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SEXTUPOLE FOCUSING OF COLD ATOMIC BEAMS

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Abstract

The use of very cold atomic beams¹ has lead to the prospect of producing high atomic hydrogen densities in the ionizers of polarized ion sources. The chief challenge in realizing the potential use of these beams in a polarized source is to capture and transmit a large fraction of the beam to the ionization volume. A simulation of a cold atomic beam has been performed for single and double sextupole systems. The optical properties of the beam have been investigated for several possible configurations.

Introduction

The atomic beam method of producing beams of polarized atoms is widely used. The method relies on the interaction of the electron's dipole moment with a nonhomogeneous magnetic field. Traditionally, sextupole magnets have been used to capture and focus the atomic flux produced by a room temperature rf dissociator since, for neutral beams, the radial restoring force is linearly proportional to the atom's distance from the axis. Subsequent refinements of the method have lead to decreasing the velocity of the beam by cooling the dissociated gas in an accommodator to liquid nitrogen temperature² and to approximately 30 °K at ETHZ³ and SIN⁴. The reduction in the mean beam velocity results in an increase in the solid angle of acceptance for a sextupole, which is given by

$$\Omega = 2.1 \frac{\mu_B B_0}{kT}$$

where μ_B is the Bohr magneton, T is the accommodator temperature and B_0 is the poletip field of the magnet.

Initial experiments at focusing cold atomic beams were performed with a superconducting solenoid.⁵ The quantity measured in the experiments was the focus factor F defined as the ratio

$$F = \rho_f / \rho_u$$

where ρ_f is the density measured with the solenoid energized and ρ_u is the density with the solenoid de-energized. At low intensity the solenoid showed substantial focusing, with a focus factor of 10; however, as the nozzle density was increased the focus factor quickly dropped to unity. This effect is due to atom-atom scattering which was much larger than expected.⁵ At high density one has losses from intrabeam scattering as well as scattering from the defocused component of the beam that remained in the cold bore of the solenoid. One possible solution to the problem was to replace the solenoid with a warm sextupole. The atomic hydrogen would recombine on the warm poletip and would be pumped away from the bore.

A permanent magnet sextupole was designed for use with the cold atomic beam source, and has been described in a previous Technical Note.⁶ The magnet was 20 cm long with a 4 cm diameter open bore with an effective poletip field of 7 kG. The magnet was mounted on a platform in the vacuum chamber that allowed the magnet to be moved longitudinally, as well as being removed from the beam. A residual gas analyzer and thermoflake were installed downstream to detect the atomic beam. A schematic of the installation is shown in Figure 1.

Single Sextupole Calculations

Two calculations were performed for this configuration. First, an estimate was made of the focus factor as a function of the nozzle-magnet separation distance. The second calculation was a determination of the length of the second drift required for point-to-point focusing as a function of velocity.

The first calculation was an estimate of the focus factor, F , as the magnet was moved along the beam axis. The focus factor was calculated first passing atoms through the sextupole using the program STRAHL. STRAHL is a ray-tracing program that uses a variation of the transfer matrix formalism developed by Zhang, et al.⁷ They wrote the transfer matrix for atoms with $m_j = 1/2$ passing through a sextupole of length L_s as

$$S_{1/2} = \begin{bmatrix} \cos (pL_S) & 1/p \sin (pL_S) \\ -p \sin (pL_S) & \cos (pL_S) \end{bmatrix}$$

and the transfer matrix for atoms in the $m_j = -1/2$ state as

$$S_{-1/2} = \begin{bmatrix} \cosh (pL_S) & 1/p \sinh (pL_S) \\ -p \sinh (pL_S) & \cosh (pL_S) \end{bmatrix}$$

The parameter p , which will be called the sextupole strength parameter, is

$$p = \left[\frac{2\mu_B B_0}{mv^2 R^2} \right]$$

where μ_B is the Bohr magneton, B_0 is the poletip field, v is the velocity of the atom, R is the poletip radius and m is the atomic mass.

A beam of atoms was generated by the Monte Carlo method. The initial radius of each atom was determined such that the rays originated uniformly across a disk of radius F_{noz} . The initial starting angle was chosen to produce a distribution that was given by

$$g(\theta) d\theta \propto \cos^n (\theta) d\theta$$

where n is determined by the initial density at the nozzle. The initial velocity distribution was assumed to be a supersonic distribution. In this calculation the velocity distribution parameters used were taken from the published values⁸ for the cold beam source: $V_{mp} = 685$ m/s, $V_d = 673$ m/s, and $T_b = 0.5$ °K. The separation of the nozzle and magnet was varied between 13 cm and 38 cm. The focus factor is

$$F = N_F/N_U$$

where N_F is the number of atoms that pass through a 5 mm diameter disk placed 75 cm from the nozzle, and N_U is the number of atoms that pass through the same disk when the sextupole fields are removed. Figure 2 shows the result of this calculation. A second set of focus factors were determined for $V_{mp} = 628$ m/s and are included on Figure 2.

