RHIC/RF-15

RHIC Project
BROOKHAVEN NATIONAL LABORATORY

RHIC/RF Technical Note No. 15

Remnant Field Levels in the RHIC Storage System During Acceleration

R. Connolly

October 1993

REMNANT FIELD LEVELS IN THE RHIC STORAGE SYSTEM DURING ACCELERATION

Roger Connolly

Brookhaven National Laboratory
October 1993

INTRODUCTION

Beam bunches passing through the storage rf cavities in RHIC will cause the cavities to resonate. The amplitude of the remnant oscillation depends strongly on the bunch width. Studies of beam stability during transition crossing [1,2] show that the total remnant voltage acting on each bunch should be less than 10kV. This paper gives the results of calculations to estimate the voltage levels in the RHIC storage system. The calculations show the threshold of 10kV can be met for the design beams if the full widths of the bunches are greater than about 6.5 ns.

BEAM BUNCH EXCITATION OF A SINGLE CAVITY

The cavity is modeled as a parallel RLC circuit driven by an ideal current source as shown in fig. 1. The response of this circuit to a beam pulse is the convolution of the beam pulse with the impulse response of the circuit. The impulse response is the solution of,

$$\frac{V(t)}{R} + \frac{1}{L}\int_{o}^{t} V(t)\;dt + C\,\frac{dV(t)}{dt} = \delta(t) \qquad (1)$$

The Laplace transform of eq. 1 is,

$$\frac{V(s)}{R} + \frac{1}{L}\frac{V(s)}{s} + C\,[sV(s) - V(o)] = 1 \tag{2}$$

Solving eq. 2 for V(s) gives,

$$V(s) = \frac{s/C\,[1+CV(0)]}{s^2+\dfrac{s}{RC}+\dfrac{1}{LC}} \tag{3}$$

The inverse transform is,

$$V(t) = \left(\frac{1}{C}+V(0)\right) e^{-t/2RC}\left[\cos\omega t - \frac{1}{2RC\omega}\sin\,\omega t\right]$$

$$\omega = \sqrt{\frac{1}{LC}-\frac{1}{4R^2C^2}} \tag{4}$$

Since $Q = \omega CR$ the coefficient of the sine term and the correction to ω are small making the impulse response,

$$V(t) = \left(\frac{1}{C}+V(0)\right) e^{-\frac{Qt}{2\omega}} \cos\,\omega\,t$$

$$\omega = \sqrt{\frac{1}{LC}} \tag{5}$$

<u>Parabolic Beam-current distribution</u>

To calculate the cavity response to a beam pulse, a parabolic beam-current distribution is used. The beam current within a bunch of full width D_b and centered at t=0 is,

$$I(t) = \frac{3q}{2D_b}\left[1-\frac{t^2}{(D_b/2)^2}\right], \qquad -\frac{D_b}{2} < t < \frac{D_b}{2}$$

$$= 0 \qquad\qquad |\,t\,| > \frac{D_b}{2} \tag{6}$$

where q is the total charge in the bunch.

Bead pull measurements, fig. 2, show the effective gap width to be about 30 cm. Therefore each beam particle can be represented as a constant current source of 1 - ns duration. The rms effective width of the beam bunch is the quadrature sum of the rms widths of the beam bunch and the current pulse from one particle,

$$\begin{aligned}\sigma_{eff} &= \left[\sigma^2_{bunch} + \sigma^2_{particle}\right]^{1/2} \\ &= \left[\frac{D_b^2}{20} + \frac{1}{3}\right]^{1/2} ns\end{aligned} \tag{7}$$

The effective full width of the beam pulse is,

$$D = \left(D_b^2 + \frac{20}{3}\right)^{1/2} ns \tag{8}$$

The contribution to the total cavity voltage by a single beam bunch is given by,

$$V_{bunch}\,(t) = \frac{3q}{2CD}\int_{t-D/2}^{t+D/2}\left[1 - \frac{(t-z)^2}{(D/2)^2}\right] e^{-\frac{Qz}{2\omega}} \cos\,\omega\,z\,d\,z \tag{9}$$

where $t = 0$ is when the peak of the effective beam bunch distribution is in the center of the gap. This is a messy integral but it can be greatly simplified by making two observations. First the exponential damping term can be removed to solve for the oscillation amplitude immediately after bunch passage. Second the integral that is left is zero for values of t for which $\omega t = \left(\frac{2n-1}{2}\right)\pi$, because at these times the integrand is an odd function.

