<p style="text-align: center;">AGS Complex Machine Studies (AGS Studies Report No. 305) Stopband Correction of the AGS Booster Down Feed Matrix</p>
Study Period: April-July 1993
Participants: C. Gardner and Y. Shoji
Reported by: Y. Shoji
Machine: AGS Booster
Beam:
Tools:
Aim: We had already reported about our studies and data. Here we report the progress of the data analysis. Through the analysis we found some mistakes and show what was wrong.

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I Mistake in SR-293 (9th Normal Sextupole Correction Test)

There was a small mistake in Table II of AGS SR-293.[1] Then some results become different.

Table II

	SH3	SV3	dRset	N(cos9*)	N(sin9*)	speed	loss
< Wrong >							
2Qx=9	0	40	-0.6	-30±10	280±10		53
			1.4	320±10	80±10		43
< Correct >							
2Qx=9	0	40	1.4	320±10	80±10		43
			-0.6	-30±10	280±10		53

Table III

resonance	SH3	SV3	$\delta N(\cos 9)/\delta R_{set}$	$\delta N(\sin 9)/\delta R_{set}$	loss
< Wrong >					
2Qx=9	0	40	145±10	-100± 8	48
< Correct >					
2Qx=9	0	40	-175±10	100± 8	48

Equation on page 3

$$\begin{aligned}
 & \text{< Wrong >} \\
 & \begin{bmatrix} \delta N(\cos 9X)/\delta dR_{set} \\ \delta N(\sin 9X)/\delta dR_{set} \end{bmatrix} = \begin{bmatrix} 6.69 \pm 0.33 & 6.31 \pm 0.33 \\ 2.93 \pm 0.29 & -1.45 \pm 0.29 \end{bmatrix} \begin{bmatrix} SH3 \\ SV3 \end{bmatrix} \\
 & \text{< Correct >} \\
 & \begin{bmatrix} \delta N(\cos 9X)/\delta dR_{set} \\ \delta N(\sin 9X)/\delta dR_{set} \end{bmatrix} = \begin{bmatrix} 6.69 \pm 0.33 & -1.68 \pm 0.33 \\ 2.93 \pm 0.29 & 3.55 \pm 0.29 \end{bmatrix} \begin{bmatrix} SH3 \\ SV3 \end{bmatrix}
 \end{aligned}$$

II Correction Matrix of 9th Normal Sextupole Strings

We had already mentioned that the relation between the 9th harmonic sextupole strings; SH3 and SV3 and the slopes of half integer correction; $\delta N(\cos 9X)/\delta dR_{set}$, $\delta N(\sin 9X)/\delta dR_{set}$, $\delta N(\cos 9Y)/\delta dR_{set}$ and $\delta N(\sin 9Y)/\delta dR_{set}$ can be reliably calculated.[2] In this section we will compare the calculated and observed results.

The connection scheme of 9th normal sextupoles strings basically follow the 9th quadrupole strings refereed as J2 and J4 in ref. [3]. They are

SV3: SHA2+SHA8-SHB2-SHB8+SHC2+SHC8-SHD2-SHD8+SHE2+SHE8-SHF2-SHF3
SH3: SHA4-SHA6-SHB4-SHB6+SHC4-SHC6-SHD4-SHD6+SHE4-SHE6-SHF4-SHF6.[4]

They use monitor windings (1 turn for each magnet) of the sextupole magnets. Then the sextupole field strength of each magnet S is $S=8.215E-4$ T/mA.[5] The dispersion h , beta functions β_x and β_y and betatron phase advance μ_x and μ_y from the beginning of a superperiod to the sextupole magnets are listed in Table I.[6] The polarities of correction strings were checked on October 28 by C.Whalen and the authors. The polarity of SH3 string was different from the standard. According to the standard a positive correction supplies defocusing sextupole field ($d^2\phi_x/dx^2 > 0$).

