedure for computations of correlators in curved space-time

- For the vacuum energy this definition was first advocated by Zeldovich, 1967 with eq: $\rho_{\rm vac} \sim G m_p^6$

- <u>Heavy ions</u>: we compute the deviation $\Delta E \equiv (E - E_{\rm Mink})$ which is a result of external sources (collisions).

- <u>DE</u>: we compute the deviation $\Delta E \equiv (E - E_{\rm Mink})$ which is a result of expansion on scale $L \sim H^{-1} \sim 10^{10}$ years.

- The crucial point is the Casimir like scaling for QCD with a gap: $1/L$ instead e^{-L}. The power like correction $\Delta E \sim L^{-1} \sim H$ would lead to an estimate

$$\Delta E \sim H\Lambda_{QCD}^3 \sim (10^{-3}\mathrm{eV})^4, \quad \text{Hubble constant } H \simeq 10^{-33}\mathrm{eV}$$

- It is very close to the observed DE value today without adjusting any parameters.

- testing the scaling on the lattice. Casimir like effects are predicted for 4D QCD: $1/L$ instead e^{-L}. Lattice confirmation (B. Holdom, Phys. Lett B 2011).

- Naive argument with $\exp(-L)$ fails because $1/L$ correction is related to non-dispersive topological term, not related to any physical propagating degrees of freedom ("wrong sign" in susceptibility)

- The evolution of then universe approaches a de-Sitter state with constant expansion rate $H \to G\Lambda_{QCD}^3$ as follows from Friedman equation

$$H^2 = \frac{8\pi G}{3}\left(\rho_{DE} + \rho_M\right), \quad \rho_{DE} \sim H\Lambda_{QCD}^3, \quad \rho_c = \frac{3H^2}{8\pi G}$$

- A number of other fine tuning issues ("coincidence problem", "drastic separation of scales" etc) are automatically resolved without a single new field. For example, the extra Casimir type energy becomes relevant at $t_0 \sim H^{-1} \sim (G\Lambda_{QCD}^3)^{-1} \sim 10$ Gyr.

- The long range order in gapped QCD is similar to Aharonov -Casher effect. If one inserts an external charge into superconductor when electric field is screened $\sim \exp(-r/\lambda)$ a neutral magnetic fluxon will be still sensitive to external charge at arbitrary large distances.

Conclusion-1: Heavy Ion Collisions

- We interpret a relatively large observed intensities of the asymmetries in heavy ion collisions as a coherent phenomenon. It is a direct manifestation of the Casimir 1/L scaling when a measured asymmetry is accumulated on large scales $L \sim 10 \text{ fm}$.

- This long range order should be (eventually) tested using different observables, such as EM probes, or explicit CP violating processes from P-odd domains.

Conclusion-2: Dark Energy

- Long range order may have profound consequences for cosmology when a relatively small parameter (from heavy ion collisions) is replaced by drastically smaller parameter in expanding universe, $L^{-1} \Rightarrow H \sim 10^{-33}\mathrm{eV}$

- The DE in this framework emerges as mismatch between the energies of the system in a nontrivial FRW background and Minkowski space-time, and entirely rooted into strongly coupled QCD. <u>Most intricate properties of the vacuum can be tested in heavy ion collisions at RHIC at LHC.</u>

- A large number of hints (observations) indeed suggest that the Universe on the Hubble scales ~Gpc is not P-symmetric....

CPODD 2012 AGENDA JUNE 25-27 2012

Chemistry Department, Bldg. 555

Session I. Chair: Dmitri Kharzeev

June 25 Monday 8:50-12:30

8:20-8:50		Registration
8:50-9:00	N. Samios	"Welcome"
9:00-9:30	E. Shuryak	"Can the Standard Model explain the cosmological baryon asymmetry?"
9:30-10:00	T. Vachaspati	"Sphalerons, turbulence, and magnetic fields"
10:00-10:30	K. Fukushima	"Photons in the Chiral Magnetic Effect"
10:30-11:00	COFFEE BREAK	
11:00-11:30	Y. Burnier	"Anomaly induced charge dependence of the elliptic flow in heavy ion collisions"
11:30-12:00	G. Wang (STAR)	"Search for effects related to Chiral Magnetic Wave at STAR"
12:00-12:30	S. Esumi (PHENIX)	"Charge asymmetric correlation measurement in 200GeV Au+Au collisions at RHIC-PHENIX"

Session II. Chair: Roy Lacey

June 25 Monday 14:00-17:00

14:00-14:30	K. Riley (STAR)	"Charge dependent azimuthal correlations of K-PI pairs"
14:30-15:00	P. Christakoglou (ALICE)	"Charge dependent azimuthal correlations with the ALICE detector at the LHC"
15:00-15:30	Dragos Velicanu (CMS)	"Di-hadron correlations measurements from CMS."
15:30-16:00	COFFEE BREAK	
16:00-16:30	M. Polikarpov	"Numerical study of the monolayer graphene phase diagram"
16:30-17:00	V. Miransky	"Magnetic catalysis and quantum Hall effect in monolayer and bilayer graphene"