The voltage antinodes are therefore at $t = \frac{n\pi}{\omega}$. The maximum induced voltage is,

$$
\begin{aligned}
V_{bunch} &= \frac{3q}{2CD} \int_{-D/2}^{D/2} \left[1 - \frac{z^2}{(D/2)^2}\right] \cos\, \omega\, z\, d\, z \\
&= \frac{q}{C} \left[\frac{3}{\omega D} \left\{\sin\left(\frac{\omega D}{2}\right) - \frac{4}{\omega D} \cos\left(\frac{\omega D}{2}\right) - \left(1 - \frac{8}{\omega^2 D^2}\right) \sin\left(\frac{\omega D}{2}\right)\right\}\right]
\end{aligned}
\tag{10}
$$

The value of the expression in square brackets is plotted in fig. 3 for the case of

$\omega = 2\pi \ x \ 196 \ x \ 10^6 \ s^{-1} = 1.23 \ x \ 10^9 \ s^{-1}$.

Gaussian Beam-current Distribution

If the beam bunch is modeled with a Gaussian distribution the voltage maximum is,

$$
\begin{aligned}
V_{bunch} &= \frac{q}{C} \int_{\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{t^2}{2\sigma^2}\right) \cos\, \omega t\, dt \\
&= \frac{q}{C} \exp\left[-\frac{\omega^2 \sigma^2}{4}\right]
\end{aligned}
\tag{11}
$$

A time width of $4\,\sigma$ contains 0.95 of the beam particles. To compare the result of using a Gaussian beam distribution to the results of the previous section, the value of the exponential is plotted as a function of the time interval $4\,\sigma$ in fig. 4. Because of the flight time across the gap, the effective $4\,\sigma$ width is,

$$
W = \left(W_b^2 + \frac{16}{3}\right)^{1/2} ns \tag{12}
$$

where W_b is the actual $4\ \sigma$ beam width in ns.

Voltage level of cavity

Bead pull measurements on the 200-MHz cavity give a measured value for $R/Q = \sqrt{L/C} = 213\ \Omega$. Using this together with the resonant frequency of

$$\omega = \frac{1}{\sqrt{LC}} = 1.26 \times 10^{9}\ s^{-1},$$ the capacitance of the cavity is calculated to be 3.73 pF.

The cavity will be tuned to 196 MHz by increasing the capacitance so the calculated final capacitance is 3.88 pF.

Page 14 of the Project Scope section of the RHIC Design Manual shows expected bunch charges between 1.1×10^{-8} C for oxygen to 1.6×10^{-8} C for protons. The remnant cavity voltage after bunch passage will be between 2.8 kV and 4.1 Kv times the value given on fig. 3 or 4. A true bunch width of 4 ns will give an effective width of about 4.7 ns. Therefore a single pass of a 4-ns bunch will excite a cavity to voltages of 1.0 to 1.4 kV.

TOTAL REMNANT VOLTAGE OF STORAGE SYSTEM

Ten cavities will be used for beam storage. Six will be common to both beams and two will be on each individual line. The counter-rotating bunches pass through the common cavities in opposite directions and separated in phase by 180°. In one pass of two bunches each of the six common cavities will be excited to twice the voltage calculated above. Therefore, each pass of two bunches will leave a total remnant voltage per ring of 14 times this voltage.

During acceleration damping loops will be in the cavities. With these loops inserted the measured Q is 420 giving an exponential damping time of 0.68 μs. With 57 bunches in each ring the time interval between successive bunches is 0.22 μs. The cavity field will decay by 28% during this time. Each bunch through a cavity will increase the field by the amount V_{bunch} calculated above. The field in a cavity on a single beam line will attain a maximum value of V_{max} after each bunch passage. It will decay to 0.72 V_{max} before the next bunch arrives. Thus 0.28 $V_{max} = V_{bunch}$. Each bunch will see a cavity voltage of $V = 0.72\ V_{max} + 0.5\ V_{bunch} = 3.1\ V_{bunch}$.