Table I Parameters at the sextupole magnets. $Q_x=4.633676$,
 $Q_y=4.583271$. [6]

magnets	h (m)	β_x (m)	β_y (m)	$9\mu_x/Q_x$	$9\mu_y/Q_y$
SHA2	1.321	11.725	4.580	1.654	1.129
SHA4	2.640	12.253	4.579	$0.916+ \pi$	$0.344+ \pi$
SHA6	2.776	12.206	4.584	$0.014+2\pi$	$2.694+ \pi$
SHA8	1.398	11.957	4.573	$2.400+2\pi$	$1.915+2\pi$

The half integer stop band width dQ is

$$dQ = (1/2\pi) \int \delta K_1 \beta e^{j9\theta} ds \quad (II-1)$$

$$= (1/2\pi) \sum 2 S h \beta e^{j9\mu/Q} (dP/P) Br(e/cp) I_s \quad (II-2)$$

Here I_s is the current of the correction string; SH3 or SV3. And $e/cp = 1/3.335641$ GeV/Tm. The sign of S should be changed as the

connection of correction magnet. From ref. [3]

$$\sqrt{N(\sin 9X)^2 + N(\cos 9X)^2} = 10^5 dQx \quad (\text{II-3})$$

and from reference [7]

$$dRset = 318.9 (dP/P) \quad (\text{cm}) \quad (\text{II-4})$$

Insert these equations into the equation (II-2) and we get

$$\begin{aligned} \delta N(\cos 9X) / \delta dRset &= (10^5 / 2\pi) (e/cp) / 319 \sum 2Sh\beta x \cos(9\mu x/Q) \text{ Is} \\ &= (10^5 / 2\pi) (e/cp) / 319 6\Sigma_A 2Sh\beta x \cos(9\mu x/Q) \text{ Is} \\ \delta N(\sin 9X) / \delta dRset &= (10^5 / 2\pi) (e/cp) / 319 6\Sigma_A 2Sh\beta x \sin(9\mu x/Q) \text{ Is} \\ \delta N(\cos 9Y) / \delta dRset &= (10^5 / 2\pi) (e/cp) / 319 6\Sigma_A 2Sh\beta y \cos(9\mu y/Q) \text{ Is} \\ \delta N(\sin 9Y) / \delta dRset &= (10^5 / 2\pi) (e/cp) / 319 6\Sigma_A 2Sh\beta y \sin(9\mu y/Q) \text{ Is} \end{aligned} \quad (\text{II-5})$$

Insert parameters in Table II-I into equations (II-5) we obtain the matrix of down feed correction;

$$\begin{bmatrix} \delta N(\cos 9X) / \delta dRset \\ \delta N(\sin 9X) / \delta dRset \\ \delta N(\cos 9Y) / \delta dRset \\ \delta N(\sin 9Y) / \delta dRset \end{bmatrix} = \begin{bmatrix} 7.90 & -2.01 \\ 3.85 & 3.94 \\ 3.37 & 0.06 \\ -0.21 & 1.69 \end{bmatrix} \begin{bmatrix} SH3 \\ SV3 \end{bmatrix} \quad (\text{II-6})$$

On the other hand the observed matrix was [2]

$$\begin{bmatrix} \delta N(\cos 9X) / \delta dRset \\ \delta N(\sin 9X) / \delta dRset \\ \delta N(\cos 9Y) / \delta dRset \\ \delta N(\sin 9Y) / \delta dRset \end{bmatrix} = \begin{bmatrix} 6.69 \pm 0.33 & -1.68 \pm 0.33 \\ 2.93 \pm 0.29 & 3.55 \pm 0.29 \\ 3.38 \pm 0.29 & 0.68 \pm 0.31 \\ 0.08 \pm 0.29 & 2.28 \pm 0.29 \end{bmatrix} \begin{bmatrix} SH3 \\ SV3 \end{bmatrix} \quad (\text{II-7})$$

The matrix elements are roughly the same as the theoretical calculations. We know that the errors of equation (II-7) were underestimated. And it seems that there is an additional systematic error. The ratios of matrix elements of equation (II-7) to equation (II-6) are listed in Table II. The calculated effect to $N(\cos 9X)$ and $N(\sin 9X)$ was about 20% larger than the observed effect. The errors do not explain the systematic error as much as 20%.

