Effects of Leakage Fields From Polarimeter Toroid Magnets

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The present conceptual design for the RHIC polarimeter¹ uses 8 toroid magnets to steer pions produced from a carbon target toward Cerenkov counters. The iron in these toroid magnets has 7° openings in the horizontal and vertical planes to allow the pions to escape, and the symmetry of this arrangement produces an octupolar field in the beam pipe of RHIC. The effects of this octupole field on the RHIC circulating beam are discussed below.

1 Tune shift with amplitude

To see the magnitude of the effect of the octupolar field on the particle dynamics, we compute the horizontal tune shift experienced by a particle with betatron oscillation amplitude a , measured at a location with amplitude function β_0 , due to a single octupole magnet:

$$\Delta\nu = \frac{1}{2\pi} \frac{3}{8} \left(\frac{B''' L_0}{6(B\rho)} \right) \beta_0 a^2$$

where $B''' \equiv \partial B_y / \partial x^3$. Rewriting this equation in terms of the beam emittance and integrating over a distribution of octupole fields, we find that

$$\Delta\nu = \frac{1}{(2\pi)^2} \frac{\epsilon_N}{8\gamma} \left(\sum_i \int_{s_{1i}}^{s_{2i}} \frac{B_i'''}{6(B\rho)} \beta(s)^2 ds \right) u^2$$

where $u \equiv a/\sigma$, and $\epsilon_N \equiv 6\pi\gamma\sigma^2/\beta_0$ is the normalized 95% emittance.

Table 1 shows the powering sequence for the 8 toroids for four different beam energies, in terms of a toroid's full strength, as provided by Mariam.² The original design shows an octupole field in the beampipe region with strength along the midplane of approximately $B_y \approx (600 \text{ T/m}^3) x^3$. While the magnetic design continues to be optimized, this octupole strength will be used in the following discussions. The tune

¹See F. G. Mariam, SN/27, May 1996.

²Ibid.

Toroid Number:	1	2	3	4	5	6	7	8
25 GeV	-1	-1	-1	-1	-1	-0.5	+1	+1
100 GeV	0	0	-1	-1	-1	+1	+1	+1
175 GeV	0	0	0	-0.5	-0.3	+0.5	+0.95	+1
250 GeV	+1	+1	+1	+1	+1	+1	+1	+1

Table 1: Powering sequence of the 8 polarimeter toroid magnets for various beam energies in terms of the design field.

shift with amplitude is shown in Figure 1 for the nominal 25 GeV case, for 25 GeV with all toroids at full strength, and for the 250 GeV case.

Figure 1: Tune shift vs. betatron amplitude (in units of rms beam size).

Table 2 shows the average tune shift of the beam particles, as well as the tune shift of a particle at the 95% amplitude of the beam distribution. The table entries correspond to the magnet settings given in Table 1, plus one extra entry which is for the condition where all magnets are set at full strength at the injection energy of 25 GeV, a worst case.

	$\Delta\nu_{ave}$	$\Delta\nu_{95\%}$
25 GeV	-3×10^{-5}	-1.4×10^{-4}
100 GeV	5×10^{-5}	2×10^{-4}
175 GeV	2×10^{-5}	1×10^{-4}
250 GeV	2×10^{-5}	1×10^{-4}
25 GeV, all +1	2×10^{-3}	1.11×10^{-2}

Table 2: Tune shifts with amplitude generated by the octupolar fields in the polarimeter toroids.

2 Feed-down effects from orbit errors

If the trajectory of the proton beam passes through the toroid system off center, then there will be feed-down effects such as non-local orbit distortion, tune shift, and chromaticity. Assume that the closed orbit is adjusted to pass off center by an amount X_{off} through the “perfectly aligned” toroid system, and assume that the phase advance doesn’t change much between the correctors (actually, it varies by about 40°), then the orbit distortion around the ring will have an amplitude in the arcs of

$$\Delta x = \frac{\sqrt{\hat{\beta}}}{2 \sin \pi \nu} X_{off}^3 \sum_i \int_{s_{1i}}^{s_{2i}} \frac{B_i'''}{6(B\rho)} \sqrt{\beta(s)} ds.$$

There will be a tune shift induced by the orbit offset by an amount

$$\Delta\nu_q = \frac{1}{4\pi} X_{off}^2 \sum_i \int_{s_{1i}}^{s_{2i}} \frac{B_i'''}{2(B\rho)} \beta(s) ds,$$

and the chromaticity will be altered by an amount

$$\Delta\xi = \frac{1}{2\pi} X_{off} \sum_i \int_{s_{1i}}^{s_{2i}} \frac{B_i'''}{2(B\rho)} D(s) \beta(s) ds.$$

where $D(s)$ is the dispersion function.

Since the dispersion function is small in the vicinity of the polarimeter, the chromaticity change due to the feed-down sextupole term is negligible. The closed orbit distortions will also be small since the field is small at millimeter-scale displacements through the system. The tune shifts induced by an orbit offset can be noticeable only for X_{off} = several millimeters. Table 3 shows the results using $X_{off} = 1$ mm computed for the various operational scenarios of the polarimeter discussed previously.

	$\Delta x(\mu\text{m})$	$\Delta\nu_q$	$\Delta\xi$
25 GeV	1	-2×10^{-4}	5×10^{-2}
100 GeV	0.07	3×10^{-5}	3×10^{-3}
175 GeV	0.1	4×10^{-5}	-8×10^{-4}
250 GeV	0.4	9×10^{-5}	-7×10^{-3}
25 GeV, all +1	4	9×10^{-4}	-7×10^{-2}

Table 3: Feed-down effects generated by orbit offsets in the octupolar fields due to the polarimeter toroids.

3 Octupole correction

The effects of the toroid system can be compensated for using a warm octupole corrector placed in the vicinity of the toroid magnets in the Q3-Q4 warm straight section. Supposing we only wish to correct the tune shift with amplitude using an octupole magnet, the corrector would need a strength given by

$$B_c''' \times L_c = \left(\sum_i B_i''' \int_{s_{1i}}^{s_{2i}} \beta(s)^2 ds \right) / \beta_c^2.$$

The largest correction is that for all toroids operating at full strength and with equal polarities. If the corrector is in the downstream portion of the straight section (i.e., near Q3), then the amplitude function will be on the order of 100 m and thus the correction required would be approximately 8000 T/m². The field gradient of an octupole magnet can be estimated using

$$B_c''' = \frac{4! \mu_0 N I}{R^4}$$

where R is the pole tip radius, N is the number of turns per pole, and I is the current per turn. The required correction can be accomplished by a warm octupole magnet with a 4 cm pole tip radius and with, for example, 4 turns per pole operating at 425 A per turn, and an effective length of 40 cm.

4 Conclusions

The first toroid magnet design iteration for use in the polarimeters had an octupole gradient of $B''' = 3600 \text{ T/m}^3$. If polarimeter magnets with these error fields were to be energized during a store, a small tune shift could result due to misalignments or orbit offsets through the magnets. More importantly, a tune spread inherently produced by the octupole fields will be present. The tune spread can be corrected, if

necessary, with a relatively modest warm octupole corrector. It should be emphasized that the octupole fields used in this analysis came from a first estimate for a toroid design. Further iterations of a true engineering design appear to be able to reduce this field by 1-2 orders of magnitude. This work is ongoing.