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Accelerator Division Technical Note

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THE OPTIMUM CENTRAL ORBIT IN THE AGS

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I. Introduction

The basic layout of the AGS is very simple. It is possible to select a coordinate system in which an orbit of the appropriate momentum will be centered in all of the straight sections and off momentum orbits will be symmetrically displaced from this optimum central orbit. Adopting this coordinate system should lead to a simplification of our understanding of the AGS and to an easier mode of operation. This proposed coordinate system was used by the original designers of the AGS but it has been lost from memory with the passage of time.

II. Basic Design

From Technical Note 215 (Where Are the AGS Magnets, E.J. Bleser, May 20, 1985) we have Figure 1 which shows schematically the path of an orbit through a magnet. The AGS is laid out for an orbit whose entrance and exit positions and angles are given in Table 1. This orbit was found by requiring that the total bend through a magnet be:

$$\theta = 2\pi L / \sum L$$

where L is the effective length of a magnet and the sum is taken over all the magnets around the ring. For a given value of B_0 the momentum of the orbit is given by:

$$p = eB_0 L/\theta$$

We shall call this orbit the Optimum Central Orbit (OCO). It is a unique orbit. The machine is designed so that when this relationship exists between B and p there is only one central orbit and it is this smooth simple one. Off momentum orbits display oscillatory behavior and are symmetrically displaced from the central orbit. From TN 215 we know that in the "BEAM PROGRAM" coordinate system this orbit is displaced toward the center of the ring by 0.191 inches in straight sections between long magnets and 0.131 inches in straight sections between short magnets. In transition sections it is the mean of these.

In order to demonstrate this orbit we go to EDC-53 (Where is Zero? E.D. Courant, November 21, 1963). Figure 2 shows orbits 4 and 5 from that paper plotted in the BEAM coordinate system for the intermediate field case ($B_0 = 4965$ Gauss). They have momentum values of 16.67 and 16.75 Megagauss inches. In TN 215 the magnetic fringe field added 4 inches to the magnet length; in EDC-53 it adds 4.1 inches. This has inconsequential effects on the geometry but we must keep track of it. For EDC-53 the bend angle in a long magnet is:

$$\begin{aligned}\theta &= 2\pi \times 94.1 / (144 \times 94.1 + 96 \times 79.1) \\ &= 27.963 \text{ mr}\end{aligned}$$

and the required momentum is:

$$\begin{aligned}p &= eB_0 L/\theta \\ &= 16.708 \text{ megagauss inches}\end{aligned}$$

(See also EDC-37, Location of the Nominal Equilibrium Orbit in AGS, E.D. Courant, January 14, 1960, which comes to the same results as the present note). This momentum is very conveniently just the mean of orbits 4 and 5. Figure 3 shows orbits 4 and 5 and their mean, which clearly corresponds to the optimum central orbit we have been seeking. Figure 4 transforms these three orbits into the OCO coordinate system and we see a very nice symmetry appear for the off momentum orbits while the central orbit is absolutely flat in this coordinate system. Figure 5 shows orbits 2, 4, 5, and 7 from EDC-53 and the symmetry clearly persists. In this coordinate system we can easily write down a momentum dispersion function which is just the orbit displacement divided by the percentage momentum shift. Figure 6 shows the dispersion function and Figure 7 shows the dispersion function and a scaled value of the beta function (as tabulated in EDC-28).

III. Conclusions

The AGS as originally designed is conceptually a very simple machine. The optimum central orbit goes around the ring just as though it were going through a string of dipole magnets. The off momentum orbits and the beta function behave as though they were seeing a continuous string of uniform quadrupole doublets. This was the intent of the original machine designers. Many drawings still show a coordinate identified as "Theoretical Beam Center Line" or as " R_0 ". This coordinate is identical with what we have called the "OCO" coordinate system. The AGS standards book clearly specifies that the trim quadrupole and sextupole magnets are to be located on the OCO center line. TN 212 (Maximum Limiting Aperture in the AGS, K.A. Brown and K.M. Brown, April 12, 1985) shows that the old pick up electrodes and many of the vacuum chambers are centered on the OCO system. However, the new pickup electrodes are thought to be centered on the "BEAM" program coordinate system (L. Ahrens, private communication). Thus we may be using a system that is not optimal conceptually, and we may be using two different systems unknowingly (J.P. Potier, private communication).

IV. Recommendations

1. Choose one coordinate system. The OCO system is conceptually the most desirable.

2. Most groups, such as the control room, device designers, mechanical engineers, and design room, will use only that system.

