The RHIC Lattice

A. G. Ruggiero

March 1992
1. Introduction

Hadron storage rings typically involve collisions between the same type of particles. There are proton-antiproton colliders where both beams share the same storage ring, and proton-proton colliders where two intersecting rings are required. RHIC is a different type of collider with a broader range of applications and experiment requirements. There is a range of particles involved since the experimental program calls for collisions between all beams of fully stripped ions which can be made available from the source. The main goal is collisions between two beams of very heavy ions; gold ions (Au) are taken as the reference; but the program also requires collisions between lighter ions all the way down to protons. Another design goal is to provide collisions between beams of unequal species, most notably protons against Au-ions, but also with other species. The ions are completely stripped and thus the charge state equals the atomic number \( Z \). In summary, whereas in all other types of hadron collider projects one deals with a well defined mass and charge state, in the RHIC the mass number ranges between 1 and about 200 and the atomic number between 1 and about 80. The parameter of relevance which enters in the design of heavy-ion storage rings is the ratio \( A/Z \) which in RHIC ranges between one (for protons) and 2.5 (for Au). It is customary to describe the performance of a heavy-ion storage ring by quoting the maximum magnetic rigidity \( B_\rho \); when this is known, one can then calculate the energy and all the other kinematic parameters for all ion species. For RHIC a value of \( B_\rho = 840 \text{ T}\cdot\text{m} \) is the nominal design which yields the specific kinetic energy of 100 GeV/u for Au-ions. Consequently the kinetic energy for protons is 250 GeV.

An additional requirement in the design of the RHIC is the need of providing collisions between beams of Au-ions over an energy range of 30-100 GeV/u. This requirement is
unique, since all the other colliders essentially emphasize the design for the operation at top energy only. Operation of RHIC at lower energies has some crucial consequences for the design of the lattice.

RHIC operates as an accelerator, a storage ring and a collider. With up to 114 bunches to inject and a relatively long acceleration time, beam lifetime effects at the injection energy cannot be ignored. The injection energy depends on the injector complex which is made of the Tandems, the HITL, the Booster and the AGS. Because of the variety of the particles used, injection is specified by a common value of magnetic rigidity of $B\rho = 97.5$ T-m. Thus, independently of the species being injected, accelerated and stored, the whole accelerating process takes always the same step from 97.5 T-m to a maximum of 840 T-m. Good storage performance is similarly required in the range of 300 to 840 T-m. The injection value of 97.5 T-m corresponds exactly to the maximum rigidity of the AGS.

Finally, an important constraint for the design of the RHIC lattice is the existence of a tunnel where the collider is to fit in size and shape. This approach is not new; as in the Tevatron, SpPSS and more recently the LHC; but it is a fact that the existence of a tunnel does restrict the choices of a design. The circumference of the RHIC is 3834 m. With a packing factor of 40% (that is the ratio of bending radius to average radius), which we shall explain later, we derive a bending radius of 250 m against an average radius of 610 m. Given the requirement of 840 T-m on the maximum magnetic rigidity, this in turn gives the operational field in the bending magnets of 3.45 T, which is a relatively modest field for present superconducting magnet technology. A summary of general parameters for RHIC is given in Table 1. A layout of the geometry of the tunnel which will house RHIC is shown in Fig. 1. From a geometric point of view, there is a sixfold periodicity with six arcs and six straight sections; the average bending radius in the arcs is 380 m and the length of a straight section 280 m which then leads to the packing factor of 40% mentioned above. This is the lowest value among all the hadron colliders with 75% in the Tevatron and SpPSS, 73% in the SSC and 61% for the LHC. This low value for the packing factor has some consequences on the lattice performance especially on the chromaticity and the required sextupole strength. The straight sections are longer probably than needed, though they include also some modest bending; their length is comparable to that of the arcs. Similar to proton-proton colliders, RHIC is made of two intersecting storage rings.
because the two beams in collision are made of particles with the same sign of the electric charge.

The chosen layout is horizontal, with the two rings side-by-side; the reasons of the choice are more historical than really because of need. Today, a re-examination of the issue would suggest the vertical layout, with one ring on top of the other, as in the SSC, as more advantageous; but also the horizontal geometry is acceptable with essentially no consequences to the lattice performance.

