AD/RHIC-AP-48

Vacuum Pipe Heating in RHIC

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Vacuum Pipe Heating

Assume M equally spaced bunches of the same shape and population with gaussian longitudinal distribution

Let

N, number of particles per bunch

σ_L, rms bunch length

P, total power dissipated

$$P = \frac{1}{2} \sum_{n=1}^{\infty} R_{nM} I_{nM}^2$$

where

R_{nM} wall resistance at nM harmonic

I_{nM} beam current at nM harmonic

$$I_{nM} = 2\, I_{ave} \exp\left(-\tfrac{1}{2} n^2 \alpha^2\right)$$

$$\alpha = M \sigma_L / R$$

R, average radius $= 610.18$ m

$$I_{ave} = Z N e \beta c M / 2\pi R$$

Z charge state

Skin Depth

$$\delta = \frac{\delta_1}{\sqrt{n}} \qquad \delta_1 = \sqrt{\frac{2\rho R}{M Z_0}}$$

$Z_0 = 377$ ohm

	S.S.	Copper
ρ @ 42 °K	50 $\mu\Omega \times$ cm	0.055 $\mu\Omega \times$ cm
δ_1, $M = 57$	0.17 mm	0.0056 mm
$M = 114$	0.12 mm	0.0040 mm
σ/ℓ	1.37×10^{15} /ohm·m^2	1.54×10^{15} /ohm·m^2

ℓ , free path length

$\sigma = 1/\rho$, conductivity

σ/ℓ is an invariant which depends on the material density

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1 For those frequencies such that

$\ell/\delta < 1$ Normal Skin Depth

$$R_{nM} = R_1 \, n^{1/2}$$

$$R_1 = \sqrt{(Z_0 \rho R M)/2b^2}$$

2 For those frequencies such that

$\ell/\delta > 1$ Abnormal Skin Depth

$$R_{nM} = R_1 \, n^{2/3}$$

$$R_1 = \left\{ \left(\frac{\ell}{\sigma}\right) \frac{\sqrt{3}\, Z_0^2 M^2 R}{4\pi b^3} \right\}^{1/3}$$

