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Bias Current Geometry for the AGS Large Ferrite Test Stand

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Introduction

A new ferrite test stand is being designed by the AGS radiofrequency group in order to characterize the magnetic properties of ferrite rings for the AGS and AGS Booster acceleration cavities. The stand is to be used for the following tasks:

- (1) to define and characterize magnetic properties of ferrite toroids,
- (2) to allow comparative evaluation of different ferrite materials,
- (3) to provide a gauge for acceptance or rejection of rings for actual cavity use, and
- (4) to allow selection of matched sets of rings to be stacked in the cavities.

In order to perform its intended tasks, the test stand should closely simulate the DC tuning and RF excitation magnetic field geometries and intensities. It should provide regulation and monitoring of ferrite temperature. At the same time, it should be constructed so as to permit rapid change of ferrite test rings, because hundreds of rings may have to be evaluated.

The requirements, above, of easy toroid replacement and accurate simulation of the synchrotron acceleration cavity's magnetic geometry are somewhat in conflict. The requirement of easy ring interchangeability pushes the design of the DC ferrite tuning coil of the test stand to a configuration with a few

turns and high DC current. The requirement of accurate DC field geometry pushes the design to an axisymmetric configuration of one turn and even higher currents to provide adequate ampere-turns magnetomotive force.

Two tuning coil designs have been proposed:

(1) a four-turn configuration surrounding two ferrite rings (Fig. 1),
and

(2) a single-turn axisymmetric geometry (Fig. 2).

Both designs are of comparable cost and delivery time.

The four-turn design is more accessible for ferrite ring replacement and requires $1/4$ of the drive current required for the single loop design (250 amperes and 1000 amperes, respectively). The $\underline{H}$ field magnetic excitation of the four-turn design would, however, be significantly different than the axisymmetric excitation produced in an axisymmetric accelerator cavity geometry. Axial and azimuthal variations of the azimuthal magnetic field intensity within the ferrite would be expected.

While a four-turn tuning coil geometry has in fact been used elsewhere, for the ferrite test cavity for the GSI heavy ion synchrotron project at Darmstadt, West Germany,¹ it is not clear that such a design accurately simulates the magnetic environment in an accelerator cavity.

In this note we present some results of a calculation of the magnetic field intensity $\underline{H}$ generated by an N-turn tuning coil geometry and we compare it to the $\underline{H}$ field generated by an axisymmetric configuration having the same ampere-turn product. We calculate the exciting field only, before insertion of the ferrite rings. We have calculated the field $\underline{H}$ for tuning coils with 4, 8, 16 loop turns (Fig. 3) and for a single axisymmetric turn (Fig. 2). In all cases we have chosen the same NI product $NI = 2\pi$ for the excitation.

Figure 1. Four-turn tuning coil geometry.

Configuration may
be slit axially for
mechanical assembly
purposes

Figure 2. Near-axisymmetric tuning coil geometry.

Figure 3. N-turn tuning coil geometry.

The calculation was repeated for several variations of the tuning coil dimensions. In each case large azimuthal and axial variations of the azimuthal $\underline{H}$ -field component H_ϕ were found, in the four-turn geometry, in addition to the radial $1/\text{radius}$ variation appearing in the axisymmetric case. The case of 16 turns was run as a check on the calculations, and results for this configuration closely approach those for the axisymmetric configuration.

The calculations indicate that the magnetic excitations for a four-turn test stand configuration are sufficiently different from those for an axisymmetric cavity configuration (i.e. a single distributed turn) that the axisymmetric test stand geometry is desirable.

Method of Calculation

The magnetic field intensity (in air) $\underline{H}(\underline{P})$ at a field point $\underline{P}$, generated by a set of N turns each carrying a current I , is given by the Biot-Savart law as

$$\underline{H}(\underline{P}) = \frac{I}{4\pi} \sum_{\text{turns}}^N \oint \frac{d\underline{s} \times \underline{r}}{|\underline{r}|^3} \quad (1).$$

where $\underline{r}$ is the radius vector from line element $d\underline{s}$ of a turn to the field point.