We could not check the polarity of quadrupole correction strings on October 28. But from the above agreement we can identify the polarity of the quadrupole correction strings. The left side of the equation (II-6) presents an applied change. On the other hand the left side of the equation (II-7) presents a residual slope. Then they should be opposite if the polarity used

in equation (II-5) were correct; a positive quadrupole correction supplied horizontally defocusing quadrupole field ($d\phi x/dx > 0$). Then in the real machine the positive correction supplies horizontally focusing quadrupole field ($d\phi x/dx < 0$).

Table II Ratio of measured coefficients to the calculated coefficients.

slope\9th sextupole	SH3	SV3
$\delta N(\cos 9X)/\delta dR_{set}$	-0.85 ± 0.04	0.84 ± 0.17
$\delta N(\sin 9X)/\delta dR_{set}$	-0.76 ± 0.09	0.90 ± 0.08
$\delta N(\cos 9Y)/\delta dR_{set}$	-1.00 ± 0.09	11 ± 5
$\delta N(\sin 9Y)/\delta dR_{set}$	0.4 ± 1.4	1.35 ± 0.17

III B and dB/dt dependence of 9th sextupole correction for $2Q_x=9$

We can calculate the 9th sextupole correction for $2Q_x=9$. From equation (10) of ref.[8] the slopes of $2Q_x=9$ correction are

$$\begin{aligned} \delta N(\cos 9X)/\delta dR_{set} &= (-75 \pm 40) + (3 \pm 12)B + (-1.06 \pm 0.29)(dB/dt) \\ \delta N(\sin 9X)/\delta dR_{set} &= (-52 \pm 40) + (-13 \pm 12)B + (-0.45 \pm 0.29)(dB/dt). \end{aligned} \quad (III-1)$$

And from equation (II-7)

$$\begin{bmatrix} SH3 \\ SV3 \end{bmatrix} = 0.001 \begin{bmatrix} 124 \pm 16 & 58 \pm 13 & 298 \pm 27 & -89 \pm 41 \\ -102 \pm 11 & 233 \pm 13 & -10 \pm 38 & 442 \pm 57 \end{bmatrix} \begin{bmatrix} \delta N(\cos 9X)/\delta dR_{set} \\ \delta N(\sin 9X)/\delta dR_{set} \\ \delta N(\cos 9Y)/\delta dR_{set} \\ \delta N(\sin 9Y)/\delta dR_{set} \end{bmatrix}. \quad (III-2)$$

The results are

$$\begin{aligned} SH3 &= (12.3 \pm 5.4) + (0.4 \pm 1.7)B + (0.158 \pm 0.044)(dB/dt) \\ SV3 &= (4.5 \pm 8.2) + (3.3 \pm 3.1)B + (-0.003 \pm 0.075)(dB/dt) \end{aligned} \quad (III-3)$$

IV Correction Matrix of 9th Skew Sextupole Strings

As we did in section II we can calculate the effect of skew sextupole strings to the slopes of $Q_x+Q_y=9$ and $Q_x-Q_y=0$ correction; $\delta N(\cos 9XY)/\delta dR_{set}$, $\delta N(\sin 9XY)/\delta dR_{set}$ and $\delta N(\cos 0XY)/\delta dR_{set}$.