For each bunch to see a combined voltage of less than 10 kV, each cavity must be excited by each bunch to field levels of less than 230V. As shown on Figs. 3 and 4, this condition will be met for effective bunch lengths greater than about 7 ns. Because the particle flight time across the cavity gap increases the effective bunch width slightly, eqs. 8 and 12, the threshold of 7 ns corresponds to a true bunch width of about 6.5 ns.

REFERENCES

1. J. Wei, "Transition Crossing in the RHIC", AD/RHIC-84, (Brookhaven National Laboratory, 1991).

2. D. P. Deng, "Longitudinal Emittance Growth in the Presence of Transient Beam Loading", RHIC/RF-2, (Brookhaven National Laboratory, 1992).

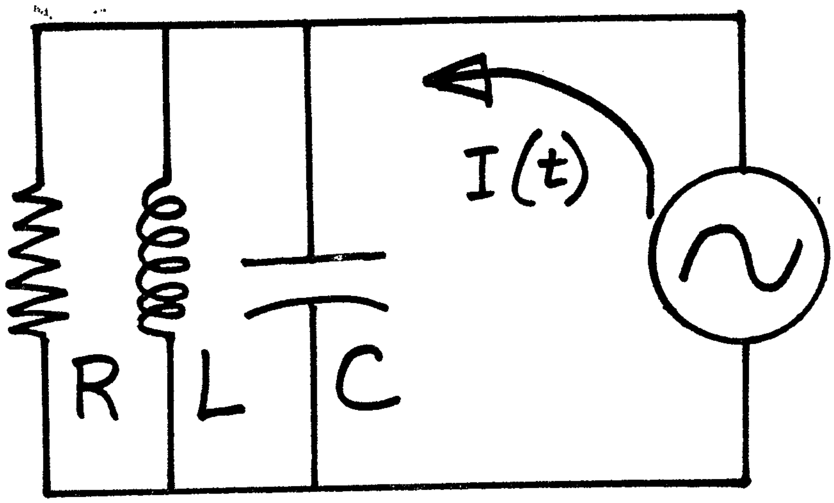

Fig. 1. The cavity is modeled as a parallel RLC circuit driven by the beam current, I(t).

