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Recent Investigations and Findings about the 2d- and 3d- Neutron Strength Functions and the p-wave Scattering Radius

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Comprehensive and detailed investigations were recently undertaken due to the scarcity of available data on the neutron d-wave strength functions and the need to obtain precise average resonance parameters for advanced technologies. This information is derived by fitting available total cross section data from few keV to 10 MeV neutron energy using the least-squares method, where the total cross section is described in terms of the scattering radius R_ℓ for various partial waves, the s-, p-, d-, f- and g- neutron strength functions, neutron penetration factors and phase shifts. The present results reveal the following findings: (i) the peak of the 2d strength function is observed for the first time at A=50; (ii) due to nuclear deformation effects, the 3d-peak is depressed producing two peaks located at 152 and 170, similar to the s-wave case; (iii) except for the deformed mass region, a spherical optical model calculation for the d-wave strength function well describes the present data; (iv) the derived s- and p-wave strength functions and the s-wave scattering radii are in good agreement with those obtained from the resolved energy region, as reported in the Atlas; (v) the d-wave strength functions are however generally discrepant with available data; (vi) several d-wave strength functions are derived here for the first time; (vii) a new finding is that the p-wave scattering radii for 92 nuclei in the mass region ^{16}O - ^{242}Pu exhibit minima roughly at A $\simeq$ 80 and 230 and maxima at A $\simeq$ 50 and 160.

I. INTRODUCTION

Comprehensive, detailed investigations were undertaken because of the scarcity of available data on the neutron d-wave strength functions and the need to obtain precise average resonance parameters for advanced technologies. Such information was previously obtained by the author [4] for s- and p-wave neutron resonances from resolved resonance regions by the Porter-Thomas method, supplemented by Bayesian analysis when the neutron orbital angular momentum was not determined experimentally. However, such p-wave assignments can be incorrectly made as demonstrated for the strong p-wave resonances in ^{98}Mo leading to an incorrect p-wave strength function determination. Alternatively, the neutron strength functions can be derived from average capture and total cross section measurements. The major d-wave strength function data cited in [4] are obtained from cross section measurements for neutron energies below about 650 keV. Because of the availability of high-precision (1% - 5%) total cross section measurements into the 10 - 20 MeV range, it is instructive to derive average resonance parameters from these measurements for the purpose of comparison with values obtained from the low energy region. Furthermore, in the later stages of these investigations, it became clear that the scattering radius for p-wave neutrons, R_1, can be extracted reliably from such an analysis. For the first time, these quantities are derived for nuclei ranging from ^{16}O to ^{242}Pu and their systematic trend with atomic mass number was found.

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II. METHODOLOGY

Averaging over many resonances described by the single-level Breit-Wigner formula, one obtains the following expression for the average total neutron cross section

$$\langle\sigma_t\rangle = \sum_\ell (2\ell+1)4\pi\lambda^2(\sin^2\delta_\ell + S'_\ell\cos 2\delta_\ell), \qquad (1)$$

where

$$S'_\ell = \frac{\pi}{2}S_\ell V_\ell\sqrt{E}. \qquad (2)$$

In Eq. (1), δ_ℓ is the hard sphere phase shift for partial wave ℓ , which is a known function of kR_ℓ, k is the neutron wave vector, R_ℓ is the scattering radius for partial wave ℓ and V_ℓ is related to the neutron penetration factor (see for example, [4]). In this derivation, it is assumed that interference effects between resonance and potential scattering, as well as between resonances, cancel out in the averaging process. The first term in Eq. (1) is known as the shape elastic cross section and the second is the compound resonance cross section.

In the present least-squares procedure of fitting measured total neutron cross sections, up to the 10 MeV neutron energy s-, p-, d-, f-, and g-wave neutrons are considered. Because of lack of information for higher partial waves, simple assumptions are made. Specifically, $S_3 = S_1$ and the scattering radii for partial waves other than $\ell = 0, 1$ (which are determined here; see below) are represented by $1.35A^{1/3}$ fm. To take into consideration other reaction mechanisms at high neutron energies, such as contributions of orbital angular momenta higher than $\ell = 4$ or the Ramsauer effect, S_4 is considered as a variable parameter without much physical content as regards neutron strength function. It is noted that similar analysis [7] was previously restricted to $\ell \leq 2$ neutrons with energies less than 640 keV.

III. TESTS OF THE METHODOLOGY

The present methodology is tested with high-precision (1% - 5%) total cross section measurements [1], [2] and available data obtained from EXFOR [3] by the least-squares method by searching for the parameters R_0, R_1, S_0, S_1, and S_2 and then comparing these values with those reported in the Atlas [4]. The selected nuclei for these tests are ^{58}Ni, ^{139}La, ^{182}W, ^{184}W, ^{186}W, ^{206}Pb, ^{209}Bi, ^{233}U, and ^{240}Pu.

