**QUADRUPOLE CORRECTORS FOR THE
HALF-INTEGER STOPBANDS IN THE BOOSTER**

**AD
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Half-Integer Stopbands

$$\begin{cases} 2Q_H = 9 \\ 2Q_H = 10 \end{cases} \quad \text{and} \quad \begin{cases} 2Q_V = 9 \\ 2Q_V = 10 \end{cases}$$

Driving Terms ($\kappa = B'/B_F$)

$$J = \frac{1}{2\pi} \oint \beta(s) \kappa(s) e^{ip\phi(s)} ds$$

H and V $p = 9$ and 10

Expectation Values of the Stopbands

* Booster TN # 112 (Feb. 1988)

Assume $(\Delta\kappa/\kappa)_{rms} = 10^{-3}$ then the rms values of the stopband widths are

$$\delta\nu_H = 0.0035$$

$$\delta\nu_V = 0.0038$$

* Similar contribution is expected from the product

sextupoles $\times$ uncorrected closed orbit

Booster TN # 107

* Once the closed orbit is corrected, the corresponding contribution is about 10 times smaller.

* It is not possible to know in advance the errors and the stopbands they cause.

The correction scheme should be capable to generate a set of driving terms with selected amplitudes and phases

There are 8 conditions:

$$\sum_s \beta_{H_s} \kappa_s \cos \theta \phi_{H_s} = \frac{2\pi}{L} |J_{Hq}| \cos \psi_{Hq}$$

$$\sum_s \beta_{H_s} \kappa_s \sin \theta \phi_{H_s} = \frac{2\pi}{L} |J_{Hq}| \sin \psi_{Hq}$$

$$\sum_s \beta_{H_s} \kappa_s \cos 10 \phi_{H_s} = \frac{2\pi}{L} |J_{H10}| \cos \psi_{H10}$$

$$\sum_s \beta_{H_s} \kappa_s \sin 10 \phi_{H_s} = \frac{2\pi}{L} |J_{H10}| \sin \psi_{H10}$$

$$\sum_s \beta_{V_s} \kappa_s \cos \theta \phi_{V_s} = -\frac{2\pi}{L} |J_{Vq}| \cos \psi_{Vq}$$

$$\sum_s \beta_{V_s} \kappa_s \sin \theta \phi_{V_s} = -\frac{2\pi}{L} |J_{Vq}| \sin \psi_{Vq}$$

$$\sum_s \beta_{V_s} \kappa_s \cos 10 \phi_{V_s} = -\frac{2\pi}{L} |J_{V10}| \cos \psi_{V10}$$

$$\sum_s \beta_{V_s} \kappa_s \sin 10 \phi_{V_s} = -\frac{2\pi}{L} |J_{V10}| \sin \psi_{V10}$$

Moreover, we require that the betatron tunes do not change, that is

$$\sum_s \beta_{H_s} K_s = 0$$

$$\sum_s \beta_{V_s} K_s = 0$$

QF, focussing horizontally $\beta_H = 13.5\text{m}$ $\beta_V = 3.5\text{m}$

QD, focussing vertically $\beta_H = 3.5\text{m}$ $\beta_V = 13.5\text{m}$

Define the origin of ϕ_H and ϕ_V at the location of any QF.

Because of the symmetry build-up in the lattice

$$\phi_{Hs} = \phi_{Vs} = \frac{2\pi}{48} (s-1)$$

$$s = 1, 2, \dots, 48$$

Corrector

κ_{je}

$$\vec{K}_e = L \begin{pmatrix} \kappa_{1e} & \kappa_{2e} & \kappa_{3e} & \kappa_{4e} & \dots & \kappa_{pe} \end{pmatrix}$$

$QF_1 \quad QD_1 \quad QF_2 \quad QD_2 \quad \dots \quad QD_4$

$$l = 1, 2, \dots, 6$$

$$\vec{K}_e = \vec{K}_1 \cos \frac{\pi}{5} (l-1)$$

This will make $\Delta Q_H = \Delta Q_V = 0$

Thus we are concerned only with the calculation of the components of the vector $\vec{K}_1$

$$M \vec{K}_1 = \vec{J}$$

$$\vec{K}_1 = M^{-1} \vec{J}$$

$$M^{-1}$$

$$\begin{bmatrix} -0.3533 & -0.0000 & 0.4217 & -0.0175 & 0.0916 & 0.0000 & -0.1093 & 0.0045 \\ 0.0351 & 0.0846 & -0.0327 & -0.1044 & -0.1352 & -0.3264 & 0.1260 & 0.4028 \\ -0.8619 & -0.2499 & 0.9703 & 0.1710 & 0.2234 & 0.0648 & -0.2515 & -0.0443 \\ 0.0257 & 0.2312 & -0.0223 & -0.2544 & -0.0990 & -0.8919 & 0.0860 & 0.9814 \\ -0.7861 & -0.4328 & 0.9505 & 0.2593 & 0.2038 & 0.1122 & -0.2464 & -0.0672 \\ -0.0257 & 0.2312 & 0.0012 & -0.2554 & 0.0990 & -0.8919 & -0.0044 & 0.9852 \\ -0.2499 & -0.2499 & 0.3739 & 0.1957 & 0.0648 & 0.0648 & -0.0969 & -0.0507 \\ -0.0351 & 0.0846 & 0.0239 & -0.1068 & 0.1352 & -0.3264 & -0.0923 & 0.4118 \end{bmatrix}$$

