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Raytracing Based Spin Tracking

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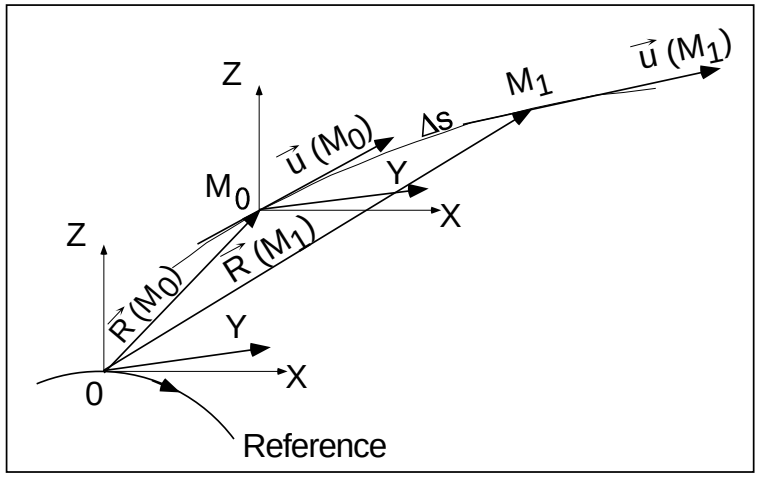
1 A short introduction to the Zgoubi method



Motion, from M_0 to M_1 is computed

by solving

$$d(m\vec{v}) \;=\; q\,(\vec{e}+\vec{v}\times\vec{b})\;dt$$

- **using truncated Taylor expansions of $\vec{R}$ and $\vec{u}=\vec{v}/v$:**

$$\begin{aligned}\vec{R}(M_1) &\approx \vec{R}(M_0)+\vec{u}(M_0)\,\Delta s+\vec{u}'(M_0)\,\frac{\Delta s^2}{2!}+\ldots+\vec{u}'''''(M_0)\,\frac{\Delta s^6}{6!}\\ \vec{u}(M_1) &\approx \vec{u}(M_0)+\vec{u}'(M_0)\,\Delta s+\vec{u}''(M_0)\,\frac{\Delta s^2}{2!}+\ldots+\vec{u}'''''(M_0)\,\frac{\Delta s^5}{5!}\end{aligned} \tag{1}$$

- **In non-zero $\vec{E}$ environment, rigidity at M_1 is re-computed :**

$$(B\rho)(M_1)\approx(B\rho)(M_0)+(B\rho)'(M_0)\Delta s+\ldots+(B\rho)'''''(M_0)\frac{\Delta s^5}{5!} \tag{2}$$

- **When necessary, time-of-flight is computed in a similar manner :**

$$T(M_1)\approx T(M_0)+\frac{dT}{ds}(M_0)\,\Delta s+\frac{d^2T}{ds^2}(M_0)\,\frac{\Delta s^2}{2}+\ldots+\frac{d^5T}{ds^5}(M_0)\,\frac{\Delta s^5}{5!} \tag{3}$$

- **In a general manner, computation of the truncated Taylor series**

$$
\begin{aligned}
position\ :\quad & \vec{R}(M_1) &&= \vec{R}(M_0) + \vec{u}(M_0)\,\Delta s + ... \\
velocity\ :\quad & \vec{u}(M_1) &&= \vec{u}(M_0) + \vec{u}'(M_0)\,\Delta s + ... \\
rigidity\ :\quad & (B\rho)(M_1) &&= (B\rho)(M_0) + (B\rho)'(M_0)\Delta s + ... \\
time\ :\quad & T(M_1) &&= T(M_0) + \tfrac{dT}{ds}(M_0)\,\Delta s + ...
\end{aligned}
\tag{4}
$$

requires that of the derivatives

$$\vec{u}^{(n)} = d^n\vec{u}\ /\ ds^n$$

$$(B\rho)^{(n)} = d^n(B\rho)\ /\ ds^n$$

$$d^n(T)\ /\ ds^n$$

Integration in magnetic field

Simplified notation :

$$\textbf{Given} \quad \vec{u} = \frac{\vec{v}}{v}\,, \quad ds = v\,dt, \quad \vec{u}' = \frac{d\vec{u}}{ds}\,, \quad m\vec{v} = mv\vec{u} = q\,B\rho\,\vec{u}\,, \vec{B} = \frac{\vec{b}}{B\rho} \tag{5}$$

- $d(m\vec{v})/dt \;=\; q\,\vec{v}\times\vec{b}$ **then writes**

$$\boxed{\vec{u}' = \vec{u}\times\vec{B}}$$

This yields the $\vec{u}^{(n)} = d^n\vec{u}\;/\;ds^n$ **needed in the Taylor expansions :** $\vec{u}' = \vec{u}\times\vec{B}$

$$\begin{aligned}
\vec{u}' &= \vec{u}\times\vec{B} \\
\vec{u}'' &= \vec{u}'\times\vec{B} + \vec{u}\times\vec{B}' \\
\vec{u}''' &= \vec{u}''\times\vec{B} + 2\vec{u}'\times\vec{B}' + \vec{u}\times\vec{B}'' \\
\vec{u}'''' &= \vec{u}'''\times\vec{B} + 3\vec{u}''\times\vec{B}' + 3\vec{u}'\times\vec{B}'' + \vec{u}\times\vec{B}''' \\
\vec{u}''''' &= \vec{u}''''\times\vec{B} + 4\vec{u}'''\times\vec{B}' + 6\vec{u}''\times\vec{B}'' + 4\vec{u}'\times\vec{B}''' + \vec{u}\times\vec{B}''''
\end{aligned} \tag{6}$$

where $\vec{(*)}^{(n)} = d^n\vec{(*)}\;/\;ds^n$**.**

Integration in electrostatic field

- **Using the same notations as earlier,**

$d(m\vec{v}) \;=\; q\,(\vec{e}+\vec{v}\times\vec{b})\,dt$ **with** $\vec{b}=0$ **can be written**

$$\boxed{(B\rho)'\vec{u}+B\rho\,\vec{u}' = \frac{\vec{e}}{v}}$$

This yields the $\vec{u}^{(n)} = d^n\vec{u}\;/\;ds^n$ **needed in the Taylor expansions :**

$$\begin{aligned}
&(B\rho)'\vec{u}+B\rho\vec{u}' && = \tfrac{1}{v}\vec{e}\\
&(B\rho)''\vec{u}+2(B\rho)'\vec{u}'+B\rho\vec{u}'' && = \left(\tfrac{1}{v}\right)'\vec{e}+\tfrac{1}{v}\vec{e}'\\
&(B\rho)'''\vec{u}+3(B\rho)''\vec{u}'+3(B\rho)'\vec{u}''+B\rho\vec{u}''' && \\
& && = \left(\tfrac{1}{v}\right)''\vec{e}+2\left(\tfrac{1}{v}\right)'\vec{e}'+\left(\tfrac{1}{v}\right)\vec{e}''\\
&(B\rho)''''\vec{u}+4(B\rho)'''\vec{u}'+6(B\rho)''\vec{u}''+4(B\rho)'\vec{u}'''+B\rho\vec{u}'''' && \\
& && = (B\rho)''''\vec{u}\left(\tfrac{1}{v}\right)'''\vec{e}+3\left(\tfrac{1}{v}\right)''\vec{e}'+3\left(\tfrac{1}{v}\right)'\vec{e}''+\tfrac{1}{v}\vec{e}'''
\end{aligned} \tag{7}$$

with $\quad d^n(B\rho)/ds^n = d^{n-1}\left(\frac{1}{v}\left(\vec{e}\cdot\vec{u}\right)\right)/ds^{n-1}$

Integration in combined magnetic+electrostatic field

The Lorentz equation in "normalized form" writes

$$(B\rho)'\vec{u} + B\rho\vec{u}' = \frac{\vec{e}}{v} + \vec{u} \times \vec{b}$$

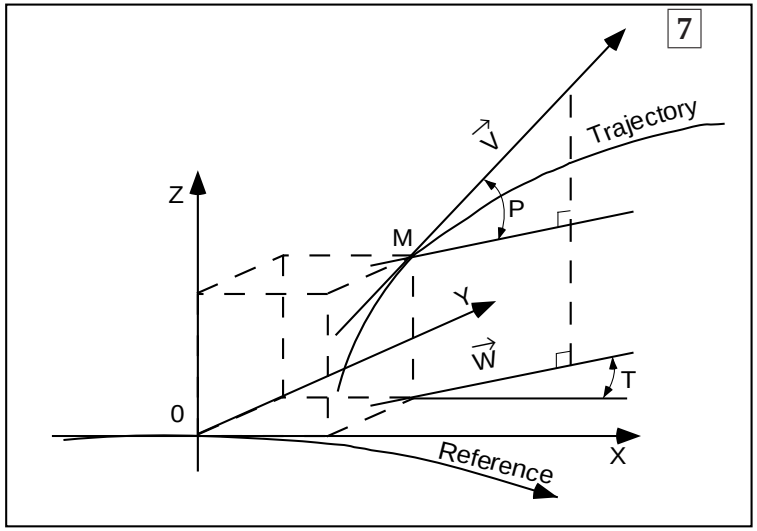
which by differentiation yields

$$\vec{B}\rho\,\boxed{\mathbf{u'}} = \left(\tfrac{1}{v}\right)\,\vec{e} - (B\rho)'\vec{u} \qquad + \qquad (\vec{u} \times \vec{b})$$

$$\vec{B}\rho\,\boxed{\mathbf{u''}} = \underbrace{\left(\frac{1}{v}\right)'\vec{e} + \left(\frac{1}{v}\right)\,\vec{e}'\,|_{B\rho} - 2(B\rho)'\vec{u}' - B\rho)''\vec{u}}_{\textbf{E field contribution}} + \underbrace{(\vec{u} \times \vec{b})'\,|_{B\rho}}_{\textbf{B field contribution}}$$

$$etc. \tag{8}$$

with the $(B\rho)^{(n)}$ **obtained using** $d^n(B\rho)/ds^n = d^{n-1}\left(\frac{1}{v}\left(\vec{e}\cdot\vec{u}\right)\right)/ds^{n-1}$**.**



- **Finally, field derivatives wrt. path length,**

$$\vec{E}^{(n)} = d^n\vec{E} \;/\; ds^n, \qquad \vec{B}^{(n)} = d^n\vec{B} \;/\; ds^n$$

- **are computed from the derivatives in magnet frame :**

$$d\vec{B}\ (\textbf{or}\ \ d\vec{E}) \;=\; \frac{\partial\vec{B}}{\partial X}dX + \frac{\partial\vec{B}}{\partial Y}dY + \frac{\partial\vec{B}}{\partial Z}dZ \;=\; \sum_{i=1,3}\frac{\partial\vec{B}}{\partial X_i}dX_i$$

- **following**

$$\begin{aligned}
\vec{B}' &= \sum_i \frac{\partial\vec{B}}{\partial X_i}u_i \\
\vec{B}'' &= \sum_{ij}\frac{\partial^2\vec{B}}{\partial X_i\partial X_j}u_iu_j + \sum_i\frac{\partial\vec{B}}{\partial X_i}u_i' \\
\vec{B}''' &= \sum_{ijk}\frac{\partial^3\vec{B}}{\partial X_i\partial X_j\partial X_k}u_iu_ju_k + 3\sum_{ij}\frac{\partial^2\vec{B}}{\partial X_i\partial X_j}u_i'u_j + \sum_i\frac{\partial\vec{B}}{\partial X_i}u_i'' \\
&etc.
\end{aligned} \tag{9}$$