The final results of the search procedures, illustrated by the solid lines, are shown in Fig. 1 - 5 for nuclei ^{58}Ni, ^{139}La, ^{182}W, ^{209}Bi, and ^{233}U, which represent different important atomic mass regions. The calculated shape elastic cross sections for the various partial waves are shown below the total cross section measurements. Several iterations had to be carried out to achieve the final best-fit result. In some cases, because of the unavailability or lack of precision data at low neutron energies, R_0 and/or S_0 were fixed to values from the Atlas [4] or [5] (shown in parentheses in Table I) in order to obtain more reliable results for the other parameters. What is unprecedented with the present aspect of this study is the fact that such a simple prescription for the total cross section can describe the data so well over an extended energy range.

The results for the parameter searches for these nuclei are summarized in Table I. Because of recent interest in a new ^{16}O evaluation, the results of the analysis for this nucleus are also included. We note that the reported uncertainties in this analysis are those due to the fitting procedure and do not include those of the input parameters. Quantities enclosed in parentheses represent assumed values. For comparison, Atlas recommendations are shown in the second row for each nucleus. Because of space limitations, a full list of the obtained parameters will be reported later in a detailed publication.

Once the viability of the method is verified, this procedure is applied to other nuclei, ranging from ^{16}O to ^{242}Pu, for which total cross section measurements are available from a few hundred keV to a few MeV in the EXFOR system.

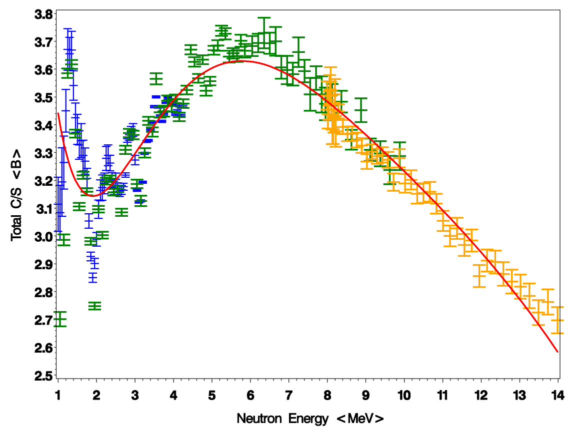

FIG. 1. ^{58}Ni(n,tot) cross section from 1 MeV to 14 MeV. The solid line is a least-squares fit to the measurements. See Table I for fitted parameters.

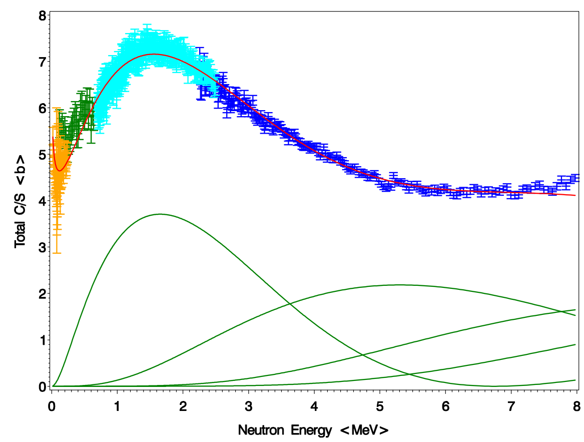

FIG. 2. ^{139}La(n,tot) cross section from a few keV to 8 MeV. The solid line is a least-squares fit to the measurements. Below the fit are shown the calculated shape elastic cross sections for partial waves $\ell = 1 - 4$.

IV. THE 2D AND 3D NEUTRON STRENGTH FUNCTIONS

The many d-wave neutron strength functions thus obtained in the present analysis are plotted versus mass number in Fig. 6 and are compared with a spherical optical model calculation carried out for a neutron energy of 600 keV (solid line) [8]. As is clearly evident in Fig. 6, the prediction does not describe the data well. On the other hand, when the optical model prediction is shifted to the left by 10 mass units, as indicated by the dashed line, it accounts for the data fairly well. Several significant features emerge from this study:

- With the exception of the deformed mass region, A = 145-180, the shifted spherical optical model calculation (dashed line) well describes the present S_2 data.

TABLE I. Neutron s-, p-, and d-wave strength functions and s- and p-wave scattring radii for some representative nuclei obtained from total cross section in the present work. For details, see the text.