For turns composed of linear segments this reduces to a sum of contributions from individual segments

$$\underline{H}(\underline{P}) = \sum_{\substack{\text{turns} \\ (M)}} \sum_{\substack{\text{vertices} \\ (K) \text{ of} \\ \text{each turn}}} \underline{H}_{MK, MK+1} \quad (2)$$

Let V_{MK} denote the vertex point K of the turn M and let V_{MK} denote the position vector from the origin of coordinates to V_{MK} .

Let $\underline{P}$ denote the position vector from the origin coordinates to the field point P .

If the m -th coil is an n -gon K will run from 1 to $n+1$, with the convention $V_{M N+1} = V_{M1}$. Let d be the perpendicular distance from $\underline{P}$ to the segment $V_{MK}V_{M K+1}$ (Fig. 4). Let θ_1, θ_2 denote the angles between the segment and position vectors from the vertex end points of the segment to the field point $\underline{P}$.

Define position vectors

$$\underline{L} = \underline{V}_{M K+1} - \underline{V}_{MK}$$

$$\underline{R} = \underline{P} - \underline{V}_{MK}$$

$$\underline{R}' = \underline{P} - \underline{V}_{M K+1}$$

$$\underline{e}_H = \frac{\underline{R} \times \underline{R}'}{|\underline{R} \times \underline{R}'|} \quad (3)$$

The vector $\underline{e}_H$ is a unit normal to the plane defined by the current-carrying segment and the field point.

The contribution to the $\underline{H}$ field at $\underline{P}$ from the segment $V_{MK}V_{M K+1}$ is:

$$\underline{H}_{MK, M K+1}(P) = (\underline{e}_H) \left(\frac{I}{4\pi d} \right) (\cos\theta_1 + \cos\theta_2) \quad (4)$$

Expressing this in terms of the vectors $\underline{L}, \underline{R}, \underline{R}'$ this reduced to

$$\underline{H}_{MK, M K+1}(P) = \left(\frac{I}{4\pi} \right) \left(\frac{\underline{R} \times \underline{R}'}{|\underline{R} \times \underline{R}'|^2} \right) \left\{ \frac{\underline{L} \cdot \underline{R}}{|\underline{R}|} - \frac{\underline{L} \cdot \underline{R}'}{|\underline{R}'|} \right\} \quad (5)$$

in vector notation.

Summing terms (5) over all segments of the tuning coil gives the desired field $\underline{H}(\underline{P})$ in (2).

Figure 4

The terms in (5) are easily computed in rectangular cartesian coordinates. The results of the calculation are expressed in cylindrical polar coordinates.

In comparing the fields due to the case of N individual loops and a single axisymmetric loop, we assume that the exciting magneto-motive force NI is equal to 2π ampere-turns in each case.

For each case we compute the radial field component H_r , the azimuthal field component H_ϕ , and the axial field component H_z for each of 9 test field points $\underline{P}$.

For the axisymmetric configuration the $\underline{H}$ field is purely azimuthal and is equal to $NI/2\pi r(\underline{P})$ for $NI = 2\pi$.

The 9 test points are chosen as follows:

Field Point $\underline{P}$	Radial Coordinate r	Azimuthal Coordinate ϕ	Axial Coordinate Z
1	$r = \text{ferrite ring}$	field point lies in	$Z = Z^+$
2	inner radius	plane of first turn	$Z = Z^-$
3	$r = \text{ferrite ring}$	$\phi = 0$	$Z = Z^+$
4	outer radius		$Z = Z^-$
5	$r = \text{ferrite ring}$	field point lies half-	$Z = Z^+$
6	inner radius	way between the planes	$Z = Z^-$
7	$r = \text{ferrite ring}$	of adjacent turns	$Z = Z^+$
8	outer radius	$\phi = \pi/N$	$Z = Z^-$
9	$r = \text{ferrite ring}$ mean radius	$\phi = \pi/2 N$	$Z = \frac{1}{2}(Z^+ + Z^-)$

$Z^+ =$ axial coordinate of top surface of ferrite ring.

$Z^- =$ axial coordinate of bottom surface of ferrite ring.