• Zgoubi is an old code (early 1970s, dedicated to spectrometer design), thus ways to simulate optical elements have accumulated over the years :

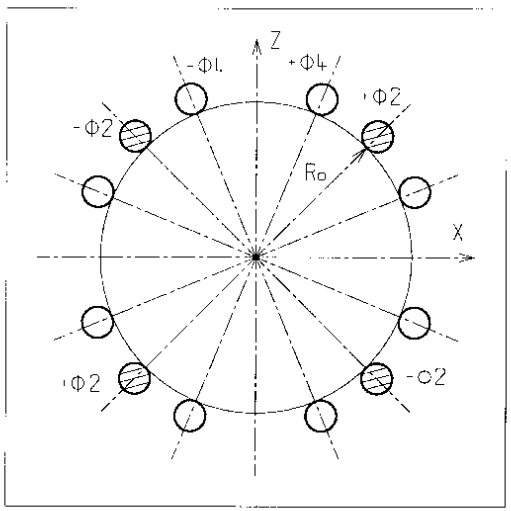
Magnetic elements :	*Keyword :*
Cyclotron magnet	DIPOLE[S], DIPOLE-M, FFAG, FFAG-SPI
Decapole	DECAPOLE, MULTIPOL
Dipole[s]	AGSMM, AIMANT, BEND, DIPOLE[S], DIPOLE-M, MULTIPOL, QUADISEX
Dodecapole	DODECAPO, MULTIPOL
FFAG magnet	DIPOLES, FFAG, FFAG-SPI, MULTIPOL
Helical dipole	HELIX
Multipole	MULTIPOL, QUADISEX, SEXQUAD
Octupole	MULTIPOL, OCTUPOLE, QUADISEX, SEXQUAD
Quadrupole	AGSQUAD, MULTIPOL, QUADRUPO, SEXQUAD
Sextupole	MULTIPOL, QUADISEX, SEXTUPOL, SEXQUAD
Skew multipoles	MULTIPOL
Solenoid	SOLENOID
Undulator	UNDULATOR

Field maps

1-D, cylindrical symmetry	BREVOL
2-D, mid-plane symmetry	CARTEMES, POISSON, TOSCA
2-D, no symmetry	MAP2D
2-D, polar mesh, mid-plane symmetry	POLARMES
3-D, no symmetry	TOSCA
EMMA dipole doublet	EMMA

• There is also a variety of electrostatic elements, and possibility of using electric field maps.

Electrostatic elements :	*Keyword :*
2-tube (bipotential) lens	EL2TUB
3-tube (unipotential) lens	UNIPOT
Decapole	ELMULT
Dipole	ELMULT
Dodecapole	ELMULT
Multipole	ELMULT
N-electrode mirror/lens, straight slits	ELMIR
N-electrode mirror/lens, circular slits	ELMIRC
Octupole	ELMULT
Quadrupole	ELMULT
R.F. (kick) cavity	CAVITE
Sextupole	ELMULT
Skew multipoles	ELMULT

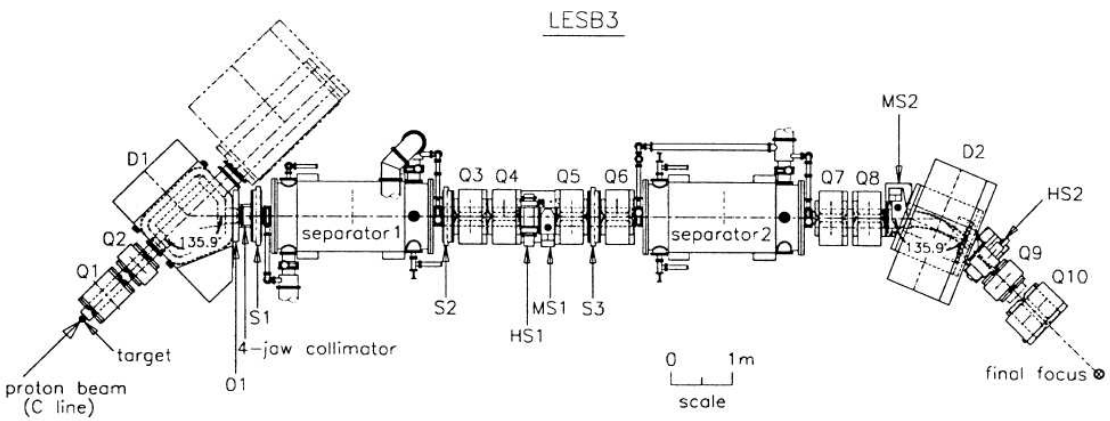


Field maps

1-D, cylindrical symmetry	ELREVOL
2-D, no symmetry	MAP2D-E
3-D field maps	TOSCA-E

• ... and combination of the two,

$E \times B$ ***elements :***	***Keyword :***
Decapole	**EBMULT**
Dipole	**EBMULT**
Dodecapole	**EBMULT**
Multipole	**EBMULT**
Octupole	**EBMULT**
Quadrupole	**EBMULT**
Sextupole	**EBMULT**
Skew multipoles	**EBMULT**
Wien filter	**SEPARA, WIENFILT**



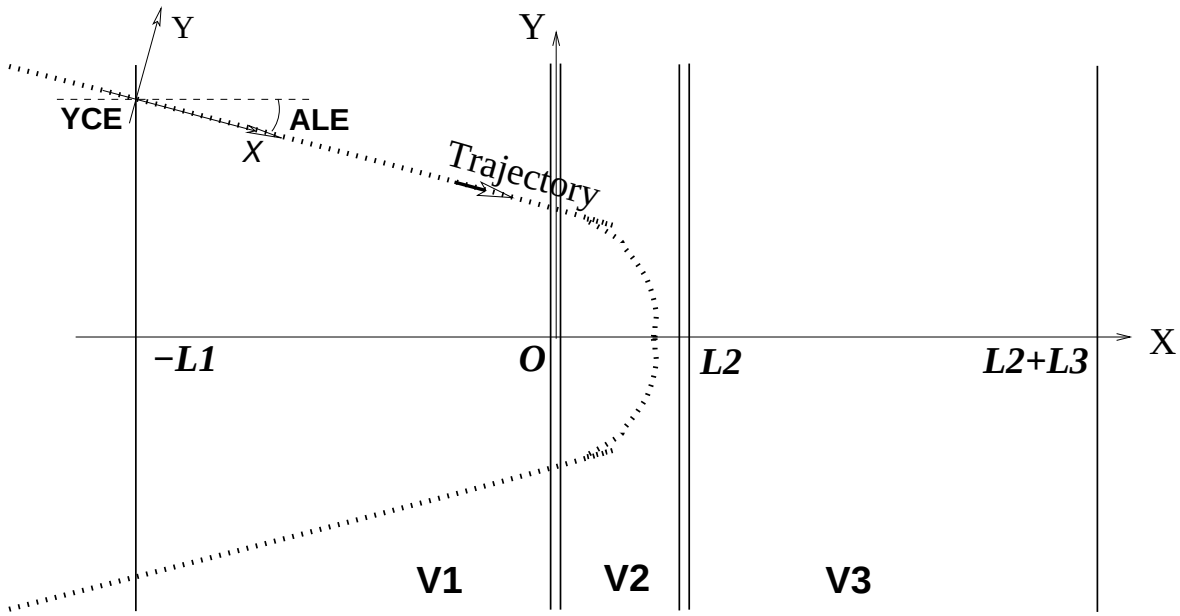
Layout of LESB3 beamline.

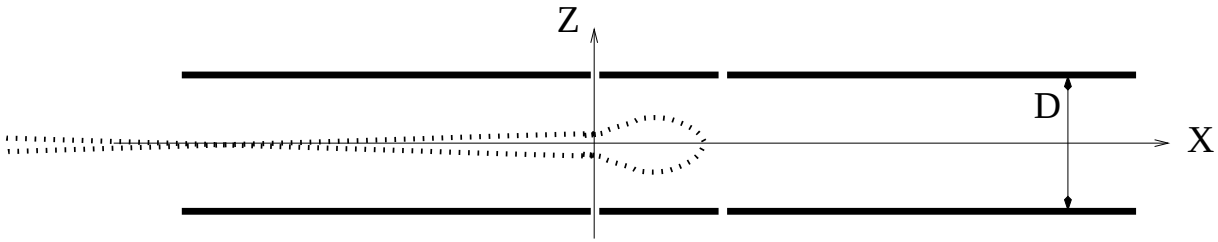
Example #1 : "ELMIR"

• **Mirror or lens, depending on the potentials on the plates, horizontal or vertical, depending on the orientation.**

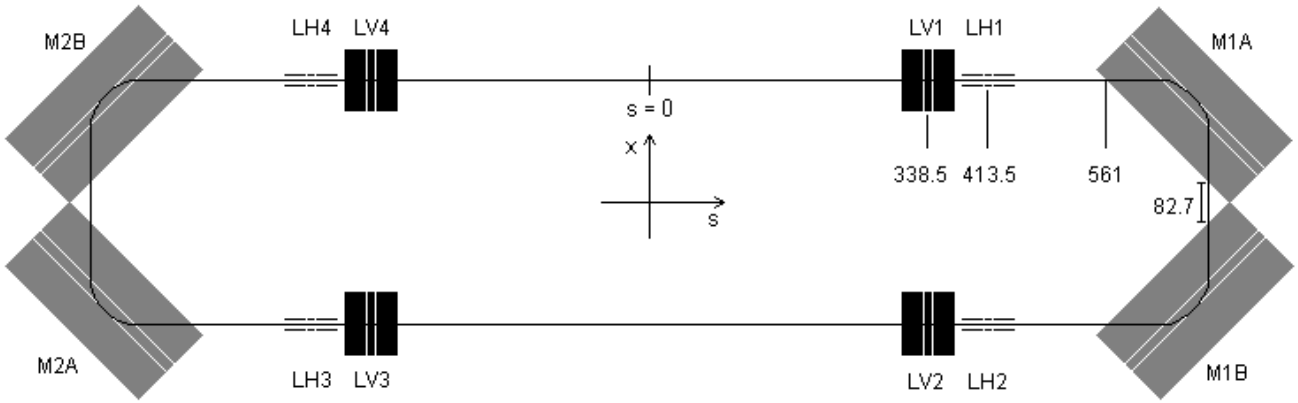
$$V(X,Z) = \sum_{i=2}^{N} \frac{V_i - V_{i-1}}{\pi} \arctan \frac{\sinh(\pi(X - X_{i-1})/D)}{\cos(\pi Z/D)}$$