Isotope	R_0 (fm)	R_1 (fm)	S_0	S_1	S_2
^{16}O	(4.8)	2.50 ± 0.34	(0.80)	1.53 ± 0.87	(0.80)
	4.80 ± 0.10	-	-	1.47 ± 0.59	-
^{58}Ni	6.42 ± 0.14	4.00 ± 0.12	1.32 ± 0.05	(0.50)	3.41 ± 0.15
	6.40 ± 0.20	-	3.79 ± 0.69	0.50 ± 0.05	3.18 ± 0.32
^{98}Mo	6.68 ± 0.08	5.09 ± 0.08	(0.58)	4.67 ± 0.10	1.31 ± 0.67
	6.90 ± 0.20	-	0.58 ± 0.13	5.55 ± 0.76	-
^{182}W	7.83 ± 0.01	7.80 ± 0.03	2.36 ± 0.03	0.66 ± 0.06	2.47 ± 0.03
	8.30 ± 0.20	-	2.51 ± 0.30	0.72 ± 0.06	1.80 ± 0.10
^{184}W	8.12 ± 0.02	8.00 ± 0.07	2.25 ± 0.07	(0.58)	2.18 ± 0.07
	8.00 ± 0.20	-	2.52 ± 0.32	0.58 ± 0.07	1.40 ± 0.10
^{186}W	8.46 ± 0.01	7.81 ± 0.03	2.02 ± 0.03	(0.43)	2.29 ± 0.03
	7.64 ± 0.05	-	2.32 ± 0.30	0.37 ± 0.05	0.96 ± 0.05
^{206}Pb	(9.54)	5.90 ± 0.14	(1.06)	(0.26)	(1.47)
	9.54 ± 0.02	-	1.06 ± 0.26	0.26 ± 0.03	1.47 ± 0.15
^{209}Bi	9.77 ± 0.03	5.82 ± 0.32	0.65 ± 0.22	(0.23)	(1.47)
	9.65 ± 0.09	-	0.65 ± 0.15	0.23 ± 0.05	-
^{233}U	9.64 ± 0.04	6.73 ± 0.22	(0.98)	1.68 ± 0.07	1.92 ± 0.09
	9.75 ± 0.15	-	0.98 ± 0.09	-	-
^{240}Pu	9.49 ± 0.02	7.33 ± 0.43	(1.03)	1.68 ± 0.03	1.87 ± 0.56
	9.60 ± 0.20	-	1.11 ± 0.08	2.80 ± 0.80	-

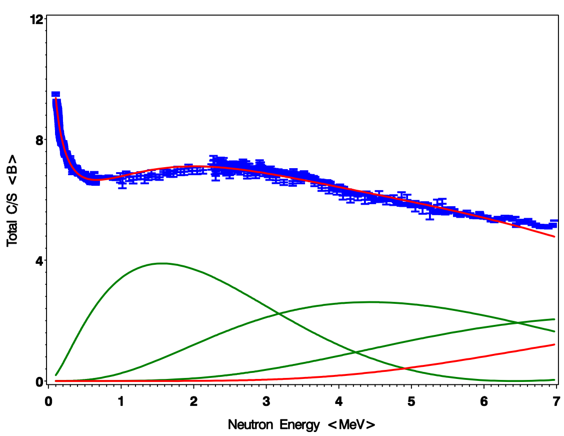

FIG. 3. ^{182}W(n,tot) cross section from a few keV to 7 MeV. The solid line is a least-squares fit to the measurements. Below the fit are shown the calculated shape elastic cross sections for partial waves $\ell = 1 - 4$.

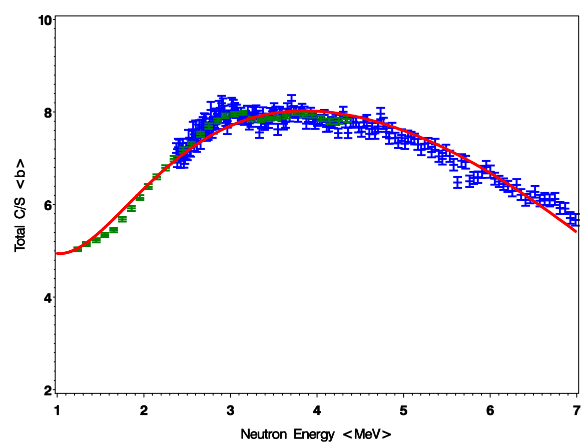

FIG. 4. ^{209}Bi(n,tot) cross section from 1 to 7 MeV. The solid line is a least-squares fit to the measurements.

- The 2d peak is observed for the first time at A=50.
- Similar to the s-wave neutron strength functions, deformation plays a significant role for the d-waves, quenching the spherical optical predictions by a factor of about two.
- The required shift of 10 mass units to smaller A values indicates that the optical model calculations have to be carried out for a neutron energy higher than 600 keV.