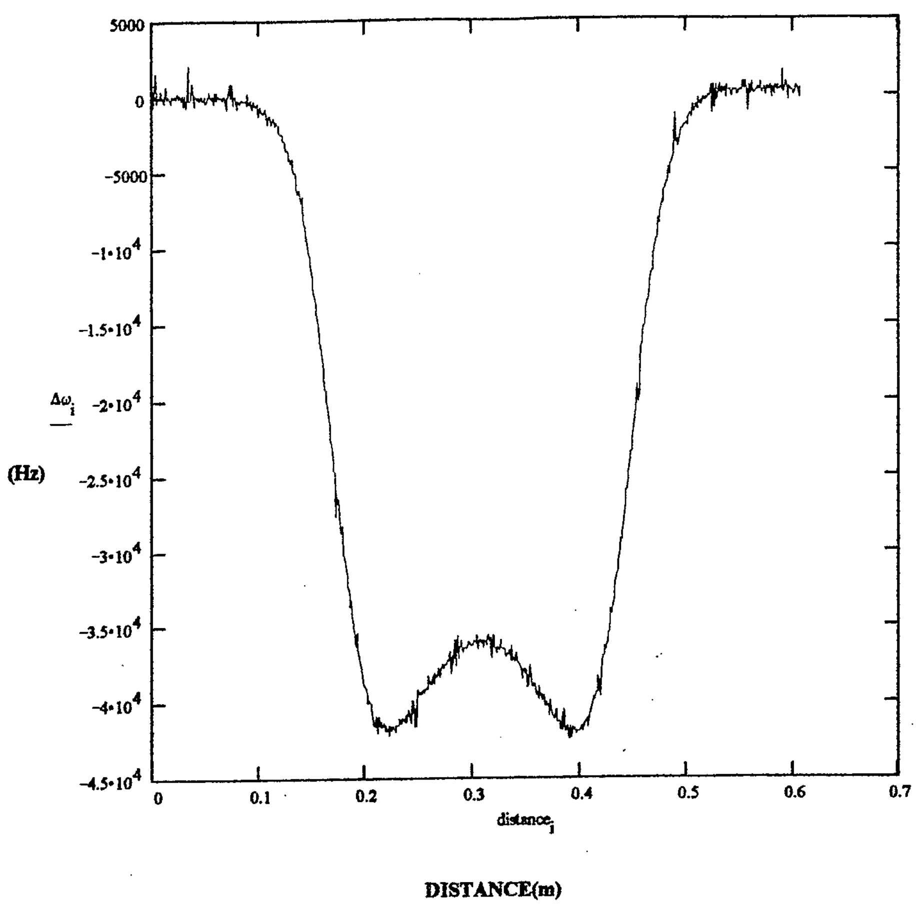

Fig. 2. Plot of resonant frequency change of a storage cavity as a function of bead position along cavity axis. The measurement was made by Christine Quiery and Steve Ellerd.