3. For the surveyors prepare one drawing relating the OCO system to the socket system.

4. For tracking program users prepare one drawing relating the OCO system to the "BEAM" program coordinate system.

5. Require that the output of all tracking programs be in the OCO coordinate system.

6. Establish the coordinates of all the ring devices in the OCO system.

7. All future devices and drawings should be prepared in the OCO system.

8. Consider presenting device information to the control room in the OCO system.

Table 1

MAGNET PARAMETERS

	<u>Long Magnet</u>	<u>Short Magnet</u>	<u>Units</u>
Magnet Length	90.000	75.000	Inches
Magnet Effective Length	94.000	79.000	Inches
Y_0	0.21906	0.15472	Inches
$\theta/2 = \tan^{-1}(Y_0')$	13.9825	11.7512	mr

Straight Section Lengths

2 foot	28.000	Inches
5 foot	64.000	Inches
10 foot	123.9907	Inches

Figure 1

ORBITS IN BEAM PROGRAM SYSTEM

Intermediate Field

Figure 2

ORBITS IN BEAM PROGRAM SYSTEM

Intermediate Field

Figure 3

ORBITS IN OCO SYSTEM

Intermediate Field

Figure 4

ORBITS IN OCO SYSTEM

Intermediate Field

Figure 5

Figure 6

DISPERSION FUNCTION AND BETA FUNCTION

Intermediate Field

Figure 7

APPENDIX I

THE OCO COORDINATE SYSTEM

Figure A1 attempts to make clear the differences between the various systems being discussed here. The "BEAM" program coordinate system goes along the socket line of the magnets and in the straight sections goes from magnet steel to magnet steel. The system used in laying out the positions of the magnets in TN 215 (called here "Layout"), which has not really been referred to as a coordinate system, also goes along the socket line of the magnets, but in the straight sections bends at the edge of the fringe field. This line is at a radius outside of the "BEAM" line and the steel to steel length along this line is a fraction of a mill longer than along the "BEAM" program line. The Socket system consists of the lines connecting the magnet sockets. The OCO system is characterized by being displaced a distance y_0 inward from the magnet socket line at the edge of the fringe field. Thus it is at a significantly smaller radius than any of the other systems. Table A1 gives the lengths and angles for the various systems. Table 1 specifies L_i , y_0 , and the angles. The other dimensions are calculated from the figure.

In Table A1 for arithmetic purposes we assume very good precision. In particular we assume the lengths are accurate to five decimal places and the angles to four decimal places. Thus the fractions of a mill added to the effective lengths in the "Layout" system look ridiculous, but are essential to give consistency with the "BEAM" system and must be included in the calculations of TN 215 to give good agreement with the AGS standards book. The actual tolerances on the survey and the construction are of course much less severe.

Inside a magnet the OCO system is ill defined and probably not needed. However in Figure A2 we show a possible system.

From a practical point of view, when we are dealing with straight sections between two similar magnets, we need only consider Figure A3 and Table A2, which show the OCO, the Socket, and the "BEAM" Program coordinate systems and the offsets between the different systems. These numbers are easily calculated from Fig. A1. For transition sections the geometry, while still trivial, is a little more complicated. For the transition from a long to a short magnet the layout is given by Figure A4 where ϵ is 2.14 mr. For the transition from a short to a long magnet the layout is given by Figure A5. This geometry has been incorporate into the calculation used to develop Table A1. Table A3 relates the type of straight section to the straight section number.

Figures A6 and A7 show the OCO system in the "BEAM" system for straight sections where the transition from long to short or from short to long magnets is made. Table A4 gives a complete summary of the transformation from the "BEAM" program coordinate system to the OCO system.

Figure A8 attempts to clarify a subtle point of geometry. For the simple case shown in Fig. A1 all the systems are parallel. However for the more complicated cases shown in Figures A4 and A5 the separation between the bend points matters, and it makes a difference if we are bending at the edge of the fringe field as in TN 215, or if we are bending at the edge of the magnet steel, as in the "BEAM" program coordinate system. From Figure A8:

$$\delta = \frac{\alpha_L - \alpha_S}{24}$$

$$= 2.6806 \text{ mr}$$

$$\epsilon = \frac{[\alpha_L - 2 \tan(\theta_L/2)] - [\alpha_S - 2 \tan(\theta_S/2)]}{28}$$

$$= 2.1384 \text{ mr}$$

The one half milliradian difference in the entrance and exit angles of the transition section shown in Table A1 for "BEAM" and "Layout" results from the difference between ϵ and δ . In Appendix V of TN 215 we said that due to the rounding down of the angle values the "BEAM" program did not close by 1.5 milliradian. A more correct statement is that due to the rounding down of the angle values the "BEAM" program would not close by 1.0 milliradians if it had not been fixed up by adding 0.02 milliradians to the calculated bend angle of 9.61 milliradians at the entrance to the short magnet wherever there is a transition between long and short magnets. Table AV-1 of TN 215 is confused as a result of some confusion on the part of the author. Table A5 is more useful. It shows the angles used in the "BEAM" program and a more exact set of values which are closer to the actual construction of the AGS. We believe that at some time these angles should be tried out in "BEAM".