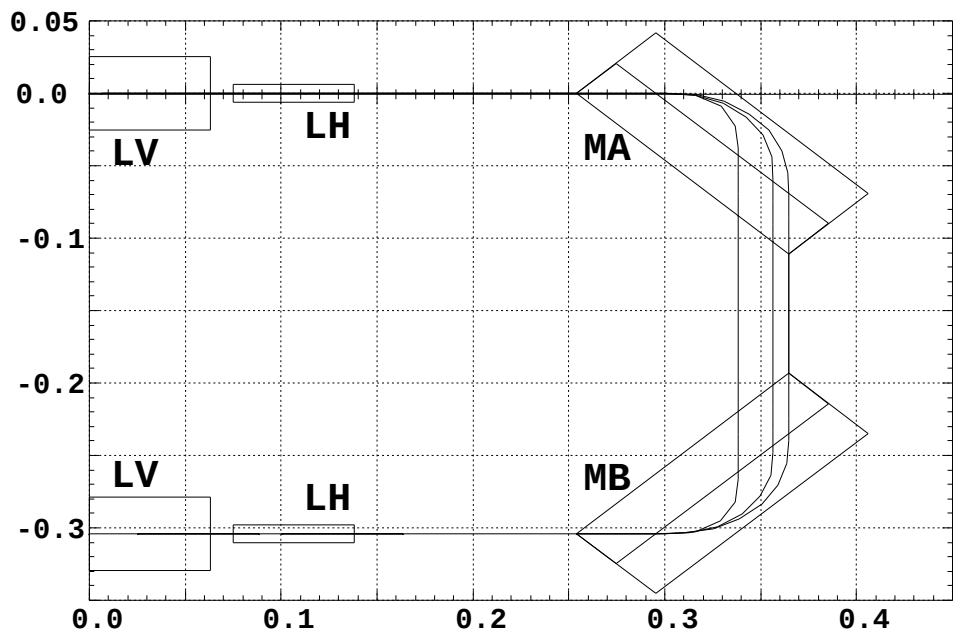
V_i **are the potential at the** N **electrodes,** $V_1 = 0$ **refers to the incident beam energy.**





Tracking in an electrostatic TOFMS ring (1999)

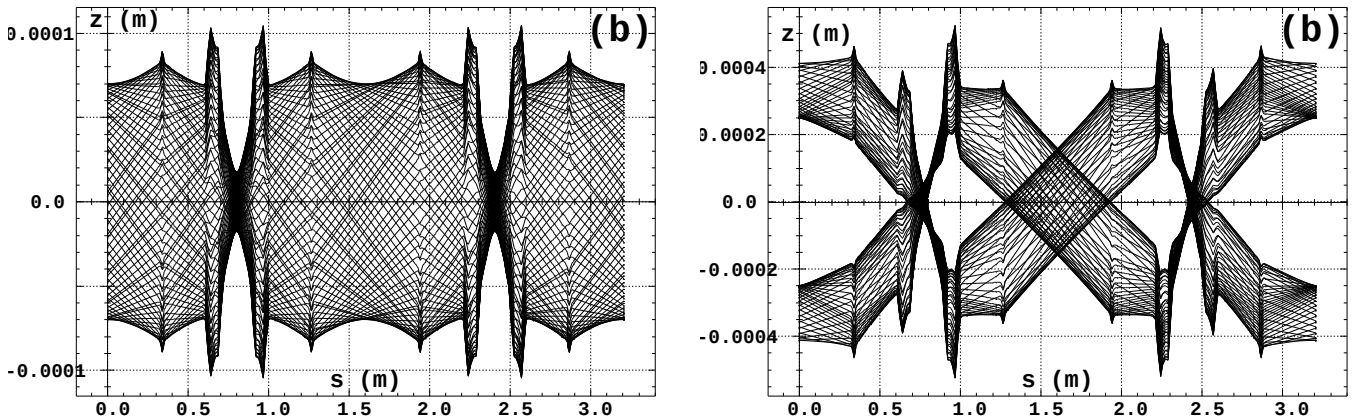




Trajectories over the half-ring (1 cell), of three ions having $\delta E/E = -20\%,\ 0$ and $+10\%$ energy dispersion. This is also a representation of the dispersion function $D_x(s) = \delta x(s)/\delta E/E$.

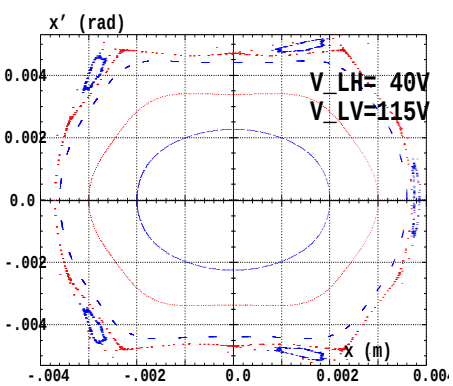
TOFMS ring, envelopes and long-term (DA) tracking

- **Vertical envelope as generated by multiturn tracking of a single particle :**

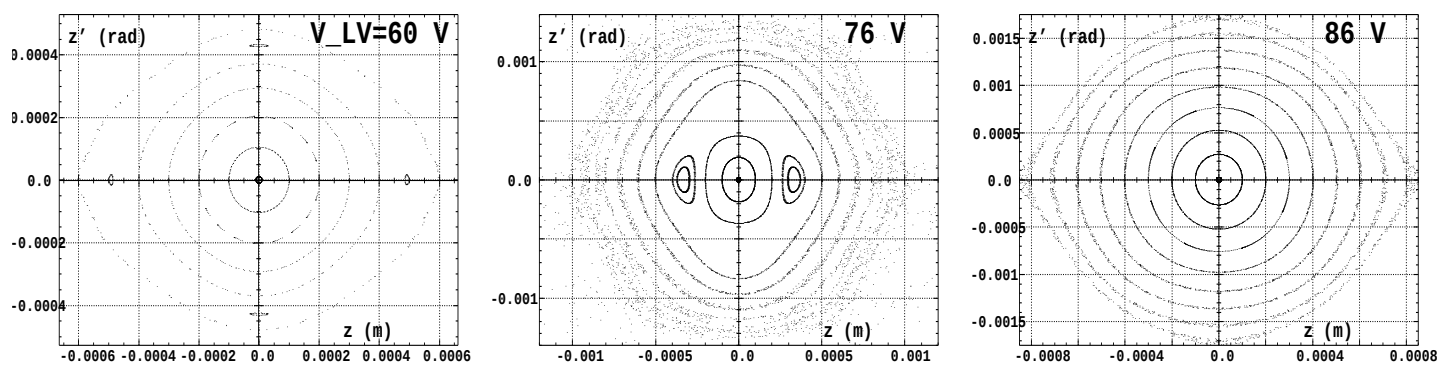


Initial coordinates at $s = 0$ are $z'_0 = 0$ and,
left : paraxial case, $z_0 = 0.07\ 10^{-3}$ m,
right : large amplitude case, $z_0 = 0.25\ 10^{-3}$ m, vertical tune $\nu_z \approx$ half integer.

- **Dynamical acceptance as observed at $s = 0$, a few 1000 turns :**



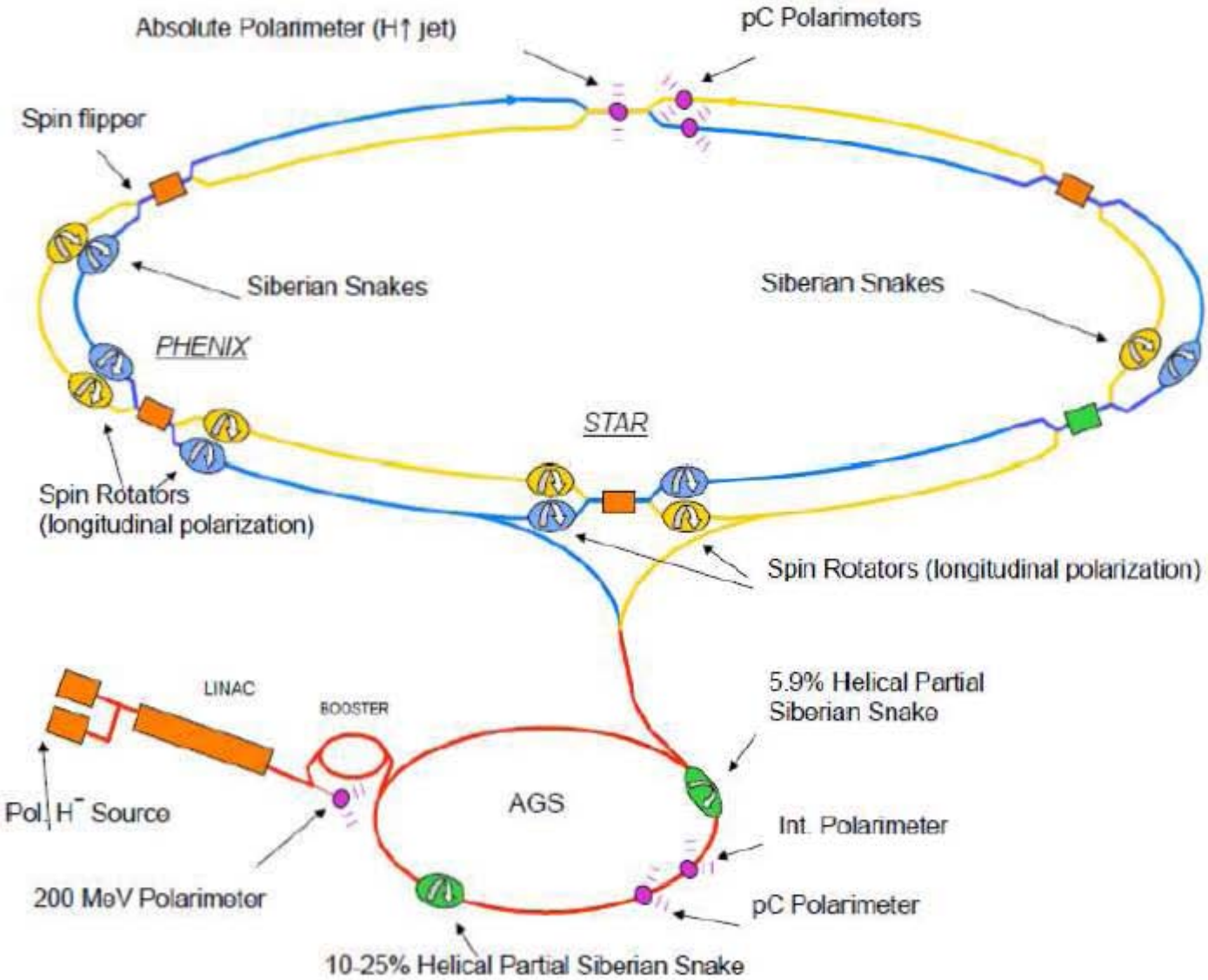
Horizontal.



Vertical.