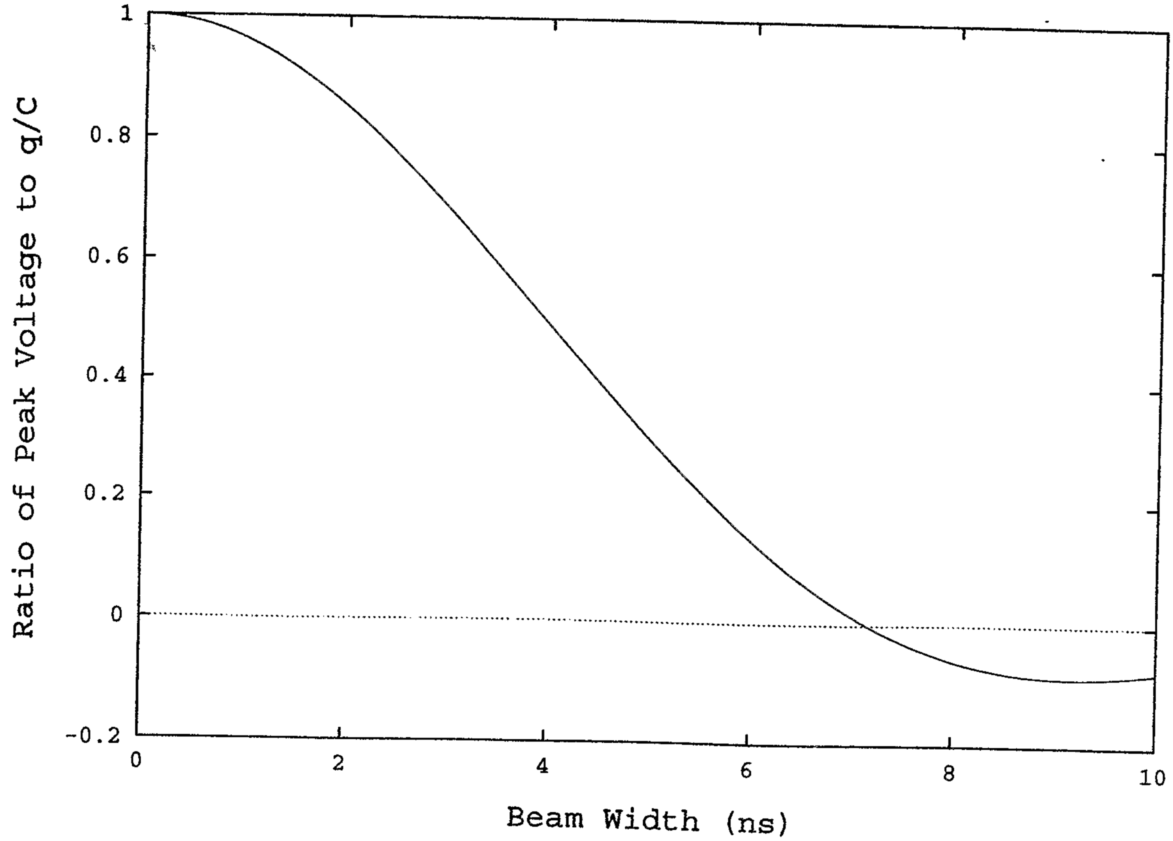

Fig. 3. Calculated single bunch excitation voltage of a storage cavity by a parabolic beam distribution plotted as a function of the full width of the bunch. The vertical axis is the ratio of the peak voltage divided by (q/C) where q is the bunch charge and C is the cavity capacitance.

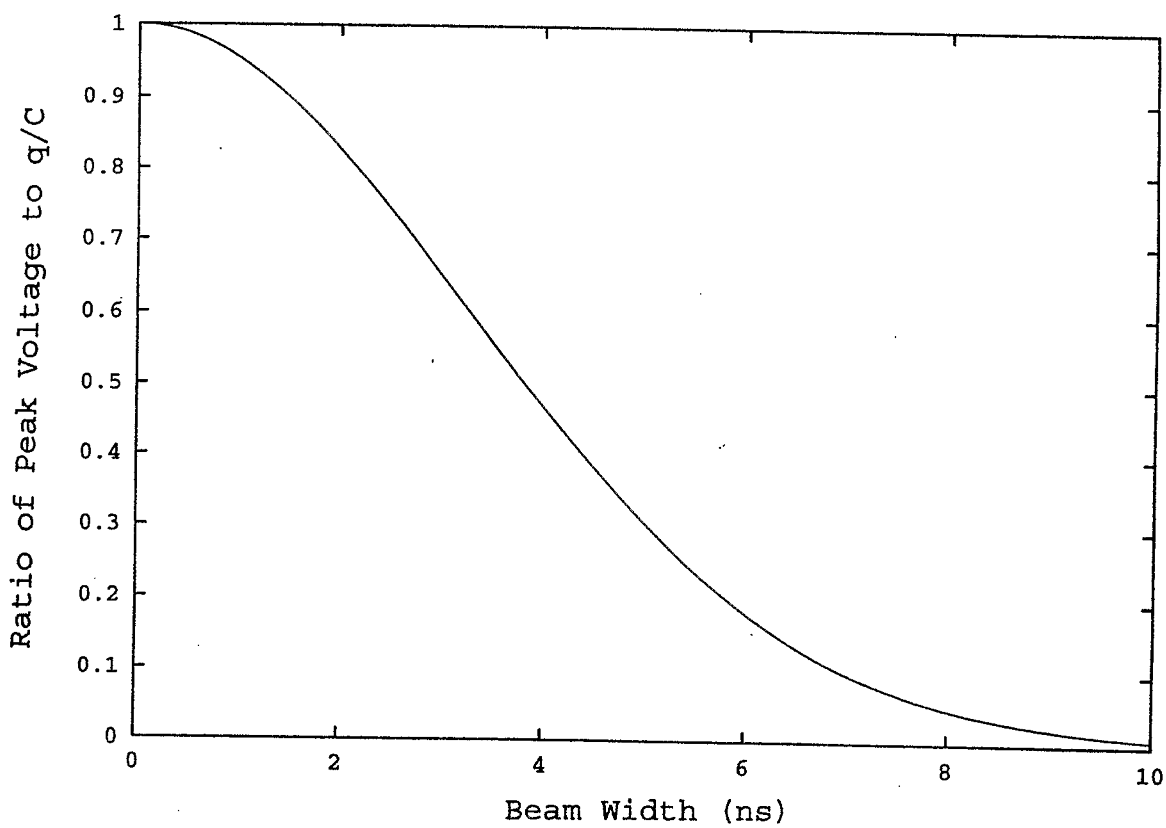

Fig. 4. Calculated single bunch excitation voltage of a storage cavity by a Gaussian beam distribution plotted as a function of the four-sigma width of the bunch.