Results, Conclusions

Program TESTAND•FOR

Ferrite Ring Outer Radius = .25
 Ferrite Ring Inner Radius = .10
 Ferrite Ring Thickness = .028
 Ferrite Midplane Axial Offset = .053
 Turn Height = .181
 Turn Inner Radius = .0508
 Turn Outer Radius = .316

All distances are in meters

All magnetic field intensi-
 ties are in ampere turns/
 meter

Turns	Point	Hr	H ϕ	H _z	H	H ϕ axisymmetric case, 1 turn
4	1	0.0	17.18	0.0		10.0
4	2	0.0	12.45	0.0		10.0
4	3	0.0	19.61	0.0		4.0
4	4	0.0	9.42	0.0		4.0
4	5	0.0	6.59	0.0		10.0
4	6	0.0	8.03	0.0		10.0
4	7	0.0	1.56	0.0		4.0
4	8	0.0	1.78	0.0		4.0
4	9	.835	4.785	1.756	5.165	5.714
16	1	0.0	10.25	0.0		10.0
16	2	0.0	10.00	0.0		10.0
16	3	0.0	5.22	0.0		4.0
16	4	0.0	4.26	0.0		4.0
16	5	0.0	9.76	0.0		4.0
16	6	0.0	10.00	0.0		10.0
16	7	0.0	3.20	0.0		10.0
16	8	0.0	3.77	0.0		4.0
16	9	.0403	5.708	.187	5.711	5.714

Program TESTAND•FOR

Ferrite Ring Outer Radius = .25
 Ferrite Ring Inner Radius = .10
 Ferrite Ring Thickness = .028
 Ferrite Midplane Axial Offset = .053
 Turn Height = .22
 Turn Inner Radius = .0508
 Turn Outer Radius = .316

All distances are in meters

All magnetic field intensi-
 ties are in ampere turns/
 meter

Turns	Point	Hr	H ϕ	H z	$ \underline{H} $	H ϕ axisymmetric case, 1 turn
4	1	0.0	13.05	0.0		10.0
4	2	0.0	11.60	0.0		10.0
4	3	0.0	10.03	0.0		4.0
4	4	0.0	8.22	0.0		4.0
4	5	0.0	7.77	0.0		10.0
4	6	0.0	8.60	0.0		10.0
4	7	0.0	1.83	0.0		4.0
4	8	0.0	2.04	0.0		4.0
4	9	.857	5.261	1.211	5.466	5.714
16	1	0.0	10.01	0.0		10.0
16	2	0.0	10.00	0.0		10.0
16	3	0.0	4.36	0.0		4.0
16	4	0.0	4.14	0.0		4.0
16	5	0.0	9.99	0.0		10.0
16	6	0.0	10.00	0.0		10.0
16	7	0.0	3.68	0.0		4.0
16	8	0.0	3.87	0.0		4.0
16	9	.0109	5.714	.032	5.714	5.714

The bias magnetic field intensities calculated for the four-turn configuration are significantly different than for the axisymmetric configuration. Large axial and azimuthal gradients of the azimuthal field component occur. The axial field gradients may be improved by increasing the axial dimension of the coil turns, as demonstrated for example by the calculations for .18 and .22 m turn height, but this does not appreciably improve the large azimuthal variation of H_ϕ with azimuthal coordinate. In particular, near the outer ferrite radius, where most of the ring material is located, azimuthal variations of over 5 to 1 in H_ϕ occur when H_ϕ is compared at a field point lying in the plane of a turn to its value for a field point lying at an azimuth halfway between the planes of adjacent turns. The significance of the test conditions and their relationship to the axisymmetric cavity is no longer clear.

Let us note, finally, that the present calculations were performed in the absence of ferrite, and the results will be modified by the presence of high permeability material. However, it is not at all obvious that the effects of high permeability material will overcome the nonuniform excitation conditions associated with use of a 4-turn coil.

For this reason a single turn axisymmetric geometry is recommended, provided that sufficient bias current can be supplied to drive the ferrite rings through their intended range of operation.

Acknowledgment

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