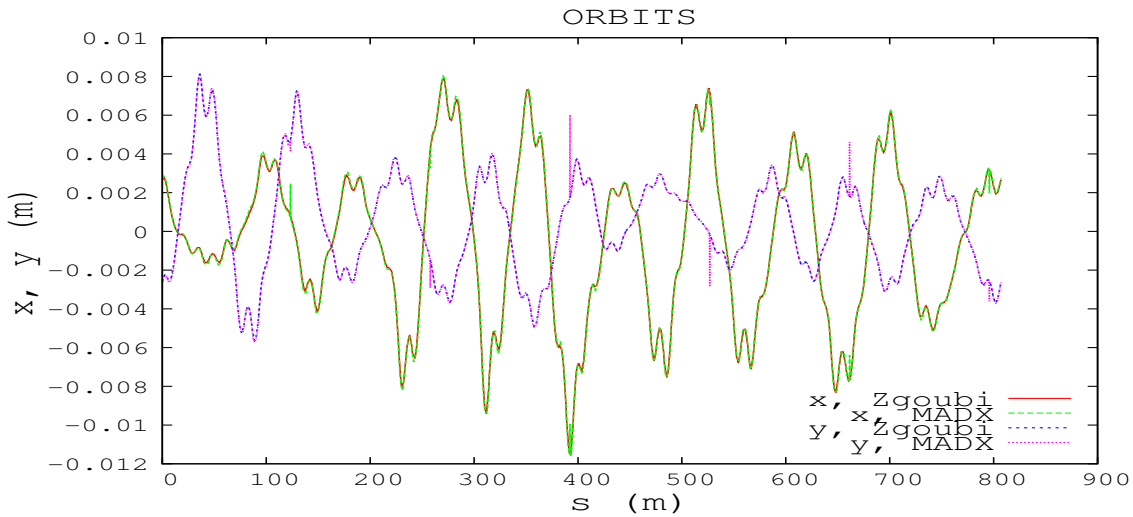
Example #2 : AGS

A sophisticated model of the AGS has been developped in Zgoubi, including 240 combined function main dipoles, tuning quadrupoles, sextupoles, orbit correctors, survey data, the two helical snakes, pickups, etc.

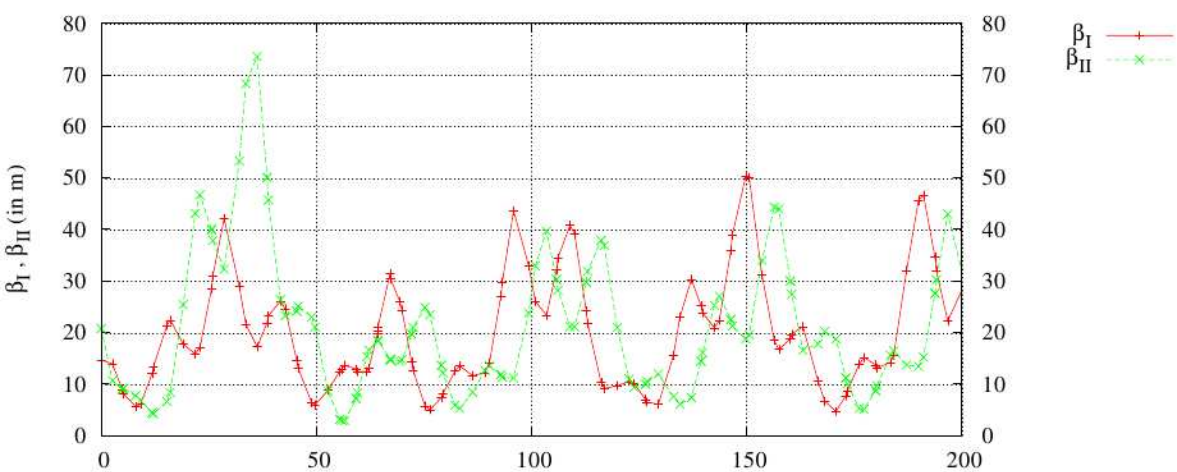


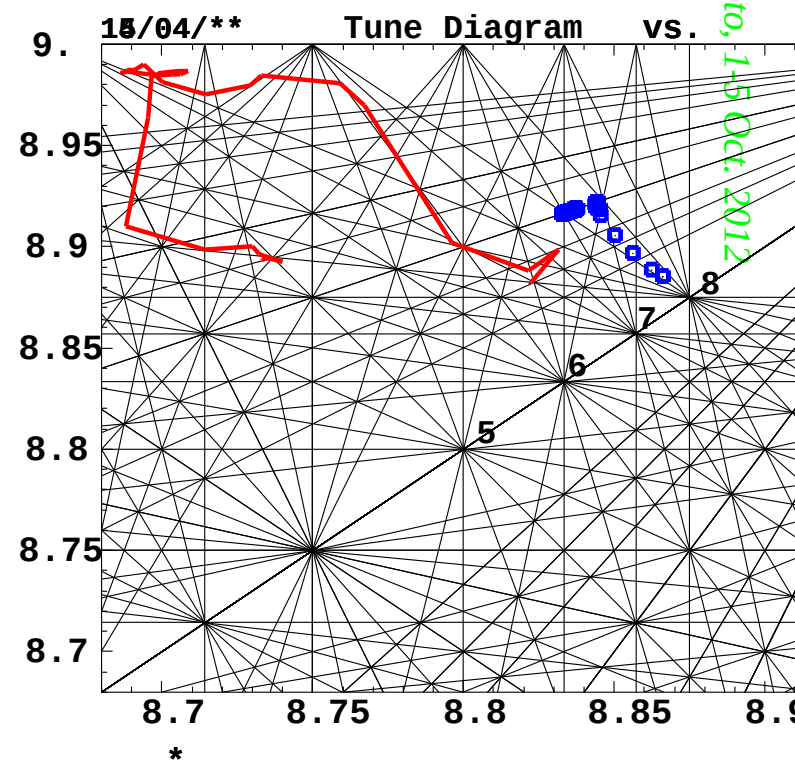
Typical outputs from Zgoubi

- **Orbit, bare AGS with survey data.**



- **Coupled envelopes (a quarter of the ring), AGS with snakes, at 162 ms : injection, $G\gamma = 4.5$. The coupling source is the cold snake essentially.**





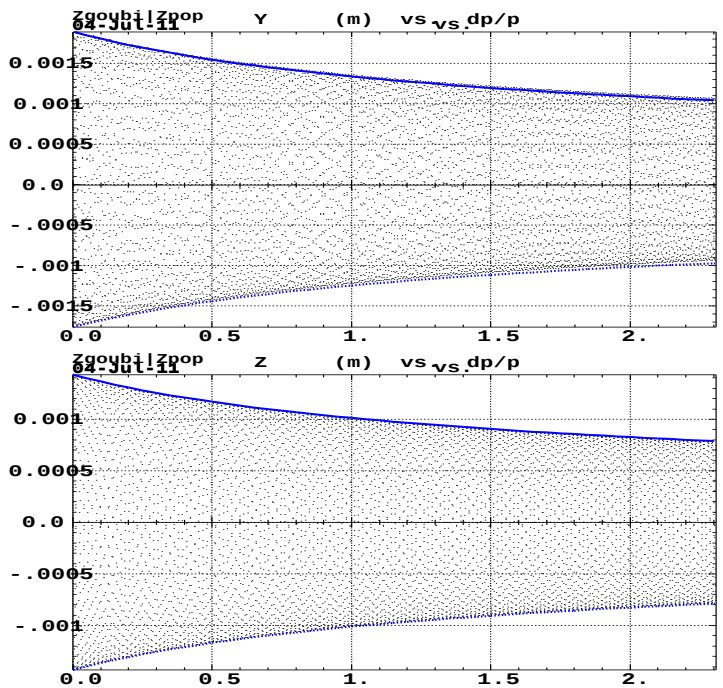
"Symplecticity" ?

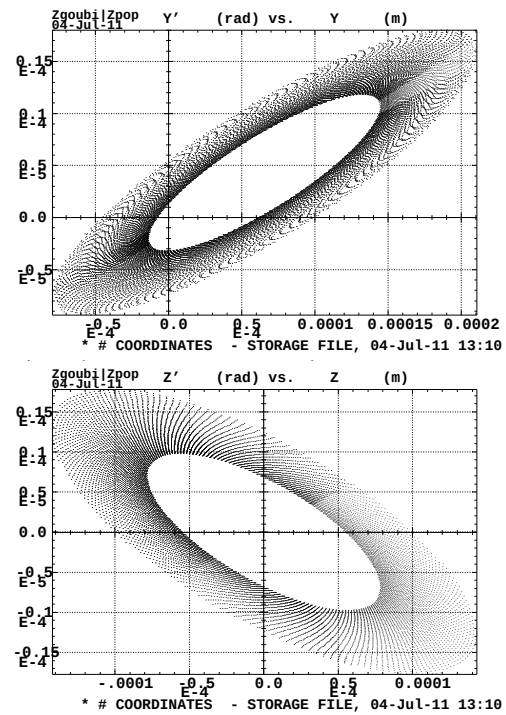
- **Zgoubi does not use any simplectization method, accuracy of numerical integration is controled essentially via (i) step size, (ii) completness of coefficients in $\vec{R}$, $\vec{u}$, $\vec{S}$ Taylor series (*"ORDRE"* command).**

- **150000 turns acceleration is performed here. Ion optics (no snakes), no coupling, includes tuning quads, chromaticity sextupoles. Basic AGS parameters :**

circumference	(m)	807.0909
transition γ		8.472
tunes x, y		8.721, 8.785
chromaticities x, y		-14.8, -2.16
ΔE per turn	(kV)	150

- **CPU time is 30 minutes about (regardless of the number of particles, on NERSC computers).**





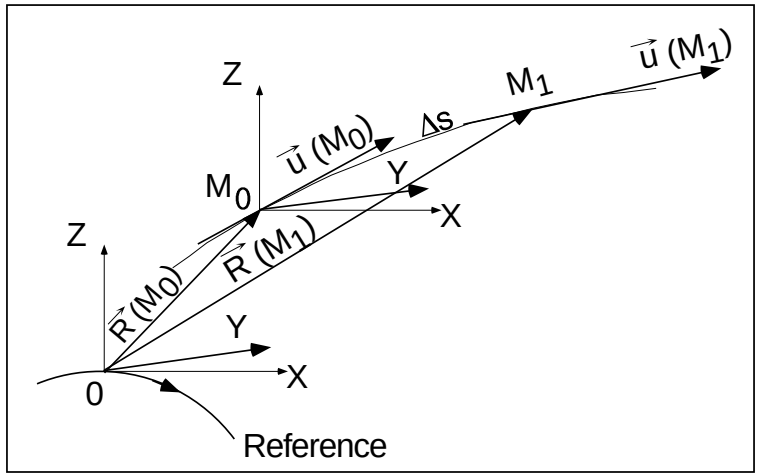
$B\rho/B\rho_0$ =**3.308**

ϵ_{x0}/ϵ_x = **3.325**

ϵ_{y0}/ϵ_y = **3.328**

Damped H (top) and V (bottom) motion over 9→32 GeV. The blue curves on the envelopes (left) are theoretical values, $\hat{x}(0) \times \sqrt{B\rho_0/B\rho}$.