2. The Arcs

Each arc is made of twelve regular FODO cells. The layout of a cell is displayed in Fig. 2 showing the amplitude and dispersion functions. The maximum lattice values, in the middle of the regular quadrupoles QF and QD, are \( \beta = 49.2 \, \text{m} \) and \( \eta = 1.7 \, \text{m} \). The vacuum chamber is a circular pipe of the same cross-section throughout all the arcs; the internal diameter is 69 mm. RHIC is the collider with the shortest and strongest focusing FODO cell of all the hadron machines. The most important consideration in this regard was the effect of intrabeam scattering. The Coulomb cross-section which is responsible for this effect scales like \( Z^4/A^2 \), which for Au ions results in an enhanced sensitivity of 3 orders of magnitude compared to protons. The nominal bunch densities for Au ions are reduced by a factor of 100 compared to protons but this still means that Au ion bunches experience a very rapid emittance growth in both transverse and longitudinal planes. After several hours of storage conditions the result is physically large beams with large energy spreads. Ensuring that the dynamic aperture of the machine is sufficient to result in acceptable luminosity lifetime under these extreme conditions was the main factor in adopting these cell parameters.

The relatively low values of \( \beta_{\text{max}} \) and \( \eta_{\text{max}} \) arising from the strong focusing result in a large betatron acceptance in the standard cell. At low energies (\( \gamma \sim 30 \)) the normalized beam emittance is 60 \( \pi \) mm-mrad whereas the injected beam is in the range of 10-20 \( \pi \) mm-mrad depending on the particle species. The arc regions should therefore allow sufficient aperture and beam lifetime not only for the injection process but also during storage.
The phase-advance across each cell is about $84^\circ$ close to $90^\circ$. A choice of $60^\circ$ would have also been acceptable, and actually preferable from the chromatic behavior point of view. The choice of $90^\circ$ is a trade-off between intrabeam scattering effects which can be reduced with a diluted beam size, that is weak focusing lattice; a relatively small dispersion to control the momentum exchange between betatron motion and longitudinal motion, and a reasonable magnet aperture. Driven by economic considerations, a 3-inch coil i.d. has been found acceptable for both main dipole and quadrupole magnets.

There is a benefit in selecting 12 regular FODO cells per arc, with either $60^\circ$ or $90^\circ$ phase-advance; to first order, the total transfer matrix is unitary, and the arcs are thus transparent from one insertion to another. There is only a single dipole magnet per half-cell which precludes the use of mid cell correctors. Correction elements are located next to each quadrupole in the arcs. Lumped corrections are perfectly adequate in the absence of strong persistent current fields in the dipoles. Because of the short length of the cells and the presence of inner and outer arcs, it was found convenient to place the dipole magnets exactly half-way from quadrupoles.

3. The Long-Straight Sections

The design of the long-straight sections was only obtained after a sequence of successive iterations. Because of horizontal configuration, side-by-side, each ring has a threefold periodicity. Each period begins with the outer arc, followed by an insertion which takes the beam from the outer to the inner side, the inner arc, and finally the insertion which takes the beam this time from the inner to the outer side. A vertical layout, as the one adopted for the SSC, would have eliminated the distinction of inner and outer arcs; but the complications due to the horizontal crossing are found to be minor and of no essential consequences to the other design considerations. The separation of the two rings is 90 cm between beam axis in the arcs.

Another major difference between the RHIC lattice design concept and that of the other hadron colliders, is the absence of distinction between utility insertions and experiment insertions, which are independently designed with different application requirements. In RHIC, all insertions have the same design; though they may be tuned at somewhat different
values. This has the advantage of allowing up to six fully developed experimental areas, and to dump the beam in any of six different locations around the ring. At the same time, lattice requirements for beam injection are enforced also in all straight-sections, though in reality the beam will be transferred from the AGS from only one direction at one point in each ring.

Figure 3 shows the sequence of magnets in one half of a long straight section, demonstrating the merging of the two beams at the collision point. Typical behavior of the lattice functions, amplitude and dispersion are plotted in Fig. 4. Each arc begins and ends with the same type of quadrupoles, QF or QD. Both symmetric and antisymmetric insertions are possible; but we have preferred antisymmetric insertions since they result in equal phase advance in both planes and introduce less chromatic effects. Thus each insertion will begin with a QF and ends up with a QD quadrupole and vice versa. The arcs accordingly will alternate from QF-to-QF to QD-to-QD configuration.