We have two skew sextupole correction strings; SH4 and SV4. Each of these strings is connected to two correction magnets such

as

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SV3;      +B2  +E2
SH3;      +B8  +E8
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to correct 14th skew sextupole [14]. To correct 9th skew sextupole we changed the polarity of one of the magnet of each string. But we lost the record about which ones were changed. Here we assume that the polarities were

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SV3;      +B2  -E2
SH3;      +B8  -E8 .
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The integrated skew sextupole field per one magnet; SQ was measured to be [9]

$$SQ = 13.3 \text{ E-4 T/m/A} . \quad (\text{IV-1})$$

A positive current (SH3>0 or SV3>0) supplies a skew sextupole kick ($d^2\phi y/dx^2 > 0$). The dispersion function h , beta functions B_x and B_y and betatron phase advance μ_x and μ_y at the correction packages are listed in Table IV.[6]

Table IV Twiss parameters at the skew sextupole correction magnets. $Q_x=4.633676$, $Q_y=4.583271$. [6]

string	magnet	$h(m)$	$B_x(m)$	$B_y(m)$	$\sqrt{B_x B_y}$	$\mu_x/2\pi$	$\mu_y/2\pi$	9θ
SH4	B2	1.291	11.337	4.757	7.3437	0.906	0.851	$1.353+3\pi$
SV4	B8	1.384	11.560	4.748	7.4086	1.482	1.424	$2.120+5\pi$

The correction parameter of $Q_x+Q_y=9$ is

$$\begin{aligned} N(\cos 9XY) &= (10^5/16\pi) (e/cp) \int A_1 \sqrt{B_x/B_y} \cos(9\theta) ds \\ &= (10^5/16\pi) (e/cp) 4 SQ h(dP/P) \sqrt{B_x/B_y} \cos(9\theta) Isq \end{aligned} \quad (\text{IV-2})$$

here

$$9\theta = 4.5\mu_x/Q_x + 4.5\mu_y/Q_y . \quad (\text{IV-3})$$

A positive correction supplies a skew quadrupole kick ($d\phi y/dx > 0$). Notice that the normalization constant of the correction strings had been changed from $N/2=(10^5/8\pi)$ [3] to $N/2=(10^5/16\pi)$. [10] The slope is

$$\begin{aligned} \delta N(\cos 9XY) / \delta dR_{set} \\ = (10^5/16\pi) (e/cp) / 319 4SQ h \sqrt{B_x/B_y} \cos(9\theta) Isq . \end{aligned} \quad (\text{IV-4})$$

$$= (10^5/16\pi) (e/cp)/319 \ 4SQ \ h/\beta_x/\beta_y \cos(9\theta) \ Isq . \quad (IV-4)$$

The matrix of down feed correction is calculated to be

$$\begin{bmatrix} \delta N(\cos 9XY)/\delta dRset \\ \delta N(\sin 9XY)/\delta dRset \end{bmatrix} = \begin{bmatrix} -0.021 & 0.058 \\ -0.092 & -0.087 \end{bmatrix} \begin{bmatrix} SV4 \\ SH4 \end{bmatrix} . \quad (IV-5)$$

The polarities of correction strings were not known. So we can change sign of SH4 and SV4 to compare the calculated matrix with the measured matrix. The observed matrix was [11]

$$\begin{bmatrix} \delta N(\cos 9XY)/\delta dRset \\ \delta N(\sin 9XY)/\delta dRset \end{bmatrix} = \begin{bmatrix} -0.13 & -0.12 \\ -0.06 & 0.04 \end{bmatrix} \begin{bmatrix} SV4 \\ SH4 \end{bmatrix} . \quad (IV-6)$$

The orders of the matrix elements were the same. The agreement will be better if we exchange $N(\cos 9XY)$ and $N(\sin 9XY)$ but it could not happen. A large error of equation (IV-6) could be the explanation of the disagreement. But something could be wrong in the skew quadrupole strings. The currents of correction power supplies was different from the programmed currents on August 4.[12] We shall check skew quadrupole correction strings. The matrix elements of the $Q_x-Q_y=0$ correction string had been also changed [10] but it did not matter.

It is impossible to calculate the correction current of the SH4 and SV4 to correct the slope of $Q_x+Q_y=9$. Because we could not set the matrix with enough accuracy. The disagreement between the theory and experiment was so large to identify the polarity of equation (IV-5).

References

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