2 Spin tracking



- **Spin is pushed, from M_0 to M_1,**

  by solving

$$\boxed{(B\rho)\,\vec{S}' = \vec{S} \times \vec{\omega}}$$

  where

$$\vec{\omega} = (1+\gamma G)\vec{b} + G(1-\gamma)\vec{b}_{//} + \frac{\beta\gamma}{c}(G + \frac{1}{1+\gamma})\vec{e} \times \vec{u}$$

$$\vec{S}' = \frac{d\vec{S}}{ds} = \frac{1}{v}\frac{d\vec{S}}{dt},$$

- **using truncated Taylor expansion at M_0**

$$\vec{S}(M_1) \approx \vec{S}(M_0) + \frac{d\vec{S}}{ds}(M_0)\,\Delta s + \frac{d^2\vec{S}}{ds^2}\,(M_0)\frac{\Delta s^2}{2} + \ldots + \frac{d^5\vec{S}}{ds^5}\,(M_0)\frac{\Delta s^5}{5!} \qquad (10)$$

• **The differential equation is the same as for particle motion, thus it is treated the same way.**

Noting, first, that the precession vector can be decomposed in two components, proper to respectively pure magnetic and pure electric field :

$$\vec{\omega} = \vec{\omega}_b + \vec{\omega}_e$$

it results that one can superimpose the two contributions

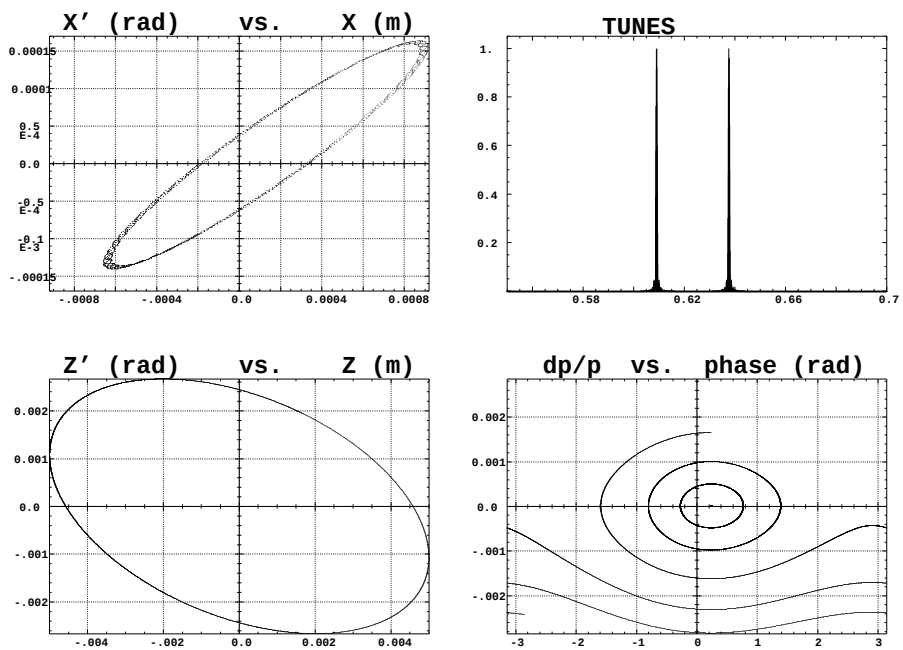
$$\begin{aligned}
B\rho\,\vec{S}' &= \\
&\vec{S} \times \vec{\omega}_b + \vec{S} \times \vec{\omega}_e \\
B\rho\vec{S}'' &= \\
&\vec{S}' \times \vec{\omega}_b + \vec{S} \times \vec{\omega}_b' + \vec{S}' \times \vec{\omega}_e + \vec{S} \times \vec{\omega}_e' - (B\rho)'\vec{S}' \\
B\rho\vec{S}''' &= \\
&\underbrace{\vec{S}'' \times \vec{\omega}_b + 2\vec{S}' \times \vec{\omega}_b' + \vec{S} \times \vec{\omega}_b''}_{\textbf{Magnetic field component}} + \underbrace{\vec{S}'' \times \vec{\omega}_e + 2\vec{S}' \times \vec{\omega}_e' + \vec{S} \times \vec{\omega}_e'' - \Big((B\rho)''\vec{S}' + 2(B\rho)'\vec{S}''\Big)}_{\textbf{Electric field component}}
\end{aligned} \tag{11}$$

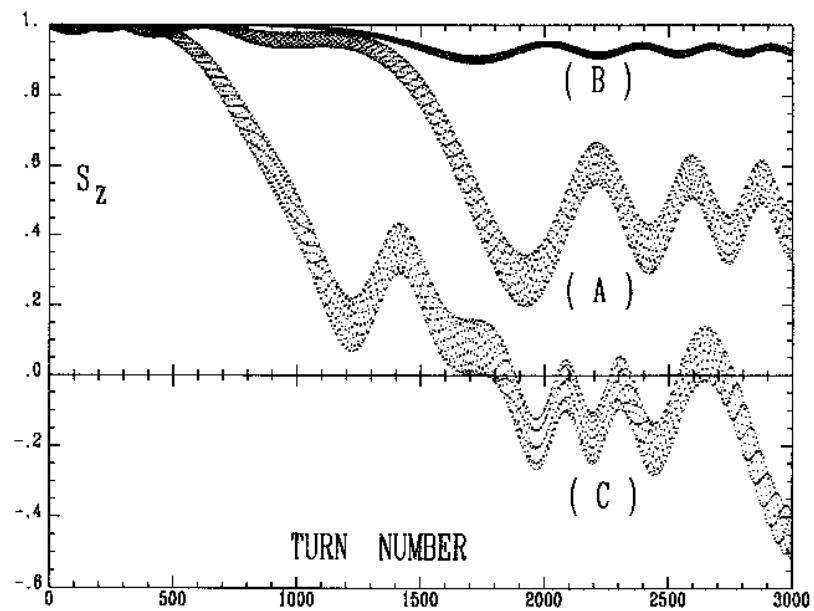
etc.

The process is then completed by applying the Δs push, following the earlier Taylor series development.

2.1 First spin precessions : SATURNE ring, early 1990s

• **These were as well the first experiments of multiturn tracking using stepwise ray-tracing, as a prerequisite for spin resonance crossing simulations. That accompanied the project of a partial (solenoidal) snake in SATURNE.**





Crossing of $\gamma G = 7 - \nu_Z$, at $\dot{B} = 2.1$ T/s.

(A) $\varepsilon_Z/\pi = 12.2\ 10^{-6}$ m.rad. The strength of the resonance is $\mid \varepsilon \mid = 3.3\ 10^{-4}$. As expected from the Froissart-Stora formula the asymptotic polarization is about 0.44.

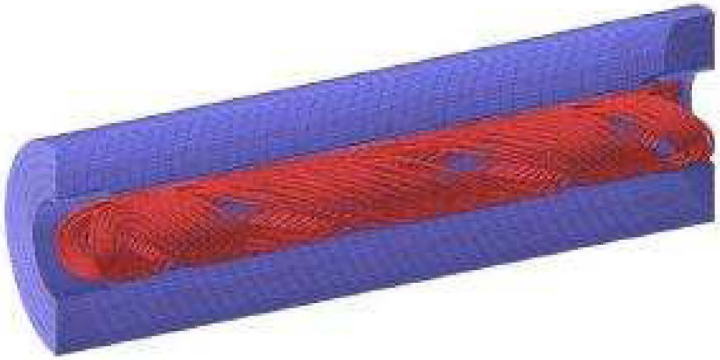
(B) $\varepsilon_Z/\pi = 1.2\ 10^{-6}$ m.rad; comparison with (A) shows that $\mid \varepsilon \mid$ is proportional to $\sqrt{\varepsilon_Z}$.

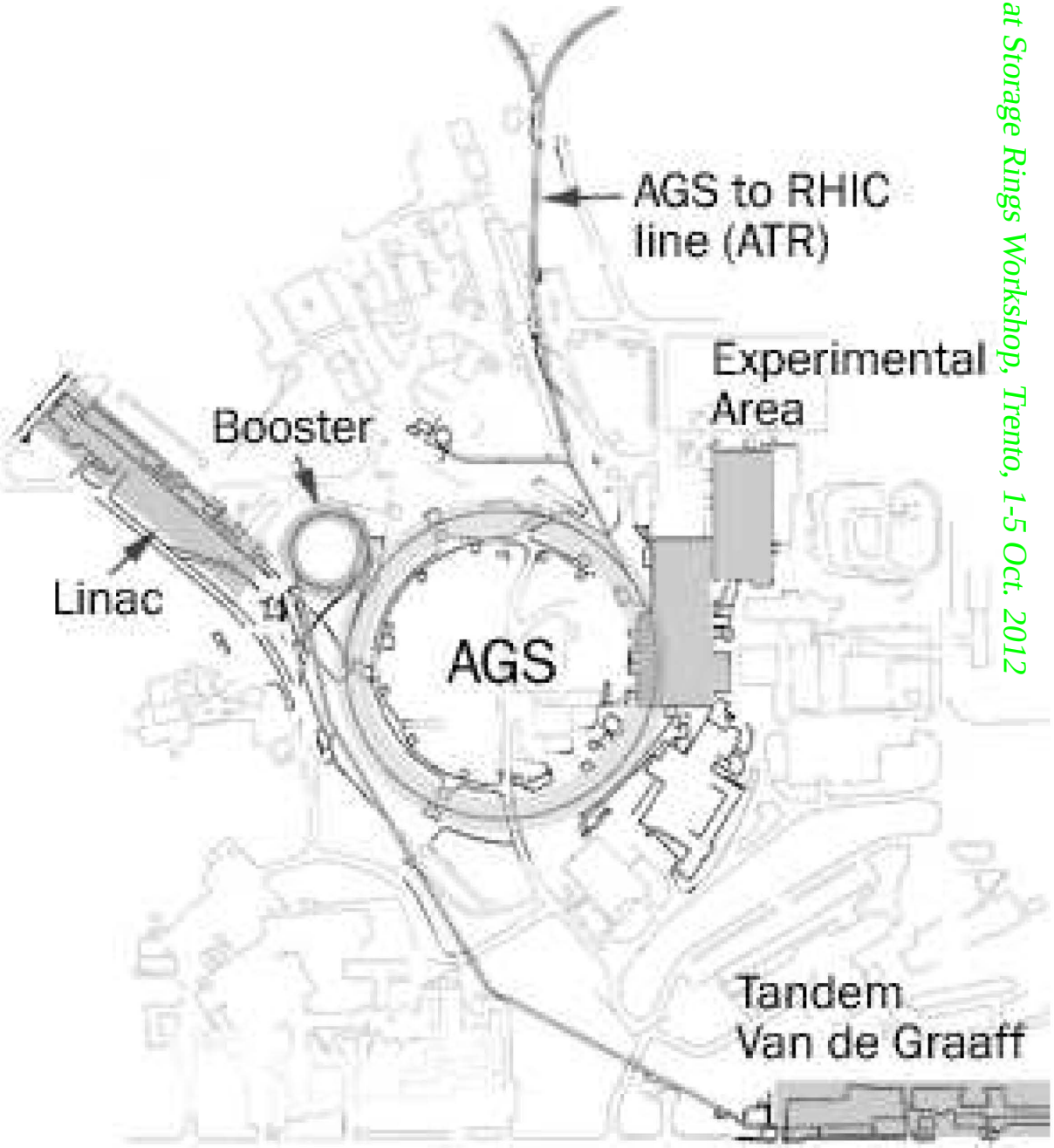
(C) Multiple crossing of $\gamma G = 7 - \nu_Z$ for a particle with with momentum dispersion 10^{-3}.