V. THE P-WAVE SCATTERING RADIUS

Traditionally the p-wave scattering radius is obtained from the interference effects between resonance and potential scattering which are observed in total or differential elastic scattering cross section measurements. By carrying out a shape analysis for strong p-wave resonances, reliable values for R_1 can be deduced, as in [6]. Extensive experience with the fitting procedure outlined here demonstrated that it is possible to extract the p-wave scattering radius from total cross section measurements. As an example, for ^{206}Pb, the present results give R_0= 5.90 $\pm$ 0.14 fm . This is to be compared with R_1 = 6.3 $\pm$ 0.3 by shape analysis of strong p-wave resonances de-

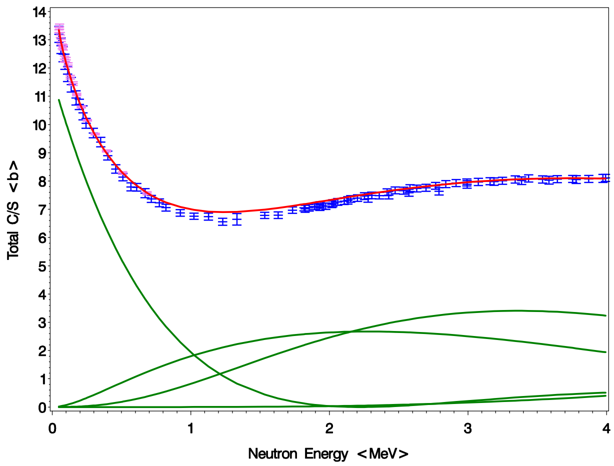

FIG. 5. ^{233}U(n,tot) cross section from a few keV to 4 MeV. The solid line is a least-squares fit to the measurements. Below the fit are shown the calculated shape elastic cross sections for partial waves $\ell = 0 - 3$.

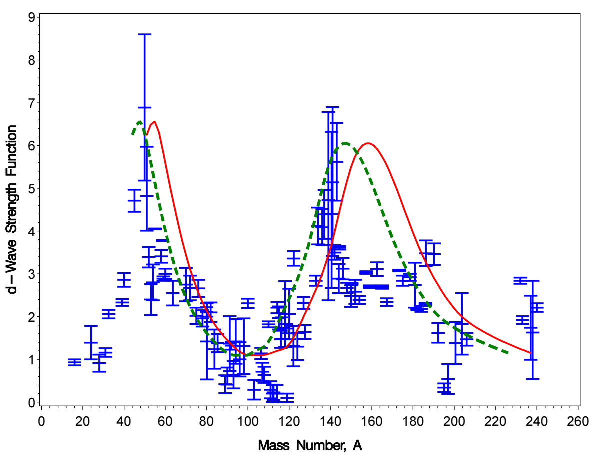

FIG. 6. The d-wave neutron strength functions are derived from total cross section measurements. The solid line is a spherical optical model calculation [8]. The dashed line corresponds to the solid line, shifted 10 mass units to smaller A values.

termined by Horen *et al.* for ^{207}Pb [6].

The various R_1 quantities derived here are plotted versus mass number in Fig. 7. As an aid to the eye, the solid line is a spline fit to the data which shows minima roughly at A $\simeq$ 80, 230 and peaks at about A $\simeq$ 50,160.

VI. CONCLUSIONS

Progress in describing total neutron cross sections in MeV region in terms of average resonance parameters is achieved. In applying this formalism to 90 nuclei and elements average resonance parameters were obtained. The systematic trend of the d-wave strength function with mass number was determined. A peak at A $\simeq$ 50 is observed and the influence of deformation around mass A $\simeq$ 160 is found. An unprecedented result is the determination of the p-wave scattering radius for a host of nuclei in the mass region from ^{16}O to ^{242}Pu.

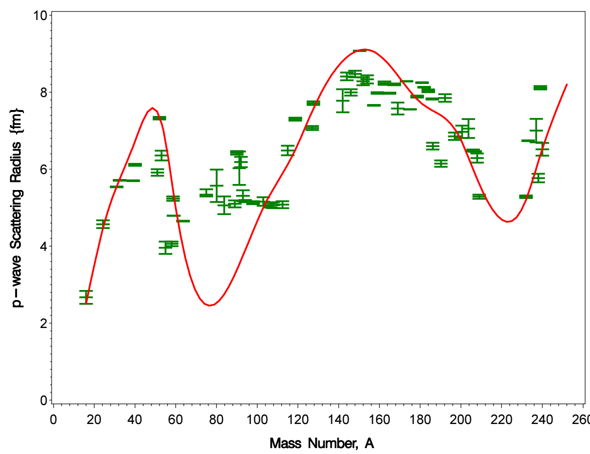

FIG. 7. The p-wave scattering radius, R_1. The spline fit to the data shows two minima at about A $\simeq$ 80 and 228 and two maxima at about A $\simeq$ 50 and 160 where the peaks of the s-wave neutron strength functions are observed.

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