b, vacuum chamber radius

assuming circular geometry = 3.6 cm

Thus

$$P = 2 I_{ave}^2 \sum_{n=1}^{\infty} R_{1_n} n^{p_n} \exp(-n^2 \alpha^2)$$

* For $n = 1$ to n_c

$$R_{1_n} = R_{1_{normal}}$$
$$p_n = 1/2$$

* For $n = (n_c + 1)$ to ∞

$$R_{1_n} = R_{1_{abnormal}}$$
$$p_n = 2/3$$

There is also a cut-off due to the finite bunch length

$$n_{cut\text{-}off} \sim \frac{1}{\alpha} = \frac{R}{M \sigma_L}$$

$$n_c \sim \left(\frac{a}{\ell}\right)^2 \left(\rho\, \delta_1\right)^2$$

	S.S.	Copper
n_c	1.3×10^{10}	22

$n_{cut\text{-}off}$

M	σ_L	
57	0.4	26
114	0.4	13
57	0.2	52

We have summed up to $\hat{n}$ given by

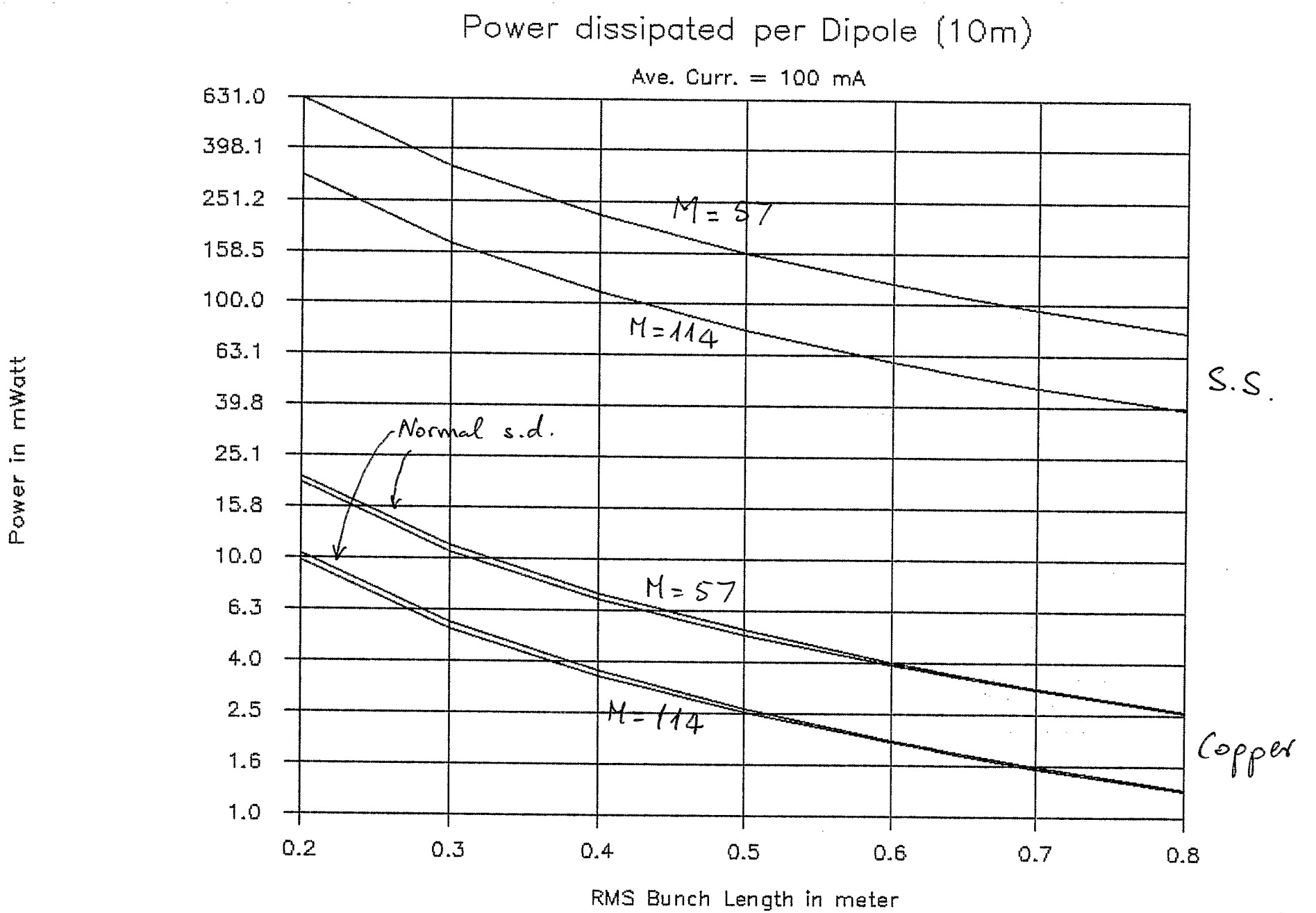
$$\hat{n}\alpha = 15$$

Assuming the Beam Parameters from CDR

	Z	N	I_{ave}
proton	1	100×10^9	71 mA (electric)
Carbon	6	22	94
Sulfur	16	6.4	73
Copper	29	4.5	93
Iodine	53	2.6	98
Gold	79	1.1	62

with $M = 57$ bunches

As reference $\sigma_L = 40$ cm


Power dissipated per Dipole (10m)
Ave. Curr. = 100 mA
631.0
398.1
251.2
158.5
100.0
63.1
39.8
25.1
15.8
10.0
6.3
4.0
2.5
1.6
1.0
Power in mWatt
0.2
0.3
0.4
0.5
0.6
0.7
0.8
RMS Bunch Length in meter
M = 57
M = 114
S.S.
Normal s.d.
M = 57
M = 114
Copper

Power Dissipated per 10m Dipole in Watts
Reference Case: Gold, Average Current = 62 mA
Stainless Steel

M	N		sigma-L 0.40 m	0.20 m
57	1.1	x 10^9	0.085	0.242
114	1.1		0.170	0.484
57	2.2		0.340	0.968
114	2.2		0.680	1.936
57	3.3		0.765	2.178
114	3.3		1.530	4.356

Power Dissipated per 10m Dipole in milli-Watts
Reference Case: Gold, Average Current = 62 mA
Copper, Normal + Abnormal Skin Depth

M	N		sigma-L 0.40 m	0.20 m
57	1.1	x 10^9	2.7	7.6
114	1.1		5.4	15.2
57	2.2		10.8	30.4
114	2.2		21.5	60.7
57	3.3		24.2	68.3
114	3.3		48.4	136.6

Power Dissipated per 10m Dipole in Watts
Reference Case: Protons, Average Current = 71 mA
Stainless Steel

M	N		sigma-L 0.80 m	0.40 m	0.20 m
57	1	x 10^11	0.039	0.112	0.317
114	1		0.078	0.224	0.634
57	3		0.351	1.008	2.853
114	3		0.702	2.016	5.706
57	8		2.496	7.168	20.288
114	8		4.992	14.336	40.576

Power Dissipated per 10m Dipole in milli-Watts
Reference Case: Protons, Average Current = 71 mA
Copper, Normal + Abnormal Skin Depth

M	N		sigma-L 0.80 m	0.40 m	0.20 m
57	1	x 10^11	1.3	3.5	10.0
114	1		2.6	7.1	19.9
57	3		11.7	31.8	89.6
114	3		23.4	63.5	179.1
57	8		83.2	225.9	636.8
114	8		166.4	451.8	1273.6