2.2 Polarization studies in AGS

• **With those concerning RHIC, these are the most recent spin dynamics exercises (2009 et seq.).**

Layout :



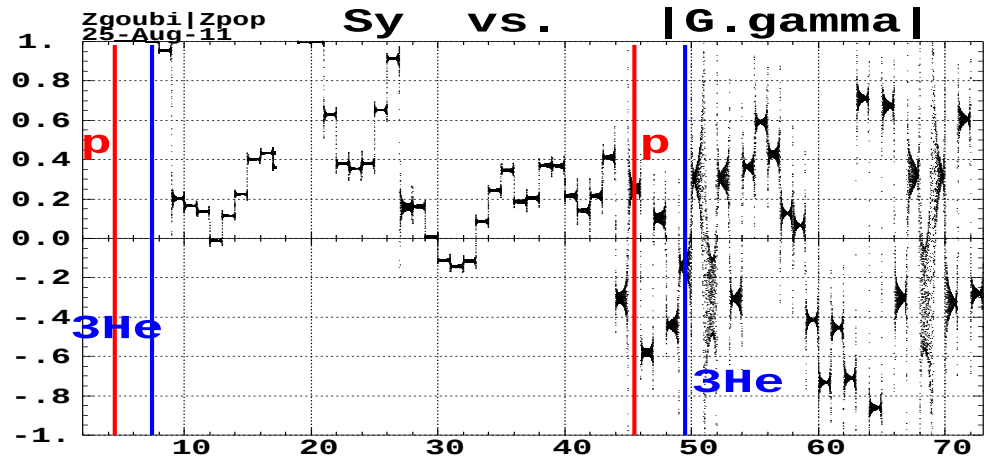


Imperfection resonances in AGS, $G\gamma - n = 0$ (Qx / Qy = 8.71 / 8.77 optics) :

- Proton, acceleration range : $G\gamma\ : 4.5 \rightarrow 45.5$, 41 integer resonances crossed.

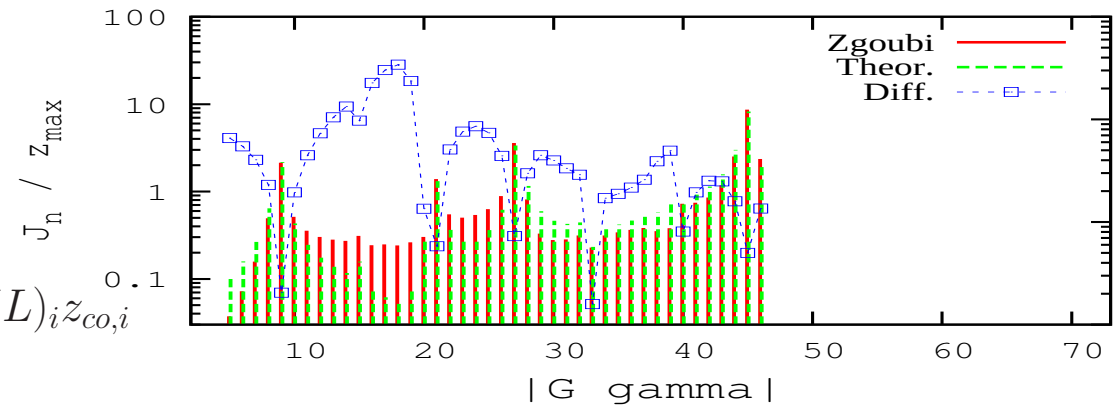
Strengths are extracted using Froissart-Stora :

$$J_n = \sqrt{\frac{-2\alpha}{\pi} \ln\left(\frac{1+P_f/P_i}{2}\right)}$$



and compared with thin-lens model :

$$J_n = \frac{1+G\gamma}{2\pi} \times \Sigma_{Qpoles} \left(\cos(G\gamma\alpha_i) + i\ \sin(G\gamma\alpha_i)\right) (KL)_i z_{co,i}$$



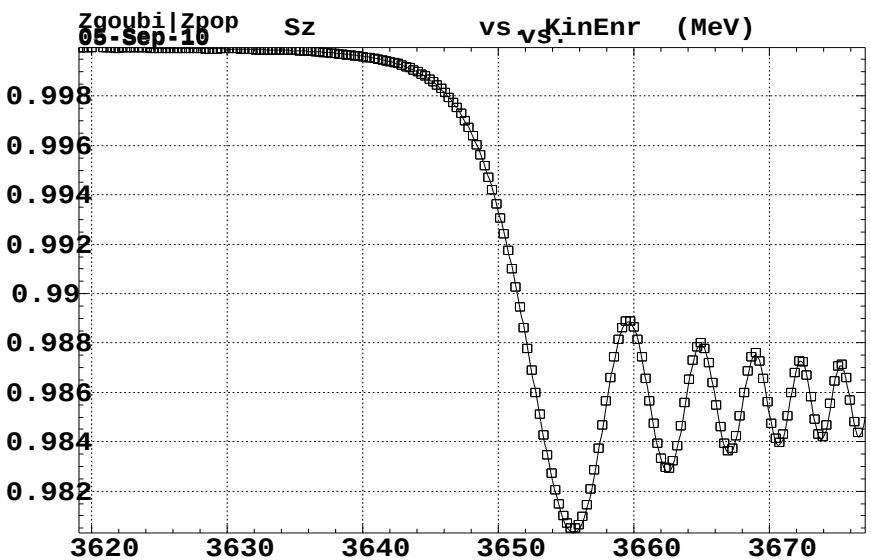
- **Top : S_y motion, $\hat{y}_{co} \approx 1.53$ mm closed orbit, zero H and V invariants.**
  Pf/Pi value is correlated to resonance strength by Froissart-Stora formula.
  Red, blue lines show injection, extraction $G\gamma$ values for p and $^3He^{2+}$.
- **Bottom : Strengths from tracking & thin-lens, superposed, and (right axis) their relative difference.**

• Fresnel integrals approximation of weak resonances

Matching the tracking by the Fresnel integral approximation yields the crossing speed α and the resonance strength $|J_n|$.

• $\gamma G = \nu_z$ (3.648013 GeV) - $\epsilon_z/\pi = 0.002\,10^{-6}$

$\pi|J_n|^2/\alpha \approx 0.014$, higher order terms have negligible effect.



Superimposition of Fresnel integral model (squares) and zgoubi tracking (solid line), $\epsilon_z/\pi = 0.002\,10^{-6}$.

•$\theta < 0$:

$$\left(\frac{p(\theta)}{p_i}\right)^2 = 1 - \frac{\pi}{\alpha}|J_n|^2 \left[(0.5 - C(-\theta\sqrt{\frac{a}{\pi}}))^2 + (0.5 - S(-\theta\sqrt{\frac{a}{\pi}}))^2\right]$$

• $\theta > 0$:

$$\left(\frac{p(\theta)}{p_i}\right)^2 = 1 - \frac{\pi}{\alpha}|J_n|^2 \left[(0.5 + C(\theta\sqrt{\frac{a}{\pi}}))^2 + (0.5 + S(\theta\sqrt{\frac{a}{\pi}}))^2\right]$$

$$C(x) = \int_0^x \cos(\frac{\pi}{2}t^2)\,dt,\ S(x) = \int_0^x \sin(\frac{\pi}{2}t^2)\,dt$$

• Overcoming imperfection resonances with snakes

The snake strength $\phi/2\pi$ dominates over strength of imperfection resonances :

$$\frac{\phi}{2\pi} \gtrsim |J_n| + 2\sqrt{\alpha}$$

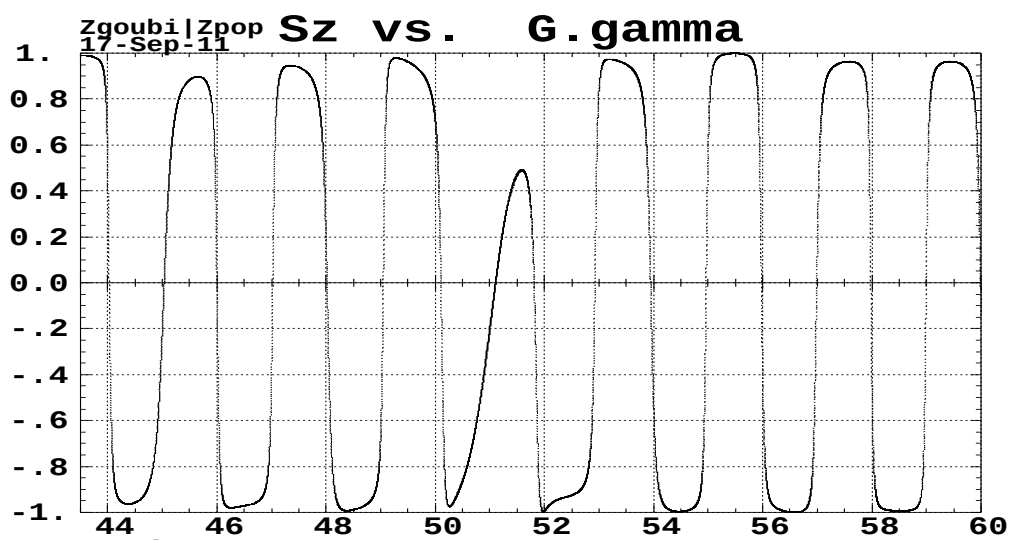
	Warm snake	Cold snake	Comments
B max (T)		3	
B nominal (T)	1.5	2.1	*Case of tracking below*
snake angle (deg.)	11	21	

• Example :

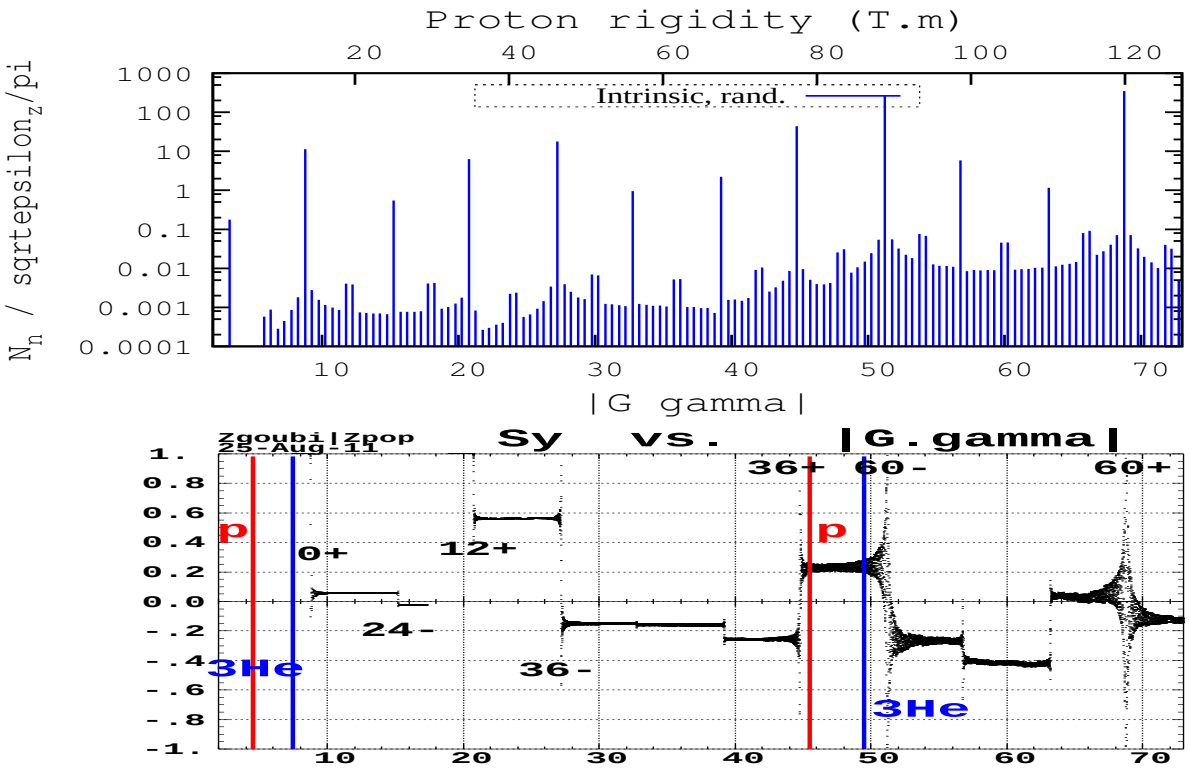
Flipping across imperfection resonances, $G\gamma \ : \ 43.5 \rightarrow 60$:

- Single particle on vertical orbit $\hat{y}_{co} \approx$ 8 mm

- H orbit < 1 mm, H and V invariants ≈zero, $\delta p/p|_{synch.} < 10^{-4}$.