Like all the modern hadron colliders, also RHIC makes use of bunched beams with the same horizontal and vertical betatron emittance, that is circular. The optimum luminosity performance is then reached with head-on collision and the same focusing at the crossing region in both horizontal and vertical direction. In all other hadron colliders the method used is to bring first the two beams together in a common vacuum chamber with the use of special dipole magnets and then to focus both beams to the same spot size with common quadrupoles. This method is very effective in obtaining very low $\beta^*$ values, for instance 0.5 m, by placing the innermost quadrupoles as close as possible to the interaction point. This, in turn, minimizes the maximum beta functions obtained in these elements. This standard technique cannot be used at RHIC since common quadrupoles cannot be employed when the two beams in collision are particles of different species and therefore magnetic rigidity. Thus the basic design approach of the RHIC low-beta insertion is inverted: the last focusing quadrupoles are placed outside the special dipoles that get the two beams merging to each other path. This results in a large physical separation of 55 m between the final set of focusing quadrupoles. This separation sets a limit on the lowest value of $\beta^*$ one can obtain without encountering too large values of $\beta_{\text{max}}$. It is because of this reason that the dynamic aperture of the machine is defined by the separation dipoles and focusing quadrupoles in the region of the highest beta values when operating at the lowest beta values of 1-2 m.
It is also because of this fact that the magnets in this region have a larger aperture (11-13 cm) than those in the rest of the ring.

Another consequence of this approach is that one now requires also a different way to match the dispersion behavior to the arcs. The antisymmetric insertion and the particular set up of the crossing magnet adopted do not allow for dispersion suppressors at the two sides of the insertion, following and preceding the arcs. The innermost triplets are adjusted then to provide the required focus but also to adjust the dispersion at the crossing point to the zero value; control of the dispersion slope is more difficult to attain. Intertwining functions in this fashion does not allow modularity in the design of the long-straight section. To some degree, modularity can be recovered with a symmetric insertion when the last-focus triplet is comprised also entirely by the crossing dipole magnets.

Neglecting for the moment the dispersion behavior, the final focus is obtained with a telescope, that is an ensemble of five quadrupoles divided in two groups: a compact triplet (Q1, Q2, Q3) by the crossing region and a doublet (Q4, Q5); these two groups are separated by about 40 m free of any magnets, which has the function of a utility section which is not otherwise available in RHIC. We have thus a telescope to tune the crossing point to a waist with $\beta^*$ ranging from 1 to about 16 m. Figure 4 displays the lattice functions for reference case $\beta^* = 2$ m and Fig. 5 the same for $\beta^* = 10$ m. The lattice tuning in Fig. 5 is used for injection, acceleration and unused crossing points. At the end of the acceleration, at the beginning of storage and collision, those regions with experiments will be tuned to a value of $\beta^*$ as low as desired to enhance luminosity. The free space available to the detectors is about $\pm 10$ m.

Between the end of the arc and Q5, there are a number of quadrupoles (Q6, Q7, Q8, Q9) equally spaced at the same distance from each other as in the arcs; they represent the natural extension of the arc by adding two regular FODO cells, a most effective way to make use of such a long straight section. It is in this section that injection takes place. Special dipoles are located with the goal of controlling dispersion and providing dispersion matching; the quadrupoles are then set independently for the overall matching of the amplitude functions. Magnets are distributed symmetrically around the crossing point and the analogous quadrupoles at the two sides are set at exactly the same excitation but with opposite sign. These quadrupoles provide the variable matching section between
the final focus and the arc section. Thus in addition to QF and QD there are nine more quadrupoles Q1 to Q9 needed for the overall tuning.

A technical difficulty with the lattice described here is the design and construction of the BC2 dipoles. As shown in Fig. 6, they are side-by-side, too close to each other to obtain a large bore diameter; one can allow again an inner vacuum chamber diameter of 72 mm, but at their location the amplitude function \( \beta \sim 250 \) m; thus they now represent locations with a considerable smaller physical betatron acceptance, of only \( 5 \pi \) mm-mrad. The nearby elements, Q1 and Q2 are seriously limiting the acceptance of the ring, since at their location the amplitude function reaches the largest value which is maintained to 670 m for \( \beta^* = 2 \) m. Thus, especially in the centermost part, the long straight section are not matched in aperture to the arcs. This effect was also amply revealed during particle tracking.

