BNL-99131-2013-CP

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Presented at the 20th International Symposium on Spin Physics (SPIN2012)
Dubna, Russia
September 17-22, 2012

Collider-Accelerator Department
Brookhaven National Laboratory

U.S. Department of Energy
DOE Office of Science

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Calculation of spin resonance harmonics in an accelerator with Snakes [1]

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Abstract

In this work, the ASPIRRIN code has been extended to provide the calculation of resonance harmonics for the case of machine with Siberian Snakes and spin rotators. Examples are shown for RHIC accelerator, including the case of the configuration with six Snakes per RHIC ring.

1 Algorithm Description

With the application of the various kind of spin rotating devices (like Snakes and spin rotators) the stable spin axis on the design closed orbit may deviate from the vertical, and an algorithm should be realized which calculates the spin resonance harmonics with this complex configuration of the magnetic field on the design orbit. We implemented the algorithm of the intrinsic resonance harmonic calculation in a program code which was added to the existing spin dynamics calculation code ASPIRRIN [1].

We consider the spin motion in a special periodical coordinate system in which the spin rotation on the design orbit happens with the constant spin tune ν. This coordinate system is defined by the right-handed orthonormal triad $(\mathbf{l}, \mathbf{m}, \mathbf{n_0})$, where $\mathbf{n_0}$ is the stable spin direction on the design orbit, and the vectors $\mathbf{l}, \mathbf{m}$ can be derived using the eigen-solutions of the one turn spin map on the design orbit. We also will use the complex vector $\mathbf{k} = \mathbf{l} + i\mathbf{m}$. In fact, as the spin tune on the design orbit can be defined up to the integer number, the choice of special periodic system is also unlimited. In the accelerator with the Snakes, we will use the periodical coordinate system in which the spin tune is 1/2 (or other constant value between 0 and 1). To describe the invariant spin field related with the orbital oscillations, we use the complex variable α defined as:

$$\mathbf{n} = \sqrt{1 - |\alpha|^2}\mathbf{n_0} + \Re(\alpha \mathbf{k}^*) \tag{1}$$

The spin motion equation can be written as:

$$\frac{d\alpha}{d\theta} = i\nu\alpha - i\mathbf{w} \cdot \mathbf{k}\sqrt{1 - |\alpha|^2} \tag{2}$$

[1] Work supported by Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with the U.S. Dept. of Energy

where $\mathbf{w}$ describes the spin perturbation by the orbital oscillations. We omit the term $\mathbf{w} \cdot \mathbf{n_0}$ which is not of interest in this paper. We consider the spin perturbation as the linear form in the orbit variables: $\mathbf{w}_j = T_{jq} x_q$. And the linear orbital motion can be presented as: $x_q = \mathcal{F}_{qr} A_r$, where $\mathcal{F}_{qr}$ is the matrix compiled from the eigen-vectors of the orbital motion, and A_r is the vector of the orbital amplitude. Let's note that currently we have implemented the algorithm for the transverse betatron motion, therefore q and r indexes run from 1 to 4. But, it should be no problem to extend it to 6D motion, if needed.

The first-order spin resonance harmonics are defined as the coefficients of the Fourier decomposition of the spin perturbation term:

$$\mathbf{w} \cdot \mathbf{k} = \sum_{r,p} \epsilon_{rp} e^{i\nu_{rp}\theta} \tag{3}$$

where $\nu_{rp} = p + Q_r$. The Q_r are the components of the betatron tune set: $(Q_x, -Q_x, Q_y, -Q_y)$. In the case of strong betatron coupling, one should use the tunes of betatron modes instead. Since we only consider linear fields and linear orbital motion, the resonance harmonics are called first-order in both the spin motion and the orbital motion sense.

Keeping only the terms corresponding to the betatron motion:

$$\mathbf{w} \cdot \mathbf{k} = \sum_{j,q,r} T_{jq} \mathcal{F}_{qr} A_r k_j = \sum_r A_r e^{iQ_r\theta} \tilde{V}_r \tag{4}$$

Here the index j runs from 1 to 3, while the indices q and r run from 1 to 4. And we introduced a four-component vector $\tilde{V}$, which is the periodical function of the azimuth θ and, therefore, can be expanded into the Fourier series:

$$\tilde{V}_r = \left[e^{-iQ_r\theta} \sum_{j,q} T_{jq} \mathcal{F}_{qj} k_j \right] = \sum_p \tilde{v}_{rp} e^{ip\theta} \tag{5}$$

Comparing the expressions (4) and (5) with (3) we see that $\tilde{v}_{rp}$ presents the spin resonance harmonic ϵ_{rp} normalized by the corresponding betatron amplitude A_r. To calculate $\tilde{v}_{rp}$ we need to take the integral over one turn:

$$\tilde{v}_{rp} = \frac{1}{2\pi} \int_0^{2\pi} \tilde{V}_r e^{-ip\theta} d\theta = \frac{1}{2\pi} \int_0^{2\pi} \left[e^{i(\nu - \nu_{rp})\theta} \sum_{j,q} T_{jq} \mathcal{F}_{qr} k_{0j} \right] d\theta \tag{6}$$

In the program the integral over one turn is taken doing the element-by-element integration.