THE PICK UP ELECTRODES IN THE OCO SYSTEM

The new pick up electrodes are located in the "BEAM" program coordinate system at (L. Ahrens, private communication):

$$y_B = 0.00$$

$$x_B = 13.43 \text{ inches}$$

From the transformations given in Table A4, we can find that the beam position as read by the PUE's is given in the OCO system by the transforms of Table A6. We believe that at some time these transforms should be tried out in the PUE program.

CIRCUMFERENCE

From Table A1 it is easy to calculate the sums of the chord lengths around the ring in the various systems. These are shown in Table A7. The actual path length of an orbit is somewhat greater since the path is curved in a magnet, adding about 0.003 inches per magnet, as is discussed in TN 215, Appendix V.

Table A1

System Dimensions

System	Magnet No.	SS L _e /2 inches	Entrance $\theta/2$ mr	Magnet L _e inches	Exit $\theta/2$ mr	SS L _e /2 inches
Layout	6	30.00020	13.9825	94	13.9825	12.00020
	7	12.00020	13.9825	94	13.9825	30.00020
	8	30.00020	13.9825	94	16.6631	12.00018
	9	12.00018	9.0706	79	11.7512	12.00014
	10	12.00014	11.7512	79	11.7512	59.99549
"BEAM"	6	32	13.98	90	13.98	14
	7	14	13.98	90	13.98	32
	8	32	13.98	90	16.12	14
	9	14	9.63	75	11.75	14
	10	14	11.75	75	11.75	61.99535
Socket	6	34.99970	13.9825	84	13.9825	16.99970
	7	16.99970	13.9825	84	13.9825	34.99975
	8	34.99975	13.9825	84	15.5466	16.99975
	9	16.99975	10.1870	69	11.7512	16.79980
	10	16.99980	11.7512	69	11.7512	64.99515
OCO	6	29.99714	13.9825	94	13.9825	11.99714
	7	11.99714	13.9825	94	13.9825	29.99714
	8	29.99714	13.9825	94	13.9825	11.99774
	9	11.99774	11.7512	79	11.7512	11.99832
	10	11.99832	11.7512	79	11.7512	59.99367

Table A2

Parameters for Fig. A2

	$\frac{a}{10^{-3} \text{ inches}}$	$\frac{b}{10^{-3} \text{ inches}}$
5' SS	42.0	191.1
2' SS	42.0	191.1
between Long Magnets		
10' SS	35.2	131.2
2' SS	35.2	131.2
between Short Magnets		

Table A3

Classification of Straight Sections

<u>Type</u>	<u>Length</u> feet	<u>SS No</u>
Long to Long	2	4, 6, 14, 16
	5	3, 5, 7, 13, 15, 17
Short to Short	2	1, 9, 11, 19
	10	10, 20
Short to Long	2	2, 12
Long to Short	2	8, 18

Table A4

Transformations

If (X_B, Y_B) are coordinates in the "BEAM" program coordinate system, then in the OCO system (X_{OCO}, Y_{OCO}) are given by:

<u>ELEMENT</u>	$\frac{X_{OCO}}{\text{inches}}$	$\frac{Y_{OCO}}{\text{inches}}$
Long Magnet	X_B	$Y_B + 0.2191$
Short Magnet	X_B	$Y_B + 0.1547$
5 Foot SS	$X_B - 2.0033$	$Y_B + 0.1911$
10 Foot SS	$X_B - 2.0014$	$Y_B + 0.1312$
2 Foot SS (Long to Long)	$X_B - 2.0033$	$Y_B + 0.1911$
2 Foot SS (Short to Short)	$X_B - 2.0014$	$Y_B + 0.1312$
2 Foot SS (Long to Short)	$X_B - 2.0033$	$Y_B + 0.1911 - 0.00214X_B$
2 Foot SS (Short to Short)	$X_B - 2.0014$	$Y_B + 0.1312 + 0.00214X_B$

Table A5

Magnet Entrance and Exit Angles

<u>As built in "Layout"</u>	<u>As built in "BEAM" system</u>	<u>Used in "BEAM" program</u>	<u>Units</u>
13.9825	13.9825	13.98	mr
11.7512	11.7512	11.75	mr
16.6631	16.1209	16.12	mr
9.0706	9.6128	9.63	mr

Table A6

Transformation for the PUE's from the "BEAM" to the OCO system

<u>Straight Section</u>	<u>SS No</u>	<u>$Y_{OCO} = Y_{BEAM} +$</u>
Long to Long	4, 14	0.191 inches
Long to Short	8, 18	0.167 inches
Short to Long	2, 12	0.156 inches

Table A7

Circumference

<u>System</u>	<u>Sum of Chord Lengths</u>	<u>Difference from BEAM</u>	<u>Effective radius $\Sigma L/2\pi$</u>	<u>Units</u>
Layout	31,775.862	+0.085	5,057.286	inches
BEAM	31,775.777	0	5,057.272	inches
Socket	31,775.669	-0.108	5,057.255	inches
OCO	31,774.631	-1.146	5,057.090	inches

Figure A1

Figure A2

Figure A3

Figure A4

Figure A5

Figure A6

Figure A7

Figure A8