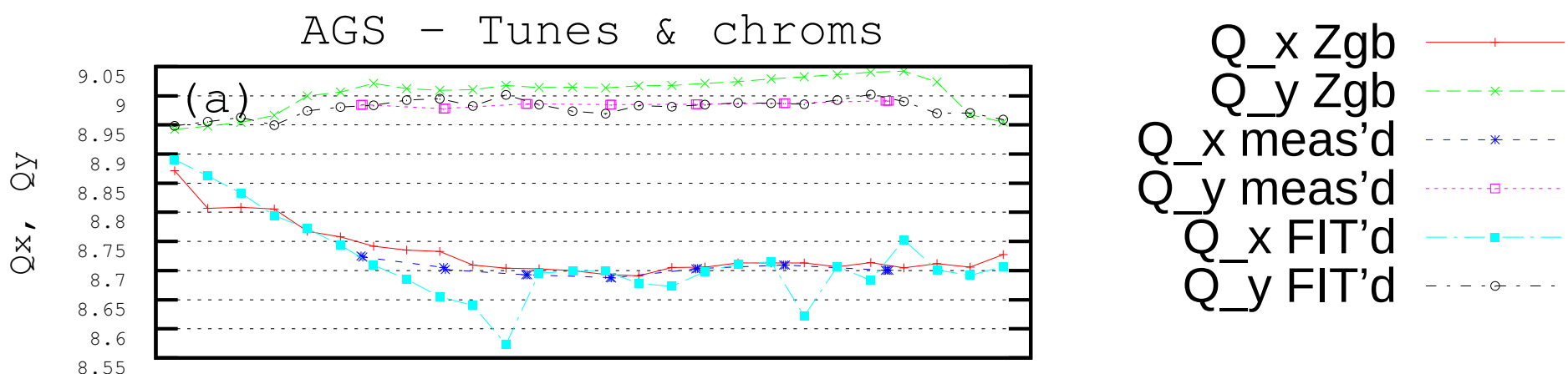


Resonances in AGS, intrinsic, $G\gamma \pm Q_y - n = 0$:



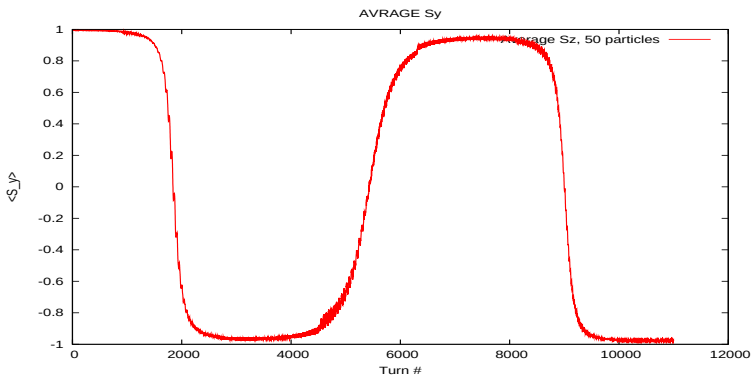
- Proton, acceleration range :
$G\gamma$: $4.5 \rightarrow 45.5$,
4 strong resonances crossed.

Intrinsic resonances are overcome by working with Q_y very close to 9, in the spin tune forbiden gap caused by the snake around $G\gamma$ =integer. In that manner, the spin tune Q_s can never satisfy $Q_s \pm Q_y - n = 0$.

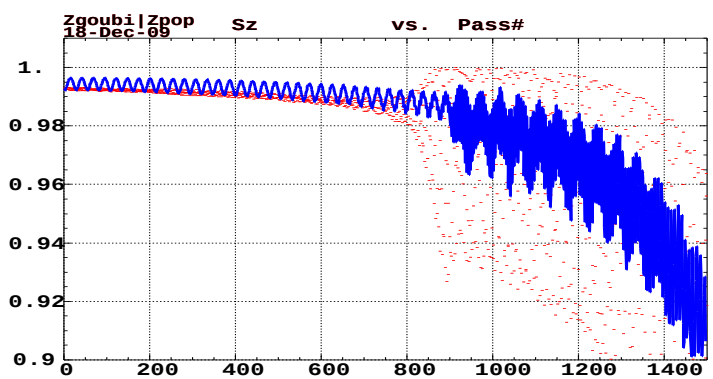


- **Intrinsic horizontal resonances** $G\gamma \pm Q_x - n = 0$

They are crossed using tune-jump (using "jump quadrupoles") at Q_s = integer $\pm Q_x$, so increasing the resonance crossing speed thus mitigating the effect (by virtue of Froissard-Stora).



Vertical projection of $\vec{S}$. Average over a few 10s of particles, turn by turn.



A zoom on the $\nu_s = 35 + \nu_x$ region, single particle, with (blue dots) and without (red dots) Q-jump.

Concluding comments

- **These simulations, including 10000s turns tracking across resonances, are performed using the OPERA field maps of the two snakes.**

- **In many cases the measured field maps of the AGS main magnets are used as well (rather than efficient semi-analytical models that we also have). They naturally include fringe fields. Symplecticity behaves well, over 1000s of turns, thanks to the smoothing interpolation methods used.**

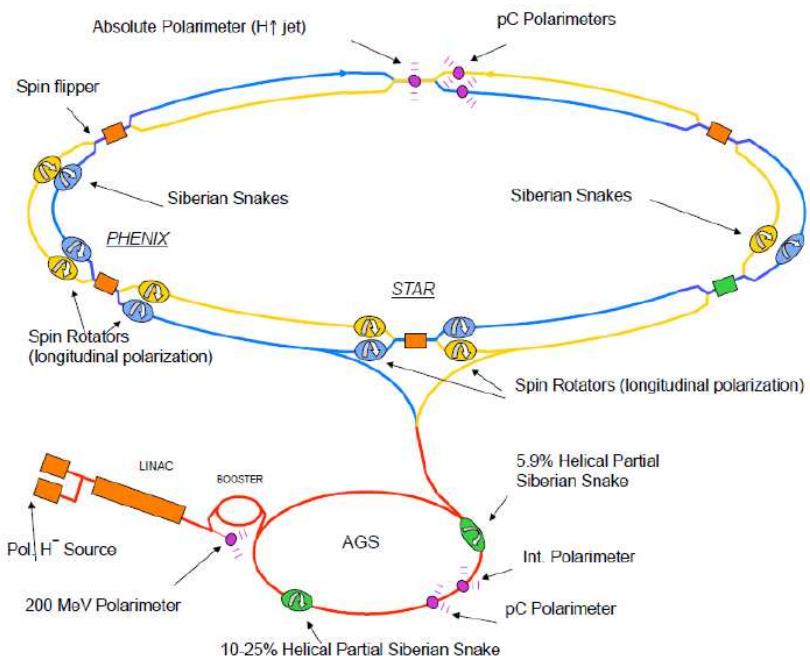
We plan to use OPERA field maps as well, that should improve symplecticity.

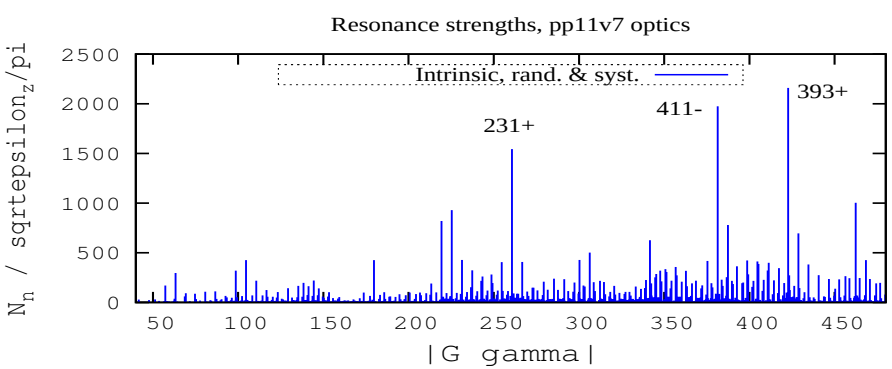
- **A full AGS cycle, 150000 turns, takes about 30 minutes on a 2.5GHz CPU.**

We routinely send 1000s of particles in 6-D phase-space on NERSC CPU farms in polarization transmission studies - still $\simeq$30 minutes !

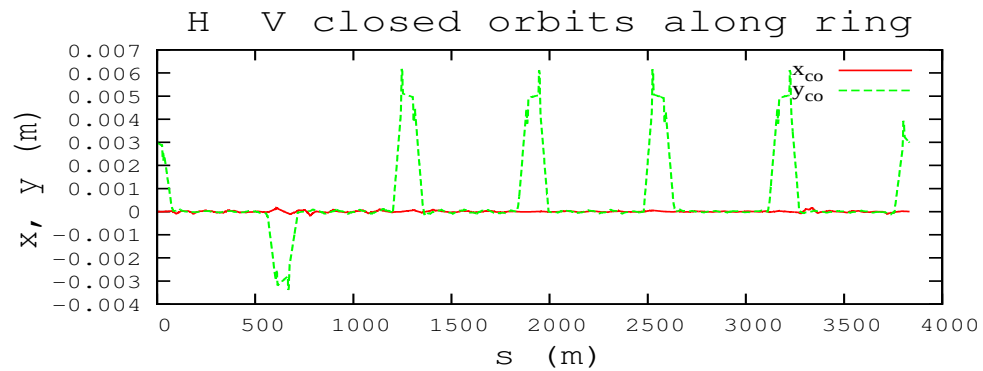
2.3 Polarization studies in RHIC

An example: simulations regarding polarization transmission with the so-called "pp11v7" optics.