2 Resonance harmonics in RHIC with two Snakes

Here we show some results of the calculation done for the RHIC collider with two Siberian Snakes. The calculations were done for the lattice which had $Q_y = 29.21$. The spin tune is independent on the energy and is equal to 1/2, and one is mostly interested in the resonance harmonics, related with the vertical

betatron motion, which are close to the spin tune value. Let's name those harmonics w_{-1} and w_1. In the periodical coordinate system, those harmonics rotate correspondingly with the rate 0.21 and 0.79. In terms of the definition of the formula (3): $w_{-1} = \epsilon_{3,-29}$; $w_1 = \epsilon_{4,30}$. Figure 1 shows the calculated values of the harmonic amplitudes versus the $G\gamma$ in the region of strong resonances in RHIC for the vertical betatron amplitude corresponding to 3.3 μm emittance. As clearly seen the harmonic amplitudes are very similar at most of the energies. It would be natural to use in this case the linear harmonics, defined in the coordinate system rotating together with the spin, that is at 1/2 rotation rate relative to the periodical system. The amplitudes of the linear harmonics are then :

$$|w_{l1m}| = |w_1| + |w_{-1}|; \qquad |w_{l1s}| = ||w_1| - |w_{-1}|| \tag{7}$$

Here w_{l1m} presents main linear harmonic, which amplitude is large. And w_{l1s} denotes secondary linear harmonic, which is small and can be considered as a perturbation. Figure 2 presents the linear harmonics amplitudes in RHIC.

In the similar way the program can calculate the intrinsic resonance harmonics, which are located further away from the spin tune (w_2,w_{-2},w_3,w_{-3}, and so on) and corresponding linear harmonics. It is also able to calculate the harmonics of the resonances related with the horizontal betatron motion, in the case when this motion has effect on the spin dynamics (the betatron coupling, spin rotators, ...).

3 Resonance harmonics in RHIC with six Snakes

In the accelerator with multiple Snakes there is a question about the optimal configuration of the Snake axes. Using the resonance harmonics approach, one can consider the choice of the Snake axis configuration, which minimizes the harmonics values for the intrinsic resonances. For instance, we can find the snake configurations that nullify the w_{l1} harmonics, which have the largest effect on the spin motion. Let's consider the system of six Snakes, placed equidistantly in RHIC, and assuming 6-fold periodicity of the orbital motion. And ϕ_i is the Snake axis angle of the ith Snake. We found that the following Snake configuration nullifies the w_{l1} harmonics:

$$\phi_{i+1} - \phi_i = \pi/2 \tag{8}$$

for all ith. In fact, as shown in Figure 3, such the Snake configuration cancels also w_{l3} and w_{l4} harmonics, so that the spin motion is governed mainly by the w_{l2} harmonics.

References

[1] *Ptitsyn V., Mane S.R. and Shatunov Yu.M.,* Nucl. Instr. and Meth. A, V.608 2009 P.225;

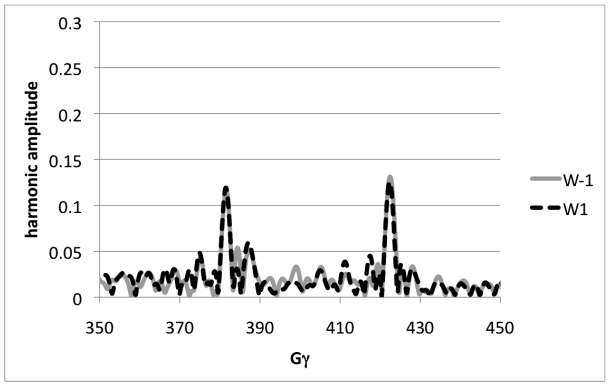

Figure 1: Resonance harmonics throughout the RHIC strong resonance region.

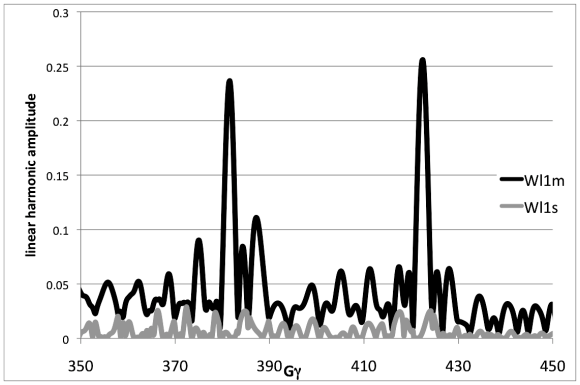

Figure 2: Linear resonance harmonics throughout the RHIC strong resonance region.

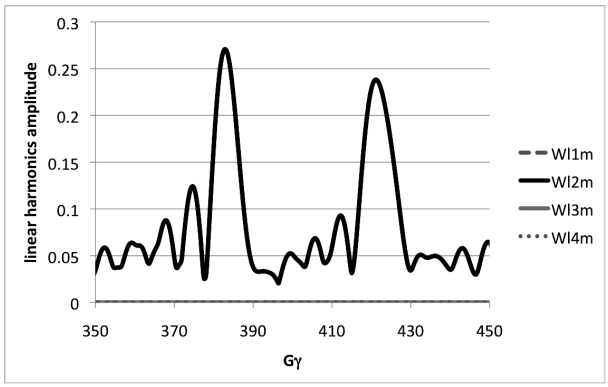

Figure 3: Linear resonance harmonics in RHIC with six Snake configuration: $(-45°; 45°; -45°; 45°; -45°; 45°)$