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Session III. Chair: Ho-Ung Yee

June 26 Tuesday 9:00-12:30

9:00-9:30	J. Erdmenger	"Holographic hydrodynamics with broken rotational symmetry"
9:30-10:00	A. Yarom	"Hydrodynamics without an entropy current"
10:00-10:30	O. Ruchayskiy	"Chiral anomaly and magnetic fields in the Standard Model plasma"
10:30-11:00	COFFEE BREAK	
11:00-11:30	I. Kirsch	"Anisotropic hydrodynamics, holography and the Chiral Magnetic Effect"
11:30-12:00	R. Loganayagam	"Anomalies and the helicity of the thermal state"
12:00-12:30	P. Surowka	"Towards kinetic theory with anomalies"

Session IV. Chair: Aihong Tang

June 26 Tuesday 14:00-17:00

14:00-14:30	A. Zhitnitsky	"P-odd fluctuations and long range order in heavy ion collisions and cosmology"
14:30-15:00	H.-C. Ren	"Some field theoretic issues regarding the Chiral Magnetic Effect"
15:00-15:20	COFFEE BREAK	
15:20-15:45	G. Ma	"AMPT results on charge azimuthal correlations at RHIC and LHC"
15:45-16:10	A. Vossen	"Prospects of Detecting local P-odd effects at Belle"
16:10-16:35	V. Incera	"Spirals of chiral and spin density waves in dense QCD in a magnetic field"
16:35-17:00	T. Kojo	"Interweaving chiral spirals"

Workshop Dinner: 6PM @ Brookhaven Center, South Room

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Session V. Chair: Jinfeng Liao

June 27 Wednesday 9:00-12:30

9:00-9:30	M. Stephanov	"Chiral Kinetic Theory"
9:30-10:00	M. Chernodub	"Spontaneous electromagnetic superconductivity of QCDxQED vacuum in strong magnetic field"
10:00-10:30	M. D'Elia	"Topological effects and the deconfinement transition"
10:30-11:00	COFFEE BREAK	
11:00-11:30	I. Shovkovy	"Chiral shift in renormalizable theories in magnetic field"
11:30-12:00	A. Andrianov	"P breaking effects in a quark (nuclear) medium with axial charge"
12:00-12:30	D. Espriu	"Dilepton excess from local parity breaking in baryon matter"

Session VI. Chair: Kirill Tuchin

June 27 Wednesday 14:00-17:00

14:00-14:30	B. Mueller	"Strong interaction effects of strong magnetic fields"
14:30-15:00	G. Basar	"Instantons and sphalerons in magnetic field"
15:00-15:30	A. Schmitt	"Inverse magnetic catalysis"
15:30-16:00	COFFEE BREAK	
16:00-16:30	Y. Qian	"P-odd effects through QCD instantons"
16:30-17:00	T. Sulejmanpasic	"Chiral symmetry breaking at finite T, and the dyonic vacuum in QCD like theories"

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P- and CP-odd Effects in Hot and Dense Matter (2012)

June 25-27, 2012

List of Registered Participants

Name	Affiliation
Alexander Andrianov	Saint Petersburg State Univ & Institut of Cosmic Sciences (ICCUB)
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Vladimir Shevchenko ..... National Research Center "Kurchatov Institute"
Igor Shovkovy ..... Arizona State University
Edward Shuryak ..... Stony Brook University
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Mikhail Stephanov..... University of Illinois
Tin Sulejmanpasic..... Universitaet Regensburg
Piotr Surowka..... Vrije Universiteit Brussel
Aihong Tang..... Brookhaven National Laboratory
Oleg Teryaev ..... JINR
Jim Thomas ..... Lawrence Berkeley National Laboratory
Kirill Tuchin..... Iowa State University
Tanmay Vachaspati ..... Arizona State University
Dragos Velicanu..... Massachusetts Institute of Technology
Anselm Vossen..... Indiana University/CEEM
Fuqiang Wang ..... Purdue University
Gang Wang ..... UCLA
Nu Xu ..... Lawrence Berkeley National Laboratory
Amos Yarom..... Technion
Ho-Ung Yee ..... Stony Brook University
Yi Yin ..... U Illinois at Chicago
Ismail Zahed ..... Stony Brook University
Ariel Zhitnitsky ..... University of British Columbia

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Volume 107 – Future Directions in High Energy QCD, October 20-22, 2011 – BNL-98119-2011
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Link: PHENIXSpinfestSchool2009@BNL
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Li Keran

Nuclei as heavy as bulls
Through collision
Generate new states of matter.
T.D. Lee

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