The focus factor for both cases exhibits a minimum at a separation of 27 cm. The reason this minimum occurs can be seen by an examination of Figure 3. Figure 3a shows the velocity distribution used to calculate the focus factors of Figure 2. In Figure 3b, the focus factor, F_m , is plotted as a function of velocity for a monochromatic beam. The relationship between the focus factor of Figure 2 and the monochromatic focus factor is

$$F = \frac{\int F_m(v) f(v) dv}{\int f(v) dv}$$

where $f(v)$ is the normalized velocity distribution for the beam. The increased values of F are seen to be due to the match between $F_m(v)$ and $f(v)$ being better for the extrema than for the center positions where only the high velocity particles will be focused at the detector.

The second calculation is a study of the position of the stigmatic image of the nozzle as a function of particle velocity. The overall transfer matrix for the system is

$$M = D_2 S_1 D_1$$

where D is the 2×2 transfer matrix for a drift and S is the corresponding transfer matrix for the focused component of the atomic beam. Requiring that $M'_{1,2} = 0$ and fixing the length of the first drift and strength of the sextupole, one can then determine the length l_2 at which the image is formed:

$$l_2 = \frac{l_1 c + s/p}{p s l_1 - c}$$

where l_1 is the distance from the nozzle to the magnet entrance, p is the magnet strength parameter given in the Introduction, $c = \cos(pL_s)$ and $s = \sin(pL_s)$.

Figure 4 shows the predicted image positions for three separate lengths of the sextupole. The magnet was located 25 cm from the nozzle. The curves show that as the sextupole is shortened, the velocity at which the image is formed at the detector position of 75 cm shifts from 775 m/s, at a 20 cm sextupole length, to 565 m/s, at 10 cm length. The 15 cm case is interesting in that the image is formed at 75 cm when the velocity is 680 m/s, the most probable velocity quoted in Ref. 7.

Two Sextupole Calculations

A set of calculations similar to those described above were carried out for a two sextupole system. The system consisted of

three drifts and two sextupole magnets. A design for a two sextupole atomic beam system must define a minimum of sixteen different parameters such as poletip fields, magnet tapers, drift lengths, etc.; therefore, only one geometry will be used to illustrate the nature of these calculations. The first drift was set at a distance of 18 cm. The second drift was set to 20 cm, while the third drift was 30 cm. The constraints on the geometry were that the bores of the magnets remained 4 cm diameter, and both magnets were 10 cm long. The magnetic field of the first magnet was kept at 7 kG, the field of the permanent magnet sextupole, while the second magnet was varied to focus 680 m/s atoms onto the detector plane. The final poletip field for the second magnet was 3 kG.

Figure 5 shows a comparison of F_m for the two sextupole system described above with the single sextupole system described in the previous section. The dashed line is the two sextupole case while the solid line is a single sextupole. The maximum F_m for the two sextupole case occurred when the atomic velocity was approximately 675 m/s. This was lower than for a single sextupole. It is also clear that the maximum focus factor was 13.5 rather than the 9.5 obtained for a single sextupole. The two sextupole system, overall, had higher monochromatic focus factors than did the single sextupole system for comparable velocity bites. The system still shows substantial sensitivity to the atomic velocity: an achromatic solution to the problem should be found for the final source design.

Conclusion

The behavior of cold atomic beams focussed by sextupoles has been simulated by the Monte Carlo program STRAHL. The initial estimates of a single sextupole system show that the position of the waist will be extremely sensitive to the mean velocity of the atomic beam. A two sextupole system based on the BNL permanent magnet sextupole yields a larger peak F_m ; however, it still shows sensitivity to the velocity distribution. Further work is needed to fully investigate the possible final configuration of the source since the example given may not produce the absolute maximum attainable flux.

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Figure 1 Schematic drawing of the single sextupole test stand.

Figure 2 Estimate of the focus factor as a function of the nozzle sextupole separation distance. The length of the sextupole was 20 cm. The observation point was 75 cm from the nozzle.

Figure 3a Velocity distribution used in the calculations shown in Figure 2.

Figure 3b Monochromatic focus factor F_m as a function of atomic velocity for a single sextupole.

Figure 4 Image position for monochromatic beams passed through a single sextupole of 10, 15, and 20 cm length. The nozzle-sextupole separation is 25 cm.

Figure 5 Comparison of F_m for a single (solid) and double (dashed) sextupole system.