Betatron tunes are adjusted with the regular quadrupole QF and QD over a range of ±1 unit. During the \( \beta^* \) squeeze the phase-advance across the insertion was not deliberately kept constant, but the change was absorbed in the regular FODO cells in the arcs.

4. Transition Energy and Chromaticity

Table 1 gives the summary of the most important and global parameters for RHIC. The betatron tunes are about equal; the fractional part is chosen close to the diagonal in a triangular region between third and fifth order resonance, with an extension of about 0.033, free from any other major resonances up to tenth order included. The integral part is chosen half-way between 27 and 30 which are multiples of the ring superperiodicity. The tunes are considerably larger than other hadron colliders, due to the relatively strong focusing properties. As a result the transition energy \( \gamma_T \) is also high; since all ion species are injected into RHIC with \( \gamma \sim 12 \) transition crossing occurs for all ions, except in the case of protons which are injected at \( \gamma \sim 30 \). Since the acceleration rate is low, crossing transition energy at slow speed can cause a number of effects, some of them intensity-dependent, like microwave instabilities and space charge mismatch. The design of RHIC includes a scheme for a fast \( \gamma_T \)-jump which increases the crossing rate by a factor of ten thus controlling those effects which ultimately will cause a bunch area growth. The jump is
triggered by quadrupole correctors placed next to each QF quadrupole in the arc, powered so to excite the integral resonance and to enhance the dispersion function. The value of $\gamma_T$ is indeed essentially given by the integral of the dispersion in the arc bending; the value can be varied by controlling the dispersion. The jump occurs in about 60 msec for a depression of $\Delta \gamma_T \sim 0.8$.

The chromaticity in RHIC is corrected with six families of sextupoles. The magnets are located next to each regular quadrupole in the arcs; there are no sextupoles in the long straight sections. There are SF-type of sextupoles next to QF quadrupoles and SD-type next to QD quadrupoles. Two families would be sufficient to adjust the chromaticities in the two planes to the desired values, usually in proximity of zero; but to flatten the dependence over a range of momentum deviation as large at $\pm 0.5\%$, as many as six families are required; they are set as follows. In the outer arc there is only one family of SF-type but two families of SD-type intertwined with each other; the opposite is applied to the inner arc, where now is a single SD-type and two SF-type families. The number of families and the strength of the sextupoles is higher in RHIC because most of the chromaticity is introduced by the long-straight sections where the sextupole correctors are not present and by relatively stronger focusing of the overall lattice.
### Table 1: RHIC General Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference</td>
<td>3833.8450 m</td>
</tr>
<tr>
<td>Average Radius</td>
<td>610.1754 m</td>
</tr>
<tr>
<td>Bending Radius</td>
<td>242.78 m</td>
</tr>
<tr>
<td>Periodicity</td>
<td>3</td>
</tr>
<tr>
<td>Dipole Field, max</td>
<td>3.46 T</td>
</tr>
<tr>
<td>Betatron Tunes: Hor.</td>
<td>28.827</td>
</tr>
<tr>
<td>Vert.</td>
<td>28.823</td>
</tr>
<tr>
<td>Transition Energy, $\gamma T$</td>
<td>24.343</td>
</tr>
<tr>
<td>Natural Chromaticity: Hor.</td>
<td>-78.12</td>
</tr>
<tr>
<td>Vert.</td>
<td>-78.43</td>
</tr>
<tr>
<td>$\beta^*$ (reference value)</td>
<td>2.0 m</td>
</tr>
<tr>
<td>Dispersion at crossing point</td>
<td>0.0 m</td>
</tr>
<tr>
<td>Crossing Angle</td>
<td>head-on</td>
</tr>
</tbody>
</table>
Figure 1: Geometry of the RHIC Tunnel.
\[ \nu^x = 0.2307 \]
\[ \nu^y = 0.2308 \]

Figure 2: Regular FODO Cell in the Arcs.
Figure 3: RHIC Long-Straight Insertion.
Figure 4: Lattice Functions for $\beta^* = 2$ m.
\[ \nu = 28.8270 \]
\[ \nu^* = 28.8230 \]
\[ \beta^* = 10.000 \]

**Figure 5:** Lattice Functions for \( \beta^* = 10 \text{ m} \).
Figure 6: Layout of the Crossing Region.