$$\vec{J} = \begin{bmatrix} J_{Hq} \cos \psi_{Hq} \\ J_{Hq} \sin \psi_{Hq} \\ J_{H10} \cos \psi_{H10} \\ J_{H10} \sin \psi_{H10} \\ -J_{Vq} \cos \psi_{Vq} \\ -J_{Vq} \sin \psi_{Vq} \\ -J_{V10} \cos \psi_{V10} \\ -J_{V10} \sin \psi_{V10} \end{bmatrix}$$

$$|J| = 0.01$$

ψ

0°	30°	60°	90°	120°	150°
-0.02220	-0.01923	-0.01110	-0.00000	0.01110	0.01923
0.00220	0.00457	0.00571	0.00532	0.00350	0.00075
-0.05415	-0.05475	-0.04067	-0.01570	0.01348	0.03905
0.00161	0.00866	0.01339	0.01453	0.01178	0.00587
-0.04939	-0.05637	-0.04824	-0.02719	0.00115	0.02918
-0.00161	0.00587	0.01178	0.01453	0.01339	0.00866
-0.01570	-0.02144	-0.02144	-0.01570	-0.00575	0.00575
-0.00220	0.00075	0.00350	0.00532	0.00571	0.00457

$$2Q_H = 9$$

0°	30°	60°	90°	120°	150°
0.02649	0.02240	0.01230	-0.00110	-0.01420	-0.02349
-0.00205	-0.00506	-0.00671	-0.00656	-0.00466	-0.00150
0.06096	0.05817	0.03979	0.01074	-0.02118	-0.04742
-0.00140	-0.00921	-0.01455	-0.01599	-0.01315	-0.00678
0.05972	0.05986	0.04397	0.01629	-0.01575	-0.04357
0.00007	-0.00796	-0.01386	-0.01605	-0.01393	-0.00809
0.02349	0.02649	0.02240	0.01230	-0.00110	-0.01420
0.00150	-0.00205	-0.00506	-0.00671	-0.00656	-0.00466

$$2Q_H = 10$$

0°	30°	60°	90°	120°	150°
0.00576	0.00498	0.00288	0.00000	-0.00288	-0.00498
-0.00850	-0.01761	-0.02201	-0.02051	-0.01352	-0.00290
0.01404	0.01419	0.01054	0.00407	-0.00349	-0.01012
-0.00622	-0.03341	-0.05164	-0.05604	-0.04542	-0.02263
0.01281	0.01461	0.01251	0.00705	-0.00030	-0.00756
0.00622	-0.02263	-0.04542	-0.05604	-0.05164	-0.03341
0.00407	0.00556	0.00556	0.00407	0.00149	-0.00149
0.00850	-0.00290	-0.01352	-0.02051	-0.02201	-0.01761

$$2Q_v = 9$$

0°	30°	60°	90°	120°	150°
-0.00687	-0.00581	-0.00319	0.00028	0.00368	0.00609
0.00792	0.01951	0.02588	0.02531	0.01796	0.00580
-0.01581	-0.01508	-0.01031	-0.00279	0.00549	0.01230
0.00540	0.03551	0.05611	0.06167	0.05070	0.02616
-0.01548	-0.01552	-0.01140	-0.00422	0.00408	0.01130
-0.00028	0.03071	0.05347	0.06190	0.05375	0.03119
-0.00609	-0.00687	-0.00581	-0.00319	0.00028	0.00368
-0.00580	0.00792	0.01951	0.02588	0.02531	0.01796

$$2Q_v = 10$$

At Injection

$$B_0 = 2.15 \text{ T.m}$$

$$\frac{B'_L}{B_0} < 0.1 \text{ m}^{-1}$$

$$L = 0.5 \text{ m}$$

$$B' < 43 \text{ G/cm}$$

$$\text{Pole Tip Radius} = 8.255 \text{ cm}$$

$$(a) \text{ Pole Tip Field} < 355 \text{ Gauss}$$

$$(b) \text{ Maximum Pole Tip Field} = 8.3 \text{ KG}$$

$$(a)/(b) < \frac{0.355}{8.3} = 4.3\%$$

*) Eliminate the zero-tune-shift conditions

still require 8 conditions for a total of 4 stopbands

$$\vec{K}_l = \vec{K}_1 \cos \frac{\pi}{2} (l-1)$$

$$l = 1, 2, \dots, 6$$

$$\frac{K}{K_{\max}} \sim 1.8\%$$

$$\text{Small tune-shift} \sim 0.02$$

(*) Only Four conditions for

$$\begin{aligned} 2Q_H &= 9 \\ 2Q_V &= 9 \end{aligned} \quad \text{stop bands}$$

$$\vec{K}_l = \vec{K}_1 \cos \pi(l-1) \quad l = 1, 2, \dots, 6$$

$$\frac{K}{K_{\max}} \sim 0.3 \%$$

No tune-shift

(*) Only Four conditions for

$$2 Q_H = 10$$

$$2 Q_V = 10$$

stop bands

$$\vec{K}_l = \vec{K}_1 \cos \frac{\pi}{3} (l-1)$$

$$l = 1, 2, \dots, 6$$

$$\frac{K}{K_{\max}} \sim 0.45 \%$$

No tune-shift