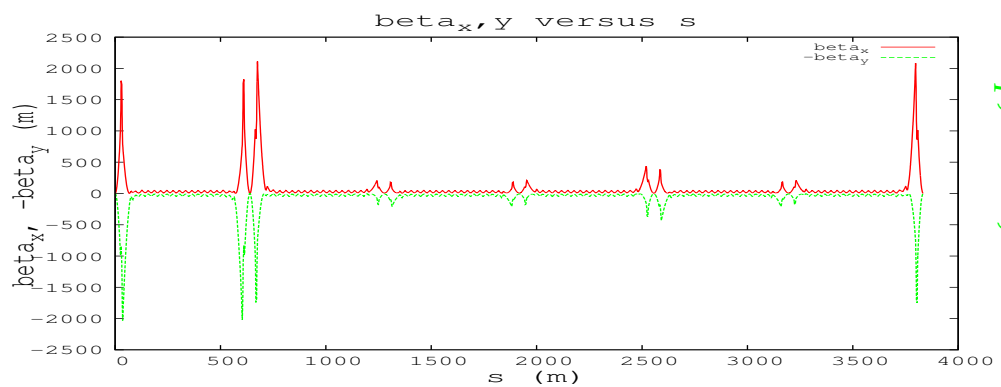




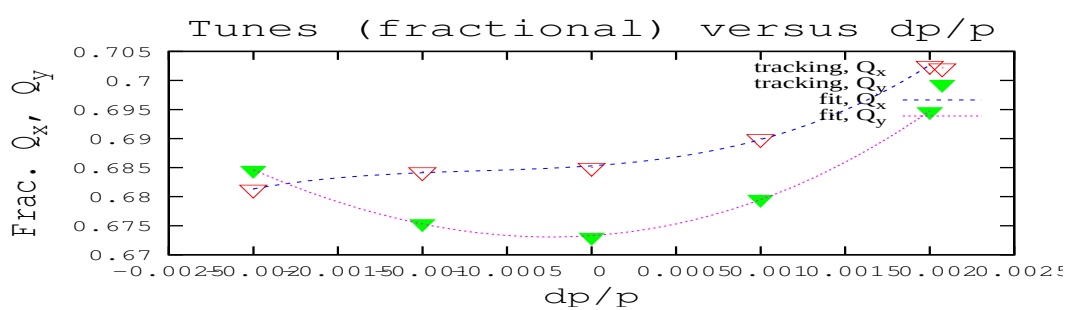
Strengths normalized to $\sqrt{\epsilon_y/\pi}$, intrinsic resonances, pp11v7 optics.



Orbits, H and V, vertical separation at IPs during the ramp.



Optical functions, H and V.



Momentum dependence of tunes.

Horizontal : $Q_x = 0.68530$, $Q'_x = 2.0370$

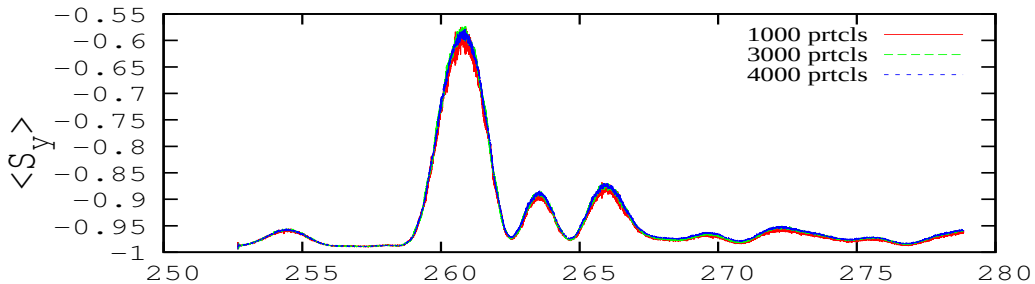
Vertical : $Q_y = 0.67329$, $Q'_y = 1.9499$

- **The ramp is covered with $\dot{\gamma} = 2.28$, twice the actual physics run conditions. This saves a factor 2 on CPU : wall clock time 35 hours about for 500000 turns, instead of 70 hours, 10^6 turns.**

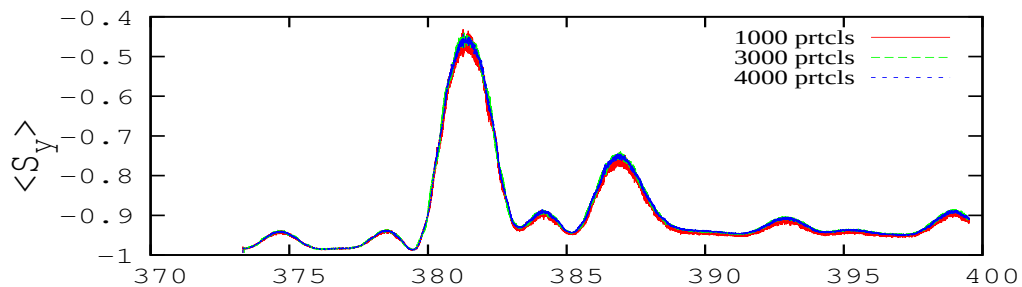
- **4032 particles are sorted at random in 2-D Gaussian densities (x,x'), (y,y') and initial $\delta p/p$, independent.**

Turn-by-turn average of S_y. Observation location is first snake :

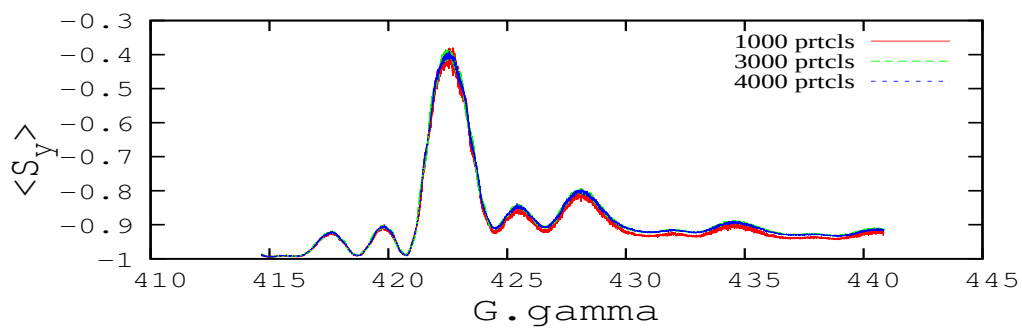
231+Qy :



411-Qy :



393+Qy :



	Orbits set		No orbit	
	Start	End	Start	End
(1) 231+Qy :				
$G\gamma$	257.2	276.8	257	276.8
Polarization	-0.98894	-0.98344	-0.98910	-0.98897
Polar. ratio	1	0.99444	1	0.99987
(2) 411-Qy :				
$G\gamma$	376.5	397	376.4	397
Polarization	-0.98646	-0.94980	-0.98713	-0.97777
Polar. ratio	1	0.96284	1	0.99052
(3) 393+Qy :				
$G\gamma$	415.1	438.8	415	438.9
Polarization	-0.99390	-0.93246	-0.99439	-0.96245
Polar. ratio	1	0.93818	1	0.96788
Product 1*2*3		0.89829		0.95858

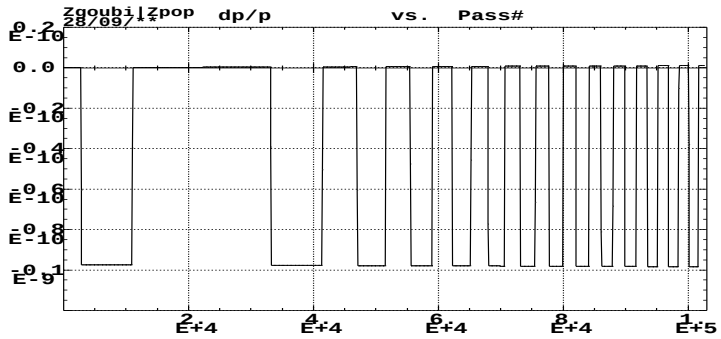
3 p-EDM ring, prospects

- **Tracking of spin motion in electric field has been installed - about a month ago**
- **A cylindrical deflector, hard edge, $E \propto (r_0/r)^n$, has been installed - about a week ago - to assess conservation of energy and transverse symplecticity of Zgoubi integrator, to start with.**

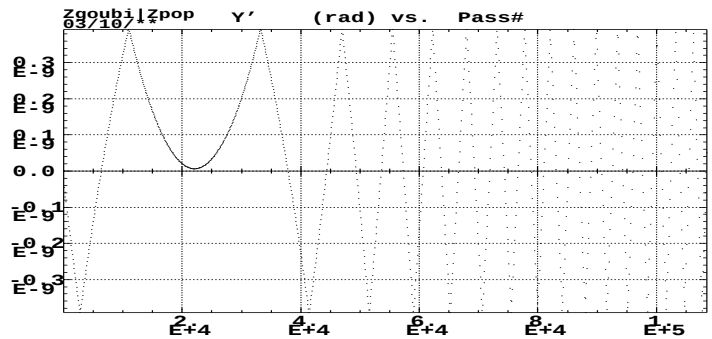
Typical data :

```
 'OBJET'
2.33741810885e3
2
1   1
4000. 0. 0. 0. 0. 1. 'O'
1 1 1
 'PARTICUL'
938.2720 1.6021764E-19  1.792847 0. 0.
 'SPNTRK'
4
1. 0. 0.
'FAISTORE'
 b_zgoubi.fai
 100
'ORDRE'
4
 'ELCYLDEF'       ==   deviation 22.5deg,
0                      r=40m, n=1 (E~1/r)
0.392699081699 40. -10482708.58082  1.0
0. 0. 5. 2.
4  .1455   2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 5. 2.
4  .1455   2.2670 -.6395 1.1558 0. 0. 0.
.001
2
0. 0. 0. 0.
 'SPNPRT'
'REBELOTE'
 999999999 0.3 99
 'END'
```

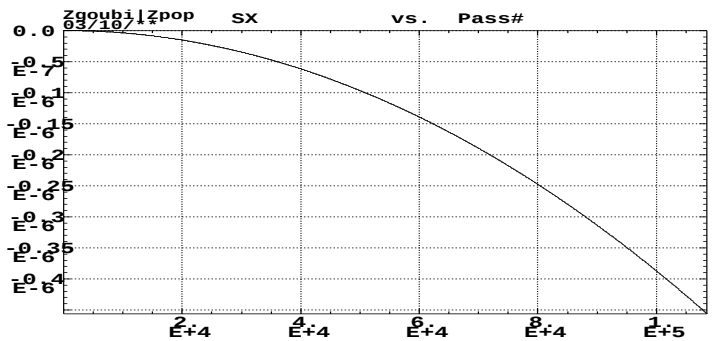
Typical result, for the moment : About 7000 turns, 7 ms,



$$dp/p < 2\,10^{-11}$$



$$x' \in \pm 4\,10^{-10}$$



$$|1 - S_l| < 4.5\,10^{-6}$$

- **Plans for the near future are to benchmark all that.**

- **Next, fringe fields will be included :**

Plans (*a priori*) :
- a straigth fall-off $G(s)$ at both ends of bend as discussed by Wollnik (Ref. : "Optics of Charged Particles"), and as practiced in RAYTRACE (S. KOWALSKI, 1988),
- either using H.A.Enge's $G(s) = \frac{G_0}{1+exp(P(d))}$ or (simpler) second order shape.

THANK YOU FOR YOUR ATTENTION